$$(*) \Leftrightarrow y' = y^2 e^{-\frac{x}{3}} \Leftrightarrow \frac{dy}{dx} = y^2 e^{-\frac{x}{3}} \Leftrightarrow \left[\begin{array}{ll} y = 0 & (1) \\ \frac{dy}{y^2} = e^{-\frac{x}{3}} dx & (2) \end{array} \right.$$

$(2) \Leftrightarrow \int \frac{dy}{y^2} = \int e^{-\frac{x}{3}} dx \Leftrightarrow -\frac{1}{y} = -3e^{-x/3} + C \Leftrightarrow y = \frac{1}{3e^{-x/3}+C}$

Ответ: $\boxed{y = \dfrac{1}{3e^{-x/3} + C} \text{ или } y = 0}$