Homework 2

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Problem 8

a) Implementing the code offered by Wille-E Coyote results in the infinite loop. Upon analyzing the formula $z(t + \Delta t) = z(t) - \Delta t(\sqrt{2g(z(t) - z_0)})$, we can see that at the very first time step, $z(t) - z_0 = 0$, resulting in $z(t + \Delta t) = z(t)$ for the first time step and every subsequent step after. While the energy is conserved, it is never converted into kinetic energy, making Wille-E hang in the air at the same height z_0 indefinitely – similar to the cartoon!

b) At the initial point: v(0) = z'(0) = 0, z(0) = h.

The governing equation: $m\frac{d^2z}{dt^2} = -mg$. Divide both sides by m : $\frac{d^2z}{dt^2} = -g$.

Expressing it as a system of differential equations:

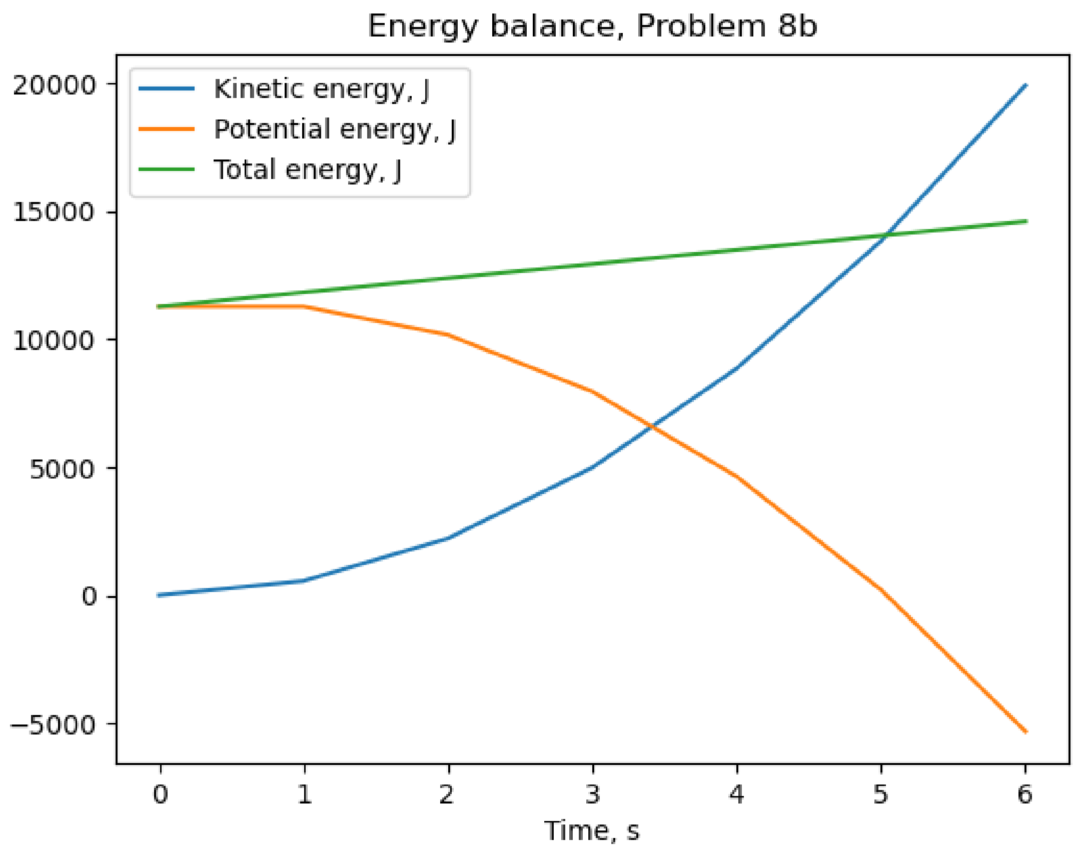
$$\begin{cases} \dfrac{d^2z}{dt^2} = z_t'' = \dfrac{dz'}{dt} = \dfrac{dv}{dt} = -g \\ \dfrac{dz}{dt} = v(t) \\ v(0) = 0, \qquad z(0) = h \end{cases}$$

Written as the Euler forward scheme:

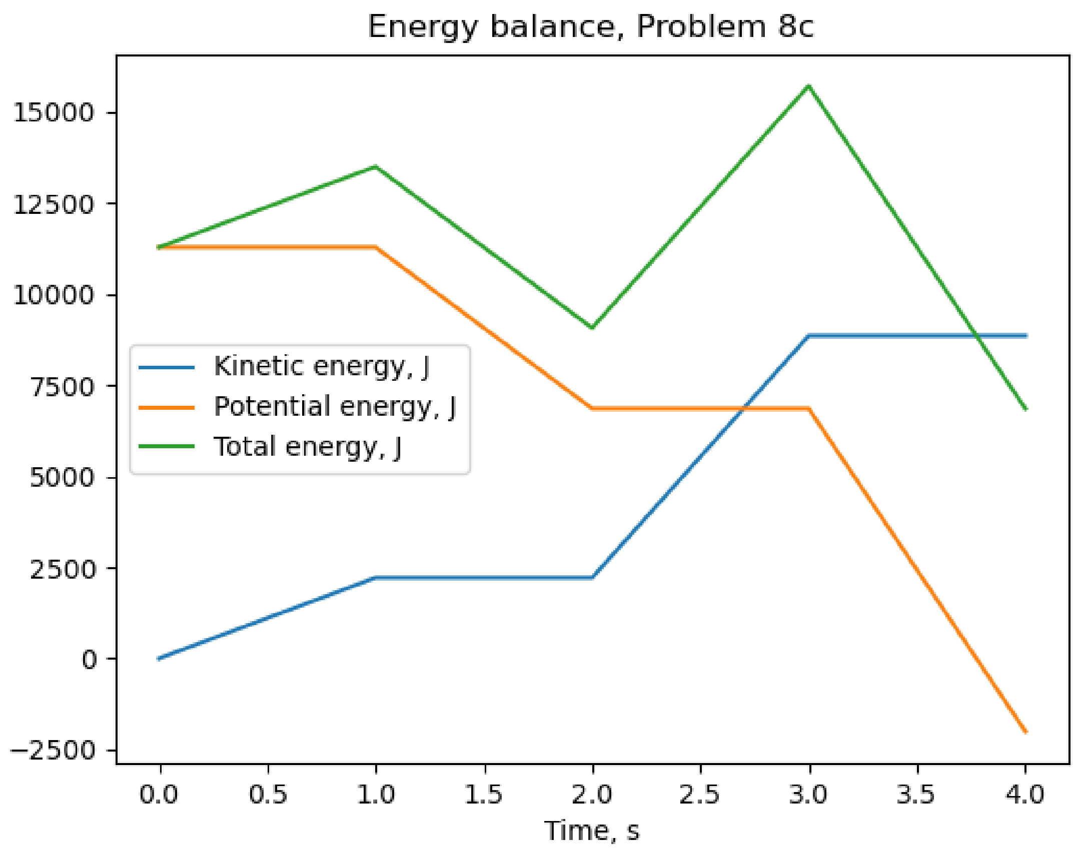
$$\begin{cases} v(t+1) = v(t) - g\Delta t \\ z(t+1) = z(t) + v(t)\Delta t \\ v(0) = 0, \qquad z(0) = h \end{cases}$$

The average mass of a southern coyote is 11.5 kg (needed as m in the energy formula).

The total energy is not exactly conserved, but slightly rises due to the kinetic energy outgrowing the loss of potential energy.


Energy balance, Problem 8b
Kinetic energy, J
Potential energy, J
Total energy, J
20000
15000
10000
5000
0
−5000
0
1
2
3
4
5
6
Time, s

c) Here, the total energy oscillates around the initial value of the energy; I assume this to be the artefact from the way the algorithm is executed (there is a delay in calculating z), but if we average all values of total energy (provided in the code), the average comes up close to the initial value of total energy, which confirms its conservation for the most part.



Code for different problems

All three files have been edited to correct the height.

Problem 8a: *p8_1.py*

Problem 8b: *p8_2.py*

Problem 8c: *p8_3.py*