Economic Systems Research

ISSN: 0953-5314 (Print) 1469-5758 (Online) Journal homepage: https://www.tandfonline.com/loi/cesr20

Adjustment of Input–Output Tables from Two Initial Matrices

Esteban Fernandez Vazquez, Geoffrey J. D. Hewings & Carmen Ramos Carvajal

To cite this article: Esteban Fernandez Vazquez, Geoffrey J. D. Hewings & Carmen Ramos Carvajal (2015) Adjustment of Input–Output Tables from Two Initial Matrices, Economic Systems Research, 27:3, 345-361, DOI: 10.1080/09535314.2015.1007839

To link to this article: https://doi.org/10.1080/09535314.2015.1007839

Economic Systems Research, 2015
Vol. 27, No. 3, 345–361, http://dx.doi.org/10.1080/09535314.2015.1007839

Routledge
Taylor & Francis Group

ADJUSTMENT OF INPUT–OUTPUT TABLES FROM TWO INITIAL MATRICES

ESTEBAN FERNANDEZ VAZQUEZ[a*], GEOFFREY J. D. HEWINGS[b] and CARMEN RAMOS CARVAJAL[a]

[a]*Department of Applied Economics, Faculty of Economics, University of Oviedo, Oviedo, Spain;*
[b]*Regional Economics Application Laboratory, University of Illinois at Urbana-Champaign, Urbana, IL, USA*

(*Received 22 June 2010; in final form 11 January 2015*)

The compilation of the information required to construct survey-based input–output (I–O) tables consumes resources and time to statistical agencies. Consequently, a number of non-survey techniques have been developed in the last decades to estimate I–O tables. These techniques usually depart from observable information on the row and column margins, and then the cells of the matrix are adjusted using as a priori information a matrix from a past period (updating) or an I–O table from the same time period (regionalization). This paper proposes the use of a composite cross-entropy approach that allows for introducing both types of a priori information. The suggested methodology is suitable to be applied only to matrices with semi-positive interior cells and margins. Numerical simulations and an empirical application are carried out, where an I–O table for the Euro Area is estimated with this method and the result is compared with the traditional projection techniques.

Keywords: Entropy econometrics; Data-weighted prior estimation; Cross-entropy; Non-survey techniques

1. INTRODUCTION

Input–output (I–O) modeling at both the national and regional scales is a topic that has gained much attention in recent decades, given the huge number of potential applications (including Social Accounting Matrices, SAMs, and Computable General Equilibrium, CGE, models) for economic researchers and policy-makers. A well-known problem is that the compilation of the information required to build a survey-based I–O table is expensive and time-consuming for statistical agencies, causing a lag between the compilation of the information in the surveys and the publication of the table. Therefore, the use of non-survey methods to adjust I–O tables has become increasingly popular.

Essentially, the non-survey techniques for obtaining an I–O table take as point of departure an initial I–O table that is assumed to be similar to the table to be estimated, together with known information on the row and column margins of the target matrix. The basic idea of the estimation process is to choose as a solution the table that is closest to the initial matrix according to some divergence criteria, being consistent with the

*Corresponding author. E-mail: evazquez@uniovi.es

observable information of the target matrix.[1] One of the most frequently used adjustment procedures is the cross-entropy (CE) technique, which is based on the Kullback–Leibler (KL) (1959) divergence criterion. This adjustment technique has been proved to generate a solution equivalent to the popular RAS scaling algorithm (McDougall, 1999), although some authors claim that the CE procedure is preferable to some other alternatives because it allows a wide range of initial information to be utilized efficiently in the estimation process (Golan et al., 1994; Golan and Vogel, 2000; Robinson et al., 2001).

This paper explores the role played by the initial information in the adjustment of non-survey I–O tables from a new approach. Traditionally, the adjustment problem has taken as point of departure either a matrix from a past period (updating) or an I–O table from another economy (national or regional) for the same time period (regionalizing).[2] The discussion of the characteristics of the different adjustment techniques has attracted much attention in the literature (Mello and Teixeira, 1993; Jackson, 1998; Gilchrist and St Louis, 1999; Jalili, 2000; Jackson and Murray, 2004; Bonfiglio and Chelli, 2008; Oosterhaven et al., 2008). In this field of analysis, some attempts to employ multiple initial matrices in the estimation process have been made. Recent examples can be found in Minguez et al. (2009, for updating) and Oosterhaven and Escobedo (2011, for regionalizing), who suggested the so-called cell-corrected RAS (CRAS) method to choose the best option between several alternative I–O tables. Geschke et al. (2014) explore the possibilities of combining initial estimates and constraints for the specific case of harmonizing multi-regional I–O databases. The novelty of our proposal is that it considers the possibility of including several heterogeneous initial matrices in the adjustment of an I–O table. The suggested methodology is suitable for application only to matrices with semi-positive interior cells and margins.

The rest of the paper is structured as follows. Section 2 presents the basis of the CE solution for the estimation problem of a matrix with unknown cells but with information on its margins. In Section 3, the details of the composite CE technique proposed in this paper are introduced. Section 4 shows a numerical Monte Carlo experiment in which the performance of the proposed method is compared with that of other competing techniques. In Section 5, an empirical application is included, where an I–O table for the Euro Area is estimated based on contemporaneous and previous priors. Finally, Section 6 concludes the paper.

2. THE CE SOLUTION FOR THE MATRIX-BALANCING PROBLEM

We will base our explanations on the matrix-balancing problem depicted in Golan (2006, p. 105), where the goal is to fill the (unknown) cells of a matrix of dimension $(n \times c)$ utilizing the information contained in the row and column sums. The z_{ij} cells of the matrix are the unknown quantities we would like to estimate (shaded in gray), where the aggregates $\sum_{j=1}^{c} z_{ij} = z_{i\cdot}$, $\sum_{i=1}^{n} z_{ij} = z_{\cdot j}$ and $\sum_{i=1}^{n} \sum_{j=1}^{c} z_{ij} = z$ are known. This is a familiar situation in the context of I–O tables estimation, where it is usual to have available aggregate

[1] The information of the target matrix is normally assumed to result in non-conflicting constraints. Variants of adjusting procedures that deal with conflicting information can be found in van der Ploeg (1982) or, more recently, in Lenzen et al. (2009).

[2] The same choice must be made with the RAS algorithm.

information of the target matrix (intermediate inputs and outputs per industry) earlier than the flows.

The z_{ij} elements can be expressed as sets of (column) probability distributions, simply by dividing the quantities of the matrix by the corresponding column sums $z_{\cdot j}$. In such a case, the previous matrix can be rewritten in terms of a new matrix **P** that is composed of c column probability distributions, where p_{ij} is the proportions $z_{ij}/z_{\cdot j}$, and the new row and column margins are now defined as $v_j = z_{\cdot j}/Z$ and $y_i = z_{i\cdot}/Z$, respectively.

Consequently, the followings equalities are fulfilled by the p_{ij} elements:[3]

$$\sum_{i=1}^{n} p_{ij} = 1; \quad j = 1, \ldots, c \tag{1}$$

$$\sum_{j=1}^{c} p_{ij} v_j = y_i; \quad i = 1, \ldots, n \tag{2}$$

These two sets of equations reflect the information available about the elements of matrix **P**. Equation 2 shows the cross-relationship between the (unknown) p_{ij} in the matrix and the (known) sums of each row and column. Additionally, Equation 1 indicates that the p_{ij} can be assumed as (column) probability distributions. Note that we have only $n + c$ pieces of information to estimate the $n \times c$ elements of matrix **P**, which makes the problem ill posed.

A solution to this type of problems can be obtained by minimizing a divergence measure with a prior probability matrix **Q**, subject to the set of constraints (1) and (2). This is called a CE problem, which can be written in the following terms:

$$\underset{\mathbf{P}}{\text{Min}}\, D(\mathbf{P}||\mathbf{Q}) = \sum_{i=1}^{n} \sum_{j=1}^{c} p_{ij} \ln \left(\frac{p_{ij}}{q_{ij}} \right). \tag{3}$$

This expression is subject to the same restrictions given by the set of Equations 1 and 2. The divergence measure $D(\mathbf{P}||\mathbf{Q})$ is the KL entropy divergence between the posterior and prior distributions. The Lagrangian function for the CE problem is as follows:

$$L = D(\mathbf{P}||\mathbf{Q}) + \sum_{i=1}^{n} \lambda_i \left[y_i - \sum_{j=1}^{c} p_{ij} v_j \right] + \sum_{j=1}^{c} \mu_j \left[1 - \sum_{i=1}^{n} p_{ij} \right]. \tag{4}$$

And the solutions can be expressed as follows:

$$\tilde{p}_{ij} = \frac{q_{ij} \exp[\tilde{\lambda}_i v_j]}{\sum_{i=1}^{n} q_{ij} \exp[\tilde{\lambda}_i v_j]}, \quad i = 1, \ldots n; j = 1, \ldots c. \tag{5}$$

The CE estimation procedure can be seen as an extension of the maximum entropy (ME) principle, given that the solutions of both approaches are the same when the M a priori probability distributions contained in **Q** are all uniform.[4] It is well known that depending

[3] Note that in such a case, these p_{ij} elements can be seen as conditional probabilities for each column.

[4] In other words, the ME solutions are obtained by minimizing the KL divergence $D(\mathbf{P}||\mathbf{Q})$ between the unknown p_{ij} and the probabilities $q_{ij} = (1/n)$; $\quad i = 1, \ldots n; \quad j = 1, \ldots k$.

on the choice made when specifying $\mathbf{Q}$, the adjustment problem can be posed as one of updating (if we take as prior a previous matrix denoted as $\mathbf{Q^r}$) or of regionalizing (we denote as $\mathbf{Q^n}$ to the contemporaneous matrix used as a prior in the adjustment).[5] In some cases, there is no room for this choice simply because only one of these two priors is available, but it may well be that the prior matrices $\mathbf{Q^r}$ and $\mathbf{Q^n}$ are both available.

3. A COMPOSITE CE METHOD: THE DATA-WEIGHTED PRIOR ESTIMATION TECHNIQUE

The procedure introduced above can be extended to develop a more flexible estimator that allows for the inclusion of both prior matrices, $\mathbf{Q^r}$ and $\mathbf{Q^n}$, simultaneously in the estimation process. The technique suggested here has been proposed in Golan (2001) in the context of linear regression models. A recent application of this method can also be found in Bernardini (2008).

For the sake of simplicity, let us assume that the target matrix is a symmetric industry-by-industry I–O table ($n = c$). If we denote the two a priori (column) distributions with $\mathbf{q^r}$ and $\mathbf{q^n}$, the proposed objective could be achieved by modifying the previous CE program in the following way:

$$\begin{aligned}\underset{\mathbf{P},\mathbf{P}^{\gamma}}{\mathrm{Min}}\, D(\mathbf{P},\mathbf{P}^{\gamma}||\mathbf{Q^r},\mathbf{Q^n},\mathbf{Q}^{\gamma}) = &\sum_{j=1}^{c}(1-\gamma_j)\sum_{i=1}^{n} p_{ij}\ln\left(\frac{p_{ij}}{q_{ij}^{r}}\right)\\ &+\sum_{j=s1}^{c}\gamma_j\sum_{i=1}^{n} p_{ij}\ln\left(\frac{p_{ij}}{q_{ij}^{n}}\right)\\ &+\sum_{h=1}^{g}\sum_{j=1}^{c} p_{hj}^{\gamma}\ln\left(\frac{p_{hj}^{\gamma}}{q_{hj}^{\gamma}}\right)\end{aligned} \tag{6}$$

s. t.:

$$\sum_{j=1}^{c} p_{ij} v_j = y_i; \quad i = 1,\ldots,n \tag{7}$$

$$\sum_{i=1}^{n} p_{ij} = 1; \quad j = 1,\ldots,c \tag{8}$$

$$\sum_{h=1}^{g} p_{hj}^{\gamma} = 1, \quad j = 1,\ldots,c. \tag{9}$$

To understand the logic of this data-weighted prior (DWP) estimator, an explanation on the objective function of the previous minimization program is required. Note that Equation 6 is divided into three terms. The first term quantifies the divergence between

[5] See Hewings (1984) for a detailed discussion on the role played by prior information in such estimation problems.

the target and the a priori probabilities where the matrix $\mathbf{Q^r}$ is chosen as the prior, with this divergence weighted by $(1 - \gamma_j)$ for each industry. The second element of Equation 6 measures the divergence with the prior $\mathbf{Q^n}$, now weighted by γ_j.

Each parameter γ_j measures the weight given to the prior $\mathbf{q^n}$ for each industry, and they are estimated simultaneously with the unknown probabilities p_{ij} of the target matrix. Because the γ_j parameters represent weights, they should take values within the interval $(0, 1)$. We treat each γ_j as a discrete random variable and assume that its g possible values between 0 and 1 are contained in the support vector $\mathbf{b_j'} = [b^{\gamma}_{1j}, b^{\gamma}_{2j}, \ldots, b^{\gamma}_{gj}]$. For example, setting $\mathbf{b_j'} = [0,\ 0.25,\ 0.5,\ 0.75, 1]$ would define a case with $g = 5$. These support vectors are set by the researcher by providing the natural lower and upper bounds $b^{\gamma}_{1j} = 0$ and $b^{\gamma}_{gj} = 1$, respectively. The unknown probabilities $\mathbf{p'}^{\gamma}_{\mathbf{j}} = [p^{\gamma}_{1j}, p^{\gamma}_{2j}, \ldots, p^{\gamma}_{gj}]$ corresponding to these points define the expected value of the parameter as $\gamma_j = \sum_{h=1}^{g} b^{\gamma}_{gj} p^{\gamma}_{gj}$, and they must be recovered to estimate the parameter.

The third element in Equation 6 relates to the Kullback divergence between the target distributions $\mathbf{p}^{\gamma}_{\mathbf{j}}$ and the corresponding a priori distribution $\mathbf{q}^{\gamma}_{\mathbf{j}}$. Given our uncertainty about a sensible value for γ_j in the absence of additional information, a natural distribution is the uniform $(q^{\gamma}_{hj} = (1/g);\ \ j = 1, .., c)$. In other words, the CE criterion for the γ_j parameter can be transformed into a ME criterion. For example, in the case with $g = 5$ possible values for γ_j in the support vector $\mathbf{b'}_{\mathbf{j}} = [0,\ 0.25,\ 0.5,\ 0.75, 1]$, the uninformative prior would be $\mathbf{q'}^{\gamma}_{\mathbf{j}} = [0.25,\ 0.25,\ 0.25,\ 0.25, 0.25]$. This means that prior to the estimation, we assume that $\gamma_j = \sum_{h=1}^{g} b^{\gamma}_{hj} q^{\gamma}_{hj} = 0.5$, which reflects the a priori belief that both priors $\mathbf{Q^r}$ and $\mathbf{Q^n}$ should be equally weighted. Equation 6 can now be written in terms of the unknown probabilities $\mathbf{p}^{\gamma}_{\mathbf{j}}$ as follows:

$$\begin{aligned}\underset{\mathbf{P},\mathbf{P}^{\gamma}}{\text{Min}}\, D(\mathbf{P}, \mathbf{P}^{\gamma} || \mathbf{Q^r}, \mathbf{Q^n}, \mathbf{Q}^{\gamma}) &= \sum_{j=1}^{c} \left(1 - \sum_{h=1}^{g} b^{\gamma}_{hj} p^{\gamma}_{hj}\right) \sum_{i=1}^{n} p_{ij} \ln\left(\frac{p_{ij}}{q^{r}_{ij}}\right) \\ &+ \sum_{j=1}^{c} \sum_{h=1}^{g} b^{\gamma}_{hj} p^{\gamma}_{hj} \sum_{i=1}^{n} p_{ij} \ln\left(\frac{p_{ij}}{q^{n}_{ij}}\right) \\ &+ \sum_{h=1}^{g} \sum_{j=1}^{c} p^{\gamma}_{hj} \ln\left(\frac{p^{\gamma}_{hj}}{q^{\gamma}_{hj}}\right). \end{aligned} \tag{10}$$

The solutions of this minimization program are as follows:[6]

$$\tilde{p}_{ij} = \frac{q_{ij}^{r(\tilde{\gamma}_j/A_j)} \exp[(A_h^{-1})\tilde{\lambda}_i v_j]}{\sum_{i=1}^{n} q_{ij}^{r(\tilde{\gamma}_j/A_j)} \exp[(A_h{}^{-1})\tilde{\lambda}_i v_j]}, \quad i = 1, \ldots, n,\ j = 1, \ldots, c, \tag{11}$$

where

$$\tilde{\gamma}_j = \sum_{h=1}^{g} b^{\gamma}_{hj} \tilde{p}^{\gamma}_{hj},$$

[6] Details about the derivation of the solution can be found in Golan (2001, p. 175).

$$A_j = \frac{[1 - \tilde{\gamma}_j]}{\left[(\tilde{\gamma}_j - 1) - \tilde{\gamma}_j \sum_{i=1}^{n} q_{ij}^r \ln(q_{ij}^r) + \tilde{\gamma}_j\right]},$$

and $\tilde{\lambda}_i$ are the Lagrangian multipliers associated with restrictions (Equation 7).

Simultaneous to the estimation of the p_{ij} cells, the DWP estimator discriminates for each industry j between the two priors considered. The proposed estimation strategy provides estimates of the weighting parameters γ_j, obtained as follows:

$$\tilde{\gamma}_j = \sum_{h=1}^{g} b_{hj}^{\gamma} \tilde{p}_{hj}^{\gamma}. \tag{12}$$

Note that as $\tilde{\gamma}_j \rightarrow 0$, the prior $\mathbf{q^r}$ gain weights for industry j and the estimates will be similar to those in a CE updating process. In contrast, for large values of $\tilde{\gamma}_j$, the adjustment from prior $\mathbf{q^n}$ takes over. Consequently, relatively large values of $\tilde{\gamma}_j$ ($\tilde{\gamma}_j \geq 0.5$) will be an indication of an industry j characterized by a high weight of the prior $\mathbf{q^n}$. In other words, in this industry, an adjustment based on this prior would be preferable, rather than updating the distribution contained in the prior $\mathbf{q^r}$.

Note that this idea can be easily generalized to the case where more priors are considered, which is the situation described in Minguez et al. (2009) or Oosterhaven and Escobedo (2011). Assume that we have s possible $\mathbf{Q^t}$ matrices to be considered as possible priors; the divergence to minimize in Equation 6 would be transformed into the following expression:

$$\sum_{t=1}^{s}\left[\sum_{j=1}^{c} \gamma_j^t \sum_{i=1}^{n} p_{ij} \ln\left(\frac{p_{ij}}{q_{ij}^t}\right)\right] + \sum_{t=1}^{s}\left[\sum_{h=1}^{g}\sum_{j=1}^{c} p_{thj}^{\gamma} \ln\left(\frac{p_{thj}^{\gamma}}{q_{thj}^{\gamma}}\right)\right], \tag{13}$$

maintaining the conditions that the γ_j^t parameters are bounded between 0 and 1 and $\sum_{t=1}^{s}\left[\sum_{j=1}^{c} \gamma_j^t\right] = 1$. For the specific case where $s = 2$, the composite Kullback divergence reduces to expression (6).

4. EVALUATING THE DWP ESTIMATION TECHNIQUE WITH NUMERICAL EXPERIMENTS

To test the performance of the proposed estimation technique, we have conducted a numerical simulation where the DWP estimation is compared with traditional adjustment techniques that only consider one prior matrix.

In the experiment, we have fixed a target matrix $\mathbf{Z}$ of inter-industry flows, with its column and row sums being the only known information. The underlying assumption in the experiment is that the vectors with total purchases and sales are perfectly known. As Oosterhaven and Escobedo (2011) note, this could cause one to assume unfairly that the RAS-type adjustments perform better than other competing non-survey techniques (the location quotient method, for example), with the latter truly non-survey ones, whereas the RAS includes the additional information contained in the margins of the target matrix.

Even when having perfect information regarding vectors **v** and **y** could be considered unrealistic, it has usually been applied in the literature for comparison purposes, and it does not affect our comparisons because all the adjustment techniques to be evaluated are based on the same available information.

The target matrix has been set as the domestic industry-by-industry symmetric table for the Euro Area[7] in 2007 published by Eurostat, which distinguishes 58 different industries, detailed in Appendix A.[8] Additionally, the matrix **Z** has been transformed into a matrix of column coefficients **P** to be estimated from the information contained in vectors **v** and y. We have defined several a priori matrices **Q** to be used in the estimation of **P**. First, we have generated a matrix $\mathbf{Q^r}$ that plays the role of a previous matrix for the Euro Area. The values of this matrix have been obtained as $q_{ij}^r = p_{ij} \cdot u_{ij}^r$, where u_{ij}^r is a perturbation term with a distribution $u_{ij}^r \tilde{N}(1, \sigma)$, where $\sigma = 0.1$.

Additionally, we have also generated a matrix $\mathbf{Q^n}$ that can be assumed to play the role of an alternative prior matrix. The elements of $\mathbf{Q^n}$ have been obtained as follows:

$$q_{ij}^n = \begin{cases} q_{ij1}^n = p_{ij1} \cdot u_{ij1}^n & \text{and} \quad u_{ij1}^n \tilde{N}(1, 2\sigma), \; j_1 = 0, \ldots, k, \\ q_{ij2}^n = p_{ij2} \cdot u_{ij2}^n & \text{and} \quad u_{ij2}^{bn} \tilde{N}(1, 0.5\sigma), \; j_2 = k+1, \ldots, 58. \end{cases}$$

In other words, this new a priori matrix is characterized by having k columns – from industry number 1 to number k – more dissimilar to the target matrix **P** than the competing prior $\mathbf{Q^r}$, which means that in these cases it would be preferable taking $\mathbf{Q^r}$ as the initial matrix. However, for the remaining industries – number $k+1$ to 58 – the opposite holds, given that the distribution is closer to the target matrix than the prior $\mathbf{Q^r}$.

In this scenario, we have estimated matrix **P** by three different methods: by adjusting from matrix $\mathbf{Q^r}$, by adjusting from matrix $\mathbf{Q^n}$, and by applying the proposed DWP estimation technique that takes both matrices as possible priors. These three estimation strategies correspond, respectively, to the minimization of the following three divergence measures:

$$\underset{\mathbf{P}}{\text{Min}}\, D(\mathbf{P}||\mathbf{Q^r}) = \sum_{i=1}^{58} \sum_{j=1}^{58} p_{ij} \ln\left(\frac{p_{ij}}{q_{ij}^r}\right), \tag{14}$$

$$\underset{\mathbf{P}}{\text{Min}}\, D(\mathbf{P}||\mathbf{Q^n}) = \sum_{i=1}^{58} \sum_{j=1}^{58} p_{ij} \ln\left(\frac{p_{ij}}{q_{ij}^n}\right), \tag{15}$$

[7] The Euro Area in 2007 consisted of the following 17 countries belonging to the EU-27: Austria, Belgium, Cyprus, Estonia, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Malta, the Netherlands, Portugal, Slovakia, Slovenia, and Spain.

[8] The branch 59, corresponding to 'Private households with employed persons' has been not considered in the numerical simulation. Data can be downloaded from: http://epp.eurostat.ec.europa.eu/portal/page/portal/esa95_supply_use_input_tables/data/workbooks

$$\begin{aligned}\underset{\mathbf{P},\mathbf{P}^{\gamma}}{\text{Min}}\, D(\mathbf{P},\mathbf{P}^{\gamma}||\mathbf{Q^r},\mathbf{Q^n},\mathbf{Q}^{\gamma}) &= \sum_{j=1}^{58}\left(1-\sum_{h=1}^{2} b_{hj}^{\gamma}p_{hj}^{\gamma}\right)\sum_{i=1}^{58} p_{ij}\ln\left(\frac{p_{ij}}{q_{ij}^{r}}\right)\\ &+\sum_{j=1}^{58}\sum_{h=1}^{2} b_{hj}^{\gamma}p_{hj}^{\gamma}\sum_{i=1}^{58} p_{ij}\ln\left(\frac{p_{ij}}{q_{ij}^{n}}\right)\\ &+\sum_{h=1}^{2}\sum_{j=1}^{58} p_{hj}^{\gamma}\ln\left(\frac{p_{hj}^{\gamma}}{0.5}\right),\end{aligned} \tag{16}$$

subject to the same type of constraints explained previously. Note that in the last term in Equation 16 we explicitly set support vectors for the γ_j parameters with $g = 2$ values (i.e. $\mathbf{b}'^{\gamma}_{\mathbf{j}} = [0,\ 1]$), which are assumed as equally probable ($q_{hj}^{\gamma} = 0.5$).[9]

To evaluate the performance of these estimation approaches, 1,000 trials have been conducted, and we have computed the average of a measure of overall deviation between the target matrix and the estimates. Table 1 shows the weighted absolute percentage error – WAPE –, which is an indicator frequently used when evaluating the adjustment of I–O tables (see Jiang et al., 2010 and 2012 for recent examples). This measure averages the percentage error, giving larger weights to errors in large cells than to errors in small cells (Oosterhaven et al., 2008). It is defined as follows:

$$\text{WAPE} = \sum_{i=1}^{58}\sum_{j=1}^{58} 100\frac{|z_{ij}-\hat{z}_{ij}|}{\sum_{i=1}^{58}\sum_{j=1}^{58} z_{ij}}, \tag{17}$$

where the $\hat{z}_{ij}$ elements denote the estimated flows. To extend this evaluation, we also obtained the deviation WAPE measure between the actual and estimated technical coefficients a_{ij}, where $a_{ij} = z_{ij}/x_j$ and x_j is the total output of industry j in the target I–O matrix. These output values are assumed known in the experiment, and they have been set as the actual output values of the inter-industry I–O table of the Euro Area in 2007. Finally, we also considered the accuracy in the estimation of the output multipliers, obtaining the WAPE for the l_{ij} elements of the matrix $\mathbf{L}$, with $\mathbf{L} = [\mathbf{I}-\mathbf{A}]^{-1}$ the inverse of Leontief.[10] Additionally, the table reports the so-called *Degrees of approximation*, indicating the average percentage of estimated elements (z_{ij}, a_{ij} and l_{ij}) that lie within 5% of the corresponding true values. Finally, an additional entropy-based criterion as the KL divergence is computed between the actual and estimated cells:

$$\text{KL} = \sum_{i=1}^{58}\sum_{j=1}^{58} z_{ij}\ \ln\left(\frac{z_{ij}}{\hat{z}_{ij}}\right). \tag{18}$$

The comparative performance of the competing techniques would depend to a great extent on the degree of relative similarity of the priors utilized with the target matrix $\mathbf{P}$.

[9] Because, by definition, the weighting parameters range between 0 and 1, we opted to set the minimum number of points required to obtain a solution ($g = 2$), to alleviate the computational burden in the experiment. In Golan et al. (1996, chapter 8), several simulation experiments are conducted to conclude that the outcomes of this type of estimator are not sensitive to the number of discrete points considered in the supporting vectors.

[10] We also follow the usual procedure of replacing the elements of $\mathbf{L}$ (l_{ij}) with $l_{ij} - \delta_{ij}$, where δ_{ij} is the Kronecker delta ($\delta_{ij} = 1$ if $i = j$; $\delta_{ij} = 0$ otherwise).

Note that k can be taken as an indicator of the comparative similarity between the prior $\mathbf{Q^n}$ and the target matrix. Small values of k would indicate a higher closeness between $\mathbf{Q^n}$ and $\mathbf{P}$; however, large values of k would be a signal of a greater similarity between $\mathbf{Q^r}$ and $\mathbf{P}$. Table 1 summarizes the results obtained for several values of k along 1,000 simulations for each k-value:

Table 1 provides an interesting perspective about the performance of the adjustment techniques. If the prior $\mathbf{Q^n}$ were closer to $\mathbf{P}$ for each and every column than the prior $\mathbf{Q^r}$ ($k = 58$), there would be no gain from using the composite prior because it would always be better to take $\mathbf{Q^n}$ as prior than the competing $\mathbf{Q^r}$, and it would also be preferable to

TABLE 1. Evaluation of the adjustment for flows, technical coefficients and output multipliers.

		WAPE (%)		
	Technique (prior used)	z_{ij}	a_{ij}	l_{ij}
$k = 0$	Adjustment from $\mathbf{Q^n}$	3.35	3.54	2.41
	Adjustment from $\mathbf{Q^r}$	6.65	7.03	4.79
	DWP ($\mathbf{Q^n}$, $\mathbf{Q^r}$)	3.73	4.00	2.72
$k = 15$	Adjustment from $\mathbf{Q^n}$	4.59	6.57	4.49
	Adjustment from $\mathbf{Q^r}$	6.65	7.03	4.79
	DWP ($\mathbf{Q^n}$, $\mathbf{Q^r}$)	4.21	5.10	3.47
$k = 30$	Adjustment from $\mathbf{Q^n}$	7.67	9.61	6.63
	Adjustment from $\mathbf{Q^r}$	6.65	7.03	4.79
	DWP ($\mathbf{Q^n}$, $\mathbf{Q^r}$)	5.34	6.25	4.29
$k = 45$	Adjustment from $\mathbf{Q^n}$	11.15	12.36	8.44
	Adjustment from $\mathbf{Q^r}$	6.65	7.03	4.79
	DWP ($\mathbf{Q^n}$, $\mathbf{Q^r}$)	6.44	6.96	4.66
$k = 58$	Adjustment from $\mathbf{Q^n}$	13.42	14.04	9.57
	Adjustment from $\mathbf{Q^r}$	6.65	7.03	4.79
	DWP ($\mathbf{Q^n}$, $\mathbf{Q^r}$)	7.53	7.91	5.39
		Degrees of approximation (%)		
	Technique (prior used)	z_{ij}	a_{ij}	l_{ij}
$k = 0$	Adjustment from $\mathbf{Q^n}$	80.59	80.59	94.93
	Adjustment from $\mathbf{Q^r}$	66.96	66.96	82.01
	DWP ($\mathbf{Q^n}$, $\mathbf{Q^r}$)	78.35	78.35	93.54
$k = 15$	Adjustment from $\mathbf{Q^n}$	73.67	73.67	86.51
	Adjustment from $\mathbf{Q^r}$	66.96	66.96	82.01
	DWP ($\mathbf{Q^n}$, $\mathbf{Q^r}$)	74.13	74.13	89.27
$k = 30$	Adjustment from $\mathbf{Q^n}$	67.81	67.81	78.87
	Adjustment from $\mathbf{Q^r}$	66.96	66.96	82.01
	DWP ($\mathbf{Q^n}$, $\mathbf{Q^r}$)	70.74	70.74	85.61
$k = 45$	Adjustment from $\mathbf{Q^n}$	62.58	62.58	73.33
	Adjustment from $\mathbf{Q^r}$	66.96	66.96	82.01
	DWP ($\mathbf{Q^n}$, $\mathbf{Q^r}$)	67.88	67.88	82.79
$k = 58$	Adjustment from $\mathbf{Q^n}$	59.30	59.30	68.82
	Adjustment from $\mathbf{Q^r}$	66.96	66.96	82.01
	DWP ($\mathbf{Q^n}$, $\mathbf{Q^r}$)	65.37	65.37	79.51

(*Continued*).

TABLE 1. Continued.

	Technique (prior used)	Theil divergence z_{ij}	a_{ij}	l_{ij}
$k = 0$	Adjustment from $\mathbf{Q^n}$	7562.8	0.029	0.025
	Adjustment from $\mathbf{Q^r}$	29,808.1	0.116	0.109
	DWP ($\mathbf{Q^n}$, $\mathbf{Q^r}$)	9375.9	0.037	0.038
$k = 15$	Adjustment from $\mathbf{Q^n}$	21,209.1	0.162	0.139
	Adjustment from $\mathbf{Q^r}$	29,808.1	0.116	0.109
	DWP ($\mathbf{Q^n}$, $\mathbf{Q^r}$)	12,960.8	0.070	0.061
$k = 30$	Adjustment from $\mathbf{Q^n}$	56,727.3	0.291	0.258
	Adjustment from $\mathbf{Q^r}$	29,808.1	0.116	0.109
	DWP ($\mathbf{Q^n}$, $\mathbf{Q^r}$)	21,691.7	0.103	0.092
$k = 45$	Adjustment from $\mathbf{Q^n}$	97,806.4	0.409	0.362
	Adjustment from $\mathbf{Q^r}$	29,808.1	0.116	0.109
	DWP ($\mathbf{Q^n}$, $\mathbf{Q^r}$)	30,020.9	0.115	0.112
$k = 58$	Adjustment from $\mathbf{Q^n}$	128,530.1	0.487	0.423
	Adjustment from $\mathbf{Q^r}$	29,808.1	0.116	0.109
	DWP ($\mathbf{Q^n}$, $\mathbf{Q^r}$)	39,434.7	0.151	0.128

any possible combination of $\mathbf{Q^n}$ and $\mathbf{Q^r}$. A similar conclusion would be obtained in an alternative case when, for each column, $\mathbf{Q^r}$ is more similar to $\mathbf{P}$ than $\mathbf{Q^n}$ ($k = 0$).

These two extreme cases illustrate when the DWP yields deviation measures that lie between the two adjustments based on one single prior: the DWP estimator selects the industries (columns) where utilizing one prior is preferable than employing the other. If one of them were superior for all columns, the best DWP possible solution would be an adjustment from this superior prior: in the limit $\tilde{\gamma}_j \to 0$ or $\tilde{\gamma}_j \to 1$, $\forall j$ depending on the case. In these situations, the results of the experiment suggest that, even when the DWP estimator is not the best option to utilize the available prior information, the deviation measures are not much larger than the best competing option. It is in intermediate situations when the DWP estimator seems to outperform the adjustment from one single prior, given that it manages to include in the estimation process a mix of columns from both priors composed of those most similar to the target in each case. Computing the rate of change of the estimates of γ_j with respect to the case where the two priors are equally preferable ($\gamma_j = 0.5$) provides a picture of how the DWP estimator adjusts the weights between the two priors. Table 2 shows the average percentage of change with respect to the case of equally weighted priors along the 1,000 simulations in the experiment.

Table 2 reports this rate of change for different values of k (more specifically, $k = 0, 15, 30, 45$ and 58). If, for example, $k = 15$, this means that the first 15 industries of $\mathbf{Q^n}$ are generated as $q^n_{ij1} = p_{ij1} \cdot u^n_{ij1}$, and the rest as $q^n_{ij2} = p_{ij2} \cdot u^n_{ij2}$. A positive value in Table 2 indicates that for that particular industry, the average $\tilde{\gamma}_j$ estimate is larger than 0.5, and the opposite is true if the cell is negative. The cells marked in gray are the cases where we have industries with $\mathbf{Q^n}$ more dissimilar than $\mathbf{Q^r}$to the target matrix. This makes prior $\mathbf{Q^r}$ preferable in these industries, and one would expect that in these cases, the weighting parameters were lower than the reference 0.5. In the gray cells, we would expect to have negative corrections, whereas in the rest of the cells, the percentage should be to be positive on average. Even when there are some exceptions, Table 2 shows that when k increases,

TABLE 2. Average rate of change (%) of the γ_j parameters with respect to the uniform solution 0.5.

	$k = 0$	$k = 15$	$k = 30$	$k = 45$	$k = 58$
ind1	– 0.0339	0.3018	0.1513	– 0.8277	– 0.3519
ind2	– 0.9247	– 0.5707	0.5509	– 0.9191	0.619
ind3	– 0.2546	0.3054	0.4521	0.92	0.8639
ind4	– 0.2638	0.0442	0.8301	0.9909	0.0287
ind5	– 0.3802	0.0973	1.3981	0.7574	– 0.7180
ind6	0.6544	– 1.0725	0.5262	1.2371	0.0890
ind7	0.2772	0.1724	– 0.2159	0.3267	– 0.0418
ind8	– 0.2962	– 0.5767	– 0.8453	0.4742	0.5066
ind9	– 0.7535	– 0.8077	– 0.9003	– 0.4688	0.2884
ind10	– 0.1352	0.1115	– 0.0521	0.3206	0.9798
ind11	0.4596	– 1.0688	– 0.6946	– 1.1194	0.0875
ind12	– 0.1290	0.0919	0.4150	– 0.2489	1.3766
ind13	– 0.2460	– 1.4147	– 0.2744	0.6358	0.9236
ind14	0.4312	0.7494	0.5896	0.3756	– 0.5022
ind15	– 0.3918	– 0.6825	0.5862	0.7178	– 0.7587
ind16	0.0060	– 0.3076	– 0.9798	0.5216	0.0021
ind17	– 0.3364	0.6658	0.4158	0.0042	– 0.4738
ind18	– 0.1323	0.0574	– 0.1209	0.2024	– 0.4335
ind19	0.2617	– 0.0097	0.6775	– 0.6658	1.3143
ind20	– 0.0833	0.5483	– 0.4145	0.6457	0.4697
ind21	0.1152	0.1572	– 1.5620	– 1.5425	– 0.2656
ind22	0.2943	– 0.2628	– 0.8703	– 0.0254	– 0.9455
ind23	0.1227	– 0.3963	– 0.9279	0.0989	– 0.8108
ind24	– 0.3299	– 0.3874	0.2454	– 1.2175	1.2178
ind25	0.1273	– 0.4470	– 0.3769	0.388	– 0.1074
ind26	0.0693	– 0.1144	– 0.9288	0.0917	– 0.8070
ind27	0.3011	– 0.1880	0.0282	– 0.5897	– 0.5133
ind28	0.0192	0.3821	– 0.4485	1.2281	0.0306
ind29	0.8667	– 0.2407	– 1.3790	– 0.841	0.0922
ind30	0.0663	0.5843	– 0.0172	0.6769	– 0.1588
ind31	– 0.1727	0.5843	– 0.0262	0.4189	0.7899
ind32	1.5221	0.0793	– 0.5684	– 1.1144	0.5355
ind33	0.3362	– 0.6864	0.3771	– 0.1487	– 0.4222
ind34	0.2094	0.5174	– 0.7766	– 1.9137	– 0.0568
ind35	– 0.0373	– 0.7417	0.4282	– 0.5074	0.2765
ind36	0.1376	0.5252	– 0.1308	0.4304	0.5343
ind37	0.1829	– 0.1558	– 0.3759	– 1.1854	– 0.7209
ind38	– 0.2943	– 0.2418	– 0.8523	– 0.5341	– 1.0157
ind39	0.2963	– 0.5658	– 0.0008	– 0.4570	– 0.6298
ind40	– 0.9762	– 0.0733	0.4016	– 0.1299	– 0.0607
ind41	– 0.1365	0.2046	– 0.1456	– 0.5593	– 0.2699
ind42	– 0.1980	– 0.4678	0.3538	0.1968	– 0.4105
ind43	0.5354	0.1211	0.3579	– 0.7247	0.3116
ind44	0.1707	0.0984	0.2010	0.0372	0.0519
ind45	– 0.8135	0.0657	– 0.8499	– 3.1290	– 2.327
ind46	0.2711	– 0.3482	– 0.0757	– 0.0566	0.272

(*Continued*).

TABLE 2. Continued.

	$k = 0$	$k = 15$	$k = 30$	$k = 45$	$k = 58$
ind47	0.2564	0.1581	1.1336	1.0374	– 0.962
ind48	– 0.7434	1.0690	– 0.0841	– 0.0406	0.2259
ind49	0.0418	– 0.1562	– 0.1502	– 0.2729	1.8199
ind50	– 0.0886	– 0.2032	– 0.5301	– 0.5104	0.3558
ind51	– 0.0972	– 0.2357	– 0.0030	0.1439	– 0.8393
ind52	0.3048	0.1992	0.2426	0.2414	0.7416
ind53	– 0.0930	0.1464	0.1723	0.5849	– 0.2954
ind54	0.4900	0.4992	– 0.0399	0.0104	– 0.7255
ind55	– 0.8389	– 0.0645	0.4800	0.4328	1.0077
ind56	0.2201	0.5445	– 0.4026	– 0.1587	0.4747
ind57	0.7716	– 0.2045	– 0.3426	0.0333	– 0.2741
ind58	0.3396	– 0.0850	0.2686	0.3202	– 0.0965
Mean	0.0940	0.0269	– 0.1604	– 0.1752	– 0.2160

Note: The mean is calculated as a weighted mean, weighting the % changes by the relative output of the industry in the target I–O table.

there is a tendency in the DWP adjustment to produce $\tilde{\gamma}_j$ estimates that are, on average, lower than 0.5.

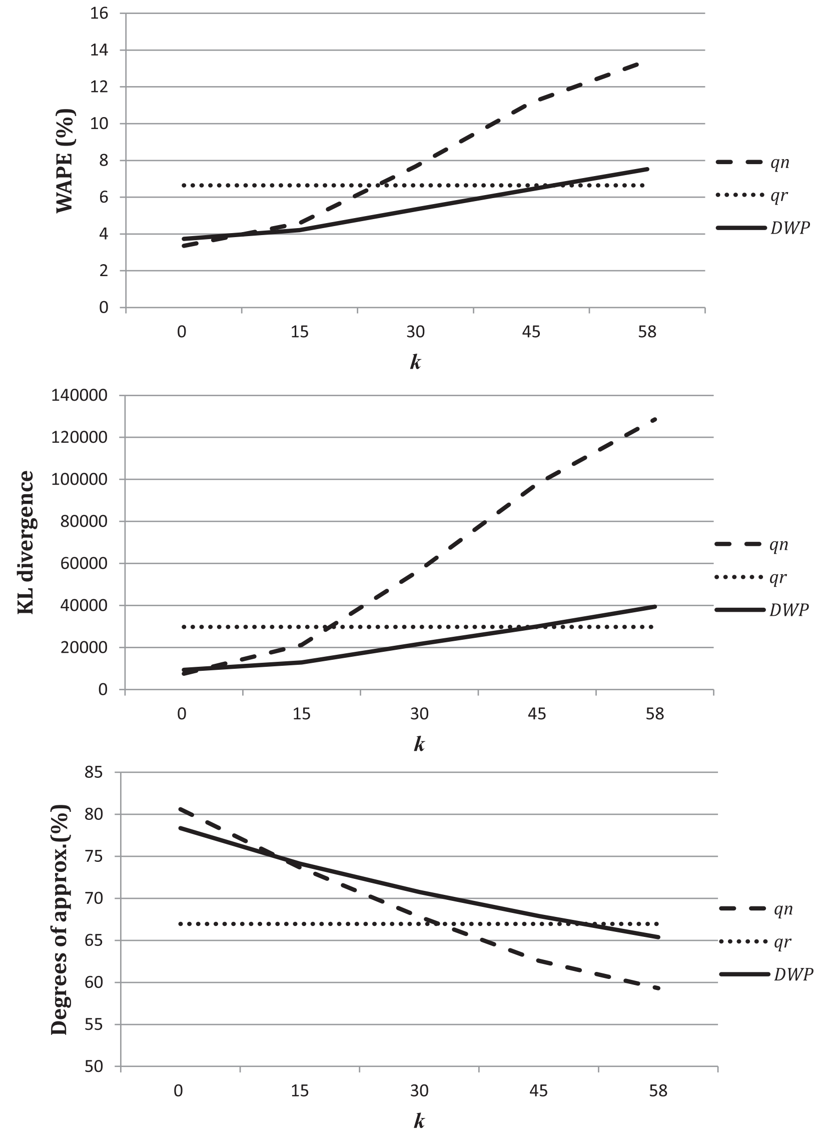
The following figure extends this idea, presenting the deviations between the actual and estimated inter-industry flows for the values of k in the previous numerical simulation. The same three divergence criteria have been taken as references: the WAPE, the degrees of approximation and the KL divergence between actual and estimated flows (Figure 1).

The horizontal axis of both figures contains the number of columns that behave like $q^n_{ij1} = p_{ij1} \cdot u^n_{ij1}$ in the prior $\mathbf{Q^n}$. The vertical axis in the upper and middle figure presents the mean of the absolute errors and the KL divergence, respectively, corresponding to the three adjusting techniques along 1,000 simulations for each value of k. Finally, the lower graph reports the average percentage of cells where the estimated flows lie within 5% of the true flows. The discontinuous line represents the results when they are obtained by adjusting from $\mathbf{Q^r}$. Because it does not depend on the characteristics of $\mathbf{Q^n}$, it is a constant value. The dotted line represents the results obtained by adjusting employing $\mathbf{Q^n}$ as prior. Not surprisingly, the deviations are lower and the degrees of approximation higher when all the columns are more similar to $\mathbf{P}$ than the other prior $\mathbf{Q^r}$, producing worse results as the number of more dissimilar columns also increases.

The solid line represents the results for the estimates obtained by the DWP technique. When the prior $\mathbf{Q^n}$ has very few columns more dissimilar to $\mathbf{P}$ than $\mathbf{Q^r}$, the DWP estimation yields worse divergence indicators than an adjustment from $\mathbf{Q^n}$. However, if $\mathbf{Q^n}$ has many columns more dissimilar to $\mathbf{P}$ than $\mathbf{Q^r}$, the DWP adjustment produces higher deviations than an adjustment from $\mathbf{Q^r}$. It is in intermediate situations where the DWP approach obtains better results than any adjustment from only one of the priors, given that in such situations taking a composite prior allows for choosing the most valuable information contained in each one.

The results suggest that combining two priors for the estimation of I–O tables could be preferable than basing the adjustment on only one prior. Although one of them could

FIGURE 1. Weighted absolute deviations, degrees of approximation and KL divergence for the inter-industry flows.



be more appropriate for the adjustment in general terms, this does not mean that all the information contained in the other should be neglected because some part of the column coefficients can distribute more similarly with the target matrix. If we incorporate both matrices of a priori information in the adjustment process by employing the DWP estimation, we let the data speak for themselves and choose the most appropriate prior for each industry, which in the end obtains smaller deviation measures.

5. AN ILLUSTRATION: ESTIMATING THE INDUSTRY-BY-INDUSTRY TABLE FOR THE EURO AREA, 2007

As a complement to the numerical simulation conducted in the previous section, this section presents an empirical application of the proposed DWP technique and compares the results obtained with other techniques. For this purpose, we again take the industry-by-industry symmetric I–O table for the Euro Area in 2007, elaborated by Eurostat and with a sector classification consisting of 58 industries. We assume fully correct information on the row and column margins (intermediate outputs and inputs, respectively) and the vector $\mathbf{x}$ of total output, and we aim to estimate the inter-industry flows matrix $\mathbf{Z}$, the matrix $\mathbf{A}$ of technical coefficients and the Leontief inverse $\mathbf{L}$. The adjustment will be made from different prior matrices, all of them with the same 58 industry classifications. Details of the industry classification are given in Appendix A.

Specifically, we considered the symmetric I–O table for the whole set of 27 countries that formed the European Union (UE-27) in 2007 ($\mathbf{Q^n}$) and the previous I–O table for the Euro Area in 2002 ($\mathbf{Q^r}$), as the two prior matrices assumed available. A time lag of five years is usually considered reasonable because the I–O relations of an economy generally hold constant in such a period. However, it might happen that some relevant changes happened only for a group of industries. For the case of these industries, taking the 2002 table for the Euro Area as prior would not be the best option if their structure had changed significantly with respect to 2002, and the contemporaneous I–O table for the EU-27 in 2007 could be preferable as prior in these cases.

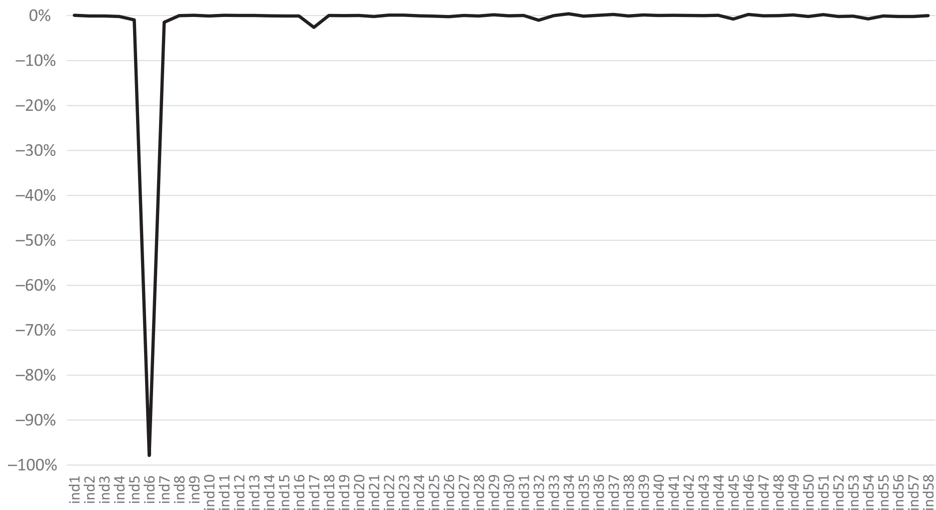
Table 3 summarizes the results obtained in this study case.

In line with the results obtained in the numerical experiments on the previous section, the proposed DWP technique produces lower absolute error measures by combining the two a priori matrices for the flows, the technical coefficients and the Leontief multipliers. This happens because, in our empirical problem, one of the priors is not always preferable to the other. Reporting the percentage change of the estimates of γ_j with respect to the case where the two priors are equally preferable ($\gamma_j = 0.5$) is also illustrative (Figure 2):

TABLE 3. WAPE and degrees of approximation (%) for flows, technical coefficients and output multipliers.

	WAPE (%)		
Technique (prior used)	z_{ij}	a_{ij}	l_{ij}
Adjusting from $\mathbf{Q^n}$	9.03	15.11	10.32
Adjusting from $\mathbf{Q^r}$	8.67	14.30	9.74
DWP ($\mathbf{Q^n}$, $\mathbf{Q^r}$)	6.63	12.16	8.05
	Degrees of approximation (%)		
Technique (prior used)	z_{ij}	a_{ij}	l_{ij}
Adjusting from $\mathbf{Q^n}$	64.56	64.56	78.33
Adjusting from $\mathbf{Q^r}$	61.03	61.03	77.79
DWP ($\mathbf{Q^n}$, $\mathbf{Q^r}$)	63.64	63.64	82.28

FIGURE 2. Percentage of change in the weighting parameters (reference: $\gamma_j = 0.5$).



In general, the estimates obtained weigh approximately equal to the prior distributions contained in the column coefficients of both matrices. However, in some industries, such as ‘Coke, refined petroleum products and nuclear fuels’ (ind17), ‘Metal ores’ (ind7) and, remarkably, ‘Uranium and thorium ores’ (ind6), the weights assigned to the contemporaneous prior $\mathbf{Q^n}$ are lower than those estimated for $\mathbf{Q^r}$. This should be interpreted as a signal of industries with larger similarities between the target matrix and the 5-year lagged Euro Area I–O table compared to the contemporaneous EU-27 table.

6. CONCLUDING REMARKS

I–O modeling often requires using some non-survey methods for estimating I–O tables. Traditionally, these techniques take an initial I–O table, similar to the table to be estimated, which is somehow adjusted to be consistent with the constraints imposed by the known information, and at the same time is the closest to the prior matrix according to some divergence criterion (the CE technique is a well-known example of such a procedure). The adjustment problem takes as point of departure either a previous matrix from a past period (updating) or an I–O table from the same time period (regionalizing).

This paper proposes a new approach for dealing with this initial information. Based on Golan (2001), the so-called DWP estimation strategy considers the possibility of including several a priori matrices in the adjustment of an I–O table, allowing for combining different types of prior information. By means of a Monte Carlo simulation, the performance of the proposed DWP method is compared with other adjustment techniques. The findings of this experiment suggest that the proposed DWP estimator can be useful in the situation where none of the available prior matrices is preferable to the other for all cases (industries). The empirical application, where the I–O table for the Euro Area in 2007 is estimated, confirms this conclusion. The results of both the simulation and the empirical example are encouraging for developing extensions of the DWP adjustment. For example, it would be possible to consider a priori weights different from the case where the two initial I–O tables are equally weighted, if previous experience suggests favoring one of the priors that performs consistently better than the other. Additionally, modifications of the objective function are also possible when, rather than the industries, the individual cells, blocks with submatrices or whole matrices are weighted in each of the priors.

SUPPLEMENTAL DATA

Supplemental material for this article is available via the supplemental tab on the article's online page at http://dx.doi.org/10.1080/09535314.2015.1007839.

References

Bernardini, R. (2008) A Composite Generalized Cross Entropy Formulation in Small Samples Estimation. *Econometric Reviews*, 27, 596–609.

Bonfiglio, A. and F. Chelli (2008) Assessing the Behavior of Non-Survey Methods for Constructing Regional Input–Output Tables through a Monte Carlo Simulation. *Economic Systems Research*, 20, 243–258.

Geschke, A., R. Wood, K. Kanemoto, M. Lenzen and D. Moran (2014) Investigating Alternative Approaches to Harmonise MRIO Data. *Economic Systems Research*, 26, 354–385.

Gilchrist, D. A. and L. V. St Louis (1999) Completing Input-Output Tables Using Partial Information, with an Application to Canadian Data. *Economic Systems Research*, 11, 185–193.

Golan, A. (2001) A Simultaneous Estimation and Variable Selection Rule. *Journal of Econometrics*, 101, 165–193.

Golan, A. (2006) Information and Entropy Econometrics. A review and Synthesis. *Foundations and Trends in Econometrics*, 2, 1–145.

Golan, A. and S. Vogel (2000) Estimation of Non-Stationary Social Accounting Matrix Coefficients with Supply-Side Information. *Economic Systems Research*, 12, 447–471.

Golan, A., G. Judge and S. Robinson (1994) Recovering Information from Incomplete or Partial Multisectoral Economic Data. *The Review of Economics and Statistics*, 76, 541–549.

Golan, A., G. Judge and D. Miller (1996) *Maximum Entropy Econometrics: Robust Estimation with Limited Data*. New York, John Wiley & Sons.

Hewings, G.J.D. (1984) The Role of Prior Information in Updating Input-Output Models. *Socio-Economic Planning Science*, 18, 319–339.

Jackson, R.W. (1998) Regionalizing National Commodity-by-Industry Accounts. *Economic Systems Research*, 10, 223–238.

Jackson, R.W. and A. Murray (2004) Alternative Input-Output Matrix Updating Formulations. *Economic Systems Research*, 16, 135–148.

Jalili, A.R. (2000). Evaluating Relative Performances of Four Non-Survey Techniques of Updating Input-Output Coefficients. *Economics of Planning*, 33, 221–237.

Jiang, X., E. Dietzenbacher and B. Los (2010) Targeting the Collection of Superior Data for the Estimation of the Intermediate Deliveries in Regional Input -Output Tables. *Environment and Planning A*, 42, 2508–2526.

Jiang, X., E. Dietzenbacher and B. Los (2012) Improved Estimation of Regional Input–Output Tables using Cross-Regional Methods. *Regional Studies*, 46, 621–637.

Kullback, J. (1959) *Information Theory and Statistics*. New York, Wiley.

Lenzen, M., B. Gallego and R. Wood (2009) Matrix Balancing under Conflicting Information. *Economic Systems Research*, 21, 23–44.

McDougall, R.A. (1999) "Entropy Theory and RAS are Friends." GTAP Working Paper No. 6. Center for Global Trade Analysis, Purdue University.

Mello, L.R.J. and J.R. Teixeira (1993) The Updating of Input-Output Matrices: A Systematic Survey. *Economics Letters*, 41, 241–245.

Minguez, S., J. Oosterhaven and F. Escobedo (2009) Cell-Corrected RAS Method for Updating or Regionalizing an Input-Output Table. *Journal of Regional Science*, 49, 329–348.

Oosterhaven, J. and F. Escobedo (2011) A New Method to Estimate Input-Output Tables by Means of Structural Lags, Tested on Spanish Regions. *Papers in Regional Science*, 90, 829–844.

Oosterhaven, J., D. Stelder and S. Inomata (2008) Estimating International Interindustry Linkages: Non-Survey Simulations of the Asian-Pacific Economy. *Economic Systems Research*, 20, 395–414.
van der Ploeg, F. (1982) Reliability and the Adjustment of Sequences of Large Economic Accounting Matrices. *Journal of the Royal Statistical Society A*, 145, 169–194.
Robinson, S., A. Cattaneo and M. El-Said (2001) Updating and Estimating a Social Accounting Matrix Using Cross Entropy Methods. *Economic Systems Research*, 13, 47–64.