L4 Quantum information and computing (QIC)

Lecture 18: Rydberg CNOT gate (DVC2)

January 6, 2023

Aims of Lecture 18: To understand how Rydberg blockade allows us to realise a CNOT gate. Together with single-qubit rotations, the CNOT gate is sufficient to realise a universal quantum computer.

Introduction:

In Lectures 17 we saw how we can exploit the strong (van der Waals) interaction between nearby Rydberg atoms to create entanglement. In this lecture, we shall look at how these interactions are used to realise a CNOT gate.

Rydberg CNOT gate: In this section, we shall look at how to use the Rydberg blockade mechanism to realise a CNOT gate. The CNOT gate employs a control $|\mathrm{C}\rangle$ and target $|\mathrm{T}\rangle$ atom which can be individually addressed and read-out as illustrated in Fig. 1.

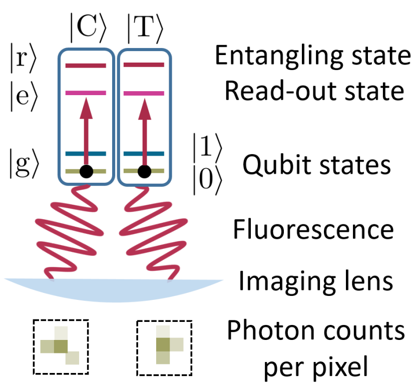

Figure 1: The level scheme for implementing two-qubit gates using alkali-metal atoms such as Rb and Cs.

As discussed in QIC.9, see Fig. 2, a CNOT gate can be realised by performing Ramsey interferometry on the target atom and introducing a π phase shift or -1-factor condition on the state of the control. In the Rydberg CNOT the -1-factor is introduce by performed a 2π pulse on the Rydberg transition which is blocked unless the control is in state $|1\rangle$.

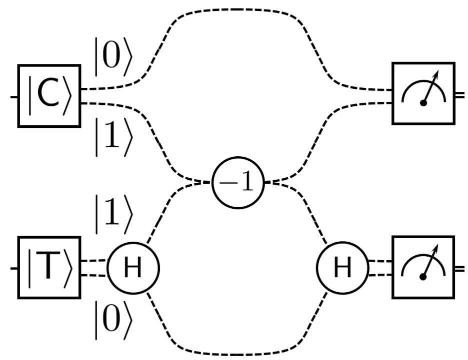

Figure 2: The CNOT as a conditional Ramsey interferometer.

The Rydberg CNOT gate is based on the following 5-pulse sequence.

1. $\pi/2$-pulse (or Hadamard) using $|0\rangle \to |1\rangle$ transition on target.

2. π-pulse ($|0\rangle \to |\mathrm{r}\rangle$) on control.

3. 2π-pulse ($|0\rangle \to |\mathrm{r}\rangle$) on target.

4. π-pulse ($|0\rangle \to |\mathrm{r}\rangle$) on control.

5. $\pi/2$-pulse (or Hadamard) using $|0\rangle \to |1\rangle$ transition on target.

The first and last pulse are the Ramsey interferometer and the middle three implement the conditional phase shift. The effect of the pulses, see Fig. 3. The 2nd and 4th pulses execute a π pulse on the control, only if it is state $|0\rangle$ which is described by the matrix

$$\mathsf{U}_{2,4} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

The 3rd pulse does a 2π-pulse on the target but only if it is state $|0\rangle$ and the control is in state $|1\rangle$.

$$\mathsf{U}_3 \quad = \quad \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} .$$

The combination of pulses 2, 3, and 4 is given by the product of $U_{2,4}$ and U_3 (as both matrices are diagonal in this basis the order does not matter):

$$\mathsf{U}_{2,3,4} = \mathsf{U}_2\mathsf{U}_3\mathsf{U}_4 \quad = \quad \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} .$$

The combination of pulses 2, 3, and 4 performs a controlled-phase operation, i.e. apply a different phase to the $|0\rangle$ and $|1\rangle$ states of the target, if the control is in state $|1\rangle$. We can convert this controlled phase into a CNOT by using a Ramsey sequence of the target, e.g. apply a Hadamard before and after. The Hadamards are implemented by pulses 1 and 5.

The complete pulse sequence can be written as $\mathsf{H}_\mathsf{T}\mathsf{U}_{2,3,4}\mathsf{H}_\mathsf{T}$ where H_T is a Hadamard on the target.

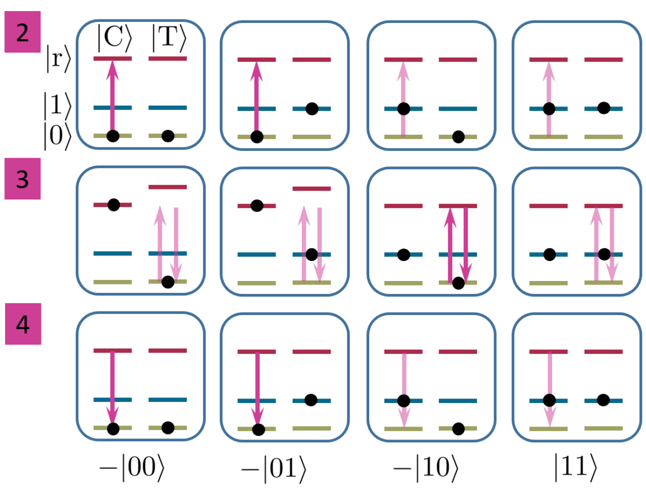

Figure 3: The middle three pulses in a Rydberg CNOT gate for each of the four input states $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$.

Finally, we derive a matrix to describe the 5-pulse Rydberg CNOT sequence. What is the difference compared to a standard CNOT? How could you correct this?

$$\mathsf{H}_\mathsf{T} \quad = \quad \sigma_0 * \mathsf{H} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix} .$$

$$\begin{aligned} \mathsf{U}_{2,3,4}\mathsf{H}_\mathsf{T} \quad &= \quad \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix} , \\ &= -\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \end{pmatrix} . \end{aligned}$$

The complete 5-pulse sequence is

$$\mathsf{H}_\mathsf{T}\mathsf{U}_{2,3,4}\mathsf{H}_\mathsf{T} = -\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \end{pmatrix} ,$$

$$= \quad -\frac{1}{2} \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{pmatrix} = - \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} .$$

The result is $-\mathsf{CNOT}$. Another 2π pulse on either the control or target converts this to CNOT.

Summary:

What do you need to be able to do?

1. Understand the five pulse sequence used to realise a Rydberg CNOT gate.
2. Be able to write matrices to describe any of the pulses in a prescribed basis.

Errata (updated 2023-01-04):

- 2022 Lecture videos
    1. QIC.3 @49.40 When talking about the state with energy fluctuations and taking out the global phase factor, the expression should be

$$\mathrm{e}^{-\mathrm{i}[E_0+\delta E_0(t)]t/\hbar}(a|0\rangle + \mathrm{e}^{-\mathrm{i}[\omega_0+\delta\omega_0(t)]t}b|1\rangle) .$$

Appendix: Review of Part II Lectures 11–18

Six key concepts and equations (Lecture numbers are specified using a superscript):

1. Light forces[(12)]:

$$\overline{\langle \boldsymbol{F} \rangle} = -\tfrac{\hbar}{2}(\boldsymbol{\nabla}\Omega u + \boldsymbol{k}\Omega v) \; .$$

   The dipole force (proportional to u) and spontaneous force (proportional to v) predominantly used for trapping and cooling, respectively.

2. Optical tweezer[(13)]

$$U(x, y, z) = -\frac{U_0}{1 + z^2/z_\mathrm{R}^2}\mathrm{e}^{-2(x^2+y^2)/w^2} \; ,$$

   where

$$U_0 = \frac{\hbar\Omega_0^2}{4|\Delta|} \; ,$$

   is the light shift. Note that the Rabi frequency can be related to the laser intensity via $\Omega = \Gamma\sqrt{\mathcal{I}/2\mathcal{I}_\mathrm{s}}$.

3. Spontaneous scattering force[(14)]

$$\boldsymbol{F} = \hbar\boldsymbol{k}R \; ,$$

   where

$$R = \Gamma P_{|\mathrm{e}\rangle} = \Gamma\frac{\Omega^2/4}{\Omega^2/2 + \Gamma^2/4 + \Delta^2} \; ,$$

4. Polarization selection rules, $\Delta m = 0, \pm 1$, and optical pumping[(14)].

5. Stimulated Raman transitions[(15)]: Effective Rabi frequency

$$\Omega_\mathrm{eff} = \frac{\Omega_1\Omega_2}{2|\Delta_{|\mathrm{e}\rangle}|} \; ,$$

   and residual spontaneous scattering rate

$$R = \frac{\Omega_1\Omega_2}{4\Delta_{|\mathrm{e}\rangle}^2} \; .$$

6. Rydberg blockade[(17)]

$$\mathcal{H}_\mathrm{Ryd} = \hbar\begin{pmatrix} 0 & \Omega & \Omega & 0 \\ \Omega & 0 & 0 & \Omega \\ \Omega & 0 & 0 & \Omega \\ 0 & \Omega & \Omega & V_\mathrm{vdW} \end{pmatrix} \; ,$$

   where $V_\mathrm{vdW} = C_6/R^6$. Rydberg CNOT gate (5 pulses)[(18)].