Практикум 3

(1.1) $A = \begin{bmatrix} 1 & 3 \\ 5 & 7 \end{bmatrix}$ $A^{-1} = -\frac{1}{8} \cdot \begin{bmatrix} 1 & 5 \\ 3 & 7 \end{bmatrix}^{*}$

$A_{11} = 7$ $A^{-1} = \begin{bmatrix} -\frac{7}{8} & \frac{3}{8} \\ \frac{5}{8} & -\frac{1}{8} \end{bmatrix}$
$A_{21} = -3$
$A_{12} = -5$
$A_{22} = 1$

(2. 4)

$$X \begin{pmatrix} 5 & -6 & 4 \\ 3 & -3 & 2 \\ 4 & -5 & 2 \end{pmatrix} = (3 \quad 2 \quad 1)$$

$XA = B \mid \cdot A^{-1}$

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$A^{-1} = \begin{pmatrix} 5 & -6 & 4 \\ 3 & -3 & 2 \\ 4 & -5 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \sim$

$\{ I + (-1) \cdot II \} \sim \begin{pmatrix} -1 & 0 & 0 \\ 3 & -3 & 2 \\ 4 & -5 & 2 \end{pmatrix} \begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$\sim \left\{ \begin{array}{l} \underline{I \cdot (-1)} \\ \underline{III - II} \end{array} \right\} \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 0 \\ -1 & 2 & 0 & -1 & 3 & -1 \\ 4 & -5 & 2 & 4 & -8 & 1 \end{array} \right)$$

$$\sim \left\{ \begin{array}{l} \underline{III + 2,5\,II} \\ \underline{II} \cdot 0,5 \end{array} \right\} \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & -0,5 & 1,5 & -0,5 \\ 0 & 0 & 2 & -1,5 & -0,5 & 1,5 \end{array} \right)$$

$$\sim \left\{ \underline{III} \cdot 0,5 \right\} \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & -0,5 & 1,5 & -0,5 \\ 0 & 0 & 1 & -0,75 & -0,25 & 0,75 \end{array} \right)$$

$$X = (3\ 2\ 1) \cdot \left(\begin{array}{ccc} -1 & 1 & 0 \\ -0,5 & 1,5 & -0,5 \\ -0,75 & -0,25 & 0,75 \end{array} \right)$$

$$\sim (\,-5,75 \quad 8,375 \quad -1,375\,)$$

(8.1)

$$A = \begin{pmatrix} 2 & 1 & 4 & -4 & 7 \\ 0 & 0 & 5 & 7 & 9 \\ 2 & 1 & -1 & 3 & -2 \\ 2 & 1 & 9 & -11 & 16 \\ 8 & 4 & 1 & 5 & 1 \end{pmatrix}$$

$M_1 = 2 \Rightarrow r(A) \geq 1$

$M_2 = \begin{vmatrix} 2 & 1 \\ 8 & 4 \end{vmatrix} = 0$

$M_1 = 1 \Rightarrow r(A) \geq 1$

$M_2 = \begin{vmatrix} 1 & -1 \\ 1 & 9 \end{vmatrix} = 10 \neq 0 \Rightarrow r(A) \geq 2$

$M_3 = \begin{vmatrix} 0 & 0 & 5 \\ 2 & 1 & -1 \\ 2 & 1 & 9 \end{vmatrix} = 0 + 0 + 10 - 10 = 0$

$M_3 = \begin{vmatrix} 0 & 5 & 7 \\ 1 & -1 & 3 \\ 1 & 9 & -11 \end{vmatrix} = 0 + 5 + 63 + 7 + 55 = 130 \neq 0 \Rightarrow r(A) \geq 3$

$$M_4 = \begin{vmatrix} 0 & 5 & 7 & 9 \\ 1 & -1 & 3 & -2 \\ 1 & 9 & -11 & 16 \\ 4 & 1 & 5 & 1 \end{vmatrix} = \{\overline{III} - \overline{II}\} =$$

$$= \begin{vmatrix} 0 & 5 & 7 & 9 \\ 1 & -1 & 3 & -2 \\ 0 & 10 & -14 & 18 \\ 4 & 1 & 5 & 1 \end{vmatrix} = \{\overline{IV} - 4 \cdot \overline{II}\} =$$

$$= \begin{vmatrix} 0 & 5 & 7 & 9 \\ 1 & -1 & 3 & 2 \\ 0 & 10 & -14 & 18 \\ 0 & 5 & -7 & 9 \end{vmatrix} = 0, \quad \text{т.к. } \overline{III} \sim \overline{IV}$$

Ответ: $r(A) = 3$