Question 1:

A1) 10011011_2

=

1	0	0	1	1	0	1	1
128	64	32	16	8	4	2	1

= $[128 + 16 + 8 + 2 + 1]_{10}$ = $\mathbf{155_{10}}$

A2) 456_7

= $[(4 \times 7^2) + (5 \times 7^1) + (6 \times 7^0)]_{10}$ = $\mathbf{237_{10}}$

A3) $38A_{16}$

= $[(3 \times 16^2) + (8 \times 16^1) + (10 \times 16^0)]_{10}$ = $\mathbf{906_{10}}$

A4) 2214_5

= $[(2 \times 5^3) + (2 \times 5^2) + (1 \times 5^1) + (4 \times 5^0)]_{10}$ = $\mathbf{309_5}$

B1) 69_{10}

→ 69 / 2 = 34 R 1

→ 34 / 2 = 17 R 0

→ 17 / 2 = 8 R 1

→ 8 / 2 = 4 R 0

→ 4 / 2 = 2 R 0

→ 2 / 2 = 1 R 0

→ 1 / 2 = 0 R 1

= $\mathbf{1000101_2}$

B2) 485_{10}

→ 485 / 2 = 242 R 1

→ 242 / 2 = 121 R 0

→ 121 / 2 = 60 R 1

→ 60 / 2 = 30 R 0

→ 30 / 2 = 15 R 0

→ 15 / 2 = 7 R 1

→ 7 / 2 = 3 R 1

→ 3 / 2 = 1 R 1

→ 1 / 2 = 0 R 1

= $\mathbf{111100101_2}$

B3) $6D1A_{16}$

Using the hexadecimal to binary chart

6 = 0110 D = 1101 1 = 0001 A = 1010

$6D1A_{16} = 0110110100011010_2$

= $\mathbf{110110100011010_2}$

Hex	Binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

C1) 1101011_2

Going from right to left

→ 1011 = B   0110 = 6

1101011_2 = $\mathbf{6B_{16}}$

C2) 895_{10}

→ 895 / 16 = 55 R 15 $(F)_{16}$

→ 55 / 16 = 3 R 7 $(7)_{16}$

→ 3 / 16 = 0 R 3 $(3)_{16}$

= $\mathbf{37F_{16}}$

Question 2:

1) $7566_8 + 4515_8$

=

+	7	5	6	6
	4	5	1	5

Starting from right to left

→ 6 + 5 = 3 carry 1

→ 6 + 1 + 1 = 0 carry 1

→ 5 + 5 + 1 = 3 carry 1

→ 7 + 4 + 1 = 4 carry 1

→ +1 = 1

= $\mathbf{14303_8}$

2) $10110011_2 + 1101_2$

=

+	1	0	1	1	0	0	1	1
					1	1	0	1

→ 1 + 1 = 0 carry 1

→ 1 + 0 + 1 = 0 carry 1

→ 0 + 1 + 1 = 0 carry 1

→ 0 + 1 + 1 = 0 carry 1

→ 1 + 1 = 0 carry 1

→ 1 + 1 = 0 carry 1

→ 0 + 1 = 1

→ 1 = 1

= $\mathbf{11000000_2}$

3) $7A66_{16} + 45C5_{16}$

=

+	7	A	6	6
	4	5	C	5

→ 6 + 5 = B

→ 6 + C = 2 carry 1

→ A + 5 + 1 = 0 carry 1

→ 7 + 4 + 1 = C

= $\mathbf{C02B_{16}}$

4) $3022_5 - 2433_5$

=

−	3	0	2	2
	2	4	3	3

→ 2 - 3 = 12 - 3 = 4 borrow 1

→ (2 - 1) - 3 = 11 - 3 = 3 borrow 1

→ (0 - 1) - 4 = 4 - 4 = 0 borrow 1

→ (3 - 1) - 2 = 0

= $\mathbf{34_5}$

Question 3:

A1) 124_{10}

→ 124 / 2 = 62 R 0

→ 62 / 2 = 31 R 0

→ 31 / 2 = 15 R 1

→ 15 / 2 = 7 R 1

→ 7 / 2 = 3 R 1

→ 3 / 2 = 1 R 1

→ 1 / 2 = 0 R 1

= $\mathbf{01111100}_{\text{8-bits two's complement}}$

A2) -124_{10} =

	0	1	1	1	1	1	0	0
+	?	?	?	?	?	?	?	?
1	0	0	0	0	0	0	0	0

Starting from right to left

→ 0 + ? = 0 → ? = 0

→ 0 + ? = 0 → ? = 0

→ 1 + ? = 0 → ? = 1 carry 1

→ 1 + ? + 1 = 0 → ? = 0 carry 1

→ 1 + ? + 1 = 0 → ? = 0 carry 1

→ 1 + ? + 1 = 0 → ? = 0 carry 1

→ 1 + ? + 1 = 0 → ? = 0 carry 1

→ 0 + ? + 1 = 0 → ? = 1 carry 1

= $\mathbf{10000100}_{\text{8-bits two's complement}}$

A3) 109_{10}

→ 109 / 2 = 54 R 1

→ 54 / 2 = 27 R 0

→ 27 / 2 = 13 R 1

→ 13 / 2 = 6 R 1

→ 6 / 2 = 3 R 0

→ 3 / 2 = 1 R 1

→ 1 / 2 = 0 R 1

= $\mathbf{01101101}_{\text{8-bits two's complement}}$

A4) -79_{10}

First convert into positive 8-bits two's complement

→ 79 / 2 = 39 R 1

→ 39 / 2 = 19 R 1

→ 19 / 2 = 9 R 1

→ 9 / 2 = 4 R 1

→ 4 / 2 = 2 R 0

→ 2 / 2 = 1 R 0

→ 1 / 2 = 0 R 1

= $\mathbf{01001111}_{\text{8-bits two's complement}}$

Find the inverse of the positive

	0	1	0	0	1	1	1	1
+	?	?	?	?	?	?	?	?
1	0	0	0	0	0	0	0	0

→ 1 + ? = 0 → ? = 1 carry 1

→ 1 + ? + 1 = 0 → ? = 0 carry 1

→ 1 + ? + 1 = 0 → ? = 0 carry 1

→ 1 + ? + 1 = 0 → ? = 0 carry 1

→ 0 + ? + 1 = 0 → ? = 1 carry 1

→ 0 + ? + 1 = 0 → ? = 1 carry 1

→ 1 + ? + 1 = 0 → ? = 0 carry 1

→ 0 + ? + 1 = 0 → ? = 1 carry 1

= $\mathbf{10110001}_{\text{8-bits two's complement}}$

B1) $00011110_{\text{8-bit two's complement}}$

Leftmost bit = 0 → positive

=

0	0	0	1	1	1	1	0
128	64	32	16	8	4	2	1

= 16 + 8 + 4 + 2 = $\mathbf{30}_{10}$

B2) $11100110_{\text{8-bit two's complement}}$

Leftmost bit = 1 → negative → find the inverse

	1	1	1	0	0	1	1	0
+	?	?	?	?	?	?	?	?
1	0	0	0	0	0	0	0	0

→ 0 + ? = 0 → ? = 0

→ 1 + ? = 0 → ? = 1 carry 1

→ 1 + ? + 1 = 0 → ? = 0 carry 1

→ 0 + ? + 1 = 0 → ? = 1 carry 1

→ 0 + ? + 1 = 0 → ? = 1 carry 1

→ 1 + ? + 1 = 0 → ? = 0 carry 1

→ 1 + ? + 1 = 0 → ? = 0 carry 1

→ 1 + ? + 1 = 0 → ? = 0 carry 1

= $\mathbf{00011010}_{\text{8-bit two's complement}}$

=

0	0	0	1	1	0	1	0
128	64	32	16	8	4	2	1

= 16 + 8 + 2 = 26_{10}

→ $11100110_{\text{8-bit two's complement}}$ = $\mathbf{-26}_{10}$

B3) $00101101_{8\text{-bit two's complement}}$

Leftmost bit = 0 → positive

=

0	0	1	0	1	1	0	1
128	64	32	16	8	4	2	1

= 32 + 8 + 4 + 1 = $\mathbf{45_{10}}$

B4) $10011110_{8\text{-bit two's complement}}$

Leftmost bit = 1 → negative → find inverse

	1	0	0	1	1	1	1	0
+	?	?	?	?	?	?	?	?
1	0	0	0	0	0	0	0	0

→ 0 + ? = 0 → ? = 0

→ 1 + ? = 0 → ? = 1 carry 1

→ 1 + ? + 1 = 0 → ? = 0 carry 1

→ 1 + ? + 1 = 0 → ? = 0 carry 1

→ 1 + ? + 1 = 0 → ? = 0 carry 1

→ 0 + ? + 1 = 0 → ? = 1 carry 1

→ 0 + ? + 1 = 0 → ? = 1 carry 1

→ 1 + ? + 1 = 0 → ? = 0 carry 1

= $\mathbf{01100010_{8\text{-bit two's complement}}}$

=

0	1	1	0	0	0	1	0
128	64	32	16	8	4	2	1

= 64 + 32 + 2 = 98_{10}

→ $10011110_{8\text{-bit two's complement}}$ = $\mathbf{-98_{10}}$

Question 4:

1b)

P	Q	¬(p ∨ q)
T	T	F
T	F	F
F	T	F
F	F	T

1c)

R	P	Q	r ∨ (p ∧ ¬q)
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	F
F	T	F	T
F	F	T	F
F	F	F	F

2b)

p	q	(p → q) → (q → p)
T	T	T
T	F	T
F	T	F
F	F	T

2d)

p	q	(p ↔ q) ⊕ (p ↔ ¬q)
T	T	T
T	F	T
F	T	T
F	F	T

Question 5:

1b) (B ^ D) v (B ^ M) v (D ^ M)

1c) B v (D ^ M)

2b) (s v y) → p

2c) p → y

2d) p <→ (s ^ y)

2e) p → (s v y)

3c) c → p

3d) c → p

Question 6:

1b) If Joe is eligible for the honors program, then he has maintained a B average.

1c) If Rajiv can go on the roller coaster, then he is at least 4 feet tall.

1d) If Rajiv is at least 4 feet tall, then he can go on the roller coaster.

2c) False. The conclusion is false

2d) False. The conclusion is false

2e) Unknown. If r is true then the expression is true and vice versa

2f) Unknown. If r is true then the expression is true and vice versa

Question 7:

1b)

¬j → (l v ¬r)

(r ^ ¬l) → j

j	l	r	¬j -> (l v ¬r)	(r ^ ¬l) -> j
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	T
F	T	T	T	F
F	T	F	T	T
F	F	T	T	T
F	F	F	F	T

Not logically equivalent

1c)

j → ¬l

¬j → l

j	l	j -> ¬l	¬j -> l
T	T	F	T
T	F	T	T
F	T	T	F
F	F	T	F

Not logically equivalent

1d)

(r v ¬l) → j

j → (r ^ ¬l)

j	l	r	(r v ¬l) -> j	j -> (r ^ ¬l)
T	T	T	T	F
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	F	T
F	T	F	T	T
F	F	T	F	T
F	F	F	T	F

Not logically equivalent

Question 8:

1c) $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$

$\rightarrow (\neg p \vee q) \wedge (\neg p \vee r)$ | Conditional Identity

$\rightarrow \neg p \vee (q \wedge r)$ | Distributive Laws

$\rightarrow$ $\mathbf{p \rightarrow (q \wedge r)}$ | Conditional Identity

1f) $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$

$\rightarrow \neg p \wedge \neg(\neg p \wedge q)$ | De Morgan's Laws

$\rightarrow \neg p \wedge \neg\neg p \vee \neg q$ | De Morgan's Laws

$\rightarrow \neg p \wedge p \vee \neg q$ | Double Negation Law

$\rightarrow (\neg p \wedge p) \vee (\neg p \wedge \neg q)$ | Distributive Laws

$\rightarrow F \vee (\neg p \wedge \neg q)$ | Complement Laws

$\rightarrow$ $\mathbf{\neg p \wedge \neg q}$ | Identity Laws

1i) $(p \wedge q) \rightarrow r \equiv (p \wedge \neg r) \rightarrow \neg q$

$\rightarrow \neg(p \wedge q) \vee r$ | Conditional Identity

$\rightarrow \neg p \vee \neg q \vee r$ | De Morgan's Laws

$\rightarrow (\neg p \vee r) \vee \neg q$ | Associative Laws

$\rightarrow \neg(\neg p \vee r) \rightarrow \neg q$ | Conditional Identity

$\rightarrow (\neg\neg p \wedge \neg r) \rightarrow \neg q$ | De Morgan's Laws

$\rightarrow$ $\mathbf{(p \wedge \neg r) \rightarrow q}$ | Double Negation Law

2c) $\neg r \vee (\neg r \rightarrow p)$

$\rightarrow \neg r \vee \neg\neg r \vee p$ | Conditional Identity

$\rightarrow \neg r \vee r \vee p$ | Double Negation Law

$\rightarrow T \vee p$ | Complement Laws

$\rightarrow$ $\mathbf{T}$ | Domination Laws

2d) ¬(p → q) → ¬q

→ ¬(¬p v q) → ¬q | Conditional Identity

→ ¬¬p ^ ¬q → ¬q | De Morgan's Laws

→ (p ^ ¬q) → ¬q | Double Negation Law

→ ¬(p ^ ¬q) v ¬q | Conditional Identity

→ ¬p v ¬¬q v ¬q | De Morgan's Laws

→ ¬p v q v ¬q | Double Negation Law

→ ¬p v T | Complement Laws

→ **T** | Domination laws

Question 9:

1c) $\exists x(x = x^2)$

1d) $\forall x(x \leq x^2)$

2b) $\forall x(\neg S(x) \wedge W(x))$

2c) $\forall x(S(x) \rightarrow \neg W(x))$

2d) $\exists x(S(x) \wedge W(x))$

Question 10:

1c) True, $R(c) = F$ and $F \rightarrow F = T$

1d) True, b and e

1e) True

1f) True

1g) False, c

1h) True

1i) True, a, c, d, e

2b) True, $x = 2$

2c) True, $y = 1$

2d) True

2e) False, no y exists where $Q(1, y)$, $Q(2, y)$, and $Q(3, y) = T$

2f) True, $y = 1$

2g) False

2h) True

2i) True

Question 11:

1c) $\exists x\ \exists y(x + y = xy)$

1d) $\forall x\ \forall y(x,\ y > 0 \rightarrow x/y > 0\ \wedge\ y/x > 0)$

1e) $\forall x(0 < x < 1 \rightarrow 1/x > 1)$

1f) $\forall x\ \forall y(x < y)$

1g) $\forall x(x \neq 0 \rightarrow x * 1/x = 1)$

2c) $\exists x(N(x)\ \wedge\ D(x))$

2d) $\forall y(P(Sam,\ y)\ \wedge\ D(y))$

2e) $\exists x\ \forall y(P(x,\ y)\ \wedge\ (N(x))$

2f) $\exists x\ (D(x)\ \wedge\ \forall y\ (y \neq x \rightarrow \neg D(y))$

3c) $\forall x\ \exists y(y \neq \text{Math } 101 \rightarrow T(x,\ y))$

3d) $\exists x\ \forall y(y \neq \text{Math } 101 \rightarrow T(x,\ y))$

3e) $\forall x\ \exists y\ \exists z(x \neq Sam\ \wedge\ y \neq z \rightarrow T(x,\ y)\ \wedge\ T(x,\ z))$

3f) $\exists x\ \exists y\ \forall z(x \neq y \neq z \rightarrow T(Sam,\ x)\ \wedge\ T(Sam,\ y)\ \wedge\ \neg T(Sam,\ z))$

Question 12:

1b) $\forall x(D(x) \vee P(x)) \vee (D(x) \wedge P(x))$

→ Negation: $\neg\forall x(D(x) \vee P(x)) \vee (D(x) \wedge P(x))$

→ De Morgan's: $\exists x \neg[(D(x) \vee P(x)) \vee (D(x) \wedge P(x))]$

→ De Morgan's: $\exists x \neg(D(x) \vee P(x)) \wedge \neg(D(x) \wedge P(x))$

→ De Morgan's: $\exists x \neg D(x) \wedge \neg P(x) \wedge \neg D(x) \vee \neg P(x)$

→ **English: There is a patient who did not get the placebo or not given the medication, or both.**

1c) $\exists x(D(x) \wedge M(x))$

→ Negation: $\neg\exists x(D(x) \wedge M(x))$

→ De Morgan's: $\forall x \neg D(x) \vee \neg M(x)$

→ **English: Every patient either was not given the medication or did not get migraines**

1d) $\forall x(P(x) \rightarrow M(x))$

→ Negation: $\neg\forall x(P(x) \rightarrow M(x))$

→ De Morgan's: $\exists x \neg(P(x) \rightarrow M(x))$

→ Conditional: $\exists x \neg(\neg P(x) \vee M(x))$

→ De Morgan's: $\exists x \neg\neg P(x) \wedge \neg M(x)$

→ Double Negation: $\exists x\ P(x) \wedge \neg M(x)$

→ **English: There is a patient who received the placebo but did not get migraines**

1e) $\exists x(M(x) \wedge P(x))$

→ Negation: $\neg\exists x(M(x) \wedge P(x))$

→ De Morgan's: $\forall x \neg M(x) \vee \neg P(x)$

→ **English: Every patient either did not get migraines or did not receive the placebo**

2c) $\forall x\ \exists y(P(x,y) \wedge \neg Q(x, y))$

2d) $\forall x\ \exists y\ \neg(P(x, y) \rightarrow P(y, x) \wedge P(y, x) \rightarrow P(x, y))$

$\rightarrow \forall x\ \exists y\ \neg(P(x, y) \rightarrow P(y, x)) \vee \neg(P(y, x) \rightarrow P(x, y))$

$\rightarrow \forall x\ \exists y\ [P(x, y) \vee \neg P(y, x)] \vee [P(y, x) \vee \neg P(x, y)]$

2e) $\forall x\ \forall y\ \neg P(x,y) \wedge \exists x\ \exists y\ \neg Q(x, y)$