# SQL Query evaluation on Incomplete Databases with correctness guarantees

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## The general context

It is a fact of life that data we need to handle on an everyday basis is rarely complete. Handling incomplete information is one of the oldest topics in database research, we have a good theoretical understanding of what is needed and what it takes to produce correct results over incomplete databases. However most of those results have limited applications due to their high complexity. The standard way of answering queries on incomplete databases is to compute certain answers: those that do not depend on the interpretation of unknown data.

Computing certain answers is coNP-hard for most reasonable semantics [2], however it is not yet a reason for undesirable behavior. Indeed one could expect from an evaluation over incomplete databases to either miss some certain answers, or returning some that are not, but not both at the same time.

The problem with SQL is that it produces both kinds of errors. [5]

## The research problem

My internship focuses on creating an algorithm based upon SQL evaluation in order to ensure that only certain answers are return. We will focus on rewriting the queries before proceed with there evaluation. A translation on relational algebra having this property is already available [5], and is our main source of work.

The translation on relational algebra has proven to be efficient, however DBMS do not translate queries in relational algebra. Moreover relational algebra is based on sets semantic while SQL databases are based on bags. The question that naturally arises is can we find a direct translation from SQL to SQL which ensure correctness regarding bags semantic. Moreover SQL scheme might contain constraints such as as keys and foreign keys that we want to take in account.

The aim of my internship is to propose a direct translation from SQL to SQL as efficient as possible which has correctness guarantee, and implement it.

## Your contribution

*Naive evaluation has correctness guarantees for positive relational algebra* [4]. This result implies that false positives come from the negation in queries. In order to deal with negation, the idea is to compute an over-approximation of the correct answers. However computing such a set can be really expensive, so we tried to avoid it as much as possible and to make it fast when we had to. A simple static analyse of the query and database scheme allowed us to deal with constraints such as keys and foreign keys.

The contributions of my internship are the following: a syntax and a semantic for SQL evaluation (section 1.3); the SQL to SQL translation (section 2); methods to help the planner while computing disjunctive queries answers (section A.2,4); a Postgres extension implementing the translation.

### Arguments supporting its validity

The translation is proven to have correctness guarantee on a semantic of SQL evaluation, however such semantic has to be prove right. We also have to work on a proper definition of certain answers for SQL Nulls in order to formally extends our results to commercial DBMS.

First results show that the various methods propose increase performances, and it's proven that those transformations does not change answers. Moreover many of the methods can be applied to optimize more general query with only a few precautions.

### Summary and future work

During my internship, i have proposed a direct translation from SQL to SQL with correctness guarantee, and i was able to implement a proof of concept which is an Extension to Postgres. This extension is able to rewrite the query and optimize it thanks to the scheme constraints. Our results confirm the feasibility as our algorithm take at most 6 times longer than the initial query to compute a subset of certain answers.

On a practical point of view, we miss only a few steps in order to implement a new functionality to SQL DBMS. The extension have to be link with the parser, the algorithm used to rewrite the queries have to be improve. Finally the methods we use to help the planner should be implemented directly in DBMS as most of them can improve the computation of general disjunctive queries.

On a theoretical point of view, one can always improve the translation of a query in a better/-faster approximation of certain answers. Moreover we have to figure out and formalize what are certain answers on SQL nulls.

Relational databases are just a small part of the issue, indeed dealing with missing information is a problem in every database, and it might be interesting to see which results can be used in other database systems.

### Acknowledgments

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Last but not least thanks to Leonid Libkin for the opportunity, i would not be able to express how much i am grateful for his availability and help throughout every step of my internship.

## 1 Preliminaries

Evaluation of SQL queries over incomplete databases is a well known problem since 1970(s), however people commonly say "You can never trust the answers you get from a database with nulls." [3]. Indeed, the way SQL handle incomplete information still produce counter-intuitive and just plain incorrect answers.

### 1.1 SQL behavior

When we evaluate the query 2 on the database 1 we obtain $[\![ord2, ord3]\!]$. However this answer does not fit the notion of correctness, indeed $NULL$ can be interpret as $ord2$ or $ord3$, meaning that none of them is a correct answer.

Such "false-positive" answers occur in real life queries [5]. Then it is natural to find a way to prevent them.

Figure 1: SQL Database

ORDERS

| ord_id | ord_date |
|---|---|
| ord1 | 2015-06-12 |
| ord2 | 2015-07-11 |
| ord3 | 2015-07-20 |

PAYMENTS

| pay_id | ord_id | pay_date |
|---|---|---|
| pay1 | ord1 | 2015-06-14 |
| pay2 | NULL | 2015-07-25 |

Figure 2: SQL Query

```
SELECT ord_id FROM Orders
WHERE NOT EXISTS
        (SELECT * FROM Payments WHERE Payments.ord_id = Orders.ord_id)
```

## 1.2 Correctness

In order to give a formal definition of correctness we need to give a few definitions. Much of the following is standard in the literature on databases with incomplete information, see, e.g., [[1], [8], [4], [7], [6]]. We consider incomplete databases with nulls interpreted as missing information. Databases are populated by two types of elements: constants and nulls, coming from countably infinite sets denoted by $Const$ and $Null$, respectively. Elements in $Null$ are denoted $\perp_t$, where $t$ is a stamp, and behave accordingly to the marked nulls semantics. We call a valuation of nulls on an incomplete database $D$ a map $v : Null(D) \to Const$ assigning a constant value to each null. According to this definition it's easy to understand that each element $\perp_t$ which might appears multiple times in the database will all be assign to the same constant by a valuation.

A typical definition of certain answers are those independent of the interpretation of missing information denoted $cert(Q, D) = \bigcap\{Q(v(D))|v \text{ is a valuation}\}$ where $Q$ is a query and $D$ a database. Such definition is a bit too restrictive as tuples with nulls can not be returned. A more general notion called certain answers with nulls, noted $cert_\perp(Q, D)$ is defined as :

$$cert_\perp(Q, D) = [\![a^n | \forall v, v(a) \in^n Q(v(D))]\!]$$

Those definition are closely related as if we remove tuples with null from $cert_\perp(Q, D)$ we obtain $cert(Q, D)$.

We say that a query evaluation algorithm, $Eval$, has correctness guarantees for a query $Q$ if for every database $D$ it returns a subset of $cert_\perp(Q, D)$.

We will focus on having an evaluation procedure for the basic fragment of SQL (i.e., first-order queries) which have correctness guarantee. Our evaluation algorithm will translate a query $Q$ in $Q'$ such that

$$\forall D, \forall a, (a \in^n Eval_{SQL}(Q', D) \implies a \in^n cert_\perp(Q, D))$$

.

## 1.3 SQL Query syntax

In this subsection we introduce a syntax in order to describe the basic fragment of SQL. Note that this is a personal choice as there is no real standard in the literature.

**Definition.** *We denote the set of well formed query without null test by* $[\![SQL]\!]$
*We denote the Set of well formed query by* $[\![SQL]\!]_\perp$

**Definition.** *Let's a Select query $Q \in [\![SQL]\!]_\perp$ a tuple $(\Sigma, R, H, P)$ such that*

$\Sigma \subseteq \{r_i.a_j | \exists r_i \in R \wedge \exists a_j \in r_i\}$
$P \subseteq \{p_i = r_j.a_k | r_j \notin R\}$ *a set of external parameter.*
$R$ *a set of relation*
$H$ *belongs to the following grammar.*

$$\begin{aligned} H ::= & \ r_i.a_j = c_k \mid r_i.a_j \neq c_k \mid r_i.a_j = r_k.a_l \mid r_i.a_j \neq r_k.a_l \mid r_i.a_j = p_k \mid r_i.a_j \neq p_k \mid \\ & r_i.a_j > c_k \mid r_i.a_j < c_k \mid r_i.a_j > r_k.a_l \mid r_i.a_j < r_k.a_l \mid r_i.a_j > p_k \mid r_i.a_j < p_k \mid \\ & null(r_i.a_j) \mid const(r_i.a_j) \mid null(p_i) \mid const(p_i) \mid \\ & exists(Q) \mid notexists(Q) \mid H \wedge H \mid H \vee H \end{aligned}$$

The query in 2 would be express as

$$(ord\_id, Orders, notexists((*, Payments, Payments.ord\_id = Order.ord\_id, \emptyset)), \emptyset)$$

**Definition.** *Let $x$ a labeled-tuple we denote by $x[a]$ the value of the column labeled as $a$ in the tuple.*
*We denote $(\Sigma, R, H, P)[x]$ the query $(\Sigma, R, H, P \cup x)$.*
*We denote $(\Sigma, R, H, P)_*$ the query $(*, R, H, P)$.*

$$\sigma_\Sigma(x) = (x[r_i.a_i] | r_i.a_i \in \Sigma)$$

$$\sigma_*(x) = x$$

$$\sigma_\Sigma(B) = [\![y^n | n = \sum_{x \in \{z | z \in \{B\} \wedge \sigma_\Sigma(z) = y\}} B(x)]\!]$$

**Definition.** *Let $R$ be a set of relations then $R^\times$ denote the bag coming from the cartesian product of $R$.*

$$R^\times = \prod_{r \in R} r$$

**Proposition 1.** *Let $R$ be a set of relation, and $z$ a tuple then:*

$$(\forall v \text{ valuation}, v(R^\times)(v(z)) \geq k) \Leftrightarrow R^\times(z) \geq k$$

This property is ensure due to the fact that a valuation $v$ have to evaluate every marked null with the same stamp by the same constant that may not already appear in the database. (Here we assume that each attribute as an infinite number of possible value). The proof is straightforward but require a lot of notation, then it won't appear in this report. However this property does not hold for SQL nulls, which is one of the reason we choose to work with marked nulls.

## 1.4 SQL Evaluation semantic

In this sub-section we propose a reasonable semantic for the SQL evaluation. Keep in mind that it is only a reasonable one as it has yet to be proven that it fits the SQL evaluation. Such work have to be done either thanks to a deep code analysis, or more realistically an high number of practical tests.

**Definition.**

$$
\begin{aligned}
Eval_{SQL}((\Sigma, R, \emptyset, P), D) &= \sigma_\Sigma(R^\times)\\
Eval_{SQL}((\Sigma, R, H_1 \wedge H_2, P), D) &= \sigma_\Sigma(Eval_{SQL}((*, R, H_1, P), D) \cap Eval_{SQL}((*, R, H_2, P), D))\\
Eval_{SQL}((\Sigma, R, H_1 \vee H_2, P), D) &= \sigma_\Sigma(Eval_{SQL}((*, R, H_1, P), D) \cup Eval_{SQL}((*, R, H_2, P), D))\\
Eval_{SQL}((\Sigma, R, null(r_i.a_i), P), D) &= \sigma_\Sigma([\![x^n | R^\times(x) = n \wedge \exists t, x[r_i.a_i] = \bot_t]\!])\\
Eval_{SQL}((\Sigma, R, const(r_i.a_i), P), D) &= \sigma_\Sigma([\![x^n | R^\times(x) = n \wedge \forall t, x[r_i.a_i] \neq \bot_t]\!])\\
Eval_{SQL}((\Sigma, R, null(p_i), P), D) &= \sigma_\Sigma([\![x^n | R^\times(x) = n \wedge \exists t, P[p_i] = \bot_t]\!])\\
Eval_{SQL}((\Sigma, R, const(p_i), P), D) &= \sigma_\Sigma([\![x^n | R^\times(x) = n \wedge \forall t, P[p_i] \neq \bot_t]\!])\\
Eval_{SQL}((\Sigma, R, r_i.a_i = c_i, P), D) &= \sigma_\Sigma([\![x^n | R^\times(x) = n \wedge x[r_i.a_i] = c_i]\!])\\
Eval_{SQL}((\Sigma, R, r_i.a_i = r_j.a_j, P), D) &= \sigma_\Sigma([\![x^n | R^\times(x) = n \wedge x[r_i.a_i] = x[r_j.a_j]]\!])\\
Eval_{SQL}((\Sigma, R, r_i.a_i = p_i, P), D) &= \sigma_\Sigma([\![x^n | R^\times(x) = n \wedge x[r_i.a_i] = P[p_i]]\!])\\
Eval_{SQL}((\Sigma, R, r_i.a_i \neq c_i, P), D) &= \sigma_\Sigma([\![x^n | R^\times(x) = n \wedge x[r_i.a_i] \neq c_i \wedge \forall t, x[r_i.a_i] \neq \bot_t]\!])\\
Eval_{SQL}((\Sigma, R, r_i.a_i \neq r_j.a_j, P), D) &= \sigma_\Sigma([\![x^n | R^\times(x) = n \wedge x[r_i.a_i] \neq x[r_j.a_j] \wedge \forall t, x[r_i.a_i] \neq \bot_t \wedge \forall t, x[r_j.a_j] \neq \bot_t]\!])\\
Eval_{SQL}((\Sigma, R, r_i.a_i \neq p_i, P), D) &= \sigma_\Sigma([\![x^n | R^\times(x) = n \wedge x[r_i.a_i] \neq P[p_i] \wedge \forall t, x[r_i.a_i] \neq \bot_t \wedge \forall t, P[p_i] \neq \bot_t]\!])\\
Eval_{SQL}((\Sigma, R, r_i.a_i > c_i, P), D) &= \sigma_\Sigma([\![x^n | R^\times(x) = n \wedge x[r_i.a_i] > c_i]\!])\\
Eval_{SQL}((\Sigma, R, r_i.a_i > r_j.a_j, P), D) &= \sigma_\Sigma([\![x^n | R^\times(x) = n \wedge x[r_i.a_i] > x[r_j.a_j]]\!])\\
Eval_{SQL}((\Sigma, R, r_i.a_i > p_i, P), D) &= \sigma_\Sigma([\![x^n | R^\times(x) = n \wedge x[r_i.a_i] > P[p_i]]\!])\\
Eval_{SQL}((\Sigma, R, r_i.a_i < c_i, P), D) &= \sigma_\Sigma([\![x^n | R^\times(x) = n \wedge x[r_i.a_i] < c_i]\!])\\
Eval_{SQL}((\Sigma, R, r_i.a_i < r_j.a_j, P), D) &= \sigma_\Sigma([\![x^n | R^\times(x) = n \wedge x[r_i.a_i] < x[r_j.a_j]]\!])\\
Eval_{SQL}((\Sigma, R, r_i.a_i < p_i, P), D) &= \sigma_\Sigma([\![x^n | R^\times(x) = n \wedge x[r_i.a_i] < P[p_i]]\!])\\
Eval_{SQL}((\Sigma, R, exists(Q), P), D) &= \sigma_\Sigma([\![x^n | R^\times(x) = n \wedge Eval_{SQL}(Q[x], D) \neq \emptyset]\!])\\
Eval_{SQL}((\Sigma, R, notexists(Q), P), D) &= \sigma_\Sigma([\![x^n | R^\times(x) = n \wedge Eval_{SQL}(Q[x], D) = \emptyset]\!])
\end{aligned}
$$

## 2 Evaluation with correctness guarantee

In order to compute a subset of certain answers with nulls for a Query $Q \in [\![SQL]\!]$, we translate it in $Q^+ \in [\![SQL]\!]_\bot$ where the evaluation of $Q^+$ approximates certain answers. Consider the query such that $Q_1 = (\Sigma, R, notexists(Q_2), P)$ we want a translation that ensure that we only obtain certain answers, then we have to be sure to also have a translation that guarantee to capture at least all certain answer in order to translate $Q_2$, otherwise we might create false positives. Let note $Q^?$ such translation.

Figure 3: Translation $Q \to (Q^+, Q^?)$

$$
\begin{aligned}
(\Sigma, R, H, P)^+ &\to (\Sigma, R, H^*, P)\\
(\Sigma, R, H, P)^? &\to (\Sigma, R, H^{**}, P)
\end{aligned}
$$

Figure 4: Translation $H \rightarrow H^*$

$$
\begin{aligned}
(H_1 \wedge H_2)^* &\rightarrow H_1^* \wedge H_2^* \\
(H_1 \vee H_2)^* &\rightarrow H_1^* \vee H_2^* \\
(r_i.a_i = c_i)^* &\rightarrow r_i.a_i = c_i \\
(r_i.a_i \neq c_i)^* &\rightarrow r_i.a_i \neq c_i \wedge const(r_i.a_i) \\
(r_i.a_i > c_i)^* &\rightarrow r_i.a_i > c_i \\
(r_i.a_i < c_i)^* &\rightarrow r_i.a_i < c_i \\
(r_i.a_i = r_j.a_j)^* &\rightarrow r_i.a_i = r_j.a_j \\
(r_i.a_i \neq r_j.a_j)^* &\rightarrow r_i.a_i \neq r_j.a_j \wedge const(r_i.a_i) \wedge const(r_j.a_j) \\
(r_i.a_i > r_j.a_j)^* &\rightarrow r_i.a_i > r_j.a_j \\
(r_i.a_i < r_j.a_j)^* &\rightarrow r_i.a_i < r_j.a_j \\
(r_i.a_i = p_j)^* &\rightarrow r_i.a_i = p_j \\
(r_i.a_i \neq p_j)^* &\rightarrow r_i.a_i \neq p_j \wedge const(r_i.a_i) \wedge const(p_j) \\
(r_i.a_i > p_j)^* &\rightarrow r_i.a_i > p_j \\
(r_i.a_i < p_j)^* &\rightarrow r_i.a_i < p_j \\
exists(Q)^* &\rightarrow exists(Q^+) \\
notexists(Q)^* &\rightarrow notexists(Q^?)
\end{aligned}
$$

Figure 5: Translation $H \rightarrow H^{**}$

$$
\begin{aligned}
(H_1 \wedge H_2)^{**} &\rightarrow H_1^{**} \wedge H_2^{**} \\
(H_1 \vee H_2)^{**} &\rightarrow H_1^{**} \vee H_2^{**} \\
(r_i.a_i = c_i)^{**} &\rightarrow r_i.a_i = c_i \vee null(r_i.a_i) \\
(r_i.a_i \neq c_i)^{**} &\rightarrow r_i.a_i \neq c_i \\
(r_i.a_i > c_i)^{**} &\rightarrow r_i.a_i > c_i \vee null(r_i.a_i) \\
(r_i.a_i < c_i)^{**} &\rightarrow r_i.a_i < c_i \vee null(r_i.a_i) \\
(r_i.a_i = r_j.a_j)^{**} &\rightarrow r_i.a_i = r_j.a_j \vee null(r_i.a_i) \vee null(r_j.a_j) \\
(r_i.a_i \neq r_j.a_j)^{**} &\rightarrow r_i.a_i \neq r_j.a_j \\
(r_i.a_i > r_j.a_j)^{**} &\rightarrow r_i.a_i > r_j.a_j \vee ((null(r_i.a_i) \vee null(r_j.a_j)) \wedge r_i.a_i \neq r_j.a_j) \\
(r_i.a_i < r_j.a_j)^{**} &\rightarrow r_i.a_i < r_j.a_j \vee ((null(r_i.a_i) \vee null(r_j.a_j)) \wedge r_i.a_i \neq r_j.a_j) \\
(r_i.a_i = p_j)^{**} &\rightarrow r_i.a_i = p_j \vee null(r_i.a_i) \vee null(p_j) \\
(r_i.a_i \neq p_j)^{**} &\rightarrow r_i.a_i \neq p_j \\
(r_i.a_i > p_j)^{**} &\rightarrow r_i.a_i > p_j \vee ((null(r_i.a_i) \vee null(p_j)) \wedge r_i.a_i \neq p_j) \\
(r_i.a_i < p_j)^{**} &\rightarrow r_i.a_i < p_j \vee ((null(r_i.a_i) \vee null(p_j)) \wedge r_i.a_i \neq p_j) \\
exists(Q)^{**} &\rightarrow exists(Q^?) \\
notexists(Q)^{**} &\rightarrow notexists(Q^+)
\end{aligned}
$$

The key to understand this algorithm, is that $Q^?$ actually compute every answers that might be a certain answer. Indeed as soon as there exist a valuation such that $x \in Q(v(D))$ then $x \in Q^?(D)$. This property is ensure by the fact that the equality is replace with a disjunction checking if the attributes are nulls. As soon as one of the attribute is null then a valuation might exists which validate the equality. Conversely $Q^+$ ensure that difference are verify for every valuation, by replacing them with a conjunction ensuring that no nulls are involve. Then it's easy to understand that we can demonstrate that: if we assume that $Q^+$ evaluation only return certain answers, then $Q^?$ return at least all of them. And if we assume that $Q^?$ computes at least every certain answers, then $Q^+$ will only return certain answers.

**Lemma 1.**

$$\forall Q \in [\![SQL]\!], \sigma_\Sigma(cert_\perp(Q_*, D)) = cert_\perp(Q, D)$$

*Proof.*

$$\begin{aligned}
&Q = (\Sigma, R, H, P)\\
x \in^n \sigma_\Sigma(cert_\perp(Q_*, D)) \Rightarrow &\sum_{z \in \{y | y \in cert_\perp(Q_*, D) \wedge \sigma_\Sigma(y) = x\}} cert_\perp(Q*, D)(z) \geq n\\
&Moreover\\
\forall g, cert_\perp(Q_*, D) \subseteq &\, Eval(Q_*, g(D))\\
&Then\\
x \in^n \sigma_\Sigma(cert_\perp(Q_*, D)) \Rightarrow \forall g, &\sum_{z \in \{y | y \in Eval(Q_*, g(D)) \wedge \sigma_\Sigma(y) = x\}} Eval(Q_*, g(D))(z) \geq n\\
&\Rightarrow \forall g, Eval(Q, g(D))(x) \geq n\\
&\Rightarrow x \in^n cert_\perp(Q, D)
\end{aligned}$$

$$\begin{aligned}
&Q = (\Sigma, R, H, P)\\
x \notin \sigma_\Sigma(cert_\perp(Q_*, D)) \Rightarrow &\sum_{z \in \{y | y \in cert_\perp(Q_*, D) \wedge \sigma_\Sigma(y) = x\}} cert_\perp(Q_*, D)(z) = 0\\
&\Rightarrow \forall y, \sigma_\Sigma(y) \neq x \vee cert_\perp(Q_*, D)(y) = 0\\
&\Rightarrow \forall y, \sigma_\Sigma(y) \neq x \vee \exists v, v(y) \notin Eval_{SQL}(Q_*, v(D))\\
&\Rightarrow \exists v, v(x) \notin \sigma_\Sigma(Eval_{SQL}(Q_*, v(D)))\\
&\Rightarrow \exists v, v(x) \notin Eval_{SQL}(Q, D)\\
&\Rightarrow x \notin cert_\perp(Q, D)
\end{aligned}$$

□

**Proposition 2.**

$$\forall Q \in [\![SQL]\!], Eval_{SQL}(Q^+, D) \subseteq cert_\perp(Q, D)$$

**Proposition 3.**

$$\forall Q \in [\![SQL]\!], cert_\perp(Q, D) \subseteq Eval_{SQL}(Q^?, D)$$

*Proof.* The proof of (2) is made by induction over the query $Q$ assuming only (3).

We only detail critical case a more complete proof can be found in appendix.

$$
\begin{aligned}
&Q = (\Sigma, R, notexists(Q'), P)\\
&\quad x \in^n Eval_{SQL}(Q^+, D) \Rightarrow \sum_{z \in \{y | y \in Eval_{SQL}(Q_*^+, D) \wedge \sigma_\Sigma(y) = x\}} Eval_{SQL}(Q_*^+, D)(z) \geq n\\
&\qquad\qquad Moreover\\
&Eval_{SQL}(Q_*^+, D)(z) \geq k \Rightarrow Eval_{SQL}((*, R, notexists(Q')^*, P), D)(z) \geq k\\
&\qquad\qquad \Rightarrow Eval_{SQL}((*, R, notexists(Q'^?), P), D)(z) \geq k\\
&\qquad\qquad \Rightarrow R^\times(z) \geq k \wedge Eval_{SQL}(Q'[z]^?, D) = \emptyset\\
&\qquad\qquad \Rightarrow R^\times(z) \geq k \wedge cert_\perp(Q'[z], D) = \emptyset\\
&\qquad\qquad \Rightarrow R^\times(z) \geq k \wedge \forall h, \forall w, h(w) \notin Eval_{SQL}(Q'[h(z)], h(D))\\
&\qquad\qquad \Rightarrow R^\times(z) \geq k \wedge \forall h, Eval_{SQL}(Q'[h(z)], h(D)) = \emptyset\\
&\qquad\qquad \Rightarrow R^\times(z) \geq k \wedge \forall h, h(z) \in Eval_{SQL}((*, R, notexists(Q'), h(P)), h(D))\\
&\qquad\qquad \Rightarrow cert_\perp(Q_*, D)(z) \geq k\\
&\qquad\qquad Then\\
&\quad x \in^n Eval_{SQL}(Q^+, D) \Rightarrow \sum_{z \in \{y | y \in Eval_{SQL}(Q_*^+, D) \wedge \sigma_\Sigma(y) = x\}} cert_\perp(Q_*, D)(z) \geq n\\
&\qquad\qquad \Rightarrow x \in^n cert_\perp(Q, D)
\end{aligned}
$$

□

*Proof.* The proof of (3) is made by induction over the query $Q$ assuming only (2).
We only detail critical case a more complete proof can be found in appendix.

$$
\begin{aligned}
&Q = (\Sigma, R, notexists(Q'), P)\\
&\quad x \in^n cert_\perp(Q, D) \Rightarrow \sum_{z \in \{y | y \in cert_\perp(Q_*, D) \wedge \sigma_\Sigma(y) = x\}} cert_\perp(Q_*, D)(z) \geq n\\
&\qquad\qquad Moreover\\
&cert_\perp(Q_*, D)(z) \geq k \Rightarrow R^\times(z) \geq k \wedge \forall h, h(z) \in Eval_{SQL}((*, R, notexists(Q'), h(P)), h(D))\\
&\qquad\qquad \Rightarrow R^\times(z) \geq k \wedge \forall h, Eval_{SQL}(Q'[h(z)], h(D)) = \emptyset\\
&\qquad\qquad \Rightarrow R^\times(z) \geq k \wedge cert_\perp(Q'[z], D) = \emptyset\\
&\qquad\qquad \Rightarrow R^\times(z) \geq k \wedge Eval(Q'[z]^+, D) = \emptyset\\
&\qquad\qquad \Rightarrow R^\times(z) \geq k \wedge z \in Eval_{SQL}((*, R, notexists(Q'^+), P), D)\\
&\qquad\qquad \Rightarrow R^\times(z) \geq k \wedge z \in Eval_{SQL}((*, R, notexists(Q')^{**}, P), D)\\
&\qquad\qquad \Rightarrow Eval_{SQL}((*, R, notexists(Q'), P)^?, D)(z) \geq k\\
&\qquad\qquad Then\\
&\quad x \in^n cert_\perp(Q, D) \Rightarrow \sum_{z \in \{y | y \in posi_\perp(Q_*, D) \wedge \sigma_\Sigma(y) = x\}} Eval_{SQL}(Q_*^?, D)(z) \geq n\\
&\qquad\qquad \Rightarrow x \in^n Eval_{SQL}(Q^?, D)
\end{aligned}
$$

□

We have proven that the evaluation algorithm such that we translate a query $Q$ in $Q^+$ and then evaluate $Q^+$ with standard SQL semantics has *correctness guarantee*. Indeed we have even

proven that such translation is valid as long as $Q^+$ computes a subset of certain answers and $Q^?$ computes at least all of them. It's easy to understand that work may be done in order to improve the over approximation of the certain answers without any loose.

# 3 Removing redundant null check

The evaluation of a query $Q^+$ which come from our translation may have a huge cost, as most DBMS such as Postgres do not behave well with disjunction. In our translation we introduce disjunctions however some of them might be redundant.

**Definition.** *For each Query and nested Query we maintain a set of attributes that have to be not null in order for the query to be true resp. false we denote this set $\bot^T_Q$ resp.$\bot^F_Q$ without taking null check in account.*

$$x \in Eval_{SQL}(Q_*, D) \Rightarrow \forall r_i.a_i \in \bot^T_Q, x[r_i.a_i] \neq \bot$$

$$x \notin Eval_{SQL}(Q_*, D) \Rightarrow \forall r_i.a_i \in \bot^F_Q, x[r_i.a_i] \neq \bot$$

*(see appendix for a proper inductive definition, which show how to compute those).*

**Definition.** *For a query Q we also compute the set of constraint which have to be true resp.false , in order for the tuples to be returned. We denote this set $nested^+(Q)$ resp. $nested^-(Q)$.*

$$x \in Eval_{SQL}(Q, D) \Rightarrow \forall c \in nested^+(Q), x \vdash c$$

$$x \in Eval_{SQL}(Q, D) \Rightarrow \forall c \in nested^-(Q), x \vdash \neg c$$

*(see appendix for a proper inductive definition, which show how to compute those).*

Now we can offer a translation $Q \to Q^\bot_Q$

Figure 6: Translatation $(H, Q) \to H^\bot_Q$

$$
\begin{aligned}
(\Sigma, R, H \vee null(r_i.a_i), P)^\bot_Q &\to (\Sigma, R, H^\bot_Q, P) \\
&\text{if } \exists Q' \in nested^+(Q), (H \vee null(r_i.a_i)) \in nested(Q'), r_i.a_i \in \bot^T_{Q'} \\
&\text{if } \exists Q' \in nested^-(Q), (H \vee null(r_i.a_i)) \in nested(Q'), r_i.a_i \in \bot^F_{Q'} \\
(\Sigma, R, H \vee null(p_i), P)^\bot_Q &\to (\Sigma, R, H^\bot_Q, P) \\
&\text{if } \exists Q' \in nested^+(Q), (H \vee null(p_i)) \in nested(Q'), p_i \in \bot^T_{Q'} \\
&\text{if } \exists Q' \in nested^-(Q), (H \vee null(p_i)) \in nested(Q'), p_i \in \bot^F_{Q'} \\
(\Sigma, R, H \wedge const(r_i.a_i), P)^\bot_Q &\to (\Sigma, R, H^\bot_Q, P) \\
&\text{if } \exists Q' \in nested^+(Q), (H \wedge const(r_i.a_i)) \in nested(Q'), r_i.a_i \in \bot^T_{Q'} \\
&\text{if } \exists Q' \in nested^-(Q), (H \wedge const(r_i.a_i)) \in nested(Q'), r_i.a_i \in \bot^F_{Q'} \\
(\Sigma, R, H \wedge const(p_i), P)^\bot_Q &\to (\Sigma, R, H^\bot_Q, P) \\
&\text{if } \exists Q' \in nested^+(Q), (H \wedge const(p_i)) \in nested(Q'), p_i \in \bot^T_{Q'} \\
&\text{if } \exists Q' \in nested^-(Q), (H \wedge const(p_i)) \in nested(Q'), p_i \in \bot^F_{Q'}
\end{aligned}
$$

To give an intuition to understand why such translation preserve the evaluation, one have to understand that in order for a tuple to be return each sub-conditions have to be fulfill. Then if some part of the query require an attribute to be a constant, we know that this attribute can not be null in the whole query. Then we are allowed to remove any null test as they can not be involved in the computation of a valid tuple. (Actually it is a bit more complicated due to the presence of disjunction but that's the main idea).

**Proposition 4.**

$$\forall Q \in [\![SQL]\!]_{\perp} Eval_{SQL}(Q, D) = Eval_{SQL}(Q^{\perp}_{Q}, D)$$

*Proof.* In order to prove the property we have to introduce an analogue query translation when we assume that the query is false. Such translations would verify:

**Lemma 2.**

$$\forall Q, \forall Q' \in nested^{-}(Q), Eval(Q, D) = Eval(Q^{\perp^F}_{Q'}, D)$$

**Lemma 3.**

$$\forall Q, \forall Q' \in nested^{+}(Q), Eval(Q, D) = Eval(Q^{\perp}_{Q'}, D)$$

Then we can prove inductively that assuming 2, 3 holds and assuming 3, 2 holds. (see appendix for a proper inductive definitions,and a full proof). Then as prop 4 is only a restriction of lemma 2 it also holds. □

This rewriting has prove to be efficient, as it reduce the number of disjunction is the query moreover it allow us to take care of SQL constraint such as NOT NULL attributes, indeed if we add those attributes to $\perp^{T}_{Q}$ every null check on those attributes will be remove.

# 4 Query Optimization

In this section we offer rewriting methods that help the planner computing disjunctive query in a more efficient way. Such optimization rely on already existing DBMS like Postgres, those DBMS does not implement a marked nulls version yet, that is why we explain how our results can be used in the case of SQL nulls.

## 4.1 SQL null

The semantic usually used to deal with SQL nulls is that there should be no distinction when applying a marking function on the database nulls, and when applying a marking function on the result of a query. Formally, let $M$ be a marking function which mark each null value with a different stamp. Then there should exist an isomorphism between : $Q(M(D))$ and $M(Q(D))$. However it's not always the case if we consider the SQL-fragment we offer in our syntax.

Moreover as mention before the property : Let $R$ be a set of relation, and $z$ a tuple then:

$$(\forall v \text{ valuation}, v(R^{\times})(v(z)) \geq k) \Leftrightarrow R^{\times}(z) \geq k$$

does not hold. Indeed Consider a relation $r = [\![\perp; \perp]\!]$ then $R^{\times}(\perp) \geq 2$ where as if $v(r) = [\![2; 3]\!]$ then $v(r)(v(\perp)) < 2$ Each SQL nulls can be evaluated differently by the valuation $v$. Then certain answers with nulls of a query $Q = (*, r, TRUE, \emptyset)$ would be $[\![\perp]\!]$ which does not seem to be satisfactory. A proper definition has not been found yet, and we are currently working on it.

We will only consider a fragment where the isomorphism holds. And we will compute the certain answers as if nulls in the answers where marked nulls, and that the multiplicity of them where the sum of each marked nulls. Just a few change to the translation (details in the appendix) allow us to have a valid evaluation algorithm.

## 4.2 Split the query

The main blow-up in computation time come from the fact that most DBMS give up using hash-join, as soon as they meet a disjunction, and choose to use nested-loop. In order to force the planner into using hash-join the way they should, we split the query in the various form it may have due to disjunction. Formally, the negative sub queries's conditions of a query $Q$ are put in DNF, and each part are evaluate separately. We are aware that such algorithm have a exponential complexity however, it work well in practice.

**Proposition 5.** *Let* $Q = (\Sigma, R, notexists((\Sigma', R', (l_1 \vee l_2) \wedge H, P'), P)$

$Eval_{SQL}(Q, D) = Eval_{SQL}((\Sigma, R, notexists(\Sigma', R', (l_1 \wedge H), P') \wedge notexists(\Sigma', R', (l_2 \wedge H), P'), P), D)$

### 4.3 Optimize FROM relations

We can avoid computing the whole bag $R'^{\times}$. First we build the set of relation that are not linked to an upper-level query by any of there attributes denotes $R'_s$. Then we can rewrite the query $Q'$.

**Proposition 6.** *Let* $Q = (\Sigma, R, notexists((\Sigma', R', H_r \wedge H, P'), P)$ *where* $H_r$ *denote every conditions which are on attributes of* $r$ *and such that* $r \in R'_s$ *then :*

$$Eval_{SQL}(Q, D) = Eval_{SQL}((\Sigma, R, notexists(\Sigma', R' \setminus r, H \wedge exists(*, r, H_r, P'), P'), P), D)$$

We have to add an exists sub-query in order to check if the selection over the relation $r$ is not empty. Indeed if the selection is empty then the Cartesian product $R'\times$ will be empty.

This methods help the DBMS because, it won't have to work on rows that are useless. Indeed as we are in a not exists condition, we only care about having one represent for each tuple that might be linked with the upper-level query.

### 4.4 Improve computation

The methods propose in this sub-section is the most efficient, when the tuples compute fit in the memory. Indeed it offers the fastest way to compute the query $Q^?$ however, it really computes each of them and have to store them. As soon as the Database involved does not fit in the ram memory, this method will be really expensive as each computation will involve a slow write on the hard drive. We won't be able to present the property formally here as it uses a fragment of SQL which is not supported in our syntax ie. (UNION ALL). The idea is to replace disjunctive query with UNION ALL, as soon as $H = l_1 \vee l_2$ verify the property such that:
$(l_1 \implies \neg l_2) \wedge (l_2 \implies \neg l_1)$ then we have :

$$Eval_{SQL}((\Sigma, R, D, P), D) = Eval_{SQL}((\Sigma, R, l_1, P), D) \text{ UNION ALL } Eval_{SQL}((\Sigma, R, l_2, P), D)$$

## 5 Results

Every result presented here are based on TCP-H database instance, in this report we present a simplified query coming from TCP-H. TCP-H database schema guarantee that $o\_orderkey, p\_partkey, l\_orderkey$ are not null attributes. Time were obtain on Postgres DBMS on a 10 Go instance with 0.5% of nulls randomly spread among each null-able attributes. Each step of the rewriting where done thanks to our Postgres Extension, Queries obtain after each steps can be found in appendix.

Figure 7: Initial SQL Query

```
SELECT o_orderkey
FROM orders
WHERE NOT EXISTS(
        SELECT *
        FROM lineitem, part
        WHERE l_orderkey = o_orderkey
        AND l_partkey = p_partkey);
```

Figure 8: Query after Split and optimize FROM

```
SELECT o_orderkey
FROM orders
WHERE NOT EXISTS(
        SELECT *
        FROM lineitem, part
        WHERE l_orderkey = o_orderkey
        AND l_partkey = p_partkey)
AND NOT EXISTS(
        SELECT *
        FROM lineitem
        WHERE l_orderkey = o_orderkey
        AND EXISTS(SELECT * FROM part WHERE l_partkey IS NULL));
```

After the applying of the translation we obtain a query without any optimization, when trying to evaluate this query PSQL estimate time is $10^3$ times longer than for $Q$. Moreover when we actually run it, it seems to be true as we never manage to get to the end on big instances (nested-loop). The result is exactly the same after we remove redundant null checks, this result is explain by the fact that the planner try to compute the result in the exact same way because there is still disjunctions (nested-loop). As soon as we split the query, PSQL manage to compute the query and it take twice the time that we need to compute the initial query, it's easily understandable as we split the query in 2 different not exists ($2 \times$ Hash-Join). Finally on more complicated queries which might split in multiple not-exists queries, we try to mix UNION ALL and split. In worst case the computation takes 6 times longer than the computation of the initial query. However we did not have enough time to implement an heuristic to choose when to use UNION-ALL and when to split, this heuristic should use meta-data such as null-percent by attributes in order to predict if the tuples will fit the memory.

The result present before are not as reliable as they should be, they were produce on a restraint number of queries, and just a few different instances of TCP-H Database. Moreover PSQL was running on my personal computer which is not that stable. Proper benchmarks will be generated during next week, and will be presented during the defense.

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# A Appendix

## A.1 Bags

**Definition.** *We call a bag $B$ a function $D \to \mathbb{N}$ such that $B(x)$ represents the multiplicity of $x$ in the bag $B$.*

**Definition.**

$$
\begin{aligned}
\forall x, \emptyset(x) &= 0 \\
\forall x, (B_1 \cap B_2)(x) &= min(B_1(x), B_2(x)) \\
\forall x, (B_1 \cup B_2)(x) &= max(B_1(x), B_2(x)) \\
\forall x, (B_1 \uplus B_2)(x) &= B_1(x) + B_2(x) \\
\forall x, (B_1 \setminus B_2)(x) &= max(0, B_1(x) - B_2(x)) \\
\forall x, [\![a]\!](x) &= \begin{cases} 1 \textit{ if } x = a \\ 0 \textit{ otherwise} \end{cases} \\
\forall x, [\![a^n]\!](x) &= \begin{cases} n \textit{ if } x = a \\ 0 \textit{ otherwise} \end{cases} \\
\forall x, [\![y^n | P(y,n)]\!](x) &= max(\{i | P(x,i)\}) \\
x \in B &\iff B(x) \geq 1 \\
x \in^n B &\iff B(x) \geq n \\
x \notin B &\iff B(x) = 0 \\
B_1 = B_2 &\iff \forall x, B_1(x) = B_2(x) \\
B_1 \subseteq B_2 &\iff \forall x, B_1(x) \leq B_2(x) \\
\{B\} &= \{x | B(x) \geq 1\}
\end{aligned}
$$

## A.2 Inductive definition

In this section we develop the definition, and building of the sets used in section 3.

**Definition.**

$$
\begin{aligned}
\bot^{T}_{(\Sigma,R,H_1 \wedge H_2,P)} &= \bot^{T}_{(\Sigma,R,H_1,P)} \cup \bot^{T}_{(\Sigma,R,H_2,P)} \\
\bot^{T}_{(\Sigma,R,H_1 \vee H_2,P)} &= \bot^{T}_{(\Sigma,R,H_1,P)} \cap \bot^{T}_{(\Sigma,R,H_2,P)} \\
\bot^{T}_{(\Sigma,R,r_i.a_i=c_i,P)} &= \{r_i.a_i\} \\
\bot^{T}_{(\Sigma,R,r_i.a_i\neq c_i,P)} &= \{r_i.a_i\} \\
\bot^{T}_{(\Sigma,R,r_i.a_i>c_i,P)} &= \{r_i.a_i\} \\
\bot^{T}_{(\Sigma,R,r_i.a_i<c_i,P)} &= \{r_i.a_i\} \\
\bot^{T}_{(\Sigma,R,r_i.a_i=r_j.a_j,P)} &= \emptyset \\
\bot^{T}_{(\Sigma,R,r_i.a_i\neq r_j.a_j,P)} &= \{r_i.a_i, r_j.a_j\} \\
\bot^{T}_{(\Sigma,R,r_i.a_i>r_j.a_j,P)} &= \emptyset \\
\bot^{T}_{(\Sigma,R,r_i.a_i<r_j.a_j,P)} &= \emptyset \\
\bot^{T}_{(\Sigma,R,r_i.a_i=p_i,P)} &= \emptyset \\
\bot^{T}_{(\Sigma,R,r_i.a_i\neq p_i,P)} &= \{r_i.a_i, p_i\} \\
\bot^{T}_{(\Sigma,R,r_i.a_i>p_i,P)} &= \emptyset \\
\bot^{T}_{(\Sigma,R,r_i.a_i<p_i,P)} &= \emptyset \\
\bot^{T}_{(\Sigma,R,exists(Q),P)} &= \bot^{T}_{Q[?]} \\
\bot^{T}_{(\Sigma,R,notexists(Q),P)} &= \bot^{F}_{Q[?]}
\end{aligned}
$$

$$
\begin{aligned}
\bot^{F}_{(\Sigma,R,H_1 \wedge H_2,P)} &= \bot^{F}_{(\Sigma,R,H_1,P)} \cap \bot^{F}_{(\Sigma,R,H_2,P)} \\
\bot^{F}_{(\Sigma,R,H_1 \vee H_2,P)} &= \bot^{F}_{(\Sigma,R,H_1,P)} \cup \bot^{F}_{(\Sigma,R,H_2,P)} \\
\bot^{F}_{(\Sigma,R,r_i.a_i=c_i,P)} &= \emptyset \\
\bot^{F}_{(\Sigma,R,r_i.a_i\neq c_i,P)} &= \emptyset \\
\bot^{F}_{(\Sigma,R,r_i.a_i>c_i,P)} &= \emptyset \\
\bot^{F}_{(\Sigma,R,r_i.a_i<c_i,P)} &= \emptyset \\
\bot^{F}_{(\Sigma,R,r_i.a_i=r_j.a_j,P)} &= \emptyset \\
\bot^{F}_{(\Sigma,R,r_i.a_i\neq r_j.a_j,P)} &= \emptyset \\
\bot^{F}_{(\Sigma,R,r_i.a_i>r_j.a_j,P)} &= \emptyset \\
\bot^{F}_{(\Sigma,R,r_i.a_i<r_j.a_j,P)} &= \emptyset \\
\bot^{F}_{(\Sigma,R,r_i.a_i=p_i,P)} &= \emptyset \\
\bot^{F}_{(\Sigma,R,r_i.a_i\neq p_i,P)} &= \emptyset \\
\bot^{F}_{(\Sigma,R,r_i.a_i>p_i,P)} &= \emptyset \\
\bot^{F}_{(\Sigma,R,r_i.a_i<p_i,P)} &= \emptyset \\
\bot^{F}_{(\Sigma,R,exists(Q),P)} &= \bot^{F}_{Q[?]} \\
\bot^{F}_{(\Sigma,R,notexists(Q),P)} &= \bot^{T}_{Q[?]}
\end{aligned}
$$

**Definition.**

$$
\begin{aligned}
nested^+(H_1 \wedge H_2) &= \{H_1 \wedge H_2\} \cup nested^+(H_1) \cup nested^+(H_2) \\
nested^+(H_1 \vee H_2) &= \{H_1 \vee H_2\} \cup nested^+(H_1) \cup nested^+(H_2) \\
nested^+(r_i.a_i = c_i) &= \{r_i.a_i = c_i\} \\
nested^+(r_i.a_i \neq c_i) &= \{r_i.a_i \neq c_i\} \\
nested^+(r_i.a_i > c_i) &= \{r_i.a_i > c_i\} \\
nested^+(r_i.a_i < c_i) &= \{r_i.a_i < c_i\} \\
nested^+(r_i.a_i = r_j.a_j) &= \{r_i.a_i = r_j.a_j\} \\
nested^+(r_i.a_i \neq r_j.a_j) &= \{r_i.a_i \neq r_j.a_j\} \\
nested^+(r_i.a_i > r_j.a_j) &= \{r_i.a_i > r_j.a_j\} \\
nested^+(r_i.a_i < r_j.a_j) &= \{r_i.a_i < r_j.a_j\} \\
nested^+(r_i.a_i = p_j) &= \{r_i.a_i = p_j\} \\
nested^+(r_i.a_i \neq p_j) &= \{r_i.a_i \neq p_j\} \\
nested^+(r_i.a_i > p_j) &= \{r_i.a_i > p_j\} \\
nested^+(r_i.a_i < p_j) &= \{r_i.a_i < p_j\} \\
nested^+(exists(Q)) &= \{exists(Q)\} \cup nested^+(Q) \\
nested^+(notexists(Q)) &= \{notexists(Q)\} \cup nested^-(Q)
\end{aligned}
$$

$$
\begin{aligned}
nested^-(H_1 \wedge H_2) &= nested^-(H_1) \cup nested^-(H_2) \\
nested^-(H_1 \vee H_2) &= nested^-(H_1) \cup nested^-(H_2) \\
nested^-(r_i.a_i = c_i) &= \emptyset \\
nested^-(r_i.a_i \neq c_i) &= \emptyset \\
nested^-(r_i.a_i > c_i) &= \emptyset \\
nested^-(r_i.a_i < c_i) &= \emptyset \\
nested^-(r_i.a_i = r_j.a_j) &= \emptyset \\
nested^-(r_i.a_i \neq r_j.a_j) &= \emptyset \\
nested^-(r_i.a_i > r_j.a_j) &= \emptyset \\
nested^-(r_i.a_i < r_j.a_j) &= \emptyset \\
nested^-(r_i.a_i = p_j) &= \emptyset \\
nested^-(r_i.a_i \neq p_j) &= \emptyset \\
nested^-(r_i.a_i > p_j) &= \emptyset \\
nested^-(r_i.a_i < p_j) &= \emptyset \\
nested^-(exists(Q)) &= nested^-(Q) \\
nested^-(notexists(Q)) &= nested^+(Q)
\end{aligned}
$$

$$nested(Q) = nested^-(Q) \cup nested^+(Q)$$

We do not prove that those inductive definitions verify the property needed in section , however such a proof is fairly straight forward with an induction.

## A.3 Proof of proposition 2

**Proposition.**

$$\forall Q \in [\![SQL]\!], Eval_{SQL}(Q^+, D) \subseteq cert_\perp(Q, D)$$

**Proposition.**

$$\forall Q \in [\![SQL]\!], cert_\perp(Q, D) \subseteq Eval_{SQL}(Q^?, D)$$

*Proof.* The proof of prop 2 is made by induction over the query $Q$ assuming only prop 4.

$$\begin{aligned}
Q &= (\Sigma, R, \emptyset, P) \\
x \in^n Eval_{SQL}(Q^+, D) &\Rightarrow x \in^n \sigma_\Sigma(R^\times) \\
&\Rightarrow \forall v, v(x) \in^n \sigma_\Sigma(v(R)^\times) \text{ by 1} \\
&\Rightarrow \forall v, v(x) \in^n \sigma_\Sigma(Eval_{SQL}(Q_*, v(D))) \\
&\Rightarrow x \in^n cert_\perp(Q, D)
\end{aligned}$$

$$\begin{aligned}
Q &= (\Sigma, R, r_i.a_i \neq p_i, P) \\
x \in^n Eval_{SQL}(Q^+, D) &\Rightarrow \sum_{z \in \{y | y \in Eval_{SQL}(Q_*^+, D) \wedge \sigma_\Sigma(y) = x\}} Eval_{SQL}(Q_*^+, D)(z) \geq n \\
&Moreover \\
Eval_{SQL}(Q_*^+, D)(z) \geq k &\Rightarrow Eval_{SQL}((*, R, (r_i.a_i \neq p_i)^*, P), D)(z) = k \\
&\Rightarrow Eval_{SQL}((*, R, r_i.a_i \neq p_i \wedge const(r_i, a_i) \wedge const(p_i), P), D)(z) = k \\
&\Rightarrow R^\times(x) \geq \wedge x[r_i.a_i] \neq P[p_i] \wedge \forall t, x[r_i.a_i] \neq \perp_t \wedge \forall t, P[p_i] \neq \perp_t \\
&Moreover \\
\forall t, x[r_i.a_i] \neq \perp_t \wedge \forall t, P[p_i] \neq \perp_t &\Rightarrow \forall h, h(x)[r_i.a_i] = x[r_i.a_i] \wedge h(P)[p_i] = P[p_i] \\
&Then \\
Eval_{SQL}(Q_*^+, D)(z) \geq k &\Rightarrow R^\times(x) \geq k \wedge \forall h, h(x)[r_i.a_i] \neq h(P)[p_i] \\
&\Rightarrow cert_\perp(Q_*, D)(z) \geq k \\
&Then \\
x \in^n Eval_{SQL}(Q^+, D) &\Rightarrow \sum_{z \in \{y | y \in Eval_{SQL}(Q_*^+, D) \wedge \sigma_\Sigma(y) = x\}} cert_\perp(Q_*, D)(z) \geq n \\
&\Rightarrow x \in^n cert_\perp(Q, D)
\end{aligned}$$

$$
\begin{aligned}
&Q = (\Sigma, R, H_1 \wedge H_2, P)\\
&x \in^n Eval_{SQL}(Q^+, D) \Rightarrow \sum_{z \in \{y \mid y \in Eval_{SQL}(Q_*^+, D) \wedge \sigma_\Sigma(y) = x\}} Eval_{SQL}(Q_*^+, D)(z) \geq n\\
&\qquad\qquad Moreover\\
&Eval_{SQL}(Q_*^+, D)(z) \geq k \Rightarrow Eval_{SQL}((*, R, H_1^* \wedge H_2^*, P), D)(z) \geq k\\
&\qquad\qquad \Rightarrow (Eval_{SQL}((*, R, H_1^*, P), D) \cap Eval_{SQL}((*, R, H_2^*, P), D))(z) \geq k\\
&\qquad\qquad \Rightarrow (Eval_{SQL}((*, R, H_1, P)^+, D) \cap Eval_{SQL}((*, R, H_2, P)^+, D))(z) \geq k\\
&\qquad\qquad \Rightarrow (Eval_{SQL}((*, R, H_1, P)^+, D))(x) \geq k \wedge (Eval_{SQL}((*, R, H_2, P)^+, D))(z) \geq k\\
&\qquad\qquad \Rightarrow (cert_\perp((*, R, H_1, P), D))(x) \geq k \wedge (cert_\perp((*, R, H_2, P), D))(z) \geq k\\
&\qquad\qquad \Rightarrow (cert_\perp((*, R, H_1, P), D) \cap cert_\perp((*, R, H_2, P), D))(z) \geq k\\
&\qquad\qquad \Rightarrow (cert_\perp((*, R, H_1 \wedge H_2, P), D))(z) \geq k\\
&\qquad\qquad \Rightarrow (cert_\perp(Q_*, D))(z) \geq k\\
&\qquad\qquad Then\\
&x \in^n Eval_{SQL}(Q^+, D) \Rightarrow \sum_{z \in \{y \mid y \in Eval_{SQL}(Q_*^+, D) \wedge \sigma_\Sigma(y) = x\}} cert_\perp(Q_*, D)(z) \geq n\\
&\qquad\qquad \Rightarrow x \in^n cert_\perp(Q, D)
\end{aligned}
$$

$$
\begin{aligned}
&Q = (\Sigma, R, H_1 \vee H_2, P)\\
&x \in^n Eval_{SQL}(Q^+, D) \Rightarrow \sum_{z \in \{y \mid y \in Eval_{SQL}(Q_*^+, D) \wedge \sigma_\Sigma(y) = x\}} Eval_{SQL}(Q_*^+, D)(z) \geq n\\
&\qquad\qquad Moreover\\
&Eval_{SQL}(Q_*^+, D)(z) \geq k \Rightarrow Eval_{SQL}((*, R, H_1^* \vee H_2^*, P), D)(z) \geq k\\
&\qquad\qquad \Rightarrow (Eval_{SQL}((*, R, H_1^*, P), D) \cup Eval_{SQL}((*, R, H_2^*, P), D))(z) \geq k\\
&\qquad\qquad \Rightarrow (Eval_{SQL}((*, R, H_1, P)^+, D) \cup Eval_{SQL}((*, R, H_2, P)^+, D))(z) \geq k\\
&\qquad\qquad \Rightarrow (Eval_{SQL}((*, R, H_1, P)^+, D))(x) \geq k \vee (Eval_{SQL}((*, R, H_2, P)^+, D))(z) \geq k\\
&\qquad\qquad \Rightarrow (cert_\perp((*, R, H_1, P), D))(x) \geq k \vee (cert_\perp((*, R, H_2, P), D))(z) \geq k\\
&\qquad\qquad \Rightarrow (cert_\perp((*, R, H_1, P), D) \cup cert_\perp((*, R, H_2, P), D))(z) \geq k\\
&\qquad\qquad \Rightarrow (cert_\perp((*, R, H_1 \vee H_2, P), D))(z) \geq k\\
&\qquad\qquad \Rightarrow (cert_\perp(Q_*, D))(z) \geq k\\
&\qquad\qquad Then\\
&x \in^n Eval_{SQL}(Q^+, D) \Rightarrow \sum_{z \in \{y \mid y \in Eval_{SQL}(Q_*^+, D) \wedge \sigma_\Sigma(y) = x\}} cert_\perp(Q_*, D)(z) \geq n\\
&\qquad\qquad \Rightarrow x \in^n cert_\perp(Q, D)
\end{aligned}
$$

□

*Proof.* The proof of prop 4 is made by induction over the query $Q$ assuming only prop 2.

$$
\begin{aligned}
Q &= (\Sigma, R, \emptyset, P) \\
x \in^n cert_\perp(Q, D) &\Rightarrow \forall v, v(x) \in^n Eval_{SQL}(Q, v(D)) \\
&\Rightarrow \forall v, v(x) \in^n \sigma_\Sigma(Eval_{SQL}(Q_*, v(D))) \\
&\Rightarrow \forall v, v(x) \in^n \sigma_\Sigma(v(R)^\times) \\
&\Rightarrow x \in^n \sigma_\Sigma(R^\times) \text{ by 1} \\
&\Rightarrow x \in^n Eval_{SQL}(Q^?, D)
\end{aligned}
$$

$$
\begin{aligned}
&Q = (\Sigma, R, r_i.a_i = r_j.a_j, P) \\
&x \in^n cert_\perp(Q, D) \Rightarrow \sum_{z \in \{y | y \in cert_\perp(Q_*, D) \wedge \sigma_\Sigma(y) = x\}} cert_\perp(Q_*, D)(z) \geq n \\
&\qquad Moreover \\
&cert_\perp(Q_*, D)(z) \geq k \Rightarrow \forall h, h(z) \in^k Eval_{SQL}(Q_*, h(D)) \\
&\qquad \Rightarrow \forall h, \forall t, R^\times(z) \geq k \wedge h(z)[r_i.a_i] = h(z)[r_j.a_j] \wedge h(z)[r_i.a_i] \neq \perp_t \wedge h(z)[r_j.a_j] \neq \perp_t \\
&\qquad Moreover \\
&\text{TRUE} \Rightarrow \forall g, \forall t, g(z)[r_i.a_i] \neq \perp_t \\
&\forall h, h(z)[r_i.a_i] = h(z)[r_j.a_j] \Rightarrow \exists h, h(z)[r_i.a_i] = h(z)[r_j.a_j] \\
&\qquad \Rightarrow z[r_i.a_i] = z[r_j.a_j] \vee (\exists t, z[r_i.a_i] = \perp_t) \vee (\exists t, z[r_j.a_j] = \perp_t) \\
&\qquad Then \\
&cert_\perp(Q_*, D)(z) \geq k \Rightarrow R^\times(z) \geq k \wedge z[r_i.a_i] = z[r_j.a_j] \vee (\exists t, z[r_i.a_i] = \perp_t) \vee (\exists t, z[r_j.a_j] = \perp_t) \\
&\qquad \Rightarrow R^\times(z) \geq k \wedge z \in Eval_{SQL}((*, R, (r_i.a_i = r_j.a_j \vee null(r_i.a_i) \vee null(r_j.a_j)), P), D) \\
&\qquad \Rightarrow R^\times(z) \geq k \wedge z \in Eval_{SQL}((*, R, (r_i.a_i = r_j.a_j)^{**}, P), D) \\
&\qquad \Rightarrow Eval_{SQL}((*, R, r_i.a_i = r_j.a_j, P)^?, D)(z) \geq k \\
&\qquad Then \\
&x \in^n cert_\perp(Q, D) \Rightarrow \sum_{z \in \{y | y \in cert_\perp(Q_*, D) \wedge \sigma_\Sigma(y) = x\}} Eval_{SQL}(Q_*^?, D)(z) \geq n \\
&\qquad \Rightarrow x \in^n Eval_{SQL}(Q^?, D)
\end{aligned}
$$

$$
\begin{aligned}
&Q = (\Sigma, R, H_1 \wedge H_2, P)\\
&\quad x \in^n cert_\perp(Q, D) \Rightarrow \sum_{z \in \{y | y \in cert\perp(Q_*, D) \wedge \sigma_\Sigma(y) = x\}} cert_\perp(Q_*, D)(z) \geq n\\
&\qquad\qquad Moreover\\
&cert_\perp(Q_*, D)(z) \geq k \Rightarrow R^\times(z) \geq k \wedge \forall h, h(z) \in Eval_{SQL}((*, R, H_1 \wedge H_2, h(P)), h(D))\\
&\qquad\qquad \Rightarrow R^\times(z) \geq k \wedge \forall h, h(z) \in Eval_{SQL}((*, R, H_1, h(P)), h(D)) \wedge h(z) \in Eval_{SQL}((*, R, H_1, h(P)), h\\
&\qquad\qquad \Rightarrow cert_\perp((*, R, H_1, P), D)(z) \geq k \wedge cert_\perp((*, R, H_2, P), D)(z) \geq k\\
&\qquad\qquad \Rightarrow Eval_{SQL}((*, R, H_1, P)^?, D)(z) \geq k \wedge Eval_{SQL}((*, R, H_2, P)^?, D)(z) \geq k\\
&\qquad\qquad \Rightarrow Eval_{SQL}((*, R, H_1^{**}, P), D)(z) \geq k \wedge Eval_{SQL}((*, R, H_2^{**}, P), D)(z) \geq k\\
&\qquad\qquad \Rightarrow Eval_{SQL}((*, R, H_1^{**} \wedge H_2^{**}, P), D)(z) \geq k\\
&\qquad\qquad \Rightarrow Eval_{SQL}((*, R, H_1 \wedge H_2, P)^?, D)(z) \geq k\\
&\qquad\qquad \Rightarrow Eval_{SQL}(Q_*^?, D)(z) \geq k\\
&\qquad\qquad Then\\
&\quad x \in^n cert_\perp(Q, D) \Rightarrow \sum_{z \in \{y | y \in posi_\perp(Q_*, D) \wedge \sigma_\Sigma(y) = x\}} Eval_{SQL}(Q_*^?, D)(z) \geq n\\
&\qquad\qquad \Rightarrow x \in^n Eval_{SQL}(Q^?, D)
\end{aligned}
$$

□

## A.4 Proof of proposition 4

Figure 9: Translation $(H, Q) \rightarrow H_Q^{\perp^F}$

$$
\begin{aligned}
(\Sigma, R, H \vee null(r_i.a_i), P)_Q^{\perp^F} &\rightarrow (\Sigma, R, H_Q^{\perp^F}, P)\\
&\quad \text{if } \exists Q' \in nested^-(Q), (H \vee null(r_i.a_i)) \in nested(Q'), r_i.a_i \in \perp_{Q'}^T\\
&\quad \text{if } \exists Q' \in nested^+(Q), (H \vee null(r_i.a_i)) \in nested(Q'), r_i.a_i \in \perp_{Q'}^F\\
(\Sigma, R, H \vee null(p_i), P)_Q^{\perp^F} &\rightarrow (\Sigma, R, H_Q^{\perp^F}, P)\\
&\quad \text{if } \exists Q' \in nested^-(Q), (H \vee null(p_i)) \in nested(Q'), p_i \in \perp_{Q'}^T\\
&\quad \text{if } \exists Q' \in nested^+(Q), (H \vee null(p_i)) \in nested(Q'), p_i \in \perp_{Q'}^F\\
(\Sigma, R, H \wedge const(r_i.a_i), P)_Q^{\perp^F} &\rightarrow (\Sigma, R, H_Q^{\perp^F}, P)\\
&\quad \text{if } \exists Q' \in nested^-(Q), (H \wedge const(r_i.a_i)) \in nested(Q'), r_i.a_i \in \perp_{Q'}^T\\
&\quad \text{if } \exists Q' \in nested^+(Q), (H \wedge const(r_i.a_i)) \in nested(Q'), r_i.a_i \in \perp_{Q'}^F\\
(\Sigma, R, H \wedge const(p_i), P)_Q^{\perp^F} &\rightarrow (\Sigma, R, H_Q^{\perp^F}, P)\\
&\quad \text{if } \exists Q' \in nested^-(Q), (H \wedge const(p_i)) \in nested(Q'), p_i \in \perp_{Q'}^T\\
&\quad \text{if } \exists Q' \in nested^+(Q), (H \wedge const(p_i)) \in nested(Q'), p_i \in \perp_{Q'}^F
\end{aligned}
$$

**Lemma.**

$$\forall Q' \in nested^+(Q), Eval(Q, D) = Eval(Q_{Q'}^\perp, D)$$

**Lemma.**

$$\forall Q' \in nested^{-}(Q), Eval(Q, D) = Eval(Q^{\perp^F}_{Q'}, D)$$

*Proof.* Let suppose $H_1 \wedge H_2 \in nested^{+}(Q)$.
The critical case is when $\exists r_i.a_i \in (\perp^T_{H_1} \setminus \perp^T_{H_2})$ and $(H \vee null(r_i.a_i) \in nested(H_2)$.
Let's assume there exists $x$ such that $x \in Eval_{SQL}(Q, D)$ and $x \notin Eval(Q^{\perp}_{H_1 \wedge H_2}, D)$. Moreover we know by induction that $x \in Eval(Q^{\perp}_{H_1}, D) \wedge x \in Eval(Q^{\perp}_{H_2}, D)$. Then if $x[r_i.a_i] = \perp$ we know that $x \notin Eval_{SQL}(Q, D)$ as $x \in \perp^T_{H_1}$ and $H_1 \in nested^{+}(Q)$. So $x[r_i.a_i] \neq \perp$ then the rewriting is correct.
Let's assume there exists $x$ such that $x \notin Eval_{SQL}(Q, D)$ and $x \in Eval(Q^{\perp}_{H_1 \wedge H_2}, D)$. Moreover we know by induction that $x \in Eval(Q^{\perp}_{H_1}, D) \wedge x \in Eval(Q^{\perp}_{H_2}, D)$. Then if $x[r_i.a_i] = \perp$ we know that $x \notin Eval_{SQL}(H_1, D)$ as $x \in \perp^T_{H_1}$. So $\forall H, x \notin Eval_{SQL}(H_1 \wedge H, D)$ especially $Eval(Q^{\perp}_{H_1 \wedge H_2}, D)$. Then $x[r_i.a_i] \neq \perp$ then the rewriting is correct.

Let suppose $H_1 \vee H_2 \in nested^{-}(Q)$.
The critical case is when $\exists r_i.a_i \in (\perp^F_{H_1} \setminus \perp^F_{H_2})$ and $(H \vee null(r_i.a_i) \in nested(H_2)$.
Let's assume there exists $x$ such that $x \in Eval_{SQL}(Q, D)$ and $x \notin Eval(Q^{\perp}_{H_1 \vee H_2}, D)$. Morover we know by induction that $x \in Eval(Q^{\perp}_{H_1}, D) \wedge x \in Eval(Q^{\perp}_{H_2}, D)$. Then if $x[r_i.a_i] = \perp$ we know that $x \notin Eval_{SQL}(Q, D)$ as $x \in \perp^F_{H_1}$ and $H_1 \in nested^{-}(Q)$. So $x[r_i.a_i] \neq \perp$ then the rewriting is correct.
Let's assume there exists $x$ such that $x \notin Eval_{SQL}(Q, D)$ and $x \in Eval(Q^{\perp}_{H_1 \vee H_2}, D)$. Morover we know by induction that $x \in Eval(Q^{\perp}_{H_1}, D) \wedge x \in Eval(Q^{\perp}_{H_2}, D)$. Then if $x[r_i.a_i] = \perp$ we know that $x \in Eval_{SQL}(H_1, D)$ as $x \in \perp^F_{H_1}$. So $\forall H, x \in Eval_{SQL}(H_1 \vee H, D)$ especially $Eval_{SQL}(Q^{\perp}_{H_1 \vee H_2}, D)$. Then $x[r_i.a_i] \neq \perp$ then the rewriting is correct.

Other case are trivial as, $\perp^T$ resp.$\perp^F$ only increase with this$\wedge$ resp $\vee$. In case of negation we only have to assume that lemma 3 is true.
The proof for lemma 3 is analogue □

## A.5 Proof of proposition 5

**Proposition.** *Let* $Q = (\Sigma, R, notexists((\Sigma', R', (l_1 \vee l_2) \wedge H, P'), P)$

$$Eval_{SQL}(Q, D) = Eval_{SQL}((\Sigma, R, notexists(\Sigma', R', (l_1 \wedge H), P') \wedge notexists(\Sigma', R', (l_2 \wedge H), P'), P), D)$$

*Proof.*

$$\begin{aligned}
x \in^n Eval_{SQL}(Q, D) &\Leftrightarrow x \in^n Eval_{SQL}((\Sigma, R, notexists((\Sigma', R', (l_1 \wedge H) \vee (l_2 \wedge H), P'), P), D) \\
&\Leftrightarrow R^{\times}(\sigma^{-1}_{\Sigma}(x)) \geq n \wedge Eval_{SQL}((\Sigma', R', (l_1 \wedge H) \vee (l_2 \wedge H), P' \cup x]), D) = \emptyset \\
&\Leftrightarrow R^{\times}(\sigma^{-1}_{\Sigma}(x)) \geq n \wedge Eval_{SQL}((\Sigma', R', l_1 \wedge H, P' \cup x]), D) \\
&\cup Eval_{SQL}((\Sigma', R', l_1 \wedge H, P' \cup x]), D) = \emptyset \\
&\Leftrightarrow R^{\times}(\sigma^{-1}_{\Sigma}(x)) \geq n \wedge Eval_{SQL}((\Sigma', R', l_1 \wedge H, P' \cup x]), D) = \emptyset \\
&\wedge Eval_{SQL}((\Sigma', R', l_1 \wedge H, P' \cup x]), D) = \emptyset \\
&\Leftrightarrow x \in^n Eval_{SQL}((\Sigma, R, notexists(\Sigma', R', (l_1 \wedge H), P'), P), D) \\
&\wedge x \in^n Eval_{SQL}((\Sigma, R, notexists(\Sigma', R', (l_2 \wedge H), P'), P), D) \\
&\Leftrightarrow x \in^n Eval_{SQL}((\Sigma, R, notexists(\Sigma', R', (l_1 \wedge H), P') \wedge notexists(\Sigma', R', (l_2 \wedge H), P'), P), D)
\end{aligned}$$

□

## A.6 Proof of proposition 6

**Proposition.** *Let* $Q = (\Sigma, R, notexists((\Sigma', R', H_r \wedge H, P'), P)$ *where* $H_r$ *denote every conditions which are on attributes of* $r$ *and such that* $r \in R'_s$ *then :*

$$Eval_{SQL}(Q, D) = Eval_{SQL}((\Sigma, R, notexists(\Sigma', R' \setminus r, H \wedge exists(*, r, H_r, P'), P'), P), D)$$

*Proof.* If $Eval_{SQL}((*, r, H_r, P'), D) = \emptyset$ then the proof is immediate. As $Eval_{SQL}(Q, D)$ will be equal to $\sigma_\Sigma(R^\times)$ in deed $Eval_{SQL}((\Sigma', R', H_r \wedge H, P'), D) = \emptyset$ as there is no element of $r$ which are able to fulfill $H_r$ condition. We found exactly the same for $Eval_{SQL}((\Sigma, R, notexists(\Sigma', R' \setminus r, H \wedge exists(*, r, H_r, P'), P'), P), D)$ in deed $exists(*, r, H_r, P')$ will never be true.

If $Eval_{SQL}((*, r, H_r, P'), D) \neq \emptyset$ then $exists(*, r, H_r, P')$ will always be true. Then $Eval_{SQL}((\Sigma, R, notexists(\Sigma', R' \setminus r, H \wedge exists(*, r, H_r, P'), P'), P), D) = Eval_{SQL}((\Sigma, R, notexists(\Sigma', R' \setminus r, H, P'), P), D)$ Moreover as $r$ is not connected to any other relation. We have $\{Eval_{SQL}((\Sigma', R' \setminus r, H, P'), D)\} = \{Eval_{SQL}(\Sigma', R', H, P'), D)\}$. Indeed we are only multiplying the number of row by the number of tuples in $r$. Then as we are only checking if the set is empty it does not change anything because each attributes linked with a upper-level query will still have at least one representative. □

## A.7 SQL nulls Translation

Here we present the translation for SQL Nulls. The only variation is due to the fact that $\bot_t = \bot_t$ is evaluate to true in case of marked nulls, while $\bot = \bot$ is evaluate to unknown in case of SQL nulls.

Figure 10: Translatation $Q \to (Q^+, Q^?)$

$$(\Sigma, R, H, P)^+ \to (\Sigma, R, H^*, P)$$
$$(\Sigma, R, H, P)^? \to (\Sigma, R, H^{**}, P)$$

Figure 11: Translatation $H \to H^*$

$$(H_1 \wedge H_2)^* \to H_1^* \wedge H_2^*$$
$$(H_1 \vee H_2)^* \to H_1^* \vee H_2^*$$
$$(r_i.a_i = c_i)^* \to r_i.a_i = c_i$$
$$(r_i.a_i \neq c_i)^* \to r_i.a_i \neq c_i$$
$$(r_i.a_i = r_j.a_j)^* \to r_i.a_i = r_j.a_j$$
$$(r_i.a_i \neq r_j.a_j)^* \to r_i.a_i \neq r_j.a_j$$
$$null(r_i.a_i)^* \to null(r_i.a_i)$$
$$const(r_i.a_i)^* \to const(r_i.a_i)$$
$$exists(Q)^* \to exists(Q^+)$$
$$notexists(Q)^* \to notexists(Q^?)$$

Figure 12: Translatation $H \rightarrow H^{**}$

$$
\begin{aligned}
(H_1 \wedge H_2)^{**} &\rightarrow H_1^{**} \wedge H_2^{**} \\
(H_1 \vee H_2)^{**} &\rightarrow H_1^{**} \vee H_2^{**} \\
(r_i.a_i = c_i)^{**} &\rightarrow r_i.a_i = c_i \vee null(r_i.a_i) \\
(r_i.a_i \neq c_i)^{**} &\rightarrow r_i.a_i \neq c_i \vee null(r_i.a_i) \\
(r_i.a_i = r_j.a_j)^{**} &\rightarrow r_i.a_i = r_j.a_j \vee null(r_i.a_i) \vee null(r_j.a_j) \\
(r_i.a_i \neq r_j.a_j)^{**} &\rightarrow r_i.a_i \neq r_j.a_j \vee null(r_i.a_i) \vee null(r_j.a_j) \\
null(r_i.a_i)^{**} &\rightarrow null(r_i.a_i) \\
const(r_i.a_i)^{**} &\rightarrow const(r_i.a_i) \\
exists(Q)^{**} &\rightarrow exists(Q^{?}) \\
notexists(Q)^{**} &\rightarrow notexists(Q^{+})
\end{aligned}
$$

## A.8 Query rewriting step

Figure 13: Translated Query

```
SELECT o_orderkey
FROM orders
WHERE NOT EXISTS(
        SELECT *
        FROM lineitem, part
        WHERE l_orderkey = o_orderkey OR l_orderkey IS NULL OR o_orderkey IS NULL
        AND l_partkey = p_partkey OR l_partkey IS NULL OR p_partkey IS NULL);
```

Figure 14: Query after remove redundant

```
SELECT o_orderkey
FROM orders
WHERE NOT EXISTS(
        SELECT *
        FROM lineitem, part
        WHERE l_orderkey = o_orderkey
        AND l_partkey = p_partkey OR l_partkey IS NULL);
```

Figure 15: Query after split

```
SELECT o_orderkey
FROM orders
WHERE NOT EXISTS(
        SELECT *
        FROM lineitem,part
        WHERE l_orderkey = o_orderkey
        AND l_partkey = p_partkey)
AND NOT EXISTS(
        SELECT *
        FROM lineitem,part
        WHERE l_orderkey = o_orderkey
        AND l_partkey IS NULL);
```

Figure 16: Query after optimize FROM

```
SELECT o_orderkey
FROM orders
WHERE NOT EXISTS(
        SELECT *
        FROM lineitem,part
        WHERE l_orderkey = o_orderkey
        AND l_partkey = p_partkey)
AND NOT EXISTS(
        SELECT *
        FROM lineitem
        WHERE l_orderkey = o_orderkey
        AND EXISTS(SELECT * FROM part WHERE l_partkey IS NULL));
```

Figure 17: Query optimize with UNION ALL

```
SELECT o_orderkey
FROM orders
WHERE NOT EXISTS(
        SELECT *
        FROM ((SELECT * FROM lineitem,part WHERE l_partkey = p_partkey)
        UNION ALL (SELECT * FROM lineitem,part WHERE l_partkey IS NULL)) as lp
        WHERE l_orderkey = o_orderkey);
```