

Пример 6

3.6

$$\begin{array}{l} A = (-2; -3; 1) \\ B = (-1; -1; 0) \\ C = (4; 1; 1) \\ D = (-5; 5; 3) \end{array} \left| \begin{array}{l} \vec{AC} = (6; 4; 0) \\ \vec{BD} = (-4; 6; 3) \\ (\vec{AC} \cdot \vec{BD}) = -4 \cdot 6 + 6 \cdot 4 + 0 \cdot 3 = 0 \\ \Rightarrow AC \perp BD // \text{пр.} \end{array} \right.$$

$$\begin{array}{l} \textcircled{6} |\vec{a}| = 4 \\ |\vec{b}| = 5 \\ (\vec{a}, \vec{b}) = \frac{\pi}{4} \\ \vec{I} = \vec{a} - 2\vec{b} \\ \vec{II} = 3\vec{a} + 2\vec{b} \\ S_{\Delta} = ? \end{array} \quad \begin{array}{l} S_{\Delta} = \frac{|\vec{I} \times \vec{II}|}{2} = \frac{|\vec{a} - 2\vec{b} \times 3\vec{a} + 2\vec{b}|}{2} = \\ = \frac{1}{2} |\vec{a} \times 3\vec{a} - \vec{a} \times 6\vec{b} + 6\vec{b} \times \vec{a} + 2\vec{b} \times 2\vec{b}| = \\ = \frac{1}{2} |0 - 6[\vec{a} \times \vec{b}] + 4[\vec{b} \times \vec{a}] + 0| = \\ = \frac{1}{2} | -6[\vec{a} \times \vec{b}] - 4[\vec{a} \times \vec{b}] | = \\ = \frac{1}{2} | -10[\vec{a} \times \vec{b}] | = 5 |\vec{a} \times \vec{b}| = 5 \cdot |\vec{a}| \cdot |\vec{b}| \cdot \sin \frac{\pi}{4} = \\ = 5 \cdot 4 \cdot 5 \cdot \frac{\sqrt{2}}{2} = 50\sqrt{2} \\ // \text{ответ} \end{array}$$

$$\begin{aligned} 4.1) \quad \vec{a} &= \vec{i} + 2\vec{j} + \vec{k} = (1; 2; 1) \\ \vec{b} &= 9\vec{i} - 11\vec{j} + 3\vec{k} = (9; -11; 3) \\ \vec{c} &= 2\vec{i} + 4\vec{j} - 2\vec{k} = (2; 4; -2) \end{aligned}$$

Если $\vec{a}, \vec{b}, \vec{c}$ колл., то

$$\vec{d} = [\vec{a}, \vec{b}] \perp \vec{c}, \text{ т.е. } (\vec{d}, \vec{c}) = 0$$

$$\begin{aligned} \vec{d} &= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 2 & 1 \\ 9 & -11 & 3 \end{vmatrix} = 6\vec{i} - 9\vec{j} - 11\vec{k} - 18\vec{k} - \\ &\quad - 11\vec{i} - 3\vec{j} = \\ &= -5\vec{i} - 12\vec{j} - 29\vec{k} = (-5; -12; -29) \end{aligned}$$

$$\begin{aligned} (\vec{d}, \vec{c}) &= d_x c_x + d_y c_y + d_z c_z = \\ &= -5 \cdot 2 - 12 \cdot 4 + 29 \cdot 2 = -10 - 48 + 58 = 0 \Rightarrow \\ &\Rightarrow \vec{a}, \vec{b}, \vec{c} - \text{компланарны} // \text{Ответ} \end{aligned}$$