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Bayesian Survival Approach to Analyzing the Risk of Recurrent Rail Defects

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Abstract

This paper develops a Bayesian framework to explore the impact of different factors and to predict the risk of recurrence of rail defects, based upon datasets collected from a US Class I railroad between 2011 and 2016. To this end, this study constructs a parametric Weibull baseline hazard function and a proportional hazard (PH) model under a Gaussian frailty approach. The analysis is performed using Markov chain Monte Carlo simulation methods and the fit of the model is checked using a Cox–Snell residual plot. The results of the model show that the recurrence of a defect is correlated with different factors such as the type of rail defect, the location of the defect, train speed limit, the number of geometry defects in the last three years, and the weight of the rail. First, unlike the ordinary PH model in which the occurrence times of rail defects at the same location are assumed to be independent, a PH model under *frailty* induces the correlation between times to the recurrence of rail defects for the same segment, which is essential in the case of recurrent events. Second, considering Gaussian frailties is useful for exploring the influence of unobserved covariates in the model. Third, integrating a Bayesian framework for the parameters of the Weibull baseline hazard function as well as other parameters provides greater flexibility to the model. Fourth, the findings are useful for responsive maintenance planning, capital planning, and even preventive maintenance planning.

Rail is an efficient and safe mode of transportation and also plays a vital role in freight movement in the United States. According to the *National Transportation Statistics* report (*1*), freight railroads operating in the United States earned close to $70 billion in revenue in 2016. However, freight train services might be interrupted occasionally by derailments caused by an existing track defect. Train derailment has severe consequences and potential for loss of both life and infrastructure. It also results in huge costs and notoriety for railway companies.

According to the existing literature, track defects could be either a track geometry type or track structural type of defect (*2, 3*). Track geometry defects are generated from the geometry conditions of the track, including profile, alignment, gage, and more (*3*, *4*). Track structural defects (rail defects), which is the main focus of this study, indicate poorly conditioned structural parameters such as rails, sleepers, ties, subgrades, and so on, as shown in Figure 1.

Rail defects are the leading cause of major derailments in the United States (*5*). They are initiated in rail by fatigue or other failure mechanisms. They could be also developed in any type of rail or welds as a result of the rail manufacturing process, dynamic effects, rail wear, and plastic flow (*6*). These defects could be of different types. More details on each type of rail defect are given by Nordco Rail Services and Inspection Technologies (*6*).

Rail defects can grow in size through regular rail operations via tonnage accumulation on the rail and might lead to complete rail breakage if this continues to go unnoticed (*7*). When railways notice a defect with size above the threshold value, they take that rail segment out of service as part of responsive track maintenance (Figure 2). However, because of the intrinsic features of a rail segment, as well the complex interactions between the factors related to the environment, traffic,

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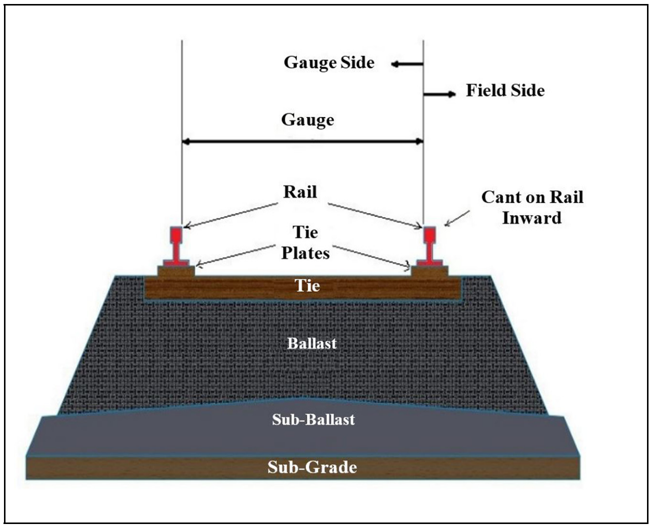

Figure 1. Rail structural components.

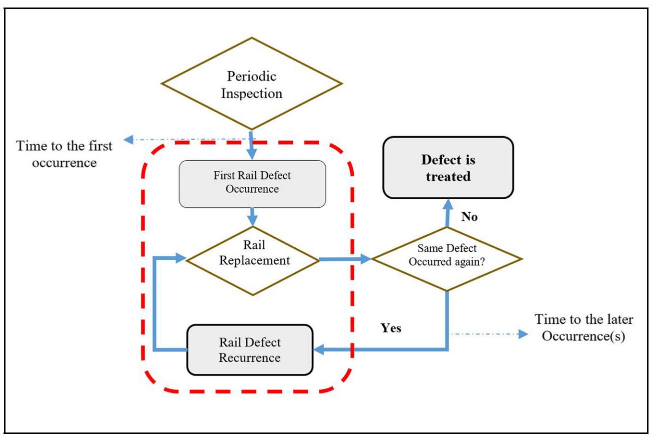

Figure 2. Decision-making procedure for recurrent rail defects.

infrastructure, and more, it is still probable another defect will appear later in the same location as tonnage accumulates on the rail segment. In other words, because of some specific known or unknown features, some segments are more susceptible to the recurrence of a specific type of defect (or within a shorter tonnage accumulation interval) than others.

The objective of this paper is to develop a comprehensive methodology for obtaining a reliable model that accounts for the influence of various factors on the recurrence of rail defects and further predicts the time to the next probable occurrence of a defect in the same location. This objective is achieved first by gathering data from different resources in a North American Class I railroad between 2011 and 2016. The next step includes data processing to reshape the data into the desired format for our analysis. Later, in the modeling step, a specific variant of survival analysis model (*8–10*) (known as the frailty Cox PH model (*11–13*)) in the context of a Bayesian framework is adopted. The independent and identically distributed (*i.i.d*) assumption of the standard Cox model for observations (*9*) means that this model would result in significant bias and inefficiency with regard to recurrent events as no correlation is assumed for the recurrent rail defects at the same location. Instead, we take advantage of a frailty model in which the association between the recurrent rail defects is induced by considering the random effect variable for different occurrences of the same observation. This model is able not only to induce the connectivity between multiple occurrences of a specific rail defect in the same location, but also to account for the heterogeneity arising from unknown or unavailable factors being included in the model.

The paper provides several key contributions:

- A comprehensive logical methodology framework for data collection, preprocessing, and modeling based on a comprehensive collection of datasets from different resources in a Class I railroad is designed.
- For the first time, the correlated event times of survival analysis are applied in the context of railway transportation for recurrent rail defects.
- A Bayesian framework is developed by performing Markov chain Monte Carlo (MCMC) simulation to optimize the parameters of the model, which can provide greater flexibility and different shapes of hazard function for the model.
- The fit of the model is verified by using a Cox–Snell residual plot and the results of the model are discussed.

The rest of the paper is organized as follows. The next section briefly presents the previous research on the application of statistical and data-driven modeling in the context of rail defects and service failures. Then, we present a comprehensive overview of the proposed methodology, followed by a discussion of the structure of the data and the steps taken to prepare the final data table, while presenting a few insights from the data. In the fifth section, the modeling approach is explained, before the results of the model and discussions are provided. The final section concludes.

Literature Review

Some previous studies have considered the statistical analysis of rail breakage as a result of existing defects in the structure of the rail. A comprehensive review of the data-driven studies in the context of railway engineering including rail track maintenance was conducted by Ghofrani et al. (*14*). To name a few, Schafer and Barkan (*4*) apply an artificial neural network to estimate the occurrence of rail breaks. In another study, a fuzzy logic method to model the interaction between the factors affecting rail breakage was conducted (*15*). From the probability distribution statistical methods, Vesković et al. and Kumar fitted the normal and Weibull distributions for rail breaks and the important affecting parameters were estimated by maximum likelihood estimation (*15, 16*). Other examples appear in the literature (*17–20*).

Tyler Dick et al. present a parametric discrete choice logit model to predict the locations of the rail breaks by using a two-year period of rail data (*17*). A step-wise regression was used to select the variables, and these authors found that the data on characteristics of rail, operational information, and infrastructure were significant in predicting rail breaks. Average dynamic loading, which was expected to have a positive correlation with breaks, had a negative correlation in this model as it lacks the consideration of a few other significant factors. This model did not provide the desired accuracy when tested on data for the latest service failures as a result of the limited availability of data. Another model was later proposed by the author which considered a few other factors that had a considerable correlation with rail breaks, using four years of rail data (*5*). These factors include data on inspection, infrastructure, past geometric, and past rail defects. Four techniques of variable selection were evaluated and the simple regression model was chosen for the prediction of rail breaks as it was found to fit best with the test data. While the cumulative tonnage itself well explains the breaks, consideration of other factors that also correlate with the breaks will improve the accuracy of predictions. For example, considering the rail characteristics is important as crack propagation per fatigue cycle is greater in lighter rail than in heavier rail. Inspection data are helpful in increasing the accuracy of predicting rail breaks, as the repair of detected cracks limits the growth of cracks and increases the useful rail life. So, the later model could better predict rail breaks than could the former.

Although several studies have been conducted to predict rail breaks, most of these studies did not have complete rail, inspection, and maintenance data. Both the studies of Tyler Dick et al. (*17*) and Schafer and Barkan (*5*) had included some, but only rail data for a very short range of about 2–4 years. The studies by Orringer and colleagues considered 10 years of data (*21, 22*). However, their research was based on the mechanical properties of the rail rather than statistical models.

Survival analysis models have been widely used in the literature on highway accidents to estimate the time to event of interest (*23, 24*). Although Sadeghi and Askarinejad implemented survival analysis to assess the dynamic derailment risk (*3*), to the best of our knowledge no research has been conducted to-date to estimate the risk of reoccurrence of rail defects using survival analysis and, more specifically, frailty models.

Research Methodology

The recurrence of rail defects is usually caused by complex interactions between different factors. Most often, lack of a comprehensive method to process rail defects and difficulty in associating these with other data sources is a major challenge for researchers. To overcome this challenge, we have designed a logical methodology framework for data collection, processing, and analysis of our research. Figure 3 shows this framework and its four main components, which we have used for our study.

Data Description

Data Preprocessing

Defects-related data include rail defects and geometry defects obtained from a North American Class I railroad. Rail defects comprise the main dataset, which is the basis for any later data processing, data fusion, and analysis in our research. The features of the rail defect information include date and location of occurrence, type, size of the defect, and the type of remedial action taken for the defect. From the literature, we know that existing geometry defects might influence the occurrence of rail defects (*5*), so it is assumed that it would have the same effect on the recurrence of defects as well. What we are mostly interested in from the track geometry defect data is the number of geometry defects that occurred in the three years after a rail defect was found.

Infrastructure-related datasets mainly include three major static datasets: features related to the rail curves, grades of the rail track, and rail layout over the entire rail network, as shown in Figure 3. For traffic-related data, the dataset related to the traffic (i.e., tonnage) moved across the network during the years of study were collected. The maintenance dataset mainly includes the history of the ultrasonic testing for rail defects during the aforementioned years. Moreover, the rail patch dataset includes the information on the replacement of rail sections along the network.

As mentioned in the previous section, in the second stage of the methodology we aim to prepare a unique dataset consisting of all corresponding important attributes from different data sources. To handle this, we have devised the following procedure:

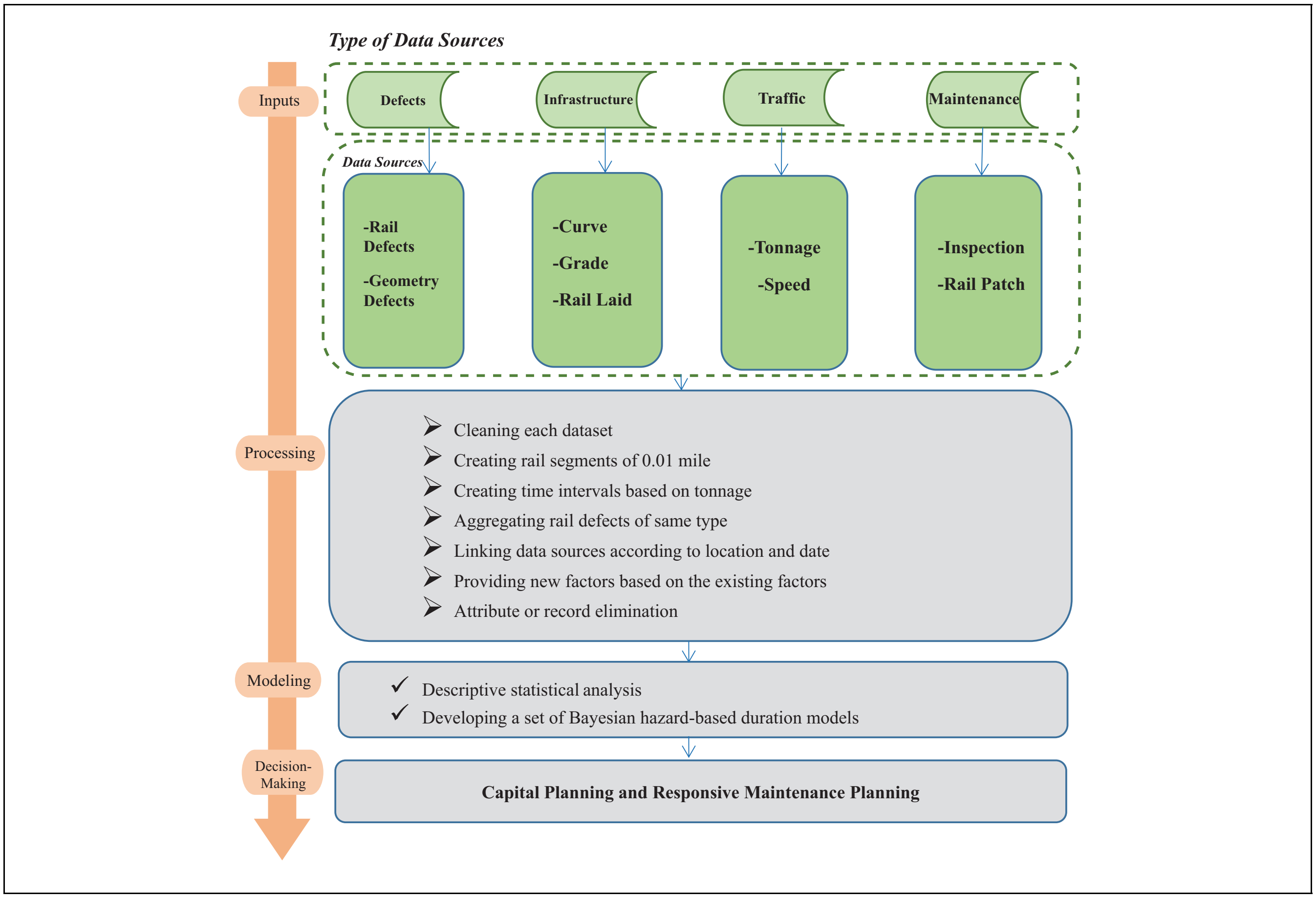

Figure 3. The methodology framework for this study.

- The spatial coordinates for a rail defect are defined by track number, position within a curved track (i.e., whether it is the low or high side of the curve), direction of the rail with respect to the direction of travel (i.e., right or left), and milepost location. To generate consistent spatial units and accommodate different modeling purposes, we have divided the rail network into segments of length 0.01 mi. Dividing the network into short sections for data analysis purposes has been used extensively in the literature (*3, 5*).
- Then we created tonnage intervals, or the cumulative tonnage between two consecutive rail defects of the same type on each segment. If a defect occurs n_i times in the same segment (i), tonnage intervals should be calculated as many as (n_i + 1) times for that defect. The first intervals include the accumulated tonnage, from the beginning of the study horizon (February 2011) until the first occurrence of the defect. The (n_i + 1) tonnage intervals include the calculated tonnage load from the occurrence of the last defect until the end of the study horizon. In this case, it is said that the defect occurrence at this interval is censored (and therefore it is not observed) (*25*).
- Three new variables were created based on the tonnage interval. The first one is *interval_number*, which shows the interval number for each defect (1, 2, …, n_i + 1). The second is *status*, which is 1 when the tonnage interval is associated with a defect occurrence, and 0 if censored. The third is defect *ID*, which would be the same for different intervals of each defect to distinguish the intervals of the same segment from other segments. An example of the tonnage intervals as well as the created new variables based on those intervals in the dataset is provided in Figure 4. Figure 5 also depicts the recurrence of defects and censoring time for three sample segments.
- Considering the features of spatial units for rail defects as well as its temporal condition (i.e., date of occurrence of the defect), all the other aforementioned datasets are linked with the rail defect dataset.

Prefix	Track Type	Milepost	Defect Type	ID	Interval_Number	Time_from	Time_to	Status	Start_Tonn	End_Tonn	Tonn_Gap
0	1	390.89	TDD	351	1	1/3/2011	2/12/2015	1	0	798768707	798768707
0	1	390.89	TDD	351	2	2/12/2015	5/14/2015	1	798768707	873199155	74430447.9
0	1	390.89	TDD	351	3	5/14/2015	5/14/2015	1	873199155	875232759	2033604.1
0	1	390.89	TDD	351	4	5/14/2015	12/30/2016	0	875232759	1221722965	346490206
00C	SG	249.14	CH	7791	1	1/3/2011	8/8/2011	1	0	18186815	18186815
00C	SG	249.14	CH	7791	2	8/8/2011	10/13/2011	1	18186815	25629468	7442652.99
00C	SG	249.14	CH	7791	3	10/13/2011	12/13/2011	1	25629468	35059404.5	9429936.53
00C	SG	249.14	CH	7791	4	12/13/2011	2/17/2012	1	35059404.5	43349418.8	8290014.27
00C	SG	249.14	CH	7791	5	2/17/2012	12/30/2016	0	43349418.8	240865279	197515860
0	1	2.023	SD	101	1	1/3/2011	9/9/2016	1	0	75235419	75235419
0	1	2.024	SD	101	2	9/9/2016	10/7/2016	1	75235419	77930334.7	2694915.71
0	1	2.024	SD	101	3	10/7/2016	12/30/2016	0	77930334.7	81864102.3	3933767.58

Figure 4. A sample view of the data restructuring based on tonnage interval.

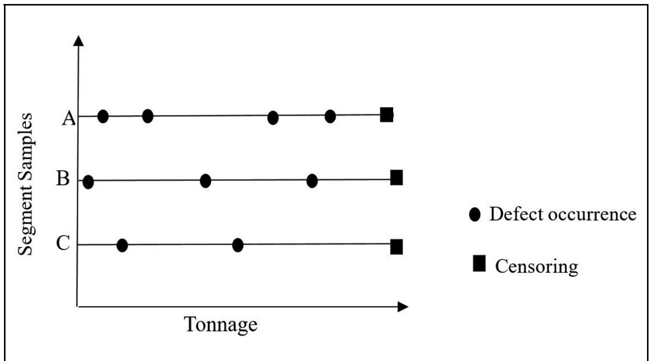

Figure 5. Three examples of segments with recurrent rail defects; Segments A, B, and C have four, three, and two occurrences of defects before being censored, respectively.

- By constructing a new integrated data framework, new useful features (such as "frequency of geometry defects in the last three years") could be created by other attributes to be used in the modeling process. Moreover, some attributes do not need to be included in the model and they could be removed from the final dataset.
- Record elimination was used according to the needs of our study. As an example, we have removed all the observations with the "non-replacement" remedy action as we are only interested in the analysis of defects which have been replaced.

Considering all the mentioned steps, an integrated dataset including four groups of variables (defects, infrastructure, traffic, and remedial action taken) was captured for our modeling purposes.

Descriptive Statistical Analysis

As mentioned before, our models are based on field data of 21,000 mi of rail network from a North American Class I railroad, including six years of rail defect-related data, infrastructure-related data, traffic data, and maintenance-related data, as presented in Table 1. As most traffic is carried by mainline tracks, we focus our analysis on mainline tracks in this study. Table 1 presents variable descriptions for attributes in the dataset.

Table 1. Variable Description

Attribute	Explanation
Def_Type	Type of defects (such as Bolt Hole Break [BHB], Detail Fracture [TDD], and so forth)
Def_Size	Size of defect
Geo_Def	Number of geometry defects in the last three years
Remedy_Type	Type of remedy action taken to treat the defect
Def_Remedy_Gap	Number of days between the date of defect found and the date it is treated
Freight_Speed	Freight train speed at the location of the defect
Def_Freq	The frequency of defects that occurred in the same location
Inspection_Freq	Number of inspections in the last three years
Curve/Tang	Whether the occurred defect is on the low side of a curve, high side of a curve, or on tangents
Weight	The weight of rail at the location of the defect
MGT	The tonnage of freight (in MGT) at the location of the defect

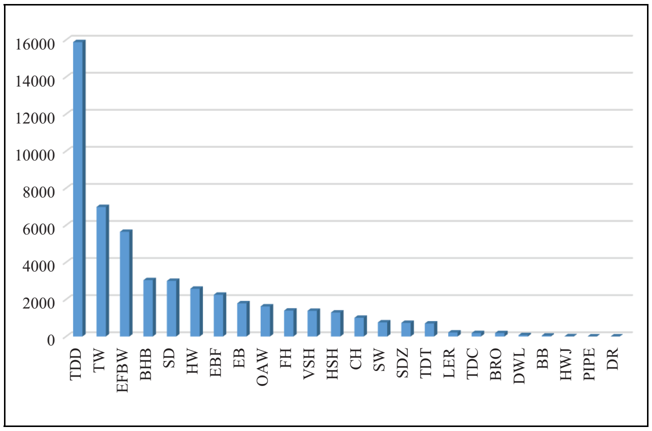

Figure 6. Different types of defects in a US Class I railroad.
Note: y-axis = number of defects; x-axis = defect type.

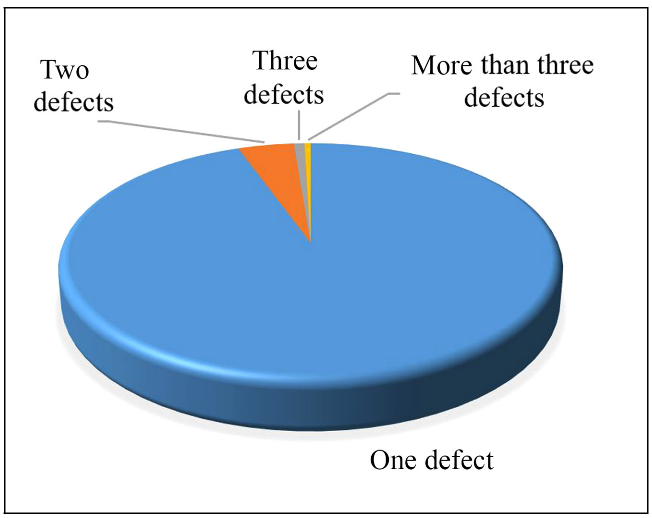

Figure 7. The proportion of recurrent defects versus non-recurrent defects.

Type of rail defect is one important characteristic of the rail defect dataset. Some of the most frequent types of defects in the dataset under study, including TDD (detail fracture), TW (thermite weld), EFBW (electric flash butt weld [plant weld]), SSC (shells, spalling, or corrugation), SD (shelling), are depicted in Figure 6.

According to Figure 7, the majority of rail segments (95%) in our study have only one defect, meaning the recurrent-defect segments account for only 5% of the total. Some summary statistics for the variables in two separate groups corresponding to recurrent defects (occurred more than once) and non-recurrent defects (occurred only once) are presented in Table 2.

Figure 8 depicts the location of recurrent and non-recurrent defects on the rail. As seen in this figure, for both groups the majority of defects occur on the tangent. For the curves, defects have occurred more frequently on the high side of the curve rather than its low side.

Modeling Approach

Correlated event times have been mainly studied in the context of health and medical sciences (*13, 25–28*). However, the nature of railway assets means this problem could be easily extended to the health and monitoring of railway assets. Rail defects refer to the ill-conditioned parameters of rail structure components for which remedial actions could be taken. However, these remedial actions are not always able to treat defects completely and there is still a chance for a defect to appear multiple times in the same location. For these repeated defects, which we would refer to as recurrent rail defects, the correlation between times of occurrence of the same defect at the same location may arise from two distinct sources:

1. *Heterogeneity across track segments*: Some segments of a rail track have a higher or lower rate of rail defect than other segments because of the unknown or unmeasurable effects. Rail segments might have different features, material traits, and so forth, which influence the likelihood that they will be subject to additional defect development, but this is either unknown or cannot be measured easily. This introduces heterogeneity across the observations and generates within-subject

Table 2. Summary Statistics for Numerical Variables

	Min.		Max.		Avg.	
Attribute/group of defects	Recurrent	Non-recurrent	Recurrent	Non-recurrent	Recurrent	Non-recurrent
Freight train speed	20	10	60	60	33.06	41.76
Number of geometry defects in the last three years	0	0	4	24	0.080	0.167
Rail weight	115	75	141	155	134.1	129.5

Note: Min. = minimum; Max. = maximum; Avg. = average.

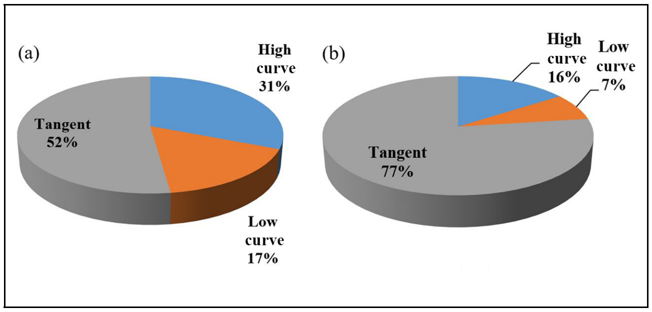

Figure 8. The location of defects for each category of defects: (*a*) Recurrent defects and (*b*) Non-recurrent defects.

correlation in the occurrence and timing of recurrent defects within a specific track segment.

2. Occurrence dependence: The occurrence of one defect may make further defects of the same type more or less likely. This occurrence dependence may be produced by a weakening effect or strengthening effect. Either of these implies that the risk for a rail defect is a function of the occurrence of previous defects. This also creates a within-subject correlation.

Our work here attempts to address the effects of heterogeneity and occurrence dependence for recurrent rail defects. The proportional hazards model (*29*) and its extensions have been widely used to model correlated events in the context of medical sciences; however, no studies could be found for survival analysis of recurrent events in railway studies. Moreover, we are using this model in the context of Bayesian analysis to make a more realistic configuration of the distribution of the parameters used in the model.

As previously mentioned, the correlation between defect occurrence gap times violates the Cox model's assumption that the timing of defects is independent. This makes the standard Cox model biased and inefficient for modeling recurrent defects (*13, 25*). Consequently, a variation of the Cox model, known as the frailty Cox model, has been proposed for estimation with recurrent events to account for the correlation (*13*).

Frailty or random effects models integrate heterogeneity into the estimator by making assumptions about the frailty distribution and incorporating it into the model estimates. The underlying logic of frailty models comes from the fact that some rail segments are intrinsically more or less prone to experiencing the defects of a specific type than are others, and that the distribution of these effects can be at least approximated. In this case, there is only a single individual for each value of the random effect so it is said to be shared over time by a single individual. Since the hazard is necessarily positive, the distribution of random effects is usually selected from the positive class of distributions, including gamma, Gaussian, and t distributions.

Shared Frailty Model Specification

As mentioned before, most survival analysis studies have been implemented under the assumption that recurrent events are independent and identically distributed. However, Cox proportional hazard models cannot be used because of the dependence of data when data come from multiple records which actually belong to the same location (i.e., the same rail segment in our study) but at a different time.

Modeling dependence on recurrent survival data has received considerable attention recently. The main development in modeling this kind of data is to consider frailty models, in which the data are considered to be conditionally independent. When frailties are considered, the dependence between recurrent rail defects can be considered as an unknown and unobservable risk factor (or explanatory variable) of the hazard function.

In the case of the proportional hazard frailty model, the hazard rate of a segment is given by

$$h(t) = h_0(t)e^{\beta' X + Z_i} \tag{1}$$

where X is a vector covariate; $\beta = (\beta_1, \beta_2, \ldots, \beta_p)'$ is a vector of regression parameters to be estimated; Z_i is an unobserved frailty for the ith segment; and $h_0(t)$ is the baseline hazard function, which has been assumed to be similar for all the segments in the study population, respectively. For the purpose of this paper, we have assumed a Weibull distribution with parameters μ and γ for the baseline hazard function. The second parameter γ allows great flexibility in the model and different shapes of the hazard function. The respective baseline hazard function, survival function, and the density in the case of a Weibull distribution are given by the following equations (*30*):

$$h(t) = \mu\gamma t^{\gamma-1} \tag{2}$$

$$S(t) = e^{-\mu t^{\gamma}} \tag{3}$$

$$f(t) = \mu\lambda t^{\gamma-1} e^{-\mu t^{\gamma}}, \mu>0, \gamma>0 \tag{4}$$

Let's consider right-censored survival data (t_{ij}, δ_{ij}) in which $i = 1, 2, \ldots, n$ corresponds to the individuals and $j = 1, 2, \ldots, m$ is related to the interval number for each segment and t_{ij} is the gap time (accumulative tonnage here) for interval j of the segment i. δ_{ij} is the indicator variable taking value 1 if the jth interval for the ith segment corresponds to the occurrence of a rail defect and value 0 otherwise. In other words, t_{ij} is related to a defect

Table 3. Posterior Inference of Regression Coefficients and Frailty Variance

Variables	Mean	Median	SD	95% CI lower	95% CI upper
Weight of rail	−0.0006	−0.0005	0.0026	−0.0058	0.0044
Frequency of geometry defects	0.0123	0.0124	0.0082	−0.0036	0.0283
Location of rail (L)	−0.0320	−0.0306	0.0527	−0.1426	0.0689
Location of rail (T)	−0.0551	−0.0546	0.0320	−0.1187	0.0083
Freight speed limit	0.0014	0.0015	0.0012	−0.0010	0.0038
Type of defect (Detail Fracture [TDD])	0.1693	0.1691	0.0674	0.0369	0.3074
Type of defect (Thermite Weld [TW])	0.1999	0.1994	0.0721	0.0603	0.3370
Type of defect (Vertical Split Head [VSH])	0.2956	0.2976	0.1202	0.0549	0.5222
Posterior inference of frailty variance	**0.491**	**0.271**	**0. 040**	**0.006**	**0.891**

occurrence if $\delta_{ij} = 1$ and it is a censoring time if $\delta_{ij} = 0$. Therefore, the triplet (t_{ij}, δ_{ij}, x_{ij}) is observed for all occurrences of defects in all segments. Given the unobserved frailty z_i, t_{ij} are independent. Therefore, the complete data likelihoods for the proportional hazard (PH) model under the frailty approach is given by (*30*):

$$L_{\mathrm{PHFM}} = \prod_{i=1}^{n} \left[\mu\gamma t_i^{\gamma-1} e^{\beta' x + z_i}\right]^{\delta_i} e^{-\mu t^{\gamma} e^{\beta' x + z_i}} \tag{5}$$

In the case of frailty models, the most important thing is to assign an appropriate probability distribution to the frailty variable. Several researchers have used different distributions for this purpose. Following the concept of using a normal prior probability distribution (or, simply, "prior" from this point forward) for frailty variable by Ruktiari et al. and Zhou and Hanson (*31, 32*), in this paper we consider an independent normal frailty prior which is defined as $z_1, \ldots, z_m \sim N(0, \tau^2)$. The density of the frailty variable, in this case, would be given by (*30*):

$$f(Z) = \frac{1}{\tau\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{Z}{\tau}\right)^2}; \ -\infty < Z < \infty, \tau > 0 \tag{6}$$

Prior Specification

The wide acceptability of gamma distribution as a conjugate prior in Bayesian inference means a gamma prior is considered for τ and therefore $\tau^{-2} \sim \Gamma(a_\tau, b_\tau)$. In line with other researchers (*33–35*), we consider a normal prior for the regression parameters as $\beta \sim N(0,m)$. Owing to the flexibility and simplicity of the gamma distribution (*33, 36*), the hyper parameters of the baseline hazard function are considered with a gamma prior; therefore it is assumed that $\mu \sim \Gamma(\rho, \rho)$ and $\gamma \sim \Gamma(a, b)$.

Posterior Calculation and Parameter Configuration

The joint posterior distribution for all the parameters of the PH model would be

$$P_{\mathrm{PH}} = \prod_{i=1}^{n} \left[\mu\gamma t_i^{\gamma-1} e^{\beta X + Z_i}\right]^{\delta_i} e^{-\mu t^{\gamma} e^{\beta X + Z_i}} \pi(z)\pi(\mu)\pi(\gamma)\pi(\beta)\pi(\tau^2) \tag{7}$$

where π (.) is the corresponding prior distribution (*30*).

To obtain the data likelihood of the various parameters, we have to integrate z_i with the independent gamma prior density, which was discussed earlier. The form of the likelihood of data after integration is too complicated to work with. Consequently, it is not easy to analytically evaluate the marginal posterior distributions. To overcome this difficulty, we resorted to Monte Carlo integration, which draws samples from the defined distribution and then forms sample averages to approximate expectations. In MCMC these samples are drawn by running a cleverly constructed Markov chain for a long time. There are plenty of ways of constructing these chains. One of the simplest and most popular MCMC sampling algorithms found in the Bayesian computational literature is the Gibbs sampler. The literature corresponding to the MCMC approach by using a Gibbs sampler is too vast to be listed here (*37*). In this paper, the method is used to integrate over the posterior distribution of model parameters, given the data to make inference for the desired model parameters.

At this point, the MCMC calculations have been implemented using R software version 3.4.3. A burn-in period of 5000 iterates was considered and the Markov chain was subsampled to get a final chain size of 4000 iterations.

Results and Discussions from Bayesian Frailty Model

Following the convergence diagnostics (see (*38*)), we consider the following posterior distribution summaries in Table 3. Statistics summaries include the parameters' posterior distribution mean, standard deviation, MC error, and the 95% highest posterior distribution density interval.

In Table 3, the negative sign of the mean of a variable means that the variable is associated negatively with the hazard function, that is, lower risk of recurrence of defect to reoccur in the same location.

One important infrastructure-related variable is "weight of rail," which according to the model is associated with lower risk of recurrence of the defects. This is expected as usually the crack propagation per fatigue cycle is greater in the lighter rail than in the heavier rail. Another defect-related variable that negatively affects the survival time (and therefore positively affects the risk of the recurrence of a defect on the same location), is the number of geo defects in the last three years. This finding agrees with previous studies that considered the significant covariates on the occurrence of the first defect (*39*).

The negative signs for the location of rail (L, T) means that defects that occur on the low side of the rail (L) and tangents (T) have a lower risk for recurrence compared to the high side of the curve. In other words, the MGT interval for a defect of the same type to reoccur on the tangent (or low side of the curve) of a segment is longer compared to the high side of the curve on the same segment.

The results also depict that the "freight train speed limit" is also positively associated with the risk of recurrent defects. This could be mainly referred to the higher speeds on the track being associated with stronger dynamic forces on the rail, which increases the probability of the recurrence of the rail defect in the same location.

Moreover, according to the presented results of our model, defects of types Detail fracture, Termite Weld, and Vertical Slit Head are negatively correlated with the survival function of defect type SD, indicating that the presence of each of these types of defects increases the risk of recurrence compared to defect type SD. We do not present all types of defect in Table 3.

The posterior inference of frailty variance is likely to be around 0.5, which means there is a positive relationship between the occurrences of defects on the same rail segment; in other words, there is strong posterior evidence of a high degree of heterogeneity in the population of segments. Some segments are more prone to shorter times until the recurrence of rail defects than are others.

The set of plots in Figure 9 shows the trace plots of some of the parameters for the fitted model. According to these trace plots, the priori distribution is well calibrated, which is indicated by the parameters having sufficient state changes as the MCMC algorithm runs.

The set of plots in Figure 10 shows the posterior density plots of the parameters of the fitted model. As is obvious from this figure, the estimates of the posterior marginal distribution for all the coefficients have smooth and unimodal shapes.

Model Diagnostics

After the model fitting, the next task is to check the goodness of fit of the model. This is mainly because the model fitting is only according to a certain set of assumptions. Utilizing regression diagnostics procedures employed to evaluate the model assumptions, we investigate the existence of observations with a large, undue influence on the analysis. In regression analysis, residuals have a very powerful impact on diagnostic checking procedures (*35*). In the case of survival analysis, where we are faced with the problem of censored data, special consideration must be given to the residuals of the censored observations. The idea of one such kind of residual plot was given by Cox and Snell (*40*), where they showed that a plot of estimated cumulative hazard function (based on Cox and Snell's residual and the censored data) versus the Cox and Snell residual is able to check the overall goodness of fit in survival models. We evaluated the presumed relationship of unit exponentially distributed residuals for a good model fit. This is done graphically with the graphs of the Cox–Snell residual and formally using the Kolmogorov–Smirnov goodness-of-fit test. It is observed that residuals from a correctly fitted model follow a unit exponential distribution. For the above fitted frailty model, the Cox–Snell plot is given in Figure 11, where it can be seen that the data fit the proposed models quite well.

Conclusion

In this paper, a PH model has been developed by using a parametric Weibull baseline hazard function under the frailty approach in the context of a Bayesian mechanism. The model is fitted by real-life survival datasets related to rail defects, and diagnostics checking is conducted using the Cox–Snell plot. It is observed that the frequency of the geometry defects on rail, the high side of the curves, higher freight train speeds, and lighter-weight rail are all associated with higher risk of recurrence of defects of the same type at the same location. Moreover, compared to defect type SD, certain types of defect, including TDD, TW, and VSH, are significantly associated with higher risk of rail defect recurrences.

As the recurrence of rail defects is not a desirable event, the railroad should focus on reducing the risk of defect recurrence by minimizing the impacts of these covariates. It is suggested to take additional caution on the rail segments having higher values for these variables. It is recommended to record the data on lighter segments of rail as well as on the high side of the curves on rail segments to be more certain of the risk of defect recurrence. The impact of past geometry defects can be reduced by minimizing their occurrence either by accurately

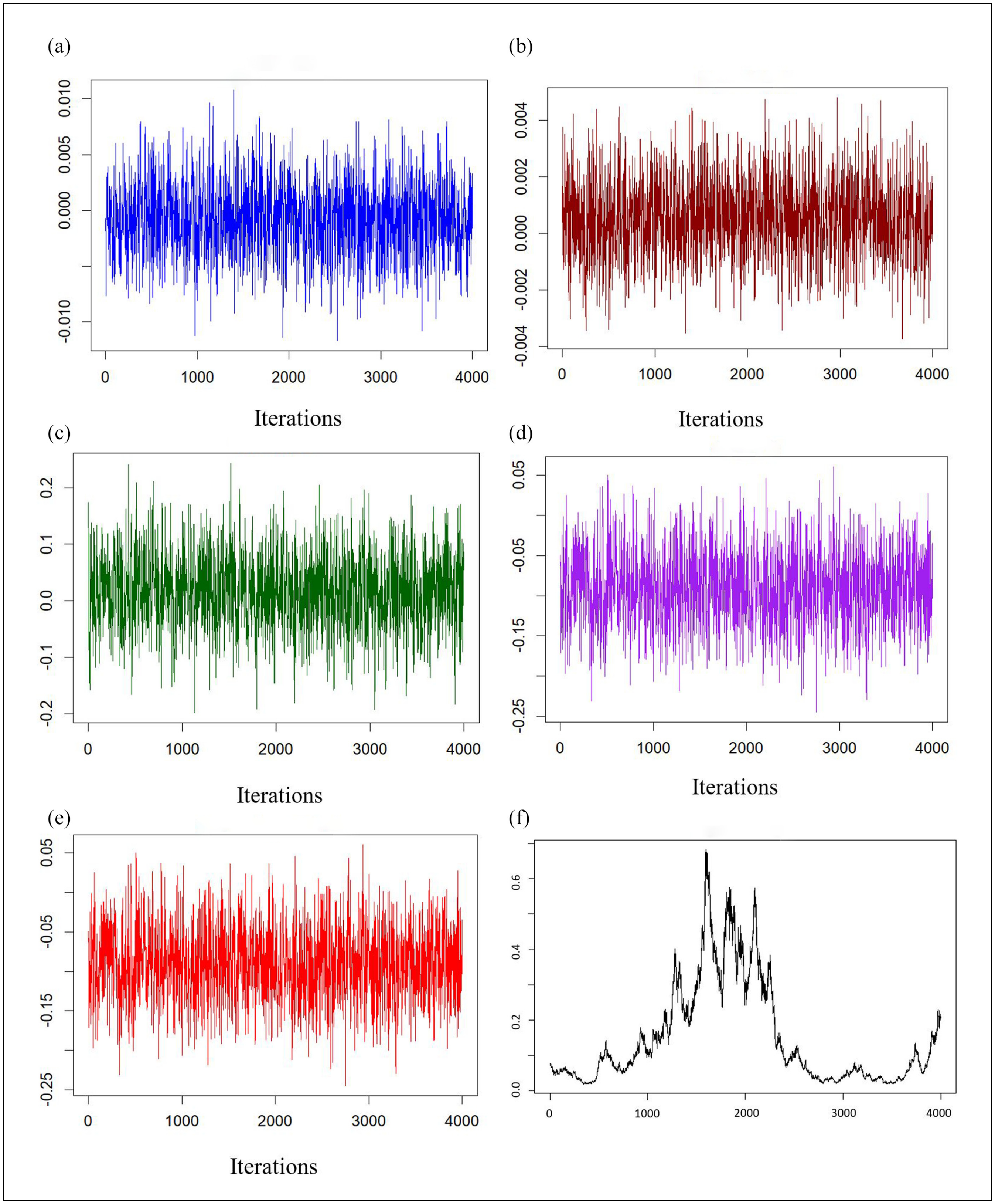

Figure 9. Trace plots for the regression coefficients and frailty variance for the PH frailty model: (*a*) weight of rail, (*b*) speed limit, (*c*) left curve, (*d*) tangent, (e) geometry defects, and (f) frailty variance.

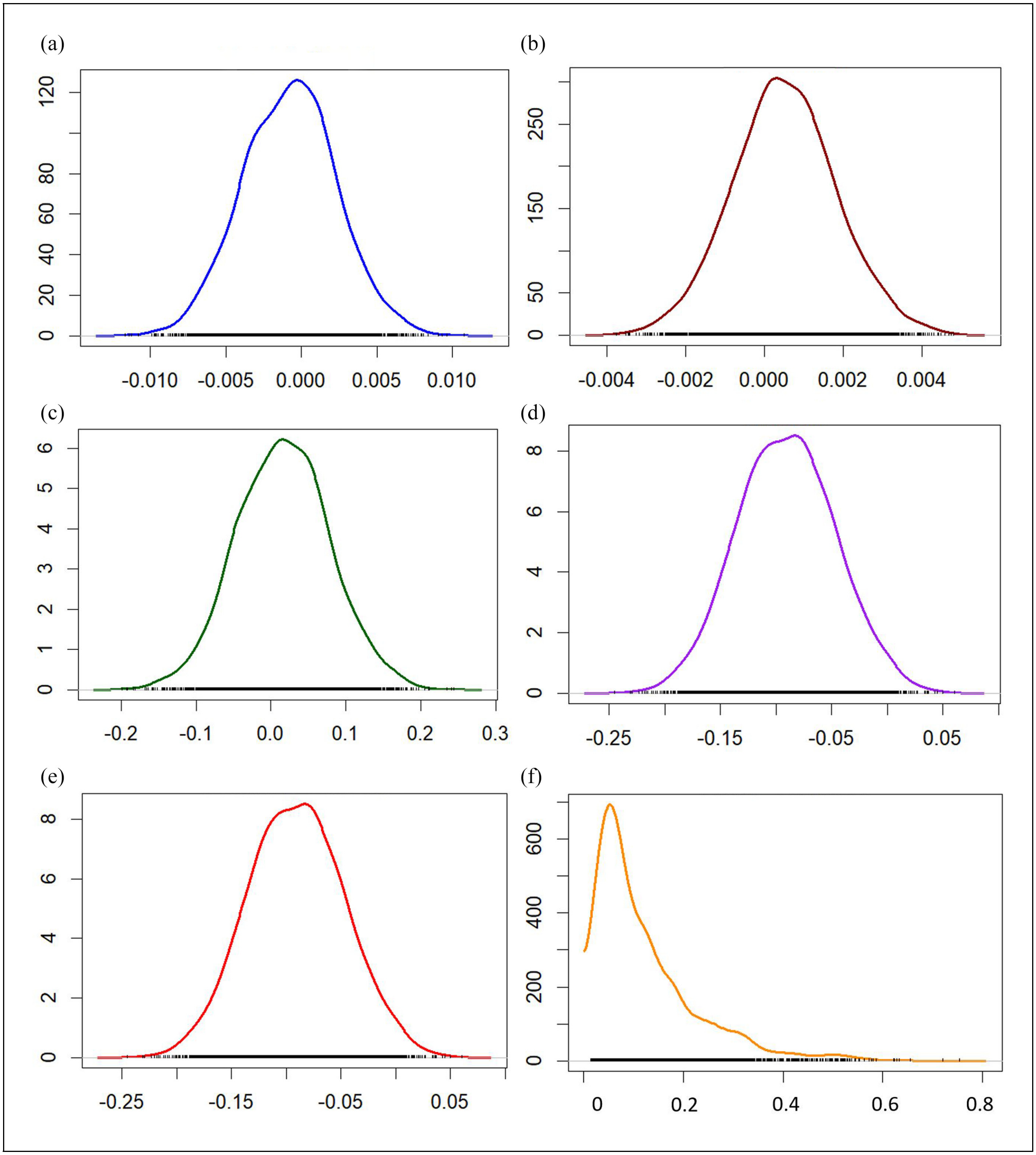

Figure 10. Density plots for posterior distribution of the regression coefficients and frailty variance for the PH frailty model: (*a*) weight of rail, (*b*) speed limit, (*c*) left curve, (*d*) tangent, (*e*) geometry defects, and (*f*) frailty variance.

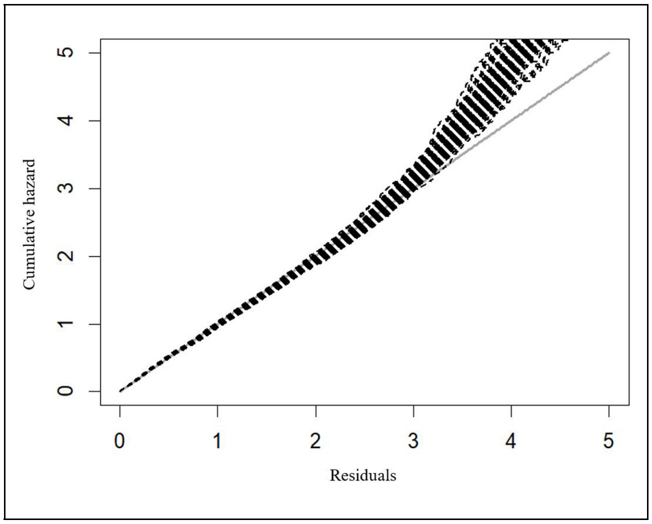

Figure 11. Cox–Snell plot for the fitted model.

predicting the defects or by decreasing the inspection intervals. The impact of the speed can be reduced by decreasing the speed limit on certain segments. This information will assist the railroad in the decision-making related to prioritizing track segments for maintenance actions to mitigate the negative impacts of these factors. The rail segments with a higher risk of frequent rail defects should be treated more frequently.

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Author Contributions

The authors confirm contribution to the paper as follows: study conception and design: FG, QH; data collection: QH, AA; analysis and interpretation of results: FG, RM, AP; draft manuscript preparation: FG, RM, AP. All authors reviewed the results and approved the final version of the manuscript.

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