## Задача о назначениях в руото

Сергей Володин, 374 гр.

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## Постановка задачи

$$\begin{cases} \sum_{ij} x_{ij} c_{ij} \to \min \\ \forall j \sum_{i} x_{ij} = 1 \\ \forall i \sum_{j} x_{ij} = 1 \\ x_{ij} \in \overline{0, 1} \end{cases}$$

- $c_{ij}$  стоимость выполнения i-м работником j-й работы
- $x_{ij} = 1 \Leftrightarrow i$ -й работник выполняет j-ю работу.

## Модель руото

```
Код на github: https://github.com/etoestja/inf/tree/master/mipt/s7/discrete/3
$ cat assignment_problem.py
from __future__ import division
from pyomo.environ import *
model = AbstractModel()
model.I = Set()
model.J = Set()
model.c = Param(model.I, model.J)
model.x = Var(model.I, model.J, domain=NonNegativeReals)
def obj_expression(model):
return summation(model.c, model.x)
model.OBJ = Objective(rule=obj_expression)
def constI(model, i):
# return the expression for the constraint for i
return sum(model.x[i,j] for j in model.J) == 1
def constJ(model, j):
# return the expression for the constraint for i
return sum(model.x[i,j] for i in model.I) == 1
# the next line creates one constraint for each member of the set model.I
model.c1 = Constraint(model.I, rule=constI)
model.c2 = Constraint(model.J, rule=constJ)
Данные
$ cat assignment_problem.dat
```

```
$ cat assignment_problem.dat
set I := 1 2 3 4;
set J := 1 2 3 4;
param c:
1 2 3 4 :=
```

```
1 1 2 1 2
2 2 3 3 4
3 1 0 0 1
4 1 1 1 1
```

## Solver glpk

Commands:

```
$ pyomo solve assignment_problem.py assignment_problem.dat --solver=glpk
     0.00] Setting up Pyomo environment
0.00] Applying Pyomo preprocessing actions
     0.00] Creating model
     0.10] Applying solver
Γ
    0.12] Processing results
    Number of solutions: 1
    Solution Information
      Gap: 0.0
      Status: feasible
      Function Value: 4.0
    Solver results file: results.json
     0.12] Applying Pyomo postprocessing actions
     0.12] Pyomo Finished
results.json:
    "Problem": [
        {
            "Lower bound": 4.0,
            "Name": "unknown",
            "Number of constraints": 9,
            "Number of nonzeros": 33,
            "Number of objectives": 1,
            "Number of variables": 17,
            "Sense": "minimize",
            "Upper bound": 4.0
        }
    ],
    "Solution": [
        {
            "number of solutions": 1,
            "number of solutions displayed": 1
        },
            "Constraint": "No values",
            "Gap": 0.0,
            "Message": null,
            "Objective": {
                "OBJ": {
                    "Value": 4.0
                }
            },
            "Problem": {},
            "Status": "feasible",
            "Variable": {
                "x[1,3]": {
                    "Value": 1.0
                },
                "x[2,1]": {
                    "Value": 1.0
                "x[3,2]": {
                    "Value": 1.0
                },
                "x[4,4]": {
                     "Value": 1.0
                }
```

```
}
        }
    ],
    "Solver": [
        {
            "Error rc": 0,
            "Statistics": {
                "Branch and bound": {
                    "Number of bounded subproblems": 0,
                    "Number of created subproblems": 0
                }
            },
            "Status": "ok",
            "Termination condition": "optimal",
            "Time": 0.006086111068725586
        }
    ]
Solver bonmin
  Commands:
$ pyomo solve assignment_problem.py assignment_problem.dat --solver=bonmin
     0.00] Setting up Pyomo environment
     0.00] Applying Pyomo preprocessing actions
     0.00] Creating model
     0.10] Applying solver
     0.48] Processing results
    Number of solutions: 1
    Solution Information
      Gap: None
      Status: optimal
      Function Value: 4.0000011978
    Solver results file: results.json
     0.48] Applying Pyomo postprocessing actions
     0.48] Pyomo Finished
results.json:
    "Problem": [
        {
            "Lower bound": -Infinity,
            "Number of constraints": 0,
            "Number of objectives": 1,
            "Number of variables": 16,
            "Sense": "unknown",
            "Upper bound": Infinity
        }
    ],
    "Solution": [
        {
            "number of solutions": 1,
            "number of solutions displayed": 1
            "Constraint": "No values",
            "Gap": null,
            "Message": "bonmin\\x3a Optimal",
            "Objective": {
                "OBJ": {
                    "Value": 4.000001197829755
                }
            },
            "Problem": {},
            "Status": "optimal",
            "Variable": {
                "x[1,3]": {
```

```
"Value": 1.00000029930368
                },
                "x[2,1]": {
                    "Value": 1.0000000299529193
                "x[3,2]": {
                    "Value": 1.0000000299371015
                "x[4,4]": {
                    "Value": 1.0000000299467693
                }
            }
        }
    ],
    "Solver": [
        {
            "Error rc": 0,
            "Id": 3,
            "Message": "bonmin\\x3a Optimal",
            "Status": "ok",
            "Termination condition": "optimal",
            "Time": 0.3690049648284912
        }
    ]
Solver cbc
  Commands:
 $ pyomo solve assignment_problem.py assignment_problem.dat --solver=cbc
     0.00] Setting up Pyomo environment
     0.00] Applying Pyomo preprocessing actions
0.00] Creating model
0.10] Applying solver
    0.46] Processing results
    Number of solutions: 1
    Solution Information
      Gap: 0.0
      Status: optimal
      Function Value: 4
    Solver results file: results.json
     0.46] Applying Pyomo postprocessing actions
     0.46] Pyomo Finished
results.json:
{
    "Problem": [
        {
            "Lower bound": -Infinity,
            "Name": "tmpBVftNA.pyomo";
            "Number of constraints": 9,
            "Number of nonzeros": 33,
            "Number of objectives": 1,
            "Number of variables": 17,
            "Sense": "minimize",
            "Upper bound": Infinity
        }
    ],
    "Solution": [
        {
            "number of solutions": 1,
            "number of solutions displayed": 1
        },
            "Constraint": "No values",
            "Gap": 0.0,
            "Message": null,
```

```
"Objective": {
                "OBJ": {
                    "Value": 4
            },
            "Problem": {},
            "Status": "optimal",
            "Variable": {
                "x[1,3]": {
                    "Value": 1
                },
                "x[2,1]": {
                    "Value": 1
                },
                x[3,2]: {
                    "Value": 1
                "x[4,4]": {
                    "Value": 1
                }
            }
        }
    ],
    "Solver": [
            "Error rc": 0,
            "Status": "ok",
            "Termination condition": "unknown",
            "Time": 0.3434598445892334,
            "User time": -1.0
        }
    ]
Solver scip
  Commands:
$ pyomo solve assignment_problem.py assignment_problem.dat --solver=scip
     0.00] Setting up Pyomo environment
     0.00] Applying Pyomo preprocessing actions
0.00] Creating model
0.10] Applying solver
     0.47] Processing results
    Number of solutions: 1
    Solution Information
      Gap: None
      Status: optimal
      Function Value: 4.0
    Solver results file: results.json
     0.48] Applying Pyomo postprocessing actions
     0.48] Pyomo Finished
results.json:
{
    "Problem": [
        {
            "Lower bound": -Infinity,
            "Number of constraints": 0,
            "Number of objectives": 1,
            "Number of variables": 16,
            "Sense": "unknown",
            "Upper bound": Infinity
        }
    ],
    "Solution": [
        {
            "number of solutions": 1,
```

```
"number of solutions displayed": 1
        },
            "Constraint": "No values",
            "Gap": null,
            "Message": "optimal solution found",
            "Objective": {
                "OBJ": {
                    "Value": 4.0
                }
            },
            "Problem": {},
            "Status": "optimal",
            "Variable": {
                "x[1,3]": {
                    "Value": 1.0
                "x[2,1]": {
                    "Value": 1.0
                "x[3,2]": {
                    "Value": 1.0
                "x[4,4]": {
                    "Value": 1.0
                }
            }
        }
    ],
    "Solver": [
        {
            "Error rc": 0,
            "Id": 0,
            "Message": "optimal solution found",
            "Status": "ok",
            "Termination condition": "optimal",
            "Time": 0.361814022064209
        }
    ]
Solver couenne
  Commands:
 $ pyomo solve assignment_problem.py assignment_problem.dat --solver=couenne
     0.00] Setting up Pyomo environment
     0.00] Applying Pyomo preprocessing actions
     0.00] Creating model
Г
     0.10] Applying solver
     0.13] Processing results
    Number of solutions: 1
    Solution Information
      Gap: None
      Status: optimal
     Function Value: 3.9999994267
    Solver results file: results.json
     0.13] Applying Pyomo postprocessing actions
     0.13] Pyomo Finished
results.json:
{
    "Problem": [
        {
            "Lower bound": -Infinity,
            "Number of constraints": 0,
            "Number of objectives": 1,
            "Number of variables": 16,
```

```
"Sense": "unknown",
        "Upper bound": Infinity
    }
],
"Solution": [
        "number of solutions": 1,
        "number of solutions displayed": 1
    },
        "Constraint": "No values",
        "Gap": null,
        "Message": "couenne\\x3a Optimal",
        "Objective": {
            "OBJ": {
                "Value": 3.99999942673723
            }
        },
        "Problem": {},
        "Status": "optimal",
        "Variable": {
            "x[1,1]": {
                "Value": 9.157762731214199e-09
            },
            "x[1,3]": {
                "Value": 0.99999993501932
            },
            "x[1,4]": {
                "Value": -2.6596947977353125e-09
            },
            "x[2,1]": {
                "Value": 1.0
            "x[2,2]": {
                "Value": 7.958210192017798e-09
            "x[2,4]": {
                "Value": -7.958210192017798e-09
            },
            "x[3,2]": {
                "Value": 0.999999945008361
            },
            "x[3,3]": {
                "Value": 2.3544397697945758e-08
            },
            "x[3,4]": {
                "Value": -1.8045233795922773e-08
            "x[4,1]": {
                "Value": -9.157762770328759e-09
            "x[4,2]": {
                "Value": -2.4590462999185547e-09
            },
            "x[4,3]": {
                "Value": -1.704632972535231e-08
            },
            "x[4,4]": {
                "Value": 1.0
            }
   }
],
"Solver": [
    {
        "Error rc": 0,
        "Id": 3,
        "Message": "couenne\\x3a Optimal",
```

```
"Status": "ok",
            "Termination condition": "optimal",
            "Time": 0.01647496223449707
        }
   ]
}
Solver ipopt
  Commands:
$ pyomo solve assignment_problem.py assignment_problem.dat --solver=ipopt
     0.00] Setting up Pyomo environment
     0.00] Applying Pyomo preprocessing actions
     0.00] Creating model
     0.10] Applying solver
     0.47] Processing results
   Number of solutions: 1
   Solution Information
      Gap: None
      Status: optimal
      Function Value: 4.0000006493
   Solver results file: results.json
     0.47] Applying Pyomo postprocessing actions
     0.47] Pyomo Finished
results.json:
{
    "Problem": [
        {
            "Lower bound": -Infinity,
            "Number of constraints": 8,
            "Number of objectives": 1,
            "Number of variables": 16,
            "Sense": "unknown",
            "Upper bound": Infinity
        }
   ],
    "Solution": [
        {
            "number of solutions": 1,
            "number of solutions displayed": 1
        },
            "Constraint": "No values",
            "Gap": null,
            "Message": "Ipopt 3.12.4\\x3a Optimal Solution Found",
            "Objective": {
                "OBJ": {
                    "Value": 4.00000064926868
                }
            },
            "Problem": {},
            "Status": "optimal",
            "Variable": {
                "x[1,3]": {
                    "Value": 1.000000121297563
                },
                "x[2,1]": {
                    "Value": 1.00000018012586
                },
                "x[3,2]": {
                    "Value": 1.000000141865464
                "x[3,3]": {
                    "Value": 1.6100187743708992e-10
                "x[4,4]": {
```