Алгоритмы и модели вычислений.

Задание 11: DFT

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Теория

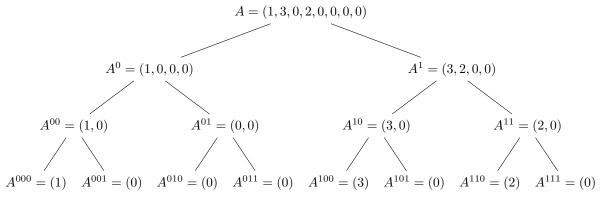
(сюда будут ссылки)

- 1. Многочлен $P_n(x) = a_0 + a_1 x + ... + a_{n-1} x^{n-1} \longleftrightarrow (a_0, ..., a_n) = P_n$ (порядок коэффициентов как на семинаре, а не как в задании). Считаем $\exists l \in \mathbb{N} \cup \{0\} : n = 2^l$.
- 2. $\omega_n^k \stackrel{\text{def}}{=} e^{\frac{2\pi k}{n}i}$
- 3. $\varphi(P) \stackrel{\text{\tiny def}}{=} (P_n(\omega_n^0), ..., P_n(\omega_n^{n-1}))$ дискретное преобразование Фурье
- 4. $P_n^0 \stackrel{\text{def}}{=} (a_0, a_2, a_4, \ldots), P_n^1 \stackrel{\text{def}}{=} (a_1, a_3, a_5, \ldots) \Rightarrow$ свойство: $P_n(x) = P_n^0(x^2) + x \cdot P_n^1(x^2)$. Следствия :
 - (a) $P_n(\omega_n^j) = P_n^0(\omega_{n/2}^j) + \omega_n^j P_n^1(\omega_{n/2}^j), \ 0 \le j < \frac{n}{2}$
 - (b) $P_n(\omega_n^{\frac{n}{2}+j}) = P_n^0(\omega_{n/2}^j) \omega_n^j P_n^1(\omega_{n/2}^j), \ 0 \leqslant j < \frac{n}{2}$
- 5. $n=1 \Rightarrow \varphi(P_n)=\varphi((a_0))=(a_0)$
- 6. Обозначаем $\varphi(A) = \alpha$, элементы кортежей как $(a_0, ..., a_{n-1})[i] = a_i$.

7. Тогда
$$4 \Rightarrow \begin{cases} \alpha[j] &= \alpha^0[j] + \omega_n^j \alpha^1[j] \\ \alpha[n/2+j] &= \alpha^0[j] - \omega_n^j \alpha^1[j] \end{cases}$$

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1. A = (1, 3, 0, 2, 0, 0, 0, 0), B = (1, 0, 1, 3, 0, 0, 0, 0). Дерево вызовов:



- (a) Для $A^{000}, A^{001}, ..., A^{111}$ результат преобразования $\alpha^{ijk} = A^{ijk}$ (см. 5)
- (b) $\alpha^{00} = (\alpha^{000}[0] + \omega_2^0 \cdot \alpha^{001}[0], \alpha^{000}[0] \omega_2^0 \alpha^{001}[0]) = |\omega_2^0 = 1| = (1, 1)$
- (c) $\alpha^{01} = (\alpha^{010}[0] + \omega_2^0 \cdot \alpha^{011}[0], \alpha^{010}[0] \omega_2^0 \alpha^{011}[0]) = |\omega_2^0 = 1| = (0, 0)$
- (d) $\alpha^{10} = (\alpha^{100}[0] + \omega_2^0 \cdot \alpha^{101}[0], \alpha^{100}[0] \omega_2^0 \alpha^{101}[0]) = |\omega_2^0 = 1| = (3, 3)$
- (e) $\alpha^{11} = (\alpha^{110}[0] + \omega_2^0 \cdot \alpha^{111}[0], \alpha^{110}[0] \omega_2^0 \alpha^{111}[0]) = |\omega_2^0 = 1| = (2, 2)$
- (g) $\alpha^0[1] = \alpha^{00}[1] + \underbrace{\omega_4^1}_{=i} \alpha^{01}[1] = 1$ (h) $\alpha^0[2+0] = \alpha^{00}[0] \underbrace{\omega_4^0}_{=1} \alpha^{01}[0] = 1$
- (i) $\alpha^0[2+1] = \alpha^{00}[1] \underbrace{\omega_4^1}_{-} \alpha^{01}[1] = 1$

$$\begin{aligned} \text{(j)} \quad & \alpha^{1}[0] = \alpha^{10}[0] + \underbrace{\omega_{4}^{0}}_{=1} \alpha^{11}[0] = 1 \\ \text{(k)} \quad & \alpha^{1}[1] = \alpha^{10}[1] + \underbrace{\omega_{4}^{1}}_{=i} \alpha^{11}[1] = 1 \\ \text{(l)} \quad & \alpha^{1}[2+0] = \alpha^{10}[0] - \underbrace{\omega_{4}^{0}}_{=1} \alpha^{11}[0] = 1 \\ \text{(m)} \quad & \alpha^{1}[2+1] = \alpha^{10}[1] - \underbrace{\omega_{4}^{1}}_{=i} \alpha^{11}[1] = 1 \end{aligned}$$

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