ENERGY TECHNOLOGY SYSTEMS ANALYSIS PROGRAMME

Energy Technology Systems Analysis Programme

TIMES Version 4.5 User Note

Linearized MACRO Formulation for ETSAP TIMES

Antti Lehtila
VTT, Finland

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1. Introduction

Both bottom-up partial equilibrium (PE) and top-down general equilibrium (GE) models are widely used for energy-economic analyses. While PE models compute the equilibrium by simultaneously configuring the production and consumption of commodities and their prices, thereby maximizing the total economic surplus, GE models are capable of describing the behavior of supply, demand, and prices in the whole economy, by harnessing the possibilities of substituting different production factors in order to maximize the overall utility. The substitution possibilities are captured in production functions, which describe the changes in the proportions in which the various factors are employed as the result of price changes under given substitution elasticities. As in PE models, prices are determined by market clearance conditions that equalize supply and demand for all commodities in the economy, both energy-related and non-energy.

The core functional form employed in GE modeling is the constant elasticity of substitution (CES) function, which offers a high degree of flexibility for the modeling of substitution options among commodities and input factors. The generic CES function covers the Leontief function and the Cobb-Douglas function as its special cases, and is used to model production, consumer utility and trade, usually in nested hierarchies. However, one challenge for energy systems modeling is that while the standard CES functions aggregate the economic quantities in their nonlinear fashion, values are conserved but physical volumes are not.

Hard-linking top-down and bottom-up type models for integrated energy systems analysis has been usually accomplished by narrowing the focus in one of the models, typically by aggregating the macro sectors of the economy. Examples are the ETA-Macro model (Manne & Richels 1992), the MESSAGE-Macro model (Messner & Schrattenholzer 2000), MARKAL-Macro (Manne & Wene 1992, Epelly et al. 2000) and the Merge model (Manne & Richels 2005). An NLP formulation equivalent to MARKAL-Macro has been integrated in the TIMES model generator since 2006, and a slightly different MSA formulation based on a decomposition algorithm has been available since 2012. Linearized CES demand functions have been available in TIMES since 2017. Implementing the full hard-linked Macro model as a linearized convex optimization problem has thus emerged as a natural generalization to consider. This user note briefly describes such a new variant introduced in v4.5.0: the Macro MLF formulation, the main benefit of which is being more effectively and robustly solved by utilizing high-performance interior-point LP solvers.

2. OVERVIEW OF TIMES-MACRO

2.1 General Design

TIMES-Macro is a reduced form general equilibrium model consisting of a simple macro model with one production versus one consumption sector for each region, and a bottom-up model to represent the energy sector of each region in detail.

The MACRO model can be characterized as a single-sector, optimal growth dynamic inter-temporal general equilibrium model, which maximizes the regional utility. The utility is a logarithmic function of the consumption of a single generic consumer in each region. The industry sectors are presented by the production function of a single representative aggregate industry sector. Inputs for the production are labor, capital, and energy services. The supply of the energy services is fully covered by the multi-sector energy system, which is represented by the TIMES bottom-up model.

Part of the annual production is used to cover the costs for energy demand. The remaining part can be used for investments in the capital stock or for consumption by the households. Thus, the linkage between the MACRO model and the TIMES model is established in two directions: 1) the TIMES model provides the costs of the energy demand to the MACRO model, while in the other direction 2) the MACRO model determines the energy demand being input factor for the TIMES model. The linkage of the two models is shown in Figure 1.

Government is not represented in TIMES-Macro, and the original implementation works only for single-region models. In a multi-regional context, all regions seek to maximize their utility, while being linked by trade. Finding the equilibrium then becomes more complex, but can be implemented by using Negishi iterations.

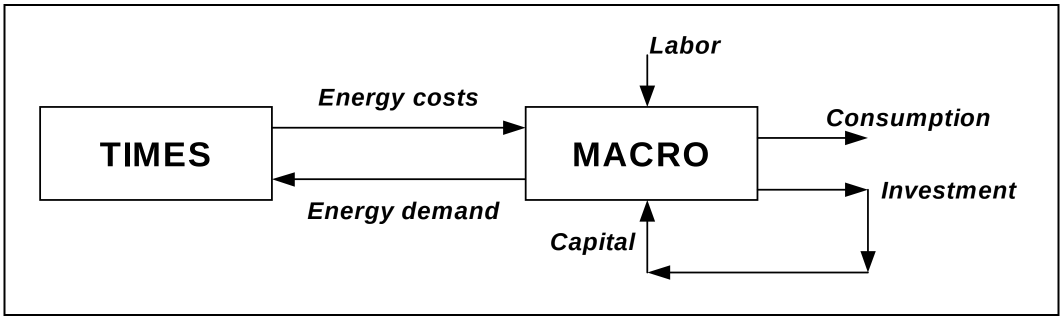

Figure 1. Conceptual illustration of the TIMES-Macro linkage for a single-region TIMES model.

2.2 Core Functional Forms

The objective function of TIMES-Macro is maximizing the sum of the cumulative discounted regional utility, which is logarithmic in absolute consumption. The core relations in the Macro module can be expressed as follows:

$$Max\,U = \sum_{t=1}^{T}\sum_{r} nwt_r \cdot pwt_t \cdot dfact_{r,t} \cdot \ln(C_{r,t}) \tag{1}$$

$$Y_{r,t} = C_{r,t} + INV_{r,t} + EC_{r,t} + NTX(nmr)_{r,t} \tag{2}$$

$$Y_{r,t} = \left(akl_r \cdot K_{r,t}^{kpvs_r \cdot \rho_r} \cdot l_{r,t}^{(1-kpvs_r)\rho_r} + \sum_{k} b_{r,k} \cdot DEM_{r,t,k}^{\rho_r} \right)^{\frac{1}{\rho_r}} \tag{3}$$

$$K_{r,t+1} = tsrv_{r,t} \cdot K_{r,t} + \frac{1}{2}(d_t \cdot tsrv_{r,t} \cdot INV_{r,t} + d_{t+1} \cdot INV_{r,t+1}) \tag{4}$$

$$K_{r,T} \cdot (growv_{r,T} + depr_r) \le INV_{r,T} \tag{5}$$

$$\sum_{r} NTX\,(trd)_{r,t} = 0 \qquad \forall \{t, trd\} \tag{6}$$

where

- $C_{r,t}$: annual consumption in period *t* (variable)
- $Y_{r,t}$: annual production in period *t* (variable)
- $K_{r,t}$: total capital in period *t* (variable)
- $INV_{r,t}$: annual investments in period t (variable)
- $DEM_{r,t,k}$: annual demand for energy commodity *k* in period *t* (variable)
- $EC_{r,t}$: annual energy system costs in Macro in period *t* (variable)
- $NTX(trd)_{r,t}$: annual net exports of commodity *trd* in period *t* (variable)
- akl_r : production function constant
- $b_{r,k}$: demand coefficient for demand commodity *k*
- $depr_r$: depreciation rate
- $dfact_{r,t}$: utility discount factor for period *t*
- $growv_{r,t}$: growth rate in period *t* (calibration parameter)
- $kpvs_r$: capital value share
- $l_{r,t}$: annual labor growth index in period *t*
- nwt_r : Negishi weight for region *r*
- pwt_t : weight multiplier for period *t*
- $tsrv_{r,t}$: capital survival factor between periods *t* and *t+1*
- ρ_r : elasticity of substitution constant ($\rho_r = 1 - 1 / \sigma_r$)
- T : number of periods in the model horizon

The production function (3) is a nested CES function with the input factors capital, labor and energy services. The input factors labor l_t and capital K_t form an aggregate, in which both can be substituted by each other. The second term in (3) indicates that the different energy services $DEM_{r,t,k}$ may also be substituted by each other. Finally, the aggregate of the energy services and the aggregate of capital and labor can substitute each other. The CES structure is thus very simple, but quite similar to that used in the Merge model (Manne & Richels 2005) and comparable to the forms commonly employed in CGE modeling (Figure 2). The substitution constant ρ_r refers to the user-defined elasticity of substitution σ_r , by the expression $\rho_r = 1 - 1 / \sigma_r$. The user-defined capital value share $kpvs_r$ describes the share of capital in the sum of all production factors. The parameters akl_r and b_k of the production are determined from the results from a Baseline TIMES model run without the Macro module.

Equation (2) describes that part of the annual production Y_t which is used to cover the costs $EC_{r,t}$ for energy demands $DEM_{r,t,k}$ in the production function. The other part can be used for investments $INV_{r,t}$ in the capital stock K_t or for consumption by the households. The production factor labor is not an endogenous model variable in Macro, but is given as index function starting with the value 1 in the first period. The labor supply is then growing over time with the growth rate $growv_{r,t}$.

The capital dynamics equations (4) describes the capital stock in the current period K_{t+1} based on the stock in the previous period and investments in the current and previous period. Depreciation of the capital is taken into account by the capital survival factor $tsrv_{r,t}$. The terminal condition (5) ensures that the investment in the last period is high enough to avoid elimination of the capital stock in the last period.

These functional forms are shared by each of the Macro formulations, despite the obvious implementation differences arising from decomposition and linearization.

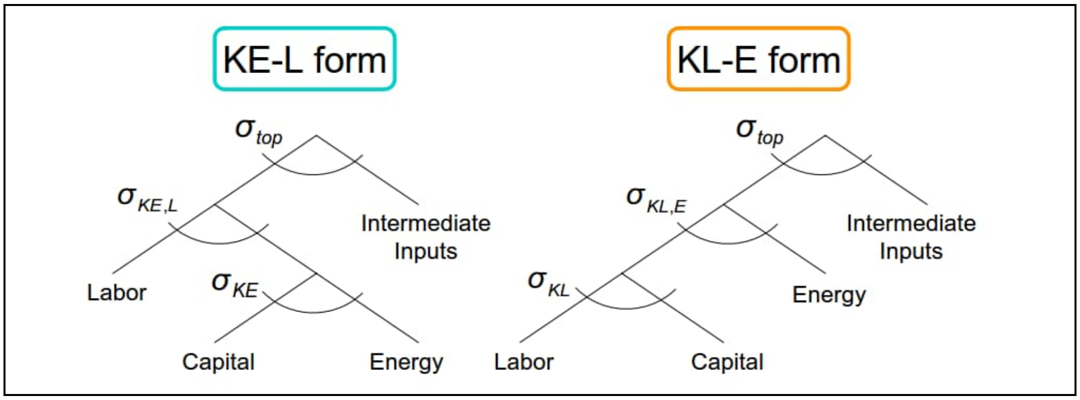

Figure 2: Two major forms of substitution structures (see Okagawa and Ban 2008).

3. DIFFERENCES BETWEEN THE MACRO FORMULATIONS

3.1 Overview

As mentioned within the introduction above, there are now three slightly different formulations of TIMES-Macro available in the model generator:

1. The original TIMES-Macro, based on solving large NLP problems;
2. The Macro MSA decomposition algorithm, based on solving large LP and small NLP sub-problems for the energy system and Macro, respectively;
3. The linearized Macro formulation (MLF), based on solving large LP problems (but is first calibrated by solving small NLP problems).

Albeit all three being in general very much similar, these formulations have notable differences in the following aspects, which are elaborated in more detail below:

- Calibration procedure
- Handling of multi-region utility maximization
- Flexibility in the CES structure
- Treatment of taxes and subsidies
- Effective discount factors

3.2 Calibration Procedure

The original TIMES-Macro and the Macro MSA formulations employ the same calibration procedure and algorithm, where a separate Baseline calibration run is required before running policy scenarios. However, in the new MLF formulation no separate calibration run is required, but the calibration is done automatically when running policy scenarios, and is also more quick and robust. The pre-requisites for the MLF calibration are quite simple and can be summarized as follows:

1. A Baseline run has been solved, producing Base Prices for the demands;
2. The Baseline must also have been run with the switch $SET ANNCOST LEV, in order to obtain consistent total annual costs and taxes for the calibration;
3. For calibration, one either needs to have a licence for an NLP solver, or the number of regions must be small enough to fit within the GAMS demo limits.

3.3 Multi-region Utility Maximization

Due to the lack of proper weights on the regional utility functions, the original NLP formulation of TIMES-Macro is in essence appropriate only for single-region analyses. On the other hand, both in the decomposed MSA algorithm and in the MLF formulation, we calculate Negishi weights for this purpose, which also balance for inter-temporal trade deficits, following the sequential optimization algorithm of Rutherford (1992).[1] The model is re-solved iteratively until the Negishi weights converge, both within the calibration algorithm and when running policy scenarios. However, as compared to the MSA algorithm, the MLF formulation inherently requires significantly fewer iterations, because unlike the individual demand variations under MSA, the Negishi weights tend to converge very quickly.

3.4 Flexibility in CES Structure

In the new MLF formulation, the demands for energy services can optionally also be aggregate demands defined by the user, using CES functions (including nested CES forms). Therefore, the energy service demands $DEM_{r,t,k}$ need not be the individual demands in the TIMES model, but can be CES aggregates with different substitution elasticities. In addition, there is a new input parameter for the substitution elasticity between the final demands $DEM_{r,t,k}$, which thus needs not be the same as the elasticity between the aggregate energy services and the capital–labor aggregate.

3.5 Treatment of Taxes and Subsidies

In the original TIMES Macro and in the MARKAL Macro formulations, taxes and subsidies are treated in the same way as costs and revenues: taxes are treated like positive costs and subsidies like negative costs (or revenues). On the other hand, in the Macro-MSA formulation, taxes and subsidies are not included in the Macro accounting of annual energy system costs, even though they of course do affect the marginal costs of the demands. In this respect, the Macro-MSA formulation gives a better representation for the macro-economic impacts of taxes and subsidies.

In the MLF formulation, taxes and subsidies need to be included in the accounting of annual energy system costs, like in the original TIMES Macro. However, in the MLF case, a lump-sum rebate of net tax revenues can be easily applied. Within the Negishi iteration loop implemented for completing each MLF policy run, we can make sure that crediting the net taxes as a lump-sum rebate revenue will be made consistently. This tax rebating approach follows closely the one implemented in the widely-known Merge energy system model (Manne & Richels 2005)[2].

[1] Rutherford, Thomas F. 1992. Sequential Joint Maximization. Discussion Papers in Economics – 92-08; Boulder, University of Colorado, September 1992.

[2] Manne A, Richels R 2005. MERGE: an integrated assessment model for global climate change. In: Loulou R, Waaub J-P, Zaccour G, (eds) Energy and Environment. GERAD 25th Anniversary Series. Springer, pp 175–189.

3.6 Effective Discounting

In PE models, the discounting of all costs and revenues is normally done according to exogenously provided general discount rates, which can be constant or time-dependent. In the Macro model, however, the effective discount factors are endogenous and therefore in general not consistent with the discount factors in the PE model. Consequently, even in a calibrated Baseline run, where all the Macro energy service demands are calibrated to coincide with the original PE demands, the results from TIMES-Macro and the PE Baseline will in general be inevitably different.

In the original TIMES-Macro formulation, no consideration was made to address this issue of discrepancy in discounting, and therefore any full Baseline replication was actually not possible. Only the energy service demands and their prices were guaranteed to be reproduced as in the PE Baseline.

In the Macro MSA formulation, the full model is decomposed into LP and NLP sub-problems, both using their own discounting methods. That makes it possible to replicate the PE Baseline results with the MSA algorithm, but in turn, any results from the MSA algorithm are to some extent inconsistent with the original Macro results. In the MSA algorithm, there is, however, also an experimental option to harmonize the discount rates used in the LP sub-problem with those in the Macro NLP sub-problems, by updating the former in each master iteration while calibrating the Baseline, and the updated discount factors are then subsequently employed in policy runs. With that experimental option the MSA results could be expected to be consistent with the original Macro results, when both were based on the same calibration.

In the new MLF formulation, a different approach for harmonizing the discount rates is employed. During the MLF calibration, the discount factors of the GE Baseline are updated to converge to the PE discount factors, which makes it easy to replicate all the PE Baseline results with the MLF algorithm, with very high levels of accuracy. In the policy scenarios, the discount factors of course remain endogenous to the policy impact, and are now again inconsistent both with the factors in the original Macro and those in the MSA algorithm. Nonetheless, the new harmonizing method for the discount factors under MLF appears to be rather convenient, as the PE Baseline results can be basically fully replicated. For benchmarking purposes, the MLF extension implements also an option for solving the model using an equivalent NLP formulation but with those same harmonized discount factors, for verifying the accuracy of the MLF linearization compared to the equivalent NLP formulation.

4. SETTING UP FOR USING MLF

The pre-requisites for using the MLF formulation are quite simple and can be summarized as follows:

1. A Baseline run has been solved, producing Base Prices for the demands as well as total annual costs and taxes (written in com_bprice.gdx).
2. Policy runs require activating elastic demands as well as Macro input data on at least the projections and Base year values for GDP in each region.

For the Baseline run, the recommended RUN file settings are the following:

- $SET TIMESED NO (required)
- $SET ANNCOST LEV (required)
- $SET OBLONG YES (recommended)

For the Policy runs, the recommended RUN file settings are the following:

- $SET TIMESED YES (required)
- $SET MACRO MLF (required)
- $SET ANNCOST LEV (recommended)
- $SET OBLONG YES (recommended)

The only required input data for the policy runs are the Base year GDP values by region (parameter TM_GDP0_r) and the Baseline growth projections for GDP by region (parameter $TM\_GR_{r,t}$). However, there are also a number of other, optional input parameters for Macro of which the most important are listed below:

- TM_ESUB_r – elasticity of substitution for the (KL)E aggregate
- TM_DESUB_r – elasticity of substitution for the demand aggregate
- $TM\_STEP_{r,c,lim}$ – number of steps for demand c in the UP/LO direction
- $TM\_VOC_{r,y,c,lim}$ – max. variance of demand c in the UP/LO direction
- TM_DEPR_r – depreciation rate (%)
- TM_KGDP_r – initial capital to GDP ratio
- TM_KPVS_r – capital value share
- *TM_DEFVAL* – default constants, including e.g. the following:
  - NEGTOL – Negishi convergence tolerance (default=0.01)
  - MACSTEP – number of steps for Macro aggregates
  - MACVOC – max. variance of Macro aggregates
  - LOGSTEP – number of steps for the logarithmic utility

Any dummy imports should of course be disabled when using any of the Macro formulations or when running a Baseline for Macro or elastic demands, because dummy imports tend to cause ill-behaving demand prices even when not active.

5. ILLUSTRATIVE DEMO MODEL

5.1 Overview

In the Github TIMES_Demo repository, there is a Demo model illustrating the basic use of the MLF Macro formulation. The Demo model is based on the Demo12 model documented in the TIMES documentation, Part IV. This is a small two-region model, including electricity and natural gas trade between the regions.

5.2 Illustrative Demo Runs

The Github repository includes three run files for the Demo12 model, illustrating the use of the MLF formulation, and the recommended and required switches:

1. demo12Base.run – The model Baseline run
   Note the required switches for an MLF Baseline in the RUN file:
   $SET TIMESED NO
   $SET ANNCOST LEV
2. demo12MLF-BaseRep.run – Baseline replication run using MLF
   Here, we use the MLF Formulation without changing any input data; only the GDP projections and Base year GDP values are added to the input data;
   Note also the switches used for the policy run (TIMESED, MLF, ANNCOST).
3. demo12MLF-PolicyEx.run – Example policy run using MLF
   Here, we add a policy scenario input file ghgpolicy.dd, defining stringent CO_2 emission constraints;
   Note again the required & recommended switches in the run file.

The Baseline run is needed for getting the Base Prices for the demands as well as the Baseline annual system costs. These data are written to the GDX file com_bprice.gdx and are subsequently read by TIMES when running MLF. The Baseline replication run is included here only for illustrating the accuracy of the MLF formulation in replicating the Baseline solution, when no changes are made to the model scenario.

Finally, the example policy run demonstrates the functionality and performance of the MLF Formulation at solving an example real policy case. The policy run is by default run with the LP formulation, but with a special switch ($SET NONLP NL) it can be run also with an equivalent NLP formulation, to see the quality (accuracy) of the LP solution in comparison to the NLP solution, as illustrated in the next section.

5.3 Performance of the MLF Formulation

The main motivation for implementing the MLF formulation was the expected performance gains. For any big multi-region models, solving a full NLP Macro formulation would be time-consuming, and also the decomposed MSA algorithm has a significant performance penalty compared to using the model in the PE mode.

Regarding the performance of the linearized Macro MLF formulation, it has two notable aspects: solution accuracy and solution time. There is of course a trade-off between the model size (mainly number of additional step variables) and the accuracy of the linearization. The defaults for the number of MLF step variables have been chosen to achieve a reasonable balance in that trade-off, such that the error due to the linearization would in general be less than 0.05% in the main Macro aggregates. The comparison of the GDP losses in the GHG policy example scenario, between the MLF and the equivalent non-linear formulation, is shown in Figure 3. One can see that the differences are indeed almost negligibly small, less than 0.03%.

As to the Demo12 model, it is a very small model where any performance benefits from using MLF still remain insignificant. However, repeated tests with a larger model with about 140,000 constraints showed an average time overhead of only about 25% from using MLF for a single solve. As the MSA algorithm typically requires 12–17 iterations on top of the initial solve, while the MLF algorithm would usually require 1–3 additional iterations, the solution time savings compared to using MSA appear to become significant already with models of moderate sizes.

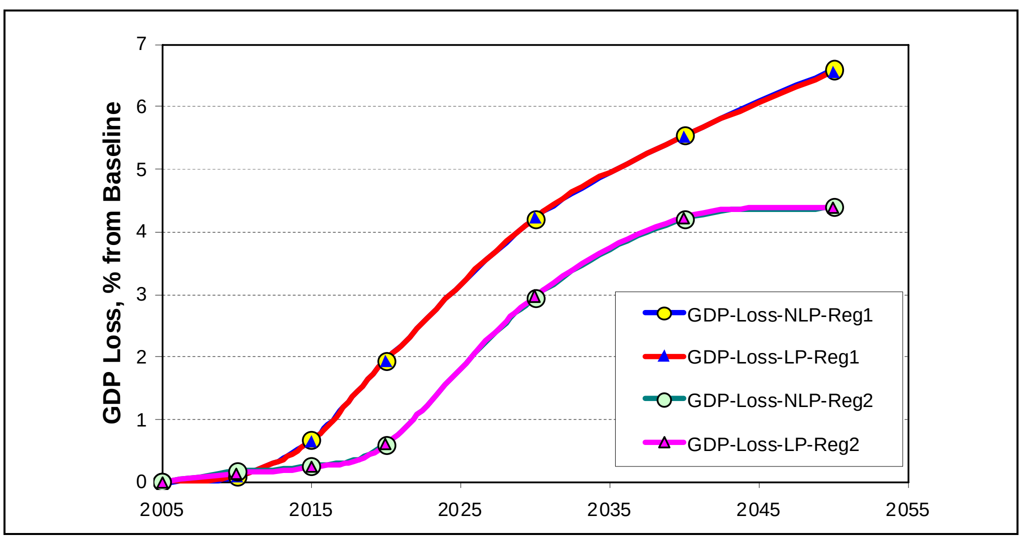

Figure 3: Differences between the GDP results obtained for the GHG policy example from the MLF forumulation and from the equivalent NLP formulation.

6. References

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