

A new method to estimate input-output tables by means of structural lags, tested on Spanish regions*

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Abstract. The RAS method extrapolates a single matrix to conform to new row and column totals. This paper proposes a cell-correction of RAS (CRAS) that uses the deviations of multiple RAS projections, to improve the projection of the input-output table (IOT) of a specific country or region. The new method is tested on eleven survey-based IOTs of Spanish regions for 1999–2005. CRAS is shown to outperform RAS when three to four survey IOTs are used that are close to the target IOT. When more IOTs are added, for most but not all regions, CRAS gradually becomes worse than applying RAS to the single best IOT.

JEL classification: C61, C67, D57, R15

Key words: RAS, spatial projection, input-output tables, Spanish regions

1 Introduction

RAS is known as an iterative technique to update semi-positive input-output tables (IOTs), given an old table and new row and column totals (Stone 1961; Bacharach 1970). In consultancy practice, RAS is also used to construct national input-output tables (IOTs) for countries that do not have their own IOT. This practice follows the construction of regional IOTs from a given national or regional IOT in combination with the row and column totals of the region at hand

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¹ Meanwhile it has been proved that the old iterative solution to the RAS updating problem (Stone 1961) is equivalent to solving the non-linear minimization of information gain (Bacharach 1970; Snickars and Weibull 1977; Bachem and Korte 1979). See Junius and Oosterhaven (2003) for a generalized RAS (GRAS) algorithm, which also covers cases with negative cells and negative constraints, while constraints may change sign.

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(Hewings 1969, 1977). Both ideas are combined when an old interregional IO table has to be updated given new regional row and column totals, and new national cell totals (Oosterhaven et al. 1986). All applications have in common that one single (old) matrix is given that needs to satisfy a set of (new) constraints.

In view of the tremendous amount of national, regional, interregional and international IOTs now readily available on the internet, it is surprising that hardly any attention has been paid to the problem of projecting a new IOT using the information from as many of the existing IOTs as possible. Early attempts (Tilanus 1966; Johansen 1968; Lecomber 1969; Barker 1975), focused on the projection of IO coefficients, mostly assuming a biproportional structure, without knowledge on the future row and column totals, but this did not led to methodologies to project whole IOTs with given row and column totals. Moreover, Tilanus (1966) and Barker (1975) reported that extrapolating a series of IO coefficients produces results that are worse than using the most recent coefficients. This may be the reason why this approach has not been pursued since (see Miller and Blair 2009).

Mínguez et al. (2009), however, provide a new theoretical and empirical perspective. They developed a cell-correction on the RAS method (CRAS), and tested their method on the temporal projection of Dutch IOTs over the period 1969–1986, using as many of the older tables that were comparable. In that setting, they conclude that CRAS outperforms RAS, whenever gradual changes need to be forecasted. Using a multitude of old tables, however, leads to worse results than using the most recent IOT when sudden shocks, such as the oil price rises of 1973–74 and 1979–80, need to be covered.

Here we adapt CRAS to perform as a non-survey estimation method for national or regional IO tables, when older IOTs are not available. The interesting difference is that the temporal projection of an economy, discussed above, is far simpler than the spatial projection of an economy. The reason is that time is one-dimensional and uni-directional (from past to future). Space, however, is at least two-dimensional and bi-directional. Moreover, distance may be defined in many ways, for example, physical or socio-economic, whereas time essentially is simply time. Hence, when only one single IOT is used for a temporal RAS, clearly the best choice is to take the most recent IOT available. When, however, a single IOT is used for a spatial RAS, the best choice is not so obvious. It is not simply the IOT of the region or country closest by physical space. Instead it is the IOT of the region or country most close by in terms of IO structure, but which region or country that is, is not clear beforehand.

To test CRAS as a spatial projection method we need a set of identically defined, survey-based IOTs. The IOTs need to be survey-based, as non-survey IOTs are not suited for testing a non-survey construction method, while they need to be identically defined across regions or countries for the obvious reasons. To test CRAS, we use the set of survey-based symmetric IOTs for 11 Spanish regions collected and harmonized for the construction of a seven region interregional semi-survey IOT for Spain (Escobedo-Cardeñoso and Oosterhaven 2009). Eight tables are for 2005, but three are for 2000 (see Figure 1). Hence, our test of CRAS as a spatial projection method also has a minor temporal dimension.

As Spain has 17 regions, it follows that six Spanish regions do not have a survey-based IO table yet. If the test on the eleven existing IOTs is successful, and it is, the obvious first application of CRAS at the regional level is the non-survey construction of the six yet non-existent Spanish regional IOTs.³

² As a spatial projection technique, CRAS may also be tested on for example, the harmonized set of European IOTs (Eurostat 2010).

³ There are two more Spanish regions with IO data, Cataluña and Canarias, but they only have a use table. The application of CRAS to Cataluña and Canarias may therefore be more accurate than that for the other four regions.



Fig. 1. Spanish regions with a regional survey IO table (italics, year in legend)

The setup of this paper is as follows. Section 2 will briefly summarize the nature of using structural lags in CRAS as a spatial non-survey IOT construction method. Section 3 will discuss the setup of the test on the existing 11 Spanish regional IOTs. The core of the problem is twofold. First, the test has to be set up such that it comes as close as possible to its potential use as a non-survey technique. Second, a solution has to be found for defining the structural IO distance between the regions at hand. Section 4 discusses the results of the comparison of the 11 survey IOTs with their non-survey estimates based on either RAS or CRAS applied to an increasing number of more and more different survey IOTs. Section 5 concludes that CRAS outperforms RAS when a limited set of three to four survey IOTs is used that are close to the IOT that has to be projected.

2 The cell-corrected RAS method (CRAS)

The goal of a conventional spatial RAS projection consists of obtaining an input-output transactions matrix Z^R for country or region R of dimension $m \times n$ as similar as possible to the input-output transactions matrix Z^S of country or region S of the same dimension, knowing only the margins (the row and column sums) of the target Z^R . CRAS tries to improve the RAS projection of Z^R , given the exogenous choice of Z^S .

⁴ The idea of minimizing the distance between a known matrix and the target is logical as no further information is assumed to be available (Miller 1998).

2.1 Statement of the programming model

The proposed new spatial projection method CRAS has two stages. In the first one, data for different regions S are used in a standard RAS approach to estimate the statistical deviations between the projected IO tables and the true IO table for region R:

$$d_{ij}^{R(S)} = \frac{z_{ij}^{R}}{\tilde{z}_{ij}^{R(S)}}; \quad i = 1, \dots, m; \quad j = 1, \dots, n; \quad R, S = 1, \dots, T; \quad S \neq R$$
 (1)

where $d^{R(S)}$ is the unexplained deviation if we use Z^S to project the target matrix Z^R by means of RAS; z^R_{ij} are the true values and $\tilde{z}^{R(S)}_{ij}$ are the values of the RAS projection of R from S; T is the number of regions or countries with a comparable IO table. Note that the problem is defined such that it also applies to rectangular matrices with $m \neq n$.⁵

From (1), the first two distribution moment vectors μ^d (mean) and σ^d (standard deviation) of the stochastic deviations d are calculated as follows:

$$\mu_{ij}^{d} = \sum_{s=1, s \neq R}^{T} d_{ij}^{R(S)}$$
 and $\sigma_{ij}^{d} = \sqrt{\sum_{s=1, s \neq R}^{T} \left(d_{ij}^{R(S)} - \mu_{ij}^{d}\right)^{2}}; \forall i, j, R$ (2)

That is, we will have T values for μ_{ij}^d and σ_{ij}^d , one for each target region R. The second stage of the method uses the data of (2) to correct the RAS projection of R from $S(\tilde{z}^{R(S)})$ by solving the following optimization problem:

Minimize
$$\sum_{i=1}^{m} \sum_{j=1}^{n} \left(\frac{d_{ij}^{R(S)} - \mu_{ij}^{d}}{\sigma_{ij}^{d}} \right)^{2}$$
. (3)

Subject to:

$$\sum_{i=1}^{n} d_{ij}^{R(S)} \tilde{z}_{ij}^{R(S)} = u_i^R; \quad i = 1, \dots, m$$
 (4)

$$\sum_{i=1}^{m} d_{ij}^{R(S)} \tilde{z}_{ij}^{R(S)} = v_{j}^{R}; \quad j = 1, \dots, n$$
 (5)

$$d_{ij}^{R(S)} \ge 0; \quad i = 1, \dots, m; \quad j = 1, \dots, n$$
 (6)

where u_i^R equal the known row sums of the target matrix Z^R , and v_i^R equal the known column sums of Z^R . Equation (6) assures that the solution is semi-positive, although this last constraint is not operational because all d values are centred on 1.

Once the optimization problem (3)–(6) is solved and the optimal values $d_{ij}^{R(S)^*}$ are obtained, the solution of CRAS, namely, the values of the transaction matrix $\hat{Z}^{R(S)}$, is obtained as:

$$\hat{z}_{ij}^{R(S)} = d_{ij}^{R(S)*} \tilde{z}_{ij}^{R(S)}; \quad i = 1, \dots, m; \quad j = 1, \dots, n$$
(7)

where (*) refers to the optimal values of $d^{R(S)}$.

⁵ Rectangular matrices are needed if both intermediate and final transactions have to be projected simultaneously, as is the case in the present paper. Rectangular matrices are also needed when supply and use tables need to be projected.

2.2 Solution of the programming model⁶

It is instructive and handy to derive an explicit solution to better understand the behaviour of the model. Consider the Lagrange function associated with problem (3)–(6):

$$L(d, \lambda, \gamma) = \sum_{i=1}^{m} \sum_{j=1}^{n} \left(\frac{d_{ij} - \mu_{ij}^{d}}{\sigma_{ij}^{d}} \right)^{2} + \sum_{i=1}^{m} \lambda_{i} \left(\sum_{j=1}^{n} d_{ij} \tilde{z}_{ij} - u_{i} \right) + \sum_{j=1}^{n} \gamma_{j} \left(\sum_{i=1}^{m} d_{ij} \tilde{z}_{ij} - v_{j} \right)$$
(8)

where λ_i and γ_i are the Lagrange multipliers.

The derivatives of the Lagrange function with respect to d, λ and γ are:

$$\frac{\partial L}{\partial d_{ij}} = 2 \frac{d_{ij} - \mu_{ij}^d}{\sigma_{ij}^d} + \tilde{z}_{ij} \left(\lambda_i + \gamma_j \right) = 0; \quad i = 1, \dots, m; \quad j = 1, \dots, n$$
(9)

$$\sum_{j=1}^{n} d_{ij} \tilde{Z}_{ij} - u_{i} = 0; \quad i = 1, \dots, m$$
 (10)

$$\sum_{i=1}^{m} d_{ij} \tilde{z}_{ij} - v_{j} = 0; \quad j = 1, \dots, n$$
(11)

Note that (9)–(11) represent a linear system with the following structure:

$$\begin{bmatrix} A_{(mxn,mxn)} & B_{(mxn,m+n)} \\ B_{(m+n,mxn)}^T & O_{(m+n,m+n)} \end{bmatrix} \begin{bmatrix} d_{(mxn)} \\ \lambda_{(m+n)} \\ \gamma_{(m+n)} \end{bmatrix} = \begin{bmatrix} c_{(mxn)} \\ u_{(m+n)} \\ v_{(m+n)} \end{bmatrix}$$

$$(12)$$

where the dimensions of the corresponding matrices are in parentheses. Matrix A is a diagonal matrix with $a_{ij} = 2/(d_{ij}^d)^2$, matrix B contains the RAS solution \tilde{z}_{ij} , and O is a zero matrix. Note that, for convenience, the deviation matrix e has been reorganized in a column vector. The elements of the vector c are $c_{ij} = 2\mu_{ij}^d/(d_{ij}^d)^2$, and u and v are the vectors with the row sums and column sums of the target matrix, respectively.

For the system (12) to have an unique solution, the rank of the coefficient matrix must be equal to its dimension (mxn+m+n). The first column block $\begin{bmatrix} A \\ B^T \end{bmatrix}$ has rank mxn if $\sigma_{ij}^d \neq 0$ and finite, \forall_{ij} , because in that case A is a full diagonal matrix. However, the rank of matrix B is m+n-1 if $\tilde{z}_{ij} \neq 0$, \forall_{ij} , because in (10) and (11) there is one redundant constraint due to the compatibility condition that the sum of the row totals should equal the sum of the column totals, u = v. This redundancy must be removed, and therefore (12) must be generated eliminating one constraint in (10) and (11), no matter which. Once this condition holds, the system of linear equations is easily solved using sparse-oriented algorithms (LU, Gauss-elimination, etc.).

⁶ This section is derived from Mínguez et al. (2009). We have deleted the superscript R(S) to simplify the notation.

3 Setting up RAS for testing CRAS

Next, we discuss how RAS and CRAS have to be applied to the 11 Spanish survey-based regional IO tables⁷ in order to test CRAS as a spatial IO projection method. The core of the problem is twofold. First, the test has to be set up such that it comes as close as possible to its potential use as a projection method. Second, the structural distance in terms of IOTs, between the regions at hand, has to be defined in order to determine the performance of each of the *T-1* possible RAS projections for each region *R*.

3.1 Setting-up RAS as a spatial IO projection method

When international and interregional transit trade is removed, the layout of the Spanish survey-based IOTs equals that shown in Table 1. The core problem is how to use the 11 survey IOTs to simulate a situation that resembles the non-survey estimation of the lacking six IOTs, as much

Table 1. Layout of the standardized Spanish regional input-output table*

	Intermediate demand and local final demand	Exports to RoS	Exports to RoW	Total output
Own region sectors	$ \begin{pmatrix} z_{11}^R & \dots & z_{1n}^R & z_{y_{11}}^R \dots \\ \dots & z_{ij}^R & \dots & \dots & z_{y_{iq}}^R \dots \\ z_{m1}^R & \dots & z_{mn}^R & \dots z_{y_{mf}}^R \end{pmatrix} $	$egin{pmatrix} e_1^{R^E} \ e_i^{R^E} \ e_m^{R^E} \end{pmatrix}$	$egin{pmatrix} e_1^{R^M} \ e_i^{R^M} \ e_m^{R^M} \end{pmatrix}$	$egin{pmatrix} x_1^R \ x_i^R \ x_m^R \end{pmatrix}$
	CRAS	CRAS	Given	Estimation
Rest of Spain sectors	$ \begin{pmatrix} p_{11}^{R^E} & \cdots & p_{1n}^{R^E} & p_{y_{11}}^{R^E} \\ \cdots & p_{ij}^{R^E} & \cdots & \cdot p_{y_{iq}}^{R^E} \\ p_{m1}^{R^E} & \cdots & p_{mm}^{R^E} & \cdots p_{y_{mf}}^{R^E} \end{pmatrix} $	0	0	
	CRAS			
Rest of World sectors	$ \begin{pmatrix} p_{11}^{R^M} & \dots & p_{1n}^{R^M} & p_{y_{11}}^{R^M} \dots \\ \dots & p_{ij}^{R^M} & \dots & p_{y_{iq}}^{R^M} \dots \\ p_{m1}^{R^M} & \dots & p_{mn}^{R^M} & \dots p_{y_{mf}}^{R^M} \end{pmatrix} $	0	0	
	Estimation			
Value added	$\left(g_1^R \cdots \cdot \cdot g_j^R \cdots g_n^R \cdots g_{y_q}^R \cdots\right)$ Given	0	0	
Total output	$\left(x_1^R \dots x_j^R \dots x_n^R \dots x_{y_q}^R \dots\right)$ Estimation			

Notes: * z = intra-regional intermediate and final demand; e = exports; x = total output = total input; p = imports; g = value added; m = number of supplying sectors; n = number of purchasing sectors; n = number of final demand categories; n = number of final demand categories; n = Spanish regions; n = Rest of Spain (RoS); n = Rest of the World (RoW).

⁷ The Spanish regional symmetric input-output tables can be found in the web sites of the Institute of Statistics of the corresponding Spanish regional government. They are also available upon request from the second author.

as possible. The solution of this problem should be based on data that in the European Union, and therefore also for Spain, are given for all regions, which are indicated with 'given' in Table 1. We will show that the data indicated with 'estimation' in Table 1 can be estimated easily from the 'given' data. The data indicated with 'CRAS' then remain to be estimated by means of either RAS or CRAS.

The arguments for selecting the 'estimation' part of Table 1 are as follows. We assume that nothing is known about either the intra-regional transactions or the exports and imports with regards to the rest of Spain (RoS), because estimating them is the core of any non-survey estimation of a regional IOT. We should not assume that problem away by using the actual survey row and column totals of these matrices while comparing either RAS or CRAS with a survey IOT.⁸

Unknown total use and total output per regional sector, however, may be estimated easily by using sector-specific ratios with the 'given' gross value added at market prices as their base. These ratios may be calculated either from the Spanish national IOT or from an appropriate average of the known regional IOTs. In order to separate the estimation error of these unknown totals from the estimation error of CRAS, we will use the actual information of each of the 11 regional survey-based IOTs while testing CRAS.

A more problematic decision is whether or not to assume that the also lacking imports from the rest of the World (RoW) can be estimated *a priori* or not, either as a full matrix or as a single row. The only regional foreign import data readily available in Spain are the totals by product by region (DataComex 2009). Hence, along each row of the foreign import matrix, it might be assumed that all purchasing sectors and all categories of final demand have the same RoW import ratio. This will of course introduce an estimation error. In order not to pollute the estimation error of CRAS with the RoW import estimation error, we will use the actual RoW survey data while testing CRAS.

When the 'given' and the 'estimated' data are taken from the survey IOT of region R, RAS and CRAS are competing to estimate the remaining data. The IO data of the ten remaining regions S that form the database to estimate the remaining IO data for region R then have the following structure:

$$Z^{S} = \begin{pmatrix} z_{11}^{S} & \dots & z_{1n}^{S} & z_{y_{11}}^{S} \dots & e_{1}^{S^{E}} \\ \dots & z_{ij}^{S} & \dots & \dots & z_{y_{iq}}^{S} \dots & e_{i}^{S^{E}} \\ z_{m1}^{S} & \dots & z_{mn}^{S} & \dots & z_{y_{mf}}^{S} & e_{m}^{S^{E}} \\ p_{\bullet 1}^{S^{E}} & \dots & p_{\bullet n}^{S^{E}} & \dots & p_{y_{ng}}^{S^{E}} \dots & 0 \end{pmatrix}$$

$$(13)$$

Note that comparing the set-up and notation of Table 1, we have aggregated the import table for the rest of Spain to a single RoS import row, p^{S^E} , as a full matrix is hardly ever required in IO applications. The zero in (13) indicates that transit trade through region S is excluded.

Next, for the target region R we only need the column and row sums of (13), namely, we need to estimate:

⁸ Unfortunately, in the past it has been assumed that the total intra-regional purchases and sales are known *a priori* for all sectors (Czamanski and Malizia 1969; Morrison and Smith 1974; Sawyer and Miller 1983). As a consequence, it was unjustly concluded that RAS, as a non-survey technique, performed far better than competing non-survey techniques, such as the location quotient method (Schafer and Chu 1969). The latter techniques, however, have been developed for the difficult estimation of precisely these intra-regional totals. So, they should not have been assumed to be known *a priori* (see also Thumann 1978).

⁹ In practice, this should be done at the furthest disaggregate level possible.

$$v^{R} = (v_{1}^{R} \dots v_{q}^{R} \dots v_{n+f+1}^{R}) \text{ and } u^{R} = \begin{pmatrix} u_{1}^{R} \\ \dots \\ u_{i}^{R} \\ \dots \\ u_{m+1}^{R} \end{pmatrix},$$
 (14)

to be substituted in (4) and (5). Note that the last entry of v^R indicates the total exports to the RoS $\left(\sum_i e_i^{R^E}\right)$, while the last entry of u^R indicates the total imports from the RoS $\left(\sum_q p_q^{R^E}\right)$. The column sums of regional intermediate and final purchases from all of Spain can simply

The column sums of regional intermediate and final purchases from all of Spain can simply be calculated from the 'given' and the 'estimated' survey data in Table 1:

$$1 \le q \le n + f \Rightarrow v_q^R = z_{\bullet q}^R + p_{\bullet q}^{R^E} = x_q^R - g_{\bullet q}^R - p_{\bullet q}^{R^M}$$
 (15)

The row totals of the regional intermediate and final sales to all of Spain are also simply calculated from the 'given' and the 'estimated' survey data in Table 1:

$$1 \le i \le m \Rightarrow u_i^R = z_{i\bullet}^R + e_i^{R^E} = x_i^R - e_i^{R^M} \tag{16}$$

The more difficult problem is how to estimate the totals of last row and column of (13), without using the RoS trade data from the IOT of the target region R. Hence, these totals have to be estimated by means of the RoS trade data of base region S.¹⁰ To make the maximum use of the 'given' sector structure of region R, the sectoral trade ratios of region S are applied to region S 'estimated' sectoral purchases (15) and sectoral sales (16). This will lead to two, most certainly conflicting, estimates of region S so total intra-regional transactions. Of the latter, we take the unweighted average to derive the required totals of the last row and last column of (13).¹¹

To clarify the need to take an average, the estimation will be formalized by means of regional purchase coefficients (RPCs; Stevens and Trainer 1980), and regional sales coefficients (RSCs; Su 1970; Boomsma and Oosterhaven 1992). These equal one minus, respectively, the sectoral import ratio and the sectoral export ratio of region *S*:

$$RPC_q^S = 1 - \frac{z_{\bullet q}^{S^E}}{z_{\bullet q}^S + z_{\bullet q}^{S^E}}; \forall q \text{ and } RSC_i^S = 1 - \frac{e_i^{S^E}}{z_{i \bullet}^S + e_{i \bullet}^{S^E}}; \forall i$$
 (17)

The intra-regional transaction total may then be estimated as the unweighted average of the estimates by means of the RPCs and the RSCs of region *S*:

¹⁰ This estimation problem is precisely the reason why a national IOT cannot be used to construct a non-survey regional IOT with any measure of accuracy, as the national IOT does not contain information on regional imports from the RoS nor on the regional exports to RoS. This is the reason why we have not added the national IOT as one of the priors in our non-survey method. For a survey-based construction of a regional IOT, however, the national IOT contains invaluable, because very detailed, information on sectoral technology and foreign imports, which should not be disregarded.

¹¹ One might argue that taking a weighted average is better (see Jensen and McGaurr 1977; Gerking 1979). However, this argument only applies to cases where conflicting survey data have to be reconciled, on the basis on which one must have developed an idea on the relative reliability of the row data versus the column data. It does not apply to the present *non-survey* estimation case, where such prior information is lacking.

$$z_{\bullet\bullet}^{R(S)} = \frac{\sum_{q} RPC_{q}^{S} \left(z_{\bullet q}^{R} + z_{\bullet q}^{R^{E}}\right) + \sum_{i} RSC_{i}^{S} \left(z_{i\bullet}^{R} + e_{i}^{R^{E}}\right)}{2}$$

$$(18)$$

The exports' column total and the imports' row total with regard to RoS, follow as the residual:

$$q = n + f + 1 \Rightarrow v_q^R = e_{\bullet}^{R^E} = x_{\bullet}^R - e_{\bullet}^{R^M} - z_{\bullet \bullet}^{R(S)}$$
(19)

$$i = m + 1 \Rightarrow u_i^R = z_{\bullet \bullet}^{RE} = x_{\bullet}^R - va_{\bullet \bullet}^R - z_{\bullet \bullet}^{RM} - z_{\bullet \bullet}^{R(S)}$$
 (20)

The calculations (15)–(20) are made 110 times. For each of the 11 RAS projections we use the survey IOTs of the 10 remaining regions S. Subsets of these ten RAS estimates for each R are then used to calculate the average deviation and the standard deviation of (2), which are used in the second stage of CRAS to produce the cell-corrected estimate of CRAS according to (7).

The next problem is which subsets of S to use. In temporal projections this choice is simple: the most recent table is the best choice. Temporal RAS and CRAS projections are then simply compared by adding more and more, less recent IOTs to the CRAS method (see Mínguez et al. 2009). In spatial projections this is far more complicated, though theoretically it is still simple. The best choice for RAS is to take the survey IOT of the region S that resembles the projection region S best, and the best choice for CRAS is to add the second best, the third best, etc., regions S. Empirically, however, it is not known beforehand which region is best, second best, etc.

To test RAS against CRAS, our choice is to compare the best choice of regions in both cases. Hence, we have to determine the rank order of the ten non-survey RAS projections of each of the 11 survey IOTs. To determine this rank order and to evaluate the performance of CRAS we will only compare the intra-regional transactions parts of the IOTs, thus excluding estimates of the trade with the RoS. We could also compare the RoS results separately, but comparing the intra-regional part is far more important as its estimation errors determine the estimation errors of the regional or national Type I or Type II multipliers for which regional or national IOTs are used most.

3.2 Accuracy of different RAS estimates for the Spanish regional IO tables

The comparisons are made by inspecting the distance between a projection z and the true value z^{true} , using different matrix distance measures (de Mesnard and Miller 2006). We only use additive measures, as their multiplicative equivalents have the same basic properties (de Mesnard 2004). Moreover, we only use cell-weighted measures, as their unweighted equivalents give undue emphasis on the errors in small cells that, percentage-wise, are usually much larger than the errors in the large cells. Thus, we only use the following matrix distance measures:

• Weighted absolute percentage error (WAPE: Butterfield and Mules 1980):

$$WAPE = \frac{\sum_{i} \sum_{j} |z_{ij} - z_{ij}^{true}|}{\sum_{k} \sum_{l} z_{kl}^{true}}$$
(21)

¹² We use the symmetric IO tables of 11 regions in current prices with 30 sectors. Note that all RAS solutions are obtained within an absolute error tolerance of $\frac{1}{2} \left(\sum_{i} \left| z_{i\bullet}^R - u_i \right| + \sum_{j} \left| z_{\bullet j}^R - v_j \right| \right) < 0.0001$.

		Target input-output table									
	ada	ara	ast	bal	clm	csl	cva	gal	mad	nav	pva
WAPE in %	22.3	30.0	30.8	48.1	41.2	26.0	44.7	28.7	50.3	32.2	36.5
Base IOT	gal	csl	gal	nav	gal	ara	gal	ada	clm	csl	nav
WNSE/1000	99	16	26	212	89	32	164	54	723	19	111
Base IOT	gal	csl	gal	nav	gal	ara	gal	ada	clm	ara	nav
MIG*1000	260	305	312	553	381	324	500	312	496	347	386
Base IOT	gal	gal	gal	nav	gal	nav	gal	ada	pva	csl	ada

Table 2. Matrix distance measures of best RAS projection per target table*

Note: * See Figure 1 for the abbreviations and locations of the regions.

• Weighted normalized squared error (WNSE; Deming and Stephan 1940):

$$WNSE = \frac{\sum_{i} \sum_{j} (z_{ij} - z_{ij}^{true})^{2}}{\sum_{k} \sum_{l} z_{kl}^{true}}$$
(22)

• Minimum information gain (MIG; Tilanus and Theil 1965):

$$MIG = \frac{\sum_{i} \sum_{j} \left| z_{ij}^{true} \ln \left(\frac{z_{ij}}{z_{ij}^{true}} \right) \right|}{\sum_{k} \sum_{j} z_{kl}^{true}}$$
(23)

Table 2 shows the result of (21)–(23) for the single best RAS projection per target IOT. As might be expected from the mathematical properties of above formulas, *MIG* shows the least variation in performance of RAS per target IOT as logarithms are taken of percentage-wise large errors, whereas WSNE shows the largest variation as squares are taken of the absolutely large errors. The WAPE nicely summarizes that the best RAS projection needs to be improved upon, as the weighted average cell-error runs from 25 percent to 50 percent, per target IOT, which should be considered high.

Next, especially when one looks at WSNE, it appears that projecting the IOT of the Madrid region is most difficult. This is not too surprising, as the small capital region, with its very strong service sector, specialized in central government and has an economic structure unlike any other Spanish region. Looking at WAPE and MIG, the Baleares Islands and the Comunidad Valenciana also appear to be difficult to project. The case of the Baleares is clear, as it is an island economy with a much stronger tourist sector than any of the other 10 regions with an IOT. The Comunidad Valenciana also has a strong tourist sector. Moreover, its IOT relates to year 2000, whereas most other IOTs relate to 2005 (see Figure 1), which makes its IOT more difficult to predict. Galicia, on the other hand, has by far the most popular IOT to use for the projection of the IOT of any other region, which indicates that it has the least peculiar characteristics. Finally, with five regions, the same base IOT is used twice in order to get the best RAS prediction, while with the other six regions the best RAS prediction is obtained with the same base IOT for all three distance measures.

The rank order of the performance of all possible 10 base IOTs, of which Table 2 only shows the best projection, does differ, but not much. Therefore, from here on we only present the

results for the average of the three measures.¹³ In fact, we might as well have presented the results for the WAPE measure, as it occupies a middle position between taking the logarithms and taking the squares of the errors, while its weighted percentage error nature makes it more easy to interpret.

To get an unweighted average, all three measures need to be normalized. To get results that are also easy to interpret, we normalize each with the minimal value of its single best RAS projection for target region R, namely, we normalize with the values shown in Table 2. This implies that the normalization is dependent upon target region R and the best base region S. Consequently, for that R(S)-combination, all RAS values of other base IOTs will be larger than one, whereas CRAS values may either be smaller or larger than one, depending on whether that CRAS projection performs better or worse than the best RAS projection at hand. The theoretical minimum is zero, indicating a CRAS projection that produces the true target IOT, while the corresponding best RAS projection does not.

Thus, we only show results for the average normalized distance measure (ANM):

$$ANM^{(C)RAS,R(S)} = \frac{1}{3} \left[\frac{WAPE^{(C)RAS,R(S)}}{\min\limits_{S \neq R} WAPE^{RAS,R(S)}} + \frac{WNSE^{(C)RAS,R(S)}}{\min\limits_{S \neq R} WNSE^{RAS,R(S)}} + \frac{MIG^{(C)RAS,R(S)}}{\min\limits_{S \neq R} MIG^{RAS,R(S)}} \right]; \tag{24}$$

The rank-order of the results of (24) for the 10 RAS-projections of each of the eleven target IOTs are shown in Table 3. Figure 1 shows the location and the size of the regions at hand. When selected data from the regional IOTs and information of Table 3 and Figure 1 is combined, four types of regions may be distinguished.

First, the regions of Andalucía, Comunidad Valenciana, and País Vasco: as can be seen in Table 3, these regions appear at most four times as one of the best four regions to predict the regional IOT of another region, so these regions do not represent a good choice as base matrix. This is most likely due to the fact that these regions have a large economic size, in the case of Andalucía a very strong specialization in agriculture and in the case of País Vasco a very strong specialization in the industrial sector.

Second, the regions of Aragón, Castilla-La Mancha and Castilla y León: also represent poor choices for a base IOT, as they also appear at most four times in the group of the best four base

		Target input-output table										
		ada	ara	ast	bal	clm	csl	cva	gal	mad	nav	pva
gu	1 st	gal	csl	gal	nav	gal	ara	gal	ada	clm	csl	gal
ranking	2^{nd}	ast	nav	nav	mad	csl	nav	ast	ast	ast	ara	nav
	3^{rd}	pva	gal	csl	gal	ast	ast	clm	pva	pva	ast	ada
acy	4^{th}	nav	ast	ara	ast	nav	gal	pva	nav	bal	gal	ast
accuracy	5 th	csl	pva	pva	clm	ada	clm	nav	clm	ara	pva	ara
	6^{th}	ara	clm	ada	ada	pva	ada	ara	ara	nav	clm	csl
matrix	7^{th}	clm	cva	cva	ara	bal	pva	ada	csl	gal	cva	clm
mat	8 th	cva	ada	clm	csl	ara	cva	csl	cva	csl	ada	cva
Se 1	9 th	bal	bal	mad	cva	cva	bal	bal	bal	ada	mad	mad
Base	10^{th}	mad	mad	bal	pva	mad	mad	mad	mad	cva	bal	bal

Table 3. Rank order of RAS projections, average matrix distance measure*

Note: * See Figure 1 for the abbreviations and locations of the regions.

¹³ These separate results are available upon request from the second author.

IOTs. In this case this outcome may partly be due to geographic characteristics, as these are all large inland regions; besides, Castilla-La Mancha and Castilla y León have a strong specialization in agriculture.

Third, the regions of Asturias, Galicia and Navarra: clearly represent the best choice as base region, as they appear, respectively, 10, nine and eight times in the group of the best four base IOTs. These are medium-sized or small regions in terms of geographic and economic size, and are coastal regions or located very close to the sea. Moreover, all these regions have a balanced economic structure, with only a weak specialization in manufacturing.

Fourth, the regions of the Baleares and Madrid: offer the worst or the second-worst base IOT for most of the other nine regions. This is most certainly due to their very specific geographic and economic characteristics, which were described above. They only serve as a good base region once, namely while projecting each other's IOT.¹⁴

This broad brush picture gives an indication of the type of characteristics that are likely to define which regions or countries may be expected to have a comparable IO structure. Geographic size and location are important, as is economic size, but sectoral structure seems to be the single most important factor that determines which IOTs represent the best base IOTs for the spatial projection of the unknown IOT of target region S. The actually best base IOTs will of course be found by comparing the structure of the row and column totals (14) of each potential base region S with those of the target region S, but precisely those data will not be available, as the target region does not have an IOT yet. Hence, base IOTs should first be selected on having a comparable sectoral structure, and second on having a comparable economic and geographic size.

Table 4 contains the values on which the rank order of the regions in Table 3 is based. Its upper part emphasizes the importance of choosing the right base matrix. In the case of Aragón and Castilla y León, for example, there are only two regions that are more or less suitable for a conventional RAS projection, and in the case of Andalucía, there is only one. In the case of these regions, picking the fourth or fifth best base IOT already leads to estimation errors that are 50 percent to 100 percent larger than that of the best base matrix, which already was not too good to start with (see Table 2). The lower part of Table 4, in addition, shows that picking a really

		Target input-output table										
		ada	ara	ast	bal	clm	csl	cva	gal	mad	nav	pva
50	1 st	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.03	1.01	1.04
kir	2^{nd}	1.41	1.12	1.29	1.18	1.20	1.14	1.10	1.16	1.16	1.09	1.09
cy ranking	3^{rd}	1.47	1.34	1.31	1.34	1.34	1.37	1.14	1.29	1.22	1.32	1.11
	4^{th}	1.74	1.49	1.34	1.37	1.34	1.53	1.20	1.36	1.23	1.40	1.24
accuracy	5 th	1.82	1.96	1.41	1.47	1.36	1.88	1.25	1.41	1.31	1.46	1.27
	6^{th}	1.86	2.06	1.70	1.51	1.38	2.05	1.26	1.46	1.37	1.74	1.35
matrix	7^{th}	1.95	2.38	1.83	1.54	1.39	2.16	1.26	1.54	1.37	2.43	1.36
nat	8^{th}	2.70	2.41	1.85	1.58	1.41	2.71	1.32	1.81	1.41	2.44	1.58
se 1	9 th	2.84	3.38	2.30	1.59	1.51	3.78	1.41	2.05	1.50	2.65	1.99
Base	10^{th}	3.93	3.68	2.68	1.87	2.23	5.05	1.64	3.12	1.56	2.73	2.04

Table 4. Numerical accuracy of RAS projections, average matrix distance measure*

Note: * See Figure 1 for the abbreviations and locations of the regions.

¹⁴ Note that this implies that being most similar is not symmetric. No region, for instance, resembles the Baleares well. Still, Navarra resembles it best, and predicts its IOT with a WAPE of 48% (Table 2). Conversely, any other region resembles Navarra more than the Baleares (Table 3), which predicts Navarra's IOT with an error of 88% (32% of Table 2 times 2.73 of Table 4).

wrong base IOT for a conventional RAS projection easily leads to errors that are 50 percent to more than 300 percent or even 500 percent larger than those of the best choice.

To conclude, this section indicates that the performance of using RAS as a non-survey technique in the old fashioned way is weak, even when the best choice of base IOT is made. Moreover, not choosing the best base matrix may lead to errors that could be up to three times larger. Finally, it shows that the non-similar unique, so-called 'non-fundamental part' of the economy is unfortunately quite large, at least in the case of the Spanish regions studied.¹⁵ Next, we consider if using CRAS can improve the quality of RAS as a spatial projection method.

4 Comparing RAS and CRAS, with more and more regions

By choosing the best region in each single projection we give RAS a head start over CRAS, but to not disadvantage CRAS unduly, we successively add the second best, the third best etc., when comparing CRAS with RAS. To make this comparison we estimate the deviations between the projected and the true IOTs for each of the regional IOTs, as indicated in (1). Then we calculate the average and standard deviation corresponding to those statistical deviations, as in (2), and apply that data in (3).

As we observe in Table 5 and Figure 2, for all regions there are four to nine CRAS combinations that produce errors that are up to 80 percent smaller than the best RAS projection.

The CRAS method which uses the two, three, four or five most similar IOTs (i.e., CRAS 2, CRAS 3, CRAS 4 and CRAS 5), always produces a better projection than the best RAS projection. On the other hand, when still more, increasingly dissimilar IOTs are added, the performance of CRAS deteriorates quite systematically. This is especially the case when the last one or two IOTs are added, which quite often are those of Madrid and the Baleares.

Still, with each number of base IOTs there are cases to be found in which CRAS outperforms RAS. The problem, of course, is how the analyst, not knowing which CRAS combinations perform better, decides on the number and the specific base IOTs to include in a CRAS projection.

	Target input-output table										
	ada	ara	ast	bal	clm	csl	cva	gal	mad	nav	pva
Best RAS*	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.03	1.01	1.04
CRAS 2	0.53	0.40	0.38	0.52	0.37	0.35	0.22	0.45	0.38	0.31	0.39
CRAS 3	0.58	0.51	0.44	0.47	0.65	0.79	0.49	0.54	0.40	0.62	0.50
CRAS 4	0.75	0.77	0.50	0.46	0.60	0.85	0.55	0.73	0.79	0.74	0.63
CRAS 5	0.86	0.78	0.60	0.57	0.74	0.94	0.71	0.99	0.81	0.97	0.84
CRAS 6	1.04	1.02	0.77	0.48	0.76	1.69	0.98	1.88	0.85	0.99	0.92
CRAS 7	1.18	1.00	0.91	0.71	0.93	1.77	1.19	1.92	0.80	2.45	1.26
CRAS 8	1.15	1.17	1.40	0.63	0.88	2.25	1.10	2.48	0.76	3.35	1.43
CRAS 9	1.20	1.35	1.67	0.70	0.91	2.25	1.21	2.39	0.85	5.30	1.96
CRAS 10	1.63	2.25	1.71	0.78	1.64	3.80	1.34	3.02	0.90	5.80	1.99

Table 5. Normalized performance of best RAS, and best CRAS by number of regions

Note: * The 'best' RAS may not be equal to 1, as it is defined as the average of three matrix distance measures, which are not always normalized with regard to the same base region (see Table 2). Best overall approach. CRAS outperforms RAS.

¹⁵ See Jensen et al. (1987) on the concept of the fundamental economic structure. See Thakur (2008) for a recent temporal application.

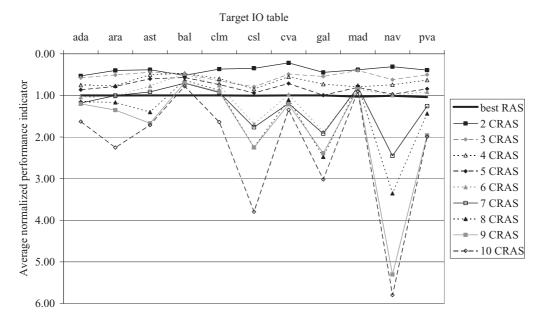


Fig. 2. Projection error of RAS versus CRAS with more and more, less alike regions *Note*: A zero indicates that CRAS produces a perfect projection, whereas RAS does not.

Table 6. Outperformance of the CRAS approach compared to the best RAS approach*

Region	ada	ara	ast	bal	clm	csl	cva	gal	mad	nav	pva
CRAS 2	47%	60%	62%	48%	63%	65%	78%	55%	63%	69%	62%
CRAS 3	42%	49%	56%	53%	35%	21%	51%	46%	61%	39%	52%
CRAS 4	25%	23%	50%	53%	40%	16%	45%	27%	23%	27%	39%
CRAS 5	14%	22%	40%	43%	26%	6%	29%	1%	22%	4%	19%

Note: * $(1 - ANM^{CRAS} / ANM^{RAS}) \times 100\%$.

The target IOTs of the Baleares and Madrid are of special interest, as in these two cases any CRAS approach, regardless of the number of IOTs used, is always better than the best RAS. There is little that these two regions have in common in terms of IO structure and spatial characteristics, but Figure 2 shows that the spread in their CRAS performance measures is relatively small, while Table 2 shows that the best RAS projection tends to be worse than that of the other regions. So, when one has a bad conventional RAS projection it is easier to improve it by adopting CRAS, but again whether one has a bad conventional RAS projection will not be known beforehand.

Finally, Table 6 summarizes the improvement of the CRAS projection with the best two, three, four and five base IOTs compared to the best RAS projection. We see that there are very relevant improvements to be made in all cases, with a minimum of 47 percent for Andalucía and a maximum of 78 percent in the case of Comunidad Valenciana. In all cases, except for the Baleares, the largest improvement is reached when the two most similar IOTs are combined in CRAS 2. The main practical problem is of course how to determine these two best regions.

Hence, in conclusion, in our opinion, choosing CRAS with the three to four most similar IOTs probably is the best strategy, as trying to choose the single best IOT for a conventional RAS projection almost certainly leads to a worse result as one can easily pick the second or third

best instead of the best. Choosing CRAS with the three to four best IOTs, including maybe one wrong choice, most certainly produces a better result.

5 Conclusion

The availability of many different regional or national input-output tables provides the researcher with extra information that should be used to improve the accuracy of any spatial IO projection method. We show that the CRAS method precisely does that by adding cell-specific corrections to RAS, which only uses one single known matrix. The cell corrections of CRAS are determined by minimizing the sum of the squared mean deviations of RAS projections between the multiple known tables, weighted by the inverse of their standard deviation.

In the test of the performance of CRAS relative to RAS with eleven Spanish regional survey IOTs for 1999–2005, it is shown that it is crucial to choose the right IO table as a base matrix to get a good performance of the RAS method as a spatial IO projection technique. When the wrong IOT is chosen, estimation errors may easily be up to three times larger compared to using the right single IOT, which already suffers from estimation errors between 25 percent and 50 percent.

This sensitivity for the right choice of base table is considerably reduced when CRAS is used with three to four IOTs of regions with an economic structure that is considered to be most similar to the IOT that has to be projected. In that case, CRAS may give a reduction in the error of 50 percent to 80 percent compared to the best RAS projection. Based on our Spanish test, however, the analyst is advised against adding more IOTs to a CRAS projection, as those IOTs may become too different from the IO structure of the target region or country.

Only when there are no base IOTs that seem similar to the target IOT, as in our test with Madrid and the Baleares, might it be advisable to use as many distantly similar IOTs as possible.

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Resumen. El método RAS extrapola una única matriz para que se ajuste a nuevos totales por fila y por columna. Este artículo propone una corrección de celdas de RAS (CRAS) que utiliza las desviaciones de proyecciones RAS múltiples, para mejorar la proyección de tablas inputoutput (TIO) de una región o país específicos. El nuevo método se prueba en once TIO basadas en muestreos de regiones españolas para el periodo 1999-2005. Se muestra como el método CRAS tiene un mejor desempeño que RAS cuando se utilizan TIO de tres a cuatro muestreos cercanos a la TIO objetivo. Para la mayoría de regiones, aunque no para todas, la aplicación de CRAS se comporta gradualmente peor cuando se añaden más TIO en comparación con la aplicación de RAS a la mejor TIO.

要約 RAS 法はある行列を新しい行和と列和に整合させる推計方法である。本稿は、特定の国または地域の産業関連表 (IOT: input-output table)の予測を改善するために、RAS 法による複数の推計の離誤差を使用したセル・コレクション RAS 法 (CRAS 法)を提案する。この CRAS 法を、1995-2005 年におけるスペインの 11 の地域サーベイデータに基づく産業関連表で検証する。この産業関連表に類似した3から4つの産業関連表を使用したところ、CRAS 法はRAS 法よりも予測精度が高く、産業関連表の数が増えるにつれ、すべてではないがほとんどの地域で、RAS 法を最も適切な産業関連表に適用した場合よりも、CRAS 法の予測精度は落ちることが示された。