# L4 Quantum information and computing (QIC) 2020-21

## Lecture 14: DVC1: Initialisation of atom arrays I

November 27, 2021

**Aims of Lecture 14**: To explain the application of the light forces to slowing and cooling atoms. To derive an expression for the minimum stopping distance, and understanding the principle of sideband cooling. In QIC.14, we shall mainly focus on the example of **Sr-87**, see Fig. 1. For Rb, Cs, see QIC.15-26

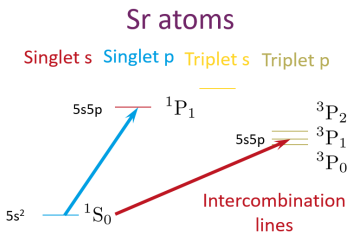


Figure 1: Energy levels of Sr-87: The $^1$S$_0$ $\rightarrow ^1$P$_1$ and $^1$S$_0$ $\rightarrow ^3$P$_1$ transitions have wavelengths of $\lambda_0 = 461$ and 689 nm, respectively. The $^1$P$_1$ and $^3$P$_1$ states have lifetimes of $\tau = 5.27$ ns and $\tau = 21.5$ $\mu$s, respectively. These correspond to decay rates, $\Gamma_{461} = 2\pi(30.2$ MHz) and $\Gamma_{689} = 2\pi(7.4$ kHz).

**Introduction:** In Lecture 13 we found that the depth of the **optical tweezer** trap is typically less than a millikelvin. Consequently, we need a means to cool atoms from room temperature (or an atomic beam) to less than 1 mK. This is achieved using laser cooling which mainly exploit the spontaneous (or scattering) force—cycle of absorption and spontaneous emission.

**Scattering force:** In this section, we consider the laser cooling effect and derive the equilibrium temperature. For a monochromatic laser beam with wave vector $\boldsymbol{k}$, the scattering force is

$$\boldsymbol{F} = \hbar \boldsymbol{k} R \,, \tag{1}$$

where $R$ is the scattering rate. The scattering rate is given by the spontaneous emission rate times the probability to be in the excited state

$$R = \Gamma P_{|\mathrm{e}\rangle} = \Gamma \frac{\Omega^2/4}{\Omega^2/2 + \Gamma^2/4 + \Delta^2} \,,$$

Often it is more convenient to rewrite the Rabi frequency in terms of the saturation intensity. Using $\Omega = \Gamma\sqrt{\mathcal{I}/(2\mathcal{I}_\mathrm{s})}$, the spontaneous scattering force for an atom in the propagation direction is

$$F = \hbar k \frac{\Gamma}{2} \frac{\mathcal{I}/\mathcal{I}_\mathrm{s}}{\mathcal{I}/\mathcal{I}_\mathrm{s} + 1 + 4(\Delta - kv)^2/\Gamma^2} \,.$$

Laser cooling require counter-propagating, red-detuned laser beams. In this case, a moving atom is Doppler shifted into resonance with a laser beam which exerts a force that opposes the motion.

**Worked example**: For a single laser beam, the spontaneous scattering force is given by $F = \hbar k \Gamma P_{|\mathrm{e}\rangle}$, where $P_{|\mathrm{e}\rangle}$ is the probability to be in the excited state and $\Gamma$ is the spontaneous scattering rate. Write an equation for $P_{|\mathrm{e}\rangle}$ in terms of the laser intensity $\mathcal{I}$, the saturation intensity $\mathcal{I}_\mathrm{s}$, the detuning $\Delta$, and the decay rate of the excited state $\Gamma$. Write expressions for $P_{|\mathrm{e}\rangle}$ in three regimes: (i) low intensity $\mathcal{I} \ll \mathcal{I}_\mathrm{s}$. Plot $P_{|\mathrm{e}\rangle}$ versus detuning in this case. What is the full-width half maximum? (ii) Intermediate intensity, $\mathcal{I} = \mathcal{I}_\mathrm{s}$ on resonance.(iii) High intensity, $\mathcal{I} \gg \mathcal{I}_\mathrm{s}$. Plot $P_{|\mathrm{e}\rangle}$ as a function of intensity for $\Delta = 0$.

The probability to be in the excited state is

$$P_{|\mathrm{e}\rangle} = \frac{\Omega^2/4}{\Gamma^2/4 + \Omega^2/2 + \Delta^2} = \frac{\Omega^2/4}{1 + 2\Omega^2/\Gamma^2 + 4\Delta^2/\Gamma^2} \,.$$

Using $\Omega^2/\Gamma^2 = \mathcal{I}/2\mathcal{I}_\mathrm{s}$ this can be rewritten in terms of the intensity

$$P_{|\mathrm{e}\rangle} = \frac{\mathcal{I}/(2\mathcal{I}_\mathrm{s})}{1 + \mathcal{I}/\mathcal{I}_\mathrm{s} + 4(\Delta/\Gamma)^2} \,.$$

Three useful cases.

1. Low intensity, $\mathcal{I} \ll \mathcal{I}_\mathrm{s}$: The excited state probability is a **Lorentzian**

$$P_{|\mathrm{e}\rangle} = \frac{\mathcal{I}/(2\mathcal{I}_\mathrm{s})}{1 + 4(\Delta/\Gamma)^2} \,,$$

with full width half maximum (FWHM) equal to $\Gamma$. Consequently, $\Gamma$ is often called the **linewidth**. For Sr, Fig. 1 there a large variation in the linewith between the singlet and triplet transition.

2. Intermediate intensity, $\mathcal{I} = \mathcal{I}_s$ and $\Delta = 0$: The time-averaged probability to be in the excited state is

$$P_{|e\rangle} = \frac{1}{4} ,$$

3. High intensity, $\mathcal{I} \gg \mathcal{I}_s$: The transition is said to be saturated and the time-averaged probability to be in the excited state is $P_{|e\rangle} = \frac{1}{2}$.

**Stopping distance:** Whether we can load atoms into the tweezer array depends on slowing them from room temperature (or hotter) to rest in practical distance. The maximum scattering force occurs at high intensity $\mathcal{I} \gg \mathcal{I}_s$. Using the previous result $P^{\max}_{|e\rangle} = \frac{1}{2}$ we have

$$F^{\max}_{\text{spont}} = \hbar k \frac{\Gamma}{2} = \frac{h}{2\lambda\tau} ,$$

where $\lambda$ is the transition wavelength and $\tau$ is the excited state lifetime. The minimum stopping distance is

$$s = \frac{u^2}{2(F^{\max}_{\text{spont}}/m)} = \frac{mu^2}{h/(\lambda\tau)} .$$

Using $\frac{1}{2}mu^2 = \frac{1}{2}k_B T$,[1] we get

$$s = \frac{k_B T}{h/\tau}\lambda .$$

Hence the stopping distance is equal to the ratio of the thermal energy to atomic resonance width times the optical wavelength. At room temperature the thermal energy is of order $h$ times terahertz whereas the resonance with is of the order of $h$ times megahertz so the stopping distance is 1 million times the wavelength, i.e. just under a meter. In the thermal distribution, a fraction of the atoms can be stopped in a shorter distance.

As an atom changes speed, the light-atom resonance condition changes due to the Doppler effect. For an atom with velocity $\boldsymbol{v}$ the detuning becomes

$$\Delta_{\boldsymbol{v}} = \Delta - \boldsymbol{k}\cdot\boldsymbol{v} .$$

As the resonance width is $\Delta = \pm\Gamma/2$, we can define a **capture** velocity $v_{\text{Doppler}} = \Gamma/k$. For Cs, with $\Gamma = 2\pi(5.0\text{ MHz})$ and $\lambda = 852$ nm, we obtain $v = 4\text{ ms}^{-1}$. Consequently, if we want to realise the minimum stopping distance we need to do something to keep the laser on resonance. On way to to do this is to use a magnetic field gradient. This is called a **Zeeman slower**.

**Sideband cooling**: After 'stopping', atoms are loaded into the tweezer using the spontaneous force to extract the excess energy. Inside the tweezer, it is possible to employ other cooling mechanisms such as **sideband cooling**.

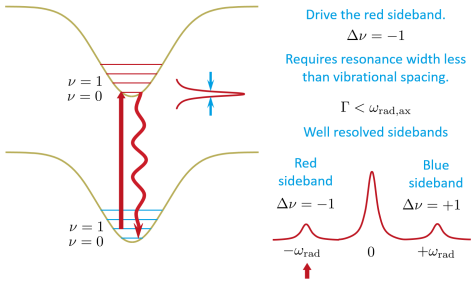


Figure 2: Principle of sideband cooling: Use laser to drive transitions that decrease vibrational quantum number—the red-sideband transition. Requires sufficiently narrow transition or high trapping frequency, $\omega_{\text{trap}}$, to satisfy the well-resolved sideband condition, $\omega_{\text{trap}} > \Gamma$.

In sideband cooling, we tune the laser to the red-sideband. To achieve this we need to be in the well-resolved sideband regime, $\omega_{\text{trap}} > \Gamma$. Using Sr, this is achieved by cooling on the intercombination line, ${}^1S_0 \rightarrow {}^3P_1$ which has a narrow linewidth, $\Gamma_{689} = 2\pi(7.4\text{ kHz})$. For alkali atoms, which do not have an intercombination line we need to find alternatives, see QIC.16. We can use the sideband spectrum to extract the temperature of the atom in a tweezer. From the property of harmonic oscillation wave function, the Rabi frequency to drive the red sideband transition from state $n$ to $n-1$ is proportional to $\sqrt{n}$, whereas the Rabi frequency to drive the blue sideband transition from state $n$ to $n+1$ is proportional to $\sqrt{n+1}$. As the excited state population is proportional to Rabi frequency squared, we obtain a ratio of the red to the blue sideband intensity of $\bar{n}/(\bar{n}+1)$ where $\bar{n}$ is the mean vibrational quantum number. From statistical physics, we can derive that

$$e^{-\hbar\omega_{\text{trap}}/k_B T} = \frac{\bar{n}}{\bar{n}+1} = \frac{I_{\text{red}}}{I_{\text{blue}}} .$$

Recent experiments, Young *et al*, Nature **588**, 466 (2020) measure $\bar{n} = 0.06$.

**Summary:** What do you need to be able to do?

1. Understanding the significant of $\Gamma$ in the context of stopping and cooling atoms.
2. Be able to calculate the stopping distance.
3. Explain the principle of sideband cooling.
4. Given input parameters such as laser wavelengths, powers and beam sizes, be able to estimate whether the condition for sideband cooling is satified.

[1] For Cs atoms, $m = 133 \cdot 1.66 \times 10^{-27}$ kg. Using $T = 293$ K, we find $u = 135\text{ ms}^{-1}$.