INCOME AND SUBSTITUTION EFFECTS

[See Chapter 5 and 6]

Two Demand Functions

- Marshallian demand $x_i(p_1,\ldots,p_n,m)$ describes how consumption varies with prices and income.
  - Obtained by maximizing utility subject to the budget constraint.

- Hicksian demand $h_i(p_1,\ldots,p_n,\underline{u})$ describes how consumption varies with prices and utility.
  - Obtained by minimizing expenditure subject to the utility constraint.

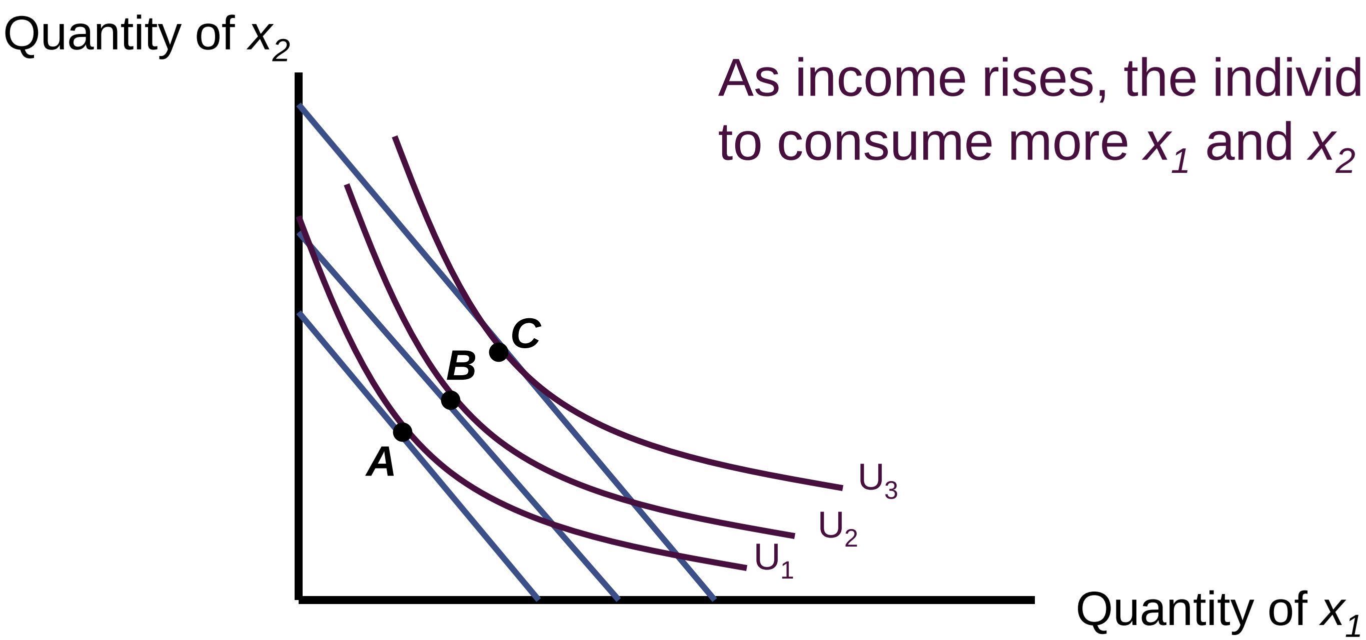
Changes in Income

Changes in Income

- An increase in income shifts the budget constraint out in a parallel fashion
- Since p_1/p_2 does not change, the optimal *MRS* will stay constant as the worker moves to higher levels of utility.

Increase in Income

- If both x_1 and x_2 increase as income rises, x_1 and x_2 are <u>normal</u> goods

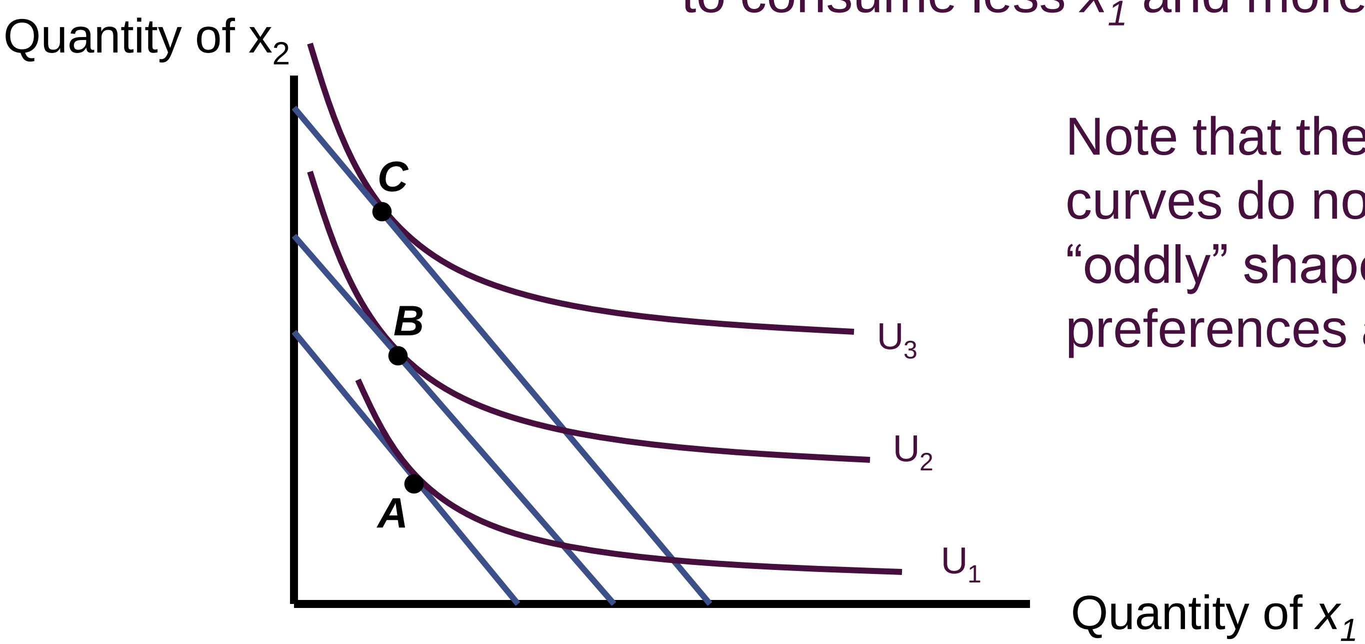


As income rises, the individual chooses to consume more x_1 and x_2

Increase in Income

- If x_1 decreases as income rises, x_1 is an inferior good

As income rises, the individual chooses to consume less x_1 and more x_2



Note that the indifference curves do not have to be "oddly" shaped. The preferences are convex

Changes in Income

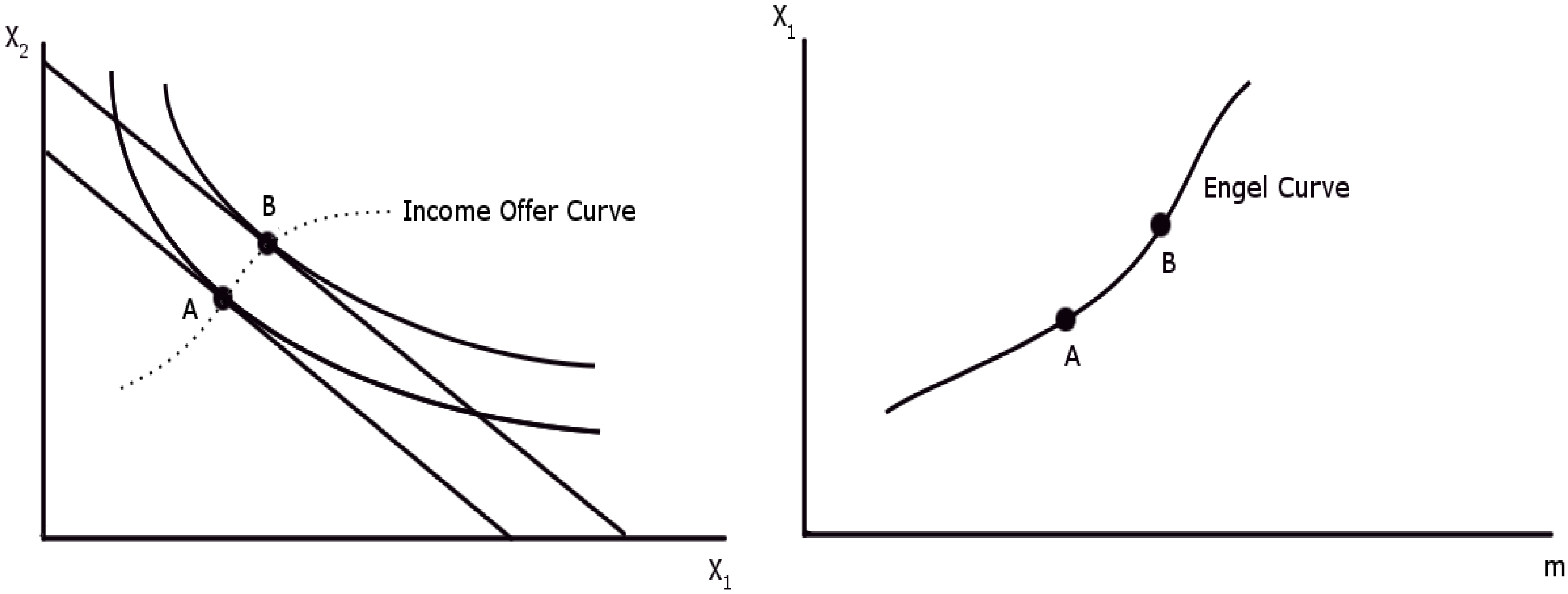
- The change in consumption caused by a change in income from m *to m'* can be computed using the Marshallian demands:

$$\Delta x_1 = x_1(p_1, p_2, m') - x_1(p_1, p_2, m)$$

- If $x_1(p_1,p_2,m)$ is increasing in m, i.e. $\partial x_1/\partial m \geq 0$, then good 1 is normal.
- If $x_1(p_1,p_2,m)$ is decreasing in m, i.e. $\partial x_1/\partial m < 0$, then good 1 is inferior.

Engel Curves

- The Engel Curve plots demand for x_i against income, *m*.



Own Price Effects

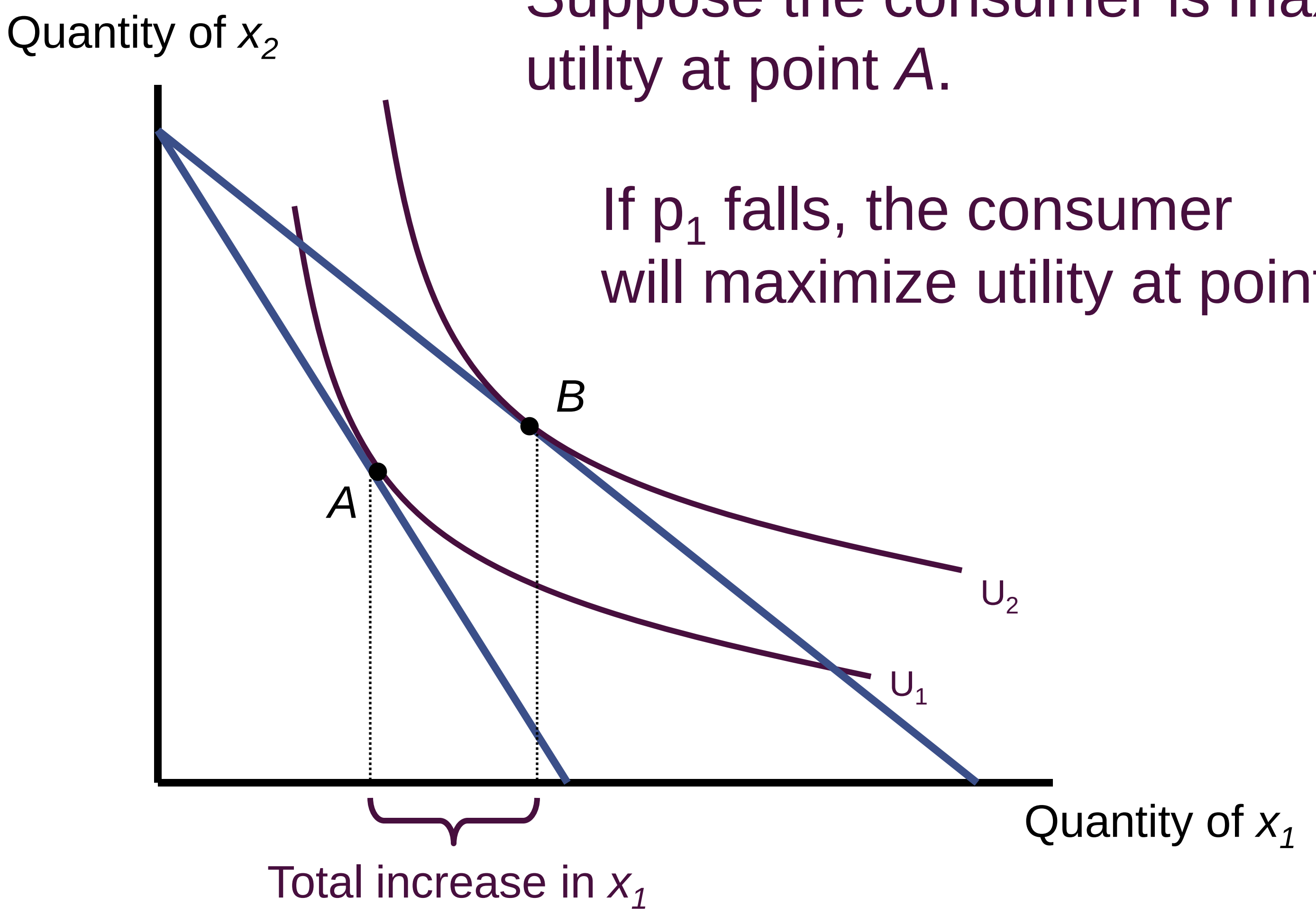
Changes in a Good's Price

- A change in the price of a good alters the slope of the budget constraint
- When the price changes, two effects come into play
  - substitution effect
  - income effect
- We separate these effects using the Slutsky equation.

Changes in a Good's Price

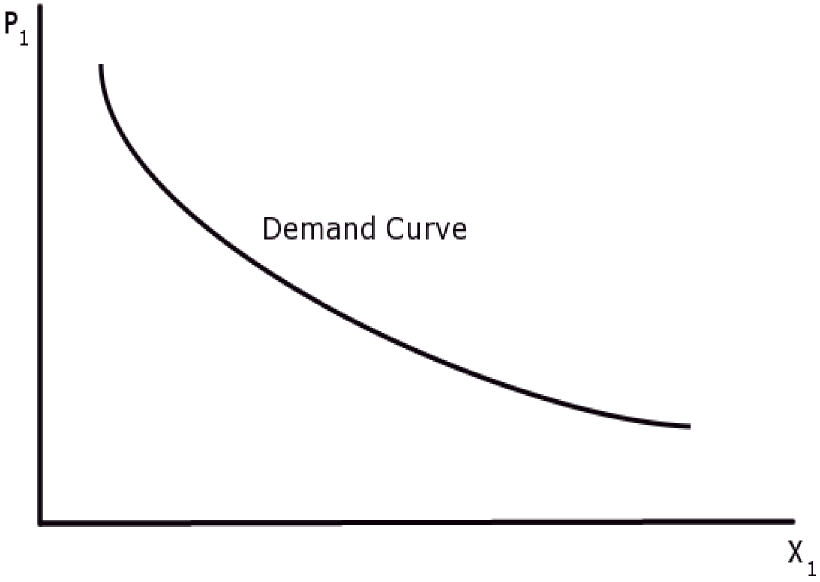
Suppose the consumer is maximizing utility at point *A*.

If p_1 falls, the consumer will maximize utility at point *B*.



Demand Curves

- The Demand Curve plots demand for x_i against p_i, holding income and other prices constant.



Changes in a Good's Price

- The total change in x_1 caused by a change in its price from p_1 to p_1' can be computed using Marshallian demand:

$$\Delta x_1 = x_1(p_1', p_2, m) - x_1(p_1, p_2, m)$$

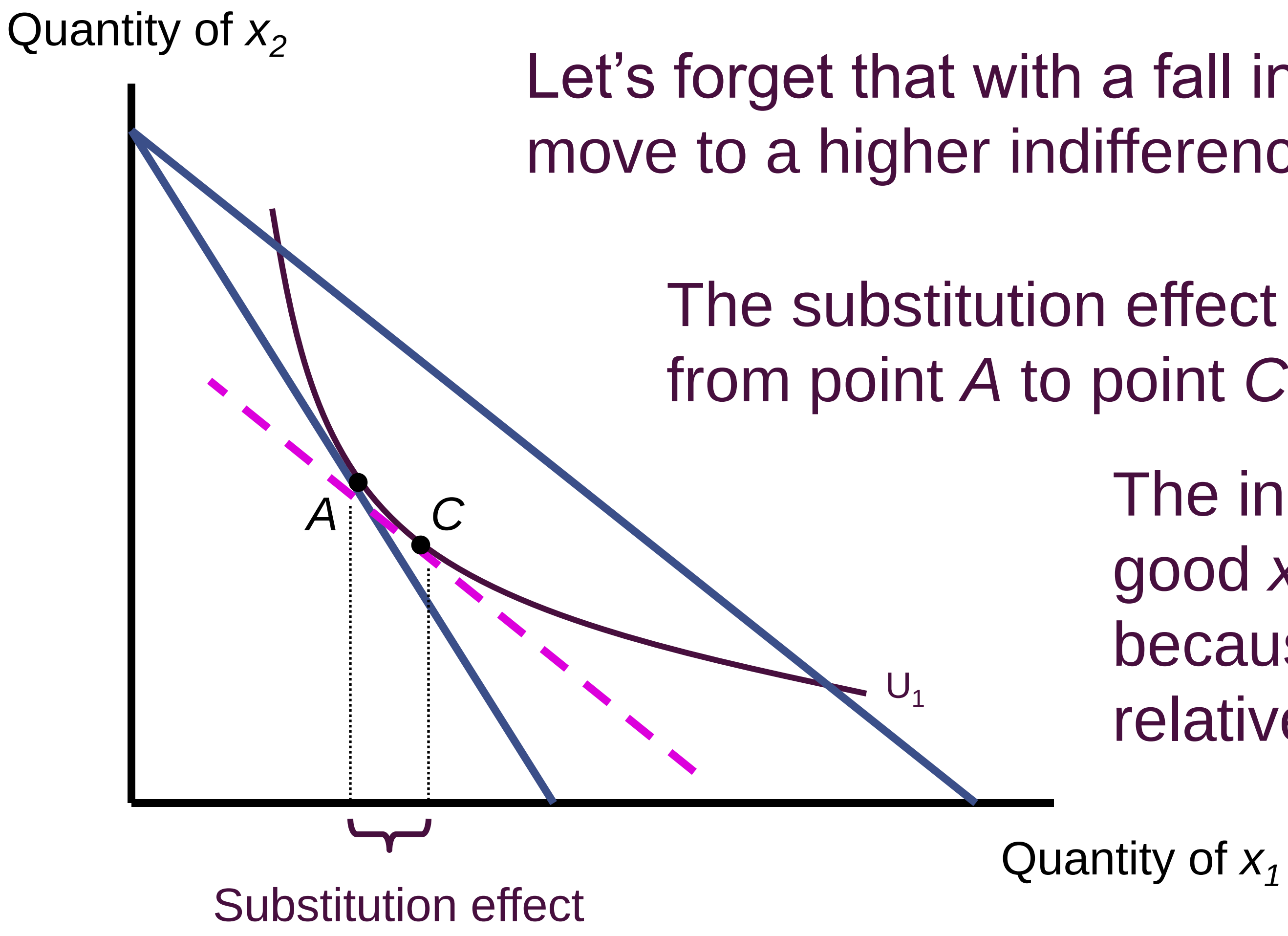
Two Effects

- Suppose p_1 falls.

1. Substitution Effect
   – The relative price of good 1 falls.
   – Fixing utility, buy more x_1 (and less x_2).

2. Income Effect
   – Purchasing power also increases.
   – Agent can achieve higher utility.
   – Will buy more/less of x_1 if normal/inferior.

Substitution Effect



Let's forget that with a fall in price we can move to a higher indifference curve.

The substitution effect is the movement from point *A* to point *C*

The individual substitutes good x_1 for good x_2 because it is now relatively cheaper

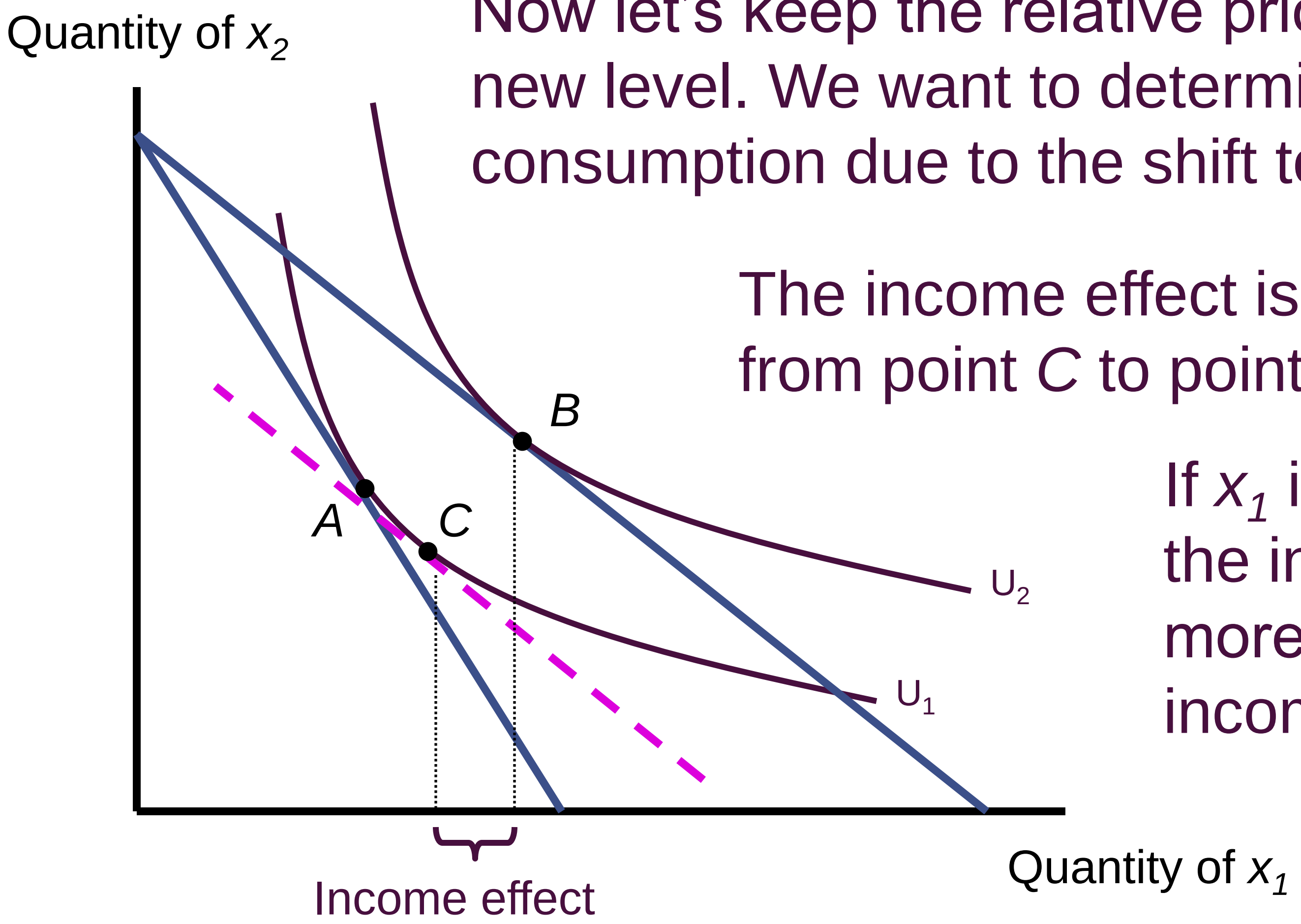
Substitution Effect

- The substitution effect caused by a change in price from p_1 to p_1' can be computed using the Hicksian demand function:

$$\text{Sub. Effect} = h_1(p_1', p_2, \underline{U}) - h_1(p_1, p_2, \underline{U})$$

Income Effect

Now let's keep the relative prices constant at the new level. We want to determine the change in consumption due to the shift to a higher curve



The income effect is the movement from point *C* to point *B*

If x_1 is a normal good, the individual will buy more because "real" income increased

Income Effect

- The income effect caused by a change in price from p_1 to p_1' is the difference between the total change and the substitution effect:

$$\text{Income Effect} =$$

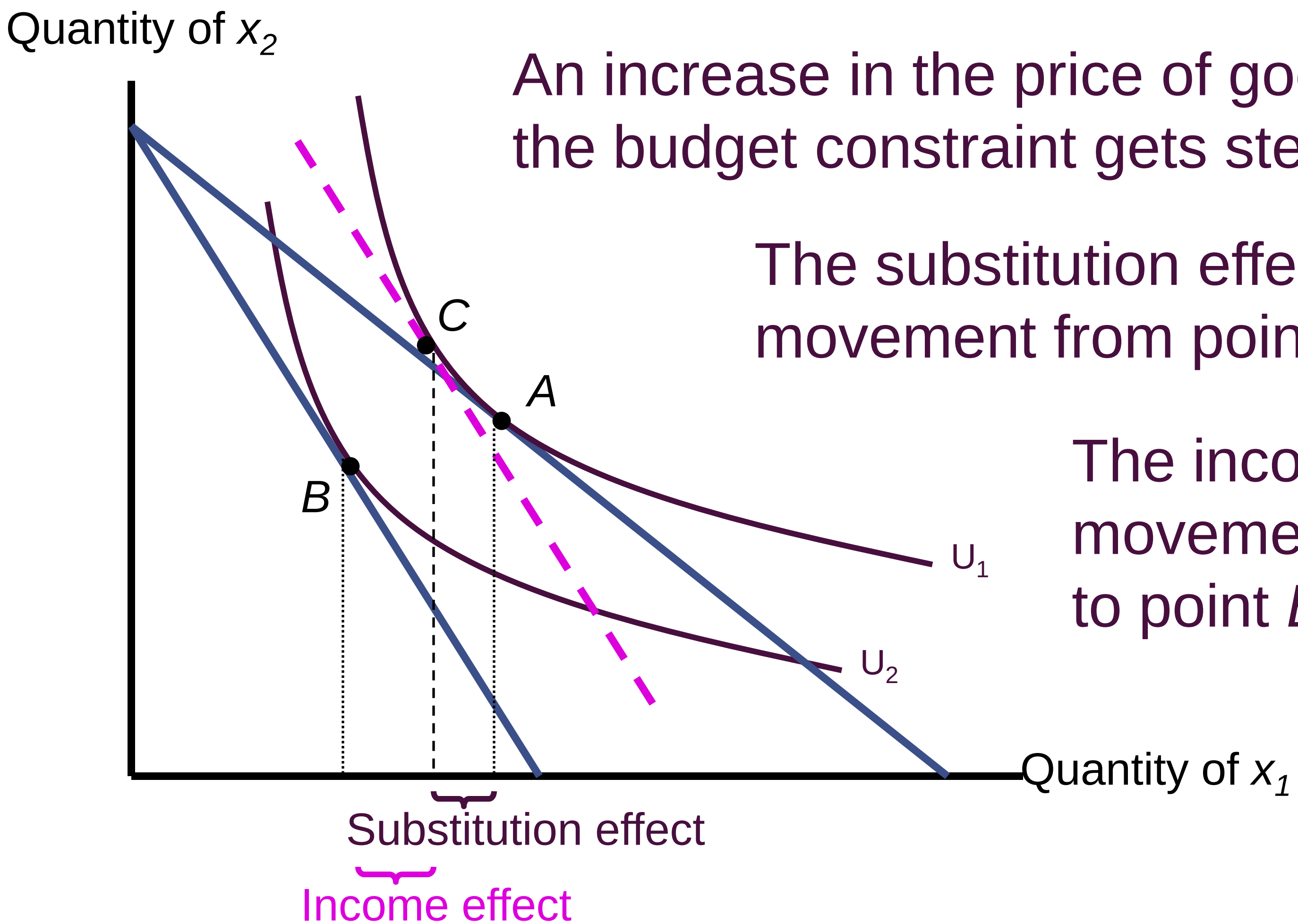
$$[x_1(p_1', p_2, m) - x_1(p_1, p_2, m)] - [h_1(p_1', p_2, \underline{U}) - h_1(p_1, p_2, \underline{U})]$$

Increase in a Good 1's Price

An increase in the price of good x_1 means that the budget constraint gets steeper

The substitution effect is the movement from point *A* to point *C*

The income effect is the movement from point *C* to point *B*



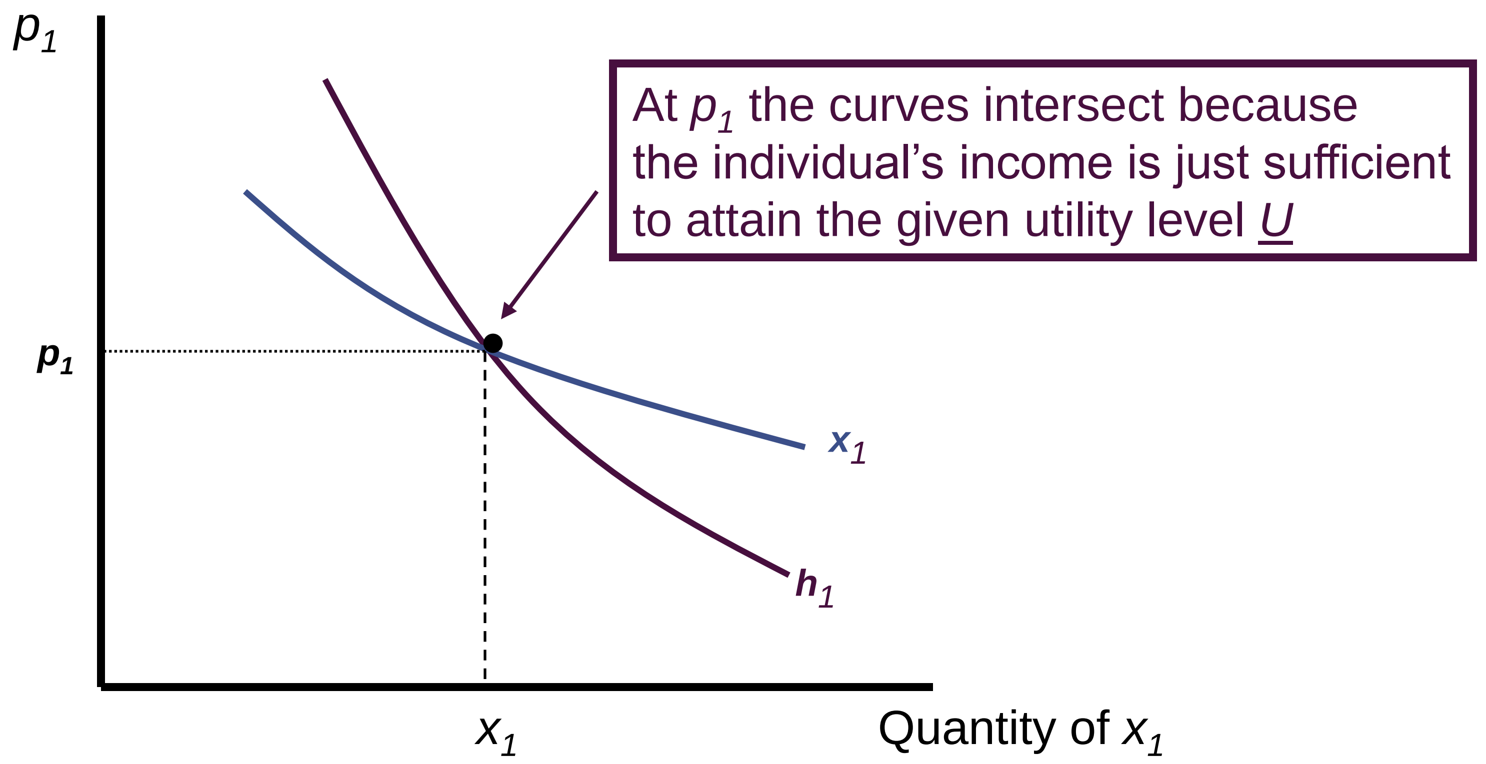
Hicksian & Marshallian Demand

- Marshallian demand
  - Fix prices (p_1,p_2) and income m.
  - Induces utility $\underline{u} = v(p_1,p_2,m)$
  - When we vary p_1 we can trace out Marshallian demand for good 1
- Hicksian demand (or compensated demand)
  - Fix prices (p_1,p_2) and utility $\underline{u}$
  - By construction, $h_1(p_1,p_2,\underline{u})= x_1(p_1,p_2,m)$
  - When we vary p_1 we can trace out Hicksian demand for good 1.

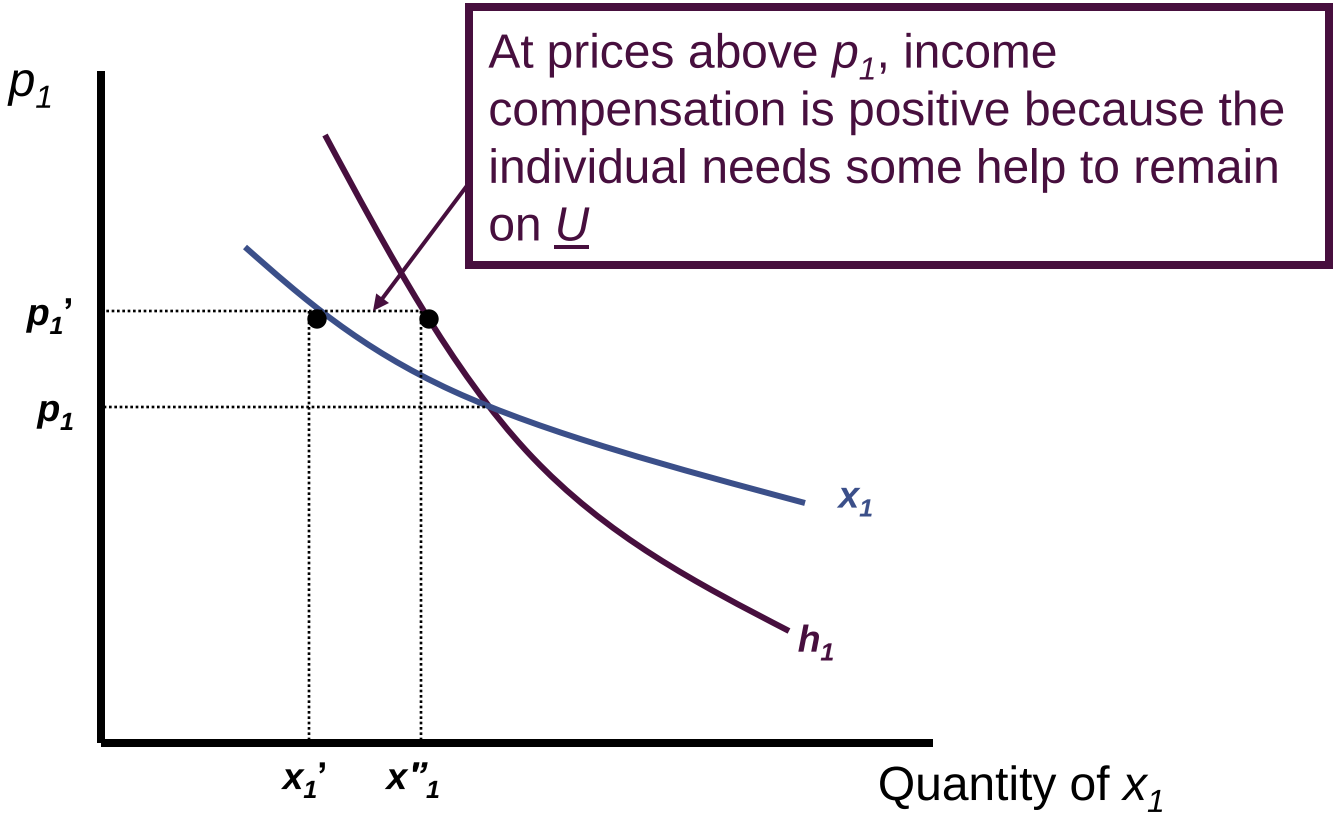
Hicksian & Marshallian Demand

- For a normal good, the Hicksian demand curve is less responsive to price changes than is the uncompensated demand curve
  - the uncompensated demand curve reflects both income and substitution effects
  - the compensated demand curve reflects only substitution effects

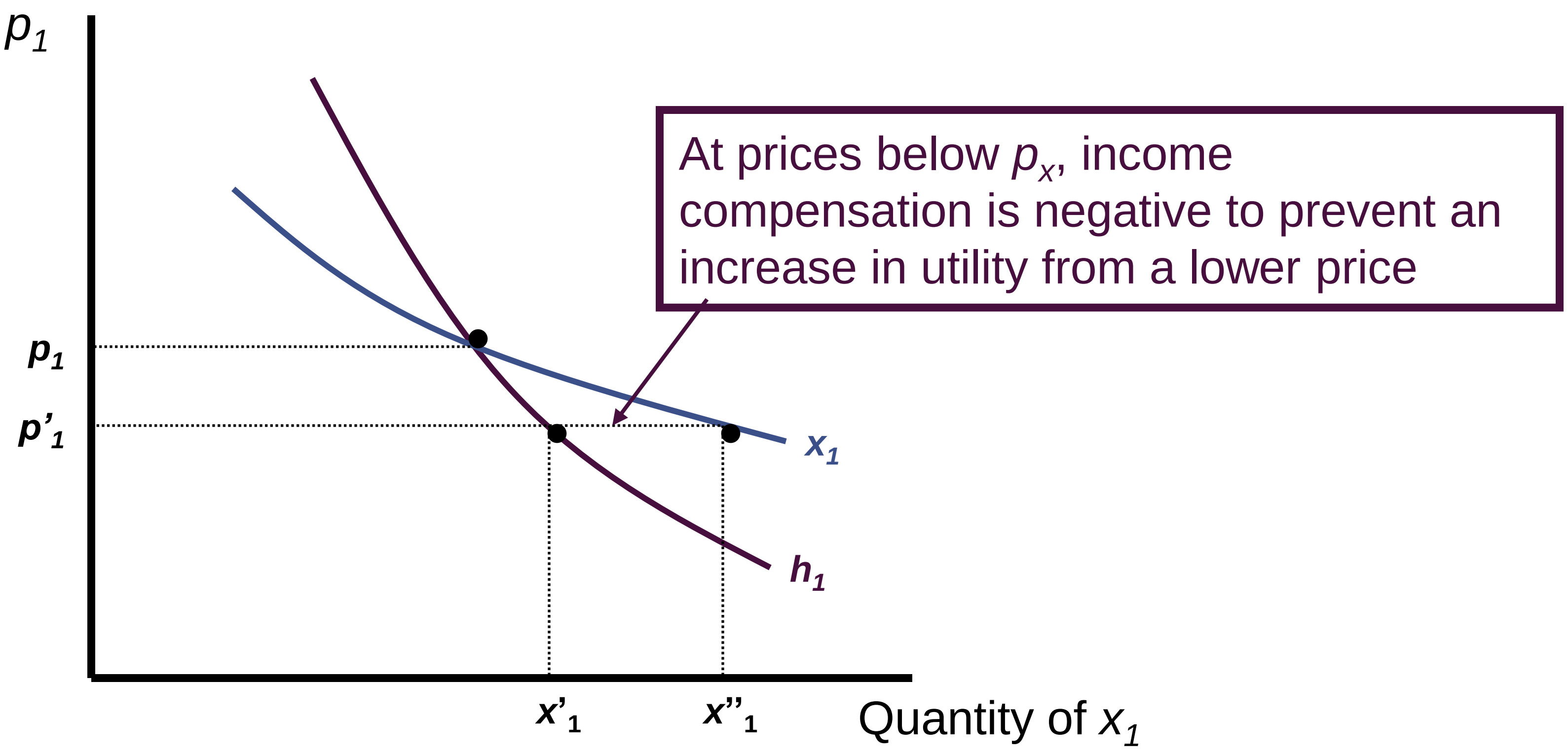
Hicksian & Marshallian Demand



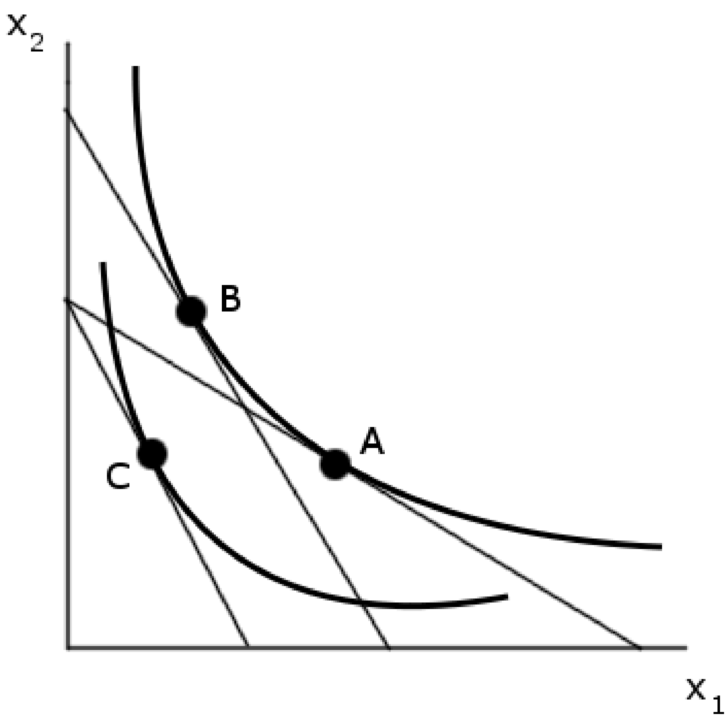
Hicksian & Marshallian Demand

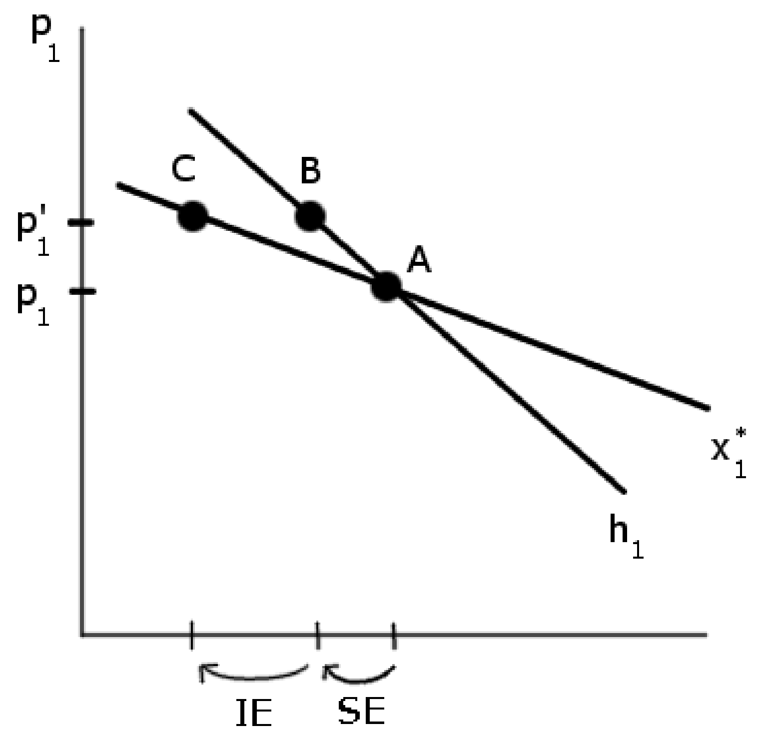


Hicksian & Marshallian Demand



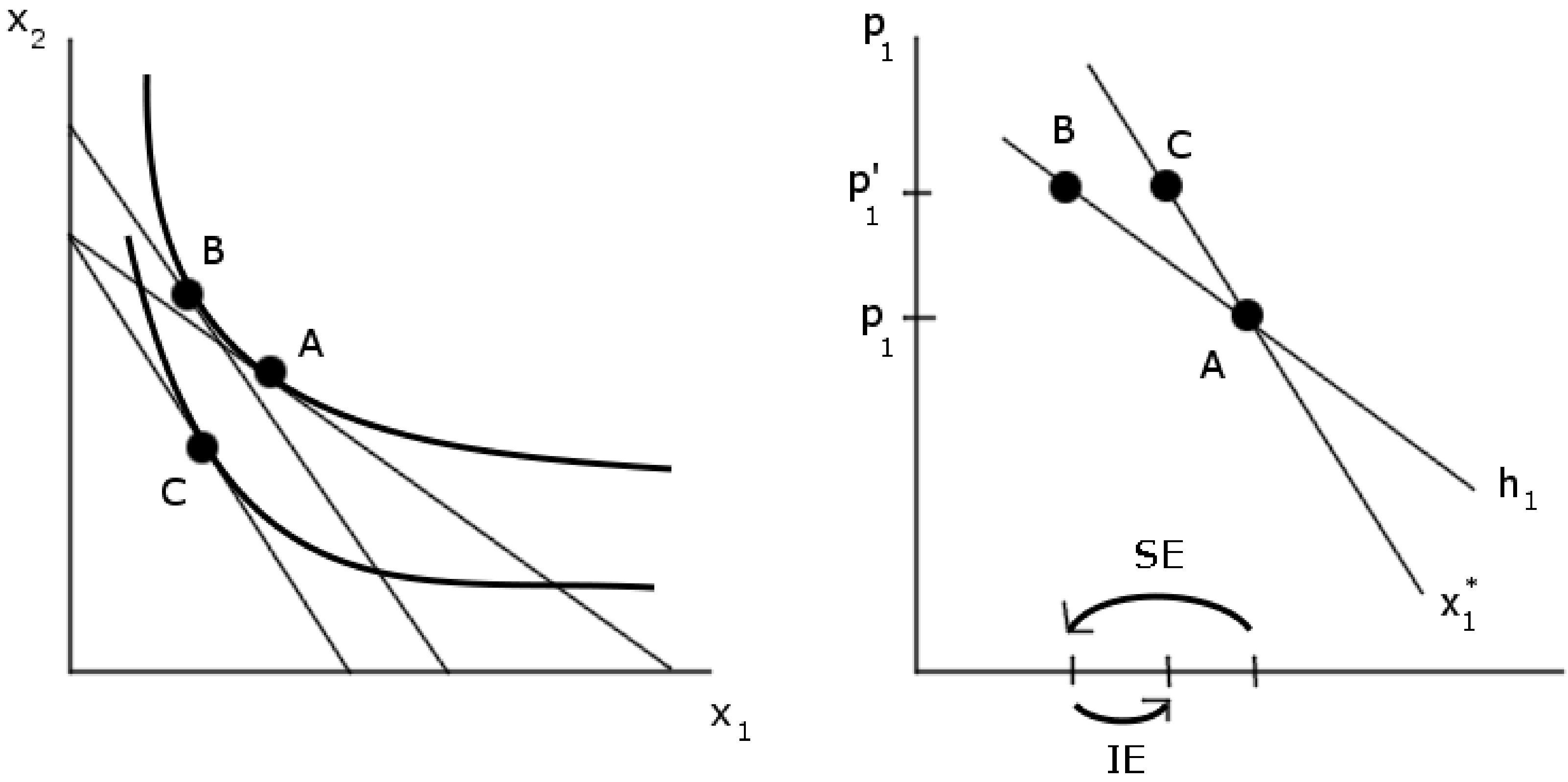
Normal Goods





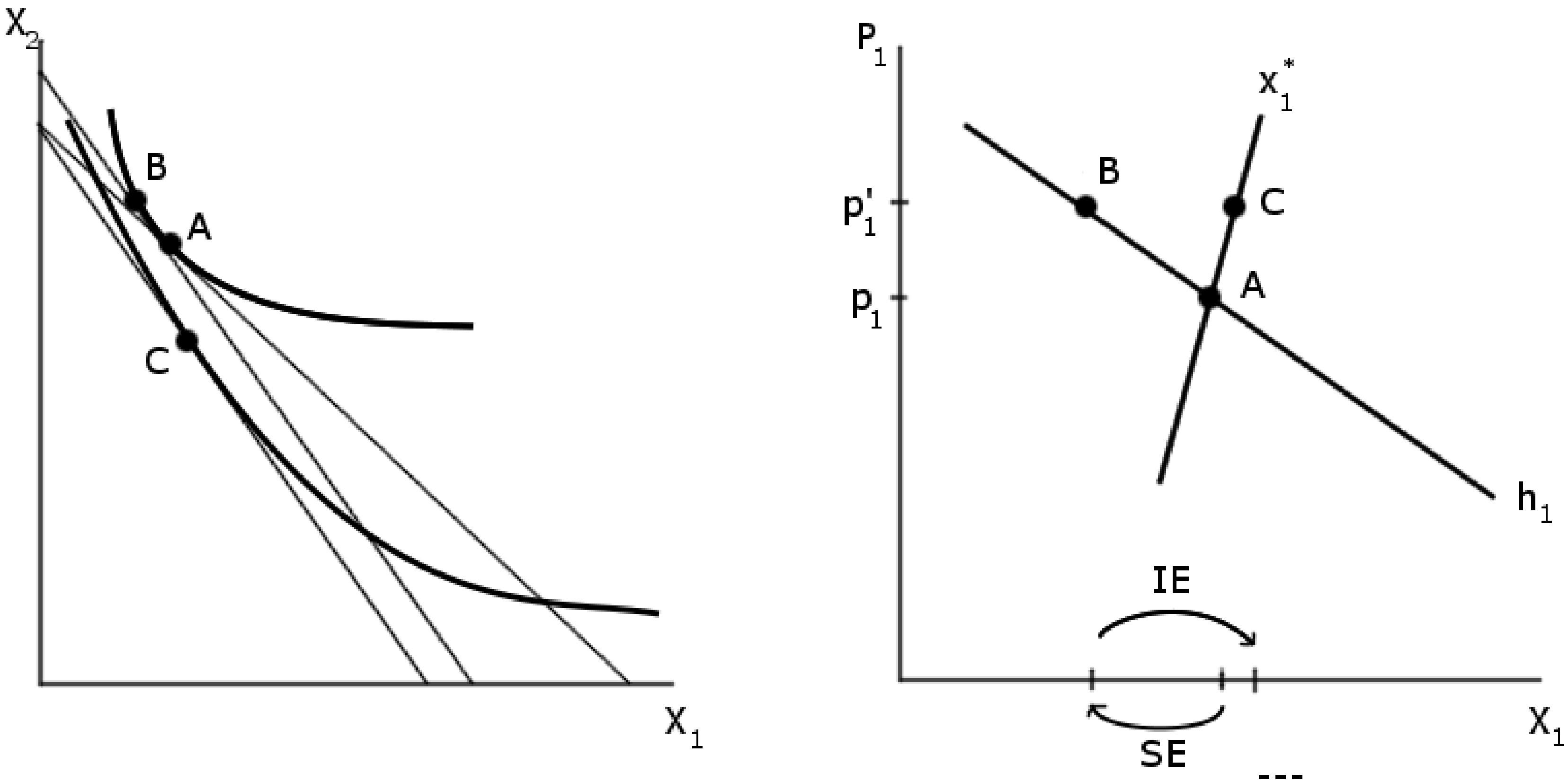
- Picture shows price rise.
- SE and IE go in same direction

Inferior Good



- Picture shows price rise.
- SE and IE go in opposite directions.

Inferior Good (Giffen Good)



- Picture shows price rise
- IE opposite to SE, and bigger than SE

SLUTSKY EQUATION

Slutsky Equation

- Suppose p_1 increase by Δp_1.

1. Substitution Effect.
   - Holding utility constant, relative prices change.
   - Increases demand for x_1 by $\frac{\partial h_1}{\partial p_1}\Delta p_1$

2. Income Effect
   - Agent's income falls by $x^*_1 \times \Delta p_1$.
   - Reduces demand by $x_1^* \frac{\partial x_1^*}{\partial m}\Delta p_1$

Slutsky Equation

- Fix prices (p_1,p_2) and income m.
- Let $\underline{u} = v(p_1,p_2,m)$.
- Then

$$\frac{\partial}{\partial p_1} x_1^*(p_1, p_2, m) = \frac{\partial}{\partial p_1} h_1(p_1, p_2, \underline{u}) - x_1^*(p_1, p_2, m) \cdot \frac{\partial}{\partial m} x_1^*(p_1, p_2, m)$$

- SE always negative since h_1 decreasing in p_1.
- IE depends on whether x_1 normal/inferior.

Example: $u(x_1,x_2)=x_1x_2$

- From UMP

$$x_1^*(p_1,p_2,m)=\frac{m}{2p_1} \quad \text{and} \quad \mathrm{v}(p_1,p_2,m)=\frac{m^2}{4p_1p_2}$$

- From EMP

$$h_1(p_1,p_2,\underline{u})=\left(\frac{p_2}{p_1}\underline{u}\right)^{1/2} \quad \text{and} \quad \mathrm{e}(p_1,p_2,\underline{u})=2(\underline{u}p_1p_2)^{1/2}$$

- LHS of Slutsky:

$$\frac{\partial}{\partial p_1}x_1^*(p_1,p_2,m)=-\frac{1}{2}mp_1^{-2}$$

- RHS of Slutsky:

$$\frac{\partial}{\partial p_1}h_1-x_1^*\cdot\frac{\partial}{\partial m}x_1^*=-\frac{1}{2}\underline{u}^{1/2}p_1^{-3/2}p_2^{1/2}-\frac{1}{4}mp_1^{-2}=-\frac{1}{4}mp_1^{-2}-\frac{1}{4}mp_1^{-2}$$

CROSS PRICE EFFECTS

Changes in a Good's Price

- The total change in x_2 caused by a change in the price from p_1 to p_1' can be computed using the Marshallian demand function:

$$\Delta x_2 = x_2^*(p_1', p_2, m) - x_2^*(p_1, p_2, m)$$

Substitutes and Complements

- Let’s start with the two-good case
- Two goods are substitutes if one good may replace the other in use
  - examples: tea & coffee, butter & margarine
- Two goods are complements if they are used together
  - examples: coffee & cream, fish & chips

Gross Subs/Comps

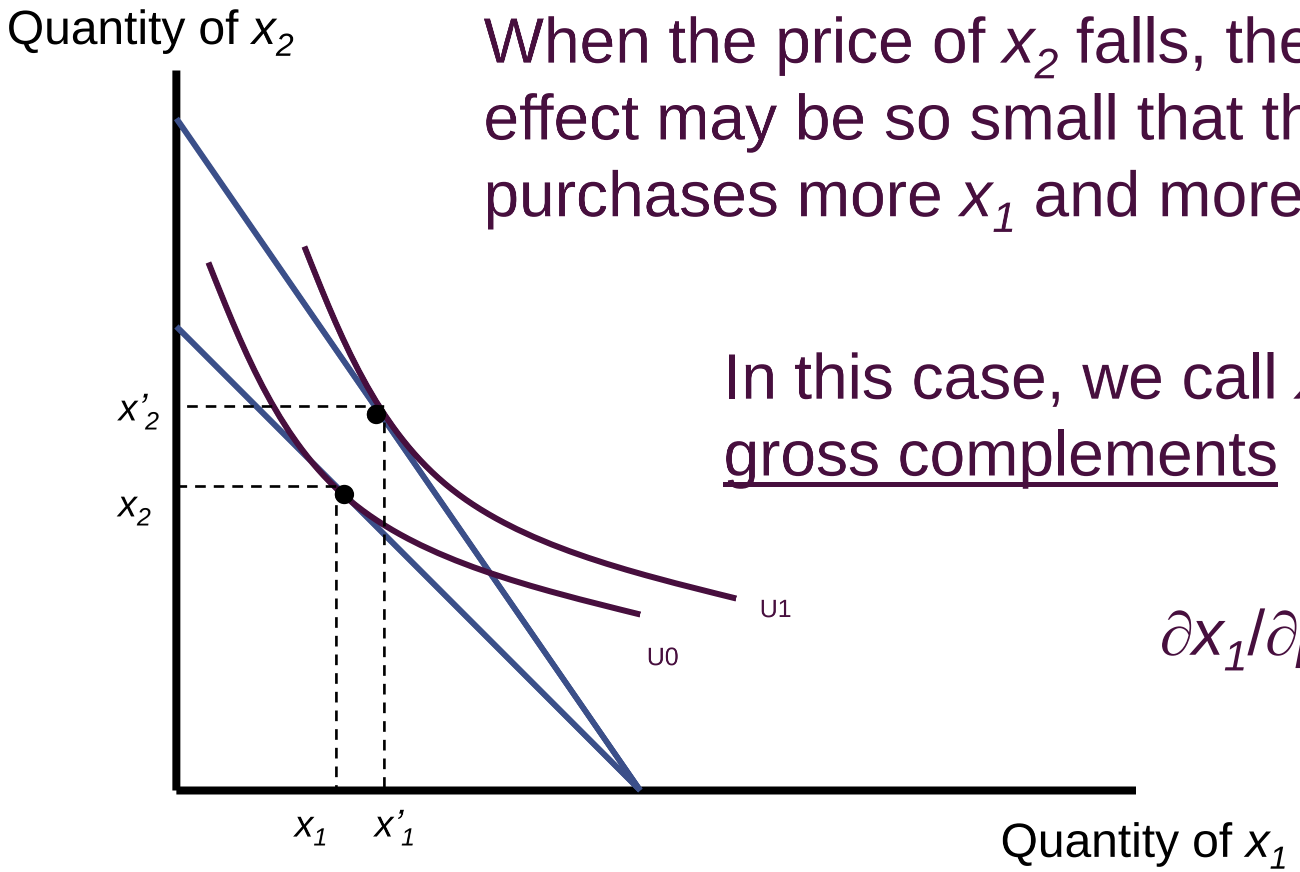
- Goods 1 and 2 are gross substitutes if

$$\frac{\partial x_1^*}{\partial p_2} > 0 \quad \text{and} \quad \frac{\partial x_2^*}{\partial p_1} > 0$$

- They are gross complements if

$$\frac{\partial x_1^*}{\partial p_2} < 0 \quad \text{and} \quad \frac{\partial x_2^*}{\partial p_1} < 0$$

Gross Complements

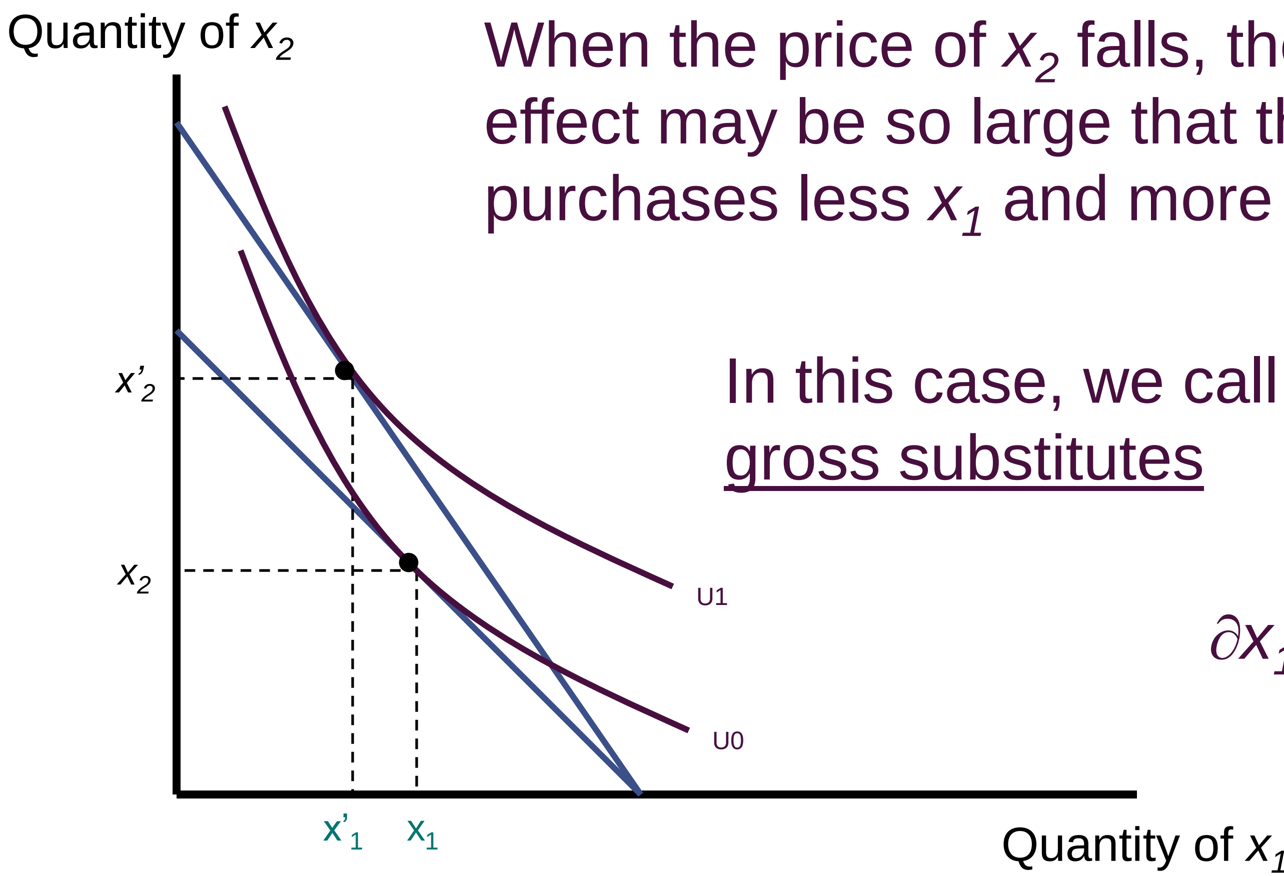


When the price of x_2 falls, the substitution effect may be so small that the consumer purchases more x_1 and more x_2

In this case, we call x_1 and x_2 <u>gross complements</u>

$\partial x_1/\partial p_2 < 0$

Gross Substitutes



When the price of x_2 falls, the substitution effect may be so large that the consumer purchases less x_1 and more x_2

In this case, we call x_1 and x_2 <u>gross substitutes</u>

$$\partial x_1/\partial p_2 > 0$$

Gross Substitutes: Asymmetry

- Partial derivatives may have opposite signs:
  - Let x_1=foreign flights and x_2=domestic flights.
  - An increase in p_1 may increase x_2 (sub effect)
  - An increase in p_2 may reduce x_1 (inc effect)

- Quasilinear Example: $U(x,y) = \ln x + y$
  - From the UMP, demands are

$$x_1 = p_2 / p_1 \text{ and } x_2 = (m - p_2)/p_2$$

  - We therefore have

$$\partial x_1 / \partial p_2 > 0 \text{ and } \partial x_2 / \partial p_1 = 0$$

Net Subs/Comps

- Goods 1 and 2 are net substitutes if

$$\frac{\partial h_1}{\partial p_2} > 0 \quad \text{and} \quad \frac{\partial h_2}{\partial p_1} > 0$$

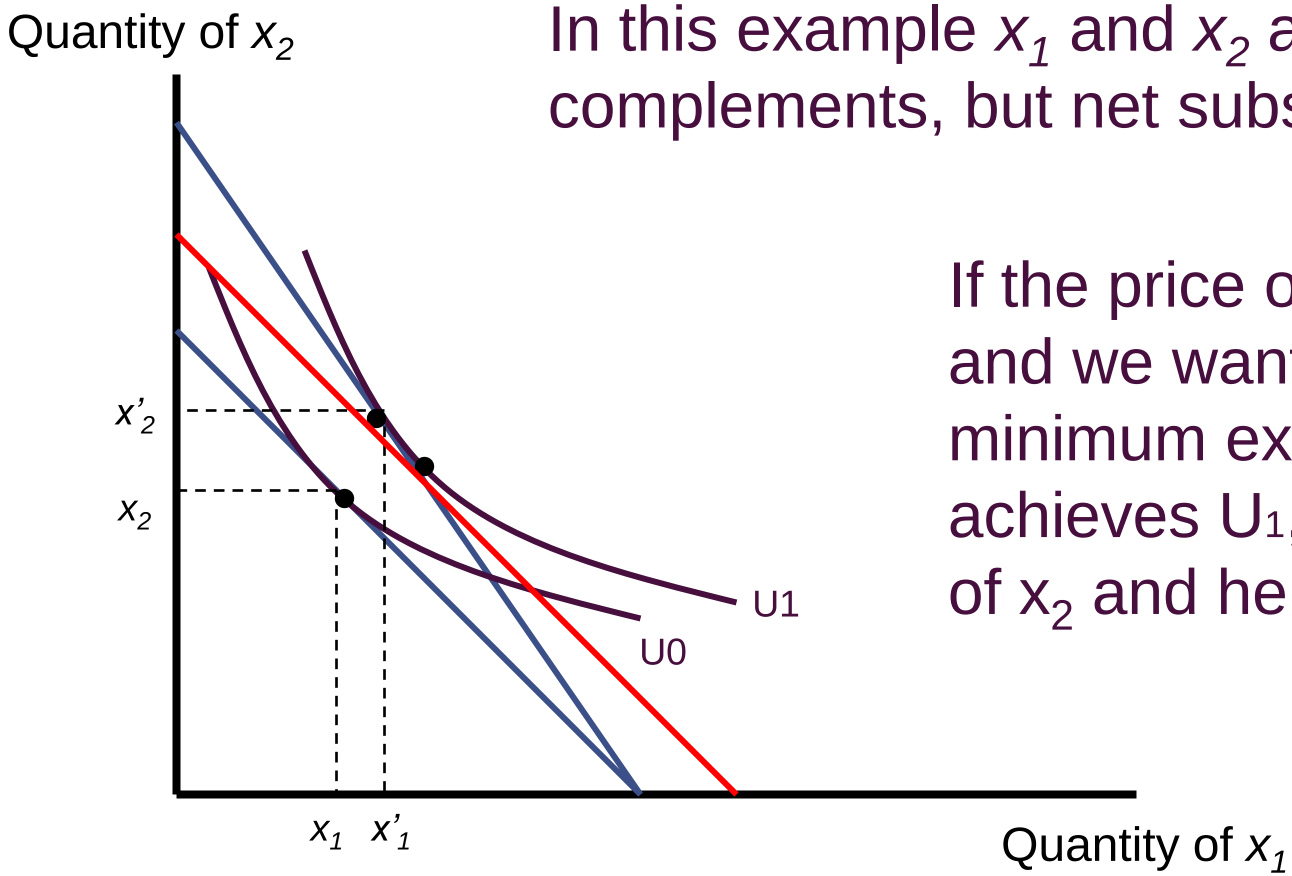
- They are net complements if

$$\frac{\partial h_1}{\partial p_2} < 0 \quad \text{and} \quad \frac{\partial h_2}{\partial p_1} < 0$$

- Partial derivatives cannot have opposite signs
  - Follows from Shepard's Lemma (see EMP notes)
- Two goods are always net substitutes.
  - Moving round indifference curve.

Gross Comps & Net Subs

In this example x_1 and x_2 are gross complements, but net substitute



If the price of x_2 increases and we want to find the minimum expenditure that achieves U_1, we buy less of x_2 and hence more of x_1.

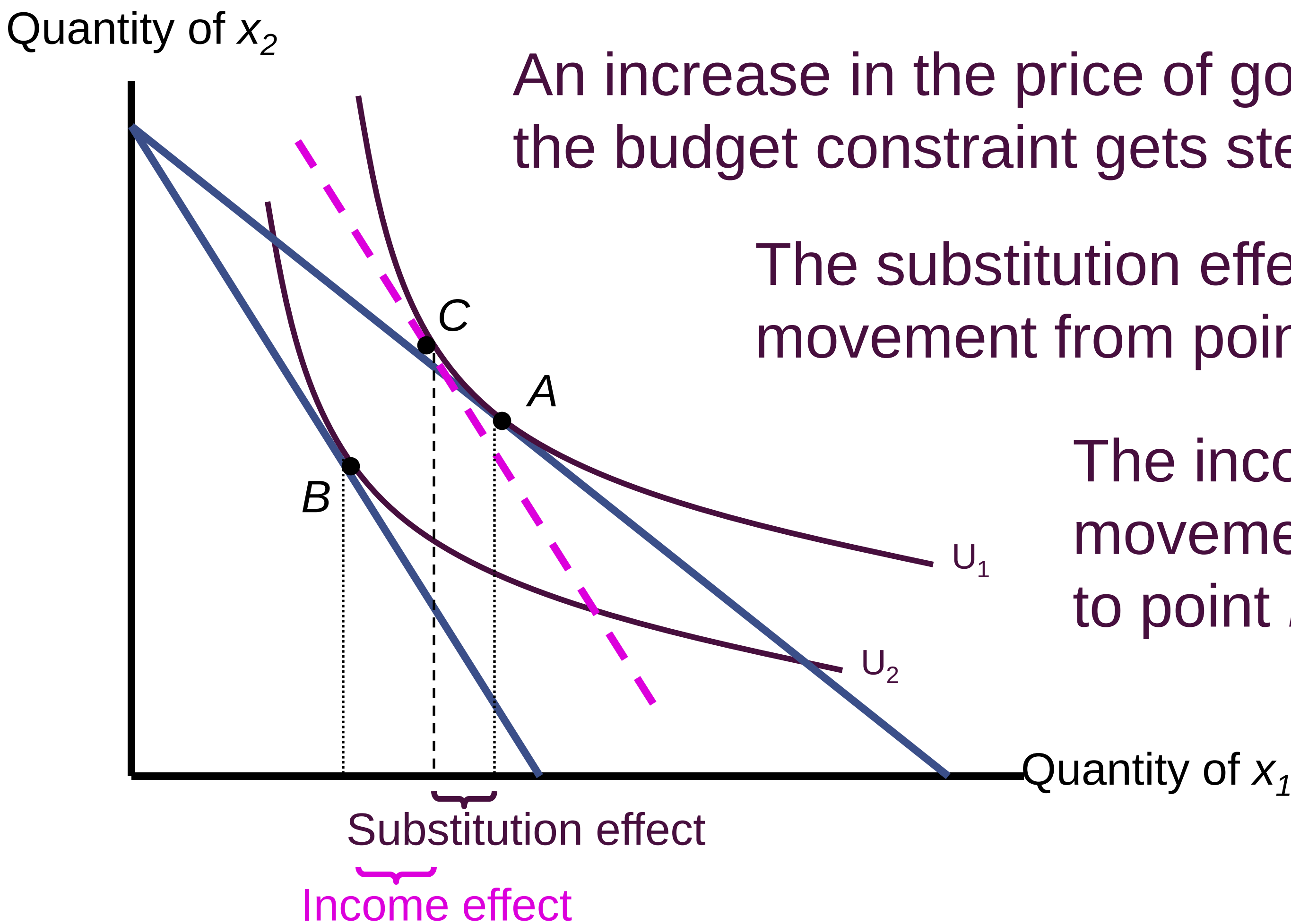
Substitution and Income Effect

- Suppose p_1 rises.

1. Substitution Effect
   – The relative price of good 2 falls.
   – Fixing utility, buy more x_2 (and less x_1)

2. Income Effect
   – Purchasing power decreases.
   – Agent can achieve lower utility.
   – Will buy more/less of x_2 if inferior/normal.

Increase in a Good 1's Price



An increase in the price of good x_1 means that the budget constraint gets steeper

The substitution effect is the movement from point *A* to point *C*

The income effect is the movement from point *C* to point *B*

Slutsky Equation

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2. Income Effect
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   - Reduces demand by $x_1^* \frac{\partial x_2^*}{\partial m} \Delta p_1$

Slutsky Equation

- Fix prices (p_1,p_2) and income m.
- Let $\underline{u} = v(p_1,p_2,m)$.
- Then

$$\frac{\partial}{\partial p_1} x_2^*(p_1, p_2, m) = \frac{\partial}{\partial p_1} h_1(p_1, p_2, \underline{u}) - x_1^*(p_1, p_2, m) \cdot \frac{\partial}{\partial m} x_1^*(p_1, p_2, m)$$

- SE depends on net complements or substitutes
- IE depends on whether x_1 is normal/inferior.

Example: $u(x_1,x_2)=x_1x_2$

- From UMP

$$x_2^*(p_1,p_2,m)=\frac{m}{2p_2} \quad \text{and} \quad \mathrm{v}(p_1,p_2,m)=\frac{m^2}{4p_1p_2}$$

- From EMP

$$h_2(p_1,p_2,\underline{u})=\left(\frac{p_1}{p_2}\underline{u}\right)^{1/2} \quad \text{and} \quad \mathrm{e}(p_1,p_2,\underline{u})=2(\underline{u}p_1p_2)^{1/2}$$

- LHS of Slutsky:

$$\frac{\partial}{\partial p_1}x_2^*(p_1,p_2,m)=0$$

- RHS of Slutsky:

$$\frac{\partial}{\partial p_1}h_2-x_1^*\cdot\frac{\partial}{\partial m}x_2^*=\frac{1}{2}\underline{u}^{1/2}p_1^{-1/2}p_2^{-1/2}-\frac{1}{4}mp_1^{-1}p_2^{-1}=\frac{1}{4}mp_1^{-1}p_2^{-1}-\frac{1}{4}mp_1^{-1}p_2^{-1}$$

Demand Elasticities

Demand Elasticities

- So far we have used partial derivatives to determine how individuals respond to changes in income and prices.
  - The size of the derivative depends on how the variables are measured (e.g. currency, unit size)
  - Makes comparisons across goods, periods, and countries very difficult.

- Elasticities look at percentage changes.
  - Independent of units.

Income Elasticities

- The income elasticity equals the percentage change in x_1 caused by a 1% increase in income.

$$e_{x_1,m} = \frac{\Delta x_1 / x_1}{\Delta m / m} = \frac{dx_1}{dm}\frac{m}{x_1} = \frac{\partial \ln x_1}{\partial \ln m}$$

- Normal good: $e_{1,m} > 0$
- Inferior good: $e_{1,m} < 0$
- Luxury good: $e_{1,m} > 1$
- Necessary good: $e_{1,m} < 1$

Marshallian Demand Elasticities

- The own price elasticity of demand $e_{x1,p1}$ is

$$e_{x_1,p_1} = \frac{\Delta x_1 / x_1}{\Delta p_1 / p_1} = \frac{\partial x_1}{\partial p_1} \cdot \frac{p_1}{x_1} = \frac{\partial \ln x_1}{\partial \ln p_1}$$

- If $|e_{x1,p1}| < -1$, demand is elastic
- If $|e_{x1,p1}| > -1$, demand is inelastic
- If $e_{x1,p1} > 0$, demand is Giffen

Marshallian Demand Elasticities

- The cross-price elasticity of demand ($e_{x2,p1}$) is

$$e_{x_2,p_1} = \frac{\Delta x_2 / x_2}{\Delta p_1 / p_1} = \frac{\partial x_2}{\partial p_1} \cdot \frac{p_1}{x_2} = \frac{\partial \ln x_2}{\partial \ln p_1}$$

Elasticities: Interesting Facts

- If demand is elastic, a price rise leads to an increase in spending:

$$\frac{\partial}{\partial p_1}[p_1 x_1^*] = x_1^* + p_1 \frac{\partial x_1^*}{\partial p_1} = x_1^*[1 + e_{x_1, p_1}] < 0$$

Elasticities: Interesting Facts

- Demand is homoegenous of degree zero.

$$x_1^*(kp_1, kp_2, km) = x_1^*(p_1, p_2, m)$$

- Differentiating with respect to k,

$$p_1 \cdot \frac{\partial x_1^*}{\partial p_1} + p_2 \cdot \frac{\partial x_1^*}{\partial p_2} + m \cdot \frac{\partial x_1^*}{\partial m} = 0$$

- Letting k=1 and dividing by x^*_1,

$$e_{x_1, p_1} + e_{x_1, p_2} + e_{x_1, \mathrm{m}} = 0$$

- A 1% change in all prices and income will not change demand for x_1.

Elasticities: Engel Aggregation

- Take the budget constraint

$$m = p_1x_1 + p_2x_2$$

- Differentiating,

$$1 = p_1 \cdot \frac{\partial x_1}{\partial m} + p_2 \cdot \frac{\partial x_2}{\partial m}$$

- Divide and multiply by x_1m and x_2m

$$1 = p_1 \cdot \frac{\partial x_1}{\partial m} \cdot \frac{x_1 m}{x_1 m} + p_2 \cdot \frac{\partial x_2}{\partial m} \cdot \frac{x_2 m}{x_2 m} = s_1 e_{x_1,m} + s_2 e_{x_2,m}$$

where $s_1=p_1x_1/m$ is expenditure share.

- Food is necessity (income elasticity<1)
  - Hence income elasticity for nonfood>1

Some Price Elasticities

- Specific Brands:
  - Coke -1.71
  - Pepsi -2.08
  - Tide Detergent -2.79

Some Price Elasticities

- Narrow Categories:
  - Transatlantic Air Travel -1.30
  - Tourism in Thailand -1.20
  - Ground Beef -1.02
  - Pork -0.78
  - Milk -0.54
  - Eggs -0.26

Some Price Elasticities

- Broad Categories:
  - Recreation -1.30
  - Clothing -0.89
  - Food -0.67
  - Imports -0.58
  - Transportation -0.56

CONSUMER SURPLUS

Consumer Surplus

- How do we determine how our utility changes when there is a change in prices.
- What affect would a carbon tax have on welfare?
- Cannot look at utilities directly (ordinal measure)
- Need monetary measure.

Consumer Surplus

- One way to evaluate the welfare cost of a price increase (from p_1 to p_1') would be to compare the expenditures required to achieve a given level of utilities $\underline{U}$ under these two situations

$$\text{Initial expenditure} = e(p_1, p_2, \underline{U})$$

$$\text{Expenditure after price rise} = e(p'_1, p_2, \underline{U})$$

Consumer Surplus

- Clearly, if p_1**'** > p_1 the expenditure has to increase to maintain the same level of utility:

$$e(p_1',p_2,\underline{U}) > e(p_1,p_2,\underline{U})$$

- The difference between the new and old expenditures is called the <u>compensating variation</u> (*CV*):

$$CV = e(p_1',p_2,\underline{U}) - e(p_1,p_2,\underline{U})$$

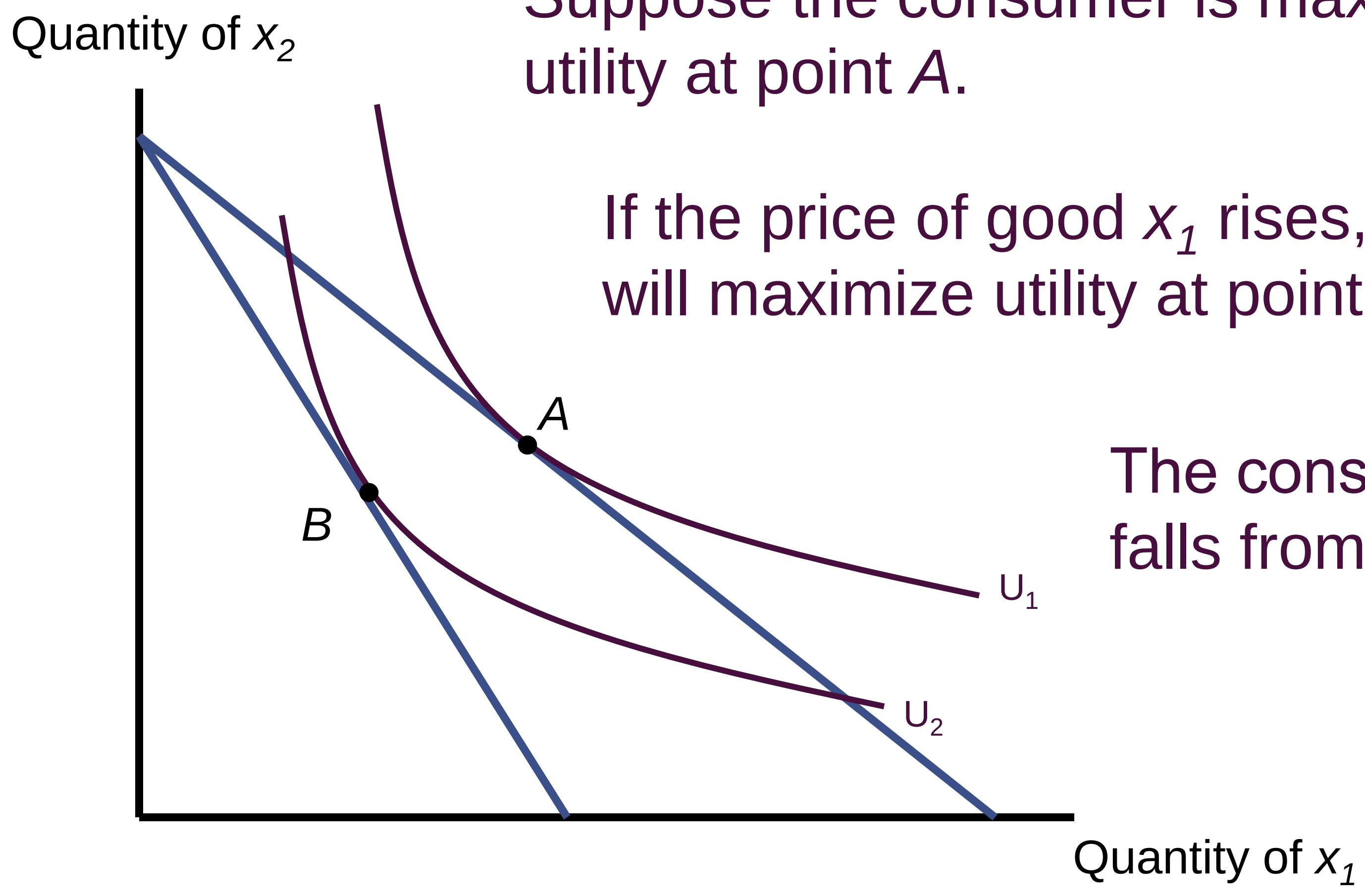
where $\underline{U} = v(p_1,p_2,,\text{m})$.

Consumer Surplus

Suppose the consumer is maximizing utility at point *A*.

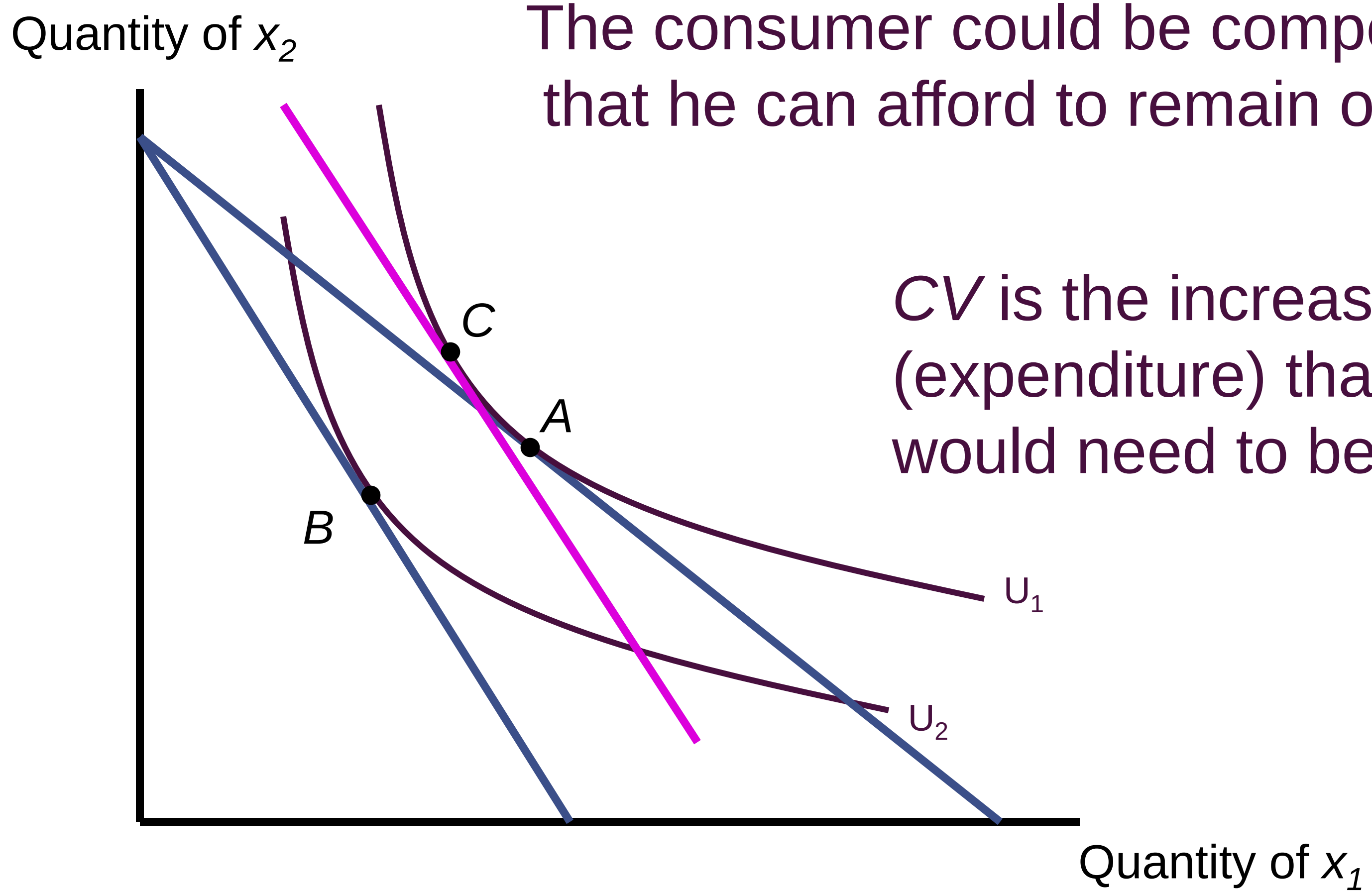
If the price of good x_1 rises, the consumer will maximize utility at point *B*.

The consumer's utility falls from U_1 to U_2



Consumer Surplus

The consumer could be compensated so that he can afford to remain on U_1



CV is the increase in income (expenditure) that the individual would need to be achieve U_1.

Consumer Surplus

- From Shepard's Lemma:

$$\frac{\partial e(p_1, p_2, \underline{U})}{\partial p_1} = h_1(p_1, p_2, \underline{U})$$

- CV equals the integral of the Hicksian demand

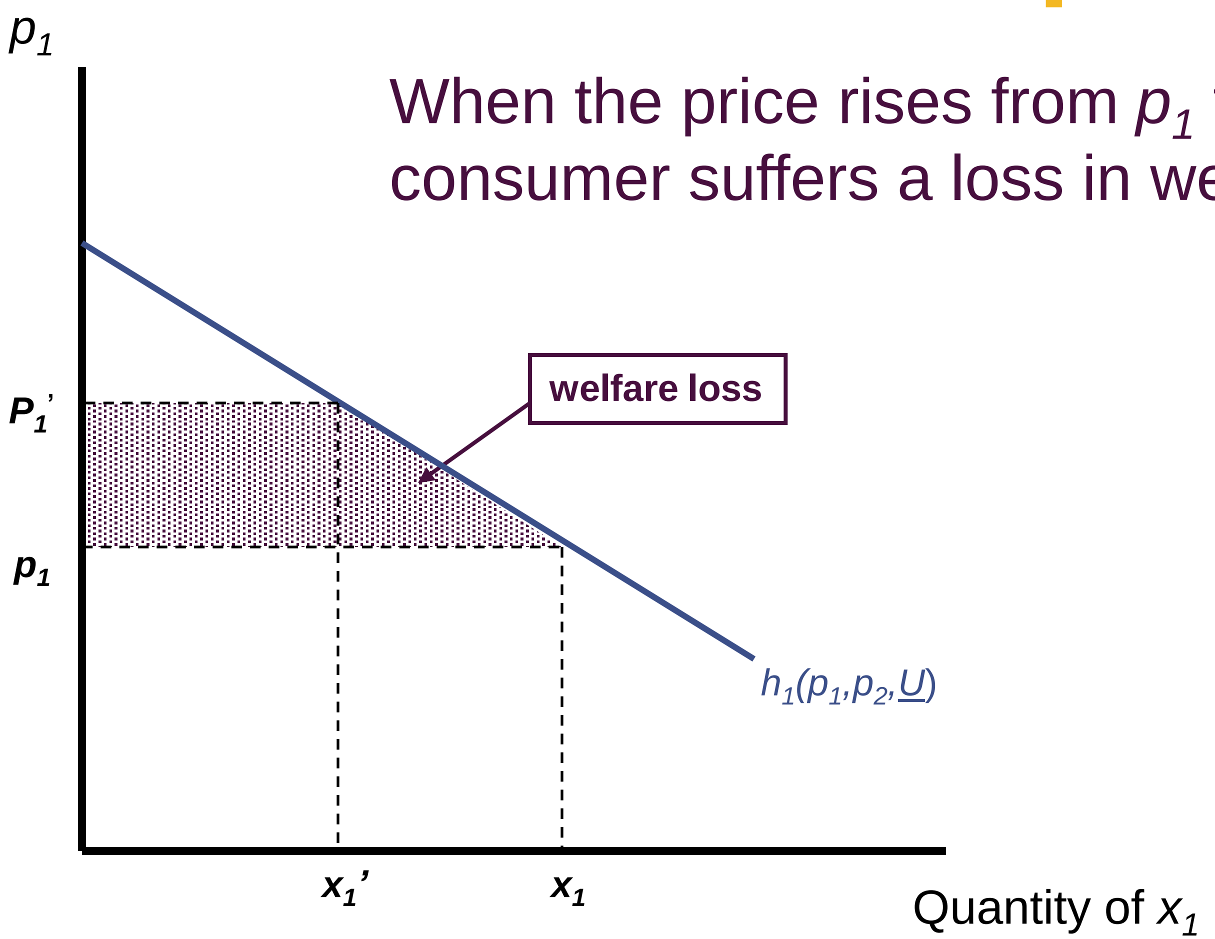
$$CV = e(p_1', p_2, \underline{U}) - e(p_1, p_2, \underline{U})$$

$$= \int_{p_1}^{p_1'} \frac{\partial}{\partial p_1} E(z, p_2, \underline{U})dz = \int_{p_1}^{p_1'} h_1(z, p_2, \underline{U})dz$$

- This integral is the area to the left of the Hicksian demand curve between p_1 and p_1'

Consumer Surplus

When the price rises from p_1 to p_1' the consumer suffers a loss in welfare



Consumer Surplus

- Consumer surplus equals the area under the Hicksian demand curve above the current price.
- CS equals welfare gain from reducing price from $p_1=\infty$ to current market price.
- That is, CS equals the amount the person would be willing to pay for the right to consume the good at the current market price.

A Problem

- Problem: Hicksian demand depends on the utility level which is not observed.

- Answer: Approximate with Marchallisn demand.

- From the Slutsky equation, we know the Hicksian and Marshallian demand functions have approximately the same slope when the good forms only a small part of the consumption bundle (i.e. when income effects are small)

Quasilinear Utility

- Suppose $u(x_1,x_2)=v(x_1)+x_2$
- From UMP, Marshallian demand for x_1

$$v'(x^*_1)=p_1/p_2$$

- From EMP, Hicksian demand for x1,

$$v'(h_1)=p_1/p_2$$

- Hence $x^*_1(p_1,p_2,m)=h_1(p_1,p_2,\underline{u})$.
- And

$$CV = \int_{p_1}^{p_1'} h_1(z, p_2, \underline{U})dz = \int_{p_1}^{p_1'} x_1^*(z, p_2, m)dz$$

Consumer Surplus

- We will define <u>consumer surplus</u> as the area below the Marshallian demand curve and above price
  - It shows what an individual would pay for the right to make voluntary transactions at this price
  - Changes in consumer surplus measure the welfare effects of price changes