L4 Quantum information and computing (QIC) 2020-21

Lecture 13: Optical tweezers

December 21, 2020

Aims of Lecture 13: To derive an expression for the potential well created by a focused laser beam, and estimate how well an atom can be localised.

Introduction:

In QIC.12 we found that the **light-shift** of the ground state due to a laser characterised by a Rabi frequency Ω and a detuning Δ is

$$U = \frac{\hbar\Omega^2}{4\Delta} .$$

In this lecture we shall look at the potential well created by a focus laser beam, i.e. an **optical tweezer**.

Optical tweezer potential

The light shift calculation tells us that if we used red-detuned light ($\Delta < 0$), $U < 0$ and atoms are attracted to high intensity (maximum Rabi frequency).[1] In this case, a single focused laser beam can be used to create a potential well and trap an atom, see Fig. 1(left). A focused laser beam trap is known as an **optical tweezer**. A array of focused laser beams produced an atom array with spacing of a few microns which is convenient for individual laser addressing, see Fig. 1(right).

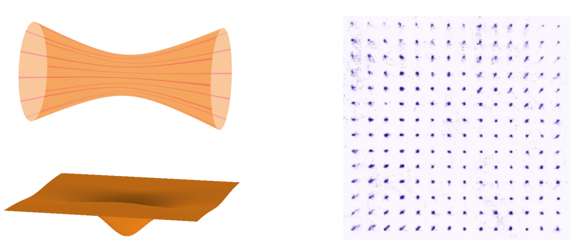

Figure 1: Left: Schematic of a focused laser beam (optical tweezer) showing the optical trapping potential below. Right: 14×14 array of single atoms (From pasqal).

[1] We can thinking of the atomic dipole as a driven oscillator. If we drive below resonance d and $\mathcal{E}$ are in-phase and $U = -d\mathcal{E} < 0$, whereas above resonance d and $\mathcal{E}$ are out-of-phase and $U = -(-d)\mathcal{E} > 0$.

The parameters particular to the atom are the transition wavelength between states $|\mathrm{g}\rangle$ and $|\mathrm{e}\rangle$, λ_0, the mass, m and the strength of the light-matter coupling which is proportional to the scattering rate, Γ, as Γ is that observable is related to the dipole matrix element $\hbar\Gamma = \mathcal{D}_0^2/[3\pi\epsilon_0(\lambda_0/2\pi)^3]$. The laser parameters that we can engineer are the laser power P_L the focus size w_0 and the detuning Δ. The electric field squared $\mathcal{E}_0^2 = 2\mathcal{I}_0/(\epsilon_0 c)$, where the intensity at the focus of a gaussian beam with a beam radius w_0 is $\mathcal{I}_0 = 2P_\mathrm{L}/(\pi w_0^2)$. Using

$$\begin{aligned}\Omega^2 &= \frac{\mathcal{D}_0^2\mathcal{E}_0^2}{\hbar^2} = \frac{\Gamma}{\hbar}3\pi\epsilon_0\left(\frac{\lambda_0}{2\pi}\right)^3\frac{2\mathcal{I}_0}{\epsilon_0 c} , \\ &= \Gamma^2\frac{\mathcal{I}_0}{2\mathcal{I}_\mathrm{s}} ,\end{aligned}$$

where $\mathcal{I}_\mathrm{s} = (\pi/3)hc\Gamma/\lambda_0^3$ is known as the **saturation intensity**, we can write the maximum trap depth as

$$U_0 = \frac{\hbar\Gamma}{2}\frac{P_\mathrm{L}}{\pi w_0^2\mathcal{I}_\mathrm{s}}\frac{\Gamma}{2\Delta} ,$$

As mentioned above the potential is negative for $\Delta < 0$ red detuning. In this case an atom is attracted to a region of high intensity such as the focus of the laser beam as illustrated in Fig. 1(left).[2] The equation for gaussian laser beams gives the full three dimensional form of the trapping potential

$$U(x,y,z) = -\frac{U_0}{1+z^2/z_\mathrm{R}^2}\mathrm{e}^{-2(x^2+y^2)/w^2} , \qquad (1)$$

where $w = w_0(1+z^2/z_\mathrm{R}^2)^{1/2}$ and $z_\mathrm{R} = \pi w_0^2/\lambda$ is the Rayleigh range. This potential is plotted in Fig. 1(left).

Atom localisation Close to the origin we can approximate the optical tweezer potential as harmonic. For the radial direction x (similar for y) we can write

$$U(x,0,0) = -U_0\mathrm{e}^{-2x^2/w_0^2} \approx -U_0(1 - 2x^2/w_0^2 + \ldots) .$$

[2] For red detuning—driving at a frequency below resonance—the field and the dipole oscillate in-phase $\boldsymbol{d}$ and $\boldsymbol{\mathcal{E}}$ are parallel and the energy $\langle U\rangle = -\langle \boldsymbol{d}\cdot\boldsymbol{\mathcal{E}}\rangle$ is negative.

To find the harmonic trapping frequency we equate the quadratic term with a harmonic oscillator potential

$$\tfrac{1}{2}m\omega_{\rm rad}^2 x^2 = U_0 2x^2/w_0^2 \ .$$

This gives a radial trapping frequency

$$\omega_{\rm rad} = \left(\frac{4U_0}{mw_0^2}\right)^{1/2} \ .$$

For the axial direction

$$U(0,0,z) = -U_0/(1+z^2/z_{\rm R}^2) \approx -U_0(1-z^2/z_{\rm R}^2) \ .$$

Equating the quadratic term to an axial harmonic potential

$$\tfrac{1}{2}m\omega_{\rm ax}^2 z^2 = U_0 z^2/w_0^2 \ .$$

gives

$$\omega_{\rm ax} = \left(\frac{2U_0}{mz_{\rm R}^2}\right)^{1/2} \ .$$

The minimum uncertainty in the position of the atom in the tweezer is given by the size of the harmonic oscillator ground state $a_0 = [\hbar/(m\omega_{\rm osc})]^{1/2}$. Note that if we want a_0 to match the standard deviation of the gaussian probability distribution (matching the Δx appearing in Heisenberg uncertainty relations) then we need to define $a_0 = [\hbar/(2m\omega_{\rm osc})]^{1/2}$.
Typical parameters for Rb atoms, $P_{\rm L} = 1$ mW, $w_0 = 1$ μm, $\lambda = 810$ nm, $U_0/k_{\rm B} = 0.6$ mK. Radial size (assuming we can cool to harmonic oscillator ground state) $a_0 = 33$ nm. For trap spacing of 3 μm uncertainty in position is 1%! An example for a Cs tweezer is shown in Fig. 2. The light-matter interaction is sensitive to the phase of the field which depends on position, $\phi_{\rm L} = kr$, so any positional uncertainty or fluctuation will lead to phase fluctuations which destroy the coherence of the light-matter interaction, see Workshop Problem 4.

Spontaneous scattering A potential problem with the optical tweezer is that the laser light can still excite the atom to state $|{\rm e}\rangle$ which destroys the qubit. To avoid this we use a large detuning $|\Delta| \gg \Omega$ but this makes the trap less deep so do we win? Using the steady-state solutions to the optical Bloch equation we found that the probability to be in the excited state is

$$P_{|{\rm e}\rangle} = \frac{\Omega^2/4}{\Delta^2+\Gamma^2/4+\Omega^2/2} \ ,$$

which for $|\Delta| \gg \Omega$ gives

$$P_{|{\rm e}\rangle} \approx \frac{\Omega^2}{4\Delta^2} \ .$$

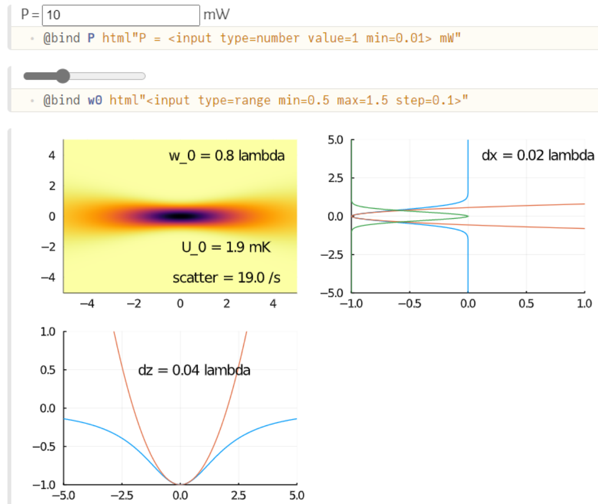

Figure 2: Heatmap of the trapping potential in xz or yz plane for Cs atoms with laser beam waist $w_0 = 0.8\lambda_0$, where $\lambda_0 = 852$ nm is the 5s–5p transition wavelength. For a laser power $P = 10$ mW and wavelength $\lambda = 1$ μm, we obtain a trap depth $U_0/k_{\rm B} = 2$ mK. The Pluto/Julia notebook to make this plot is on DUO.

The ratio of photon scattering rate $\Gamma P_{|{\rm e}\rangle}$ to the trap depth is $\Gamma/|\Delta|$ so we win by using a larger detuning but only if we can compensate the loss of trap depth by using more laser power. For Cs, a trap depth of order 2 mK scatters around 20 photons per second, see Fig. 2. To load the traps we need to make use of laser cooling. After loading we could lower the trap depth to lower the photon scattering.

Summary:

What do you need to be able to do?

1. Derive an expression for the trapping potential in 3D using gaussian beam optics.
2. Derive expressions for the spatial extent of atomic wave packet assuming it can be cooled to the motional ground state.
3. Estimate the population in the excited state and the rate of spontaneous scattering for particular trap parameters.