L4 Quantum information and computing (QIC)

Lecture 17: Quantum entanglement via Rydberg blockade

January 6, 2023

Aims of Lecture 17: To understand how to make atom interact between atoms allow us to create entanglement.

Introduction:

In QIC.16 we introduced the concept of Rydberg atoms where the valance electron is excited to a state with high pricipal quantum number, n. In this Lecture we shall look at how to exploit the strong interactions between Rydberg atoms to realise quantum entanglement. The mechanism is known as **Rydberg blockade**.

Rydberg blockade

For laser light resonant with the $|0\rangle \to |\mathrm{r}\rangle$ transition the interaction Hamiltonian, in the $\{|00\rangle, |0\mathrm{r}\rangle, |\mathrm{r}0\rangle, |\mathrm{rr}\rangle\}$ basis is

$$\mathcal{H}_{\mathrm{Ryd}} = \frac{\hbar}{2}\begin{pmatrix} 0 & \Omega & \Omega & 0 \\ \Omega & 0 & 0 & \Omega \\ \Omega & 0 & 0 & \Omega \\ 0 & \Omega & \Omega & 2V_{\mathrm{vdW}} \end{pmatrix}. \tag{1}$$

For highly-excited Rydberg atoms separated by about 10 microns or less $|V_{\mathrm{vdW}}| \gg \Omega$. Consequently, the doubly-excited Rydberg state $|\mathrm{rr}\rangle$ is blocked is shifted out of resonance and no longer excited, see Fig. 1. The suppression of more than one Rydberg atom is known as dipole or **Rydberg blockade**. Note that as $V_{\mathrm{vdW}} = \hbar C_6/R^6$, the condition $|V_{\mathrm{vdW}}| \gg \Omega$ is a condition on the spacing between the atoms $R < R_{\mathrm{b}}$, where $R_{\mathrm{b}} = (C_6/\Omega)^{1/6}$ is known as the **blockade radius**. Typically R_{b} is of the order of 10 microns.

As the $|\mathrm{rr}\rangle$ state is decoupled by the van der Waals shift we can remove it and rewrite the interaction as

$$\mathcal{H}_{\mathrm{Ryd}} = \frac{\hbar}{2}\begin{pmatrix} 0 & \Omega & \Omega \\ \Omega & 0 & 0 \\ \Omega & 0 & 0 \end{pmatrix}.$$

What happens in this case is easier to understand if we rewrite the Hamiltonian in an entangled basis $|00\rangle$,

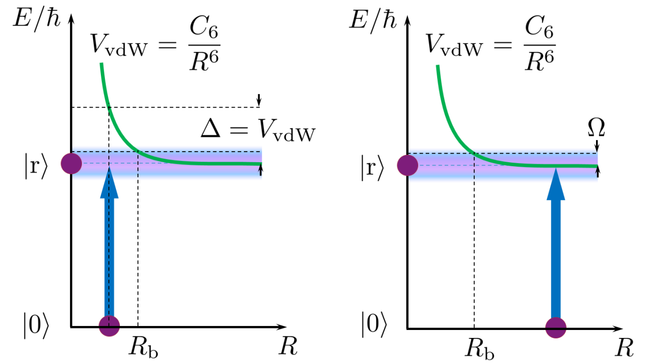

Figure 1: Energy level diagram illustrated Rydberg blockade. If one atom is in the Rydberg state $|\mathrm{r}\rangle$ then the energy needed to excite a second Rydberg atom is shifted by the van der Waals interaction, $\hbar V_{vdW}$. For two atoms with spacing $R < R_{\mathrm{b}}$ (left), the transition to drive the 2nd atom to state $|\mathrm{r}\rangle$ is shifted off-resonance, i.e., the excitation of the double excited state $|\mathrm{rr}\rangle$ is **blocked**. For two atoms with spacing $R \gg R_{\mathrm{b}}$ (right) the interactions are neglibible and the excitation of each atom is no longer correlated.

$\frac{1}{\sqrt{2}}(|0\mathrm{r}\rangle + |\mathrm{r}0\rangle)$ and $\frac{1}{\sqrt{2}}(|0\mathrm{r}\rangle - |\mathrm{r}0\rangle)$. To transform the Hamiltonian to the new basis, we use the rotation matrix

$$\mathsf{R} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}.$$

The new Hamiltonian is[1]

$$\mathcal{H}'_{\mathrm{Ryd}} = \mathsf{R}^\dagger \mathcal{H}_{\mathrm{Ryd}} \mathsf{R} = \frac{\hbar}{2}\begin{pmatrix} 0 & \sqrt{2}\Omega & 0 \\ \sqrt{2}\Omega & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

The laser drives Rabi oscillations between the states $|00\rangle$ and the maximally entangled Bell state, $\frac{1}{\sqrt{2}}(|0\mathrm{r}\rangle + |\mathrm{r}0\rangle)$ at a frequency $\sqrt{2}\Omega$. The coupling between $|00\rangle$ and

[1]

$$\frac{1}{2}\begin{pmatrix} 0 & \Omega & \Omega \\ \Omega & 0 & 0 \\ \Omega & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0 & \sqrt{2}\Omega & 0 \\ \Omega & 0 & 0 \\ \Omega & 0 & 0 \end{pmatrix}.$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}\frac{1}{2}\begin{pmatrix} 0 & \sqrt{2}\Omega & 0 \\ \Omega & 0 & 0 \\ \Omega & 0 & 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0 & \sqrt{2}\Omega & 0 \\ \sqrt{2}\Omega & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

$\frac{1}{\sqrt{2}}(|0\mathrm{r}\rangle - |\mathrm{r}0\rangle)$ is zero due to destructive interference. Consequently, the anti-symmetric Bell state $\frac{1}{\sqrt{2}}(|0\mathrm{r}\rangle - |\mathrm{r}0\rangle)$ is not populated.

The consequence of blockade is that driving the $|0\rangle \rightarrow |\mathrm{r}\rangle$ transition leads directly to entanglement. Due to interactions, the two-atom system only supports a single excitation. This excitation is shared between the atoms. We can only excite one atom but we do not know (and we cannot know, without a measurement) which one.

Summary:

What do you need to be able to do?

1. Understand the concept of Rydberg blockade, and estimate the blockade radius given the parameters.
2. Derive the interaction Hamiltonian in the entangled basis, and show how Rydberg blockade leads to entanglement.