Question 5

Solve the following questions from the Discrete Math Zybook:

1. Exercise 1.12.2, b, e

b)	$\neg q$	Hypothesis
	$\neg q \vee \neg r$	Addition, 1
	$\neg(q \wedge r)$	De Morgan's Law, 2
	$p \to (q \wedge r)$	Hypothesis
	$\neg p$	Modus Tollens, 3, 4

e)	$p \vee q$	Hypothesis
	$\neg p \vee r$	Hypothesis
	$q \vee r$	Resolution, 1, 2
	$\neg q$	Hypothesis
	r	Disjunctive Syllogism, 3, 4

2. Exercise 1.12.3 c

c)	$p \vee q$	Hypothesis
	$\neg(\neg p) \vee q$	Double Negation, 1
	$\neg p \to q$	Conditional Identity, 2
	$\neg p$	Hypothesis
	q	Modus Ponens, 3, 4

3. Exercise 1.12.5 c, d

c) j: I will get a job
c: I will buy a new car
h: I will buy a new house

The form of the argument is:

$$\begin{array}{l} (c \wedge h) \to j \\ \neg j \\ \hline \therefore \neg c \end{array}$$

The argument is invalid. When c = T and h = j = F, both hypotheses are true but the conclusion is false.

d) j: I will get a job
c: I will buy a new car
h: I will buy a new house

The form of the argument is:

$(c \wedge h) \rightarrow j$
$\neg j$
h

$\therefore \neg c$

The argument is valid

$(c \wedge h) \rightarrow j$	Hypothesis
$\neg j$	Hypothesis
$\neg(c \wedge h)$	Modus Tollens, 1, 2
$\neg c \vee \neg h$	De Morgan's Law, 3
$\neg h \vee \neg c$	Commutative Laws, 4
h	Hypothesis
$\neg(\neg h)$	Double Negation, 6
$\neg c$	Disjunctive Syllogism, 5, 7

Solve the following questions from the Discrete Math Zybook:

1. Exercise 1.13.3, b

b)

	P	Q
a	F	T
b	F	F

$\exists x\ (P(x) \vee Q(x))$ is true because $Q(a)$ is true. $\exists x\ \neg Q(x)$ is also true because $\neg Q(b)$ is true. However since $P(a) = P(b) = F$, $\exists x\ P(x)$ is false. Therefore both hypotheses are true and the conclusion is false.

2. Exercise 1.13.5, d, e

d) M(x): x missed class
D(x): x got a detention

$\forall x\ (M(x) \to D(x))$
Penelope is a student in the class
$\neg M(\text{Penelope})$

$\therefore \neg D(\text{Penelope})$

This argument is invalid, if we let M(Penelope) = F and D(Penelope) = T, then all the hypotheses are true, but the conclusion is false.

e) M(x): x missed class
D(x): x got a detention
A(x): x received an A

$\forall x\ ((M(x) \vee D(x)) \to \neg A(x))$
Penelope is a student in the class
$A(\text{Penelope})$

$\therefore \neg D(\text{Penelope})$

Penelope = P	Element definition
$\forall x\ ((M(x) \vee D(x)) \to \neg A(x))$	Hypothesis
P is a student in the class	Hypothesis
$(M(P) \vee D(P)) \to \neg A(p)$	Universal Instantiation, 2, 3
$A(P)$	Hypothesis
$\neg\neg A(P)$	Double Negation, 5
$\neg(M(P) \vee D(P))$	Modus Tollens, 4, 6
$\neg M(P) \wedge \neg D(P)$	De Morgan’s Law, 7
$\neg D(P)$	Simplification, 8

Question 6

Solve the following questions from the Discrete Math Zybook:

Exercise 2.2.1, c, d

c) Direct Proof

Proof: Assume x is a real number and $x \leq 3$

$$12 - 7x + x^2 \geq 0$$
$$(x - 4)(x - 3) \geq 0$$

Since $x \leq 3$, no matter what possible value of x is plugged in, $(x - 4)(x - 3) \geq 0$ will always be true.

d) Direct Proof

Proof: Assume that m and n are odd integers = $(2k + 1)$, $(2j + 1)$

$$x = m * n$$
$$x = (2k + 1) * (2j + 1)$$
$$x = 4kj + 2k + 2j + 1$$
$$x = 2(2kj + k + j) + 1$$

Since k and j are integers, $(4kj + 2k + 2j)$ is also an integer. Since $x = 2c + 1$ where $c = (2kj + k + j)$, x is odd.

Question 7

Solve the following questions from the Discrete Math Zybook:

Exercise 2.3.1, d, f, g, l

d) Proof by Contrapositive

Proof: Let n be an integer. Assume n is even and prove $n^2 - 2n + 7$ is odd. If n is even, then $n = 2k$ for some integer k.

$$
\begin{aligned}
&n^2 - 2n + 7 \\
&= (2k)^2 - 2(2k) + 7 \\
&= 4k^2 - 4k + 7 \\
&= 4k^2 - 4k + 6 + 1 \\
&= 2(2k^2 - 2k + 3) + 1
\end{aligned}
$$

Since k is an integer, $(4k^2 - 4k)$ is also an integer. Since $n^2 - 2n + 7 = 2c + 1$ where $c = (2k^2 - 2k + 3)$, it must be odd.

f) Proof by Contrapositive

Proof: Let x be a non-zero real number. Assume 1/x is not irrational and prove x is not irrational. Since 1/x is rational, it can be represented by a/b where a and b are both non-zero integers. Thus x = b/a which is also rational.

g) Proof by Contrapositive

Proof: For every pair of real numbers x and y, assume $x > y$ and prove that $x^3 + xy^2 > x^2y + y^3$.

$$
\begin{aligned}
&x^3 + xy^2 = x(x^2 + y^2) \\
&x^2y + y^3 = y(x^2 + y^2) \\
&x(x^2 + y^2) > y(x^2 + y^2) \\
&x > y
\end{aligned}
$$

Since $x > y$, $x^3 + xy^2 > x^2y + y^3$ must be true.

1) Proof by Contrapositive

Proof: For every pair of real numbers x and y, assume $x \le 10$ and $y \le 10$ and prove $x + y \le 20$.

$$x + y \le 10 + 10 \le 20$$

Question 8

Solve the following questions from the Discrete Math Zybook:

Exercise 2.4.1, c, e

c) Proof by Contradiction

Proof: Assume that $(a + b + c) / 3 = x < a, b, c$. This would mean that $3x < a + b + c$ which contradicts the fact that the average is the sum of a, b, and c.

d) Proof by Contradiction

Proof: Assume that x is the smallest integer. However, $x - 1$ is also an integer and clearly smaller than x, which is a contradiction.

Question 9

Solve the following questions from the Discrete Math Zybook:

Exercise 2.5.1, c

c) Proof by Cases

Proof: Let's consider two cases: x and y are even and x and y are odd.

Case 1: x and y are even. They can thus be represented by 2k and 2j respectively where k and j are integers.

$$x + y = 2k + 2j$$
$$x + y = 2k + 2j$$
$$x + y = 2(k + j)$$

Since (x + y) can be represented by 2c, where c is (k + j), it must be even.

Case 2: x and y are odd. They can thus be represented by 2k + 1 and 2j + 1 respectively where k and j are integers.

$$x + y = 2k + 1 + 2j + 1$$
$$x + y = 2k + 2j + 2$$
$$x + y = 2(k + j + 1)$$

Since (x + y) can be represented by 2c, where c is (k + j + 1), it must be even.