L4 Quantum information and computing (QIC)

15: Single-qubit gates stimulated-Raman transitions (DVC2.1) and read-out (DVC3)

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Aims of Lecture 16: To understand how to perform single-qubit gates (rotations) in an alkali-metal (Rb or Cs) Rydberg atom quantum computer. To discuss how we read-out the qubit state.

Introduction: In Lecture 15 we learnt how to initialise the atom in a particular state within the computational basis $\{|0\rangle, |1\rangle\}$. Now we need a way to drive single-qubit rotations. We shall use the example of Cs and define our computational basis as $|0\rangle = |6\mathrm{s}^2S_{1/2}(F = 3, m_F = 3)\rangle$ and $|1\rangle = |6\mathrm{s}^2S_{1/2}(F = 4, m_F = 4)\rangle$.

Stimulated Raman transitions

The disadvantage of driving the $6\mathrm{s}^2S_{1/2}(F = 3, m_F = 3) \to 6\mathrm{s}^2S_{1/2}(F' = 4, m'_F = 4)$ transition directly is that this is a microwave transition with a wavelength of a few centimeters. This makes it is difficult to address only one atom in the array. In particular, as in order to make the atoms interact such that we can create entanglement (see Lecture 17) we would like them to be as close as possible.

To allow the atoms to be spaced by a few microns and still use single site addressing we use two-photon optical transitions via the excited state $|\mathrm{e}\rangle$. However, to avoid spontaneous emission during the rotation we need to far detune the lasers from resonance with $6\mathrm{s}^2S_{1/2}(F,) \to 6\mathrm{p}^2P_{3/2}(F', m'_F)$

However to make them interact A solution this problem is to use the ground state hyperfines states, but drive transitions between them optically via an excited state $|\mathrm{e}\rangle$. Such a two-photon transition is known as a **stimulated-Raman transition**. Stimulated Raman transitions combine single-qubit addressability with low decoherence due to spontaneous emission.

A resonant stimulated-Raman transition is driven by two laser beams both detuned by Δ_e from the $|0\rangle \to |\mathrm{e}\rangle$ and $|1\rangle \to |\mathrm{e}\rangle$ transitions, where $|0\rangle$ and $|1\rangle$ are typically the lower and upper ground state hyperfine state and $|\mathrm{e}\rangle$ is an excited state, see Fig. 1.[1] To drive transitions between

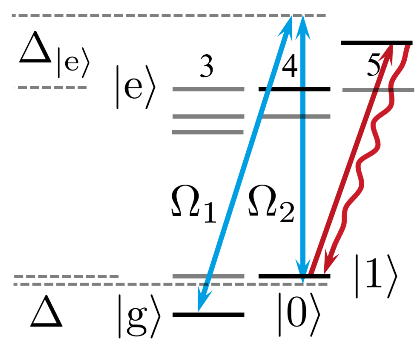

Figure 1: Example levels used for the qubit in Cs. $|0\rangle$ and $|1\rangle$ correspond to the $6\mathrm{s}^2S_{1/2}(F = 3, m_F = 3)$ and $6\mathrm{s}^2S_{1/2}(F = 4, m_F = 4)$, respectively. Single-qubit rotations are implemented using stimulated-Raman transitions, $|0\rangle \to |\mathrm{e}\rangle \to |1\rangle$ (blue lines). Read-out of $|0\rangle$ and $|1\rangle$ is implemented by driving the closed $6\mathrm{s}^2S_{1/2}(F = 4, m_F = 4)$ to $6\mathrm{p}^2P_{3/2}(F' = 5, m'_F = 5)$ transition on resonance (red lines). If the atom is in $|0\rangle$ there is dark (no fluorescence), if it is in $|1\rangle$ the atom is bright.

particular magnetic sub-levels we need to make use of the polarization selection rules introduced in QIC.15.

For a two-photon transition, the laser phase will depend on both $\boldsymbol{k}$ vectors, e.g. $\phi_\mathrm{L} = (\boldsymbol{k}_1 - \boldsymbol{k}_2) \cdot \boldsymbol{r}$ where $\boldsymbol{r}$ is the position of the atom. By using two laser beam propagating in the same direction $\boldsymbol{k}_1 \approx \boldsymbol{k}_2$, we can minimise phase errors.

We write the state vector as a superposition of all three states, i.e.,

$$|\psi\rangle = a|0\rangle + b|1\rangle + c|\mathrm{e}\rangle \ .$$

Adapting the two-level coupled equation from Lecture 4 Appendix to a 3-level system, we have

$$\begin{aligned} \mathrm{i}\dot{a} &= \tfrac{1}{2}\Omega_1 \mathrm{e}^{\mathrm{i}\Delta_{|\mathrm{e}\rangle} t} c \ , \\ \mathrm{i}\dot{c} &= \tfrac{1}{2}\Omega_1 \mathrm{e}^{-\mathrm{i}\Delta_{|\mathrm{e}\rangle} t} a + \tfrac{1}{2}\Omega_2 \mathrm{e}^{-\mathrm{i}\Delta_{|\mathrm{e}\rangle} t} b \ , \\ \mathrm{i}\dot{b} &= \tfrac{1}{2}\Omega_2 \mathrm{e}^{\mathrm{i}\Delta_{|\mathrm{e}\rangle} t} c \ , \end{aligned}$$

where $\Omega_{1,2}$ are the Rabi frequencies of beams 1 and 2, respectively. If $|\Delta_{|\mathrm{e}\rangle}| \gg \Omega_{1,2}$, then a and b are constant over a time $t > 1/|\Delta_{|\mathrm{e}\rangle}|$ and we can integrate the middle

[1] As before we might also want to consider a detuning, Δ, from the two-photon resonance, see Fig. 1. To simplify the derivation we shall put $\Delta = 0$.

equation

$$c = \frac{1}{2}\frac{\Omega_1}{\Delta_{|e\rangle}}e^{-i\Delta_{|e\rangle}t}a + \frac{1}{2}\frac{\Omega_2}{\Delta_{|e\rangle}}e^{-i\Delta_{|e\rangle}t}b \ .$$

Substituting for c in the first and third equation we have obtain

$$\begin{aligned} i\dot{a} &= \frac{1}{2}\left(\frac{\Omega_1^2}{2\Delta_{|e\rangle}}a + \frac{\Omega_1\Omega_2}{2\Delta_{|e\rangle}}b\right) \ , \\ i\dot{b} &= \frac{1}{2}\left(\frac{\Omega_1\Omega_2}{2\Delta_{|e\rangle}}a + \frac{\Omega_2^2}{2\Delta_{|e\rangle}}b\right) \ . \end{aligned}$$

These equations mean that states $|0\rangle$ and $|1\rangle$ are coupled with an effective Rabi frequency

$$\Omega_{\text{Raman}} = \frac{\Omega_1\Omega_2}{2|\Delta_{|e\rangle}|} \ ,$$

and shifted by an amount $\Omega_{1,2}^2/4\Delta_{|e\rangle}$. This shift is the same **light shift** or **ac-Stark shift** that we found for the optical tweezer. For convenience we shall put $\Omega_1 = \Omega_2 = \Omega$ such that

$$\Omega_{\text{Raman}} = \frac{\Omega^2}{2|\Delta_{|e\rangle}|} \ .$$

Spontaneous emissions per Rabi cycle: Even if the single-photon transitions are far detuned there is still a small probability of populating the excited state $|e\rangle$, and hence a chance of spontaneous emission. To estimate the number of spontaneous emission events per Rabi flop (i.e., the probability that the gate operation is interrupted by spontaneous emission), consider first the single-photon transition $|0\rangle \to |e\rangle$, which is described by the coupled equations:

$$i\dot{a} \ = \ \tfrac{1}{2}\Omega e^{i\Delta_{|e\rangle}t}c \ , \text{ and } i\dot{c} = \tfrac{1}{2}\Omega e^{-i\Delta_{|e\rangle}t}a \ .$$

For large $\Delta_{|e\rangle}$, the probability to be in the excited state is very small and we can put $a \approx 1$ and integrate the equation for c, i.e.,

$$c = \frac{1}{2}\frac{\Omega}{\Delta_{|e\rangle}}e^{i\Delta_{|e\rangle}t} \ ,$$

so the probability to be in the excited state is

$$|c|^2 = \frac{\Omega^2}{4\Delta_{|e\rangle}^2} \ .$$

The other transition $|1\rangle \to |e\rangle$ contributes an additional $\Omega^2/(4\Delta_{|e\rangle}^2)$ so the total is

$$|c|^2 = \frac{\Omega^2}{2\Delta_{|e\rangle}^2} \ .$$

The spontaneous emission rate (assuming the excited state decays at a rate Γ) for $|\Delta_{|e\rangle}| \gg \Omega$ and Γ is $R = \Gamma|c|^2 = \Gamma\Omega^2/(2\Delta_{|e\rangle}^2)$. The upper limit on the number of spontaneous emission events per π-pulse is equal to the interaction time $t_\pi = \pi/\Omega_{\text{Raman}}$ times the scattering rate R, i.e.

$$N_{\text{spont}} = \frac{\Omega^2}{2\Delta_{|e\rangle}^2}\Gamma\frac{\pi}{\Omega^2/(2|\Delta_{|e\rangle}|)} = \pi\frac{\Gamma}{|\Delta_{|e\rangle}|} \ .$$

This result shows that we can only reduce spontaneous emission by increasing $|\Delta_{|e\rangle}|$. For $\Gamma = 2\pi(6\text{ MHz})$ we need to choose $|\Delta_{|e\rangle}| = 2\pi(80\text{ GHz})$ in order to get $N_{\text{spont}} \sim 2\times10^{-4}$. However, the large detuning also reduces the two-photon Rabi frequency and hence the gate speed. So, there is still a trade-off between speed and decoherence.

Read out: The choice $|0\rangle = |6s^2S_{1/2}(F=3, m_F=3)\rangle$ and $|1\rangle = |6s^2S_{1/2}(F=4, m_F=4)\rangle$ is a compromise. The magnetically-insensitive states $|0\rangle = |6s^2S_{1/2}(F=3, m_F=0)\rangle$ and $|1\rangle = |6s^2S_{1/2}(F=4, m_F=0)\rangle$ as used in atomic clocks have the advantage requirement less magnetic noise shielding.

The advantage of using $|1\rangle = |6s^2S_{1/2}(F=4, m_F=4)\rangle$ is that we scatter photons using the closed $|6s^2S_{1/2}(F=4, m_F=4)\rangle \to |6p^2P_{3/2}(F=5, m_F=5)$ transition without the atom following into other states, see Fig. 1.

If we want to use the magnetically insensitive level, $|1\rangle = |6s^2S_{1/2}(F=4, m_F=0)\rangle$, we would need to optically pump the atom to $|6s^2S_{1/2}(F=4, m_F=4)\rangle$ in order to scatter enough photons that we can distinguish $|1\rangle$ from $|0\rangle$.

Summary:

What do you need to be able to do?

1. Understand the principles of stimulated Raman transitions, how the effective Rabi frequency depends on the laser power and detuning.

2. Derive an expression for the probability of spontaneous emission during a single-qubit rotation.

3. Identify states that could be used for stimulated Raman transitions in, for example, Rb and Cs.

4. Work out the configuration of laser beam polarizations and propagation directions needed to drive a particular transition.

5. Discuss how to detect atoms in specific states.