

# L4 Quantum information and computing (QIC) 2020-21

## Lecture 17: Rydberg atoms and Rydberg blockade

November 13, 2021

**Aims of Lecture 17:** To understand how interactions between atoms allow us to create entanglement.

### Introduction:

In Lectures 15 and 16 we learnt how to initialise the qubit using optical pumping and how to drive single-qubit rotations using stimulated Raman transitions.

**Rydberg atoms** To find the size and energy of atoms as a function of the principal quantum number  $n$ , we use the Bohr quantization rule

$$mv_n r_n = n\hbar ,$$

where the electron speed,  $v_n$  is given by equating the centripetal force and the Coulomb attraction,

$$\frac{mv_n^2}{r_n} = \frac{e^2}{(4\pi\epsilon_0)r_n^2} ,$$

i.e.  $v_n^2 = e^2/[m(4\pi\epsilon_0)r_n]$ . Substituting into the quantization formula all squared gives  $me^2/[(4\pi\epsilon_0)]r_n = n^2\hbar^2$  and therefore

$$r_n = n^2 \frac{\hbar^2}{me^2/(4\pi\epsilon_0)} = n^2 a_0 .$$

where  $a_0$  is the Bohr radius. The energy of state  $n$  (using the virial theorem equal to one half of the potential energy) is given by

$$E_n = -\frac{e^2}{2(4\pi\epsilon_0)r_n} = -\frac{R_H}{n^2} ,$$

where  $R_H$  is known as the **Rydberg constant**. This is known as the Rydberg formula. A similar formula applies to all atoms with one electron in a Rydberg state.<sup>1</sup>

<sup>1</sup>Only we need to adjust the Rydberg constant and there is an energy shift due to the interaction between the electron and the core (nucleus plus other electrons). This shift is included via a quantum defect,  $\delta_{n,\ell}$ , so we have

$$E_n = -\frac{R_\infty}{(n - \delta_{n,\ell})^2} .$$

### Interactions

The giant size of Rydberg atoms leads to giant electric dipole moments (for transitions between a Rydberg state  $|r\rangle$  and another  $|r'\rangle$ ), and strong van der Waals interactions. To estimate the van der Waals shift we consider a pair of atom in a Rydberg state  $|r\rangle$ . Each atom experiences the electric field created by the other dipole which has the form  $\mathcal{E}_d = \mathcal{D}_0^2/(4\pi\epsilon_0 R^3)$ , where  $R$  is the distance between the atoms, and  $\mathcal{D}_0 = \langle r'|\mathbf{d}|r\rangle$  is the dipole matrix element associated with a transition between Rydberg states  $|r\rangle$  and  $|r'\rangle$ . In the pair basis  $\{|rr\rangle, |r'r'\rangle\}$  the dipole-dipole interaction,  $\mathcal{H}_{dd} = -\mathbf{d} \cdot \mathcal{E}$ , is

$$\mathcal{H}_{dd} = \hbar \begin{pmatrix} 0 & V \\ V & \Delta_{\text{pair}} \end{pmatrix} ,$$

where

$$V = \frac{\mathcal{D}_0^2/\hbar}{4\pi\epsilon_0 R^3} ,$$

and  $\hbar\Delta_{\text{pair}}$  is the energy difference between the pair states  $|r'\rangle$  and  $|r\rangle$ . For  $|\Delta_{\text{pair}}| \gg V$ , the frequency shift of  $|r, r\rangle$  is

$$V_{\text{vdW}} = \frac{V^2}{\Delta_{\text{pair}}} .$$

This is known as the **van der Waals** interaction. Note it can be either positive or negative depending on whether the state  $|r'\rangle$  is below or above  $|r\rangle$ . As  $V$  is proportional to  $1/R^3$ , the van der Waals interaction is proportional to  $1/R^6$ .

As  $V$  is proportional to the dipole matrix element squared and hence the principal quantum  $n^4$ , and the energy difference  $\Delta_{\text{pair}}$  scales as  $1/n^3$ , the van Waals interaction scales as  $n^{11}$ , and the interaction between two atoms in a high-excited Rydberg state is at least 12 orders of magnitude larger than between two ground state atoms. The van der Waals shift can be either positive or negative depending on whether the nearest Rydberg states have lower or higher energy.

Next, we consider what happens when we try to excite two nearby atoms to the Rydberg state  $|rr\rangle$ .

### Rydberg blockade

For laser light resonant with the  $|0\rangle \rightarrow |r\rangle$  transition the interaction Hamiltonian, in the  $\{|00\rangle, |0r\rangle, |r0\rangle, |rr\rangle\}$  basis is

$$\mathcal{H}_{\text{Ryd}} = \frac{\hbar}{2} \begin{pmatrix} 0 & \Omega & \Omega & 0 \\ \Omega & 0 & 0 & \Omega \\ \Omega & 0 & 0 & \Omega \\ 0 & \Omega & \Omega & 2V_{\text{vdW}} \end{pmatrix}. \quad (1)$$

For highly-excited Rydberg atoms separated by about 10 microns or less  $|V_{\text{vdW}}| \gg \Omega$ . Consequently, the doubly-excited Rydberg state  $|rr\rangle$  is blocked is shifted out of resonance and no longer excited, see Fig. 1. The suppression of more than one Rydberg atom is known as dipole or **Rydberg blockade**. Note that as  $V_{\text{vdW}} = \hbar C_6/R^6$ , the condition  $|V_{\text{vdW}}| \gg \Omega$  is a condition on the spacing between the atoms  $R < R_b$ , where  $R_b = (C_6/\Omega)^{1/6}$  is known as the **blockade radius**. Typically  $R_b$  is of the order of 10 microns.

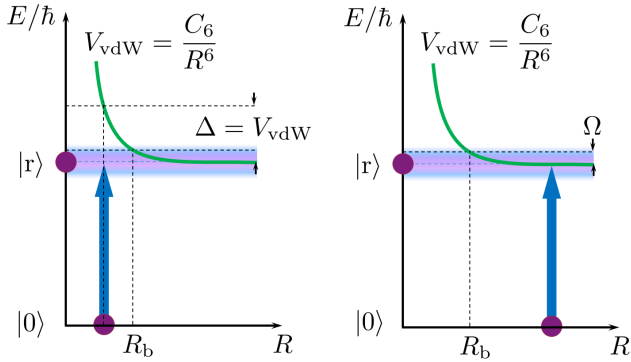


Figure 1: Energy level diagram illustrated Rydberg blockade. If one atom is in the Rydberg state  $|r\rangle$  then the energy needed to excite a second Rydberg atom is shifted by the van der Waals interaction,  $\hbar V_{\text{vdW}}$ . For two atoms with spacing  $R < R_b$  (left), the transition to drive the 2nd atom to state  $|r\rangle$  is shifted off-resonance, i.e., the excitation of the double excited state  $|rr\rangle$  is **blocked**. For two atoms with spacing  $R \gg R_b$  (right) the interactions are negligible and the excitation of each atom is no longer correlated.

As the  $|rr\rangle$  state is decoupled by the van der Waals shift we can remove it and rewrite the interaction as

$$\mathcal{H}_{\text{Ryd}} = \frac{\hbar}{2} \begin{pmatrix} 0 & \Omega & \Omega \\ \Omega & 0 & 0 \\ \Omega & 0 & 0 \end{pmatrix}.$$

What happens in this case is easier to understand if we rewrite the Hamiltonian in an entangled basis  $|00\rangle$ ,

$\frac{1}{\sqrt{2}}(|0r\rangle + |r0\rangle)$  and  $\frac{1}{\sqrt{2}}(|0r\rangle - |r0\rangle)$ . To transform the Hamiltonian to the new basis, we use the rotation matrix

$$R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}.$$

The new Hamiltonian is<sup>2</sup>

$$\mathcal{H}'_{\text{Ryd}} = R^\dagger \mathcal{H}_{\text{Ryd}} R = \frac{\hbar}{2} \begin{pmatrix} 0 & \sqrt{2}\Omega & 0 \\ \sqrt{2}\Omega & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

The laser drives Rabi oscillations between the states  $|00\rangle$  and the maximally entangled Bell state,  $\frac{1}{\sqrt{2}}(|0r\rangle + |r0\rangle)$  at a frequency  $\sqrt{2}\Omega$ . The coupling between  $|00\rangle$  and  $\frac{1}{\sqrt{2}}(|0r\rangle - |r0\rangle)$  is zero due to destructive interference. Consequently, the anti-symmetric Bell state  $\frac{1}{\sqrt{2}}(|0r\rangle - |r0\rangle)$  is not populated.

The consequence of blockade is that driving the  $|0\rangle \rightarrow |r\rangle$  transition leads directly to entanglement. Due to interactions, the two-atom system only supports a single excitation. This excitation is shared between the atoms. We can only excite one atom but we do not know (and we cannot know, without a measurement) which one.

### Summary:

What do you need to be able to do?

1. Understand the scaling of the properties of Rydberg atoms as a function of their principal quantum number,  $n$ .
2. Understand the concept of Rydberg blockade, and estimate the blockade radius given the parameters.
3. Derive the interaction Hamiltonian in the entangled basis, and show how Rydberg blockade leads to entanglement.

---


$$\begin{aligned} & \frac{1}{2} \begin{pmatrix} 0 & \Omega & \Omega \\ \Omega & 0 & 0 \\ \Omega & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 0 & \sqrt{2}\Omega & 0 \\ \sqrt{2}\Omega & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \\ & \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \frac{1}{2} \begin{pmatrix} 0 & \sqrt{2}\Omega & 0 \\ \sqrt{2}\Omega & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 0 & \sqrt{2}\Omega & 0 \\ \sqrt{2}\Omega & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \end{aligned}$$