L4 Quantum information and computing (QIC) 2020-21

Lecture 1: Qubits and the quantum advantage

October 5, 2021

Aims of Lecture 1: The central questions explored in this lecture are:

1. *What is a quantum computer?*
2. *Why might we want to build one?* Why are the better than classical—**exponential scaling**.

What is a quantum computer? A quantum computer replaces the 'bits' of a classical 'digital computer' with quantum bits—**qubits**.

What is a qubit? A 'qubit' is a system with two quantum states (or energy levels)—often called a **two-level system**).

Mathematical the two states are represented by **state vectors**[1]

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} .$$

$\{|0\rangle, |1\rangle\}$ is known as the **computational basis**. The 'qubit' state vector is

$$|\psi\rangle = a|0\rangle + b|1\rangle = a\begin{pmatrix} 1 \\ 0 \end{pmatrix} + b\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix} , \quad (1)$$

where a and b are complex coefficients that may be time dependent[2]. Many quantum systems have many levels from which we can select two to be our qubit.

The only requirements of these levels are that:

1. We can drive transitions between them, using EM fields, either lasers or microwaves. This fields are used to implement **single-qubit gates**.
2. They do not decay (or dephase) over the time scale required to complete our computation. This property is called **low decoherence**.

As we shall see below these requirements are two of the five requirements *to build a quantum computer?*

Examples of two-level systems (qubits) include:

1. 'Artificial atoms'—two-level systems in solids, e.g. **superconducting circuits**[3]. A resonant LC circuit has harmonic oscillator energy levels with frequency spacing, $\omega = 1/\sqrt{LC}$.
2. **Ions**[4] trapped by electrodes, cooled and controlled by lasers (see Nielsen and Chang, Sec. 7.6). Apart from trapping technique the same physics to Atoms.
3. **Atoms**[5] trapped by laser beams (optical tweezers), see Lectures 11–18.
4. **Photons** in superposition of two polarization states (vertical and horizontal, or left and right circular) or two waveguide channels are qubits. they interact weakly with matter resulting in robust qubits (see Nielsen and Chang, Sec. 7.4), but they are also weakly interacting with each other.[6]
5. A magnetic **spin-$\frac{1}{2}$** only has the two states—'spin up' and 'spin down', $|0\rangle = |\uparrow\rangle$ and $|1\rangle = |\downarrow\rangle$.[7] One example being explored for quantum computing, is the spin of a phosphorous ion in silicon (P in Si).[8]

Why build a quantum computer? We shall give 4 reasons:

[1] A state vector is a minimal representation. Unlike a wave function, e.g. $\langle x|\psi\rangle$ or $\langle p|\psi\rangle$, it does not tell us any details about such things as the spatial charge distribution of the state. Sometimes, the terms 'state vector' and 'wave function' are used interchangeably in the literature.

[2] Note that a and b can have any values that satisfy the normalisation condition $|a|^2 + |b|^2 = 1$. They are continuous 'analogue' variables and can be specified with infinite precision. However, the read-out is digital. So unlike classical computing, quantum computing is a hydrid of analogue and digital.

[3] More information on superconducting qubits can be found in *Circuit Quantum Electrodynamics*, A Blais, AL Grimsmo, SM Girvin, A Wallraff, arXiv:2005.12667.

[4] IonQ recently announced a 32-qubit machine.

[5] See L Henriet et al, Quantum **4**, 327 (2020).

[6] Psi Quantum is aiming to build a million qubit machine.

[7] Note that we can use any labelling we prefer for the two states.

[8] The Kane quantum computer, proposed in 1998 by Bruce Kane, is being developed in Australia by former Durham PhD student Michelle Simmons at the UNSW and within the company, Silicon Quantum Computing.

1. The first important motivation is the advancement of knowledge.[9] Our understanding of quantum mechanics has been profoundly reshaped by the development of quantum information theory. A prevalent viewpoint is that quantum theory is essentially an information theory, i.e., a way of formulating what we know and what we cannot know.

2. Moore's law. As transistor size is reduced we approach atomic dimensions.

3. Energy efficiency. There are potentially two gains. First, a classical gate has two inputs and only one output so according to Boltzmanns entropy formula the minimum energy dissipated using is $T\Delta S = k_\mathrm{B}T\ln 2$.[10] A quantum gate has two input and two outputs so the minimum energy dissipated is zero. Second and more significantly is the massive parallelism, see quantum advantage. Note that energy is particularly demanding in many machine learning applications.[11]

4. **Quantum 'advantage'**[12]. Quantum computers can store more information and compute (certain problems) much faster than classical computers.

What is the 'quantum advantage'? One qubit has two states '0' and '1'. N qubits have 2^N states. This is known as **exponential scaling** The extra states are **entangled states** that do not exist in classical systems. To see how this work, we consider the state vector of two or three qubits, first two. If the two qubits, labelled A and B, do not interact then the two qubit state vector is

$$|\psi\rangle_\mathrm{AB} \;=\; (a|0\rangle_\mathrm{A} + b|1\rangle_\mathrm{A}) \otimes (c|0\rangle_\mathrm{B} + d|1\rangle_\mathrm{B}) \ ,$$

where $\otimes$ denotes a 'tensor' product. This is known as a **product state**. If we abbreviate $|i\rangle \otimes |j\rangle$ as $|ij\rangle$ and omit the subscripts we have

$$|\psi\rangle_\mathrm{AB} \;=\; ac|00\rangle + ad|01\rangle + bc|10\rangle + bd|11\rangle \ .$$

We can also write this in vector form

$$|\psi\rangle_\mathrm{AB} \;=\; \begin{pmatrix} a \\ b \end{pmatrix} \otimes \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} ac \\ ad \\ bc \\ bd \end{pmatrix} \ ,$$

where the basis states are $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$. But, this product state is not the most general state that can exist. We could have

$$|\psi\rangle \;=\; c_{00}|00\rangle + c_{01}|01\rangle + c_{10}|10\rangle + c_{11}|11\rangle \ ,$$

where the 4 coefficients are all independent. In this case we have an additional class of state known as **entangled states**, that **cannot be factorised into a product**.[13] For 3 qubits ($N = 3$), the product state

$$|\psi\rangle \;=\; (a|0\rangle + b|1\rangle) \otimes (c|0\rangle + d|1\rangle) \otimes (e|0\rangle + f|1\rangle) \ , \ ,$$

has $2N = 6$ complex coefficients, and the general state,

$$\begin{aligned} |\psi\rangle = \quad & c_{000}|000\rangle + c_{001}|001\rangle + c_{010}|010\rangle + c_{011}|011\rangle \\ + \quad & c_{100}|100\rangle + c_{101}|101\rangle + c_{110}|110\rangle + c_{111}|111\rangle \ , \end{aligned}$$

has $2^3 = 8$ basis states characterised by 8 complex coefficients. For 40 qubits, there are $2^{40} \sim 10^{12}$ basis states (cf. 10^{11} brain cells 37×10^{12} total cells in humans) and for 100 qubits we have $2^{100} \sim 10^{30}$ basis states. In going from 40 to 100, we can encode 18 orders of magnitude more information—wow! This exponential scaling means that even a relatively small quantum computer (50–100 qubits) can be enormously more powerful than any classical computer. The gains are mainly due to the **parallelism** of working with 2^N terms simultaneously, and worth remember that most of those extra states invovle quantum entanglement.

Summary: What do you need to be able to do?

1. Write down the state vector of a qubit.

2. List 5 qubit candidates.

3. Give up to 4 reasons, why quantum computing is interesting?

4. Explain quantum advantage and exponential scaling.

5. Explain, what is meant by a product state and entangled state?

6. Be able to write a general state vector for one, two and three qubits as a sum of terms or a column vector.

[9] The idea of quantum computing was proposed by Feynman in the 1980s and his initial motivation was mostly, is this something we could do? In 1985 he wrote *we are going to be even more ridiculous... and consider bits written on one atom.... Such nonsense is very entertaining to professors like me.* R. P. Feynman, *Optics News*, February 1985, reprinted in Foundations in Physics **16**, 507 (1986).

[10] This was recognised as a problem in classical computing and led to the idea of *Conservative Logic* Fredkin, Edward; Toffoli, Tommaso (1982). Reversible classical logic fed into the development of quantum logic.

[11] *Training 500 GPUs for a week consumes about the same energy as a US citizen does in 3 years.* See also *Training a single AI model can emit as much carbon as five cars in their lifetimes.* Note that the human brain has 10^{11} neurons 10^{15} synapses, runs at a frequency of 100 Hz, and consumes a modest power of 20 W.

[12] The term **quantum supremacy** is also used but remains controversial.

[13] We shall discuss entanglement in Lecture 8.