Article

International Regional Science Review
36(4) 481-491

DOI: 10.1177/0160017613486670
irsr.sagepub.com

The Average Propagation Length: Conflicting Macro, Intra-industry, and Interindustry Conclusions

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Abstract

The average propagation length (APL) has been proposed as a measure of the fragmentation and sophistication of an economy. For a one-sector economy, we show that the APL is strictly proportional to the macro multiplier of that economy. The same holds for strong intra-industry linkages. Hence, for comparing economies and comparing single industries, the concept of the APL is of no value. For pure interindustry linkages, however, we find that the length of the supply chain between two different industries is negatively related to the strength of the multiplier between those two industries, be it weakly. Hence, the APL should only be used to compare pure interindustry linkages.

Keywords

fragmentation, supply chains, interindustry linkages, input–output tables, EXIOPOL database

Introduction

Dietzenbacher, Romero, and Bosma [DRB] (2005) invented the average propagation length (APL) to measure the average number of rounds that an exogenous impulse

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starting in industry i has to go through before it impacts industry j. They consider it a good measure of the length and thus of the sophistication of the supply chain between industry i and industry j in the economy at hand (see also Dietzenbacher and Romero [DR] 2007). In addition, Romero, Dietzenbacher, and Hewings (2009) interpret low APL values at the regional level as a measure of the spatial fragmentation of the supply chain. In this article, we question whether the APL is really a good measure of the sophistication or fragmentation of the production process of different industries across regions and nations.

When we first encountered the concept of the APL, we expected its value to be infinite, as the Taylor expansion of the Leontief inverse requires an infinite number of terms before it reaches its exact values. As always, things are a little more complicated than at first sight. Looking more closely, the formula does represent a supply chain length concept, but it is related to the size of the linkage between the two industries at hand, while its empirical value is related to the cutoff point of its Taylor expansion. Here, we report our investigation into these aspects: first, for a simple one-sector macro economy and then for a multisector economy. It appears that the conclusion is diametrically opposite, which presents a fine example of the more general phenomenon that macro conclusions and relations not always translate in a one-to-one fashion to the multisector or multiregion case, and vice versa. Moreover, we find and explain an interesting difference in the APL behavior of intra-industry linkages as opposed to pure interindustry linkages. In all, we come to the conclusion that the APL should only be used to compare pure interindustry linkages and not to compare different economies or different industries.

The APL: Definition and Established Properties

When the APL is derived from the demand-driven Leontief model, the exogenous impulse relates to a final demand *quantity pull*, whereas it relates to a primary cost *price push* when the supply-driven Ghosh model is used.[1] With the Leontief model, the APL is defined as:

$$v_{ij}^{K} = \sum\nolimits_{k=1}^{K} k \left[\left\{A^{k}\right\}_{ij} \Big/ \sum\nolimits_{k=1}^{K} \left\{A^{k}\right\}_{ij} \right], \tag{1}$$

with $\left\{A^{k}\right\}_{ij}$ = the ijth element of the kth term of the Taylor expansion of the Leontief-inverse $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1} = \mathbf{I} + \mathbf{A} + \mathbf{A}^{2} + \mathbf{A}^{3} + \ldots$, wherein the input coefficient a_{ij} from $\mathbf{A}$ indicates the amount of inputs of industry i used per unit of output of the purchasing industry j. The terms between the square brackets in equation (1) function as the weights for various lengths k of the supply chains between i and j. Hence, they sum to one when aggregated over k. DRB mathematically derive that the numerator of equation (1) may be taken from the matrix $\mathbf{L}\,(\mathbf{L} - \mathbf{I})$, if $K \Rightarrow$ infinity. Since the denominator of equation (1) may be taken from the matrix $(\mathbf{L} - \mathbf{I})$, if $K \Rightarrow$ infinity, they are able to show that in that case equation (1) reduces to:

$$v_{ij} = \sum_{s=1}^{N} l_{is}\left(l_{sj} - \delta_{sj}\right) / \left(l_{ij} - \delta_{ij}\right), \tag{2}$$

where N = number of industries, l_{ij} = ijth element of the Leontief inverse, and δ_{ij} the Kronecker delta, which is 1 if $i = j$ and 0 if $i \neq j$.When the APL is estimated with the Ghosh model, instead of with the Leontief model, a_{ij} in equation (1) is replaced with b_{ij} = the output coefficient, indicating the amount of the sales of industry i to industry j per unit of output of selling industry i. In addition, l_{is}, l_{sj}, and l_{ij} in equation (2) are replaced with g_{is}, g_{sj}, and g_{ij} from the Ghosh inverse, $\mathbf{G} = (\mathbf{I} - \mathbf{B})^{-1} = \mathbf{I} + \mathbf{B} + \mathbf{B}^2 + \mathbf{B}^3 + \ldots$ DRB, show that the APL thus calculated with the Ghosh inverse, by definition, equals the APL that is calculated with the Leontief inverse. This also makes sense intuitively, as the length of the supply chain between i and j should not depend on whether it is measured from a backward perspective with the Leontief model or from a forward perspective with the Ghosh model. This is a very nice property, if only because the number of empirical outcomes to be analyzed reduces by half.

However, in both DRB and DR, the authors argue that supply chains should be analyzed by looking at the *length* and at the *strength* of the linkages, which unfortunately again doubles the amount of information to be analyzed. In describing their empirical results, they therefore only consider industry pairs with linkages that are stronger than an arbitrary threshold. As the matrix $\mathbf{V}$ defined in equation (2) is the same for the Leontief and the Ghosh inverse, DRB define the strength of the linkage between any two industries (f_{ij}) as the average of the backward and forward linkage, as measured by the Leontief and the Ghosh inverse, while excluding the direct effects, as in equation (2):

$$\mathbf{F} = \tfrac{1}{2}(\mathbf{G} - \mathbf{I}) + \tfrac{1}{2}(\mathbf{L} - \mathbf{I}). \tag{3}$$

DRB report that the correlation coefficient between the v_{ij} of equation (2) and the f_{ij} of equation (3) is -0.47 for the 6×6 Andalusian linkages that were analyzed. Thus, stronger interindustry linkages appear to have shorter APLs. This means that stronger linkages in general have both absolutely and relatively larger first-order impacts (measured by $\mathbf{A}$) than higher-order impacts (measured by $\mathbf{A}^2 + \mathbf{A}^3 + \mathbf{A}^4 + \ldots$). DR do not report the correlation between the length and the strength of the linkages in their European Union (EU) application, but we found it to be equal to -0.51 for the 8×8 linkages of the aggregate EU input–output table used by them.

When DR move from the eight EU sectors to the six EU countries at hand, they observe that the three smaller countries, with more open economies, and therefore weaker internal linkages, have shorter linkages. Note that this intercountry result runs opposite to the one found for aggregate sectors. This discrepancy remained unnoticed. To investigate this puzzle, we first look at the interregional differences in aggregate APLs and then turn to the interindustry differences in disaggregate APLs.

Table 1. Speed of Convergence of the Taylor Expansion and the Value of the APL^K.

$K =$	$a =$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1		1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
2		1.09	1.17	1.23	1.29	1.33	1.38	1.41	1.44	1.47	1.50
3		1.11	1.23	1.35	1.46	1.57	1.67	1.77	1.85	1.93	2.00
4		1.11	1.24	1.40	1.56	1.73	1.90	2.07	2.22	2.37	2.50
5		1.11	1.25	1.42	1.61	1.84	2.08	2.32	2.56	2.79	3.00
6		1.11	1.25	1.42	1.64	1.90	2.21	2.53	2.87	3.19	3.50
7		1.11	1.25	1.43	1.66	1.94	2.30	2.71	3.14	3.58	4.00
8		1.11	1.25	1.43	1.66	1.97	2.36	2.84	3.39	3.95	4.50
9		1.11	1.25	1.43	1.66	1.98	2.41	2.95	3.60	4.31	5.00
10		1.11	1.25	1.43	1.67	1.99	2.44	3.04	3.80	4.65	5.50
Limit APL		1.11	1.25	1.43	1.67	2.00	2.50	3.33	5.00	10.00	∞
at $K =$		3	5	7	10	12	16	23	41	94	∞

Note: APL = average propagation length.

Relation with the Size of the Macro Economy

We start our investigation of the relation between v_{ij} and f_{ij} by looking at a one-sector economy. In a one-sector economy, a_{ij} reduces to $a = \sum_j \sum_i z_{ij} \big/ \sum_j x_j$, while b_{ij} reduces to $b = \sum_i \sum_j z_{ij} \big/ \sum_i x_i$, where x_j indicates the total output of industry j. Hence, a and b are equal, and therefore, for a one-sector economy, equation (1) simplifies to:

$$v^K = \left(a + 2a^2 + 3a^3 \; \ldots \; + Ka^K\right) \big/ \left(a + a^2 + a^3 \; \ldots \; + a^K\right). \tag{4}$$

Clearly, the value of the one-sector APL increases with the closedness of the economy from the outside world, and therefore also with the economic size of the region or nation at hand (i.e., with a). Moreover, the value v^K increases with the number of rounds of the Taylor expansion of the APL that is taken into account (i.e., with K). Note that the Taylor expansion of equation (4) only converges for $0 \leq a < 1$. Table 1 shows these relationships for different values of a and K.

For the limit of $a \Rightarrow 0$, we get the outcome that, irrespective of the cutoff value K, the APL $v^K \Rightarrow 1$. DRB, however, define $v_{ij} = 0$ in case $l_{ij} - \delta_{ij} = 0$, whereas defining $v_{ij} = 1$ makes more sense, as it prevents a sudden drop from a value above 1 to 0 when $l_{ij} - \delta_{ij} \Rightarrow 0$. For the limit of $a \Rightarrow 1$, we get the outcome that the $v^K \Rightarrow 0.5 + 0.5\,K$ (see the last column of Table 1). Obviously, in that case, v^K goes to infinity if the cutoff point goes to infinity. This confirms our initial intuition of the APL having a value of infinity, but we wrongly expected it to reach infinity for all values of a.

The other columns of Table 1 show v^K at intermediate values of a.[2] For all intermediate values, the APL rises if the number of rounds considered rises. Moreover, the number of rounds that needs to be considered, in turn, rises with the size of the economy studied (i.e., with $a = b$), as shown in the last row of Table 1. In a larger, more closed economy, with stronger internal linkages, it takes longer for additional impacts to become negligible, and hence more weight is shifted toward the longer propagation lengths in the calculation of the average length.

The value of limit of the APL, shown in the one but last row of Table 1, may also be derived analytically. Define $l = (1 - a)^{-1}$ as the one-sector Leontief-inverse, and note that it equals $g = (1 - b)^{-1}$, the one-sector Ghosh inverse, as $a = b$. Then, for $K \Rightarrow$ infinity, using equation (2), equation (4) reduces to:

$$v = [l(l-1)]/(l-1) \Rightarrow v = l = g = f + 1. \tag{5}$$

Hence, for a one-sector economy, the APL is strictly equal to that economy's macro output multiplier from the Leontief model, as well as to its macro input multiplier from the Ghosh model. This makes the *length* of the aggregate linkage in a one-sector economy equal to its *strength*, as measured by equation (3), except for the +1.

Hence, our conclusion, on both counts, is that it is wrong to interpret the macro APL as an indication of the sophistication of the economy at hand or of the spatial fragmentation of its production chains, because its value follows from a simple sequence: the larger the economy, the larger its aggregate multiplier (i.e., $l = g$), the more rounds K are needed before its Taylor expansion converges, and thus the larger its propagation length v. This makes the APL unsuited for comparing domestic supply chains across nations and regions.

Relation with the Strength of the Linkage at Hand

But, does this also make the APL unsuited for comparisons of individual interindustry linkages? To answer that question we need to look at a multisector economy. In that case, the strict proportionality between the correctly weighted average length and the correctly weighted average strength of an interindustry linkage, shown in equation (5), does not necessarily hold for all individual pairs of v_{ij} and f_{ij}. Hence, the question is whether the strict macro relation between v and f is a good predictor of the interindustry relation between v_{ij} and f_{ij}. Since no analytical relation could be found, this question needs to be answered by the data.

We use the domestic intermediate transaction matrices of the EXIOPOL intercountry input–output table with forty-three countries and 129 industries, which for this purpose have been aggregated to the standard fifty-nine sector classification for the twenty-seven EU countries in that database.[3] These data confirm the positive correlation between the size of the macro economy, as measured by $a = b$, and the unweighted average of the 59×59 interindustry APLs. The correlation coefficient amounts to +0.78 for the forty-three countries. This strongly confirms the result of DR, who get smaller APLs for the three smaller EU countries, and larger APLs for the three bigger ones.

To investigate the relation at the level of the individual bilateral interindustry linkages, we first look at the relationship between all v_{ij} and f_{ij} pairs, irrespective of the country to which the pair belongs. Next, we investigate the correlation between the length and the strength of linkages for each of the forty-three countries separately,

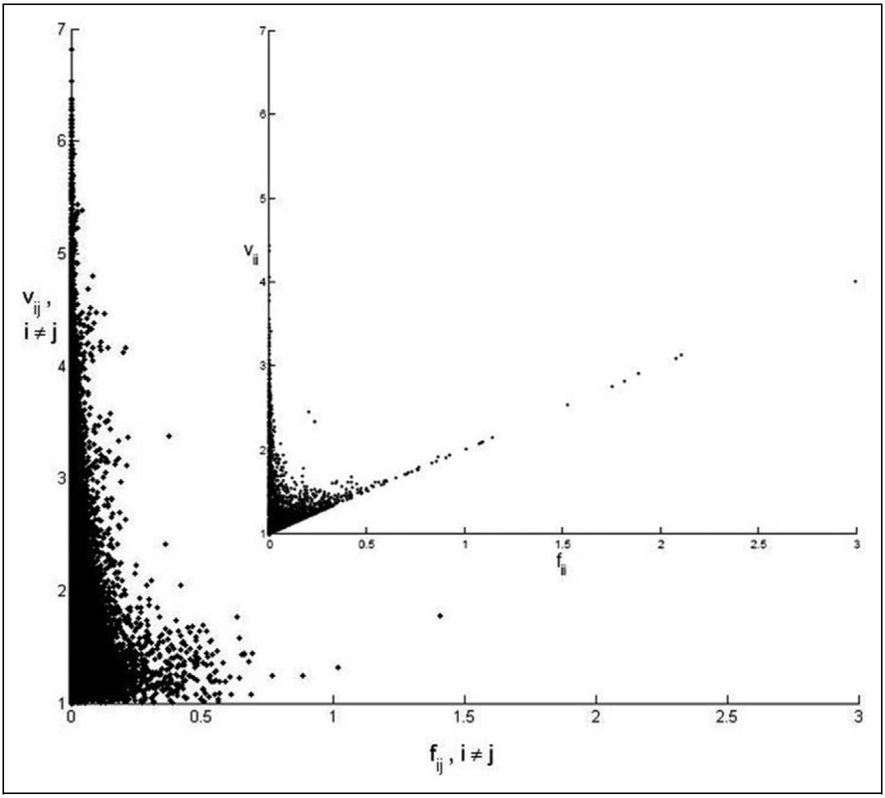

Figure 1. Scatterplot of all f_{ij}—v_{ij} pairs with $i \neq j$. The inset shows all f_{ii}—v_{ii} pairs.

and then we calculate the correlation across countries for each of the 59 $\times$ 59 interindustry linkages separately.

Figure 1 shows two scatterplots, together representing all diagonal and off-diagonal interindustry pairs of v_{ij} and f_{ij}. Note that the vertical APL axis starts at 1.00, as the length of any interindustry linkage is at least one, and not zero as in DRB. Figure 1 clearly shows that, for all interindustry linkages together, there is no positive correlation. The inset of Figure 1, however, shows that the diagonal, intra-industry linkages behave in a fashion that is quite different from that of the off-diagonal, pure interindustry linkages. Therefore, it is imperative to further investigate equation (2).

For the diagonal elements, equation (2) equals:

$$v_{ii} = \sum\nolimits_{s \neq i} l_{is} l_{si} / (l_{ii} - 1) + l_{ii} \geq l_{ii}. \tag{6}$$

Given that v_{ii} may also be calculated with the corresponding elements of the Ghosh inverse, it easily follows that

$$v_{ii} \geq l_{ii} = g_{ii} = f_{ii} + 1, \tag{7}$$

which is precisely the relationship found between v_{ii} and f_{ii} in the inset of Figure 1.

For the off-diagonal elements, equation (2) equals

$$v_{ij} = \sum\nolimits_{s \neq i \neq j} l_{is} l_{sj} / l_{ij} + l_{ii} + l_{jj} - 1 = \sum\nolimits_{s} l_{is} l_{sj} / l_{ij} - 1. \tag{8}$$

However, writing equation (8) with the corresponding elements of the Ghosh inverse and combining the result, as is done in equation (7), does not establish a formal relationship with f_{ij}, as is confirmed in Figure 1.

Besides the formal relationship between the length and the strength of the diagonal, intra-industry linkages, established in equation (7), the inset of Figure 1 also shows a surprisingly strong empirical relationship for the twenty strongest intra-industry linkages, which corresponds to the macro finding that the one-sector $v = f + 1$. These twenty strongest intra-industry linkages ($f_{ii} > 0.75$) have an intermediate input-output coefficient of $a_{ii} = b_{ii} > 0.35$ in common. For the thirty-three off-diagonal, interindustry linkages with $a_{ij} > 0.35$, however, there is no clear relationship between the length and the strength of these linkages.[4]

With hindsight, this difference is not too surprising. Relatively large diagonal cells will continually be multiplied with themselves in the second- and higher-order terms of the Taylor expansion of the Leontief inverse. Large off-diagonal cells, however, will not be multiplied with themselves in these higher order terms. Instead, they will be multiplied with generally smaller other cells (see also Oosterhaven, Eding and Stelder 2001). Thus, strong *direct* intra-industry linkages will have long supply chains that simply equal the strength of those chains $+1$. Strong *direct* inter-industry linkages, however, will tend to have shorter production chains.

These remarkable differences between the diagonal and the off-diagonal linkages lead to the conclusion that comparing the empirical values of intra-industry and pure interindustry APLs is not a sensible thing to do.

Next, we look at the relation between the length and the strength of the interindustry linkages by country. For the majority of countries, no significant correlation could be found for the diagonal, intra-industry linkages at the 10 percent confidence level, which is indicated by the dotted lines in the left-hand part of Figure 2.[5] At the 1 percent confidence level, indicated by the dashed lines, we find three significant negative correlations against nine significant positive correlations, the largest of which ($+0.76$ for Austria) resembles the strength of the theoretical relation found for a one-sector economy.

The picture of the off-diagonal, pure interindustry, linkages is both qualitatively and quantitatively quite different from that of the intra-industry linkages (compare the right- and the left-hand part of Figure 2). For off-diagonal linkages, all forty-three correlation country coefficients are negative, relatively small, but strongly significant. Our correlations coefficients are on average less than half the size of the -0.47 reported by DRB. The strong significance of our correlations supports their

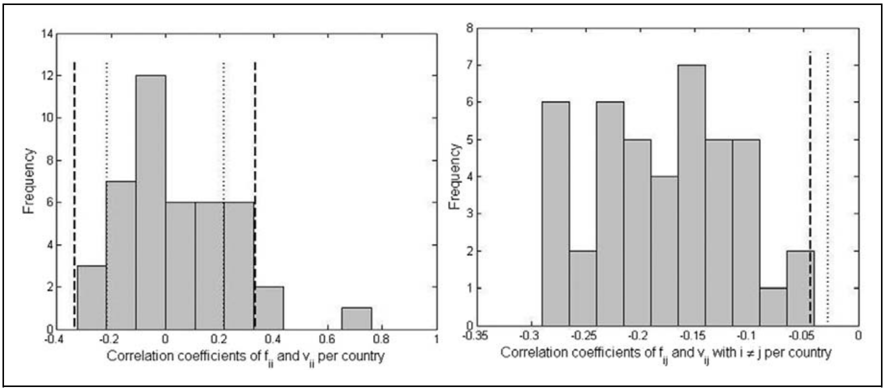

Figure 2. Correlation coefficients between length and strength of linkages, per country.

conclusion that there seems to be an inverse relationship between the length and the strength of interindustry linkages, but our result shows that this conclusion should be restricted to the off-diagonal linkages.

Thus, our conclusion that the APL is a useless concept only holds for comparisons of across regions and nations; it does not hold for pure interindustry linkages within a country. This implies that the DRB strategy of restricting the analysis to the APLs of the stronger linkages should have been limited to the off-diagonal, pure interindustry linkages. Restricting the analysis of intra-industry APLs to the stronger linkages does not make sense, as for those linkages the APL will simply be equal to the strength of that linkage $+1$.

Finally, 59×59 correlations are calculated for each interindustry pair to study whether large and more closed economies, with larger multipliers, systematically also have larger APLs or whether this relation breaks down at the level of individual interindustry pairs, as suggested by Figure 1. Figure 3 shows that the macro relation indeed breaks down, as most correlation coefficients are negative or insignificant. Again, the diagonal linkages behave different from the off-diagonal ones, but this time the difference is smaller (compare the left-hand part with the right-hand part of Figure 3).[6] The positive, right-hand tail of both distributions, however, is rather long and extends toward full correlation, especially for the intra-industry linkages. The two largest correlations relate to sector i19, leather and leather products ($+0.998$), and sector i20, wood and products of wood and cork ($+0.920$). In these cases, the countries with the strongest intra-industry linkages also have the longest intra-industry supply chains.

This confirms our earlier conclusion. Restricting the comparison of APLs of pure interindustry linkages to the stronger ones makes sense, also across nations and regions, but doing that for intra-industry linkages or for the macro APL is senseless.

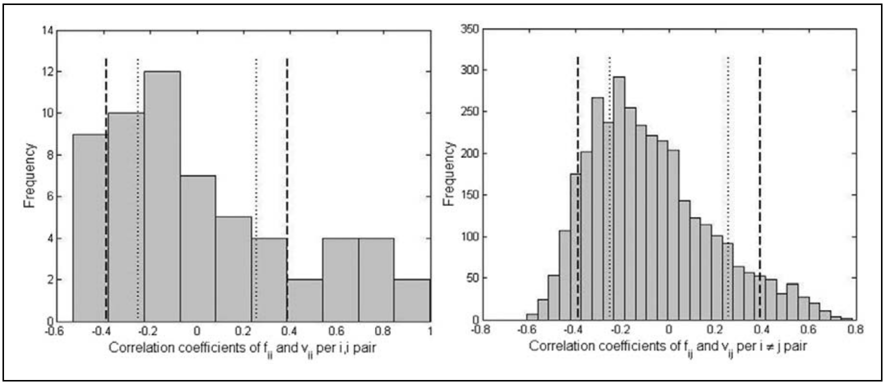

Figure 3. Correlation coefficients between length and strength of linkages, per linkage.

Conclusion

For a one-sector economy, we have shown that large and relatively closed economies, with large multipliers, have long supply chains by definition, if their length is measured by means of the APL. This also holds for the correctly weighted average of all pairs of industries i and j. These two findings render the APL a faulty measure when it is used to make comparative statements about the sophistication and fragmentation of the supply chain across nations and regions. Moreover, we found that the APL of the stronger intra-industry linkages is simply equal to the strength of that linkage $+1$. This implies that comparing intra-industry APLs to draw conclusions about the fragmentation or sophistication of these linkages also does not make sense.

However, at the pure interindustry level, from industry i and industry j, no formal relationship between the length (the APL) and the strength (the multiplier) of the supply chain between those industries could be derived. When looking at different interindustry linkages within each of the forty-three countries of the EXIOPOL database empirically, we found a weak negative, but significant correlation between the length and the strength of these linkages. Furthermore, when looking at the same interindustry linkage across the forty-three countries of the EXIOPOL database, the definitional positive correlation of $+1.00$ at the macro level turned into a negative or insignificant relation at the interindustry level.

This means that the APL only has an added informational value when pure interindustry linkages within a single region or country are compared. Our research into the properties of the APL also shows that it is quite dangerous to generalize from the macro level to the interindustry level, and vice versa. This can only be done if either theoretical or empirical proof is given.

Acknowledgments

The authors thank Erik Dietzenbacher and Bart Los for stimulating comments.

Declaration of Conflicting Interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

Notes

1. Note that Dietzenbacher, Romero, and Bosma (2005) thus follow Dietzenbacher's (1997) interpretation of the Ghosh model as the Leontief *price* model measured in terms of value changes. Consequently, the dual of Dietzenbacher's interpretation of the Ghosh model is identical to the Leontief quantity model. See Oosterhaven (1996) for the original interpretation of the Ghosh model as a supply-driven *quantity* model, which has a demand-pull, instead of a cost-push price model as its dual.
2. The values of a in the EXIOPOL data base, with forty-three uniformly defined national input–output tables for 2000, run from 0.245 for the small open economy of Ireland to 0.552 for the big and relatively closed economy of China.
3. See Tukker et al. (2009) for a general description of the EXIOPOL project, and Bouwmeester and Oosterhaven (2008) for a description of the construction of the international input–output table.
4. A further investigation of the thirty-three largest off-diagonal linkages showed that the cause of being large was the consequence of the usual definition of the output coefficients as the sales along the rows of the input–output table divided by their row total. For several industries, in several countries, this row total unfortunately includes quite sizable negative changes in stocks, which leads to economically implausible output coefficients $b_{ij} > 1$ that contradict the very idea of output coefficients functioning as allocation device in Ghosh (1958). Redefining the b_{ij} as the interindustry sales divided by total sales, instead of by total output, did not change the scatterplot qualitatively. The three off-diagonal values with the largest f_{ij} in Figure 1 get a much smaller f_{ij} when the b_{ij} are adjusted, whereas two diagonal values of f_{ii} fall below the linear relationship of $v_{ii} = f_{ii} + 1$, inter alia, because $a_{ii} = b_{ii}$ no longer holds.
5. The null hypothesis of zero correlation, with a two-tailed test, for the fifty-nine diagonal cells, gives critical values for the 1 percent and the 10 percent confidence levels of, respectively, ± 0.33 and ± 0.22, as drawn in the left-hand side of Figure 2. The critical values for the correlation coefficients that are based on the 58×59 off-diagonal cells are, respectively, ± 0.044 and ± 0.028, as drawn in the right-hand side of Figure 2.
6. The null hypothesis of zero correlation, with a two-tailed test, with forty-three observations, gives critical values for the 1 percent and the 10 percent confidence levels of ± 0.39 and ± 0.25, respectively, as drawn in Figure 3.

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