Toy model preliminaries

- **Goal:** **Form an expression for T' and $\mathrm{Var}(T)$.**

- **Tools:**
  - ◇ Surface energy budget,
  - ◇ Soil moisture budget,
  - ◇ Parameterization of land surface fluxes.

- Focus on inter-annual variability:

  — Use monthly-averaged variables,
  — Decompose each variable around monthly climatology:

$$X'_{i,j} \equiv X_{i,j} - \overline{X}_j \tag{3}$$

  where i: year index, j: month index.

Toy model preliminary results

Surface energy and soil moisture anomaly budgets:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(C_{eff}\,T'\right) = F' - LE' - H_s' - F_{lw}^{\uparrow\,\prime} - G' \qquad \text{(4a)}$$

$$\frac{\mathrm{d}\,m'}{\mathrm{d}t} = P' - E' - R' - I' \qquad \text{(4b)}$$

T : 2-meter temperature ,

C_{eff} : effective heat capacity of land surface ,

P : Precipitation , R : Surface Runoff , I : Infiltration .

Toy model preliminary results

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- **Assume that** m is the moisture content in the **top 10 cm of soil**

Toy model preliminary results

Surface energy and soil moisture anomaly budgets:

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- Scale analysis reveals that both **storage terms are negligible**.

Toy model preliminary results

Surface energy and soil moisture anomaly budgets:

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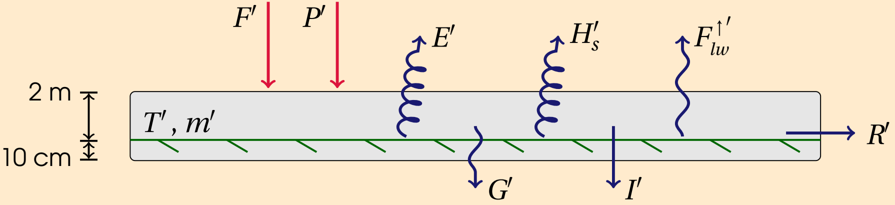
$$\cancel{\frac{\mathrm{d}m'}{\mathrm{d}t}} \;=\; P' - E' - R' - I' \qquad \text{(4b)}$$

- **Define** $G' \equiv F' - LE' - H_s' - F_{lw}^{\uparrow\,\prime}$ and $I' \equiv P' - E' - R'$

Toy model assumptions



1. F' and P' are **external forcings**, independent of $E', H'_s, F^{\uparrow\prime}_{lw}, G', I', R'$.
2. Parameterize $E', H'_s, F^{\uparrow\prime}_{lw}, G', I', R'$ as functions of T', m', F', P' **only**.

Toy model method

Ex: Consider G' (ground heat flux):

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Step 1: Regress G' as:

$$G' = G_0' + aY' \quad \text{where} \quad Y' = \{\, T', m', F', P' \,\} \tag{5}$$

Best predictor has $\min\left[\langle\, G_0', G_0' \,\rangle\right]$

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⚠ If *necessary* repeat **step 1.** with G'_0

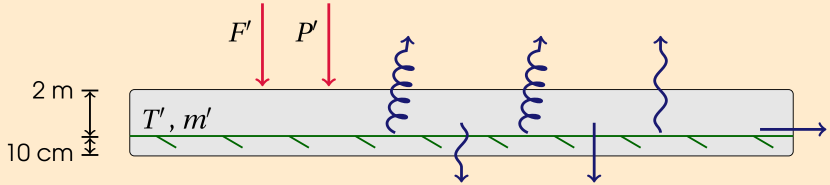
Step 2: ⚠ If T' is a *candidate*

Substitute $G' = aT'$ into surface energy anomaly budget:

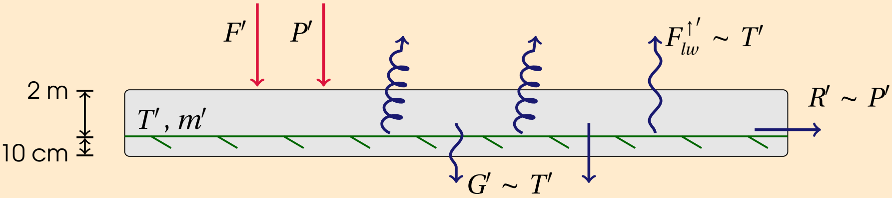
$$\widehat{T}'_{tm} = a^{-1}\left[F' - LE' - H'_s - F^{\uparrow\,\prime}_{lw}\right] \tag{6}$$

Evaluate performance with $\mathrm{RMS}\left[\widehat{T}'_{tm} - T'_{dataset}\right]$

Toy model parameterizations

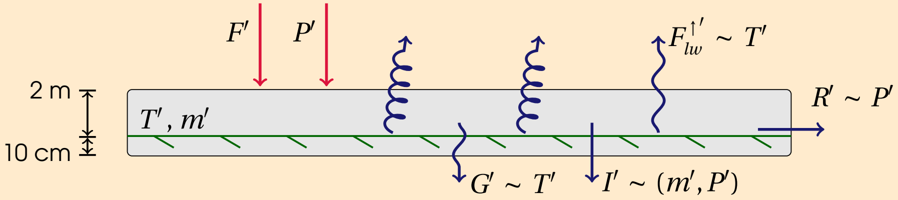


Toy model parameterizations



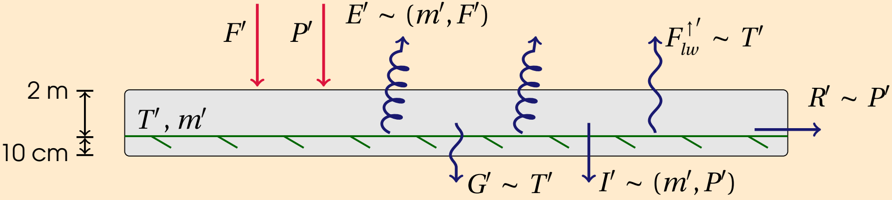
- 3 out of 6 processes were *easy* to parameterize

Toy model parameterizations



- 2 predictors for infiltration m' and P'

Toy model parameterizations

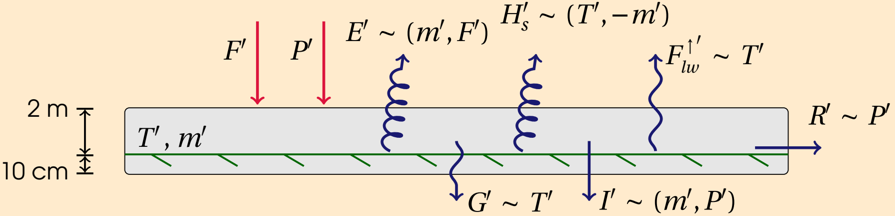


Special algorithm for E' to model both E regimes:

- **If** $\langle E', m' \rangle > 0$: 2 predictors m' and F' (moisture-limited)

  Otherwise: 1 predictor F' only (energy-limited)

Toy model parameterizations



Surprising result: H_s' is best predicted by m'!

Use 2 predictors T' and m'

Toy model expressions / coefficients

After some algebra:

$$\widehat{T}'_{tm} = \frac{1}{\gamma}\left[\left(1-\lambda\right)F'_0 - L\alpha\left(1-\lambda\right)P' - L\left(\nu_E-\nu_{Hs}\right)m'\right] \quad (7a)$$

$$\widehat{m}'_{tm} = \frac{1}{\nu_I+\nu_E}\left[\left(1+\alpha\lambda-\beta\right)P' - \left(\tfrac{\lambda}{L}\right)F'_0\right] \quad (7b)$$

Recall

$F' = F'_0 - L\alpha P'$

L : specific latent heat of vaporization

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- **Surface temperature resistance** $(\mathrm{Wm^{-2}K^{-1}})$

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Evapotranspiration efficiency (unitless)

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Fraction of P' loss to I' and R' (unitless)

Toy model expressions / coefficients

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Net effect of soil moisture anomalies on T'

All ν's have units of s^{-1}

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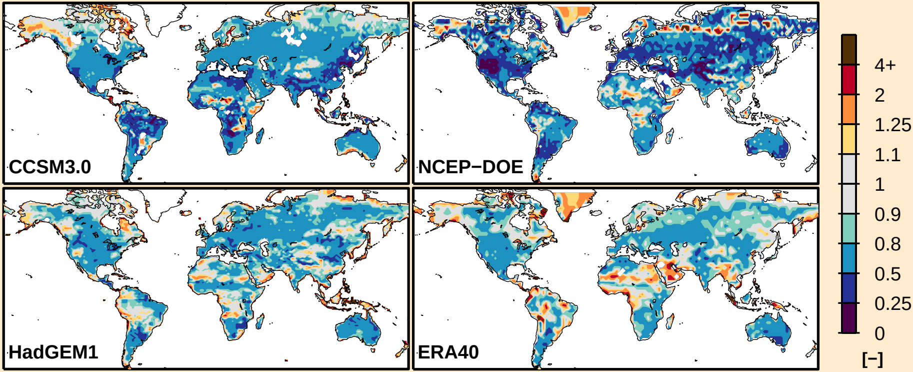
Define $\dfrac{\nu_E-\nu_{Hs}}{\nu_I+\nu_E}$ (unitless) **the coupling coefficient**

Modulates net effect of m' on T'

Toy model performance

$$\mathrm{Var}(\widehat{T}_{tm}) = \frac{1}{\gamma^2}\left[\left(1-\lambda(1-\chi)\right)^2 \mathrm{Var}(F_0) + L^2\left(\alpha(1-\lambda)+\chi(1+\alpha\lambda-\beta)\right)^2 \mathrm{Var}(P)\right]$$



- Plotted as $\dfrac{\mathrm{Var}(\widehat{T}_{tm})}{\mathrm{Var}(T_{dataset})}$

Toy model performance

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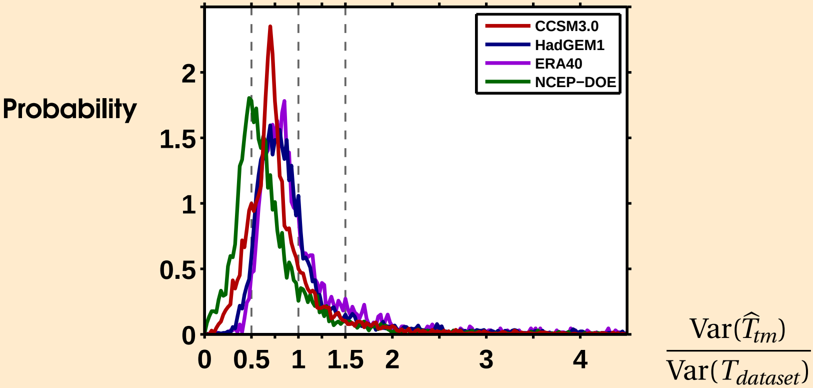

$\Longrightarrow$ Toy model underestimates dataset outputs by $\sim$ 20% to 40%.