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The efficiency of the cross-entropy method when estimating the technical coefficients of input–output tables

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ABSTRACT

Updating or estimating regional input–output tables is a challenging task addressed with non-survey methods. These can be classified into two groups: location quotient (LQ) methods and constrained matrix-balancing methods. This paper focuses on the second group and, specifically, on the performance of the cross-entropy method (CE). The most important finding is that the RAS method slightly outperforms the CE method on average, but its efficiency varies greatly from country to country. On the contrary, the performance of the CE method is more stable over countries and time. More interestingly, a fair implementation of the CE method boosted by the Flegg location quotient (FLQ) method outperforming the competing CE approach in terms of accuracy.

KEYWORDS

regional input–output tables, location quotient, entropy method

JEL C67, O18, R15

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INTRODUCTION

Building an input–output table (IOT) usually requires a vast amount of data i.e., not always completely available. To solve this problem, alternative approaches have been proposed, including the so-called non-survey. The goal is to update an existing IOT over time or adapt the IOT of a bigger geographical area to a sub-territorial level of inter-estimate.

Non-survey methods can be classified into two groups: location quotient (LQ) methods and constrained matrix-balancing methods. Several LQ methods have been suggested: the simple location quotient (SLQ) (Schaffer & Chu, 1969), the cross-industry location quotient (CILQ) (Flegg, Webber, & Elliott, 1995), the semilogarithmic location quotient (RLQ)

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(Round, 1978), the symmetric cross-industry location quotient (SCILQ) (Miller & Blair, 2009), the Flegg location quotient (FLQ) (Flegg, Webber & Elliot, 1995) and the augmented Flegg location quotient (AFLQ) (Flegg & Webber, 2000).

These methods are based on the assumption that geographical sublevels (or other reference areas) have the same productive technologies, and that sub-area input coefficients differ from those of the reference area to the extent to which goods and services are imported from other territorial areas.

In addition to the methods just mentioned, there is also a different category: the so-called constrained matrix-balancing procedures. These methods estimate unknown data based on limited initial information subject to a set of linear constraints. Among these, the RAS (Bacharach, 1970; Stone, 1961) and cross-entropy (CE) (Golan, Judge, & Robinson, 1994) methods are the most popular since they perform very well (Davis, Lofting, & Sathaye, 1977). Furthermore, this second group includes methods based on minimizing squared/absolute differences as illustrated in Pavia, Cabrer, and Sala (2009). The LQ methods are very fast and easy to implement, while methods in the second group are more time consuming and usually require solving a constrained non-linear optimization problem.

There is ample literature on the strengths and weaknesses of LQ methods, including Bonfiglio and Chelli (2008), Flegg and Tohmo (2013), Harrigan, McGilvray, and McNicoll (1980), Lamonica and Chelli (2018), Lehtonen and Tykkyläinen (2014), Kowalewksi (2015), Morrissey (2016), Oosterhaven and Cardeñoso (2011), Round (1978), and Tohmo (2004).

Likewise, earlier studies compare performances of the constrained optimization methods (Batten, 1983; Hosoe, 2014; Léony, Peeters, Quinqu, & Surry, 1999; Özçam, 2009; Pavia et al., 2009; Peters & Hertel, 2016; Robinson, Cattaneo, & El-Said, 2001).

The present study concentrates on the CE method since it is a pioneering method and its implementation makes use of the scaling procedure proposed here.

The objective of this study is twofold. The first is to analyse and improve the performance of the CE method in the context of regionalizing an IOT since the CE method is a non-linear technique that requires the solution to a system of non-linear equations. As explained below, the naive use of standard solvers for non-linear systems may fail to determine a solution via a CE approach due to the specific features of the real data. Thus, a strategy is proposed and tested to overcome this problem. The accuracy of this improved implementation of the CE method is compared with that of the constrained balancing matrix methods.

The second objective is to integrate the performance of the CE method by means of the combined use of the FLQ method. The use of this hybrid approaches to improve the estimation of regional IOTs can be found in Boero, Edwards, and Rivera (2018).

The empirical analysis performed in this study uses data from the World Input–Output Database (www.wiod.org). It provides users with a panel of data consisting of the World Inter-Country IOT (WIOT) as well the national IOTs of 40 different countries covering the period from 1995 to 2011. This data source has two main advantages. First, it is possible to compare the IOTs estimated by non-survey methods with observed IOTs, thus allowing the performance of the methods to be assessed in terms of their ability to reproduce the 'true' IOTs. Second, it is possible to measure the performance of these methods over time, thus allowing one to analyse whether changes in economic/financial country policy and/or global crisis affect this performance.

The WIOT is similar to an interregional IOT wherein countries act as regions. With respect to the national IOT, it provides a more detailed description of the interdependence of economic sectors between countries. In contrast to a national IOT, where all exports go into the final demand section and are considered exogenous to the economic system of a country, in the WIOT they are subdivided into exports of final goods and services and exports of intermediate goods and services. While the former are exogenous to the economic system, the latter depend on the final demand of other countries and are thus endogenous to the economic system.

Relevance of the WIOTs is growing rapidly as they allow analysis of crucial aspects of the global economy, focusing especially on globalization and fragmentation of production processes.

The WIOT is built with a classification for 35 economic sectors. This IOT was reaggregated into a classic IOT, treating it like the IOT for a reference territorial area where the countries act like sub-territorial areas. We realize that this working hypothesis not entirely reflect the reality because flows among countries are affected by various economic and spatial barriers that may be different compared with the flows among regions. The results of this analysis are thus influenced by this situation. However, the effects of globalization of the world economy have removed many barriers between the countries making them strongly interdependent. Therefore, investigating the performances of the above-mentioned approaches to deal with WIOT data could provide insights into the development/improvements of more specific methods for WIOT.

Moreover, with regard to the combined use of the FLQ and CE method proposed in this paper, since the FLQ method assumes the constant spatial technology hypothesis, for comparative reasons, not all 40 countries in the database are considered, but only a more cohesive subset, namely, the nations of the European Union plus Canada, China, Japan, Russia and the United States. Some economic characteristics of the countries for the same years are shown in Table A1 in Appendix A in the supplemental data online.

The comparative analysis, illustrated below, shows that the improved implementation of the CE method makes it the best-performing method when applied to countries with small economies, while the RAS is the best performing when applied to countries with large economies. Moreover, the RAS method is the fastest and the CE method is the slowest in terms of computational speed. Finally, the combined use of the CE and FLQ method provides better estimates with respect to those obtained by the CE method.

REVIEW OF THE METHODS FOR ESTIMATING AN IOT

In this section, we briefly review the most commonly used methods to estimate the regional input coefficients of an IOT. Table 1 summarizes the models illustrated below.

For simplicity and without loss of generality, we consider a two-region economy consisting of k sectors and the corresponding multiregional IOT in block matrix notation:

$$MIOT = \begin{bmatrix} \mathbf{Z}^{RR} & \mathbf{Z}^{RS} & \mathbf{f}^{R} \\ \mathbf{Z}^{SR} & \mathbf{Z}^{SS} & \mathbf{f}^{S} \\ (\mathbf{v}^{R})' & (\mathbf{v}^{S})' & 0 \end{bmatrix}, \tag{1}$$

where $\mathbf{Z}^{RR}$ and $\mathbf{Z}^{SS}$ are matrices whose entries are the flows for intermediate use from the i-th sector to the j-th sector of the same region (intraregional flows); $\mathbf{Z}^{RS}$ and $\mathbf{Z}^{SR}$ are matrices whose entries are interregional flows, that is, respectively, the exports for intermediate use from the i-th sector of region R to the j-th sector of region S (imports of region S) and the exports for intermediate use from the i-th sector of region S to the j-th sector of region R (imports of region R); $\mathbf{f}^{R}$ and $\mathbf{f}^{S}$ are vectors representing the final demand in regions R and S; $(\mathbf{v}^{R})'$ and $(\mathbf{v}^{S})'$ are row vectors whose entries are the sectorial added-value plus all the primary sectorial input of the two regions; and $(\cdot)'$ denotes the transposition operator.

Using this MIOT, the following global IOT ($\mathbf{T}^{W}$) and regional IOTs ($\mathbf{T}^{R}$ and $\mathbf{T}^{S}$) can be considered:

$$\mathbf{T}^{W} = \begin{bmatrix} \mathbf{X}^{W} & \mathbf{f}^{W} \\ (\mathbf{v}^{W})' & 0 \end{bmatrix}, \mathbf{T}^{R} = \begin{bmatrix} \mathbf{X}^{R} & \mathbf{f}^{R} + \mathbf{Z}^{RS}\mathbf{u} \\ \mathbf{I}^{R} & 0 \\ (\mathbf{v}^{R})' & 0 \end{bmatrix}, \mathbf{T}^{S} = \begin{bmatrix} \mathbf{X}^{S} & \mathbf{f}^{S} + \mathbf{Z}^{SR}\mathbf{u} \\ \mathbf{I}^{S} & 0 \\ (\mathbf{v}^{S})' & 0 \end{bmatrix}, \tag{2}$$

Table 1. Summary of regionalization methods.

Method	**References**
Simple location quotient (SLQ)	Schaffer and Chun (1969)
Cross-industry location quotient (CILQ)	Flegg et al. (1995)
Symmetric cross-industry location quotient (SCILQ)	Miller and Blair (2009)
Semilogarithmic location quotient (RLQ)	Round (1978)
Flegg location quotient (FLQ)	Flegg et al. (1995)
Augmented Flegg location quotient (AFLQ)	Flegg and Webber (2000)
RAS	Bacharach (1970); Stone (1961)
Cross-entropy (CE)	Golan et al. (1994)
Squared differences model (SSD)	Pavia et al. (2009)
Weighed squared differences model (WSD)	Pavia et al. (2009)
Normalized squared differences model (NSD)	Pavia et al. (2009)

where $\mathbf{X}^{\mathrm{R}} = \mathbf{Z}^{\mathrm{RR}}$, $\mathbf{X}^{\mathrm{S}} = \mathbf{X}^{\mathrm{SS}}$, $\mathbf{f}^{\mathrm{W}} = \mathbf{f}^{\mathrm{R}} + \mathbf{f}^{\mathrm{S}}$, $(\mathbf{v}^{\mathrm{W}})' = (\mathbf{v}^{\mathrm{R}})' + (\mathbf{v}^{\mathrm{S}})'$, $\mathbf{X}^{\mathrm{W}} = \mathbf{Z}^{\mathrm{SS}} + \mathbf{Z}^{\mathrm{RR}} + \mathbf{Z}^{\mathrm{RS}} + \mathbf{Z}^{\mathrm{SR}}$, $\mathbf{I}^{\mathrm{S}} = \mathbf{Z}^{\mathrm{RS}}$, $\mathbf{I}^{\mathrm{R}} = \mathbf{Z}^{\mathrm{SR}}$, and $\mathbf{u}$ is the unitary vector.

Moreover, let $\mathbf{x}^{b}$ ($b = \mathrm{W}$, R and S) be the vectors whose entries (x_i^{k}) are the total sectorial output/input, and $\mathbf{A}^{\mathrm{W}}$, $\mathbf{A}^{\mathrm{R}}$ and $\mathbf{A}^{\mathrm{S}}$ the matrices of global technical and regional input coefficients, respectively, whose entries are:

$$a_{ij}^{\mathrm{W}} = \frac{x_{ij}^{\mathrm{W}}}{x_j^{\mathrm{W}}},\ a_{ij}^{\mathrm{R}} = \frac{x_{ij}^{\mathrm{R}}}{x_j^{\mathrm{R}}},\ a_{ij}^{\mathrm{S}} = \frac{x_{ij}^{\mathrm{S}}}{x_j^{\mathrm{S}}}, \quad i, j = 1, 2, .., k. \tag{3}$$

Assume that only $\mathbf{T}^{\mathrm{W}}$ $(\mathbf{A}^{\mathrm{W}})$ and the vector of total sectorial output ($\mathbf{x}^{\mathrm{R}}$) of region R are known. We estimate the matrix of the regional input coefficients $\mathbf{A}^{\mathrm{R}}$ as follows:

$$\hat{a}_{ij}^{\mathrm{R}} = a_{ij}^{\mathrm{W}} q_{ij} \tag{4}$$

where q_{ij} represents the degree of modification of the national coefficient. The interregional import coefficients (the entries of $\mathbf{I}^{\mathrm{R}}$) are usually estimated from the difference between the national coefficient and the estimated regional input coefficient. Since it is hard to obtain q_{ij}, location coefficients are used as substitutes.

Brief overview of the LQ methods

The simple location quotient (SLQ) is the most popular way to estimate the intraregional input coefficients (hereafter only input coefficients). The a_{ij}^{R} is estimated as follows:

$$\hat{a}_{ij}^{\mathrm{R}} = SLQ_i a_{ij}^{\mathrm{W}} \ and \ SLQ_i = \frac{x_i^{\mathrm{R}}/\mathbf{u}'\mathbf{x}^{\mathrm{R}}}{x_i^{\mathrm{W}}/\mathbf{u}'\mathbf{x}^{\mathrm{W}}}, \tag{5}$$

where $\mathbf{u}$ is the above-mentioned unitary vector. Since SLQ_i could be > 1, the regional input coefficients are adjusted as follows:

$$\hat{a}_{ij}^{\mathrm{R}} = \begin{cases} a_{ij}^{\mathrm{W}} SLQ_i & if \quad SLQ_i < 1 \\ a_{ij}^{\mathrm{W}} & if \quad SLQ_i \geq 1 \end{cases}. \tag{6}$$

The cross-industry location quotient (CILQ) is one of the first enhancements of the SLQ method and considers both the supply and purchase sectors:

$$CILQ_{ij} = \frac{x_i^R / x_i^W}{x_j^R / x_j^W} = \frac{SLQ_i}{SLQ_j}, \tag{7}$$

$$\hat{a}_{ij}^R = \begin{cases} a_{ij}^W CILQ_{ij} & if \;\; CILQ_{ij} < 1 \\ a_{ij}^W & if \;\; CILQ_{ij} \geq 1 \end{cases}. \tag{8}$$

Contrary to the SLQ method, the CILQ method is a cell-by-cell adjustment.

The symmetric cross-industry location quotient (SCILQ) is a variant of the CILQ method. It was designed to take into account the possibility of deriving regional coefficients that exceed national values, thus overcoming the problem of asymmetric adjustments. It takes the following form:

$$SCILQ_{ij} = \quad 2 - \frac{2}{CILQ_{ij} + 1}. \tag{9}$$

The semilogarithmic location quotient (RLQ) incorporates the properties of both the SLQ and CILQ methods and takes the following form:

$$RLQ_{ij} = \frac{SLQ_i}{log_2(1 + SLQ_j)} = \frac{x_i^R / \mathbf{u}'\mathbf{x}^R}{x_i^W / \mathbf{u}'\mathbf{x}^W} \Big/ \left[log_2 \left(1 + \frac{x_j^R}{\mathbf{u}'\mathbf{x}^R} \cdot \frac{\mathbf{u}'\mathbf{x}^W}{x_j^W} \right) \right]. \tag{10}$$

The RLQ has been criticized for underestimating imports from other regions when the size of the region is small. To overcome these drawbacks, the Flegg location quotient method (FLQ) was introduced:

$$FLQ_{ij} = \begin{cases} CILQ_{ij}\lambda & for \; i \neq j \\ SLQ_{ij}\lambda & for \; i = j \end{cases}, \tag{11}$$

where λ stands for the relative size of the region and takes the following form:

$$\lambda = \left[log_2 \left(1 + \frac{\mathbf{u}'\mathbf{x}^R}{\mathbf{u}'\mathbf{x}^W} \right) \right]^{\delta}. \tag{12}$$

Here, δ $(0 \leq \delta < 1)$ is a sensitivity parameter that controls the degree of convexity in the previous equation. The larger the value of δ, the lower the value of λ, so that greater adjustments of regional imports are made:

$$\hat{a}_{ij}^R = \begin{cases} a_{ij}^W FLQ_{ij} & if \; FLQ_{ij} < 1 \\ a_{ij}^W & if \; FLQ_{ij} \geq 1 \end{cases}. \tag{13}$$

The value of parameter δ is the focus of the method. McCann and Dewhurst (1998) pointed out that regional coefficients may exceed national coefficients when there is regional specialization (i.e., the regional coefficient becomes larger than the national coefficient). Thus, the augmented FLQ (AFLQ):

$$AFLQ_{ij} = \begin{cases} FLQ_{ij}[\log_2(1 + SLQ_j)] & for \; SLQ_j > 1 \\ FLQ_{ij} & for \; SLQ_j \leq 1 \end{cases}, \tag{14}$$

where $log_2(1 + SLQ_j)$ represents the regional specialization of sector j and has been included to allow for the effects of regional specialization. If $SLQ_j > 1$ and $FLQ_{ij} \geq 1$, the national coefficients are scaled upwards. However, to avoid an excessive upward adjustment, the constraint $FLQ_{ij} \leq 1$ is imposed. Consequently, the regionalization is performed as follows:

$$\hat{a}_{ij}^{R} = \begin{cases} a_{ij}^{W} AFLQ_{ij} & if \;\; SLQ_j > 1 \\ a_{ij}^{W} FLQ_{ij} & if \;\; SLQ_j \leq 1 \end{cases}. \tag{15}$$

The computational cost of the LQ methods is approximately $5(k+1)^2 + 3(k+1)$ scalar multiplications.

Brief overview of the constrained matrix-balancing methods

In addition to methods based on the LQ, mathematical programming methods based on a constrained optimization framework can be found in the literature. These methods usually minimize a penalty function, which measures the deviation of the balanced matrix from the initial matrix subject to a set of balance conditions. In this regard, we refer to the following regional IOT, which, starting with $\mathbf{T}^{W}$, must be estimated:

$$\mathbf{T}^{R} = \begin{bmatrix} \mathbf{X}^{R} & \mathbf{f}^{R} + \mathbf{Z}^{RS}\mathbf{u} \\ (\bar{\mathbf{v}}^{R})' & 0 \end{bmatrix}, \tag{16}$$

where $(\bar{\mathbf{v}}^{R})' = (\mathbf{v}^{R})' + \mathbf{I}^{R}\mathbf{u}$.

The RAS method is one of the first and most widely used methods developed in the literature, which can be referred to for the details. To begin, the goal of the RAS procedure is to estimate a regional IOT ($\mathbf{T}^{R}$). This estimate is obtained through an iterative scaling procedure whereby the rows and columns of $\mathbf{T}^{W}$ are adjusted until their sums converge to the target vectors ($\mathbf{x}^{R}$).

In detail, let $\mathbf{s}$ be the vector with entries $s_i = x_i^{R}/x_i^{W}$, $i = 1, 2, \ldots, k+1$; and $\mathbf{D}(\mathbf{s})$ is the diagonal matrix whose entries are $d_{ii}(s) = s_i$, $i = 1, 2, \ldots, k+1$. The RAS procedure can be summarized by making two iterative rescaling of rows and columns.

For n = 0 set:

$$\hat{\mathbf{T}}^{R(n)} = \mathbf{T}^{W}\mathbf{D}(\mathbf{s}), \tag{17}$$

For n = 1, 2, 3, … ., repeat:

$$\mathbf{r}_i^{(n)} = \frac{x_i^{R}}{(\mathbf{u}'\hat{\mathbf{T}}^{R(n-1)})_i}, \quad i = 1, \; 2, \; \ldots, \; k+1, \tag{18}$$

$$\hat{\mathbf{T}}^{R(n)} = \mathbf{D}(\mathbf{r}^{(n)})\hat{\mathbf{T}}^{R(n-1)}, \tag{19}$$

$$\mathbf{s}_i^{(n)} = \frac{x_i^{R}}{(\hat{\mathbf{T}}^{R(n)}\mathbf{u})_i}, \quad i = 1, \; 2, \; \ldots, \; k+1, \tag{20}$$

$$\hat{\mathbf{T}}^{R(n+1)} = \hat{\mathbf{T}}^{R(n)}\mathbf{D}(\mathbf{s}^{(n)}), \tag{21}$$

until the following stopping criterion is satisfied:

$$|\mathbf{u}'\mathbf{T}^{R(n+1)} - \mathbf{x}^{R}| < \varepsilon \; and \; |\mathbf{T}^{R(n+1)}\mathbf{u} - \mathbf{x}^{R}| < \varepsilon, \tag{22}$$

for a positive tolerance parameter ε.

The computational cost of this iterative scheme is approximately $2(k+1)^2 M_{it}$ scalar multiplications, where M_{it} is the maximum number of iterations needed to satisfy the stopping criterion.

The CE method began with the information theory developed by Shannon (1948). Very briefly, a set of n events E_i (for $i = 1, 2, \ldots, n$) are considered, each with probability q_i

(for $i = 1, 2, \ldots, n$). The information content (also entropy) of a generic event E_i is measured as $log(1/q_i)$. As is obvious, the higher the probability of an event, the less the information obtained.

A global measure of the entropy of all n events is the mean of all q_i entropies weighted by q_i: $\sum_{i=1}^{n} q_i log(1/q_i)$.

Assume that the probability of a generic event E_i changes from q_i to p_i. The relative entropy (or information gained) that provides an informational distance between the two probability is then defined as $log(p_i/q_i)$.

A global measure of the information gained when all probabilities change from q_i to p_i is the mean of all the additional information weighted by p_i:

$$H = \sum_{i=1}^{n} p_i ln\left(\frac{p_i}{q_i}\right). \tag{23}$$

This is the Kullback and Leibler (1951) measure of divergence between two sets of probabilities, which reflects the gain in information resulting from the additional knowledge gained by changing q_i to p_i. In other words, it is an information-theoretical distance of p_i from q_i that measures the inefficiency of assuming a priori that the probabilities of E_i are q_i when the correct ones are p_i.

Following Golan et al. (1994), we start with the above-mentioned $\mathbf{T}^{W}$, treating the primary input and final demand as 'additional' intermediate input and output. The following matrix for the augmented technical coefficients $\mathbf{A}^{W}$ can be considered:

$$\mathbf{A}^{W} = \begin{bmatrix} a_{1,1}^{W} = x_{1,1}^{W}/x_1^{W} & \cdots & a_{1,k}^{W} = x_{1,k}^{W}/x_k^{W} & a_{1,k+1}^{W} = f_1^{W}/x_{k+1}^{W} \\ \vdots & & \vdots & \vdots \\ a_{k,1}^{W} = x_{k,1}^{W}/x_1^{W} & \cdots & a_{k,k}^{W} = x_{k,k}^{W}/x_k^{W} & a_{k,k+1}^{W} = f_k^{W}/x_{k+1}^{W} \\ a_{k+1,1}^{W} = v_1^{W}/x_1^{W} & \cdots & a_{k+1,k}^{W} = v_k^{W}/x_k^{W} & a_{k+1,k+1}^{W} = 0 \end{bmatrix}. \tag{24}$$

At this point it turns out that:

$$\mathbf{A}^{W}\mathbf{x}^{W} = \mathbf{x}^{W}, \tag{25}$$

$$\mathbf{u}'\mathbf{A}^{W} = \mathbf{u}. \tag{26}$$

The problem lies in generating a new matrix $\mathbf{A}^{R}$ from the existing $\mathbf{A}^{W}$ of the same dimension while respecting new row and column totals $\mathbf{x}^{R}$. That is, we look for a matrix $\mathbf{A}^{R}$ that satisfies the following consistency and addition conditions:

$$\mathbf{A}^{R}\mathbf{x}^{R} = \mathbf{x}^{R}, \tag{27}$$

$$\mathbf{u}'\mathbf{A}^{R} = \mathbf{u}. \tag{28}$$

Formally, the problem consists in minimizing the following function for $a_{ij}^{W} > 0$:

$$min_{a_{ij}^{R}} H = \sum_{i=1}^{k+1} \sum_{j=1}^{k+1} a_{ij}^{R} ln \frac{a_{ij}^{R}}{a_{ij}^{W}} \tag{29}$$

subject to:

$$\sum_{j=1}^{k+1} a_{ij}^{R} x_j^{R} = x_i^{R} \tag{30}$$

and

$$\sum_{i=1}^{k+1} a_{ij}^{R} = 1. \tag{31}$$

The solution is obtained by setting up the Lagrangian (L) for problem (29) and solving it:

$$L = \sum_{i=1}^{k+1}\sum_{j=1}^{k+1} a_{ij}^{R} ln \frac{a_{ij}^{R}}{a_{ij}^{W}} + \sum_{i=1}^{k+1} \lambda_i \left(x_i^{R} - \sum_{j=1}^{k+1} a_{ij}^{R} x_j^{R} \right) + \sum_{j=1}^{k+1} \mu_j \left(1 - \sum_{i=1}^{k+1} a_{ij}^{R} \right). \tag{32}$$

The first-order optimal conditions are:

$$\frac{\partial L}{\partial a_{ij}^{R}} = ln \frac{a_{ij}^{R}}{a_{ij}^{W}} + 1 - \lambda_i x_j^{R} - \mu_j = 0,\ i,\ j = 1, 2, \ldots, k+1, \tag{33}$$

$$\frac{\partial L}{\partial \lambda_i} = x_i^{R} - \sum_{j=1}^{k+1} a_{ij}^{R} x_j^{R} = 0,\quad i = 1,\ 2,\ \ldots, k+1, \tag{34}$$

$$\frac{\partial L}{\partial \mu_j} = 1 - \sum_{i=1}^{k+1} a_{ij}^{R} = 0,\quad j = 1,\ 2,\ ..,\ k+1. \tag{35}$$

Solving this system of equations yields:

$$a_{ij}^{R} = a_{ij}^{W} e^{(-1+\lambda_i x_j^{R} + \mu_j)},\qquad i,\ j = 1,\ 2,\ \ldots,\ k+1, \tag{36}$$

$$\sum_{j=1}^{k+1} x_j^{R} a_{ij}^{W} e^{(-1+\lambda_i x_j^{R} + \mu_j)} = x_i^{R},\qquad i = 1,\ 2,\ \ldots,\ k+1, \tag{37}$$

$$\sum_{i=1}^{k+1} a_{ij}^{W} e^{(-1+\lambda_i x_j^{R} + \mu_j)} = 1,\qquad j = 1,\ 2,\ ..,\ k+1. \tag{38}$$

The CE solution to equations (37) and (38) reads as:

$$\hat{a}_{ij}^{R} = \frac{a_{ij}^{W} e^{\lambda_i x_j^{R}}}{\sum_{p=1}^{k+1} a_{pj}^{W} e^{\lambda_p x_j^{R}}}. \tag{39}$$

Note that the estimate of a_{ij}^{R} depends on the values of the Lagrange multipliers, which must be determined by solving the non-linear system (37) and (38) for the unknowns $\lambda_1, \lambda_2, \ldots, \lambda_{k+1}$. Since no closed-form solution exists, this system is solved using numerical algorithms.

As detailed in the next section, the computational cost of the CE method is approximately $(k+1)^2\, M_f\, M_{it}$, where M_f is the number of function evaluations; and, as mentioned above, M_{it} is the number of iterations required to satisfy the stopping criterion. This is due to solving the linear system (37) and (38).

Other methods based on minimizing squared/absolute differences have been proposed (Pavia et al., 2009). In this enquiry, the following three procedures were considered.

The squared differences (SSD) model

The goal is to minimize the sum of squared differences between the entries in the estimate ($\mathbf{A}^{\mathrm{R}}$) and reference ($\mathbf{A}^{\mathrm{W}}$) matrices:

$$\min_{a_{ij}^{\mathrm{R}}} \sum_{i=1}^{\mathrm{k}+1} \sum_{j=1}^{\mathrm{k}+1} (a_{ij}^{\mathrm{R}} - a_{ij}^{\mathrm{W}})^2, \tag{40}$$

subject to:

$$\sum_{j=1}^{\mathrm{k}+1} a_{ij}^{\mathrm{R}} x_j^{\mathrm{R}} = x_i^{\mathrm{R}}, \quad i = 1, \ 2, \ .., \ k + \ 1, \tag{41}$$

$$\sum_{i=1}^{\mathrm{k}+1} a_{ij}^{\mathrm{R}} = 1, \quad j = 1, \ 2, \ .., \ k + \ 1, \tag{42}$$

$$a_{ij}^{\mathrm{R}} \geq 0, \quad i, \ j = \ 1, \ 2, \ .., \ k + 1. \tag{43}$$

In this model, all differences display the same weight.

Weighted squared differences (WSD)

An alternative to the SSD is the WSD model:

$$\min_{a_{ij}^{\mathrm{R}}} \sum_{i=1}^{\mathrm{k}+1} \sum_{j=1}^{\mathrm{k}+1} a_{ij}^{\mathrm{W}} (a_{ij}^{\mathrm{R}} - a_{ij}^{\mathrm{W}})^2, \tag{44}$$

subject to constraints (41), (42) and (43).

Since accuracy in large coefficients can be considered more important than accuracy in small coefficients, larger coefficients are weighted more than smaller ones and, consequently, changes in larger entries are weighted more heavily than those in smaller ones.

Normalized squared differences (NSD)

Another alternative to the SSD model is the NSD model:

$$\min_{a_{ij}^{\mathrm{R}}} \sum_{i=1}^{\mathrm{k}+1} \sum_{j=1}^{\mathrm{k}+1} \frac{1}{a_{ij}^{\mathrm{W}}} (a_{ij}^{\mathrm{R}} - a_{ij}^{\mathrm{W}})^2, \tag{45}$$

subject to constraints (41), (42) and (43). The weights of this model are inversely proportional to a_{ij}^{W}, so the resulting model enhances the role of small coefficients.

This formulation imposes a greater penalty on changes in small coefficients, resulting in updated matrices whose changes are more concentrated in the larger coefficients.

Moreover, in addition to these models, there are corresponding approaches where differences between a_{ij}^{R} and a_{ij}^{W} are considered as absolute values. These absolute difference models are not considered in the analysis proposed here because they are outperformed by the squared difference model.

THE RESCALING PROCEDURE FOR THE CROSS-ENTROPY (CE) METHOD

With regard to the CE method, the system of non-linear equations (37) and (38) is usually solved via iterative methods such as the Matlab *fsolve*. These solvers generally depend on a user-defined starting point, and their convergence is strongly affected by the flatness of the objective function and/or variables with different orders of magnitude (Dennis & Schnabel, 1996; Wibisono, Wilson, & Jordan, 2016).

The implementation of any standard solver fails to find a solution to equations (37) and (38) when a_{ij}^{W} and x_i^{R}, $i,j = 1, 2, \ldots, k + 1$, belongs to the data set considered in this paper.

Our specific problem shows two critical points. First, the evaluation of the exponential functions for large values of their arguments. Second, the solution to a high-dimension optimization problem (i.e., problem (29)). The first point may result in a failure of the optimization procedure, while the latter makes it difficult to determine global minimizers. This could imply inaccurate estimates of the IOT. In what follows, we define a solution method designed for our specific system (37) and (38).

The first step consists in rescaling the variables $\lambda_1, \lambda_2, \ldots, \lambda_{k+1}$. Note that equations (37) and (38) depend on the products $x_j^{R}\lambda_i$, $i,j = 1, 2, \ldots, k + 1$. Owing to the extremely high x_j^{R} values, these products could be very large if a random exploration of the feasible region is carried out, thus determining a failure in the computation of the exponential function appearing in equations (37) and (38). In order to avoid this code failure, we rewrite equations (37) and (38) as follows:

$$\sum_{j=1}^{k+1}(x_j^{r}/M)a_{ij}^{W}e^{(-1+(M\lambda_i)(x_j^{R}/M)+\mu_j)} = (x_i^{R}/M), \quad i = 1,\ 2,\ \ldots,\ k+1, \tag{46}$$

$$\sum_{i=1}^{k+1} a_{ij}^{W}e^{(-1+(M\lambda_i)(x_j^{R}/M)+\mu_j)} = 1, \quad j = 1,\ 2,\ \ldots,\ k+1, \tag{47}$$

where M is a suitable positive constant. Thus, by defining the new variables $\tilde{\lambda}_i = M\lambda_i$ and the rescaled data, $\tilde{x}_j^{R} = x_j^{R}/M$, that is:

$$\tilde{x}_j^{R} = \frac{x_j^{R}}{M}, \qquad \tilde{\lambda}_i = M\lambda_i. \quad i,\ j = 1,\ 2,\ \ldots,\ k+1 \tag{48}$$

we can reformulate equations (37) and (38) as follows:

$$\sum_{j=1}^{k+1}\tilde{x}_j^{R}a_{ij}^{W}e^{(-1+\tilde{\lambda}_i\tilde{x}_j^{R}+\mu_j)} = \tilde{x}_i^{R}, \quad i = 1,\ 2,\ \ldots,\ k+1, \tag{49}$$

$$\sum_{i=1}^{k+1} a_{ij}^{W}e^{(-1+\tilde{\lambda}_i\tilde{x}_j^{R}+\mu_j)} = 1, \quad j = 1,\ 2,\ \ldots,\ k+1. \tag{50}$$

As illustrated by Dennis and Schnabel (1996),[1] iterative methods to solve non-linear equations or optimization problems based on gradient methods are very sensitive to changes in the scale of the independent variable.

Finally, we reduce the dimension of the non-linear system of equations (49) and (50). Bearing in mind that equation (50) also reads as:

$$e^{(-1+\mu_j)} = \frac{1}{\sum_{i=1}^{k+1} a_{ij}^{W}e^{((M\lambda_i)(x_j^{R}/M))}} = \frac{1}{\sum_{i=1}^{k+1} a_{ij}^{W}e^{\tilde{\lambda}_i\tilde{x}_j^{R}}} \tag{51}$$

we use (51) in (49) to reduce the original system of $2(k + 1)$ unknown variables and equations to

[1]. To see the effect of the scaling, we consider the transform $\mathbf{y} = T\mathbf{x}$ (i.e., $\mathbf{x} = T^{-1}\mathbf{y}$). Denoting $g(\mathbf{y}) = f(T^{-1}\mathbf{y})$, we have $\nabla g(\mathbf{y}) = (T^{-1})'\nabla f(\mathbf{x})$ and $\nabla^2 g(\mathbf{y}) = (T^{-1})'\nabla^2 f(\mathbf{x})T^{-1}$. Thus, the scaling matrix affects the original gradient vector and can be used to improve the convergence of iterative schemes to optimize a scalar function $f(\mathbf{x})$ based on the gradient vector direction.

the following system of $k + 1$ unknowns and equations:

$$\sum_{j=1}^{k+1} \tilde{x}_j^{R} \frac{a_{ij}^{W}}{\sum_{p=1}^{k+1} a_{pj}^{W} e^{\tilde{\lambda}_p \tilde{x}_j^{R}}} e^{\tilde{\lambda}_p \tilde{x}_j^{R}} = \tilde{x}_i^{R} \tag{52}$$

where the rescaled data and unknown variables are given in equation (48).

In the next section, we show that there are two advantages to the rescaling approach. First, it allows one to use standard solvers to solve the optimization problem and/or non-linear system (52). Second, by choosing the scaling factor M to be the largest value of the vector x_j^R, $j = 1, 2, \dots, k + 1$, we obtain satisfactory estimates of the regional input coefficients.

In the following section, we use normally distributed random numbers as starting points of $\tilde{\lambda}_p$, $p = 1, 2, \dots, k + 1$, in order to measure the performance of the naive CE method.

THE PERFORMANCE OF THE REGIONALIZATION METHODS

Description of the data

This section illustrates the results relating to the performance of the above-mentioned methods. The analysis uses the WIOT and the national IOT ($\mathbf{T^R}$) from 1995 to 2011 from the WIOT database. A simplified pattern of a WIOT is depicted in Figure 1, where:

- $\mathbf{Z}^{R;R}$ ($R = 1, 2, \dots, 41$) is a 35×35 matrix whose elements are the flows for intermediate use from the i-th sector of country R to the j-th sector of the same country;
- $\mathbf{Z}^{R;K}$ ($R, K = 1, 2, \dots, 41$ and $R \neq K$) is a 35×35 matrix whose elements are the exports for intermediate use from the i-th sector of country R to the j-th sector of country K;
- $\mathbf{C}^{R;R}$($R = 1, 2, \dots, 41$) is a 35×5 matrix of the domestic final demand in country R;
- $\mathbf{E}^{R;K}$ ($R, K = 1, 2, \dots, 41$ and $R \neq K$) is a 35×5 matrix of the exports of country R for final demand purposes in country K;
- $\mathbf{x}^R$ ($R = 1, 2, \dots, 41$) is a 35×1 vector whose entries are the sectorial output of country R;
- $(\bar{\mathbf{v}}^R)'$ ($R = 1, 2, \dots, 41$) is a 1×35 vector whose entries are the sectorial added-value plus the primary sectorial input of country R.

The methodology adopted to analyse the behaviour of the various methods is as follows. Using the national IOT ($\mathbf{T}^R$), for every year and each country (R), the national augmented matrix of input coefficients ($A^R = [a_{ij}^R]$) was determined.

In a similar way, the world augmented matrix of input coefficients ($A^W = [a_{ij}^W]$) was determined. In this regard, let:

$\mathbf{Z}^{1;1}$	$\mathbf{Z}^{1;2}$	…	$\mathbf{Z}^{1;40}$	$\mathbf{Z}^{1;41}$	$\mathbf{C}^{1;1}$	…	$\mathbf{E}^{1;40}$	$\mathbf{E}^{1;41}$	$\mathbf{x}^{1}$
$\mathbf{Z}^{2;1}$	$\mathbf{Z}^{2;2}$	…	$\mathbf{Z}^{2;40}$	$\mathbf{Z}^{2;41}$	$\mathbf{E}^{2;1}$	…	$\mathbf{E}^{2;40}$	$\mathbf{E}^{2;41}$	$\mathbf{x}^{2}$
…	…	…	…	…	…	…	…	…	…
$\mathbf{Z}^{40;1}$	$\mathbf{Z}^{40;2}$	…	$\mathbf{Z}^{40;40}$	$\mathbf{Z}^{40;41}$	$\mathbf{E}^{40;1}$	…	$\mathbf{C}^{40;40}$	$\mathbf{E}^{40;41}$	$\mathbf{x}^{40}$
$\mathbf{Z}^{41;1}$	$\mathbf{Z}^{41;2}$	…	$\mathbf{Z}^{41;40}$	$\mathbf{Z}^{41;41}$	$\mathbf{E}^{41;1}$	…	$\mathbf{E}^{41;40}$	$\mathbf{C}^{41;41}$	$\mathbf{x}^{41}$
$(\mathbf{v}^1)'$	$(\mathbf{v}^2)'$	…	$(\mathbf{v}^{40})'$	$(\mathbf{v}^{41})'$					
$(\mathbf{x}^1)'$	$(\mathbf{x}^2)'$	…	$(\mathbf{x}^{40})'$	$(\mathbf{x}^{41})'$					

Legend: 41 is the Rest of the World

Figure 1. Pattern of a world inter-country input–output table.

- $\mathbf{X}^{\mathrm{W}} = \sum_{R=1}^{41} \sum_{R=1}^{41} \mathbf{Z}^{\mathrm{R;R}}$;
- $\mathbf{f}^{\mathrm{W}} = \left(\sum_{R=1}^{41} \mathbf{C}^{\mathrm{R;R}} + \sum_{R=1}^{41} \sum_{K=1}^{41} \mathbf{E}^{\mathrm{R;K}} \right)$;
- $\mathbf{x}^{\mathrm{W}} = \sum_{R=1}^{41} \mathbf{x}^{\mathrm{R}}$;
- $(\bar{\mathbf{v}}^{\mathrm{W}})' = \sum_{R=1}^{41} (\bar{\mathbf{v}}^{\mathrm{R}})'$.

At this point, the following world IOT ($\mathbf{T}^{\mathrm{W}}$), in block matrix notation, was considered:

$$\mathbf{T}^{\mathrm{W}} = \begin{bmatrix} \mathbf{X}^{\mathrm{W}} & \mathbf{f}^{\mathrm{W}} \\ (\bar{\mathbf{v}}^{\mathrm{W}})' & 0 \end{bmatrix}. \tag{53}$$

Thus, the world augmented matrix of input coefficients was determined:

$$\mathbf{A}^{\mathrm{W}} = \mathbf{T}^{\mathrm{W}} \mathbf{D}(\mathbf{x}^{\mathrm{W}})^{-1}. \tag{54}$$

In total there are 544 (i.e., 32 × 17) country-per-time matrices of the true national input coefficients, $\mathbf{A}^{\mathrm{R}}$, and 17 (one per year) world matrices of the true technical coefficients, $\mathbf{A}^{\mathrm{W}}$.

Using each of the 17 world matrices, $\mathbf{A}^{\mathrm{W}}$, and the national sectorial total output, we estimate the countries' input coefficient matrices $\hat{\mathbf{A}}^{\mathrm{R}}$ using the rescaling method described above. To assess the performance of the regionalization methods, the estimated input coefficient matrices are compared with the true ones, $\mathbf{A}^{\mathrm{R}}$, obtained from the national IOT.

We assess the performance of these methods with respect to three aspects: accuracy of the estimated input coefficients, variability of the approximation error, and direction of bias. With regard to the CE method, we also assess the robustness of the results to a scaling factor.

COMPARATIVE ANALYSIS OF THE CE, RAS AND SQUARED DIFFERENCE OPTIMIZATION METHODS

Accuracy of the estimated input coefficients – MAD analysis

With regard to the accuracy of the estimated input coefficients, several measures have been developed in the scientific literature. Among these, as suggested by Wiebe and Lenzen (2016), the mean absolute difference index (MAD) is used:

$$MAD = \sum_{i=1}^{35} \sum_{j=1}^{35} \frac{|a_{ij} - \hat{a}_{ij}|}{35 \cdot 35}. \tag{55}$$

Considering the group of constrained optimization methods, the values of the MAD index (multiplied by 100) are synthesized in Table 2 while detailed MAD values are shown in Tables A2–A6 in Appendix A in the supplemental data online.

Table 2 shows satisfactory values of the MAD index for these methods, according to both country and year. In addition, this index is very stable as a function of time since its time series has a low standard deviation (SD). This finding provides initial empirical evidence that these methods perform well when applied to real data. Interestingly, over the financial crisis, the performance of all methods worsens. This could be due to the combined effect of two facts. On the one hand, the links among economic sectors began to weaken, implying an increase in the number of a_{ij}^{R} close to zero. On the other hand, as shown below, the simultaneous estimation of input coefficients close to zero is a challenge for these methods.

With respect to the first aspect, Table 3 reports some descriptive statistics of regional input coefficients for the years 2007 and 2008. In general, it can be seen that the empirical distribution in 2008 shows a larger kurtosis and a larger positive asymmetry than in 2007.

Table 2. Descriptive statistics for the MAD index.

Country	RAS Mean	RAS SD	CE Mean	CE SD	SSD Mean	SSD SD	WSD Mean	WSD SD	NSD Mean	NSD SD
AUT	0.59	0.04	0.59	0.06	0.62	0.06	0.68	0.06	0.60	0.06
BEL	0.74	0.03	0.77	0.04	0.78	0.04	0.86	0.04	0.76	0.05
BGR	0.93	0.04	0.95	0.02	0.96	0.03	1.15	0.08	0.97	0.02
CAN	0.73	0.01	0.72	0.03	0.73	0.04	0.78	0.04	0.72	0.04
CHN	0.69	0.02	0.77	0.02	0.85	0.02	1.05	0.03	0.80	0.02
CYP	0.94	0.04	0.93	0.07	0.95	0.09	1.06	0.10	0.91	0.09
CZE	0.77	0.08	0.75	0.07	0.76	0.09	0.87	0.10	0.76	0.08
DEU	0.63	0.04	0.68	0.04	0.71	0.05	0.80	0.07	0.68	0.05
DNK	0.79	0.09	0.80	0.08	0.82	0.08	0.93	0.08	0.82	0.09
ESP	0.64	0.04	0.67	0.06	0.65	0.05	0.78	0.07	0.64	0.06
EST	0.86	0.09	0.80	0.05	0.79	0.05	0.88	0.06	0.80	0.05
FIN	0.76	0.02	0.77	0.03	0.77	0.03	0.86	0.03	0.77	0.03
FRA	0.55	0.05	0.62	0.07	0.62	0.08	0.68	0.08	0.60	0.07
GBR	0.61	0.03	0.70	0.07	0.72	0.08	0.79	0.10	0.71	0.08
GRC	0.81	0.04	0.80	0.06	0.81	0.07	0.91	0.08	0.79	0.06
HUN	0.67	0.04	0.65	0.04	0.69	0.04	0.80	0.04	0.66	0.04
IRL	1.02	0.06	0.87	0.05	0.91	0.05	1.04	0.05	0.89	0.05
ITA	0.64	0.04	0.65	0.07	0.63	0.06	0.70	0.06	0.63	0.06
JPN	0.66	0.05	0.65	0.06	0.63	0.04	0.69	0.05	0.62	0.05
LTU	0.88	0.03	0.93	0.05	0.96	0.06	1.09	0.08	0.94	0.06
LUX	1.25	0.09	0.96	0.08	1.01	0.09	1.11	0.10	0.97	0.09
LVA	0.92	0.02	0.89	0.04	0.88	0.03	0.99	0.03	0.90	0.04
MLT	1.09	0.08	1.06	0.06	1.08	0.05	1.21	0.06	1.04	0.05
NLD	0.68	0.03	0.72	0.06	0.73	0.06	0.80	0.06	0.72	0.05
POL	0.67	0.07	0.71	0.06	0.71	0.06	0.80	0.08	0.72	0.07
PRT	0.72	0.01	0.70	0.02	0.68	0.02	0.77	0.02	0.67	0.02
ROM	0.93	0.02	0.96	0.02	0.97	0.02	1.16	0.06	0.97	0.02
RUS	0.84	0.03	0.92	0.03	0.93	0.02	1.04	0.01	0.94	0.02
SVK	0.77	0.05	0.80	0.03	0.80	0.03	0.88	0.04	0.80	0.04
SVN	0.69	0.02	0.73	0.02	0.72	0.02	0.80	0.02	0.71	0.02
SWE	0.71	0.03	0.75	0.04	0.75	0.04	0.86	0.04	0.74	0.04
USA	0.54	0.03	0.57	0.06	0.57	0.06	0.67	0.07	0.56	0.06

Accuracy of the estimated input coefficients – the challenge of small-input coefficients' estimation

As for the second aspect, Table 2 shows that Bulgaria, Cyprus, Ireland, Latvia, Lithuania, Luxembourg and Malta are countries where the estimates obtained with all methods are unsatisfactory in that the corresponding MAD values (multiplied by 100) show the highest values (generally close to or > 1). In contrast, Austria, France and the United States are countries where the methods perform well, showing MAD values ≤ 0.60.

Table 3. Descriptive statistics for the observed input coefficients.

	2007							2008						
Country	**Mean**	**Median**	**SD**	**Minimum**	**Maximum**	**SK**	**KU**	**Mean**	**Median**	**SD**	**Minimum**	**Maximum**	**SK**	**KU**
AUT	0.015	0.004	0.036	0	0.549	7.517	84.597	0.015	0.004	0.037	0	0.587	7.890	94.255
BEL	0.016	0.004	0.039	0	0.458	5.528	42.664	0.016	0.004	0.040	0	0.463	5.667	45.002
BGR	0.016	0.004	0.040	0	0.561	6.300	55.435	0.016	0.004	0.039	0	0.396	5.517	39.486
CAN	0.014	0.003	0.035	0	0.651	8.324	115.824	0.014	0.003	0.033	0	0.506	6.731	70.280
CHN	0.017	0.004	0.046	0	0.607	6.438	54.380	0.017	0.004	0.046	0	0.635	6.457	56.045
CYP	0.013	0.002	0.039	0	0.473	6.743	60.439	0.014	0.002	0.047	0	0.796	8.821	107.367
CZE	0.017	0.004	0.044	0	0.578	5.960	49.835	0.017	0.004	0.045	0	0.570	6.002	50.272
DEU	0.015	0.004	0.035	0	0.461	5.883	50.896	0.015	0.004	0.035	0	0.448	6.081	54.264
DNK	0.015	0.004	0.040	0	0.739	9.231	132.656	0.015	0.003	0.040	0	0.719	8.907	124.803
ESP	0.015	0.004	0.037	0	0.511	5.853	50.302	0.015	0.004	0.037	0	0.575	6.256	61.512
EST	0.016	0.005	0.035	0	0.41	5.387	41.111	0.016	0.005	0.034	0	0.387	5.193	37.904
FIN	0.015	0.005	0.032	0	0.385	5.269	41.605	0.015	0.005	0.033	0	0.376	5.164	39.211
FRA	0.015	0.004	0.035	0	0.445	5.212	40.443	0.015	0.004	0.036	0	0.502	5.521	47.716
GBR	0.014	0.003	0.035	0	0.685	8.257	120.139	0.014	0.003	0.035	0	0.688	8.237	120.023
GRC	0.013	0.002	0.033	0	0.557	7.546	90.933	0.013	0.002	0.033	0	0.515	7.293	80.547
HUN	0.016	0.006	0.035	0	0.474	6.042	53.768	0.016	0.006	0.035	0	0.456	5.817	49.730
IRL	0.016	0.003	0.041	0	0.411	5.586	40.349	0.016	0.003	0.043	0	0.432	5.789	43.504
ITA	0.016	0.005	0.035	0	0.541	6.218	62.382	0.016	0.005	0.035	0	0.570	6.421	68.391
JPN	0.015	0.004	0.038	0	0.513	6.873	66.181	0.015	0.004	0.040	0	0.546	6.991	67.954
LTU	0.013	0.005	0.027	0	0.31	5.190	39.323	0.014	0.005	0.030	0	0.381	6.266	56.727
LUX	0.014	0.002	0.042	0	0.726	7.718	91.394	0.014	0.002	0.043	0	0.700	7.160	79.356
LVA	0.015	0.004	0.035	0	0.382	5.717	45.250	0.015	0.004	0.035	0	0.446	5.906	49.776

(*Continued*)

Table 3. Continued.

	2007							2008						
Country	**Mean**	**Median**	**SD**	**Minimum**	**Maximum**	**SK**	**KU**	**Mean**	**Median**	**SD**	**Minimum**	**Maximum**	**SK**	**KU**
MLT	0.015	0.003	0.041	0	0.661	7.299	78.688	0.015	0.003	0.041	0	0.616	7.051	72.383
NLD	0.015	0.004	0.035	0	0.563	6.248	66.452	0.015	0.004	0.035	0	0.510	5.663	51.827
POL	0.016	0.005	0.033	0	0.419	5.354	42.584	0.016	0.005	0.033	0	0.432	5.385	43.683
PRT	0.015	0.003	0.042	0	0.625	7.144	71.164	0.015	0.003	0.043	0	0.719	7.747	88.833
ROM	0.014	0.004	0.034	0	0.351	5.512	40.299	0.014	0.004	0.034	0	0.351	5.398	38.694
RUS	0.014	0.005	0.029	0	0.389	5.248	42.167	0.014	0.005	0.029	0	0.400	5.374	44.959
SVK	0.016	0.004	0.039	0	0.487	5.825	47.575	0.016	0.004	0.039	0	0.486	5.701	45.266
SVN	0.015	0.005	0.036	0	0.44	6.017	51.291	0.015	0.005	0.036	0	0.430	5.840	48.530
SWE	0.015	0.005	0.034	0	0.587	6.996	82.860	0.015	0.005	0.035	0	0.575	6.705	75.780
USA	0.014	0.003	0.037	0	0.575	6.493	66.477	0.015	0.003	0.038	0	0.650	7.210	83.964

Note: SK, skewness; KU, kurtosis.

Figure 2 show the density of the observed and estimated input coefficients corresponding to countries where each method considered achieves its best (left panels) and worst (right panels) performance according to the MAD index.

It is easy to see that the densities of the estimated input coefficients of France in 1999, France in 2009 and the United States in 2011 match better than those of Luxembourg in 2011, Bulgaria in 1997 and Malta in 1999.

By way of example, in Figure 3 we use the colormap technique to draw a pseudo-colour plot of the matrix entries corresponding to the matrices of the observed input coefficients (left panels) and those estimated using the RAS (upper two panel rows) and CE (lower two panel rows) methods.

Table 4 shows some descriptive statistics of the observed and estimated coefficients that provide a quantitative measure of the quality of the estimates.

It is very clear that the distributions of input coefficients in the countries where the methods achieve the worst performance are more positively skewed than the distributions where the methods achieve the best performance. This is evident from the pronounced difference in the interquartile ranges (IQRs) and the median values given that the two distributions show similar mean values.

Thus, the distribution of input coefficients for Bulgaria, Malta and Luxembourg is more concentrated around zero with fatter tails than the distributions for France and the United States. This feature seems to contribute negatively to the accuracy of the final estimation of the input coefficients obtained using any of the methods considered.

We investigate this issue in more detail, analysing for each country and year the association between the third quartile (Q3) of the distribution of the observed input coefficients (see Table A7 in Appendix A in the supplemental data online) and that of the MAD index values. The results of this analysis are shown in Figure 4. As expected, the correlations index between Q3 and MAD are negative for all the methods considered. This means that the performance of the methods worsens if the number of observed input coefficients close to zero increases. The RAS method has the highest negative correlations. Furthermore, it is worth noting that starting in 2004, the correlations corresponding to different methods exhibit a smaller degree of variability than those in the period 1995–2004, indicating changes in the world economy.

To investigate this issue further, we compute the correlations between the countries gross value added and the corresponding MAD values of each method. The mean value of the correlation coefficients obtained is equal to -0.45. This result indicates that the accuracy depends on the size of the economy in that the larger the economy, the smaller the MAD value (i.e., the better the performance).

To confirm what has just been observed, Figures 5 and 6 show MAD values and Q3 values corresponding to the CE and RAS methods, respectively, while the black horizontal line denotes the MAD value equal to 0.01.

Interestingly, countries with Q3 values < 0.01 are those with MAD values > 0.01. This is evident for Cyprus, Luxembourg and Malta, that is the country with the smallest Q3 value in 1995 and 2000 and the largest MAD in the same years.

Accuracy of the estimated input coefficients – MAD ranking

We rank the methods in term of the MAD index. For each country (k) and year (t), the rank of the MAD index by methods (j) was determined and denoted by R_{ktj}. The rank sum by year was then considered for a selected method and country:

$$R_{kj} = \sum_{t=1995}^{2011} R_{ktj} \tag{56}$$

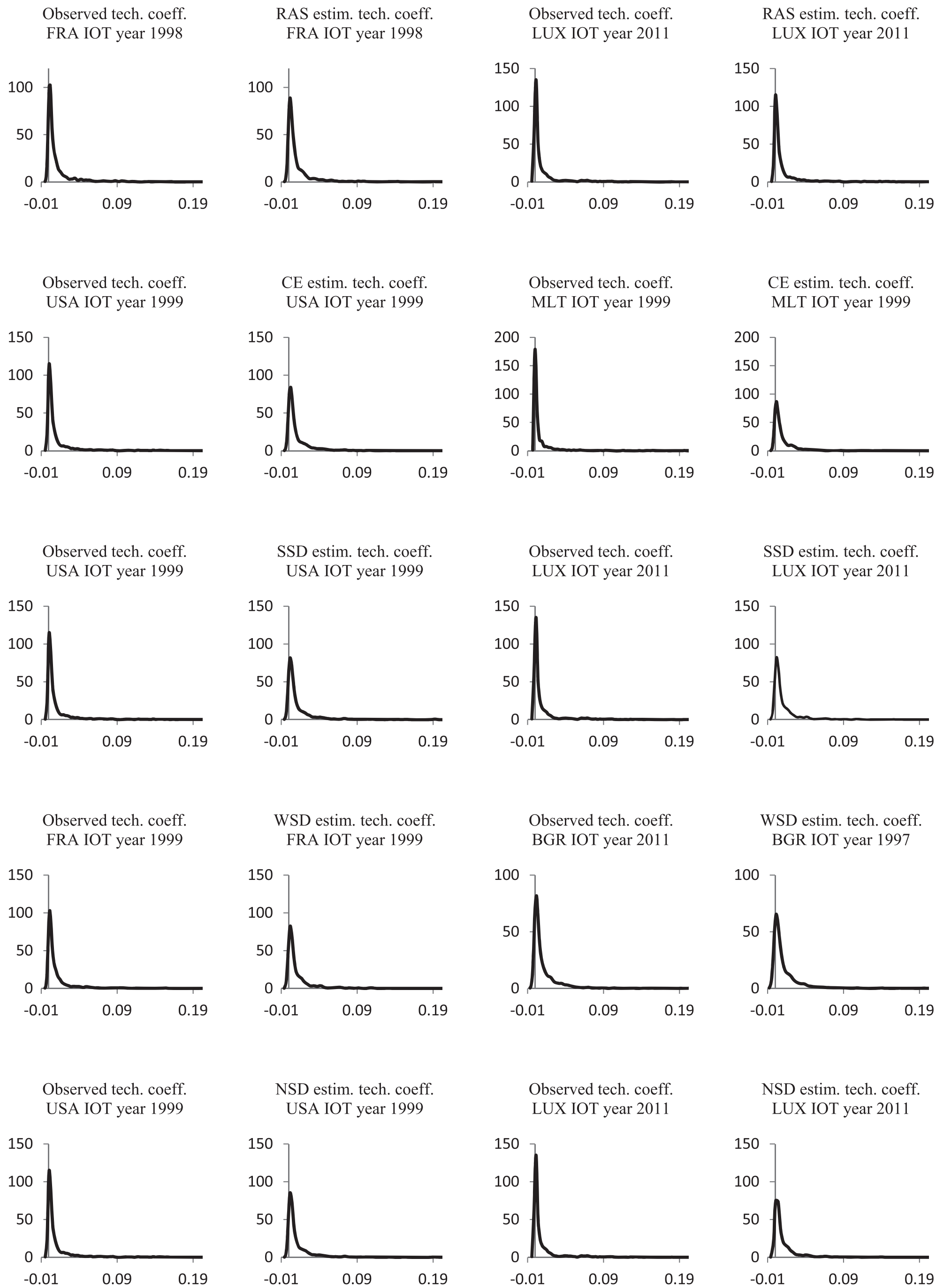

Figure 2. (left) Distribution of the observed and best estimate of input coefficients per method; and (right) distribution of observed and worst estimate of input coefficients per method.

The R_{kj} index ranges from 17 to 85, where 17 is the best value and 85 is the worst value. In fact, $R_{kj} = 17$ means that method j applied to country k attains the lowest MAD value over all years,

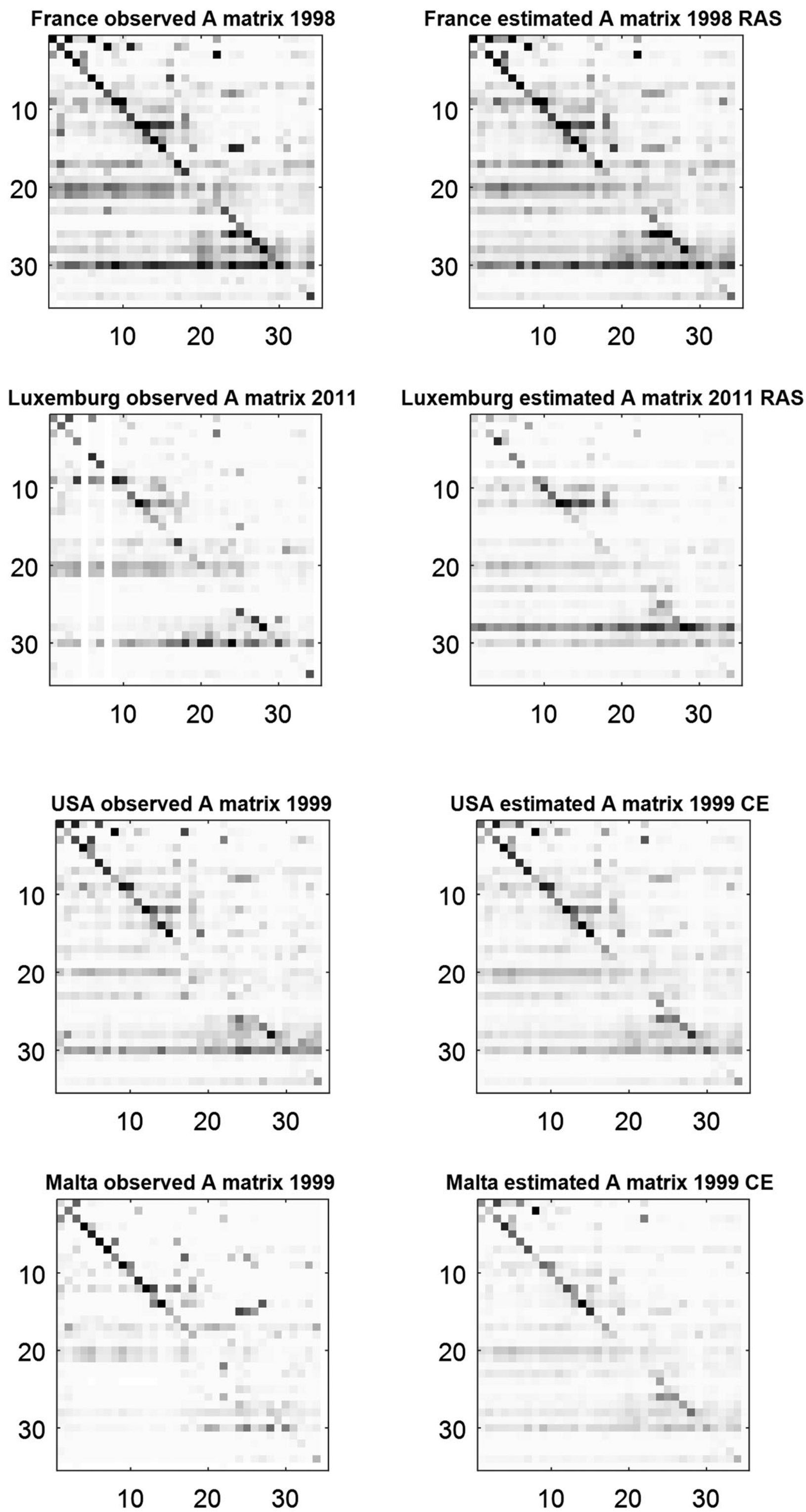

Figure 3. Examples of observed and estimated A matrices for the RAS and CE methods.

Table 4. Descriptive statistics for the observed and best/worst estimated input coefficients.

Data type	Country	Year	Minimum	Maximum	Mean	Median	SD	Q1	Q3	IQR
RAS										
Best estimate	FRA	1998	0	0.304	0.014	0.004	0.001	0.001	0.013	0.012
Observed	FRA	1998	0	0.277	0.014	0.004	0.032	0.001	0.011	0.010
Worst estimate	LUX	2011	0	0.605	0.016	0.003	0.040	0.001	0.010	0.009
Observed	LUX	2011	0	0.682	0.015	0.002	0.042	0.000	0.001	0.001
CE										
Best estimate	USA	1999	0	0.418	0.014	0.005	0.031	0.002	0.014	0.012
Observed	USA	1999	0	0.411	0.014	0.002	0.038	0.000	0.010	0.010
Worst estimate	MLT	1999	0	0.753	0.013	0.001	0.045	0.000	0.007	0.007
Observed	MLT	1999	0	0.753	0.013	0.001	0.045	0.000	0.007	0.007
SSD										
Best estimate	USA	1999	0	0.463	0.014	0.005	0.032	0.002	0.014	0.012
Observed	USA	1999	0	0.411	0.014	0.002	0.038	0.000	0.010	0.010
Worst estimate	LUX	2011	0	0.647	0.016	0.005	0.040	0.002	0.014	0.012
Observed	LUX	2011	0	0.682	0.015	0.002	0.042	0.000	0.001	0.001
WSD										
Best estimate	FRA	1999	0	0.463	0.015	0.005	0.032	0.002	0.014	0.012
Observed	FRA	1999	0	0.307	0.014	0.004	0.033	0.001	0.011	0.011
Worst estimate	BGR	1997	0	0.495	0.017	0.005	0.037	0.001	0.002	0.000
Observed	BGR	1997	0	0.378	0.017	0.005	0.038	0.001	0.018	0.017

NSD										
Best estimate	USA	1999	0	0.453	0.014	0.005	0.031	0.002	0.014	0.012
Observed	USA	1999	0	0.411	0.014	0.002	0.038	0.000	0.010	0.010
Worst estimate	LUX	2011	0	0.821	0.016	0.005	0.042	0.002	0.014	0.012
Observed	LUX	2011	0	0.682	0.015	0.002	0.042	0.000	0.001	0.001

Note: Q1, first quartile; Q3, third quartile; interquartile range (IQR) = Q3–Q1.

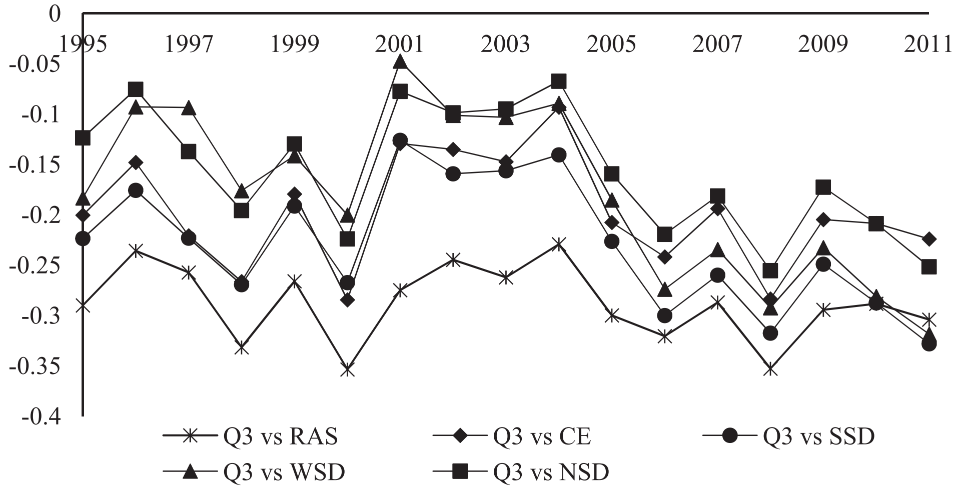

Figure 4. Correlations between observed Q3 and MAD index.

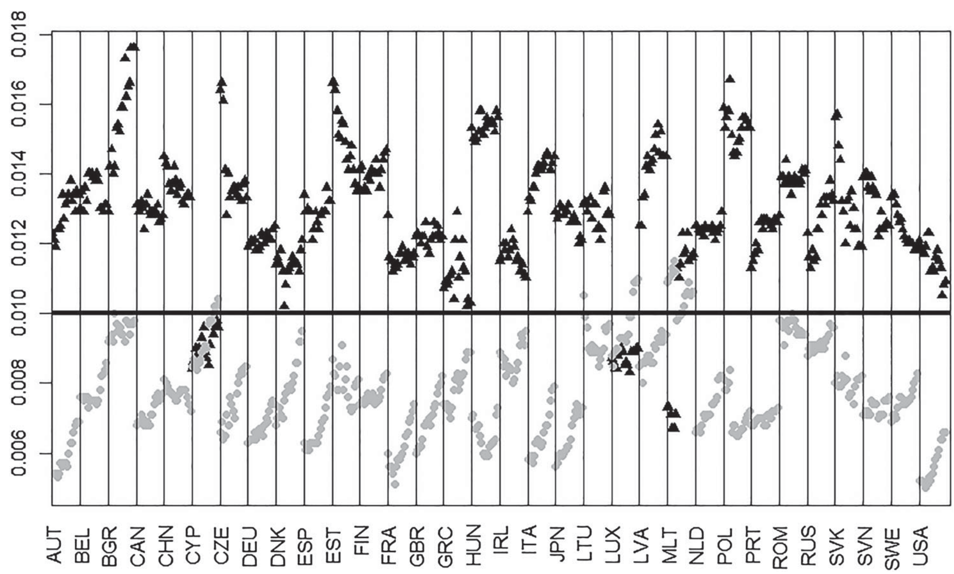

Figure 5. Observed third quartile Q3 (triangles) versus MAD index of the CE method (circles). The vertical lines identify MAD and Q3 time series for different countries. The name of the country is written at the beginning of the time series (left vertical line).

thus achieving the first position in terms of performance corresponding, to $R_{ktj} = 1$ for $t = 1995$, 1996, … , 2011. Vice versa, $R_{kj} = 85$ means that method j applied to country k attains the largest MAD value over all years, thus achieving the last position (i.e., the fifth one), corresponding to $R_{ktj} = 5$ for $t = 1995$, 1996, … , 2011.

Table 5 shows the results of the ranking while Figure 7 displays the density of ranking statistics for each method. From Figure 7 it is possible to note that the RAS method has the highest variability in the MAD index since its behaviour is very heterogeneous from country to country. It holds the first position in 56% of countries and second position in 9% of cases.

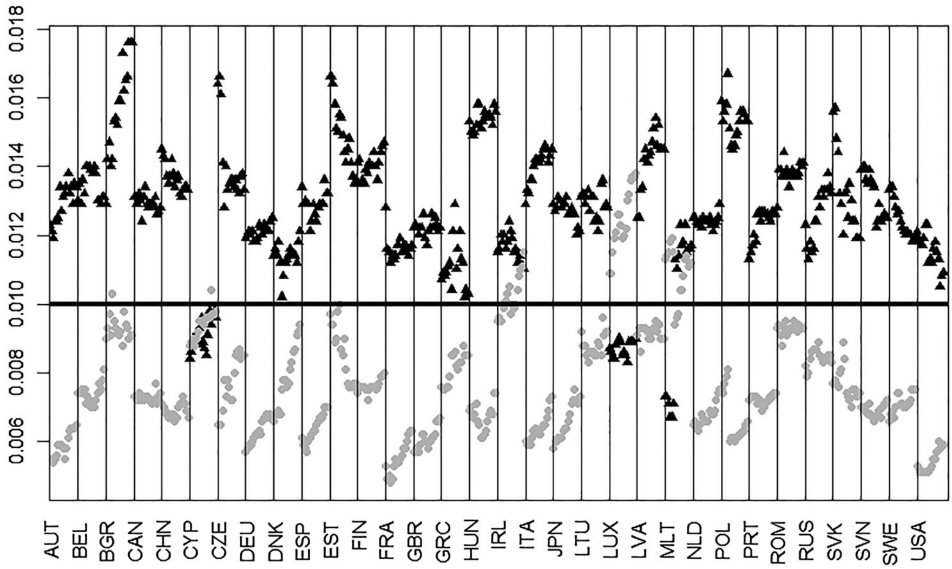

Figure 6. Observed third quartile Q3 (triangles) versus MAD index of the RAS method (circles). The vertical lines identify MAD and Q3 time series for different countries. The name of the country is written at the beginning of the time series (left vertical line).

The index is equal to 17 in Germany, the UK, the Netherlands, Poland and Russia. This means that in these countries and for each year, the RAS method always has the best performance.

The RAS method appears in the last two positions for some years (i.e., $R_{kj} \geq 55$) only for the Czech Republic, Estonia, Ireland, Japan, Luxembourg, Latvia, Malta and Portugal.

The CE method takes more homogeneous values over the countries and achieves the first two positions for some years in 66% of countries. It is followed by the NSD method, which occupies the first two positions in 50% of countries, while it is generally ranked third in the remaining countries. The SSD method holds first position in 9% of countries but is ranked between the first and third positions in 50% of countries. Finally, the WSD method occupies the last position with the lowest performance.

In conclusion, the analysis highlights that the RAS method has the best performance, but its efficiency is very heterogeneous from country to country. On the contrary, the CE and NSD methods show more stable behaviour. Their performance is greater than the RAS method in 41% of cases. The SSD and WSD methods then follow.

Focusing on RAS and CE methods, Figure 8 shows the RAS MAD index minus the CE MAD index. The MAD differences indicate that the RAS method slightly outperforms the CE method except for small countries such as Luxembourg, Greece, Ireland and Malta.

Analysis of bias of the estimated input coefficients

We now investigate in greater detail the accuracy of the estimates of the methods by comparing the decimal places of the true and estimated input coefficients. To simplify the notation, we drop the superscript R. Table A8 in Appendix A in the supplemental data online uses some descriptive statistics to show the results of this investigation when a_{ij} and $\hat{a}_{ij}$ rounded to the first, second and third decimal places. Specifically, for each country, Table A8 shows the statistics regarding the ratio of the number of times that the first decimal place of a_{ij} and $\hat{a}_{ij}$ is equal to the total number of input coefficients (i.e., 35 × 35), that is, the relative frequency (also probability) of matching the first decimal place. Tables A9 and A10 in Appendix A in the supplemental data online

Table 5. Rank sum of the various methods per year for each country.

Country	RAS	CE	SSD	WSD	NSD
AUT	36	32	65	85	37
BEL	19.5	51.5	63	85	36
BGR	23.5	38.5	46	85	62
CHN	20	72	55	74	34
CAN	28.5	23.5	68	85	50
CYP	48	37	55	85	30
CZE	51.5	22.5	49	85	47
DEU	17	40	68	85	45
DNK	22	29	64	85	55
ESP	33.5	64.5	46	85	26
EST	66	39	27	78	45
FIN	35	35	52	85	48
FRA	18	53	59	85	40
GBR	17	39	61	85	53
GRC	54.5	37.5	57	85	21
HUN	46.5	22.5	63	85	38
IRL	76	18	51	77	33
ITA	52	60	28	85	30
JPN	66.5	48.5	39	84	17
LTU	23.5	40.5	65	85	41
LUX	85	20	51	68	31
LVA	55	34	30	85	51
MLT	59	37	53	85	21
NLD	17	47	56	85	50
POL	17	47	39	85	67
PRT	68	48	33	85	21
ROM	18	33	66	85	53
RUS	17	39	56	85	58
SVK	27	49	46	85	48
SVN	20	66	45	85	39
SWE	21	55	58	85	36
USA	22.5	39.5	68	85	40
Mean	36.91	41.19	52.56	83.63	40.72
SD	20.56	13.43	12.01	3.87	12.51
Minimum	17	18	27	68	17
Maximum	85	72	68	85	67
Range	68	54	41	17	50

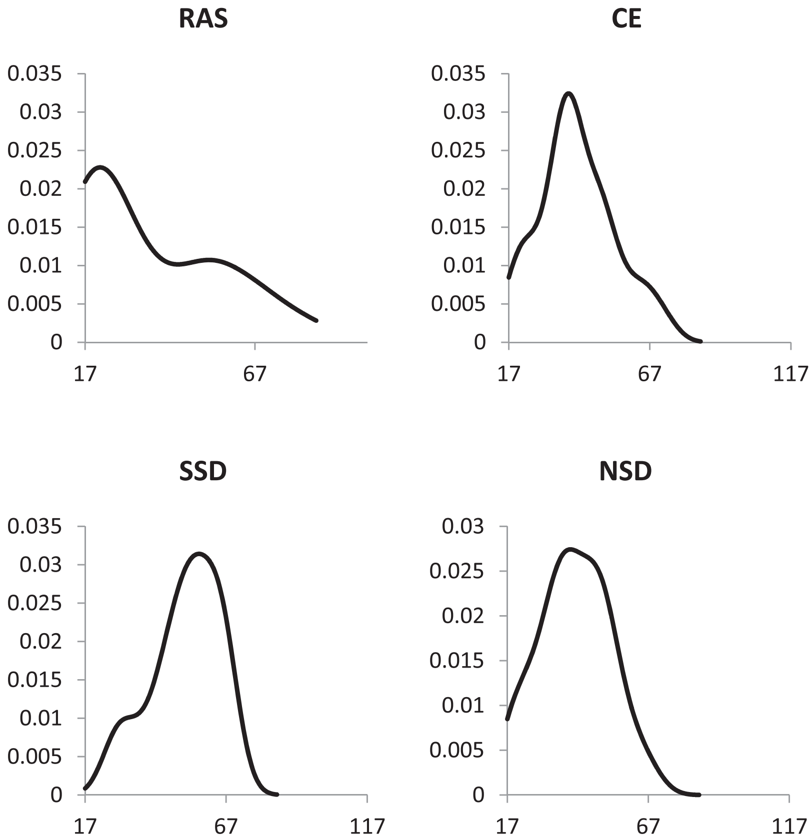

Figure 7. Density of the rank sum index per method.

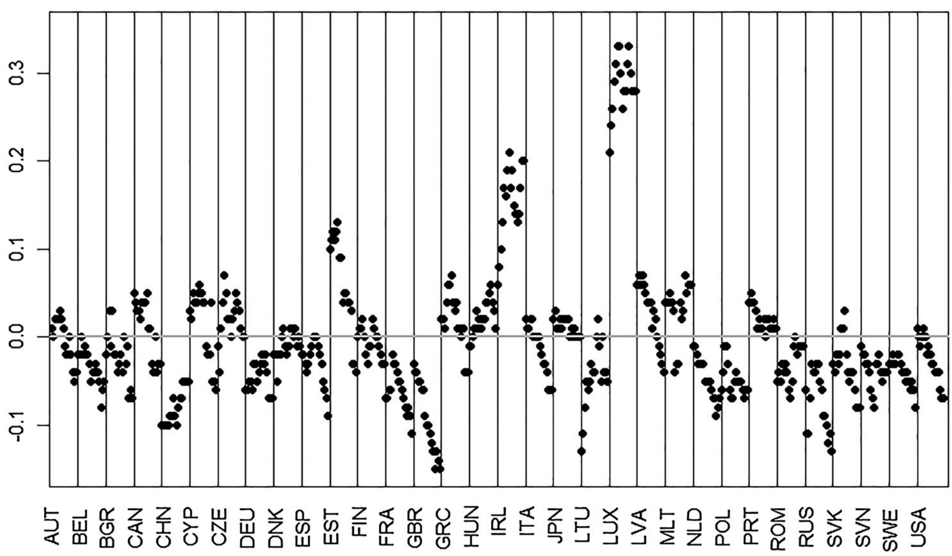

Figure 8. Differences in RAS MAD and CE MAD.

respectively display some statistics regarding the relative frequency of matching the first two and the first three decimal places.

Table A8 in Appendix A in the supplemental data online shows that the methods behave uniformly over the countries, with the frequencies of matching the first decimal place ≥ 0.97. In contrast, as Table A11 in Appendix A online shows, the frequency that the estimated first decimal place is larger than the true one (i.e., $\hat{a}_{ij} > a_{ij}$) is always < 0.03.

As shown in Table A9 in Appendix A in the supplemental data online, that is, the matching of the first two decimal places of the true and estimated input coefficients, the methods continue to behave uniformly. The frequency of matching reduces to 0.70 on average. Specifically, these frequencies vary from 0.66 to 0.76 for the RAS and CE methods, and from 0.65 to 0.75 for the SSD and NSD methods, with the worst frequencies for the WSD method ranging from 0.62 to 0.71. The probability that the coefficients estimated by using two decimal places is smaller than the true ones (i.e., $a_{ij} > \hat{a}_{ij}$) varies from 0.10 to 0.21 on average (see Table A12 in Appendix A in the supplemental data online). Finally, considering the first three decimal places of a_{ij} and $\hat{a}_{ij}$, we observe different performances among the methods. Specifically, the probability of coincidence is, on average, equal to 0.24 for the RAS method, 0.23 for the CE method, 0.22 for the SSD and NSD methods, and 0.19 for the WSD method (see Table A10 in Appendix A in the supplemental data online). The probability that $a_{ij} > \hat{a}_{ij}$ is, on average, equal to 0.43 for the RAS method, 0.46 for the CE method, 0.47 for the SSD and NSD methods, and 0.49 for the WSD method (see Table A13 in Appendix A in the supplemental data online).

In sum, the results of this analysis do not reveal relevant differences among the methods considered. The first decimal figure is usually correctly estimated in 98% of cases. The first two decimal places are correctly estimated in 70% of cases, except for the WSD method, where the percentage is about 67%. Finally, the first three decimal places are correctly estimated in about 20% of cases. The highest percentage is achieved by the RAS method (24%) while the lowest is achieved by the WSD method (19%). Moreover, the percentage of overestimation varies from 31% (CE, SSD and NSD) to 36% (RAS).

After having assessed the accuracy of the methods, we now investigate their performance in terms of variability patterns by using the SD of the absolute differences between the observed and estimated input coefficients.

Table A14 in Appendix A in the supplemental data online shows some descriptive statistics for each country of the SD index values multiplied by 100. The mean value of this index varies from 1.30 to 3.23 for the RAS method, from 1.32 to 2.75 for the CE method, from 1.27 to 2.75 for the SSD method, from 1.32 to 3.02 for the WSD method, and from 1.27 to 2.71 for the NSD method. Consequently, the NSD and CE methods have the lowest variability and the RAS method has the highest variability.

Specifically, Bulgaria, Cyprus, Denmark, Ireland, Luxembourg, Latvia and Malta have the highest variability in absolute error since they show higher SD values. Note that some of the countries with the largest SD values are those with the largest MAD values.

This finding suggests that estimating the input coefficients corresponding to Cyprus, Luxembourg and Malta is a challenging task for the optimization methods. There are three possible reasons: the fat tail of the distribution of input coefficients, the concentration of the distribution around zero and the high variability of the magnitude of the coefficient.

Finally, the low variability of the index over time must be highlighted. In fact, the largest observed range of the SD index over years studied is 1.25/100 (SSD method for Cyprus) while the smallest is 0.31/100 (RAD method for the United States).

We investigate the validation of the methods further by analysing the direction of bias. Tables A15 and A16 in Appendix A in the supplemental data online exhibit, respectively, the MD and

D_+ indices, which are defined as:

$$MD = \sum_{i=1}^{35} \sum_{j=1}^{35} \frac{a_{ij} - \hat{a}_{ij}}{35 \cdot 35}, \tag{57}$$

$$D_+ = Pr[(a_{ij} - \hat{a}_{ij}) > 0], \tag{58}$$

while D_- denotes the quantity defined as:

$$D_- = Pr[(a_{ij} - \hat{a}_{ij}) < 0]. \tag{59}$$

The MD index looks like the MAD index, but the differences between the estimated and 'true' input coefficients appear without absolute values. This index measures how much, on average, a method overestimates or underestimates the 'true' input coefficients. If MD is negative, it indicates a tendency to overestimate; if it is positive, it denotes a tendency to underestimate. The closer MD is to zero, the lower the tendency. Finally, in the extreme case where MD equals zero, there is no tendency. This can be explained by two contrasting facts: either the method exactly replicates the 'true' input coefficients or overestimation and underestimation compensate.

In contrast to the MD index, the RD_+ index measures the relative frequency of positive differences. Thus, RD_+ is a rough estimate of the probability of overestimated input coefficients. In fact, if $RD_+ = 1$ and $RD_- = 0$, the estimated input coefficients are always greater than the 'true' ones, whereas if $RD_+ = 0$ and $RD_- = 1$, the opposite holds. Finally, if $RD_+ = RD_-$, there is compensation between overestimation and underestimation.

Considering the results of the MD index (see Table A15 in Appendix A in the supplemental data online), a pattern is evident once again: since the index has values very close to zero in all the situations considered, these methods seem to have no specific tendency.

Analysing the values of RD_+, that is, the overestimation probability, shown in Table A15 in Appendix A in the supplemental data online, we can see that the index varies from 0.47 to 0.77 on average. That is, the CE method shows a tendency to overestimate the input coefficients, in contrast to the RAS, SSD, WSD and NSD methods.

Results of the comparative analysis

In sum, the analysis shows that in consideration of the MAD index values, the RAS method in general slightly outperforms the other methods but it exhibits pronounced heterogeneous behaviour among countries. In contrast to the RAS method, the CE and NDS methods are more homogeneous among countries, with performances higher than the RAS in 41% of cases. The SSD and WSD methods have the lowest performance.

With regard to the variability of the error, the CE and NSD methods have the lowest variability while the RAS method has the highest variability.

Finally, with respect to the bias direction, the CE method shows a slight tendency to overestimate the input coefficients, particularly in small countries, while the opposite is observed for the other constrained matrix-balancing methods considered. Thus, an efficient implementation of the CE method makes it very competitive when applied to the analysis of real data. It is also preferable to the RAS method when applied to the analysis of small countries such as Cyprus, Estonia, Ireland and Luxembourg.

A BRIEF LOOK AT COMPUTATIONAL COST

From the point of view of computational cost, the CE method we compare the execution time of all the methods considered in Table 6. It shows that the CE method is the most time consuming,

Table 6. Descriptive statistics for execution time of the methods.

	CE	RAS	SSD	WSD	NSD
Mean	1.825	0.018	0.588	1.082	2.341
SD	1.266	0.006	0.080	0.244	1.649
Minimum	0.719	0.016	0.469	0.797	0.672
Maximum	5.813	0.031	0.828	2.234	7.438

Note: Values are in seconds.

while the RAS method have the best performance in terms of execution time, as expected due to the theoretical computational cost mentioned in the second section.

COMPARATIVE ANALYSIS OF THE CE AND THE FLQ-BOOSTED CE METHODS

As previously mentioned, the performance of the LQ methods has already been widely analysed in the literature. Specifically, recently Lamonica and Chelli (2018) showed that the FLQ method with $\delta = 0.2$ (hereafter only FLQ) is one of the best performing LQ methods in terms of the MAD index when applied to the data set used in this paper.

We conclude our analysis by showing the performance of the combined use of the FLQ and CE methods. This is a very preliminary experiment worthy of future investigation. With equation (37), we use as starting points the values of $\tilde{\lambda}_p$ solutions to the following equations:

$$\frac{a_{ij}^{\mathrm{W}} e^{\tilde{\lambda}_p \tilde{x}_j^{\mathrm{R}}}}{\sum_{p=1}^{k+1} a_{pj}^{\mathrm{W}} e^{\tilde{\lambda}_p \tilde{x}_j^{\mathrm{R}}}} = \begin{cases} a_{ij}^{\mathrm{W}} FLQ_{ij} & if\ FLQ_{ij} < 1 \\ a_{ij}^{\mathrm{W}} & if\ FLQ_{ij} \geq 1 \end{cases}, \tag{60}$$

which also reads as:

$$\frac{e^{\tilde{\lambda}_p \tilde{x}_j^{\mathrm{R}}}}{\sum_{p=1}^{k+1} a_{pj}^{\mathrm{W}} e^{\tilde{\lambda}_p \tilde{x}_j^{\mathrm{R}}}} = \begin{cases} FLQ_{ij} & if\ FLQ_{ij} < 1 \\ 1 & if\ FLQ_{ij} \geq 1 \end{cases}. \tag{61}$$

This suggests a procedure based on three main steps:

(1) Generate the FLQ approximations to input coefficients.
(2) Solve the non-linear system (60) to determine the values of the starting point for $\tilde{\lambda}_p$.
(3) Solve equation (52) with a standard iterative algorithm by using the values of $\tilde{\lambda}_p$, generated in step 2.

We apply the procedure described in steps 1–3 to estimate the input coefficients for 2011. The results are shown in Table A18 in Appendix A in the supplemental data online. The combined use of these methods and a scaling factor of 10,000 times the maximum of the sum of row and columns provides estimates for input coefficients with a MAD index that is smaller than the corresponding index of the CE method for 17 of 32 countries. Furthermore, the quartiles of the absolute error and the maximum absolute error of the CE method result larger than or equal to those of CE-FLQ method for 28 of 32 countries. Thus, the CE-FLQ approach provides a significant improvement in the IOT estimation.

Consequently, from this simple and very partial analysis, it seems that the joint use of the two techniques should provide for better estimates.

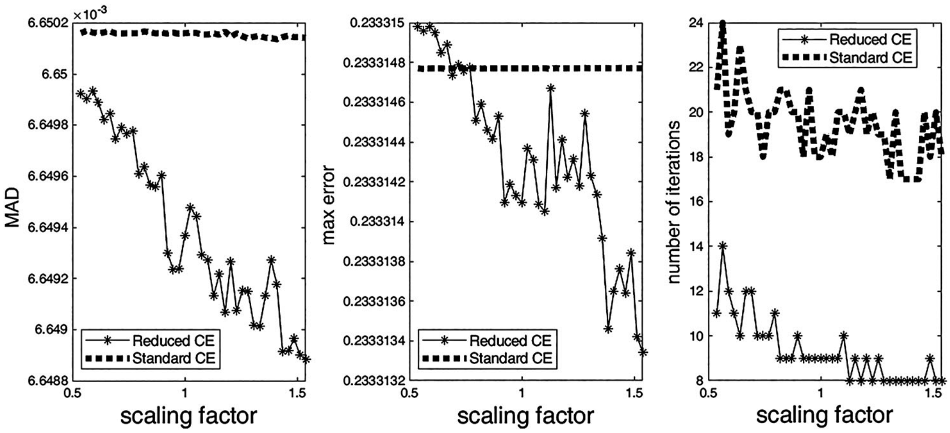

Figure 9. Robustness of the CE results to the scale factor M. Each panel shows the results relative to the CE standard method (solution to equations (37) and (38); dotted line) as well as those relative to the CE reduced method (solution to equation (52); solid line). The left panel displays the MAD index, the middle panel the largest absolute error of the input coefficient estimates and the right panel the number of iterations.

CE ROBUSTNESS TO SCALING FACTOR

We conclude our analysis by illustrating the robustness of the CE estimates to the rescaling factor used to implement the CE method. We show the robustness of the results obtained by the rescaling procedure, comparing the CE estimates obtained by solving equations (37) and (38) (standard CE for short) as well as equations (52) (reduced CE for short) for different values of the scaling factor M.

Figure 9 shows the results of this analysis. Specifically, from left to right, Figure 9 shows the MAD index, the largest relative error of the input coefficient estimates (middle panel), and the number of iterations necessary to determine the estimates of the input coefficients (right panel) as a function of the scaling factor, s_f, defined as $s_f = \dfrac{M}{max\, a_{ij}^{\mathrm{W}}}$.

The rescaled CE method displays MAD values and maximum errors that vary in the sixth decimal place, while the variation of the MAD and maximum error of the standard CE is imperceptible. In contrast, the difference in the number of iterations between these two methods is undeniable, favouring the reduced CE approach. Thus, the standard CE method seems to be completely unaffected by the scaling factor while requiring twice the number of iterations.

CONCLUSIONS

An IOT can be built on sub-territorial levels or existing IOTs can be updated over time via survey, semi-survey or non-survey methods. Since the costs and time required with survey or semi-survey methods are often prohibitive, non-survey or indirect, methods have long attracted the attention of statisticians and researchers.

Various methods have been proposed in the literature that can be classified in two major groups: LQ methods and constrained matrix-balancing methods. This paper reported the results of a variety of experiments based on real data on the performance of the constrained matrix-balancing methods. The most important results of this study are as follows:

- An efficient implementation of the CE method makes it very competitive when applied to the analysis of real data. While the RAS method slightly outperforms the CE method its efficiency varies widely from country to country. On the contrary, the behaviour of the CE method is more stable.
- The performance of the CE method is superior to the RAS method for countries with small economies and it is very similar to the performance of the NSD method.
- The CE method has very good predictive power, and in particular for countries with small economies.
- Since the method uses numerical algorithms, due to the high number of parameters to be estimated and the high values of the sectorial total output that are involved in the algorithm, it may be not convergent. This represents a limit of the method when, as in situations considered in this paper, real data are used. However, this problem, as proposed in the paper, can be solved by using the strategy of rescaling sectorial total productions.
- The combined use of FLQ and CE methods provide better estimates with respect to the CE method.

Finally, note that the performance of all the methods analysed worsens close to the financial crisis in all countries and this finding suggests that the effect of globalization and the economic crisis which has weakened the links between economic sectors have affected these non-survey approaches.

DISCLOSURE STATEMENT

No potential conflict of interest was reported by the authors.

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