First CMB Constraints on Dark Matter Effective Theory

Kimberly K. Boddy

Department of Physics & Astronomy, University of Hawaii, Honolulu, HI 96822, USA

Vera Gluscevic

School of Natural Sciences, Institute for Advanced Study, Einstein Drive, Princeton, NJ 08540, USA

Abstract.

I. INTRODUCTION

Various probes: astrophysical, cosmological, and laboratory–based, are engaged in an effort to constrain allowed interactions between dark matter (DM) and the Standard Model, and to guide DM particle theory towards the right underlying description of the dark matter sector. The most sensitive constraints typically come from direct searches for rare collisions between DM particles from the local Galactic halo and atomic nuclei in low–background underground detectors **??**. Nuclear recoil measurements from these experiments have already excluded large portions of the relevant parameter space, especially for DM particle masses above 1 GeV or so **??**. Data from new generation of direct detection (DD) experiments is forthcoming from a number of projects worldwide **??**. Experimental targets based on liquid noble gases and those based on germanium crystals promise to deliver another order of magnitude in sensitivity beyond the current state-of-the-art limits **??**.

However, information about DM gained from DD measeuments is subject to caveats. As has been pointed out by a number of previous studies, conversion of the observed nuclear recoil rate into a limit on DM–baryon interaction cross section relies on a detailed knowledge of DM astrophysical parameters—energy density and velocity distribution—on very small scales **??**; unfortunatelly, the only way to measure these parameters reliably is through DD experiments themselves. In particular, in presence of local dark structures (dark streams or disks), the common assumption of Maxwell–Boltzmann phase space distribution can severely break down **??**, misleading the interpretation of the null–signal status of DD experiments. Furthermore, due to the small relative velocity of DM particles in the local halo, the nuclear–recoil analysis is typically only sensitive to DM particle masses larger than a GeV or so. Measurements below this limit rely on tracking down DM–induced electronic recoils, which is a far more involved process of disentangling spurious radiogenic backgrounds, and is only beginning to be used in DM searches **??**.

Alternative probes can thus provide valuable complementary information about DM interactions, sidestepping some of the caveats of DD probes. In paticular, recent proliferation of precision cosmological measurements provides a testing ground for the very same interaction physics in the early universe In this study, we focus on constraining interactions with baryons using cosmic microwave background (CMB) temperature and polarization power spectra.

Previous studies have placed constraints on DM–baryon interactions by solely considering CMB temperature measurements **??**; in particular, the most recent such analysis of Ref. **??** is only valid for heavy particles with masses much greater than the proton mass **??**. In this study, we will extend this work to include the latest temperature, as well as polarization, measurements from the Planck sattelite **??**, and constrain DM particle masses down to an MeV—going three orders of magnitude below the reach of nuclear recoil measurements in DD experiments. Furthermore, in order to make contact with limits from DD, we will provide the very first constraint on complete DM effective theory (EFT). Hereby, we adopt the EFT approach in order to generalize our constraints to be relevant for any possible low–energy description of spin 1/2, 0, and 1 DM particles. By doing so, we will fully address the allowed modeling space for DM particles that interact with the Standard Model in a theoretically well–framed and motivated manner.

In Sec. II we review the nonrelativisitic effective theory of dark matter interactions with baryons. In Sec. III we revise how the Boltzmann formalism in presence of DM–baryon interactions and describe how we incorporate these effects into the CLASS Boltzmann code. In Sec. IV we show current constraints using the latest Planck 2015 data, and in Sec. V, we give projections for Stage–4 CMB experiment. In Sec. VI we discuss caveats, future avenues of investigation, and conclude.

II. DARK MATTER EFFECTIVE FIELD THEORY

The effective field theory (EFT) for dark matter interactions with nucleons enables us to systematically describe processes relevant for direct detection experiments [1, 2]. For a given contact interaction, we can construct the corresponding Galilean-invariant operators $\hat{\mathcal{O}}$ in the nonrelativistic limit. Furthermore, the operator is Hermitian if it is constructed from the following quantities [1]:

$$\frac{i\vec{q}}{m_N},\ \vec{v}^{\perp},\ \vec{S}_\chi,\ \vec{S}_N\ , \tag{1}$$

where $\vec{q}$ is the momentum transfer, m_N is the nucleon mass, and $\vec{S}_\chi$ is the dark matter spin, and $\vec{S}_N$ is the nucleon spin. We define

$$\vec{v}^{\perp} \equiv \vec{v} + \frac{\vec{q}}{2\mu_N}\ , \tag{2}$$

where $\vec{v}$ is the relative velocity of the dark matter and nucleon and μ_N is the reduced mass of the dark matter-nucleon system. Energy conservation requires $\vec{v}^{\perp} \cdot \vec{q} = 0$, and thus $|\vec{v}^{\perp}|^2 = v^2 + |\vec{q}|^2/4\mu_N^2$, where $v^2 = |\vec{v}|^2$. To extend the discussion to dark matter scattering on some target nucleus, we instead use the quantity $|\vec{v}_T^{\perp}|^2 = v^2 + |\vec{q}|^2/4\mu_T^2$ in the rest frame of the target nucleus, and μ_T is the reduced mass of the dark matter-nucleus system.

Following Ref. [1, 2], we consider dark matter-nucleon interactions of the form[1]

$$\hat{\mathcal{H}} = \sum_{\tau=0}^{1}\sum_{i=1}^{15} c_i^\tau \hat{\mathcal{O}}_i^\tau t^\tau\ , \tag{3}$$

where i labels the structure of the interaction and τ labels isospin. The isospin operators are $t^0 \equiv 1$ and $t^1 \equiv \tau_3$, and we recover the couplings to protons and neutrons via $c_i^{(p)} = c_i^0 + c_i^1$ and $c_i^{(n)} = c_i^0 - c_i^1$. Dark matter interactions with nuclei are assumed to be the sum of interactions with the individual nucleons. We assume the nuclear wave functions are of the standard shell model form, where the underlying single-particle basis is the harmoic oscillator with a parametric size

$$b = \sqrt{41.467/(45A^{-1/3} - 25A^{-2/3})}\ \mathrm{fm}\ . \tag{4}$$

Consequently, we may express the amplitude for scattering as a sum over products of the dark matter response functions, $R_k^{\tau\tau'}(\vec{v}_T^{\perp 2}, |\vec{q}|^2/m_N^2, \{c_i^\tau c_j^{\tau'}\})$, and nuclear response functions, $W_k^{\tau\tau'}(y)$, where $y = (|\vec{q}|b/2)^2$. The types of responses are $k = \{M, \Sigma'', \Sigma', \Phi'', \Phi''M, \tilde{\Phi}', \Delta, \Delta\Sigma'\}$. For convenience, we define $\tilde{W}_k^{\tau\tau'}(y) = 4\pi W_k^{\tau\tau'}(y)/(2J_N+1)$, where J_N is the spin of the nucleus. We thus write the spin-averaged amplitude squared as

$$\begin{aligned}\left\langle|\mathcal{M}|^2\right\rangle &= \frac{1}{2J_X+1}\ \frac{1}{2J_N+1}\ |\mathcal{M}|^2 \\ &= \sum_{\tau,\tau'}\sum_k \left(\frac{|\vec{q}|^2}{m_N^2}\right)^{\delta_k} R_k^{\tau\tau'}\left(\vec{v}_T^{\perp 2}, \frac{|\vec{q}|^2}{m_N^2}, \{c_i^\tau c_j^{\tau'}\}\right)\tilde{W}_k^{\tau\tau'}(y)\end{aligned} \tag{5}$$

where $\delta_k = 0$ for $k = \{M, \Sigma'', \Sigma'\}$ and $\delta_k = 1$ for $k = \{\Phi'', \Phi''M, \tilde{\Phi}', \Delta, \Delta\Sigma'\}$. The differential cross section is

$$\frac{d\sigma}{d|\vec{q}|^2} = \frac{1}{4\pi v^2}\left\langle|\mathcal{M}|^2\right\rangle\ . \tag{6}$$

By integrating over all possible values of momentum transfers, from 0 to $|\vec{q}|^2_{\max} = 4\mu_T^2 v^2$, we obtain the total cross section for dark matter-nucleus scattering. However, in the cosmological context, dark matter-baryon scattering has the most significant impact for large momentum transfers. In Section III the relevant collision term in the Boltzmann equations involves the momentum-transfer cross section (or the transfer cross section, for short), which weights the differential cross section by a factor of $(1-\cos\theta)$ to preferentially pick out scattering processes with large momentum transfer:

$$\sigma_T(v) = \frac{1}{4\pi v^2}\int_0^{|\vec{q}|^2_{\max}} d|\vec{q}|^2\,(1-\cos\theta)\left\langle|\mathcal{M}|^2\right\rangle = \frac{1}{4\pi v^2}\int_0^{|\vec{q}|^2_{\max}} d|\vec{q}|^2\,\frac{|\vec{q}|^2}{2\mu_T^2 v^2}\left\langle|\mathcal{M}|^2\right\rangle\ , \tag{7}$$

where we have used the relation between the momentum transfer and center-of-mass dark matter scattering angle θ: $|\vec{q}|^2 = 2\mu_T^2 v^2(1-\cos\theta)$. In the following subsection, we describe the procedure for algorithmically implementing this calculation into CLASS.

[1] Although we write the sum over 15 operators, the operator $\mathcal{O}_2$ is omitted by enforcing its coupling to be zero [1, 2].

A. Transfer Cross Section

The transfer cross section is a function of the relative velocity v, and it is important to isolate the terms with various v dependencies in order to perform a velocity average in Section III. In order to simplify notation, we defined a new variable $z \equiv |\vec{q}|^2/|\vec{q}|^2_{\rm max} = |\vec{q}|^2/(4\mu_T^2 v^2)$. Thus, we also have $|\vec{v}_T^{\perp}|^2 = v^2(1+z)$ and $y = zv^2(\mu_T b)^2$. The transfer cross section in Eq. (7) becomes

$$\sigma_T(v) = \frac{2\mu_T^2}{\pi} \int_0^1 dz\, z \left\langle |\mathcal{M}|^2 \right\rangle \ . \tag{8}$$

The calculation of $\sigma_T(v)$ amounts to weighting the integrand of $\sigma(v)$ by a factor of $2z$. By parametrizing the cross section in terms of v and z instead of $|\vec{v}_T^{\perp}|^2$ and $|\vec{q}|^2$, we would like to express a given term in the transfer cross section schematically as $\sigma_0 v^n$.

Let us break $\left\langle |\mathcal{M}|^2 \right\rangle$ into individual terms that depend only on z and v. Each dark matter response function has a numeric coefficient, associated with the couplings from operators $\mathcal{O}_i$ and $\mathcal{O}_j$, multiplying powers of $|\vec{v}|_T^{\perp 2}$ and $(|\vec{q}|/m_N)^2 = zv^2(2\mu_T/m_N)^2$:

$$\begin{aligned} R_k^{\tau\tau'} &= \sum_{i,j=0}^{15} c_i^\tau c_j^{\tau'} R_{k,ij} \left(|\vec{v}|_T^{\perp 2}\right)^{\alpha_{k,ij}} \left(\frac{|\vec{q}|^2}{m_N^2}\right)^{\beta_{k,ij}} \\ &= \sum_{i,j=0}^{15} c_i^\tau c_j^{\tau'} R_{k,ij} \left(\frac{2\mu_T}{m_N}\right)^{2\beta_{k,ij}} v^{2\alpha_{k,ij}+2\beta_{k,ij}} (1+z)^{\alpha_{k,ij}} z^{\beta_{k,ij}} \ , \end{aligned} \tag{9}$$

where the last line contains a choose function that appears from the binomial expansion of $(1+z)^{\alpha_{k,ij}}$. The coupling coefficients c_i^τ have a mass dimension of -2, and we set the mass scale to be at the weak scale $m_v^2 = (2G_F)^{-1}$. A different normalization of $1/\Lambda^2$ may be used by setting an overall coupling scale $\bar{c} = m_v^2/\Lambda^2$. Furthermore, we allow the possiblity for these couplings to be Wilson coefficients from a relativistic theory, such as those in Ref. [3], multiplying powers of $|\vec{q}|^2/m_N^2$. Thus, we have

$$c_i^\tau = \frac{\bar{c}}{m_v^2} \sum_\gamma c_{i,\gamma}^\tau z^\gamma v^{2\gamma} \left(\frac{2\mu_T}{m_N}\right)^{2\gamma} \ . \tag{10}$$

The nuclear response function for a given baryonic element is a function of a polynomial of y times e^{-2y} (except for hydrogen, for which there is no exponential factor):

$$\begin{aligned} \tilde{W}_k^{\tau\tau'} &= e^{-2y} \sum_\rho \tilde{W}_{k,\rho}^{\tau\tau'} y^\rho = e^{-zv^2(\mu_T b)^2} \sum_\rho \tilde{W}_{k,\rho}^{\tau\tau'} z^\rho v^{2\rho} (\mu_T b)^{2\rho} \\ &= e^{-zv^2(\mu_T b)^2} \sum_\rho \tilde{W}_{k,\rho}^{\tau\tau'} z^\rho v^{2\rho} \left(\frac{2\mu_T}{m_N}\right)^{2\rho} \left(\frac{m_N b}{2}\right)^{2\rho} \ . \end{aligned} \tag{11}$$

Thus, we may write the transfer cross section as

$$\begin{aligned} \sigma_T(v) = \frac{2\mu_T^2 \bar{c}^2}{\pi m_v^4} &\sum_k \sum_{\tau,\tau'=0}^{1} \sum_{i,j=0}^{15} \sum_\gamma \sum_\rho c_{i,\gamma}^\tau c_{j,\gamma}^{\tau'} R_{k,ij} \tilde{W}_{k,\rho}^{\tau\tau'} \left(\frac{m_N b}{2}\right)^{2\rho} \\ &\times \left(\frac{2\mu_T}{m_N}\right)^{2(\beta_{k,ij}+\delta_k+\gamma+\gamma'+\rho)} v^{2(\alpha_{k,ij}+\beta_{k,ij}+\delta_k+\gamma+\gamma'+\rho)} \mathcal{I}(v^2; \beta_{k,ij}, \delta_k, \gamma, \gamma', \rho) \end{aligned} \tag{12}$$

where

$$\mathcal{I}(v^2; \beta_{k,ij}, \delta_k, \gamma, \gamma', \rho) = \int_0^1 dz\, e^{-zv^2(\mu_T b)^2} z^{1+\beta_{k,ij}+\delta_k+\gamma+\gamma'+\rho} (1+z)^{\alpha_{k,ij}} \ . \tag{13}$$

For hydrogen, there is no exponential factor in the nuclear response function, so we can obtain the correct result by setting $b = 0$. Although we can express $\mathcal{I}$ in terms of an incomplete Gamma function, but it will be easier to leave it in this form when we average over velocities in Section III.

To make contact with direct detection constraints, we constrain the cross section (not transfer cross section) for scattering on protons. We may use the expression in Eq. (12), recalling that the transfer cross section is obtained by incorporating an additional $2z$ in the integrand of the normal cross section. For hydrogen, the nuclear response functions are constant ($\rho = 0$), and the only responses involved are $k = \{M, \Sigma'', \Sigma'\}$ (thus, $\delta_k = 0$). Furthermore, the nuclear responses for hydrogen do not depend on the isospin labels. We do not incorporate any additional factors of $|\vec{q}|^2/m_N^2$ in front of the coupling coefficients ($\gamma = \gamma' = 0$). Thus, the hydrogen/proton cross section is

$$\begin{aligned}
\sigma_p(v) &= \frac{\mu_p^2 \bar{c}^2}{\pi m_v^4} \sum_{k \in M, \Sigma'', \Sigma'} \sum_{\tau, \tau'} \sum_{ij} c_i^\tau c_j^{\tau'} R_{k,ij} \tilde{W}_k^{\tau\tau'} \left(\frac{2\mu_p}{m_N}\right)^{2\beta_{k,ij}} v^{2(\alpha_{k,ij}+\beta_{k,ij})} \int_0^1 dz\, z^{\beta_{k,ij}} (1+z)^{\alpha_{k,ij}} \\
&= \frac{\mu_p^2 \bar{c}^2}{\pi m_v^4} \sum_{k \in M, \Sigma'', \Sigma'} \sum_{ij} c_i^{(p)} c_j^{(p)} R_{k,ij} \frac{4\pi}{2J_N + 1} W_k \left(\frac{2\mu_p}{m_N}\right)^{2\beta_{k,ij}} v^{2(\alpha_{k,ij}+\beta_{k,ij})} \int_0^1 dz\, z^{\beta_{k,ij}} (1+z)^{\alpha_{k,ij}} \\
&= \frac{\mu_p^2 \bar{c}^2}{\pi m_v^4} \sum_{k \in M, \Sigma'', \Sigma'} \sum_{ij} c_i^{(p)} c_j^{(p)} R_{k,ij} \frac{4\pi W_k}{2} \left(\frac{2\mu_p}{m_N}\right)^{2\beta_{k,ij}} v^{2(\alpha_{k,ij}+\beta_{k,ij})} \int_0^1 dz\, z^{\beta_{k,ij}} (1+z)^{\alpha_{k,ij}} \; .
\end{aligned} \tag{14}$$

III. BOLTZMANN EQUATIONS

A. Density Perturbations

We work on Fourier space to express the evolution of the dark matter and baryon density fluctuations, δ_χ and δ_b, and their velocity divergences, θ_χ and θ_b as

$$\begin{aligned}
\dot{\delta}_\chi &= -\theta_\chi - \frac{\dot{h}}{2} \\
\dot{\delta}_b &= -\theta_b - \frac{\dot{h}}{2} \\
\dot{\theta}_\chi &= -\frac{\dot{a}}{a}\theta_\chi + c_\chi^2 k^2 \delta_\chi + R_\chi(\theta_b - \theta_\chi) \\
\dot{\theta}_b &= -\frac{\dot{a}}{a}\theta_b + c_b^2 k^2 \delta_b + R_\gamma(\theta_\gamma - \theta_b) + \frac{\rho_\chi}{\rho_b} R_\chi(\theta_\chi - \theta_b) \; ,
\end{aligned} \tag{15}$$

where k is the wave number of a given Fourier mode. The rate coefficient R arises from scattering processes that change the momenta (or equivalently velocity in the nonrelativistic limit) of the particles, resulting in a drag force that affects the evolution of θ. The standard term $R_\gamma = (4/3)(\rho_\gamma/\rho_b) a n_e \sigma_T$ is associated with Compton scattering, and there is a similar term R_χ that arises from dark matter-baryon scattering.

For a single dark matter-baryon collision, the dark matter momentum changes by [4]

$$|\Delta \vec{p}_\chi| = \frac{m_\chi m_B}{m_\chi + m_B} |\vec{v}_\chi - \vec{v}_B| \left(\hat{n} - \frac{\vec{v}_\chi - \vec{v}_B}{|\vec{v}_\chi - \vec{v}_B|} \right) \; , \tag{16}$$

where m_B is the baryon mass, $\hat{n}$ is the direction of the scattered dark matter particle in the center-of-mass frame, $\vec{v}_\chi$ is the dark matter velocity, and $\vec{v}_B$ is the baryon velocity. The relative velocity is $\vec{v} = \vec{v}_\chi - \vec{v}_B$. Note that quantities labeled with a capital "B" indicate a particular baryonic species and not the baryon fluid as a whole. The overall effect is obtained by integrating the momentum transfer per collision over all collisions [5], with the phase space distribution functions

$$f_\chi(\vec{v}_\chi) = n_\chi \frac{g_\chi}{(2\pi)^{3/2} \bar{v}_\chi^{3/2}} \exp\left[-(\vec{v}_\chi - \vec{V}_\chi)^2 / (2\bar{v}_\chi^2) \right] \tag{17}$$

$$f_B(\vec{v}_B) = n_B \frac{g_B}{(2\pi)^{3/2} \bar{v}_B^{3/2}} \exp\left[-(\vec{v}_B - \vec{V}_b)^2 / (2\bar{v}_B^2) \right] \; , \tag{18}$$

where $\vec{V}_\chi$ and $\vec{V}_b$ are the dark matter and baryon peculiar velocities and $\bar{v}_\chi^2 = T_\chi/m_\chi$ and $\bar{v}_B^2 = T_b/m_B$ are the dark matter and baryon velocity dispersions. The internal degrees of freedom are simply $g_\chi = 2J_\chi + 1$ and $g_B = 2J_B + 1$, where J_χ and J_b are the intrinsic spins. The resulting drag force per unit mass, or drag acceleration, on the dark matter fluid is given by

$$\frac{d\vec{V}_\chi}{dt} = -\frac{g_B g_\chi}{m_\chi}\int d^3v_\chi d^3v_B f_\chi(\vec{v}_\chi) f_B(\vec{v}_B)\int \frac{d\sigma}{d\Omega}|\vec{v}_\chi - \vec{v}_B||\Delta\vec{p}_\chi|$$
$$= -\frac{g_B g_\chi \rho_B}{m_\chi + m_B}\frac{1}{(2\pi)^{3/2}}\frac{1}{(\bar{v}_B^2+\bar{v}_\chi^2)^{3/2}}\int d^3v\,\vec{v}\left[\sigma_T^{(B)}(v)v\right]\exp\left[-\frac{(\vec{v}-(\vec{V}_\chi-\vec{V}_b))^2}{2(\bar{v}_B^2+\bar{v}_\chi^2)}\right] , \tag{19}$$

where we write a superscript on $\sigma_T^{(B)}$ to indicate that this cross section is for a particular baryonic element. We mimic the form of a single term in the transfer cross section in Eq. (12), while dropping subscripts for clarity, to write $\sigma_T^{(B)}(v) \to \sigma_n^{(B)} v^n \mathcal{I}(v^2)$, where $n = 2(\alpha+\beta+\delta+\gamma+\gamma'+\rho)$ and $\sigma_n^{(B)}$ is a constant with respect to v. Furthermore, we take the limit $(\vec{V}_\chi - \vec{V}_b)^2 \ll (\bar{v}_\chi^2 + \bar{v}_B^2)$, and the acceleration reduces to

$$\frac{d\vec{V}_\chi}{dt} = -(\vec{V}_\chi - \vec{V}_b)\frac{g_B g_\chi \rho_B \sigma_n c_n}{m_\chi + m_B}(\bar{v}_B^2+\bar{v}_\chi^2)^{(n+1)/2}\int_0^1 dz\,(1+z)^\alpha z^{1-\alpha+n/2}\left[1+4\mu_B^2 b^2(\bar{v}_B^2+\bar{v}_\chi^2)z\right]^{-(3+n/2)} , \tag{20}$$

where

$$c_n = \frac{2^{\frac{n+5}{2}}\Gamma\left(3+\frac{n}{2}\right)}{3\sqrt{\pi}} . \tag{21}$$

The remaining integral may be expressed as a sum, and the rate coefficient is

$$R_\chi = a\rho_b \sum_B \sum_n \frac{f_B g_B g_\chi}{m_\chi + m_B}\sigma_n^{(B)} c_n \left(\frac{T_b}{m_B}+\frac{T_\chi}{m_\chi}\right)^{\frac{n+1}{2}} \sum_{p=0}^{\alpha}\binom{\alpha}{p}\left(2+\frac{n}{2}-\alpha+p\right)^{-1}$$
$$\times\left[1-(2\mu_B b)^2\left(\frac{T_b}{m_B}+\frac{T_\chi}{m_\chi}\right)\right]^p\left[1+(2\mu_B b)^2\left(\frac{T_b}{m_B}+\frac{T_\chi}{m_\chi}\right)\right]^{-(2+n/2-\alpha+p)} , \tag{22}$$

where f_B is the mass fraction of the baryon B.

B. Temperature Evolution

The evolution equations for the dark matter and baryon temperatures are

$$\dot{T}_\chi = -2\frac{\dot{a}}{a}T_\chi + 2R'_\chi(T_b - T_\chi)$$
$$\dot{T}_b = -2\frac{\dot{a}}{a}T_b + \frac{2\mu_b}{m_\chi}\frac{\rho_\chi}{\rho_b}R'_\chi(T_\chi - T_b) + \frac{2\mu_b}{m_e}R_\gamma(T_\gamma - T_b) , \tag{23}$$

where $\mu_b \approx m_{\rm H}(n_{\rm H} + 4n_{\rm He})/(n_{\rm H} + n_{\rm He} + n_e)$ is the mean molecular weight of the baryons, and the rate coefficient R'_χ is the same as Eq. (22), except that there is an additional factor of $m_\chi/(m_\chi + m_B)$.

IV. CONSTRAINTS FROM PLANCK

V. FORECASTS FOR STAGE–4 EXPERIMENT

VI. DISCUSSION AND CONCLUSIONS

Appendix A: Response Functions

Element	J_N	k	τ	τ'	ρ	$W_{k,\rho}^{\tau\tau'}$
Hydrogen-1	1/2	M	0	0	0	2
		M	0	1	0	2
		M	1	0	0	2
		M	1	1	0	2
		Σ''	0	0	0	2
		Σ''	0	1	0	2
		Σ''	1	0	0	2
		Σ''	1	1	0	2
		Σ'	0	0	0	4
		Σ'	0	1	0	4
		Σ'	1	0	0	4
		Σ'	1	1	0	4
Helium-4	0	M	0	0	0	16

TABLE I: Terms in the nuclear response functions. The column k labels the response type, τ and τ' label the isospins, ρ labels the power of y, and $W_{k,\rho}^{\tau\tau'}$ labels the numerical coefficient. These numbers are from Ref. [6], except that the values of $W_{k,\rho}^{\tau\tau'}$ are a factor of 4 larger here, due to difference in changing between the proton-neutron flavor basis and the isospin basis.

k	i	j	$\alpha_{k,ij}$	$\beta_{k,ij}$	$R^{\tau\tau'}_{k,ij}$
M	1	1	0	0	1
M	5	5	1	1	$J_X(J_X+1)/3$
M	8	8	1	0	$J_X(J_X+1)/3$
M	11	11	0	1	$J_X(J_X+1)/3$
Φ''	3	3	0	1	$1/4$
Φ''	12	12	0	0	$J_X(J_X+1)/12$
Φ''	15	15	0	2	$J_X(J_X+1)/12$
Φ''	12	15	0	1	$-J_X(J_X+1)/12$
Φ''	15	12	0	1	$-J_X(J_X+1)/12$
$\Phi''M$	3	1	0	0	1
$\Phi''M$	12	11	0	0	$J_X(J_X+1)/3$
$\Phi''M$	15	11	0	1	$-J_X(J_X+1)/3$
$\tilde{\Phi}'$	12	12	0	0	$J_X(J_X+1)/12$
$\tilde{\Phi}'$	13	13	0	1	$J_X(J_X+1)/12$
Σ''	10	10	0	1	$1/4$
Σ''	4	4	0	0	$J_X(J_X+1)/12$
Σ''	4	6	0	1	$J_X(J_X+1)/12$
Σ''	6	4	0	1	$J_X(J_X+1)/12$
Σ''	6	6	0	2	$J_X(J_X+1)/12$
Σ''	12	12	1	0	$J_X(J_X+1)/12$
Σ''	13	13	1	1	$J_X(J_X+1)/12$
Σ'	3	3	1	1	$1/8$
Σ'	7	7	1	0	$1/8$
Σ'	4	4	0	0	$J_X(J_X+1)/12$
Σ'	9	9	0	1	$J_X(J_X+1)/12$
Σ'	12	12	1	0	$J_X(J_X+1)/24$
Σ'	15	15	1	2	$J_X(J_X+1)/24$
Σ'	12	15	1	1	$-J_X(J_X+1)/24$
Σ'	15	12	1	1	$-J_X(J_X+1)/24$
Σ'	14	14	1	1	$J_X(J_X+1)/24$
Δ	5	5	0	1	$J_X(J_X+1)/3$
Δ	8	8	0	0	$J_X(J_X+1)/3$
$\Delta\Sigma'$	5	4	0	0	$J_X(J_X+1)/3$
$\Delta\Sigma'$	8	9	0	0	$-J_X(J_X+1)/3$

TABLE II: Terms in the dark matter response functions. The column k labels the response type, i and j label the operators associated with the coupling coefficients, $\alpha_{k,ij}$ labels the powers of $|\vec{v}|_T^{\perp 2}$, $\beta_{k,ij}$ labels the powers of $|\vec{q}|^2/m_N^2$, and $R^{\tau\tau'}_{k,ij}$ labels the numerical coefficient. These numbers are from Ref. [2]

[1] A. L. Fitzpatrick, W. Haxton, E. Katz, N. Lubbers, and Y. Xu, JCAP **2**, 004 (2013), 1203.3542.
[2] N. Anand, A. L. Fitzpatrick, and W. Haxton, Phys. Rev. C **89**, 065501 (2014), 1308.6288.
[3] V. Gluscevic, M. I. Gresham, S. D. McDermott, A. H. G. Peter, and K. M. Zurek, JCAP **1512**, 057 (2015), 1506.04454.
[4] C. Dvorkin, K. Blum, and M. Kamionkowski, Phys. Rev. D **89**, 023519 (2014), 1311.2937.
[5] K. Sigurdson, M. Doran, A. Kurylov, R. R. Caldwell, and M. Kamionkowski, Phys. Rev. D **70**, 083501 (2004), [Erratum: Phys. Rev.D73,089903(2006)], astro-ph/0406355.
[6] R. Catena and B. Schwabe, JCAP **4**, 042 (2015), 1501.03729.