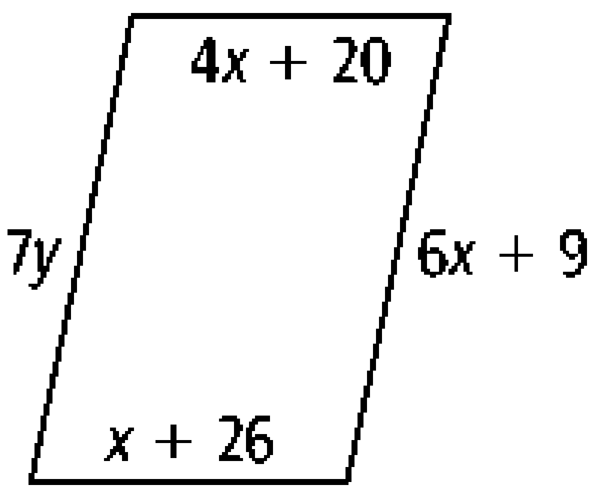
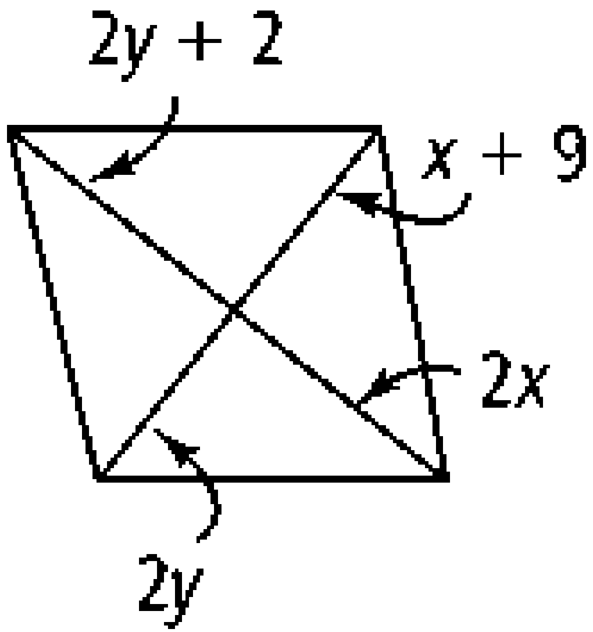
G3b Conditions for Parallelograms Coordinate Proofs

SKETCH THE PICTURE and solve.
For what values of *x* and *y* must each figure be a parallelogram?

1.



2.

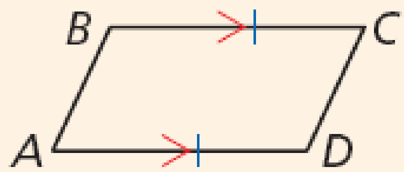
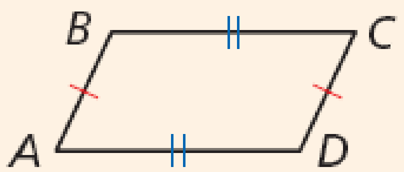


G3a Conditions for Parallelograms

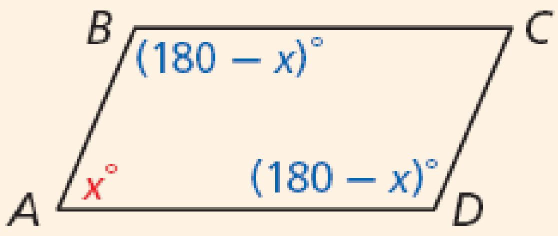
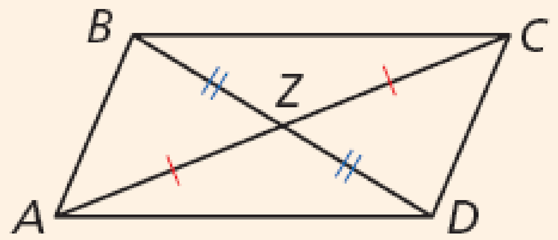
Objective: We will prove that a given quadrilateral is a parallelogram.

Homework: Page 402-403: 6-7, 14-15, 26, 27 (write as a two column proof)

Theorems Conditions for Parallelograms

	THEOREM	EXAMPLE
6-3-1	If one pair of opposite sides of a quadrilateral are parallel and congruent, then the quadrilateral is a parallelogram. (quad. with pair of opp. sides $\parallel$ and $\cong$ $\rightarrow$ ▱)	
6-3-2	If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram. (quad. with opp. sides $\cong$ $\rightarrow$ ▱)	
6-3-3	If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram. (quad. with opp. ∠s $\cong$ $\rightarrow$ ▱)	

Theorems — Conditions for Parallelograms

THEOREM		EXAMPLE
6-3-4	If an angle of a quadrilateral is supplementary to both of its consecutive angles, then the quadrilateral is a parallelogram. (quad. with ∠ supp. to cons. ⦟s → ▱)	
6-3-5	If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. (quad. with diags. bisecting each other → ▱)	

Show that quadrilateral *ABCD is a parallelogram by using the* definition.

$$A(-3,2), B(-2,7), C(2,4), D(1,-1)$$

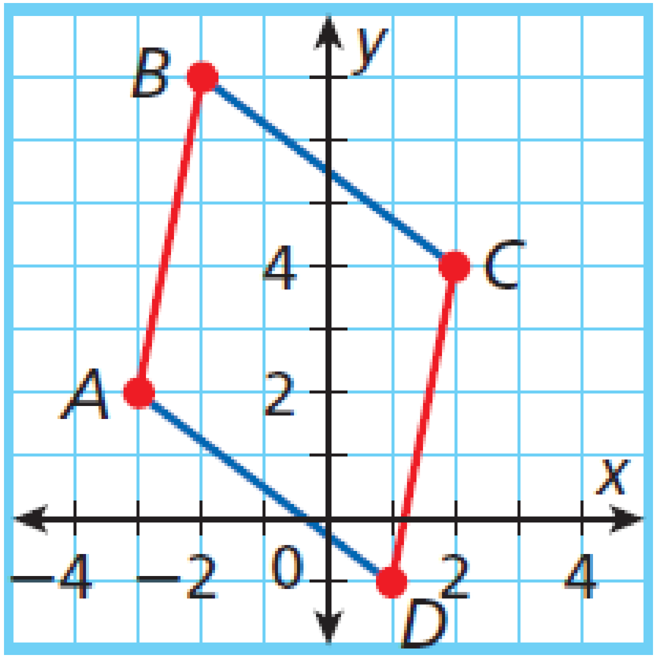
slope of $\overline{AB} = \dfrac{7-2}{-2-(-3)} = \dfrac{5}{1} = 5$

slope of $\overline{CD} = \dfrac{-1-4}{1-2} = \dfrac{-5}{-1} = 5$

slope of $\overline{BC} = \dfrac{4-7}{2-(-2)} = \dfrac{-3}{4} = -\dfrac{3}{4}$

slope of $\overline{DA} = \dfrac{2-(-1)}{-3-1} = \dfrac{3}{-4} = -\dfrac{3}{4}$

Since both pairs of opposite sides are parallel, *ABCD* is a parallelogram by definition.



Show that quadrilateral *FGHJ is a parallelogram by using the Theorem 6-3-1*. *If one pair of opposite sides of a quadrilateral is parallel and congruent, then the quadrilateral is a parallelogram.*

$$F(-4,-2), G(-2,2), H(4,3), J(2,-1)$$

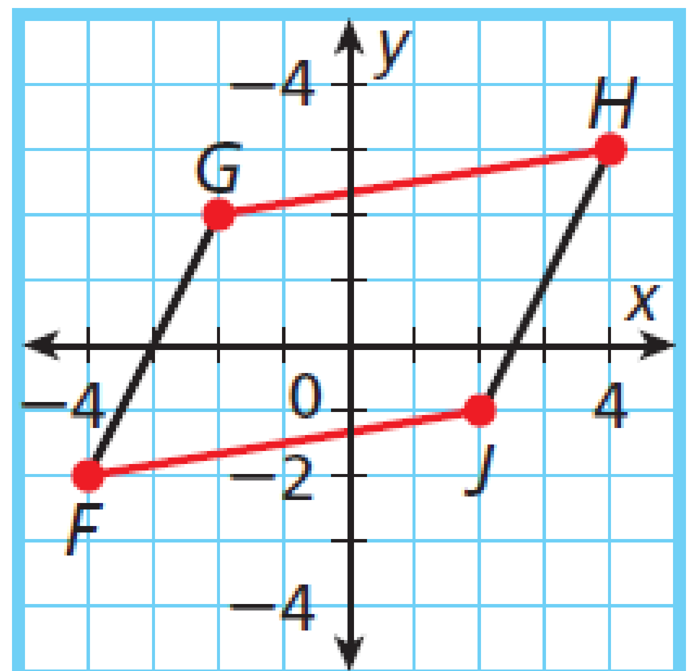
slope of $\overline{GH} = \dfrac{3-2}{4-(-2)} = \dfrac{1}{6}$

slope of $\overline{JF} = \dfrac{-2-(-1)}{-4-2} = \dfrac{-1}{-6} = \dfrac{1}{6}$

$GH = \sqrt{[4-(-2)]^2 + (3-2)^2} = \sqrt{37}$

$JF = \sqrt{(-4-2)^2 + [-2-(-1)]^2} = \sqrt{37}$

$\overline{GH}$ and $\overline{JF}$ have the same slope, so $\overline{GH} \parallel \overline{JF}$. Since $GH = JF$, $\overline{GH} \cong \overline{JF}$. So by Theorem 6-3-1 $FGHJ$ is a parallelogram.



Show that quadrilateral *NPQR is a parallelogram by using the Theorem 6-3-2*. *If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.*

$$N(5,1),\ P(2,7),\ Q(6,9),\ R(9,3)$$

Class Work

6-3 Practice B: 6-7

Summary

Name three different ways to prove a quadrilateral is a parallelogram on the coordinate plane.