L4 Quantum information and computing (QIC)

Lecture 14: Loading and state preparation of atom arrays (DVC1)

April 28, 2023

Aims of Lecture 14: To explain the application of the light forces to slowing and cooling[1] atoms in order to initialise a qubit array (DVC1). To derive an expression for the minimum stopping distance. To appreciate differences between alkali atoms like Rb and Cs and alkali earths like Sr. To understand state preparation of alkali atoms using **optical pumping**.

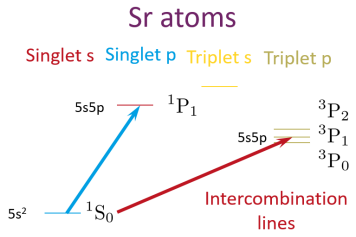

Figure 1: Energy levels of Sr-87: The $^1S_0 \rightarrow ^1P_1$ and $^1S_0 \rightarrow ^3P_1$ transitions have wavelengths of $\lambda_0 = 461$ and 689 nm, respectively. The $^1P_1$ and $^3P_1$ states have lifetimes of $\tau = 5.27$ ns and $\tau = 21.5$ μs, respectively. These correspond to decay rates, $\Gamma_{461} = 2\pi(30.2$ MHz) and $\Gamma_{689} = 2\pi(7.4$ kHz).

Introduction: In QIC.13 we found that the depth of the **optical tweezer** trap is < 1 mK. Consequently, we need a means to cool atoms from room temperature (or to slow a fast atomic beam). This is achieved using laser cooling which mainly exploits the spontaneous (or scattering) force—cycles of absorption and spontaneous emission.

Scattering force: In this section, we consider laser slowing and derive an expression for the stopping distance. For a monochromatic laser beam with wave vector $\boldsymbol{k}$, the scattering force is

$$\boldsymbol{F} = \hbar \boldsymbol{k} R \,, \tag{1}$$

where R is the scattering rate. The scattering rate is given by the spontaneous emission rate times the probability to be in the excited state, see QIC.6,

$$R = \Gamma P_{|e\rangle} = \Gamma \frac{\Omega^2/4}{\Omega^2/2 + \Gamma^2/4 + \Delta^2} \,,$$

[1]Note that **Doppler and sideband cooling** and the **magneto-optical trap (MOT)** are no longer covered.

Often it is more convenient to rewrite the Rabi frequency, $\Omega = d\mathcal{E}/\hbar$, in terms of the laser intensity, $\mathcal{I} = \frac{1}{2}\epsilon_0 c\mathcal{E}^2$. Using the relationship between decay rate and dipole moment, as in QIC.13, $\Gamma = d^2/[3\pi\epsilon_0(\lambda/2\pi)^3]$, we get $\Omega = \Gamma\sqrt{\mathcal{I}/(2\mathcal{I}_s)}$, where $\mathcal{I}_s = (\pi/3)hc\Gamma/\lambda^3$ is the **saturation intensity**, and we can re-wrtie the spontaneous scattering force for an atom moving with velocity v as

$$F = \hbar k \frac{\Gamma}{2} \frac{\mathcal{I}/\mathcal{I}_s}{\mathcal{I}/\mathcal{I}_s + 1 + 4(\Delta - kv)^2/\Gamma^2} \,.$$

Laser cooling require counter-propagating, red-detuned laser beams. In this case, a moving atom is Doppler shifted into resonance with a laser beam which exerts a force that opposes the motion.

Worked example: For a single laser beam, the spontaneous scattering force is given by $F = \hbar k \Gamma P_{|e\rangle}$, where $P_{|e\rangle}$ is the probability to be in the excited state and Γ is the spontaneous scattering rate. Write an equation for $P_{|e\rangle}$ in terms of the laser intensity $\mathcal{I}$, the saturation intensity $\mathcal{I}_s$, the detuning Δ, and the decay rate of the excited state Γ. Write expressions for $P_{|e\rangle}$ in three regimes: (i) low intensity $\mathcal{I} \ll \mathcal{I}_s$. Plot $P_{|e\rangle}$ versus detuning in this case. What is the full-width half maximum? (ii) Intermediate intensity, $\mathcal{I} = \mathcal{I}_s$ on resonance. (iii) High intensity, $\mathcal{I} \gg \mathcal{I}_s$. Plot $P_{|e\rangle}$ as a function of intensity for $\Delta = 0$.

The probability to be in the excited state is

$$P_{|e\rangle} = \frac{\Omega^2/4}{\Gamma^2/4 + \Omega^2/2 + \Delta^2} = \frac{\Omega^2/\Gamma^2}{1 + 2\Omega^2/\Gamma^2 + 4\Delta^2/\Gamma^2} \,.$$

Using $\Omega^2/\Gamma^2 = \mathcal{I}/2\mathcal{I}_s$ this can be rewritten in terms of the intensity

$$P_{|e\rangle} = \frac{\mathcal{I}/(2\mathcal{I}_s)}{1 + \mathcal{I}/\mathcal{I}_s + 4(\Delta/\Gamma)^2} \,.$$

Three useful cases.

1. Low intensity, $\mathcal{I} \ll \mathcal{I}_s$: The excited state probability is a **Lorentzian**

$$P_{|e\rangle} = \frac{\mathcal{I}/(2\mathcal{I}_s)}{1 + 4(\Delta/\Gamma)^2} \,,$$

with full width half maximum (FWHM) equal to Γ. Consequently, Γ is often called the **linewidth**. For Sr, Fig. 1 there a large variation in the linewith between the singlet and triplet transitions.

2. Intermediate intensity, $\mathcal{I} = \mathcal{I}_s$ and $\Delta = 0$: The time-averaged probability to be in the excited state is $P_{|e\rangle} = \frac{1}{4}$.

3. High intensity, $\mathcal{I} \gg \mathcal{I}_s$: The transition is said to be saturated and the time-averaged probability to be in the excited state is $P_{|e\rangle} = \frac{1}{2}$.

Stopping distance: Whether we can load atoms into the tweezer array depends on slowing them from room temperature (or hotter) to rest in practical distance. The maximum scattering force occurs at high intensity $\mathcal{I} \gg \mathcal{I}_s$. Using the previous result $P^{\max}_{|e\rangle} = \frac{1}{2}$ we have

$$F^{\max}_{\text{spont}} = \hbar k \frac{\Gamma}{2} = \frac{h}{2\lambda\tau} ,$$

where λ is the transition wavelength and τ is the excited state lifetime. The minimum stopping distance is

$$s = \frac{u^2}{2(F^{\max}_{\text{spont}}/m)} = \frac{mu^2}{h/(\lambda\tau)} .$$

Using $\frac{1}{2}mu^2 = \frac{1}{2}k_B T$,[2] we get $s = (k_B T)\tau\lambda/h$. Hence the stopping distance is equal to the ratio of the thermal energy to atomic resonance width times the optical wavelength. At room temperature the thermal energy is of order h times terahertz whereas the resonance with is of the order of h times megahertz so the stopping distance is 1 million times the wavelength, i.e. just under a meter. In the thermal distribution, a fraction of the atoms can be stopped in a shorter distance.

As an atom changes speed, the light-atom resonance condition changes due to the Doppler effect. For an atom with velocity $\boldsymbol{v}$ the detuning becomes $\Delta_{\boldsymbol{v}} = \Delta - \boldsymbol{k}\cdot\boldsymbol{v}$. As the resonance width is $\Delta = \pm\Gamma/2$, we can define a **capture** velocity $v_{\text{Doppler}} = \Gamma/k$. For Cs, with $\Gamma = 2\pi(5.0\text{ MHz})$ and $\lambda = 852$ nm, we obtain $v = 4\text{ ms}^{-1}$. Consequently, if we want to realise the minimum stopping distance we need to do something to keep the laser on resonance. On way to to do this is to use a magnetic field gradient. This is called a **Zeeman slower**.

State initialisation: For alkali atoms, we have the problem that the ground state is $(2S+1)(2I+1)$-fold degenerate for electron spin, S, and nuclear spin, I. After laser cooling and trapping in the tweezer, the atom may be in any of these states and we need a way to select only one to initialise our qubit.

For Cs with $I = 7/2$, that is 16 **hyperfine states** within the electronic ground state. These hyperfine states are split into two groups by the hyperfine interaction. The lower (upper) hyperfine state with total angular momentum $F = 3$ ($F = 4$) contains $2F+1 = 7$ ($2F+1 = 9$) states labelled $m_F = -3$ to $m_F = +3$ ($m_F = -4$ to $m_F = +4$). The m labels are known as the **magnetic quantum numbers** and denote the projection of the angular momentum onto a particular axis (the quantization axis) usually defined as the direction of an external magnetic field. [3][4] Note that for small magnetic fields (where the Zeeman shift is less than the fine or hyperfine splitting) the Zeeman shift is linearly proportional to m_F, i.e. $\Delta E = g_F m_F \mu_B B$, where μ_B is the Bohr magneton and g_F, the gyromagnetic ratio, is a number that depends on the angular momentum quantum numbers of the state. [5]

Polarization selection rules **The polarization of light allows us to drive transitions between states with different** m. Conservation of angular momentum requires that when a photon is absorbed, the angular momentum of the photon is transferred to the atom. For a conventional laser beam mode, we may assume the photons can drive transitions with $\Delta F = -1, 0, +1$. The projection of photon angular momentum along the quantization axis (parallel to an external magnetic field) is transferred to the magnetic quantum number, and we may have $\Delta m_F = -1, 0, +1$. The three case, $\Delta m_F = -1, 0, +1$, are known as σ_-, π and σ_+ transitions, respectively.

The simplest case is when the light propagate in the same direction as the magnetic field, i.e. along z. Consider

[2] For Cs atoms, $m = 133 \cdot 1.66 \times 10^{-27}$ kg. Using $T = 293$ K, we find $u = 135\text{ ms}^{-1}$.

[3] An atom like ^{88}Sr is special because the angular momentum of the ground state is zero. It has zero nuclear spin $I = 0$ and two-valence electrons which in ground state have zero orbital angular momentum, $L = 0$, and form a spin singlet, $S = 0$. Recall that states are labelled as $^{2S+1}L_J$. The ground state of Sr is $^1S_0$. However all of the excited states, apart from the singlet S series, still have angular momentum.

[4] There are magnetic quantum numbers associated with each different type of angular momentum, m_s for the electron spin, m_ℓ for the orbital angular momentum, m_j for the total electronic angular momentum, m_I for the number spin and m_F for the total atomic (electronic plus nuclear) angular momentum.

[5] Often we can take the magnetic field direction to be along the z axis and we call this the quantisation axis. However, there are cases where the magnetic field changes sign. For example, in the magneto-optical trap (MOT) used to accumulate laser cooled atoms, the magnetic field changes sign at the origin. If $B > 0$ for $z > 0$ and $B < 0$ for $z < 0$, then the state we label as $m = +1$ for $z > 0$ becomes $m = -1$ for $z < 0$.

a right-circularly polarized photon. This has angular momentum $-\hbar$ along z (see *Optics f2f*, p. 57) therefore it will drive a $\Delta m = -1$ (σ_-) transition. A left-circularly polarized photon, has angular momentum $+\hbar$ along the propagating direction and drives a $\Delta m = +1$ (σ_+) transition. However, if we reverse the magnetic field, the projection of the photon momentum onto the quantization axis changes sign and now left- and right-circularly polarized light drives $\Delta m = -1$ and $+1$ transition. Opposite to before.[6]

Linearly polarized light propagating along z drives a superposition of σ_+ and σ_- transitions. Linear light polarized along z (propagating perpendicular to z) drives $\Delta m = 0$, known π) transitions. These cases are summarised in Fig. 2.

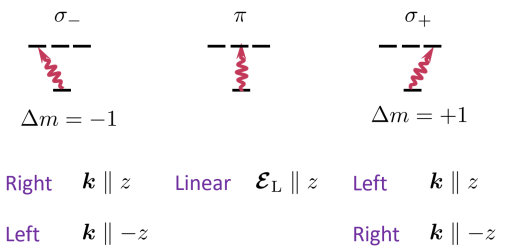

Figure 2: Polarization selection rules.

Optical Pumping: We can exploit the polarization selection to 'pump' the atoms in a particular magnetic sublevels. This technique[7] is used to initialise the atoms in a particular qubit state $|0\rangle$ or $|1\rangle$.

If we apply resonant light to drive a transition from a ground $|g\rangle$ with total atomic angular momentum F, to an excited state $|e\rangle$ with total atomic angular momentum F', we can use polarization to redistribute the atomic population amongst the ground state magnetic sublevels. An example for Cs is shown in Fig. 3. The selection rules for optical pumping are:

1. For excitation, we can select $\Delta m_F = -1, 0, +1$ transitions (σ^-, π and σ^+) by using right-circular (propagating along z), linear (polarized along z) and left-circular (propagating along z), respectively.

2. For decay, $\Delta m_F = -1, 0, +1$ and $\Delta F = -1, 0, +1$ are all allowed. Atoms may fall into the other F level in the ground state so we also need a **repump** laser.

[6] Confusingly, some books and publications label the light as $\sigma^\pm$ but light cannot be $\sigma^\pm$, light only has a particular polarization and a propagation direction. What transitions particular light polarizations drive depends on the propagation direction relative to the direction of an external magnetic field (which defines the quantization axis).

[7] Optical pumping was pioneered by Alfred Kastler who won the Nobel Prize in 1966.

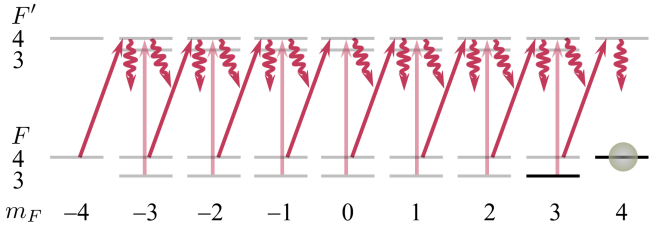

Figure 3: Example of optical pumping into the $6s^2S_{1/2}(F = 4, m_F = 4)$ state in Cs which can be used as one of the qubit states. For light propagating in the magnetic field direction, left-circularly polarized light resonant with $6s^2S_{1/2}(F = 4) \rightarrow 6p^2P_{3/2}(F = 4)$ drives $\Delta m_F = +1$ transitions. A linearly polarized **repump** resonant with the $6s^2S_{1/2}(F = 3) \rightarrow 6p^2P_{3/2}(F = 4)$ transition prevents the population accumulating in the lower hyperfine state. An advantage of this scheme is that the final state $F = 4, m_F = 4$ is dark so the pumped atoms are not heated by further scattering.

3. The exception to rules 1. and 2. is that $m_F = 0 \rightarrow m'_F = 0$ is not allowed when $F = F'$. This can be used to pump atoms into the $m_F = 0$ state, see Fig. 4.

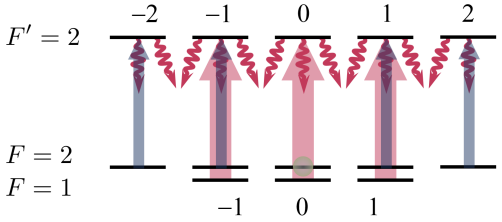

Figure 4: Example of optical pumping into the $5s^2S_{1/2}(F = 2, m_F = 0)$ state in ^{87}Rb ($I = 3/2$). For light linearly polarized parallel to the magnetic field direction, we drive $\Delta m_F = 0$ transitions. However, for a laser tuned to $5s^2S_{1/2}(F = 2) \rightarrow 5p^2P_{3/2}(F' = 2)$ transition the $m_F = 0 \rightarrow m'_F = 0$ transition is not allowed. Consequently atoms accumulate in $6s^2S_{1/2}(F = 2, m_F = 0)$. The microwave transition, $6s^2S_{1/2}(F = 1, m_F = 0) \rightarrow 6s^2S_{1/2}(F = 2, m_F = 0)$ is **magnetically insensitive**. This transition is used in atomic clocks, and is a good choice for quantum computing, see Lecture 16. The 'red' laser repumps out of the lower hyperfine level.

Summary: What do you need to be able to do?

1. Understanding the significant of Γ in the context of stopping and cooling atoms.

2. Be able to calculate the stopping distance.

3. Explain why labeling a laser beam σ_-, π or σ_+ does not make sense.

4. Explain configuration of laser polarizations, propagation and magnetic field directions, needed to drive particular transitions and optically pump atoms into particular states.

5. Sketch optical pumping diagrams for different cases.