A Computer Simulation of the Hodgkin & Huxley Membrane Patch Model

NENS 230 - Example final project

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1 Introduction

The Hodgkin and Huxley model, first described in 1952, is a mathematical model that describes how action potentials in neurons are initiated and propagated. In this project, a computer model of a patch of membrane in a neuron was simulated according to the Hodgkin Huxley equations governing membrane kinetics. Thus, the response of the membrane to various stimuli could be observed via the computer simulation.

2 Methods

The hodgkin-huxley model describes a set of four differential equations that govern the ion currents and membrane voltage of a patch of cell membrane. Using Matlab, these equations were approximately solved using a foward Euler approximation. A simple example of a simulated action potential is given in Figure 1.

Using this model, we can then explore how the model responds to different stimulus protocols. The results of these experiments are discussed below.

3 Files

1. `hhmodel.m` - Main script that generates figures
2. `hhsim.m` - function that simulates the membrane potential given an input current pulse
3. `pulsegen.m` - generates a current / stimulus pulse given some input parameters (size and length of the pulse)

4 Results

4.1 Membrane Voltage and Stimulus Current

Given a sufficiently large current pulse, the membrane patch generates an action potential. A plot of the membrane potential v_m and stimulus current i_{stim} as a function of time is located in Figure 1.

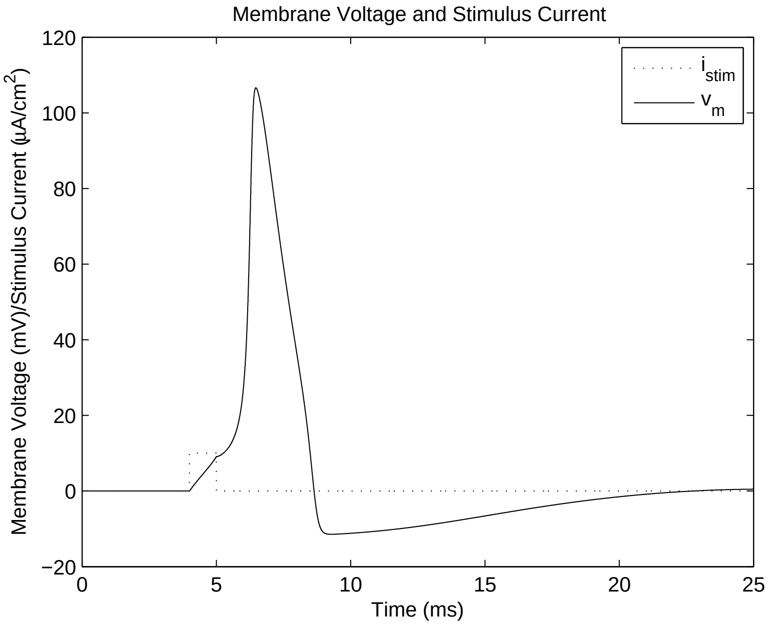

Figure 1: Membrane Voltage and Stimulus Current

4.2 Ion Conductances

The plot of g_{Na} and g_K as a function of time is located in Figure 2.

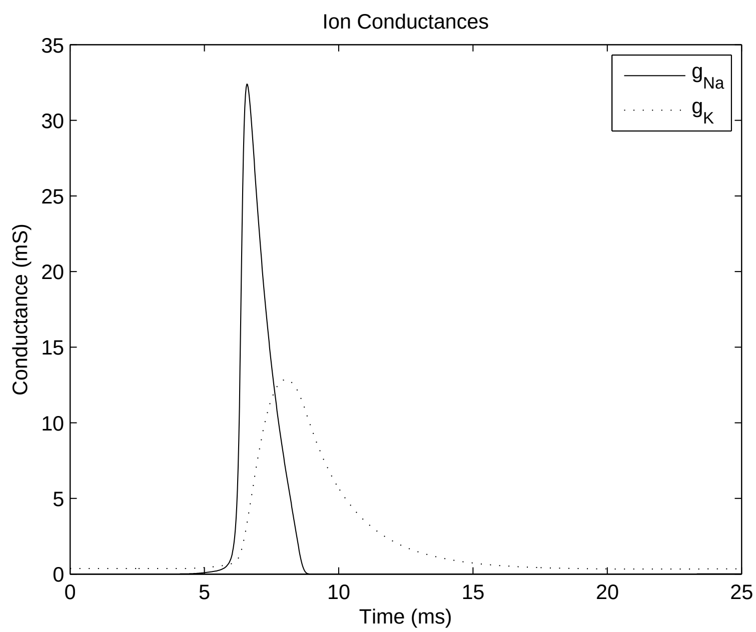

Figure 2: Ion Conductances

4.3 Ion Currents

The plot of i_{Na} and i_K as a function of time is located in Figure 3.

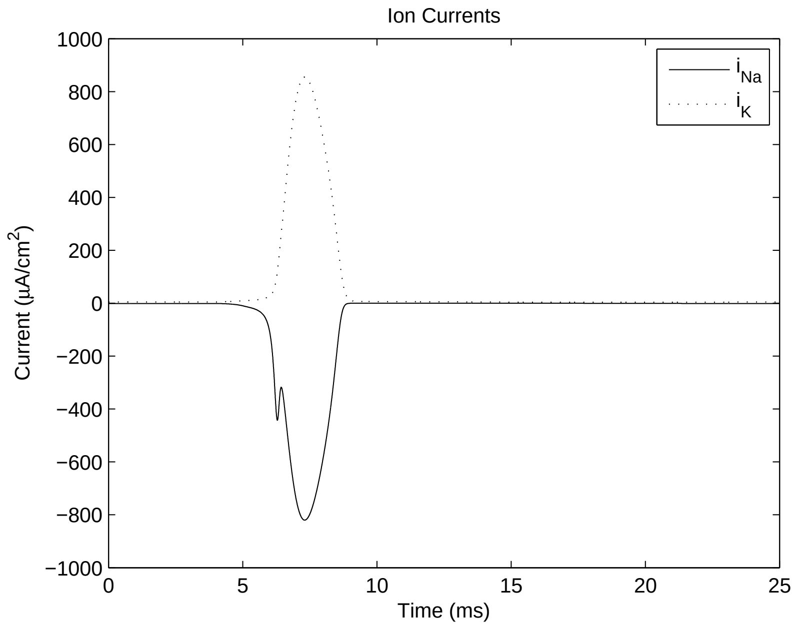

Figure 3: Ion Currents

4.4 Threshold and Strength-Duration Relationship

The minimum stimulus required to cause an action potential was found for various stimulus durations. The stimulus thresholds determined are located in Table 1. As you can see, as the stimulus duration increases, a smaller stimulus threshold is required to cause an action potential.

Stimulus Duration (ms)	Stimulus Threshold ($\frac{\mu A}{cm^2}$)
0.1	64
0.25	26
0.5	13
3	3

Table 1: Stimulus Threshold for Various Stimulus Durations

4.5 Multiple Stimuli

The `hhmodel.m` script was modified to apply a second current stimulus 15 milliseconds after the original was applied. In this case, the amplitude of the first stimulus was fixed at $15\frac{\mu A}{cm^2}$. The threshold of the second stimulus needed to produce the second action potential was found to be $\mathbf{20\frac{\mu A}{cm^2}}$. The plot of the two action potentials is located in Figure 4.

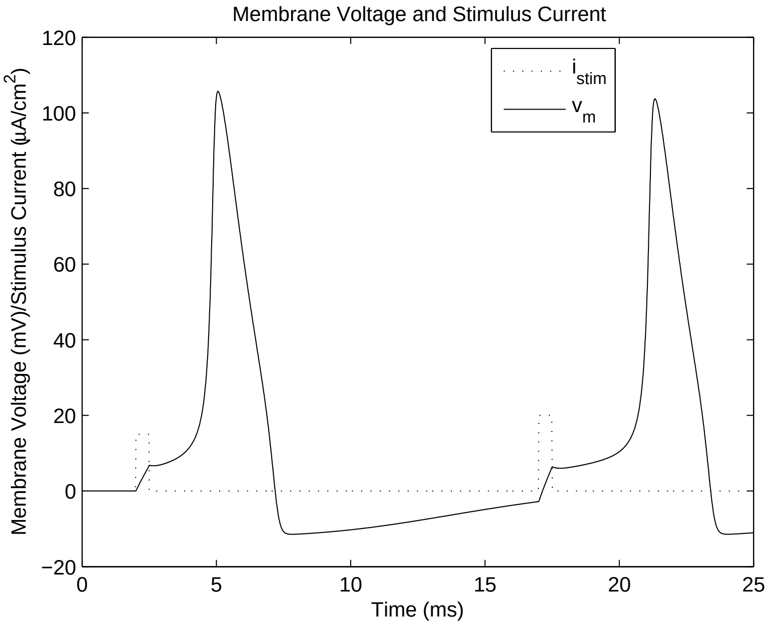

Figure 4: Dual Action Potentials

4.6 Threshold vs. Stimulus Delay

The minimum stimulus required to cause the second action potential was found for various delays between the two square pulses. The stimulus thresholds determined are located in Table 2. As you can see, as the delay between the stimuli decreases, a larger stimulus threshold is required to cause an action potential. When the delay is **7ms**, it takes a second stimulus whose strength is greater than 10x the original stimulus strength to cause the second action potential.

Stimulus Delay (ms)	Stimulus Threshold ($\frac{\mu A}{cm^2}$)
15	20
12	40
10	68
8	138
7	229

Table 2: Stimulus Threshold for Various Stimulus Delays

5 Conclusions

This project studied the behavior of a patch of membrane undergoing Hodgkin Huxley kinetics. Using Matlab, I was able to simulate a set of differential equations governing the behavior of membrane potential and ionic currents.

This model let me investigate the relationship between a the strength and length of a current stimulus pulse and the threshold required to generate an action potential. Furthermore, I examined the effect of the refractory period of the membrane on the threshold required to generate two action potentials in quick succession.