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The spatial structure debate in spatial interaction modeling: 50 years on

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Abstract
Spatial interaction and spatial structure are foundational geographical abstractions, though there is often variation in how they are conceptualized and deployed in quantitative models. In particular, the last five decades have produced an exceptional diversity regarding the role of spatial structure within spatial interaction models. This is explored by outlining the initiation and development of the notion of spatial structure within spatial interaction modeling and critically reviewing four methodological approaches that emerged from ongoing debate about the topic. The outcome is a comprehensive coverage of the past and a sketch of one potential path forward for advancing this long-standing inquiry.

Keywords
distance-decay, misspecification, spatial analysis, spatial interaction, spatial structure

I Introduction

Spatial interaction (SI) typically refers to the aggregate flows of people, information, or goods across space as they move between a set of locations (Batty, 2008; Fotheringham, 2017; O'Kelly, 2015; Wang, 2017). As such, quantitative models of SI provide a mechanism to understand and predict components of SI systems and are typically constructed upon the hypothesis that flow volumes are a function of the potential at origins, the attractiveness of destinations, and the cost of overcoming the separation between origins and destinations. The simplicity and generalizability of this framework has motivated the use of SI models in diverse settings, and consequently, such tools have consistently remained of interest to geographers and allied disciplines (Batty, 2013; Fotheringham and O'Kelly, 1989; Haynes and Fotheringham, 1984; Patuelli and Arbia, 2016; Roy, 2004; Sen and Smith, 1995; Ullman, 1980). Another important geographical concept is that of spatial structure, which characterizes the organization, distribution, or relative arrangement of entities embedded within a spatial system (Bennett and Haining, 1985; Cliff et al., 1975a; Haggett, 1965; Lloyd, 2014). Although spatial structure and SI are foundational abstractions, the manner in which they are theoretically conceived and practically deployed in quantitative models is highly varied. In particular, the assertion is put forth and examined here that there is exceptional diversity regarding the role of spatial structure in SI modeling. This is achieved by outlining the initiation and development of the notion of

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spatial structure within SI modeling and characterizing a portion of the current landscape on the topic that pertains to cross-sectional analyses.

The investigation of spatial structure in SI models appears to have been launched by Curry's 'A Spatial Analysis of Gravity Flows' (1972). In response, an active debate ensued over the following decade about the role of the spatial structure of locations in SI systems and models. However, the inquiry eventually fractured and the SI modeling framework evolved into several different methodological directions, each with unique analytical traditions and underlying assumptions. At the same time, advancement in the ability to collect, store, and disseminate data has produced many novel flow data sets, often with increasingly finer spatial and temporal resolutions (Arribas-Bel, 2014; Lovelace et al., 2015; Manley and Dennett, 2019). Though these new data provide unparalleled opportunities to increase our understanding of SI processes, the realization of these opportunities is at least partially governed by the methodological framework chosen to perform an analysis. More specifically, the stance adopted here is that how spatial structure is conceptualized and operationalized in SI models has implications for how we build knowledge about SI phenomena.

To develop this view, traditional SI models are first introduced that do not explicitly consider spatial structure in their formulation. Then, a summary of the main concepts and ideas are provided by tracing the investigation of spatial structure in SI modeling in approximate chronological order. Concurrently, several proposed approaches for incorporating spatial structure in SI models are critically reviewed, their contribution to the SI knowledge-building-process are assessed, and their limitations are highlighted. Finally, additional themes and comparisons of approaches are discussed and suggestions are made for moving the SI research agenda forward before ending with some concluding remarks.

II Gravity-Type SI Models

Early SI models were motivated through analogies to the physical law of gravity where the number of flows between two locations is given by the product of the populations of the origin and destination divided by the distance between them (Carrothers, 1956; Farmer and Oshan, 2017; Hua and Porell, 1979). The formulation was subsequently generalized to,

$$T_{ij} = k \frac{V_i^{\mu} W_j^{\alpha}}{d_{ij}^{\beta}} \tag{1}$$

where T represents an $n \times m$ matrix of flows between n origins (subscripted by i) and m destinations (subscripted by j), V and W are $n \times p$ and $m \times p$ vectors of p origin and destination attributes, respectively, d is an $n \times m$ matrix of the costs to overcome the physical separation between i and j (usually distance or time), k is a scaling factor that ensures the model reproduces the total number of flows present in the data, and μ, α, and β each represent a vector of exponential parameters. When data for T, V, W, and d are available, it is possible to calibrate or estimate the parameters, which summarize the effect that each model component contributes toward explaining the system of known flows (T). Various flow forecasts are also possible by tweaking different model components or parameters (Fotheringham and O'Kelly, 1989).

Derived through entropy maximization, Wilson's 'family' of gravity-type SI models provided further generalization and formalization (Wilson, 1967, 1971). Depending upon the information incorporated about the total inflows and outflows (i.e. constraints), the following members of the family are,

Unconstrained

$$T_{ij} = kV_i^{\mu} W_j^{\alpha} f(d_{ij}) \tag{2}$$

Production-constrained

$$T_{ij} = A_i O_i W_j^{\alpha} f(d_{ij}) \tag{3a}$$

$$A_i = (\sum_j W_j^{\alpha} f(d_{ij}))^{-1} \tag{3b}$$

Attraction-constrained

$$T_{ij} = B_j V_i^{\mu} D_j f(d_{ij}) \tag{4a}$$

$$B_j = (\sum_i V_i^{\mu} f(d_{ij}))^{-1} \tag{4b}$$

Doubly constrained

$$T_{ij} = A_i B_j O_i D_j f(d_{ij}) \tag{5a}$$

$$A_i = (\sum_j B_j D_j f(d_{ij}))^{-1} \tag{5b}$$

$$B_j = (\sum_i A_i O_i f(d_{ij}))^{-1} \tag{5c}$$

where O_i and D_j are the total number of flows emanating from an origin or terminating at a destination, A_i and B_j are balancing factors that replace k and ensure that O_i and D_j are preserved in addition to the total system flows, and d_{ij} takes on a functional form, referred to as the distance-decay function. The distance-decay function is most commonly either a power function or an exponential function, and β is expected to take on a negative value that indicates the decaying nature on the effects of increasing physical separation on the propensity for flows to occur. The exponential distance-decay function is the natural result of the max-entropy derivation, while the power distance-decay function is justified when the analyst believes there is a logarithmic evaluation of transport costs and is also the specification that arises from the original physical analogy. Furthermore, these two forms have also gained acceptance through empirical consensus where the exponential function is often more appropriate for shorter interactions such as intra-urban trips and the power function is more appropriate for longer distance trips such as migrations flows (Fotheringham and O'Kelly, 1989).

The unconstrained model (or total-trip constrained model) given in equation (2) is equivalent to equation (1) and does not conserve the total inflows or outflows. The production-constrained and attraction-constrained models are given in equations (3a) and (4a). They conserve either the number of total outflows or inflows at each location and are therefore useful for building models that allocate individuals either from a set of origins or to a set of destinations. Finally, the doubly constrained model is given in equation (5a), which conserves both the inflows and the outflows at each location during model calibration. These models may also be derived using the economic behavioral framework of utility theory (McFadden, 1974, 1977) and have been linked to discrete choice models (Anas, 1983; Fotheringham, 1986). An exposition of the family of SI models, their applications, and various extensions is provided by Wilson (2010a).

Though nonlinear optimization was initially the preferred calibration technique for the family of models, they are frequently linearized and included within an ordinary least squares regression specification by taking the logarithm of both sides of a given model. For the unconstrained model,[1] this yields the so-called log-linear or log-normal gravity model,

$$\ln T_{ij} = k + \mu \ln V_i + \alpha \ln W_j - \beta \ln d_{ij} + \epsilon \tag{6}$$

where β is still expected to take on a negative value and ϵ is a random normal error term. A Poisson log-linear regression calibration framework is also popular because it more realistically models discrete entities (i.e. flows of people), avoids the issue of zero flows that cannot be logged, and is not subject to transformation bias for the dependent variable (Flowerdew and Aitkin, 1982; Flowerdew and Lovett, 1988). Using binary indicator variables for each origin and/or destination within Poisson regression yields the constrained variants of the family of SI models, producing results equivalent to those obtained using nonlinear optimization (Tiefelsdorf and Boots, 1995).

A popular extension to the family of models involves calibrating separate models for subsets of flows from each origin to all destinations, which is also known as an origin-specific local model. A set of parameter estimates for each model term is then obtained for each origin and is mappable in order to explore potential spatial variation (Oshan, 2016). The importance of origin-specific SI models in the discovery and understanding of spatial structure effects is noted below.

III The Spatial Structure Debate

Throughout the 1970s and 1980s, there was much debate about the role of the structure of a spatial system on the results from SI models. The intense interest in the matter can be best captured in a series of publications, associated comments and replies, and subsequent reviews (Cliff et al., 1974, 1975b, 1976; Curry, 1972; Curry et al., 1975; Fotheringham, 1981; Fotheringham and Webber, 1980; Griffith, 1976; Griffith and Jones, 1980; Johnston, 1973, 1975; Sayer, 1977; Sheppard, 1984; Sheppard et al., 1976). A primary concern surfaced that somehow an unidentified aspect of the spatial system was influencing distance-decay parameter estimates and producing *unintuitive* patterns in the local distance-decay parameter estimates from origin-specific SI models (i.e. relatively weak negative or positive distance-decay). Since these unintuitive patterns manifested in a spatial manner, the unidentified effect became known as the spatial structure effect.[2] Therefore, as evidence toward a spatial structure effect accrued, a consensus formed that whatever was causing the unlikely patterns in the estimated distance-decay parameters could be skewing their behavioral interpretation. Over the next few decades, several theories pertaining to the spatial structure effect were proposed, but the SI corpus remains far from a consensus in terms of what causes spatial structure effects and how to best account for them.

Though the debate initially included a variety of culprits[3] that could be contaminating distance-decay parameters, two primary theories emerged. The first was that of Curry, Griffith, and Sheppard (CGS), who argued that it was spatial autocorrelation among the locational attributes that causes the spatial structure effect (Curry, 1972; Curry et al., 1975; Sheppard et al., 1976). However, both Cliff et al. (1974, 1975b, 1976) and Fotheringham (1981) discuss how the concept of spatial autocorrelation among locational attributes is an intermediary for the collinearity between explanatory variables and is an unnecessary concept in this context because it does not provide any explanation for the unintuitive patterns in the distance-decay parameter estimates. Second, Johnston (1973, 1976) put forth the hypothesis that the spatial pattern observed in local distance-decay parameter estimates was caused by the variation in the distributions of distances between each origin and the set of destinations, which would later be described as the *conditional distance distribution* (Tiefelsdorf, 2003). Johnston tested the conditional distance distribution hypothesis using a controlled simulation, but the experiment was not entirely convincing because the data were generated assuming that all of the flows between each origin and destination are constant, implying no relationship between the number of flows and distance (Fotheringham, 1981; Sheppard, 1979). Therefore, neither the theory of CGS nor Johnston sufficiently exposed the spatial structure effect or how to address it.

A second wave of the debate witnessed several additional approaches to further explore the spatial structure effect. Griffith and Jones (1980) doubled-down on the idea of spatial autocorrelation in locational attributes by exploring the correlations between different components of doubly constrained SI models of migration across census tracts for several individual cities. The main goal was to answer the question, 'is the rate of distance decay in SI models independent of the spatial structure associated with the corresponding origins and destinations?' Through a series of loosely connected experiments, it was surmised that an 'indirect relationship exists between distance-decay and the geometric pattern, with this relationship being reasonably sensitive to changes in the geometry of destinations'. And even though little-to-no spatial autocorrelation was found in the locational attributes for each cities' system of locations, it was concluded that 'there exists a fundamental geometric dimension relating to the geographic distribution of workers/jobs', without explicitly defining the nature of bias in the distance-decay parameter estimates. It also appears that Griffith and Jones identified but were unable to untangle three distinct issues: (i) spatial autocorrelation of the locational attributes; (ii) spatial autocorrelation among the migration flows themselves; and (iii) the uncertainty that arises from modeling individual trips as aggregate flows between areal units.

In contrast to Griffith and Jones (1980), the approach of Fotheringham and Webber (1980) proposed to account for spatial structure by explicitly modeling the interdependence between SI and locational attributes. For example, in a study of migration between urban centers, to accurately obtain parameters for the effect of urban centers' size on SI, there also needs to be a sufficient model for growth of urban centers as well. Since growth is theorized to depend upon SI, these two things should be modeled with a simultaneous equation system that is calibrated using two-stage least squares. In general, parameter estimates will be biased and inconsistent if any of the independent variables are a function of the dependent variable, which is a type of endogeneity misspecification. Though Fotheringham and Webber (1980) derived the exact nature of this misspecification, it was only for the unconstrained SI model and contained the restrictive assumptions that the system is highly sensitive to the proposed feedback mechanism, the interdependence is instantaneous, and the system is in some steady-state equilibrium.

After reviewing much of the existing literature on origin-specific SI models and spatial structure, Fotheringham (1981) brought an additional perspective to the debate by suggesting that the presence of spatial structure effects should be diagnosed by directly analyzing the unintuitive patterns in the local distance-decay parameter estimates. By doing so, it was noticed that the patterns in the parameters seem to be driven by each location's accessibility to the other locations in the system. It was then deduced that spatial autocorrelation in the locational attributes specifically served as a surrogate for multicollinearity between accessibility and distance. A discourse between Sheppard (1982) and Fotheringham (1982) refined this argument by noting that spatial autocorrelation in the residuals can indicate that there is one of several types of misspecification that can bias parameter estimates (e.g. omitted variables or incorrect functional form); however, observing spatial autocorrelation in the independent variables is not sufficient to diagnose such misspecification(s). Furthermore, the concept of spatial autocorrelation was now unnecessary in this context: the spatial structure effect described by Fotheringham, (1981) had been identified as a misspecification arising due to the omission of a model term that captures location accessibility. It was demonstrated that the inclusion of a variable that measures each destination's accessibility to all other destinations can greatly reduce the spatial variation in local distance-decay parameter estimates and remove unintuitive patterns, and was therefore dubbed the Competing Destinations (CD) model (Fotheringham, 1983).

Over the next few decades, the CD framework was expanded, reinforced, applied, and critiqued. In parallel, several additional methods to account for spatial structure were also proposed. The next section elaborates upon each methodology.

IV Accounting for Spatial Structure

In the wake of the debate, four generalizable approaches were proposed to account for the spatial structure effect: the CD model, the Box–Cox transform, spatial econometric models, and eigenvector spatial filtering (ESF). As each approach is introduced and discussed, three characteristics are highlighted to inform a contemporary research agenda. First, the misspecification a method is meant to correct in order to remove the spatial structure effect is identified. Since multiple occurrences of misspecification are possible, it is important to clarify those being addressed by each method. Second, connections are drawn from each approach to the spatial structure debate summarized hitherto and henceforth to follow the progression of theorization and conceptualization within the SI modeling paradigm. Finally, some limitations of each method for investigating SI processes are evaluated.

1 The CD Model

The core tenets of the CD model are that: (i) the relative configuration of destinations with respect to each other can affect an individual's propensity to select a particular destination; and (ii) including a destination accessibility term in SI models can capture this factor and mitigate the observed spatial structure effect (Fotheringham, 1983). Accessibility is often defined based on a combination of physical locations and attribute values. While spatial systems typically do have attribute spatial variation that may contribute to the spatial structure effect, it is ultimately driven by relative spatial organization and how individuals perceive the overall system. For example, in the event that attributes are uniform or random across observations, the spatial structure effect would be due solely to the configuration of locations in space.

The behavioral foundation of the CD model is that spatial decisions, such as location choice, often arise from a hierarchical two-stage or multi-stage decision-making process as a spatial information processing strategy when there are many choices to evaluate (Curtis and Fotheringham, 1995; Fotheringham and Curtis, 1992, 1999; Hirtle and Jonides, 1985; McNamara, 1986; Walker et al., 1990). Individuals first select one or more clusters of locations and then subsequently choose an individual destination from within these clusters. The effect is that as the accessibility of a particular destination, j, to all other potential destinations increases, j will experience an increase or decrease in the number of flows than it would otherwise expect if it existed independently (i.e. not a member of any clusters) (Fotheringham and O'Kelly, 1989).

Destination accessibility can be defined as A_{ij} in equation (8) and added to the SI models introduced here, where A_{ij} is thought of as the likelihood that other destinations are also considered along with destination j. For an unconstrained log-linear SI model, for example, this results in:

$$\ln T_{ij} = k + \mu \ln V_i + \alpha \ln W_j - \beta \ln d_{ij} + \delta \ln A_{ij} + \epsilon \tag{7}$$

$$A_{ij} = \sum_{\substack{k=1 \\ (k \neq i, k \neq j)}}^{n} W_k d_{jk}^{\sigma} \tag{8}$$

where δ is the parameter corresponding to destination accessibility, A_{ij}, which is the sum of the attractiveness, W, at each alternative destination k weighted by its distance to each alternative destination d_{jk}, and σ is a parameter that controls the scale over which destinations compete with each other, which in practice is often set to -1 or calibrated iteratively with the parameters in equation (7). Adopting the destination accessibility term, A_{ij}, is also advantageous to avoid the undesirable independence from irrelevant alternatives[4] (IIA) property that exists in many spatial choice modeling frameworks (Fotheringham, 1986). Importantly, the set of locations defining CD is likely unique for different contexts, need not be the entire set of locations, can include locations that are not included in the original SI data set, or could rely on bespoke criteria (Pellegrini et al., 1997; Thill, 1992; Thill and Horowitz, 1997).

A failure to account for destination accessibility when spatial structure effects are present generally biases origin-specific distance-decay parameter estimates upwards for accessible origins and downward for inaccessible origins. The strength of this bias is shown by Fotheringham (1984) to depend on the strength of two relationships: that between the volume of flows and destination accessibility and that between destination accessibility and distance from the origin to each destination. It is this bias that causes the unintuitive spatial patterns observed in origin-specific distance-decay parameter estimates, such as positive estimates for the most accessible origins (Fotheringham, 1981). Furthermore, the destination accessibility bias can be categorized in terms of its relationship to the volume of flows where competition effects are associated with a negative relationship (i.e. negative δ) and agglomeration effects are associated with a positive relationship (i.e. positive δ). The type of effect that arises is typically dependent upon the type of SI process being modeled, though competition is observed more often in empirical settings. The proliferation of the competition effect may also be driven by the behavioral tendency of individuals to underestimate the overall size of large objects, such as the attractiveness of clusters (Fotheringham and Curtis, 1999; Stevens, 1975).

The determinants of potential distance-decay parameter estimate bias due to destination accessibility were further outlined such that bias may arise due to a combination of

1. a relationship between A_{ij} and T_{ij};
2. a direct relationship between A_{ij} and d_{ij};
3. an indirect relationship between A_{ij} and d_{ij} due to a relationship between A_{ij} and W_j and a relationship between d_{ij} and W_j.

Bias source (3) can occur independently of bias source (2), since it can occur even when there is no relationship between A_{ij} and d_{ij}. In addition, if bias sources (2) and (3) do not occur, but source (1) does occur, then distance-decay parameter estimates will not be biased; however, A_{ij} is still a relevant explanatory variable that can increase the accuracy of the model. These sources of bias can be similarly defined for the parameter estimates on other variables, though it is likely that the bias will be stronger for distance-decay whenever there is a stronger relationship between A_{ij} and d_{ij} than between A_{ij} and locational attributes (Fotheringham, 1984).

It becomes clear that the CD model is addressing a very particular misspecification that arises organically due to collinearity among variables that would ordinarily be included in most, if not all, SI models. Therefore, it is attempting to correct for the omission of a correlated spatially patterned variable. Baxter (1983, 1985) provides a more general framework for analyzing omitted variable bias in SI models, though it is less amenable to interpretation. For example, it does not provide strong empirical evidence for any particular form, and it is not clear how each form would arise, as has been done for the CD framework. As a result, the CD model has been applied in many domains including urban modeling (Fotheringham, 1985; Fotheringham and Knudsen, 1986), the study of telecommunications flows (Guldmann, 1999), access to health facilities (de Mello-Sampayo, 2016), crime location analysis (Bernasco, 2010), determinants of trade (de Mello-Sampayo, 2017a, 2017b), and recreation and tourism (Matthews et al., 2018), though it has enjoyed particular popularity within migration modeling (Fik et al., 1992; Fotheringham et al., 2000, 2004; Ishikawa, 1987, 1990; Kalogirou, 2015; Pellegrini and Fotheringham, 1999, 2002; Yano et al., 2003), commuting-to-work research (Gitlesen et al., 2010; Gitlesen and Thorsen, 2000; Thorsen and Gitlesen, 1998), and retail analysis (Birkin et al., 2010; Fotheringham, 1988; Fotheringham and Knudsen, 1986; Guy, 1987; Pellegrini et al., 1997). The underlying theory of the CD model has also been explored via simulation studies (Fotheringham et al., 2001; Lo, 1991a; Ubøe et al., 2008) and it has been connected with other key concepts, such as central place theory (Fik and Mulligan, 1990) and trip chaining (Bernardin et al., 2009).

A criticism of the CD model's explicit focus on spatial configuration is that it overlooks other important factors. For example, Gordon (1985) suggests that further work investigating the spatial patterns of distance-decay parameters should focus on functional and economic differences between locations rather than solely physical accessibility. Lo (1991a, 1991b, 1992) takes up this call and argues that spatial structure should be renamed to *destination interdependence*, where this interdependence is composed of physical aspects and economic aspects. The physical aspect, which Lo calls *locational substitutability*, is synonymous with the effects associated with the CD model, whereas *economic substitutability* is offered as an example of the economic aspects that might still be misspecified in CD models. Economic substitutability refers to consumer preferences toward destination activities and services. If the degree that destinations provide activities that are substitutable, or conversely, that are complimentary, varies and this is not accounted for, then SI models may produce biased parameter estimates. Similarly, Pooler (1998) proposes an alternative competition effect, termed *spatial influence* which also theorizes a hierarchical decision-making process, but where macro-level groups of destinations are based primarily on attributes (i.e. aspatial) rather than their locations in space. However, in the exposition of both the theories of economic substitutability and spatial influence, no generalizable framework or solution is proposed to account for the omitted variables that are causing model misspecification. One reason may be that such misspecifications are idiosyncratic, whereas potential exists for a (locational) spatial structure effect in any SI scenario. This also suggests that some criticisms of the CD model may be overstated, such as Hu and Pooler (2002) who assert that any spatial variation within local distance-decay parameter estimates means that the model is misspecified. In contrast, spatial variation could be due to variations in how distance is perceived or due to aggregation error, both of which can occur even when the model is correctly specified. Furthermore, extensions have been proposed to account for more complex location choice relationships within the CD model framework, such as disaggregated SI retail models that include store brand and household type (Newing et al., 2015).

2 The Box–Cox Transform

It would be approximately 30 years before Johnston's argument about functional forms in SI models was picked up by again by Tiefelsdorf (2003), who illustrates how a degree of spatial variation in local distance-decay parameter estimates can arise systematically due to conditional distance distributions *when the functional form of distance is misspecified*. Recall that power and exponential functional forms of distance-decay are employed the most often. Johnston's original argument was that spatial variation in distance-decay parameter estimates can arise if the underlying data-generating process follows the intervening opportunities model but a gravity-type SI model is calibrated on the data. This is essentially an extreme case of functional form misspecification, which Tiefelsdorf (2003) generalizes upon to show that any inconsistencies between the true data-generating functional form on distance and that which is specified in the model can result in distance-decay parameter estimates with spatial variation. Subsequently, it is recommended that the correct functional form of the distance term can be obtained by using the Box–Cox transformation, which encompasses a spectrum of functions depending on a tuning parameter that is selected by maximizing the model likelihood ratio. Tiefelsdorf (2003) claims that this allows the spatial structure effect that is caused strictly by distance to be accounted for, which cannot otherwise be accounted for by the CD model. However, it is not clear that functional form misspecification causes *unintuitive* interpretations of local distance-decay.

Several additional factors generate uncertainty about the effect of conditional distance distributions and the use of the Box–Cox transformation approach. First, although the distance term receives the most attention, any of the terms within a SI model could be subject to functional form misspecification. This suggests more work is needed to understand how the Box–Cox transformation approach performs if other terms are subject to functional form misspecification or distance is correlated with other attributes. Second, there are not many empirical examples of the Box–Cox transformation approach,[5] making it difficult to gauge the prevalence of conditional distance distribution misspecification and assess the magnitude of the associated spatial variation compared to other sources of misspecification. Researchers have often opted instead to choose theoretically motivated functional forms, other selection techniques, to shift focus to travel time or monetary costs, or to include technical, cultural, or political separations that may co-influence flows along with distance (Buczkowska et al., 2019; Chun, 2008; de Dios Ortúzar et al., 2011; Fischer et al., 2006; Martnez and Viegas, 2013; Mukherji and Silberman, 2019; Sohn et al., 2019; Thomas and Tutert, 2013; Vries et al., 2009). Despite these ambiguities, Tiefelsdorf (2003) is commonly cited to caution against spatial dependence in SI models and to motivate spatial econometric or Eigenvector spatial filter techniques (Griffith and Fischer, 2013; Fischer and Griffith, 2008; Fichet de Clairfontaine et al., 2015; Margaretic et al., 2017; LeSage and Pace, 2008).

3 Spatial Econometric Models

Traditional spatial regression models incorporate spatial dependence (i.e. spatial structure) in (i) the dependent variable, (ii) the error term, (iii) the independent variables, or (iv) some combination of (i–iii) when modeling relationships across two-dimensional geographic units (Halleck Vega and Elhorst, 2015). Spatial dependence is implied by the presence of spatial autocorrelation, which may violate common independence assumptions at the core of many modeling techniques. Curry (1972) first theorized the role of spatial autocorrelation in SI models, though it was among two-dimensional location attributes and not the flows themselves. Subsequently, Griffith and Jones (1980) posited that SI itself is spatially autocorrelated due to the fact that the SI system was spatially structured. Almost three decades later, the complexity of representing dependence between SI flows compared to two-dimensional spatial units was recognized and approached (Fischer and Griffith, 2008).

The two spatial regression specifications that are most frequently used and studied in the spatial econometric literature are the spatial autoregressive model (SAR) or spatial lag model and the spatial error (SE) model (Halleck Vega and Elhorst, 2015), which satisfy (i) and (ii) above, respectively. Several early attempts were made to extend the SE model to SI models (Brandsma and Ketellapper, 1979; Bolduc et al., 1989, 1992, 1995); however, these extensions seem to have suffered from issues of interpretability, estimation, or both. Other attempts at modeling spatial structure effects in SI models using a SE framework[6] have produced more encouraging results (Fischer and Griffith, 2008; Lee and Pace, 2005; Porojan, 2001), though a standard specification has not arisen nor has a consistent spatial effect been identified. This may be due to the idiosyncrasies of the processes being modeled or because in the SE framework, spatial dependence is treated as a nuisance and is not of substantive interest. Nevertheless, without a specific underlying theoretical misspecification, it is difficult to assess the quality of any particular method or to build intuition about the role of spatial structure.

Recent work has also proposed an extension to the SAR model to accommodate flow data (LeSage and Fischer, 2014; LeSage and Pace, 2008; LeSage and Thomas-Agnan, 2015). To include a spatially dependent process in the unconstrained SI model, LeSage and Pace (2008) suggest the following specification:

$$\begin{aligned} \ln T_{ij} &= \rho_i M_i y + \rho_j M_j y + \rho_{ij} M_{ij} y + \psi X + \epsilon \\ M_i &= I_n \otimes M \\ M_j &= M \otimes I_n \\ M_{ij} &= M_i \otimes W_j = M_i \otimes M_j = M \otimes M \end{aligned} \tag{9}$$

where ψX becomes $k + \mu \ln V_i + \alpha \ln W_j - \beta \ln d_{ij}$, M_i, M_j, and M_{ij} are contiguity-based spatial weight matrices that define neighborhoods from the perspective of origins, destinations, and origin–destination pairs, respectively, ρ_i, ρ_j, and ρ_{ij} are the corresponding autoregressive parameters, I_n is an $n \times n$ identity matrix with nonzero entries on the diagonal representing n locations, and $\otimes$ denotes the Kronecker product. In creating M_i and M_j, the role of the Kronecker product is to map the spatial relations among n locations encoded in M to spatial relations between n^2 flows, which can then be combined via the Kronecker product to create M_{ij}. Note that here it is assumed that there is an equal number of origins and destinations and that all origins are also destinations and, therefore, nonzero entries in M_{ij} are denoted by scenarios where both M_i and M_j are nonzero in the case of binary contiguity. This specification is motivated by the theory that the movements that people decide to make are based upon their knowledge of neighboring flows in a previous time period. Assuming that the exogenous variables are relatively stable over time and that the cross-section of flows may be taken as the steady-state equilibrium of a long-run process, then LeSage and Pace (2008) demonstrate mathematically that a SAR data-generating process may be an adequate representation of SI. It was then subsequently applied to study migration (LeSage and Pace, 2008; Murayama and Nagayasu, 2019; Sardadvar and Vakulenko, 2020), trade and commodity flows (Chun et al., 2012; LeSage and Polasek, 2008; Metulini et al., 2018b; Moura et al., 2019; Yin et al., 2020), tourism (Alvarez-Diaz et al., 2020; de la Mata and Llano, 2013; Marrocu and Paci, 2013), commuting, transit, and labor choices (Kerkman et al., 2017; LeSage and Fischer, 2014; LeSage and Thomas-Agnan, 2015), and airline passenger travel (Kim et al., 2019; Margaretic et al., 2017).

Importantly, when locations serve as both origins and destinations and are represented by the same variable (i.e. population), a change in a single location's attribute value can influence many flows in the system, which can further diffuse across the system if observations are spatially dependent. That is, it is not possible to interpret how a change in a single origin attribute (destination attribute) would affect flows originating (terminating) from that origin (destination) without also considering how that change would also effect flows that terminate at that origin (originate at that destination). Therefore, scalar summary measures that capture the multiple changes across the system, which are known as *effects estimates*, should be used rather than treating the regression parameter estimates as representing the relations between system components as is typically done (LeSage and Fischer, 2014; LeSage and Thomas-Agnan, 2015). The SAR SI model allows for these type of feedback effects or spillovers and can be thought of as an instance of the feedback misspecification originally proposed by Fotheringham and Webber (1980). Including the spatial lag(s) of the dependent variable is theorized to account for endogenous effects that may arise due to changes in shared resources such as transportation infrastructure that can cause reactions to defuse through an entire system (i.e. global spillovers) (LeSage and Fischer, 2014). However, this interpretation is dependent upon a set of locations serving as both origins and destinations with a common set of attributes, which might not be appropriate in some SI systems.

Since the focus of the work of LeSage and Pace (2008), LeSage and Fischer (2014), and LeSage and Thomas-Agnan (2015) is on the interpretation of SI parameter estimates, it is surprising that little is mentioned about the interpretation of the parameter estimate on the distance variable (i.e. distance-decay). LeSage and Thomas-Agnan (2015) acknowledge distance and spatial dependence may be competing to explain the same variation in the dependent variable and notice that the distance-decay parameter estimate is smaller in magnitude in a SAR SI model than in a corresponding basic SI model. Similar results can be noted in additional applications of the SAR SI model (de la Mata and Llano, 2013; Kerkman et al., 2017; LeSage and Pace, 2008), especially, with increasingly complex definitions of spatial structure (LeSage and Polasek, 2008). LeSage and Pace (2008) note that the distance-decay for models with spatial lags (i.e. SAR model) are not comparable to those without spatial lags, since those with lags require the effects estimate for interpretation. Curiously, the effects estimate for distance-decay is otherwise not typically reported or discussed in these studies. In a study of air passenger data from Margaretic et al. (2017) who use a model with lags being based on either the origin or on the destinations, a counter-claim is provided that variables characterizing an origin–destination dyad, like distance, do not require the effects estimate for interpretation. Hence, it is not immediately clear how to interpret distance-decay in these spatial econometric models of SI.

A general critique of spatial econometric specifications, such as the SAR model, is that they have identification issues, rely upon strong assumptions, and are prone to overfitting (Corrado and Fingleton, 2012; Gibbons and Overman, 2012; Halleck Vega and Elhorst, 2015; McMillen, 2003, 2012; Partridge et al., 2012; Pinkse and Slade, 2010). As a result, several additional perspectives have been put forth to complement the use of these models, such as the use of natural experiments (Gibbons and Overman, 2012), incorporating stronger underlying theory for a model (Corrado and Fingleton, 2012), semi-parametric and nonparametric smoothing methods (McMillen, 2012), and the use of simpler specifications when there is no strong theoretical basis for more complex ones (Gibbons and Overman, 2012; Halleck Vega and Elhorst, 2015).

4 Eigenvector Spatial Filtering

ESF is a technique that accounts for spatial autocorrelation based on the interpretation that the eigenvectors of a projected contiguity-based[7] connectivity matrix are the set of possible orthogonal and uncorrelated map patterns (Griffith, 1996, 2011) given a particular definition of connectivity. Further, the first eigenvector, E_1, is the set of real numbers that produces the map pattern with the largest achievable Moran's I correlation coefficient (MC), the second eigenvector, E_2, is the set of real numbers that produces the map pattern with the largest achievable MC while remaining uncorrelated with E_1, and continues on such that E_n, achieves the largest negative MC and is uncorrelated with the preceding $(n-1)$ eigenvectors. The projected connectivity matrix, C, that is most frequently[8] decomposed into such eigenvectors is defined as

$$(I - 11'/n)M_n(I - 11'/n) \qquad (10)$$

where I is an $n \times n$ identity matrix, 1 is an $n \times 1$ vector of 1's, $'$ denotes the matrix transpose operation, and M_n is the binary connectivity matrix for n mutually exclusive and exhaustive spatial units (i.e. locations) that partition the study space. By selecting a subset of the eigenvectors derived from C and creating a linear combination, it is possible to produce a synthetic variable that serves as a proxy for missing spatially autocorrelated exogenous variables (Griffith 2004), which can be included in a linear regression as follows (Chun and Griffith 2011):

$$Y = X\beta + E\gamma + \epsilon \tag{11}$$

where Y is a dependent variable representing areal units, X is a set of explanatory variables, E is a set of selected eigenvectors, β and γ are coefficient vectors, and ϵ is a vector of normally distributed random errors. This specification has been shown to produce results where the error term does not violate common assumptions of independence (Griffith 2000, 2002, 2004).

After the ESF framework was established, it was subsequently extended from spatial data aggregated to n areal units to SI flow data that occur between n^2 pairs of origins and destinations such that $X\beta$ becomes $\mu \ln V_i + \alpha \ln W_j - \beta \ln d_{ij}$ for various types of spatial processes, such as migration (Chun, 2008; Chun and Griffith, 2011; Gu et al., 2019; Liu and Shen, 2017), commuting (Griffith, 2007, 2009a, 2009b, 2011; Griffith and Chun, 2015), patent citations and research collaboration (Fischer and Griffith, 2008; Griffith et al., 2016; Griffith and Fischer, 2013; Scherngell and Lata, 2012), and trade (Metulini et al., 2018a; Patuelli et al., 2015). While this cluster of work signifies a standard protocol for applying and interpreting the ESF SI methodology, a closer look at the literature indicates some ambiguities and inconsistencies with the technique in the SI context.

First, the primary motivation for using an ESF SI framework is often unclear and is stated as either that flows are a priori dependent upon each other or that the ESF's can serve as a proxy for spatially patterned omitted variables or both. Much of the research using an ESF in a SI model tends to cite Curry (1972) or Griffith and Jones (1980), which have already been discussed as convoluting several modeling violations and spatial analysis concepts. Second, Black's (1992) notion of network autocorrelation is often cited as inspiration to capture flow autocorrelation in ESF SI models but spatial contiguity is then used to define dependence. Thus, the use of Black's network autocorrelation term in the ESF SI context is a misnomer because Black actually defines proximity in terms of network connectivity and not in terms of spatial proximity. Further, network autocorrelation was examined by Black to define additional substantive geographical variables such as regional indicators or accessibility terms, which were demonstrated to successfully reduce the measured network autocorrelation to insignificant levels. Thus, the underlying concepts and motivations of using an ESF in SI models are not always clear.

The ESF framework is also dependent upon a number of specification decisions that vary across existing research within SI modeling. For example, there is variation in how spatial relationships are defined (i.e. C and M matrices) and made operational (i.e. selecting a subset of eigenvectors E), the type of SI model the ESF is applied to (i.e. equations (2) to (5a)), and the underlying probability model. Each of these issues is an important part of the modeling framework that may impact the model results. Table 1 captures the diversity of the ESF SI methodology specifications and their associated results, which presents the details of 22 SI models (with and without an ESF) that were extracted from 13 research articles produced during the first decade after the technique was introduced. Any entries of *not reported* indicate

Table 1. Characteristics of eigenvector spatial filtering methodologies applied to spatial interaction data.

Source	Process	Scale (*n*)	Spatial interaction model	Probability model	*C*	*M*	*E*	Pre-filter	Criterion	β effect	Significant	SE
Griffith (2007)	Commuting	Reported-3 (439)	UNC	Poisson	Standard	Contiguity	Separate	$>$ 0.25 MC	**Minimize RSS or MC**	Larger	Not reported	Not reported
Fischer and Griffith (2008)	Patent citations	NUTS-2 (112)	UNC	Log-normal	Standard	Contiguity	Separate	$>$ 0.25 MC	Maximize likelihood	Larger	No	Larger
Fischer and Griffith (2008)	Patent citations	NUTS-2 (112)	UNC	Poisson	Standard	Contiguity	Separate	$>$ 0.25 MC	Maximize likelihood	Larger	Yes	Larger
Chun (2008)	Migration	States (49)	UNC	Poisson	Symmetry-corrected	S-coded contiguity	Product	Not reported	Minimize *T* statistic	Smaller	No	Smaller
Chun (2008)	Migration	States (49)	PC	Poisson	Symmetry-corrected	S-coded contiguity	Product	Not reported	Minimize *T* statistic	Smaller	No	Smaller
Griffith (2009a)	Commuting	NUTS-3 (439)	DC	Poisson	Standard	Contiguity	Product	$>$ 0.5 MC	**Smallest *p* value (**$<$ 0.1**)**	Smaller	Yes	Smaller
Griffith (2009b)	Commuting	Counties (254)	DC	Poisson	Standard	Contiguity	Product	$>$ 0.5 MC	Not reported	Smaller	Yes	Not reported
Chun and Griffith (2011)	Migration	States (49)	UNC	Log-normal	Standard	Contiguity	Sum	$>$ 0.25 MC	Minimize AIC	Larger	Yes	Larger
Chun and Griffith (2011)	Migration	States (49)	UNC	Poisson	Standard	Contiguity	Sum	$>$ 0.25 MC	Minimize Quasi-AIC	Larger	Yes	Typically larger
Griffith (2011)	Commuting	Counties (73)	DC	Poisson	Standard	Contiguity	Product	$>$ 0.5 MC	**Smallest *p* value**	Smaller	Yes	Larger
Scherngell and Lata (2012)	Collaborations	NUTS-2 (255)	UNC	Neg. binomial	Standard	5 NN	Separate	$>$ 0.25 MC	Not reported	Smaller	Yes	Larger
Griffith and Fischer (2013)	Patent citations	NUTS-2 (257)	DC	Poisson	Standard	Contiguity	Product	$>$ 0.5 MC	Minimize AIC	Larger	Not reported	Not reported
Patuelli et al. (2015)	Trade	Countries (64)	UNC	Neg. binomial	Symmetry-corrected	3 NN	Separate	$>$ 0.25 MC	**Statistical sig.**	Smaller	Not reported	Not reported
Griffith and Chun (2015)	Commuting	Counties (11)	DC	Poisson	Standard	Not reported	Product	Not reported	Not reported	Larger	Not reported	Not reported
Griffith and Chun (2015)	Commuting	Tracts (38)	DC	Poisson	Standard	Not reported	Product	Not reported	Not reported	Almost identical	Not reported	Not reported
Griffith and Chun (2015)	Commuting	Counties (73)	DC	Poisson	Standard	Not reported	Product	Not reported	Not reported	Smaller	Not r eported	Not reported
Griffith and Chun (2015)	Commuting	NUTS-1 (17)	DC	Poisson	Standard	Not reported	Product	Not reported	Not reported	Smaller	Not reported	Not reported
Griffith and Chun (2015)	Commuting	NUTS-2 (40)	DC	Poisson	Standard	Not reported	Product	Not reported	Not reported	Smaller	Not reported	Not reported

(continued)

Table 1. (continued)

Source	Process	Scale (n)	Spatial interaction model	Probability model	C	M	E	Pre-filter	Criterion	β effect	Significant	SE
Griffith and Chun (2015)	Commuting	NUTS-3 (439)	DC	Poisson	Standard	Not reported	Product	Not reported	Not reported	Smaller	Not reported	Not reported
Griffith et al. (2016)	Patent citations	NUTS-2 (257)	DC	Poisson	Symmetry-corrected	8 NN	**Sum**	$> \pm 0.25$ MC	**Smallest *p* value (< 0.1)**	Larger	Yes	Smaller
Margaretic et al. (2017)	Air travel	Cities (279)	UNC	Log-normal	Symmetry-corrected	C-coded 4/5 NN	Separate	> 0.25 MC	Minimize AIC	Larger	Yes	Smaller
Margaretic et al. (2017)	Air travel	Cities (279)	UNC	Poisson	Symmetry-corrected	C-coded 4/5 NN	Separate	> 0.25 MC	Minimize AIC	Larger	Yes	Smaller

UNC: unconstrained; PC: production-constrained; DC: doubly constrained; RSS: residual sum-of-squares; AIC: Akaike Information Criterion.
Note: Standard means the projection matrix given in equation (10) rather than one that forces symmetry. The weight matrix *M* is assumed binary unless another coding scheme is noted.

that the necessary information could not be found in a particular publication, while bolded entries indicate that a particular detail of the ESF model was unclear and could not be concluded with certainty.

Since Table 1 is organized roughly in chronological order, it is possible to detect some temporal patterns. Initially the ESF methodology was applied to unconstrained SI models, though eventually the focus shifted primarily to doubly constrained models. An associated trend is that originally there was a separate ESF for origin variables and destination variables; however, this was eventually abandoned in favor of a single ESF that is specified using a combination (i.e. sum or product) of origin proximity relationships and destination proximity relationships. This seems to mark a shift in the primary motivation for using an ESF from correcting for spatial autocorrelation in the explanatory variables to spatial autocorrelation in flows themselves, though this distinction is not typically made. Further examining Table 1 reveals that outside of these patterns a standard protocol is not apparent.

Perhaps the most varied aspect of the ESF SI framework is the selection criterion employed to select a specific subset of eigenvectors. Forward or backward stepwise selection is employed in the SI literature, and the selection criterion may involve directly optimizing the model fit, indirectly optimizing a model fit statistic, minimizing spatial autocorrelation, or finding all eigenvectors that are collectively statistically significant (Table 1). In addition, the collection of all eigenvectors is typically prefiltered,[9] so that only those with higher levels of positive spatial autocorrelation can be selected. In one recent case though, both negatively and positively spatially autocorrelated eigenvectors are included (Griffith et al., 2016). Hence, it is unclear which eigenvectors should be a priori filtered from the selection process and how sensitive the model results are to various filtering schemes, which is important because Chun et al. (2016) show that too many or too few eigenvectors can cause over- or under-correction by the ESF approach.

ESF SI model results are frequently deemed more intuitive than their non-ESF counterparts, though some confusion arises based on the aggregated results observed here. For instance, the effect that adding an ESF has on the distance-decay coefficient estimate seems to vary extensively (Table 1). In some cases, it results in a stronger distance-decay and in other cases, it results in weaker distance-decay. In addition, the standard errors for these estimates with an ESF are sometimes larger and sometimes smaller than the standard errors from a model without an ESF, though sometimes they are not reported at all. When the standard errors are reported, it is not always the case that the change in distance-decay is statistically distinguishable (at the 95 percent confidence interval) from the original distance-decay estimate. This suggests some variability of the impact of the ESF on the estimated distance-decay parameter. Related to this, Hodges and Reich (2010) and Paciorek (2010) demonstrate that a spatially correlated random effect, which an ESF is theorized to approximate (Fischer and Griffith, 2008), can compete with the substantive spatial effects (i.e. main effects) in a regression model and potentially result in biased estimates. An alternative interpretation posits that ESF components may suggest that a SI system is more vulnerable or resilient to disruptions than anticipated by a traditional gravity-type model (Griffith and Chun, 2015). More work is therefore necessary to explore whether adding an ESF model makes SI model interpretation more intuitive or if it potentially obfuscates distance-decay effects.

V Further Themes and a Future Research Scheme

Several themes either persist throughout or begin to emerge from the foregoing sections.

Most prominently, that distance-decay is a quintessential concept within quantitative human geography and much energy has been put forth to understand the intricacies associated with it. However, more recent methods in SI modeling shift the focus away from distance-decay in favor of alternative but related spatial concepts, such as accessibility, spatial dependence, network dependence, and associated notions of autocorrelation. It is easy to conflate these ideas under the banner of spatial structure, but it has been shown that the frameworks proposed to make these concepts operational are not interchangeable. Rather, further juxtaposition of the frameworks helps clarify their differences.

One observation is that SI models invoked through spatial dependence and spatial autocorrelation (i.e. spatial econometrics and spatial filtering) are supported by such general mechanisms that interpretation of the results in terms of distance-decay and specific SI processes can become opaque. This may be because they were originally developed for observations across points and areal units and then subsequently adapted to flows. It may also be related to their comparatively high flexibility that may indiscriminately account for a large degree of spatial variation in data regardless of its source. A consequence is that problems of one spatial concept, that of distance-decay, become couched in a new spatial concept, that of spatial autocorrelation or spatial dependence (Sayer, 1977), rather than incrementally and iteratively developing theories and corresponding models. For example, the veracity of the spillovers assumed to exist in SAR SI processes and the diffusions quantified by the scalar summary metrics (i.e., effects estimates) have not been investigated empirically. Another example is that the ESF method generally increases model fit, but is also prone to overfitting (Helbich and Griffith, 2016; Oshan and Fotheringham, 2017), and could mask the fact that other substantive covariates may be appropriate. In contrast, the CD model was conceived in the context of flows and SI decision-making and is suited to modeling more specific geographic entities and processes. Though it does not provide a panacea for incorporating spatial structure into SI models, it does encourage the analyst to critically evaluate how spatial structure is abstracted and is less prone to overfitting. This provides evidence of trade-offs among interpretability, conceptual adaptability, model complexity, and the types of assumptions that are made across the spatial structure approaches and future inquiries could focus on collectively considering these issues in greater detail.

The different approaches are also comparable in how they extend or diverge from legacy SI modeling traditions. For example, the CD and ESF methods can augment any member of the family of models and are straightforward to adapt to different probability models. Though a log-normal model may be appropriate for some flows (i.e. retail expenditure), a Poisson distribution is natural for discrete entities (i.e. people), and sometimes a negative binomial distribution or zero-inflated extension is ideal (Burger et al., 2009; Metulini et al., 2018a; Silva and Tenreyro, 2006). A CD or ESF variation is possible in any of these circumstances, though it should be remembered that only Poisson maximum likelihood estimates will yield results equivalent to the family of multiplicative models and ensure the constraints upheld by the balancing factors are met (Arvis and Shepherd, 2013). It is also less clear how to interpret the balancing factors in constrained models in the ESF approach (Griffith and Fischer, 2013) where the balancing factors and the ESF may compete to account for spatial variation, potentially altering previous interpretations as accessibilities or rents (Cesario, 1977; Morphet and Shabrina, 2020). In contrast, the SAR SI model is generally only available using Gaussian assumptions, which does not respect the constraints nor easily extend to other distributional assumptions.[10]

There are also some important tendencies from the initial phases of the spatial structure debate that are less prevalent in the contemporary literature. Although local models and spatial non-stationarity were originally used to diagnose problems arising from spatial structure, there has been limited recent research using local SI models (e.g. Kalogirou, 2015; Kordi and Fotheringham, 2016; Nakaya, 2001; Nissi and Sarra, 2011; Zhang et al., 2019a, 2019b). Specifically, the SAR and ESF approaches have virtually ignored the possibility of spatial non-stationarity. Meanwhile, in the ESF literature, spatial autocorrelation is often distinguished as *local* distance effects in contrast to *global* distance effects that are captured by distances between locations (Griffith, 2007, 2009a, 2011; Griffith and Paelinck, 2018). An effect that occurs locally and globally implies a multiscale process that potentially varies over space (Wolf et al., 2018). However, there has yet to be a comparison or incorporation of local SI models within the context of ESFs, which could be used to shed light on the nature of scale in SI models rather than a priori assuming process stationarity. Initial work in this direction has begun but does not include SI models (Harris et al., 2017; Harris, 2019).

Another trend worth reviving in light of advances in computing capabilities is the reliance on simulations for building intuition about SI models (e.g. Cliff et al., 1974; Fotheringham et al., 2001; Johnston, 1973; Lo, 1991a). Spatial econometric and spatial filtering approaches have been explored via simulation, but not within the context of SI models. In particular, Pace et al. (2013) demonstrate a general tension between the SAR and ESF models since the ESF filters out the spatial spillovers implied by the SAR model. An interesting extension could therefore use simulation studies to explicitly compare implementations to account for spatial structure in SI models in order to verify their underlying theory and measure their performance under different types of misspecification. This might also help clarify the nature and degree of misspecification bias for different scenarios and types of spatial systems.

An area largely untapped for advancing SI modeling is how to leverage aspects of the modern data economy. The swell of new data sources with increasing spatial and temporal resolutions provide a wealth of additional information that can supplement SI models and provide novel insights. One strategy is to refine the definition of origin and destination variables and the definition of spatial structure based on spatial and temporal context (Waddington et al., 2019). Another way forward is to explore the dynamics of human behavior by calibrating parameters for temporal subsets of data to inspect how processes change over time (Batty, 2018; Oshan, 2020). There is also a history of dynamic SI models that investigate how a component of the SI system responds to the evolution of other components (Birkin and Heppenstall, 2011; Fotheringham and Knudsen, 1986; Harris and Wilson, 1978; Nijkamp and Reggiani, 1988; Wilson, 2008; Wilson, 2010b) and these could be enhanced and validated using sensors that continuously collect mobility data to construct contextualized flows (Siła-Nowicka et al., 2016; Sila-Nowicka and Fotheringham, 2019).

Ultimately, the recommendations put forth here to move the SI modeling research agenda forward are a call for more specific theories and a better understanding of the connection between theories, models, and the interpretations extracted from them. As quantitative human geography progresses, the incorporation of best practices for scientific code development, a culture of data-sharing, and the establishment of common task frameworks could also go a long way toward broadening and aligning overlapping yet sometimes diverging branches of inquiry within the SI modeling paradigm (Wolf et al., 2020).

VI Concluding Remarks

An entry point into the vast SI corpus was provided in this review through the pursuit of a number of tasks. First, SI models and the spatial structure debate were introduced, outlining some foundational concepts, the central issues, and the intellectual lineage of the main debate arguments. Four frameworks proposed to incorporate spatial structure in SI models were then considered. The typical motivations, theories, assumptions, applications, and interpretations of each framework were highlighted, as well as some criticisms and limitations. Connections were also made between each methodological approach and the debate at large. Finally, some synthesis was constructed around additional themes detected across the literature and some avenues for subsequent research were identified. These included more explicit evaluations of framework tradeoffs, a closer consideration of the repercussions of framework evolution on knowledge accumulation, the reincorporation of simulations, local models, and spatial non-stationarity, and the utilization of burgeoning data sources and technology. The overall outcome is a comprehensive coverage of the past and a sketch of one potential path forward for the treatment of spatial structure in SI models. This path advocates primarily for a tighter integration of the SI modeling ecosystem and a more holistic approach to the issue of spatial structure. It also calls for shifting the focus back toward modeling specific SI processes and spatial decisions. These suggestions are only some of the many possible directions to contribute toward the next 50 years of SI modeling.

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Notes

1. The unconstrained model is used as an example throughout this manuscript because it provides the most convenient linkage between the various extensions presented in subsequent sections. Similarly, this manuscript focuses on regression-based estimation techniques because they are often at the heart of the debate and they provide a consistent framework to discuss all of the models described here.
2. Before consensus formed around the terminology of spatial structure, many terms were used to describe the unidentified effect, including map pattern, locational structure, interdependence, and spatial autocorrelation. Furthermore, these terms would sometimes be used to refer to empirical data and other times the resulting model parameter estimates, which may have caused confusion and intensified the debate.
3. Early accounts of the unidentified spatial structure effect, such as (Curry, 1972), included a plethora of potential factors, such as the shape of the study area, aggregation and representation bias, spatial autocorrelation in the locational attributes, model estimation techniques, and interdependence between SI and locational distributions, though no specific links were made.
4. The IIA property implies that if choice a is a substitute for choice b and choice b is a substitute for choice c, then choices a and c are also substitutes for each other. However, this may be very unlikely if choice a and choice c are very far apart.
5. Two examples are Willigers et al. (2007) and Martnez and Viegas (2013), though the focus is on accuracy rather than misspecification.
6. Multilevel specifications have also been suggested to incorporate latent spatially structured effects (LeSage et al., 2007; LeSage and Llano, 2013; Zhang et al., 2020)
7. Distance-based spatial weights have been used within the ESF technique, though it requires a distance cut-

off which denotes the point at which all further relations become zero entries of the spatial weight. Furthermore, distance-based examples are based on research in the field of ecology and have not been employed in spatial interaction models (Blanchet et al., 2008; Borcard et al., 2004; Borcard and Legendre, 2002; Dray et al., 2006; Griffith and Peres-Neto, 2006; Legendre et al., 2002).

8. While equation (10) is the most commonly found projection, others have been defined that ensure symmetric spatial relationships (Chun 2008). In addition, M may be standardized using different coding schemes (Boots, 1999; Chun, 2008).
9. A LASSO routine and a random effects variant of the ESF have been proposed that suggest more parsimonious methods for selecting a subset of eigenvectors (Seya et al., 2015; Murakami and Griffith, 2015), though neither of them has been applied in the spatial interaction literature and therefore do not shed light on the variations found in existing ESF spatial interaction research. Moreover, Chun et al. (2016) show that an ideal number of eigenvectors is dependent upon the amount of spatial autocorrelation in model residuals and the size of the tessellation. A method for identifying an ideal number of eigenvectors is put forth, but has not been applied in the spatial interaction literature.
10. A semi-parametric Poisson SI model incorporating a spatial autoregressive component exists but deviates from maximum likelihood estimates (Sellner et al., 2013).

References

Alvarez-Diaz M, D'Hombres B, Ghisetti C, et al. (2020) Analysing domestic tourism flows at the provincial level in Spain by using spatial gravity models. *International Journal of Tourism Research*. DOI: 10.1002/jtr.2344.

Anas A (1983) Discrete choice theory, information-theory and the multinomial logit and gravity models. *Transportation Research Part B: Methodological* 17(1): 13–23.

Arribas-Bel D (2014) Accidental, open and everywhere: emerging data sources for the understanding of cities. *Applied Geography* 49: 45–53.

Arvis JF and Shepherd B (2013) The Poisson quasi-maximum likelihood estimator: a solution to the 'adding up' problem in gravity models. *Applied Economics Letters* 20(6): 515–519.

Batty M (2008) Spatial interaction. In: Kemp K (ed.) *Encyclopedia of Geographic Information Science*. Thousand Oaks: SAGE, pp. 416–418.

Batty M (2013) *The New Science of Cities*. Cambridge: The MIT Press.

Batty M (2018) Visualizing aggregate movement in cities. *Philosophical Transactions of the Royal Society B: Biological Sciences* 373(1753): 20170236.

Baxter M (1983) Model misspecification and spatial structure in spatial-interaction models. *Environment and Planning A* 15(3): 319–327.

Baxter M (1985) Misspecification in spatial interaction models: further results. *Environment and Planning A* 17(5): 673–678.

Bennett RJ and Haining RP (1985) Spatial structure and spatial interaction: modelling approaches to the statistical analysis of geographical data. *Journal of the Royal Statistical Society. Series A (General)* 148(1): 1.

Bernardin VL, Koppelman F and Boyce D (2009) Enhanced destination choice models incorporating agglomeration related to trip chaining while controlling for spatial competition. *Transportation Research Record: Journal of the Transportation Research Board* 2132(1): 143–151.

Bernasco W (2010) Modeling micro-level crime location choice: application of the discrete choice framework to crime at places. *Journal of Quantitative Criminology* 26(1): 113–138.

Birkin M, Clarke G and Clarke M (2010) Refining and operationalizing entropy-maximizing models for business applications. *Geographical Analysis* 42(4): 422–445.

Birkin M and Heppenstall A (2011) Extending spatial interaction models with agents for understanding relationships in a dynamic retail market. *Urban Studies Research* 2011: 1–12.

Black WR (1992) Network autocorrelation in transport network and flow systems. *Geographical Analysis* 24(3): 207–222.

Blanchet FG, Legendre P and Borcard D (2008) Modelling directional spatial processes in ecological data. *Ecological Modelling* 215(4): 325–336.

Bolduc D, Gaudry MJ and Dagenai MG (1989) Spatially autocorrelated errors in origin-destination models: a new specification applied to aggregate mode choice. *Transportation Research Part B: Methodological* 23(5): 361–372.

Bolduc D, Laferriere R and Santarossa G (1992) Spatial autoregressive error components in travel flow models. *Regional Science and Urban Economics* 22(3): 371–385.

Bolduc D, Laferriere R and Santarossa G (1995) Spatial autoregressive error components in travel flow models: an application to aggregate mode choice. In: Anselin L and Florax R (eds) *New Directions in Spatial Econometrics*. Berlin: Springer, pp. 96–108.

Boots B (1999) A variance-stabilizing coding scheme for spatial link matrices. *Environment and Planning A* 31: 165–180.

Borcard D and Legendre P (2002) All-scale spatial analysis of ecological data by means of principal coordinates of neighbour matrices. *Ecological Modelling* 153(1): 51–68.

Borcard D, Legendre P, Avois-Jacquet C, et al. (2004) Dissecting the spatial structure of ecological data at multiple scales. *Ecology* 85(7): 1826–1832.

Brandsma A and Ketellapper RH (1979) A bivariate approach to spatial autocorrelation. *Environment and Planning A* 11(1): 51–58.

Buczkowska S, Coulombel N and de Lapparent M (2019) A comparison of Euclidean distance, travel times, and network distances in location choice mixture models. *Networks and Spatial Economics* 19(4): 1215–1248.

Burger M, Van Oort F and Linders GJ (2009) On the specification of the gravity model of trade: zeros, excess zeros and zero-inflated estimation. *Spatial Economic Analysis* 4(2): 167–190.

Carrothers GAP (1956) An historical review of the gravity and potential concepts of human interaction. *Journal of the American Institute of Planners* 22(2): 94–102.

Cesario F (1977) A new interpretation of the normalizing or balancing factors of gravity-type spatial models. *Socio-Economic Planning Sciences* 11(3): 131–136.

Chun Y (2008) Modeling network autocorrelation within migration flows by eigenvector spatial filtering. *Journal of Geographical Systems* 10(4): 317–344.

Chun Y and Griffith DA (2011) Modeling network autocorrelation in space–time migration flow data: an eigenvector spatial filtering approach. *Annals of the Association of American Geographers* 101(3): 523–536.

Chun Y, Griffith DA, Lee M, et al. (2016) Eigenvector selection with stepwise regression techniques to construct eigenvector spatial filters. *Journal of Geographical Systems* 18(1): 67–85.

Chun Y, Kim H and Kim C (2012) Modeling interregional commodity flows with incorporating network autocorrelation in spatial interaction models: an application of the US interstate commodity flows. *Computers, Environment and Urban Systems* 36(6): 583–591.

Cliff A, Haggett P, Ord JK, et al. (1975a) *Elements of Spatial Structure: A Quantitative Approach*, 1st ed. Cambridge: Cambridge University Press.

Cliff A, Martin R and Ord J (1974) Evaluating the friction of distance parameter in gravity models. *Regional Studies* 8: 281–286.

Cliff A, Martin R and Ord J (1975b) Map pattern and friction of distance parameters: reply to comments by R. J. Johnston, and by L. Curry, D. A. Griffith and E. S. Sheppard. *Regional Studies* 9(3): 285–288.

Cliff A, Martin R and Ord J (1976) A reply to the final comment. *Regional Studies* 10(3): 341–342.

Corrado L and Fingleton B (2012) Where is the economics in spatial econometrics? *Journal of Regional Science* 52(2): 210–239.

Curry L (1972) A spatial analysis of gravity flows. *Regional Studies* 6(2): 131–147.

Curry L, Griffith DA and Sheppard ES (1975) Those gravity parameters again. *Regional Studies* 9(3): 289–296.

Curtis A and Fotheringham AS (1995) Large-scale information surfaces: an analysis of city-name recalls in the United States. *Geoforum* 26(1): 75–87.

de Dios Ortúzar J, Willumsen LG and Wiley InterScience (Online service) (2011) *Modelling Transport*. Oxford: Wiley-Blackwell.

de la Mata T and Llano C (2013) Social networks and trade of services: modelling interregional flows with spatial and network autocorrelation effects. *Journal of Geographical Systems* 15(3): 319–367.

de Mello-Sampayo F (2016) A spatial analysis of mental healthcare in Texas. *Spatial Economic Analysis* 11(2): 152–175.

de Mello-Sampayo F (2017a) Competing-destinations gravity model applied to trade in intermediate goods. *Applied Economics Letters* 24(19): 1378–1384.

de Mello-Sampayo F (2017b) Testing competing destinations gravity models – evidence from BRIC International. *The Journal of International Trade & Economic Development* 26(3): 277–294.

Dray S, Legendre P and Peres-Neto PR (2006) Spatial modelling: a comprehensive framework for principal coordinate analysis of neighbour matrices (PCNM). *Ecological Modelling* 196(3–4): 483–493.

Farmer C and Oshan T (2017) Spatial interaction. *The Geographic Information Science & Technology Body of Knowledge* (4th Quarter 2017 Edition).

Fichet de Clairfontaine A, Fischer MM, Lata R, et al. (2015) Barriers to cross-region research and development collaborations in Europe: evidence from the fifth European Framework Programme. *The Annals of Regional Science* 54(2): 577–590.

Fik TJ, Amey RG and Mulligan GF (1992) Labor migration amongst hierarchically competing and intervening origins and destinations. *Environment and Planning A* 24(9): 1271–1290.

Fik TJ and Mulligan GF (1990) Spatial flows and competing central places: towards a general theory of hierarchical interaction. *Environment and Planning A* 22(4): 527–549.

Fischer MM and Griffith DA (2008) Modeling spatial autocorrelation in spatial interaction data: an application to patent citation data in the European Union. *Journal of Regional Science* 48(5): 969–989.

Fischer MM, Scherngell T and Jansenberger E (2006) The geography of knowledge spillovers between high-technology firms in Europe: evidence from a spatial interaction modeling perspective. *Geographical Analysis* 38(3): 288–309.

Flowerdew R and Aitkin M (1982) A method of fitting the gravity model based on the Poisson distribution. *Journal of Regional Science* 22(2): 191–202.

Flowerdew R and Lovett A (1988) Fitting constrained Poisson regression models to interurban migration flows. *Geographical Analysis* 20(4): 297–307.

Fotheringham A and Curtis A (1992) Encoding spatial information: the evidence for hierarchical processing. In: Frank AU, Campari I and Formentini U (eds) *From Space to Territory: Theories and Methods of Spatio-Temporal Reasoning*. Pisa, Italy: Springer.

Fotheringham AS (1981) Spatial structure and distance-decay parameters. *Annals of the Association of American Geographers* 71(3): 425–436.

Fotheringham AS (1983) A new set of spatial-interaction models: the theory of competing destinations. *Environment and Planning A* 15(1): 15–36.

Fotheringham AS (1984) Spatial flows and spatial patterns. *Environment and Planning A* 16(4): 529–543.

Fotheringham AS (1985) Spatial competition and agglomeration in urban modelling. *Environment and Planning A* 17(2): 213–230.

Fotheringham AS (1986) Modelling hierarchical destination choice. *Environment and Planning A* 18(3): 401–418.

Fotheringham AS (1988) Consumer store choice and choice set definition. *Marketing Science* 7(3): 299.

Fotheringham AS (2017) Spatial interaction. In: Richardson D, Castree N, Goodchild MF, et al (eds) *International Encyclopedia of Geography*. Atlanta, GA: Wiley.

Fotheringham AS, Champion T, Wymer C, et al. (2000) Measuring destination attractivity: a migration example. *International Journal of Population Geography* 6(6): 391–421.

Fotheringham AS and Curtis A (1999) Regularities in spatial information processing: implications for modeling destination choice. *The Professional Geographer* 51(2): 227–239.

Fotheringham AS and Knudsen DC (1986) Modeling discontinuous change in retailing systems: extensions of the Harris-Wilson framework with results from a simulated urban retailing system. *Geographical Analysis* 18(4): 295–312.

Fotheringham AS, Nakaya T, Yano K, et al. (2001) Hierarchical destination choice and spatial interaction modelling: a simulation experiment. *Environment and Planning A* 33(5): 901–920.

Fotheringham AS and O'Kelly ME (1989) *Spatial Interaction Models: Formulations and Applications*. London: Kluwer Academic Publishers.

Fotheringham AS, Rees P, Champion T, et al. (2004) The development of a migration model for England and Wales: overview and modelling out-migration. *Environment and Planning A* 36(9): 1633–1672.

Fotheringham AS and Webber MJ (1980) Spatial structure and the parameters of spatial interaction models. *Geographical Analysis* 12(1): 33–46.

Fotheringham S (1982) Distance-decay parameters: a reply. *Annals of the Association of American Geographers* 72(3): 551–553.

Gibbons S and Overman HG (2012) Mostly pointless spatial econometrics? *Journal of Regional Science* 52(2): 172–191.

Gitlesen JP, Kleppe G, Thorsen I, et al. (2010) An empirically based implementation and evaluation of a hierarchical model for commuting flows. *Geographical Analysis* 42(3): 267–287.

Gitlesen JP and Thorsen I (2000) A competing destinations approach to modeling commuting flows: a theoretical interpretation and an empirical application of the model. *Environment and Planning A* 32(11): 2057–2074.

Gordon IR (1985) Economic explanations of spatial variation in distance deterrence. *Environment and Planning A* 17(1): 59–72.

Griffith DA (1976) Spatial structure and spatial interaction: a review. *Environment and Planning A* 8(7): 731–740.

Griffith DA (1996) Spatial autocorrelation and eigenfunctions of the geographic weights matrix accompanying geo-referenced data. *Canadian Geographer/Le Géographe Canadien* 40(4): 351–367.

Griffith DA (2000) A linear regression solution to the spatial autocorrelation problem. *Journal of Geographical Systems* 2(2): 141–156.

Griffith DA (2002) A spatial filtering specification for the auto-Poisson model. *Statistics & Probability Letters* 58(3): 245–251.

Griffith DA (2004) A spatial filtering specification for the autologistic model. *Environment and Planning A* 36(10): 1791–1811.

Griffith DA (2007) Spatial structure and spatial interaction 25 years later. *The Review of Regional Studies* 37(1): 28–38.

Griffith DA (2009a) Modeling spatial autocorrelation in spatial interaction data: empirical evidence from 2002 Germany journey-to-work flows. *Journal of Geographical Systems* 11(2): 117–140.

Griffith DA (2009b) Spatial autocorrelation in spatial interaction. In: Reggiani A and Nijkamp P (eds) *Complexity and Spatial Networks*. Berlin, Heidelberg: Springer, pp. 221–237.

Griffith DA (2011) Visualizing analytical spatial autocorrelation components latent in spatial interaction data: an eigenvector spatial filter approach. *Computers, Environment and Urban Systems* 35(2): 140–149.

Griffith DA and Chun Y (2015) Spatial autocorrelation in spatial interactions models: geographic scale and resolution implications for network resilience and vulnerability. *Networks and Spatial Economics* 15(2): 337–365.

Griffith DA and Fischer MM (2013) Constrained variants of the gravity model and spatial dependence: model specification and estimation issues. *Journal of Geographical Systems* 15(3): 291–317.

Griffith DA, Fischer MM and LeSage J (2016) The spatial autocorrelation problem in spatial interaction modelling: a comparison of two common solutions. *Letters in Spatial and Resource Sciences* 10(1): 75–86.

Griffith DA and Jones KG (1980) Explorations into the relationship between spatial structure and spatial interaction. *Environment and Planning A* 12: 187–201.

Griffith DA and Paelinck JHP (2018) *Morphisms for Quantitative Spatial Analysis, Advanced Studies in Theoretical and Applied Econometrics*. Cham: Springer International Publishing, vol. 51.

Griffith DA and Peres-Neto PR (2006) Spatial modeling in ecology: the flexibility of eigenfunction spatial analyses. *Ecology* 87(10): 2603–2613.

Gu H, Liu Z, Shen T, et al. (2019) Modelling interprovincial migration in China from 1995 to 2015 based on an eigenvector spatial filtering negative binomial model. *Population, Space and Place* 25(8). DOI: 10.1002/psp.2253.

Guldmann JM (1999) Competing destinations and intervening opportunities interaction models of inter-city telecommunication flows. *Papers in Regional Science* 78(2): 179–194.

Guy CM (1987) Recent advances in spatial interaction modelling: an application to the forecasting of shopping travel. *Environment and Planning A* 19(2): 173–186.

Haggett P (1965) *Locational Analysis in Human Geography*, 1st ed. London: Edward Arnold.

Halleck Vega S and Elhorst JP (2015) The SLX model. *Journal of Regional Science* 55(3): 339–363.

Harris B and Wilson AG (1978) Equilibrium values and dynamics of attractiveness terms in production-constrained spatial-interaction models. *Environment and Planning A* 10(4): 371–388.

Harris P (2019) A simulation study on specifying a regression model for spatial data: choosing between autocorrelation and heterogeneity effects. *Geographical Analysis* 51(2): 151–181.

Harris P, Brunsdon C, Lu B, et al. (2017) Introducing bootstrap methods to investigate coefficient non-stationarity in spatial regression models. *Spatial Statistics* 21: 241–261.

Haynes KE and Fotheringham AS (1984) *Gravity and Spatial Interaction Models*. Beverly Hills: SAGE, vol. 2.

Helbich M and Griffith DA (2016) Spatially varying coefficient models in real estate: eigenvector spatial filtering and alternative approaches. *Computers, Environment and Urban Systems* 57: 1–11.

Hirtle SC and Jonides J (1985) Evidence of hierarchies in cognitive maps. *Memory & Cognition* 13(3): 208–217.

Hodges JS and Reich BJ (2010) Adding spatially-correlated errors can mess up the fixed effect you love. *The American Statistician* 64(4): 325–334.

Hu P and Pooler J (2002) An empirical test of the competing destinations model. *Journal of Geographical Systems* 4(3): 301–323.

Hua CI and Porell F (1979) A critical review of the development of the gravity model. *International Regional Science Review* 4(2): 97–126.

Ishikawa Y (1987) An empirical study of the competing destinations model using Japanese interaction data. *Environment and Planning A* 19(10).

Ishikawa Y (1990) Explorations into the two-stage destination choice. *Geographical Review of Japan B* 62: 75–85.

Johnston R (1975) Map pattern and friction of distance parameters: a comment. *Regional Studies* 9(3): 281–283.

Johnston R (1976) On regression coefficients in comparative studies of the friction of distance. *Journal of Economic and Social Geography* 67: 15–28.

Johnston RJ (1973) On frictions of distance and regression coefficients. *Area* 5(3): 187–191.

Kalogirou S (2015) Destination choice of Athenians: an application of geographically weighted versions of standard and zero inflated Poisson spatial interaction models: destination choice of Athenians. *Geographical Analysis* 48(2): 191–230.

Kerkman K, Martens K and Meurs H (2017) A multilevel spatial interaction model of transit flows incorporating spatial and network autocorrelation. *Journal of Transport Geography* 60: 155–166.

Kim K, Kim V and Kim H (2019) Spatiotemporal autoregressive model for origin–destination air passenger flows. *Journal of the Royal Statistical Society: Series A (Statistics in Society)* 182(3): 1003–1016.

Kordi M and Fotheringham AS (2016) Spatially weighted interaction models (SWIM). *Annals of the American Association of Geographers* 106(5): 990–1012.

Lee ML and Pace RK (2005) Spatial distribution of retail sales. *The Journal of Real Estate Finance and Economics* 31(1): 53–69.

Legendre P, Dale MR, Fortin MJ, et al. (2002) The consequences of spatial structure for the design and analysis of ecological field surveys. *Ecography* 25(5): 601–615.

LeSage JP and Fischer MM (2014) Spatial regression-based model specifications for exogenous and endogenous spatial interaction. In: *Spatial Econometric Interaction Modelling*. Cham: Springer, pp. 15–36.

LeSage JP, Fischer MM and Scherngell T (2007) Knowledge spillovers across Europe: evidence from a Poisson spatial interaction model with spatial effects. *Papers in Regional Science* 86(3): 393–421.

LeSage JP and Llano C (2013) A spatial interaction model with spatially structured origin and destination effects. *Journal of Geographical Systems* 15(3): 265–289.

LeSage JP and Pace RK (2008) Spatial econometric modeling of origin-destination flows. *Journal of Regional Science* 48(5): 941–967.

LeSage JP and Polasek W (2008) Incorporating transportation network structure in spatial econometric models of commodity flows. *Spatial Economic Analysis* 3(2): 225–245.

LeSage JP and Thomas-Agnan C (2015) Interpreting spatial econometric origin-destination flow models. *Journal of Regional Science* 55(2): 188–208.

Liu Y and Shen J (2017) Modelling skilled and less-skilled interregional migrations in China, 2000-2005: modelling skilled and less-skilled interregional migrations in China. *Population, Space and Place* 23(4): e2027.

Lloyd CD (2014) Measuring spatial structure. In: *Exploring Spatial Scale in Geography*. John Wiley & Sons, Ltd, pp. 45–102.

Lo L (1991a) Spatial structure and spatial interaction: a simulation approach. *Environment and planning A* 23(9): 1279–1300.

Lo L (1991b) Substitutability, spatial structure, and spatial interaction. *Geographical Analysis* 23(2): 132–146.

Lo L (1992) Destination interdependence and the competing-destinations model. *Environment and Planning A* 24(8): 1191–1204.

Lovelace R, Birkin M, Cross P, et al. (2015) From big noise to big data: toward the verification of large data sets for understanding regional retail flows: from big noise to big data. *Geographical Analysis* 48: 59–81.

Manley E and Dennett A (2019) New forms of data for understanding urban activity in developing countries. *Applied Spatial Analysis and Policy* 12(1): 45–70.

Margaretic P, Thomas-Agnan C and Doucet R (2017) Spatial dependence in (origin-destination) air passenger flows: spatial dependence in air passenger flows. *Papers in Regional Science* 96(2): 357–380.

Marrocu E and Paci R (2013) Different tourists to different destinations. Evidence from spatial interaction models. *Tourism Management* 39: 71–83.

Martínez LM and Viegas JM (2013) A new approach to modelling distance-decay functions for accessibility assessment in transport studies. *Journal of Transport Geography* 26: 87–96.

Matthews Y, Scarpa R and Marsh D (2018) Cumulative attraction and spatial dependence in a destination choice model for beach recreation. *Tourism Management* 66: 318–328.

McFadden D (1974) Conditional logit analysis of qualitative choice behavior. In: Zarembka P (ed.) *Frontiers in Econometrics*. New York: Academic Press, pp. 105–142.

McFadden D (1977) *Modelling the Choice of Residential Location. Cowles Foundation Discussion Paper: report 477*.

McMillen DP (2003) Spatial autocorrelation or model misspecification? *International Regional Science Review* 26(2): 208–217.

McMillen DP (2012) Perspectives on spatial econometrics: linear smoothing with structured models. *Journal of Regional Science* 52(2): 192–209.

McNamara TP (1986) Mental representations of spatial relations. *Cognitive Psychology* 18(1): 87–121.

Metulini R, Patuelli R and Griffith D (2018a) A spatial-filtering zero-inflated approach to the estimation of the gravity model of trade. *Econometrics* 6(1): 9. Available at: http://www.mdpi.com/2225-1146/6/1/9.

Metulini R, Sgrignoli P, Schiavo S, et al. (2018b) The network of migrants and international trade. *Economia Politica* 35(3): 763–787.

Morphet R and Shabrina Z (2020) Gravity model calibration by rent. *CASA Working Paper Series* 223: 1–12.

Moura TGZ, Chen Z and Garcia-Alonso L (2019) Spatial interaction effects on inland distribution of maritime flows. *Transportation Research Part A: Policy and Practice* 128: 1–10.

Mukherji N and Silberman J (2019) Knowledge flows between universities and industry: the impact of distance, technological compatibility, and the ability to diffuse knowledge. *The Journal of Technology Transfer*. DOI: 10.1007/s10961-019-09770-9.

Murakami D and Griffith DA (2015) Random effects specifications in eigenvector spatial filtering: a simulation study. *Journal of Geographical Systems* 17(4): 311–331.

Murayama K and Nagayasu J (2019) *Spatial Dependence, Social Networks, and Economic Structures in Regional Labor Migration. Technical Report 95691*. Germany: University Library of Munich. Publication Title: MPRA Paper.

Nakaya T (2001) Local spatial interaction modelling based on the geographically weighted regression approach. *GeoJournal* 53(4): 347–358.

Newing A, Clarke GP and Clarke M (2015) Developing and applying a disaggregated retail location model with extended retail demand estimations: disaggregated retail location model. *Geographical Analysis* 47(3): 219–239.

Nijkamp P and Reggiani A (1988) Dynamic spatial interaction models: new directions. *Environment and Planning A* 20(11): 1449–1460.

Nissi E and Sarra A (2011) Detecting local variations in spatial interaction models by means of geographically weighted regression. *Journal of Applied Sciences* 11(4): 630–638.

O'Kelly ME (2015) Spatial interaction. In: Wright JD (ed.) *International Encyclopedia of the Social & Behavioral Sciences*, 2nd ed. Oxford: Elsevier, pp. 152–156.

Oshan TM (2016) A primer for working with the Spatial Interaction modeling (SpInt) module in the python spatial analysis library (PySAL). *Region* 3(2): 11.

Oshan TM (2020) Potential and pitfalls of big transport data for spatial interaction models of urban mobility. *The Professional Geographer* 72: 468–480.

Oshan TM and Fotheringham AS (2017) A comparison of spatially varying regression coefficient estimates using geographically weighted and spatial-filter-based techniques: a comparison of spatially varying regression. *Geographical Analysis* 50: 53–75.

Pace RK, Lesage JP and Zhu S (2013) Interpretation and computation of estimates from regression models using spatial filtering. *Spatial Economic Analysis* 8(3): 352–369.

Paciorek CJ (2010) The importance of scale for spatial-confounding bias and precision of spatial regression estimators. *Statistical Science* 25(1): 107–125.

Partridge MD, Boarnet M, Brakman S, et al. (2012) Introduction: Whither spatial econometrics? *Journal of Regional Science* 52(2): 167–171.

Patuelli R and Arbia G (eds) (2016) *Spatial Econometric Interaction Modelling*. Advances in Spatial Science. Springer International Publishing. DOI: 10.1007/978-3-319-30196-9.

Patuelli R, Linders GJ, Metulini R, et al. (2015) The space of gravity: spatially filtered estimation of a gravity model for bilateral trade. In: Arbia G and Patuelli R (eds) *Spatial Econometric Interaction Modelling*. Advances in Spatial Science. Berlin: Springer, pp. 145–169.

Pellegrini PA and Fotheringham AS (1999) Intermetropolitan migration and hierarchical destination choice: a disaggregate analysis from the US public use microdata samples. *Environment and Planning A* 31(6): 1093–1118.

Pellegrini PA and Fotheringham AS (2002) Modelling spatial choice: a review and synthesis in a migration context. *Progress in Human Geography* 26(4): 487–510.

Pellegrini PA, Fotheringham AS and Lin G (1997) An empirical evaluation of parameter sensitivity to choice set definition in shopping destination choice models. *Papers in Regional Science* 76(2): 257–284.

Pinkse J and Slade ME (2010) The future of spatial econometrics. *Journal of Regional Science* 50(1): 103–117.

Pooler J (1998) Competition among destinations in spatial interaction models: a new point of view. *Chinese Geographical Science* 8(3): 212–224.

Porojan A (2001) Trade flows and spatial effects: the gravity model revisited. *Open Economies Review* 12(3): 265.

Roy JR (2004) *Spatial Interaction Modelling: A Regional Science Context*. Berlin: Springer.

Sardadvar S and Vakulenko E (2020) Estimating and interpreting internal migration flows in Russia by accounting for network effects. *Socio-Economic Planning Sciences* 69: 100685.

Sayer RA (1977) Gravity models and spatial autocorrelation, or atrophy in urban and regional modelling. *Area* 9: 183–189.

Scherngell T and Lata R (2012) Towards an integrated European research area? Findings from eigenvector spatially filtered spatial interaction models using European Framework Programme data: Towards an integrated European Research Area? *Papers in Regional Science* 92: 555–577.

Sellner R, Fischer MM and Koch M (2013) A spatial autoregressive Poisson gravity model: a SAR Poisson gravity model. *Geographical Analysis* 45(2): 180–201.

Sen A and Smith T (1995) *Gravity Models of Spatial Interaction Behavior*. Advances in Spatial and Network Economics. Berlin: Springer.

Seya H, Murakami D, Tsutsumi M, et al. (2015) Application of LASSO to the eigenvector selection problem in eigenvector-based spatial filtering: application of LASSO. *Geographical Analysis* 47(3): 284–299.

Sheppard E (1982) Distance-decay parameters: a comment. *Annals of the Association of American Geographers* 72(4): 549–550.

Sheppard E (1984) The distance-decay gravity model debate. In: Gaetan C and Guyon X (eds) *Spatial Statistics and Models*. Boston: Dreidel, pp. 367–384.

Sheppard ES (1979) Gravity parameter estimation. *Geographical Analysis* 11(2): 120–132.

Sheppard ES, Griffith DA and Curry L (1976) A final comment on mis-specification and autocorrelation in those gravity parameters. *Regional Studies* 10(3): 337–339.

Sila-Nowicka K and Fotheringham AS (2019) Calibrating spatial interaction models from GPS tracking data: an example of retail behaviour. *Computers, Environment and Urban Systems* 74: 136–150.

Silva JMCS and Tenreyro S (2006) The log of gravity. *The Review of Economics and Statistics* 88(4): 641–658.

Siła-Nowicka K, Vandrol J, Oshan T, et al (2016) Analysis of human mobility patterns from GPS trajectories and contextual information. *International Journal of Geographical Information Science* 30(5): 881–906.

Sohn C, Christopoulos D and Koskinen J (2019) Borders moderating distance: a social network analysis of spatial effects on policy interaction. *Geographical Analysis*. DOI: 10.1111/gean.12218.

Stevens SS (1975) *Psychophysics*. Piscataway: Transaction Publishers.

Thill JC (1992) Choice set formation for destination choice modelling. *Progress in Human Geography* 16(3): 361–382.

Thill JC and Horowitz JL (1997) Travel-time constraints on destination-choice sets. *Geographical Analysis* 29(2): 108–123.

Thomas T and Tutert S (2013) An empirical model for trip distribution of commuters in the Netherlands: transferability in time and space reconsidered. *Journal of Transport Geography* 26: 158–165.

Thorsen I and Gitlesen JP (1998) Empirical evaluation of alternative model specifications to predict commuting flows. *Journal of Regional Science* 38(2): 273–292.

Tiefelsdorf M (2003) Misspecifications in interaction model distance decay relations: a spatial structure effect. *Journal of Geographical Systems* 5(1): 25–50.

Tiefelsdorf M and Boots B (1995) The specification of constrained interaction models using the SPSS loglinear procedure. *Geographical Systems* 2: 21–38.

Ubøe J, Petter Gitlesen J and Thorsen I (2008) Laboratory testing of spurious spatial structure in trip distribution models. *Spatial Economic Analysis* 3(3): 361–372.

Ullman EL (1980) *Geography as Spatial Interaction*. Seattle: University of Washington Press.

Vries JJD, Nijkamp P and Rietveld P (2009) Exponential or power distance-decay for commuting? An alternative specification. *Environment and Planning A* 41(2): 461–480.

Waddington T, Clarke G, Clarke MC, et al. (2019) Accounting for temporal demand variations in retail location models. *Geographical Analysis* 51(4): 426–447.

Walker R, Walker R and Calzonetti F (1990) Searching for new manufacturing plant locations: a study of location decisions in Central Appalachia. *Regional Studies* 24(1): 15–30.

Wang J (2017) Economic geography: spatial interaction. In: Richardson D, Castree N and Goodchild MF (eds) *International Encyclopedia of Geography*. Atlanta, GA: American Cancer Society, pp. 1–4.

Willigers J, Floor H and van Wee B (2007) Accessibility indicators for location choices of offices: an application to the intraregional distributive effects of high-speed rail in the Netherlands. *Environment and Planning A: Economy and Space* 39(9): 2086–2898.

Wilson A (2008) Boltzmann, Lotka and Volterra and spatial structural evolution: an integrated methodology for some dynamical systems. *Journal of The Royal Society Interface* 5(25): 865–871.

Wilson A (2010a) Entropy in urban and regional modelling: retrospect and prospect. *Geographical Analysis* 42(4): 364–394.

Wilson A (2010b) Urban and regional dynamics from the global to the local: hierarchies, 'DNA', and 'genetic' planning. *Environment and Planning B: Planning and Design* 37(5): 823–837.

Wilson AG (1967) A statistical theory of spatial distribution models. *Transportation Research* 1: 253–269.

Wilson AG (1971) A family of spatial interaction models, and associated developments. *Environment and Planning A* 3: 1–32.

Wolf LJ, Fox S, Harris R, et al. (2020) Quantitative geography III: future challenges and challenging futures. *Progress in Human Geography*. DOI: 10.1177/0309132520924722.

Wolf LJ, Oshan TM and Fotheringham AS (2018) Single and multiscale models of process spatial heterogeneity: single and multiscale models. *Geographical Analysis* 50(4).

Yano K, Nakaya T, Fotheringham AS, et al. (2003) A comparison of migration behaviour in Japan and Britain using spatial interaction models. *International Journal of Population Geography* 9(5): 419–431.

Yin Z, Wang F and Gan J (2020) Spatial spillover effects of global forest product trade. *Forest Policy and Economics* 113: 102112.

Zhang L, Cheng J and Jin C (2019a) Spatial interaction modeling of od flow data: comparing geographically weighted negative binomial regression (GWNBR) and OLS (GWOLSR). *ISPRS International Journal of Geo-Information* 8(5): 220.

Zhang L, Cheng J, Jin C, et al. (2019b) A multiscale flow-focused geographically weighted regression modelling approach and its application for transport flows on expressways. *Applied Sciences* 9(21): 4673.

Zhang XN, Wang WW, Harris R, et al. (2020) Analysing inter-provincial urban migration flows in China: a new multilevel gravity model approach. *Migration Studies* 8(1): 19–42.

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