L4 Quantum information and computing (QIC) 2020-21

Lecture 12: Initialisation I: Light forces and light shifts

October 29, 2021

Aims of Lecture 12: To derive expressions for the force on an atom in a laser field.

Introduction: Initialisation To build a quantum computer, first on our list of DiVincenzo criteria (DV1) is to initialise an array of qubits. For atoms, a regular array is achieved using an array of focused laser beams (**optical tweezers**, Ashkin, Nobel Prize 2018) that trap each atom at a defined position, see Fig. 1. The tweezer traps are loaded using **laser cooling**. Both laser cooling and trapping use **light forces**.

Conveniently, the same equations that we derived to describe a qubit (two-level system) interacting with a oscillatory electromagnetic field are also applicable to **laser cooling and trapping**. However, there are key differences. Rather than qubit states $|0\rangle$ and $|1\rangle$ we will use a ground state $|\mathrm{g}\rangle$ and an excited state $|\mathrm{e}\rangle$. For qubits we need avoid errors due to **spontaneous emission** and so choose long-lived (metastable) states. In contrast, **spontaneous emission** is essential for laser cooling and we chose an excited state $|\mathrm{e}\rangle$ with a short lifetime (of the order of 10s of nanoseconds).

Light forces Both laser cooling and trapping rely on light forces. When single-frequency laser light is incident on an atom it induces an oscillatory dipole. The expectation of the dipole moment is

$$\langle d\rangle = \langle\psi|\hat{\mathsf{d}}|\psi\rangle ,$$

where the dipole operator is equal to charge times displacement, e.g. $\hat{\mathsf{d}} = -ex$ for an electric field polarized along x. Using $|\psi\rangle = a|\mathrm{g}\rangle + b\mathrm{e}^{-\mathrm{i}\omega_0 t}|\mathrm{e}\rangle$ we obtain[1]

$$\langle d\rangle = -\mathcal{D}_0(a^*b\mathrm{e}^{-\mathrm{i}\omega_0 t} + b^*a\mathrm{e}^{\mathrm{i}\omega_0 t}) ,$$

where $\mathcal{D}_0 = -e|\langle \mathrm{e}|x|\mathrm{g}\rangle| = -e|\langle \mathrm{g}|x|\mathrm{e}\rangle|$. In the rotating frame for a field $\mathcal{E} = \mathcal{E}_0\cos(\boldsymbol{kr} - \omega t)$, see Appendix in Lecture 4,

$$\begin{aligned}\langle d\rangle &= -\mathcal{D}_0(\tilde{a}^*\tilde{b}\mathrm{e}^{-\mathrm{i}\omega t} + \tilde{b}^*\tilde{a}\mathrm{e}^{\mathrm{i}\omega t}) ,\\ &= -\mathcal{D}_0\left[u\cos\omega t - v\sin\omega t\right] .\end{aligned}$$

The energy of the dipole is

$$U = -\langle d\rangle\mathcal{E} ,$$

where $\mathcal{E} = \mathcal{E}_0\cos(\boldsymbol{kr} - \omega t)$ for a laser propagating in the direction $\boldsymbol{k}/k$. The force is given by the gradient of the energy and contains two terms, corresponding the gradient of the field amplitude and the gradient of the phase. At the origin, $\boldsymbol{r} = 0$, we have

$$\langle\boldsymbol{F}\rangle = \langle d\rangle\left[\boldsymbol{\nabla}\mathcal{E}_0\cos\omega t + \boldsymbol{k}\mathcal{E}_0\sin\omega t\right] .$$

Next, we substitute for $\langle d\rangle$ and take a time average

$$\begin{aligned}\overline{\langle\boldsymbol{F}\rangle} &= -\tfrac{1}{2}\mathcal{D}_0\boldsymbol{\nabla}\mathcal{E}_0 u - \tfrac{1}{2}\mathcal{D}_0\boldsymbol{k}\mathcal{E}_0 v ,\\ &= -\tfrac{\hbar}{2}(\boldsymbol{\nabla}\Omega u + \boldsymbol{k}\Omega v) .\end{aligned}$$

The x and y components of the Bloch vector are found from steady-state solution of the optical Bloch equations, see QIC.6. Setting

$$\begin{aligned}\dot{u} &= -(\Gamma/2)u - \Delta v = 0 ,\\ \dot{v} &= \Delta u - (\Gamma/2)v - \Omega w = 0 ,\\ \dot{w} &= \Omega v - \Gamma(w-1) = 0 .\end{aligned}$$

we obtain the steady-state probability to be in state $|1\rangle$[2]

$$P_{|\mathrm{e}\rangle} = \frac{1}{2}(1-w) = \frac{\Omega^2/4}{\Delta^2 + \Gamma^2/4 + \Omega^2/2} ,$$

[1] From QIC.3

$$u = ab^* + ba^* ,\ v = \frac{1}{\mathrm{i}}(a^*b - ab^*) ,\ w = |a|^2 - |b|^2 .$$

[2] From the first and third equation, we have

$$u = -\frac{2\Delta}{\Gamma}v \text{ and } v = \frac{\Gamma}{\Omega}(w-1) .$$

Substituting into the second equation gives

$$-\frac{2\Delta^2}{\Gamma}\frac{\Gamma}{\Omega}(w-1) - \frac{\Gamma^2}{2\Omega}(w-1) - \Omega w = 0 ,$$

and

$$w = \frac{4\Delta^2 + \Gamma^2}{2\Omega^2 + 4\Delta^2 + \Gamma^2} = \frac{1}{1+s} ,$$

and

$$s = \frac{\Omega^2/2}{\Delta^2 + \Gamma^2/4} ,$$

is known as the saturation parameter.

and we write u and v in terms of $P_{|e\rangle}$,

$$u = \left(\frac{4\Delta}{\Omega}\right) P_{|e\rangle} \ , \text{ and } v = -\left(\frac{2\Gamma}{\Omega}\right) P_{|e\rangle} \ ,$$

Substituting into the force equation we have

$$\overline{\langle \boldsymbol{F} \rangle} = \boldsymbol{F}_{\text{dipole}} + \boldsymbol{F}_{\text{spont}} \ ,$$

where the two terms correspond to the **dipole force** (or **gradient force**),[3]

$$\begin{aligned} \boldsymbol{F}_{\text{dipole}} &= -\frac{\hbar \boldsymbol{\nabla} \Omega}{2} \left(\frac{4\Delta}{\Omega}\right) P_{|e\rangle} \ , \\ &= -\hbar \boldsymbol{\nabla} \Omega \frac{\Delta\Omega/2}{\Delta^2 + \Gamma^2/4 + \Omega^2/2} \ , \end{aligned}$$

and the **spontaneous force**,[4]

$$\boldsymbol{F}_{\text{spont}} = \hbar \boldsymbol{k} \Gamma P_{|e\rangle} \ .$$

The dipole and spontaneous force are sometimes interpreted in terms of cycles of absorption and stimulated emission and absorption and spontaneous emission, respectively.

The light shift The dipole force can be rewritten as the gradient of a potential, $\boldsymbol{F}_{\text{dipole}} = -\boldsymbol{\nabla} U_{\text{dip}}$, where

$$U_{\text{dip}} = \frac{\hbar \Delta \Omega^2/4}{\Delta^2 + \Gamma^2/4 + \Omega^2/2} \ .$$

This is know as the **light shift** (or ac Stark shift). In order to realise a conservative potential, we need to minimise spontaneous scattering which means minimising the population in the excited state $P_{|e\rangle}$. This is achieved using the limit of **far-detuning**—detuning is much larger than the Rabi frequency, $|\Delta| \gg \Omega$, and much larger than the decay rate of the excited state $|\Delta| \gg \Gamma$.[5] In this case

$$U = \frac{\hbar \Omega^2}{4\Delta} \ .$$

Note that we have made two approximations that are not completely accurate. First we have used the rotating wave approximation, $1/(\omega + \omega_0) \ll 1/(\omega - \omega_0)$. This is less accurate in the limit of far detuning $\Delta \sim \omega_0$. Second, we have assumed that the atom only has two levels, $|g\rangle$ and $|e\rangle$, but in fact there are more levels that may contribute to the shift.

Alternative derivation of the light shift In the far-detuned limit where can neglect spontaneous emission we estimate the light shift using the eigenvalues of the interaction Hamiltonian. To calculate the energy change of an atom in a light field we use our two-state model, where the Schrödinger equation, $i\hbar d_t |\psi\rangle = \mathcal{H}_{\text{int}} |\psi\rangle$, has the form

$$i\hbar \frac{d}{dt} \begin{pmatrix} a \\ b \end{pmatrix} = \frac{\hbar}{2} \begin{pmatrix} \Delta & \Omega \\ \Omega & -\Delta \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \ .$$

To calculate the energy shifts we use the eigenvalues of the interaction Hamiltonian which are

$$E_\pm = \pm \frac{\hbar}{2} (\Delta^2 + \Omega^2)^{1/2} \ .$$

The change in energy is given by the energy difference between with the field on and off $\Omega = 0$, i.e.

$$U_\pm = \pm \frac{\hbar}{2} \left[(\Delta^2 + \Omega^2)^{1/2} - |\Delta| \right] \ .$$

For the case of large detuning, $|\Delta| \gg \Omega$, we can simplify this expression

$$U_\pm = \pm \frac{\hbar}{2} \left[\left(|\Delta| + \frac{\Omega^2}{2|\Delta|} \right) - |\Delta| \right] = \pm \frac{\hbar \Omega^2}{4|\Delta|} \ .$$

We choose the so-called **dressed-state basis** where the 'bare' states $|0\rangle$ and $|1\rangle$ correspond to an atom in the ground state plus N photon and an atom in the excited state plus $N-1$ photons. The effect of light on the energies of states $|0\rangle$ and $|1\rangle$ are given by the eigenvalues of $\mathcal{H}$. The change in the energy for $|\Delta| \gg \Omega$,

$$U = \frac{\hbar \Omega^2}{4\Delta} \ ,$$

is known as the **light shift**. The gradient of the energy, $F = -\boldsymbol{\nabla} U$ is the **dipole force**. For **red detuning**, $\Delta < 0$, the light shift produces a potential well, $U < 0$.

In the next lecture we shall use the light shift formula to estimate the depth of the tweezer potential well, the photon scattering rate, and how well the atoms are localised.

Summary:

What do you need to be able to do?

1. Explain the two types of light force.
2. Understand how to derive expressions for the light-shift from either the light force or the eigenvalues of the interaction Hamiltonian.

[3] See Foot, eqn (9.43).

[4] See Foot, eqn (9.42).

[5] Typical parameters for Cs atoms: $\Gamma = 2\pi(5.2\text{ MHz})$. The strongest optical transition, the D2 line $6s_{1/2} \to 6p_{3/2}$, has $\lambda_0 = 852$ nm. If we use a Nd:YAG laser with $\lambda_L = 1.06\ \mu$m and a power $P = 10$ mW then $\Delta = 2\pi(70\text{ THz})$ and $\Omega = 2\pi(89\text{ GHz})$. For a more accurate estimate we would also need to add the effect of the D1 line $6s_{1/2} \to 6p_{1/2}$ have $\lambda_0 = 895$ nm.