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231 – 338

Таблица ответов

№	Ответы
1.	359
2.	-32
3.	0,75
4.	683
5.	2
6.	40,75
7.	-42,25
8.	6
9.	$\sqrt{9109}$
10.	0,59
11.	$3240\sqrt{3}$
12.	92

① $A = \begin{pmatrix} -1 & 1 & 0 \\ 1 & 3 & 2 \\ 4 & 0 & -3 \end{pmatrix}$ $B = \begin{pmatrix} 2 & 0 & 2 \\ 0 & -3 & 3 \\ 1 & 1 & 36 \end{pmatrix}$

$C = A^2 + 8 \cdot B^T$ $B^T = \begin{pmatrix} 2 & 0 & 1 \\ 0 & -3 & 1 \\ 2 & 3 & 36 \end{pmatrix}$

$A^2 = \begin{pmatrix} 2 & 2 & 2 \\ 10 & 10 & 0 \\ -16 & 4 & 9 \end{pmatrix}$

$8B^T = \begin{pmatrix} 16 & 0 & 8 \\ 0 & -24 & 8 \\ 16 & 24 & 288 \end{pmatrix}$

$C = \begin{pmatrix} 18 & 2 & 10 \\ 10 & -14 & 8 \\ 0 & 28 & 297 \end{pmatrix}$

$sum(C) = 359$ // Ответ

(2)
$$H = \begin{pmatrix} 1 & 0 & -1 & 2 \\ 0 & 2 & 1 & -1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & -2 & 36 \end{pmatrix}$$

$\det(H) = A_{11} - A_{13} + 2\,A_{14} = -102 + 70 =$
$= -32$ // Ответ

$A_{11} = -102$
$A_{13} = -70$
$A_{14} = 0$

(3)
$$Q = \begin{pmatrix} 2 & 2 & 1 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{pmatrix} \qquad Q_* = \begin{pmatrix} 1 & 4 & -2 \\ -2 & 0 & 4 \\ 3 & -4 & 2 \end{pmatrix}$$

$$Q_*^T = \begin{pmatrix} 1 & -2 & 3 \\ 4 & 0 & -4 \\ -2 & 4 & 2 \end{pmatrix} \qquad \det(Q) = 8$$

$A_{11} = 1$ $\quad A_{21} = -2$ $\quad A_{31} = 3$
$A_{12} = 4$ $\quad A_{22} = 0$ $\quad A_{32} = -4$
$A_{13} = -2$ $\quad A_{23} = 4$ $\quad A_{33} = 2$

$$Q^{-1} = \begin{pmatrix} \frac{1}{8} & -\frac{2}{8} & \frac{3}{8} \\ \frac{4}{8} & 0 & -\frac{4}{8} \\ -\frac{2}{8} & \frac{4}{8} & \frac{2}{8} \end{pmatrix} \quad sum(Q) = 0{,}75$$

Проверка:

$$\begin{pmatrix} 2 & 2 & 1 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{8} & -\frac{2}{8} & \frac{3}{8} \\ \frac{4}{8} & 0 & -\frac{4}{8} \\ -\frac{2}{8} & \frac{4}{8} & \frac{2}{8} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

④ $A \cdot X \cdot B = C$

$$A = \begin{pmatrix} -2 & 0 \\ 1 & -1 \end{pmatrix} \quad B = \begin{pmatrix} 6 & 8 \\ 2 & 1 \end{pmatrix} \quad C = \begin{pmatrix} 2 & 3 \\ -36 & 0 \end{pmatrix}$$

$$X = C \cdot A^{-1} \cdot B^{-1}$$

$$A^{-1} = \begin{pmatrix} -1 & -1 \\ 0 & -2 \end{pmatrix} \quad B = \begin{pmatrix} 1 & -2 \\ -8 & 6 \end{pmatrix}$$

$$X = \begin{pmatrix} 150 & 21 \\ 464 & 48 \end{pmatrix}$$

$sum(X) = 683$ // Ответ

⑤ $A = \begin{pmatrix} 1 & -1 & 2 & 3 & 7 \\ 0 & 5 & 4 & 3 & 1 \\ 2 & 3 & 8 & 9 & 15 \\ 3 & -8 & 2 & 6 & 20 \end{pmatrix}$

$M_1 = 3 \Rightarrow r(A) \geq 1$

$M_2 = 72 \Rightarrow r(A) \geq 2$

$M_3 = \begin{vmatrix} 0 & 5 & 4 \\ 2 & 3 & 8 \\ 3 & -8 & 2 \end{vmatrix} = 0 \Rightarrow r(A) = 2$ // Ответ

⑥

$\begin{cases} 5x_1 - 6x_2 + 4x_3 = 3 \\ 3x_1 - 3x_2 + 2x_3 = 2 \\ 4x_1 - 5x_2 + 2x_3 = 36 \end{cases} \sim \left(\begin{array}{ccc|c} 5 & -6 & 4 & 3 \\ 3 & -3 & 2 & 2 \\ 4 & -5 & 2 & 36 \end{array}\right)$

$\Delta = \begin{vmatrix} 5 & -6 & 4 \\ 3 & -3 & 2 \\ 4 & -5 & 2 \end{vmatrix} = -4$

$x_1 = 1$

$x_2 = \frac{66}{-4}$

$x_3 = \frac{101}{-4}$

$\Delta_1 = \begin{vmatrix} 3 & -6 & 4 \\ 2 & -3 & 2 \\ 36 & -5 & 2 \end{vmatrix} = -4$

$\Delta_2 = \begin{vmatrix} 5 & 3 & 4 \\ 3 & 2 & 2 \\ 4 & 36 & 2 \end{vmatrix} = 66$

$sum(x_1, x_2, x_3) = 40,75$ // Ответ

$\Delta_3 = \begin{vmatrix} 5 & -6 & 3 \\ 3 & -3 & 2 \\ 4 & -5 & 36 \end{vmatrix} = 101$

⑦

$$\begin{pmatrix} 5 & -6 & 4 & | & 1 \\ 3 & -3 & 2 & | & 0 \\ 4 & -5 & 2 & | & 36 \end{pmatrix} \sim \{ I - 2II \} \sim$$

$$\sim \begin{pmatrix} -1 & 0 & 0 & | & 1 \\ 3 & -3 & 2 & | & 0 \\ 4 & -5 & 2 & | & 36 \end{pmatrix} \sim \{ II - III \} \sim$$

$$\sim \begin{pmatrix} -1 & 0 & 0 & | & 1 \\ -1 & 2 & 0 & | & -36 \\ 4 & -5 & 2 & | & 36 \end{pmatrix}$$

$x_1 = -1$

$x_2 = -17,5$

$x_3 = -23,75$

$sum(x_1, x_2, x_3) = -42,25$ //Ответ

⑧

$\bar{m} = (3, -2, 1)$

$\bar{S} = (-1, 1, -2)$

$\bar{p} = (2, 1, -3)$

$\bar{u} = (36, -6, 5)$

$$\begin{vmatrix} 3 & -2 & 1 \\ -1 & 1 & -2 \\ 2 & 1 & -3 \end{vmatrix} = 8 \neq 0 \Rightarrow$$

$\Rightarrow \bar{m}, \bar{S}, \bar{p}$ – базис

$$\bar{a} = a_x \cdot \bar{m} + a_y \cdot \bar{s} + a_z \cdot \bar{p} = a_x(3\bar{i} - 2\bar{j} + \bar{k}) +$$
$$+ a_y(-\bar{i} + \bar{j} - 2\bar{k}) + a_z(2\bar{i} + \bar{j} - 3\bar{k}) =$$
$$= 36\bar{i} - 6\bar{j} + 5\bar{k}$$

$$\begin{cases} 3a_m - a_s + 2a_p = 36 \\ -2a_m + a_s + a_p = -6 \\ a_m - 2a_s - 3a_p = 5 \end{cases} \qquad \left(\begin{array}{ccc|c} 3 & -1 & 2 & 36 \\ -2 & 1 & 1 & -6 \\ 1 & -2 & -3 & 5 \end{array}\right)$$

$\Delta = 8$ $\qquad a_m = \frac{207}{8}$

$\qquad a_s = -\frac{257}{8}$

$\Delta_1 = 207$ $\qquad a_p = \frac{98}{8}$

$\Delta_2 = -257$

$\Delta_3 = 98$ $\qquad sum(a_m, a_s, a_p) = 6$ // Ответ

(9)

$|\bar{m}| = 36$ $\qquad |a|^2 = |3\bar{m}|^2 + |\bar{s}|^2 -$

$|\bar{s}| = 35$ $\qquad - 2 \cdot |3\bar{m}| - |\bar{s}| \cdot \cos\varphi =$

$(\widehat{m, s}) = 60^\circ$ $\qquad = 9 \cdot 36^2 + 35^2 - 2 \cdot 3 \cdot 36 \cdot$

$\bar{a} = 3\bar{m} - \bar{s}$ $\qquad \cdot 35 \cdot \frac{1}{2} = 9109$

$|\bar{a}| = ?$ $\qquad |\bar{a}| = \sqrt{9109}$ // Ответ

(10) $A(36; -1; 2)$

$B(3; 36; 2)$

$C(7; 1; 36)$

$\cos\varphi = ?$

$\overline{BA} = (33; -37; 0)$ $\quad |\overline{BA}| = \sqrt{2458}$

$\overline{BC} = (4; -35; 34)$ $\quad |\overline{BC}| = \sqrt{2397}$

$(\overline{BA}, \overline{BC}) = 1427$

$\cos\varphi = \frac{1427}{\sqrt{2458} \cdot \sqrt{2397}} = 0{,}59$ // Ответ

(11)

$2\bar{p} + \bar{q}$ и $4\bar{p} - 3\bar{q}$

$|\bar{p}| = |\bar{q}| = 36$

$(\widehat{p, q}) = \frac{\pi}{6}$ $\quad S = |[\overline{AB} \times \overline{AD}]|$

$\overline{AB} = \frac{1}{2}\overline{AC} - \frac{1}{2}\overline{BD}$

$\overline{AD} = \frac{1}{2}\overline{AC} + \frac{1}{2}\overline{BD}$

$\overline{AB} = 2\bar{p} - \frac{3}{2}q - \bar{p} - \frac{1}{2}\bar{q} = \bar{p} - 2\bar{q}$

$\overline{AD} = 2\bar{p} - \frac{3}{2}q + \bar{p} + \frac{1}{2}\bar{q} = 3\bar{p} - \bar{q}$

$$\left|\left[(\bar{p}-2\bar{q})\times(3\bar{p}-\bar{q})\right]\right| = \left|[\bar{p}\times 3\bar{p}] - [2\bar{q}\times 3\bar{p}] -\right.$$

$$\left. -[\bar{p}\times\bar{q}] + [2\bar{q}\times\bar{q}]\right| = \left|-6[\bar{q}\times\bar{p}] + [q\times\bar{p}]\right| =$$

$$= 5[\bar{p}\times\bar{q}] = 5|\bar{p}|\cdot|\bar{q}|\cdot\sin\frac{\pi}{6} = 5\cdot 36\cdot 36\cdot$$

$$\cdot\frac{\sqrt{3}}{2} = 3240\sqrt{3}$$ // Ответ

(12) $\bar{a}(36; -1; 5)$

$\bar{b}(2; 4; -2)$

$\bar{c}(3; 0; 1)$

$$\begin{vmatrix} 36 & -1 & 5 \\ 2 & 4 & -2 \\ 3 & 0 & 1 \end{vmatrix} = 92$$

$92 \neq 0 \Rightarrow$ некомпланарны

$V = |(\bar{a}, \bar{b}, \bar{c})| = 92$ // Ответ

$\det \geq 0 \Rightarrow$ обр. первую тройку