ENERGY TECHNOLOGY SYSTEMS ANALYSIS PROGRAMME

TIMES User Note (v3.4 and above)

TIMES Grid Modeling Features

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Foreword

This User Note contains the documentation for the implementation and usage of the new grid modeling features in the TIMES model generator.

The document is divided into four chapters. Chapter 1 contains an introduction, and Chapter 2 presents a brief description of the mathematical formulation, on which the implementation is based. Chapter 3 contains the description of the GAMS implementation of the new elements, along with the input parameters, variables, and equations that have been added to the TIMES model. Finally, Chapter 4 constitutes a brief User's Reference Manual for the grid modeling features in TIMES.

The work for the first update of the TIMES model generator in 2012 was funded by JRC – Institute for Energy and Transport through the Service Contract for the "Integration of TIMES based energy system modeling with electricity grid modeling" (2012/S 21-033025). The methodology was first applied in the JRC-EU-TIMES model (for more details contact JRC-EU-TIMES@ec.europa.eu). This first update included the implementation of the phase-angle formulation for DC power flow equations and an optional add-on grid facility for electricity grids, as well as a few supplementary features for electricity grids.

The work for the second update of the TIMES model generator in 2024 was carried out within the ETSAP project *TIMES Extension for electricity, gas, hydrogen and CO2 transport infrastructures*, with the design led by Evangelos Panos (PSI) and the mathematical formulations being detailed in the corresponding ETSAP project report (Panos, E. & Lehtilä, A. 2025). This second update implemented the PTDF formulation of DC power flow equations supporting transmission expansion planning, as well as the Weymouth formulation of gas grid equations, with a generalized implementation of the optional add-on grid facility.

This documentation is a supplement to the complete documentation of the TIMES model generator.

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1. Introduction

The specific grid modeling features of TIMES are intended for modelers who wish to include basic characteristics of energy transmission grids in their energy system models. The most important grid systems are electricity transmission networks. Modeling electrical grids in TIMES implies that at least a simple representation of the geographical and spatial distribution of generators, consumer loads and the transmission network is included in the reference energy system. Other grids, such as gas grids or CO_2 grids are also supported.

The grid modeling features can be useful, for example, for analyzing potential bottlenecks in the electricity transmission system, and for evaluating the impacts of the integration of large amounts of variable renewable generation in the system. Such analyses could thus be utilized for giving an indication of the enforcements and new investments that would be needed in various parts of the main grid under different future scenarios. They can also be useful to represent gas, hydrogen or CO_2 emissions pipeline networks, by accounting for the physical properties of the gas flows in those pipelines.

The specific grid modeling features include the following components:

- Linear DC power flow equations based on a phase-angle or PTDF formulation;
- Linear gas grid equations based on the Weymouth formulation;
- A facility for including a grid representation in the model by using an add-on approach with automated allocation of generation and demand to grid nodes;
- Simplified N–1 security constraints;
- New parameter for defining costs on specific grid flows.

The PTDF formulation for DC power flow equations and the Weymouth formulation for gas flow equations both support also a MIP formulation option, which is needed for consistent modeling of transmission expansion planning and bi-directional pipelines in gas grids.

2. MATHEMATICAL FORMULATION

2.1 DC Load Flow Equations

2.1.1 Phase-angle formulation

The general form of the DC load flow equations can be written as follows:

For every grid node *i*:

$$G_i - L_i = \sum_{j=1}^{M} P_{i,j} \tag{1}$$

And for each grid connection *i,j* between nodes *i* and *j* we have the relationship:

$$P_{i,j} = B_{i,j}(\delta_i - \delta_j) = (\delta_i - \delta_j) / X_{i,j} \tag{2}$$

where:

M = the number of nodes connected with node *i*
G_i = active power injected into node *i* by generators
L_i = active power withdrawn from node *i* by consumer loads
P_{ij} = branch flow from node *i* to node *j*
B_{ij} = susceptance of the branch connecting nodes *i* and *j*
X_{ij} = reactance of the branch connecting nodes *i* and *j*
δ_i = voltage phase angles of node *i* with respect to a reference angle

Equation (2) has been implemented into TIMES using the following formulation. For each pair of two grid nodes *i* in region *r* and *j* in region *r2* connected by some grid lines *p* by TOP(r,i,r2,j,p), we have the following condition (in simplified terms):

$$\sum_{p \in TOP(r,i,r2,j,p)} VAR\_IRE(r,v,t,p,i,s,EXP) - VAR\_IRE(r,v,t,p,i,s,IMP) = \\ GR\_ADMIT(r,t,i,r2,j) \cdot (VAR\_DELTA(r,t,i,s) - VAR\_DELTA(r2,t,j,s)) \tag{3}$$

where:

- VAR_IRE(r,v,t,p,n,s,ie) = flow variable for inter-regional trade process *p* in region *r* node *n* and timeslice *s*
- VAR_DELTA(r,t,n,s) = new variable representing voltage phase angle of node *n* in region *r* and timeslice *s*
- GR_ADMIT(r,t,n,r2,m) = internal parameter representing the susceptance of the grid connection between node *n* in region *r* and node *m* in region *r2*.

The internal susceptance parameter *GR_ADMIT* can be calculated form the user input parameter *PRC_REACT* as follows:

$$GR\_ADMIT(r,t,i,r2,j) = \left(\sum_{\substack{p \in \{TOP(r,i,r2,j,p) \cap RTP\_VARA(r,t,p) | \\ PRC\_REACT(R,T,P)>0\}}} \frac{1}{PRC\_REACT(r,t,p)} \right) \quad (4)$$

where:

- PRC_REACT(r,t,p) = input parameter specifying the reactance of line *p*
- TOP(r,p,c,io) = set of 4-tuples defining the topology of process *p* in region *r* in terms of commodities *c* and IN/OUT indicator *io*;
- RTP_VARA(r,t,p) = set of 3-tuples indicating availability of process in period *t*

2.1.2 PTDF Formulation

Using the PTDF formulation allows for the modeling of transmission expansion planning by accounting for the induced changes in the grid operation. That requires calculating the PTDF factors for the whole system with all lines. With a MIP formulation one can consistently eliminate the impact of the lines that are not installed, by introducing virtual lines that carry a dummy flow when the line is not installed as follows:

$$\begin{aligned}
&EQ_GR_PTDFLO(r1,t,n1,r2,n2,p,s)\$(PRC_TS(r1,p,s)\ and\ GR_TOP(r1,n1,r2,n2,p))..\\
&\sum_{v \in VINTYR} \left(VAR_IRE(r1,v,t,p,n1,s,'EXP') - VAR_IRE(r1,v,t,p,n1,s,'IMP') \right) + VAR_GVIRT(r1,t,p,n1,s)\\
&= \sum_{\substack{(reg,k)\in N \\ (reg,com)\in nrgelc}} GR_PTDF(r1,t,p,n1,reg,k) \times VAR_GNETINJ(reg,t,k,s) +\\
&\sum_{EXP(r3,i,r4,j,b)} \left(GR_PTDF(r1,t,p,n1,r3,i) - GR_PTDF(r1,t,p,n1,r4,j) \right) \times VAR_GVIRT(r3,t,b,i,s)
\end{aligned}$$

However, due to the nodal balances being imposed, in the TIMES formulation we can use artificial variables for the net injections, and this equation can be simplified as follows:

$$\begin{aligned}
&EQ_GR_PTDFLO(r1,t,p,n1,r2,n2,s)\$(PRC_TS(r1,p,s)\ and\ GR_TOP(r1,n1,r2,n2,p))..\\
&\sum_{v \in VINTYR} \left(VAR_IRE(r1,v,t,p,n1,s,'EXP') - VAR_IRE(r1,v,t,p,n1,s,'IMP') \right) + VAR_GVIRT(r1,t,p,n1,s)\\
&= \sum_{\substack{(reg,k)\in N \\ (reg,com)\in nrgelc}} GR_PTDF(r1,t,p,n1,reg,k) \times VAR_GNETINJ(reg,t,k,s)
\end{aligned} \quad (5)$$

In addition, we need to bound the virtual flows to zero when the line is installed, which can be accomplished by first bounding the scalar norm of the flows by virtual capacity:

$$\begin{aligned}
&EQ_GR_VIRTCP(r1,t,p,s,n1,bd)\$(PRC_TS(r1,p,s)\$RPC_IRE(r,p,n1,'EXP')\$CANDID(r,t,p))..\\
&\left| VAR_GVIRT(r1,t,p,n1,s) \right| \quad \le \quad VAR_XCAP(r1,t,p) \times PRC_CAPACT(r1,p) \times G_YRFR(r1,s)
\end{aligned} \quad (6)$$

The virtual capacity is bounded by a binary variable (MIP case), or using an LP relaxation:

$$\begin{aligned}
&EQ_GR_VIRTBD(r,t,p)\$CANDID(r,p)..\\
&VAR_XCAP(r,t,p) \quad \le M \times GR_CAPUP(r,t,p) \times (1 - VAR_PTDNCAP(r,t,p))
\end{aligned} \quad (7)$$

Here, $GR\_CAPUP_{r,t,p}$ is a capacity bound for the candidate line (which is a required input) and *VAR_PTDNCAP(r,t,p)* is a virtual variable representing the cumulative sum of *VAR_DNCAP* up to year *t*, where *VAR_DNCAP* is the TIMES internal binary variable for discrete new capacity (activated by using the discrete capacity extension and NCAP_DISC). Note that equation (7) is generated only for the candidate lines for implementing the transmission grid expansion.

In the case of the LP relaxation formulation equation (7) can be written as:

$$
\begin{aligned}
&EQ\_GR\_VIRTBD(r,t,p)\$CANDID(r,p)..\\
&VAR\_XCAP(r,t,p) \quad \leq \quad M \times GR\_CAPUP(r,t,p) \times \\
&\big(1-\big(1-\big(1-VAR\_CAP(r,t,p)/GR\_CAPUP(r,t,p)\big)/M\big)\big)
\end{aligned}
\tag{8}
$$

2.2 Gas grid flow equations

For gas grids, the TIMES code implements a generalized formulation that can be applied to any network involving the transportation of gaseous fuels, including also emissions (e.g., CO_2 pipelines). Here, the following additional input parameters are referred to in the formulation:

- $GG\_DENS_{r,n}$ – density of gas in terms of standard model units (for node **n**)
- $GG\_KGF_{r,t,p,n}$ – Weymouth constant for gas flow rate of pipe **p**, input node **n**
- $GG\_KLP_{r,t,p,n}$ – Linepack constant for gas flow of pipe **p** with input node **n**
- $GG\_GAMMA_{r,t,p,n}$ – Compressor factor of pipeline p with input node **n**
- $GG\_PP_{r,t,p,n,bd,\omega}$ – Fixed pressure points $\{\omega\in\Omega\}$for pipeline **p** and node **n**, side **bd**

For the model formulation, the following additional variables are introduced:

- $VAR\_MF_{r,t,p,m,r2,u,ts}$ – mass flow rate of gas in pipeline **p** from node **m** in region **r** to node **u** in **r2**, timeslice **ts**
- $VAR\_PR_{r,t,m,ts}$ – pressure of gas flow at node **m**, timeslice **ts**
- $VAR\_PADD_{r,t,p,m,ts}$ – pressure boost of gas flow in pipeline **p** at node **m**, timeslice **ts**, due to a compressor at the input node m
- $VAR\_PDIF_{r,t,p,m,r2,u,ts}$ – pressure difference of gas flow in pipeline **p** between node **m** and node **u**, timeslice **ts**
- $VAR\_LIP_{r,t,p,ts}$ – linepack of gas flow in pipeline **p**, timeslice **ts**, in region **r**
- $VAR\_Y_{r,t,p,m,r2,u,ts}$ – auxiliary binary variables for bi-directional gas flows between node **m** and node **u**, timeslice **ts**

In addition, the following control attributes are used in the formulation:

- $GR\_TOP_{r,n,r2,n2,p}$ – grid connection between nodes n and n2 (“edge” of the grid)
- $GR\_LINK_{r,n,r2,n2,p}$ – grid link between nodes n and n2 (directed arc of the grid)

The gas grid equations can be written in simplified terms as follows:

For: $\forall\ (r,p,m,r2,u)\ \in GR\_LINK,\ ts\ \in PRC\_TS_{r,p}$:

$$VAR\_MF_{r,t,p,m,r2,u,ts} \times GG\_DENS_{r,m} \times G\_YRFR_{r,ts} \times 8760 \quad = \left(VAR\_IRE^{EXP}_{r,t,p,m,ts} + VAR\_IRE^{IMP}_{r2,t,p,u,ts}\right)/2 \tag{9}$$

$$VAR\_PADD_{r,t,p,m,ts} \le \left(GG\_GAMMA_{r,t,p,m} - 1\right) \times VAR\_PR_{r,t,m,ts} \tag{10}$$

$$\begin{aligned} VAR\_MF_{r,t,p,m,r2,u,ts} &\le M \times VAR\_Y_{r,t,p,m,r2,u,ts} \\ VAR\_MF_{r2,t,p,u,r,m,ts} &\le M \times (1 - VAR\_Y_{r,t,p,m,r2,u,ts}) \end{aligned} \tag{11}$$

$$\begin{aligned} VAR\_PDIF_{r,t,p,m,r2,u,ts} &\le M \times VAR\_Y_{r,t,p,m,r2,u,ts} \\ VAR\_PDIF_{r2,t,p,u,r,m,ts} &\le M \times (1 - VAR\_Y_{r,t,p,m,r2,u,ts}) \end{aligned} \tag{12}$$

For: $\forall\ (r,p,m,r2,u)\ \in GR\_TOP,\ ts\ \in PRC\_TS_{r,p}$:

$$VAR\_PDIF_{r,t,p,m,r2,u,ts} = VAR\_PR^{P}_{r,t,p,m,ts} - VAR\_PR^{P}_{r2,t,p,u,ts} + VAR\_PDIF_{r2,t,p,u,r,m,ts} \tag{13}$$

$$\begin{aligned} VAR\_LIP_{r,t,p,ts} &= VAR\_LIP_{r,t,p,ts--1} + \\ VAR\_IRE^{EXP}_{r,t,p,m,ts} &+ VAR\_IRE^{EXP}_{r2,t,p,u,ts} - VAR\_IRE^{IMP}_{r,t,p,m,ts} - VAR\_IRE^{IMP}_{r2,t,p,u,ts} \end{aligned} \tag{14}$$

$$\begin{aligned} VAR\_LIP_{r,t,p,ts} &= GG\_KLP_{r,t,p,m} \times GG\_DENS_{r,m} \times \\ RS\_STGPRD_{r,ts} &\times \left(VAR\_PR^{P}_{r,t,p,m,ts} + VAR\_PR^{P}_{r2,t,p,u,ts}\right)/2 \end{aligned} \tag{15}$$

For: $\forall\ (r,p,m,r2,u)\ \in GR\_TOP,\ ts\ \in PRC\_TS_{r,p}, \omega \in \Omega$:

$$VAR\_MF_{r,t,p,m,r2,u,ts} \le GG\_KGF_{r,t,p,m} \times \left(\begin{matrix} \dfrac{GG\_PP^{UP,\omega}_{r,t,p,m}}{\sqrt{\left(GG\_PP^{UP,\omega}_{r,t,p,m}\right)^2 - \left(GG\_PP^{LO,\omega}_{r2,t,p,u}\right)^2}} \times VAR\_PR^{P}_{r,t,p,m,ts} - \\ \dfrac{GG\_PP^{LO,\omega}_{r2,t,p,u}}{\sqrt{\left(GG\_PP^{UP,\omega}_{r,t,p,m}\right)^2 - \left(GG\_PP^{LO,\omega}_{r2,t,p,u}\right)^2}} \times VAR\_PR^{P}_{r2,t,p,u,ts} \end{matrix} \right) + M \times \left(1 - VAR\_Y_{r,t,p,m,r2,u,ts}\right) \tag{16}$$

$$VAR\_MF_{r2,t,p,u,r,m,ts} \le GG\_KGF_{r,t,p,m} \times \left(\begin{matrix} \dfrac{GG\_PP^{UP,\omega}_{r2,t,p,u}}{\sqrt{\left(GG\_PP^{UP,\omega}_{r2,t,p,u}\right)^2 - \left(GG\_PP^{LO,\omega}_{r,t,p,m}\right)^2}} \times VAR\_PR^{P}_{r2,t,p,u,ts} - \\ \dfrac{GG\_PP^{LO,\omega}_{r,t,p,m}}{\sqrt{\left(GG\_PP^{UP,\omega}_{r2,t,p,u}\right)^2 - \left(GG\_PP^{LO,\omega}_{r,t,p,m}\right)^2}} \times VAR\_PR^{P}_{r,t,p,m,ts} \end{matrix} \right) + M \times VAR\_Y_{r,t,p,m,r2,u,ts} \tag{17}$$

In the above, Eq. (9) establishes the link between standard *VAR_IRE* variables and the mass flow variables (we consider that the mass flow is calculated on the average from the gas inflow and outflow quantities in the pipeline – which might be different due to the linepack). Eq. (10) defines the limits for the compressor pressure boost variables according to the *GG_GAMMA* input attribute (in this formulation the compressor is assumed to always increase the pressure). Eqs. (11) and Eq. (12) are only defined for bi-directional links and enforce that the gas flow and pressure difference can be positive in only one direction. Eq. (13) defines the pressure differences from the pipeline inlet and outlet pressures $VAR\_PR^{P}_{r,t,p,n,ts}$ (which are obtained by adding *VAR_PADD* to the nodal pressure variables $VAR\_PR_{r,t,n,ts}$). Eq. (14) defines the linepack storage balance and Eq. (15) defines the linepack levels by timeslice. Finally, Eqs. (16) and (17) define the Taylor expansion approximations for the Weymouth formulas for the gas flow rates. To perform the approximation several pressure points (e.g. |Ω|=20) need to be sampled in both directions.

Note that the *VAR_Y* above is a binary variable introduced for grids with bidirectional links, where the endogenous direction of the gas flow could not otherwise be handled by the linearized formulation. In fact, like Eqs. (11) and Eq. (12), the last terms in Eqs. (16) and (17) are only taken into account for bi-directional links.

The formulation with the Taylor expansion linearization of the Weymouth equation is primarily based on the approach of Shin et al (2022) and Ordoudis et al (2019). This approach supports a pure LP formulation only for gas grids with unidirectional links. Nonetheless, the TIMES code additionally implements also an experimental alternative linearization of the Weymouth equation, which is applied when the pressure points *GG_PP* have not been defined by the user. This experimental alternative formulation supports an LP relaxation also for gas grids with bi-directional links. Both formulations can, of course, be used under the MIP option with the binary flow direction variables.

In addition to the specific equations, the formulation of the gas grids includes also a few variable bounds that can be defined by the user: Upper and lower bounds for the nodal pressure variables, which are required and can be defined with the *GG_PRBD* attribute, and lower bounds for the line-pack variables, which can be defined with the *GG_KLP* attribute.

The alternative formulation is based on linearizing the Weymouth equation in a “step-wise” manner by dividing the pressure difference into a linear combination over a number of predefined pressure-difference variables, for which the contribution to the Weymouth equation can be pre-calculated. The pressure-differences are defined over the cartesian product of fixed pressure points on both sides, from higher to lower pressures in each direction. The total value of the Weymouth function is obtained as the sum of Weymouth contribution over these stepped pressure-difference variables.

For: $\forall\ (r, p, m, r2, u) \in GR\_LINK,\ ts \in PRC\_TS_{r,p},\ j, jj \in \Omega :$

$$\begin{aligned}&\sum_{\substack{(r,t,p,m,j,r2,u,jj)\in\\ GG\_PPW_{r,t,p,m,j,r2,u,jj}}} VAR\_STEP_{r,t,p,m,ts,j,jj} \times GG\_PPM_{r,t,p,m,r2,u,j} + \sum_{\substack{(r2,t,p,u,j,r,m,jj)\in\\ GG\_PPW_{r2,t,p,u,j,r,m,jj}}} VAR\_STEP_{r2,t,p,u,ts,j,jj} \times GG\_PPM_{r,t,p,m,r2,u,jj}\\ &= \quad VAR\_PR^{P}_{r,t,p,m,ts} - VAR\_PDIF_{r,t,p,m,r,m,ts}\end{aligned} \tag{18}$$

$$\begin{aligned}&\sum_{\substack{(r,t,p,m,j,r2,u,jj)\in\\ GG\_PPW_{r,t,p,m,j,r2,u,jj}}} VAR\_STEP_{r,t,p,m,ts,j,jj} \times GG\_PPM_{r2,t,p,u,r,m,jj} + \sum_{\substack{(r2,t,p,u,j,r,m,jj)\in\\ GG\_PPW_{r2,t,p,u,j,r,m,jj}}} VAR\_STEP_{r2,t,p,u,ts,j,jj} \times GG\_PPM_{r2,t,p,u,r,m,j}\\ &= \quad VAR\_PR^{P}_{r2,t,p,u,ts} - VAR\_PDIF_{r,t,p,m,r,m,ts}\end{aligned} \tag{19}$$

$$VAR\_MF_{r,t,p,m,r2,u,ts} \le GG\_KGF_{r,t,p,m} \times \sum_{\substack{(r,t,p,m,j,r2,u,jj)\in\\ GG\_PPW_{r,t,p,m,j,r2,u,jj}}} VAR\_STEP_{r,t,p,m,ts,j,jj} \times GG\_PPW_{r,t,p,m,j,r2,u,jj} \tag{20}$$

Equations (18) and (19) ensure that the pressure at node **m** and node **u**, as calculated from the step-wise variables and pressure points *GG_PPM*, is equal to the actual nodal pressure. Equation (20) then gives the linearization for the Weymouth upper bound on the flow in each direction, by using the step-wise variables and the pre-calculated *GG_PPW*.

2.3 Allocation of Generation and Load to Grid Nodes

In the context of TIMES, it may be desirable to be able to model the electricity or gas grid as an ***add-on***, without changing the original structure of the model. Therefore, we need to determine the set of necessary equations for calculating the grid injections by generation or supply units and the grid loads from customer demands in each node and timeslice of TIMES.

Within each region, the generation and load of electricity or other commodities can be allocated to the grid nodes in that region, based on linear allocation equations.

The equations for allocating electricity generation or other commodity supply to the grid nodes are defined for each grid node ***i***, each generated supply commodity ***com*** and each DAYNITE timeslice ***s*** in region ***r***, as follows (simplified formulation):

$$
\begin{aligned}
&VAR\_GRIDIO(r,t,com,i,s,OUT) = \\
&\sum_{\substack{c \in ELG(r),\\ p \in PRC(r)}} \begin{pmatrix} GR\_GENMAP(r,p,type) \cdot GR\_GENFR(r,t,i,type) \cdot \\ TOP(r,p,c,OUT) * VAR\_FLO(r,v,t,p,c,s) \end{pmatrix} + \\
&\sum_{\substack{c \in ELG(r),\\ p \in PRC(r)}} \begin{pmatrix} GR\_GENMAP(r,p,type) \cdot GR\_GENFR(r,t,i,type) \cdot \\ TOP(r,p,c,OUT) * VAR\_SOUT(r,v,t,p,c,s) - \\ TOP(r,p,c,IN) * VAR\_SIN(r,v,t,p,c,s) \end{pmatrix} + \\
&\sum_{\substack{c \in ELG(r),\\ p \in PRC(r)}} \begin{pmatrix} GR\_GENMAP(r,p,type) \cdot GR\_GENFR(r,t,i,type) \cdot \\ RPC\_IRE(r,p,c,IMP) * VAR\_IRE(r,v,t,p,c,s,IMP) - \\ RPC\_IRE(r,p,c,EXP) * VAR\_IRE(r,v,t,p,c,s,EXP) \end{pmatrix}
\end{aligned} \tag{21}
$$

where:

- VAR_GRIDIO(r,t,c,n,s,OUT) = new variable representing generation of electricity or other supply commodity *c* to node *n* in region *r* and timeslice *s*;
- VAR_FLO(r,v,t,p,c,s) = flow variable for process *p* in region *r*, commodity *c*, timeslice *s*;
- VAR_SOUT(r,v,t,p,c,s) = storage output flow variable for process *p* in region *r*, commodity *c* and timeslice *s*;
- VAR_SIN(r,v,t,p,c,s) = storage input flow variable for process *p* in region *r*, commodity *c* and timeslice *s*;
- VAR_IRE(r,v,t,p,c,s,ie) = inter-regional exchange flow variable for process *p* in region *r*, commodity *c* timeslice *s* and exchange direction indicator *ie*.
- GR_GENMAP(r,p,type) = user input parameter specifying the mapping of processes *p* to generation or other supply unit types *type* in region *r*;
- GR_GENFR(r,t,n,type) = user input parameter specifying the fractions of generation or other supply unit types *type* to be allocated to grid nodes *n* in region *r*;
- TOP(r,p,c,io) = set of 4-tuples defining the topology of process *p* in region *r* in terms of commodities *c* and IN/OUT indicator *io*;
- ELC(r), PRC(r) = sets of electricity generation or other supply commodities and related supply or trade processes in region *r*, respectively.

In an analogous manner, the total load of consumed electricity or other supply commodities can be allocated to each grid. The equations for allocating electricity (or other commodity) demand to the grid nodes are generated for each node ***i*** and each DAYNITE timeslice ***s*** in region ***r***, as follows (simplified formulation):

$$\sum_{com \in ELG(r)} VAR\_GRIDIO(r,t,com,i,s,IN) = \tag{22}$$

$$\sum_{c \in ELD(r)} \left(\begin{array}{l} GR\_ENDFR(r,t,i,c) \cdot \\ \left(\begin{array}{l} (1 + GR\_ENDFR(r,t,c,c)) \cdot VAR\_COMPRD(r,t,c,s) - \\ GR\_ENDFR(r,t,c,c) \cdot \sum\limits_{\substack{j \in N(r), \\ c \in ELG(r)}} VAR\_GRIDIO(r,t,c,j,s,IN) \end{array} \right) \end{array} \right)$$

$$\sum_{com \in ELG(r)} VAR\_GRIDIO(r,t,com,i,s,IN) = \tag{23}$$

$$GR\_DEMFR(r,t,i,s) \cdot \left(\sum_{\substack{j \in N(r), \\ c \in ELG(r)}} VAR\_GRIDIO(r,t,c,j,s,IN) \right)$$

where:

- VAR_GRIDIO(r,t,c,n,s,IN) = new variable representing demand of electricity or other supply commodity *c* from node *n* in region *r* and timeslice *s*;
- VAR_COMPRD(r,t,c,s) = variable for total production of commodity c in region *r* and timeslice *s*;
- GR_ENDFR(r,t,n,com) = user input parameter specifying the fractions of electricity demand commodities *com* to be allocated to grid nodes *n* in region *r*;
- GR_ENDFR(r,t,com,com) = user input parameter specifying the estimated transmission losses of electricity demand commodity *com* in region *r*;
- GR_DEMFR(r,t,n,ts) = user input parameter specifying the fractions of total electricity demand to be allocated to grid nodes *n* in region *r* and timeslice *ts*;
- ELG(r) = set of electricity generation or other supply commodities in region *r*;
- ELD(r) = set of sectoral electricity or other demand commodities in region *r*;
- N(r) = set of grid nodes in region *r*.

Equation (22) is applied in region *r* only when the sectoral demand allocation approach is used, i.e. the parameter GR_ENDFR has been specified for region *r*. Equation (23) is applied in region *r* only when the total demand allocation approach is used, i.e. only the parameter GR_DEMFR has been specified for region *r*.

In addition to the equations presented above for allocating the generation and demand to the grid nodes, the VAR_GRIDIO variables must also satisfy the following conditions:

$$VAR\_GRIDIO(r,t,c,i,s,IN) \le VAR\_GRIDIO(r,t,c,i,s,OUT), \quad if\ GR\_GENLEV(r,c) = 2 \tag{24}$$

$$VAR\_GRIDIO(r,t,c,i,s,IN) = VAR\_GRIDIO(r,t,c,i,s,OUT), \quad if\ GR\_GENLEV(r,c) = 3 \tag{25}$$

where:

- GR_GENLEV(r,c) = user input parameter specifying the grid connection characteristic of electricity generation or other supply commodity *c* in region *r*, as follows:
    - GR_GENLEV(r,c) = 1 means that the commodity can be injected to the grid and withdrawn from the grid in a fully flexible manner;
    - GR_GENLEV(r,c) = 2 means that the commodity can be net-injected to the grid but can not be net-withdrawn from the grid (see (24));
    - GR_GENLEV(r,c) = 3 means that the commodity can neither be net-injected to the grid nor net-withdrawn from the grid, i.e. all local production of *c* at any grid node must be consumed locally at that node (see (25));

In order to complete the integration of the new grid variables into the original model, the VAR_GRIDIO variables must also be included in the commodity balances and peak equations of the electricity generation or other supply commodities, and in the commodity balances of the grid nodes. The terms to be included are the following:

$$\sum_{i \in N(r)} VAR\_GRIDIO(r,t,com,i,s,OUT) - VAR\_GRIDIO(r,t,com,i,s,IN) \tag{26}$$

$$\sum_{com \in ELG(r)} VAR\_GRIDIO(r,t,com,i,s,IN) - VAR\_GRIDIO(r,t,com,i,s,OUT) \tag{27}$$

In the commodity balances of the electricity generation or other supply commodities *com*, the term (26), multiplied by COM_IE, is added on the demand side, whereas in the peaking constraints, the term (26) is added on the production side with an opposite sign. In the commodity balances of the grid nodes the term (27) is added on the demand side.

<u>Remark:</u> Within TIMES, both the nodal generation and nodal load variables are internally represented in terms of energy. However, these can be easily converted into terms of power by the following simple relationships:

$$VAR\_GRIDIO(r,t,c,i,s,io)^{PW} = \frac{VAR\_GRIDIO(r,t,c,i,s,io)}{G\_YRFR(r,s)} \tag{28}$$

where:

- G_YRFR(r,s) = duration of timeslice ***s*** as fraction of a year.

2.4 Simplified N-1 Security Constraints

For N–1 security considerations for grids involving energy commodities, we may want to state that in each region the following condition holds:

$$\left|GEN(r,t,s) - LOAD(r,t,s)\right| \leq \alpha \cdot PMAX(r,t) \tag{29}$$

where:

- GEN(r,t,s) = total injection into the grid in region *r*;
- LOAD(r,t,s) = total withdrawal from the grid in region *r*;
- PMAX(r,t) = total capacity of inter-regional lines between *r* and other regions.

This constraint can be written in terms of TIMES variables as follows. For each region r, period t and timeslice s, we require that the following two conditions hold:

$$\begin{aligned} &\sum_{\substack{com \in ELG(r) \\ i \in N(r)}} VAR\_GRIDIO(r,t,com,i,s,IN) - VAR\_GRIDIO(r,t,com,i,s,OUT) \\ &\quad \leq GR\_XBND(r,t) \cdot G\_YRFR(r,s) \cdot \\ &\quad \sum_{\substack{com \in ELG(r) \\ TOP\_IRE(r2,c,r,com,p) \\ v \in RPT\_VINTYR(r,v,t,p)}} \begin{matrix} COEF\_CPT(r,v,t,p) \cdot PRC\_CAPACT(r,p) \cdot \\ (VAR\_CAP(r,v,t) + NCAP\_PASTI(r,v,p)) \end{matrix} \end{aligned} \tag{30}$$

$$\begin{aligned} &\sum_{\substack{com \in ELG(r) \\ i \in N(r)}} VAR\_GRIDIO(r,t,com,i,s,OUT) - VAR\_GRIDIO(r,t,com,i,s,IN) \\ &\quad \leq GR\_XBND(r,t) \cdot G\_YRFR(r,s) \cdot \\ &\quad \sum_{\substack{com \in ELG(r) \\ TOP\_IRE(r,com,r2,c,p) \\ v \in RPT\_VINTYR(r,v,t,p)}} \begin{matrix} COEF\_CPT(r,v,t,p) \cdot PRC\_CAPACT(r,p) \cdot \\ (VAR\_CAP(r,v,t) + NCAP\_PASTI(r,v,p)) \end{matrix} \end{aligned} \tag{31}$$

where:

- GR_XBND(r,t) = user input parameter α;
- G_YRFR(r,s) = duration of timeslice ***s*** as fraction of a year;
- COEF_CPT(r,v,t,p) = capacity transfer coefficient for vintage *v*;
- PRC_CAPACT(r,p) = conversion from capacity to activity units;
- VAR_NCAP(r,v,p) = newly installed capacity of process *p* and vintage *v*;
- NCAP_PASTI(r,v,p) = past installed capacity of process *p* and vintage *v*.

When the input attribute *GR_XBND(r,t)* dedicated for this purpose is defined, the constraints formulated above are generated for electricity add-on grids only, because we cannot assume that the same parameter *GR_XBND(r,t)* would be meant applicable to all grids. However, equivalent constraints can be introduced for other grids by defining them with user constraints. In the UC approach, one would only need to refer to the VAR_IRE variables instead of VAR_GRIDIO for getting the net injection to be bounded in each region.

2.5 Costs on Grid Flows

The TIMES implementation includes also a new cost attribute COM_CSTBAL, which can be used for defining cost coefficients on different grid flows (electricity or other gaseous commodities and emissions). As the mathematical formulation of including these cost coefficients should be quite obvious, the detailed formulation of the corresponding new objective function component *OBJBAL* is omitted here, and only a cursory formulation is presented:

$$VAR\_OBJ(r,OBJBAL,cur) = \sum_{n,t,s,type} \left(\begin{array}{c} COM\_CSTBAL(r,t,n,s,type,cur) \cdot COEF\_PVT(r,t,cur) \cdot \\ \left[\text{relevant flow variable expression} \right] \end{array} \right) \qquad (32)$$

where:

- VAR_OBJ(r,OBJBAL,cur) = new component for the TIMES objective variable VAR_OBJ;
- OBJ_CSTBAL(r,t,n,s,type,cur) = user input parameter specifying a cost of type *type* on grid flows at node *n* in region *r*, in timeslice *s* and with currency *cur*;
- COEF_PVT(r,t,cur) = present value coefficient for period t in region *r* and with currency *cur*.

The cost type index in the input parameter specifies the type of quantity the cost should be applied to, and can represent generation (*type*=PRD), imports (IMP), exports (EXP), net imports (NTX), consumption (CON), or net positive generation (NPG). See also the parameter description for more details on using this special cost parameter.

3. GAMS IMPLEMENTATION

3.1 Overview

As discussed in Section 2, certain special facilities have been implemented into TIMES for the modeling of grid systems for electricity and other gaseous commodities. These features can be useful, for example, for analyzing bottlenecks in the transmission system and integration of large amounts of variable renewable generation in the system, whereby the transmission lines may have to be particularly enforced in certain parts of the grid.

3.2 Parameters

3.2.1 Input parameters

All the parameters for the grid modeling features are available in the VEDA shell, where they may be specified. Most of the new parameters have the prefix '**GR_**' or '**GG_**' in the GAMS code of the model generator. The parameters are discussed in more detail below:

1. The parameter **PRC_REACT(r,y,p)** can be used for specifying the reactance of transmission line *p* (unit: relative).
   - For inter-regional lines, the parameter can be defined on either side of the link (if defined on both sides, the maximum value is taken)
   - The parameters also define the lines to be included in the grid topology
   - If zero reactance is specified, the line is included in the grid but is excluded from the power flow equations.

Table 1. Input parameters for the dedicated TIMES grid modeling features.

Parameter	Description
PRC_REACT(r,y,p)	Reactance of grid transmission line p in region r
GR_PTDF(r,y,p,n,r2,n2)	Power transfer distribution factor of grid transmission line p in region r subject to node n2 in region r2
GR_GENLEV (r,c)	Grid connection category for electricity generation commodity c
GR_GENMAP(r,p,u)	Mapping of generation technology p to generation of type u
GR_GENFR (r,y,n,u)	Fraction of electricity generation of type u allocated to grid node n
GR_DEMFR (r,y,n,s)	Fraction of total electricity demand allocated to grid node n
GR_ENDFR(r,y,n,c)	Fraction of sectoral electricity demand allocated to grid node n
GR_XBND(r,y)	Maximum power level of net imports to / exports from region r
COM_CSTBAL (r,y,n,s,type,cur)	Cost on specific type of grid flow (energy flow) at node n
GG_DENS(r,n)	Density of gas commodity n (flow units per mass flow units)
GG_GAMMA(r,y,p,n)	Compressor factor for pipeline p at node n ($\Gamma \geq 1$)
GG_KGF(r,y,p,n)	Weymouth constant for pipeline p at node n (mass units per *p* per *hour*)
GG_KLP(r,y,p,n)	Line-packing constant for pipeline p at node n (mass units per *p)*
GG_PRBD(r,y,n,bd)	Lower and upper bounds for the pressure variables at node n
GG_PP(r,y,p,n,bd,omg)	Fixed pressure points for node n in the Weymouth Taylor expansion

2. The parameter `GR_PTDF(r,y,p,n,r2,n2)` can be used for specifying the power transfer distribution factor of grid line *p* subject to injection into node n2 in region r2 (unit: relative).

3. The parameter `GR_GENLEV(r,c)` can be used for defining the generation level indicator for generated electricity commodity *c*:

   - GR_GENLEV(r,c) = 1 means that the commodity can be injected to the grid and withdrawn from the grid in a fully flexible manner;
   - GR_GENLEV(r,c) = 2 means that the commodity can be net-injected to the grid but can not be net-withdrawn from the grid (see (24));
   - GR_GENLEV(r,c) = 3 means that the commodity can neither be net- injected to the grid nor net-withdrawn from the grid, i.e. all local production of *c* at any grid node must be consumed locally at that node (see (25));

4. The parameter `GR_GENMAP(r,p,u)` can be used for defining the mapping of generating technology p to unit type *u*. Normally, it would have the value 1.

5. The parameter `GR_GENFR(r,y,n,u)` can be used for specifying the fraction of the generation from unit type *u* to be allocated to grid node *n*.

6. The parameter `GR_DEMFR(r,y,n,s)` can be used for defining the fraction of the total demand in timeslice *s* to be allocated to grid node *n*. The parameter is only taken into account in region *r* if no `GR_ENDFR` parameter is defined for that region.

7. The parameter `GR_ENDFR(r,y,n,com)` can be used for defining in region *r*

   - (1) the fraction of the sectoral demand *com* to be allocated to node *n* (*n*≠*com*)
   - (2) the approximate transmission losses for the sectoral demand com (*n*=*com*), as a fraction of the demand level

8. The parameter `GR_XBND(r,y)` can be used for defining the maximum power level of net imports to / exports from region *r* (i.e. total net demand from / net injection to the grid in region *r*), in proportion to the total capacity of the transmission lines between R and other regions (units: fraction).

9. The parameter `COM_CSTBAL(r,y,n,s,type,cur)` can be used for defining a cost for the generation (*type*=PRD), imports (IMP), exports (EXP), net imports (NTX), consumption (CON), or net positive generation (NPG) at node N, optionally differentiated by timeslice *s* (unit: currency unit / energy unit).

10. The parameter `GG_DENS(r,n)` should be used for defining the density of the gas flow, expressed as a ratio of the standard model units to the mass flow units.

11. The parameter `GG_GAMMA(r,y,p,n)` can be used for defining a compressor factor for pipeline **p**, increasing the pressure at node **n** (unit: relative).

12. The parameter `GG_KGF(r,y,p,n)` can be used for defining the Weymouth constant for pipeline **p** with input node **n** (unit: mass units per pressure units per hour).

13. The parameter `GG_KLP(r,y,p,n)` can be used for defining the linepack constant for pipeline **p** with input node **n**, as well as a minimum linepack level (n='`ACT`').

14. The parameter `GG_PRBD(r,y,n,bd)` should be used for defining the minimum and maximum gas pressures at node **n**, which are mandatory input attributes.

Table 2. Variables for the grid modeling features in TIMES.

Variable	Description
VAR_COMAUX(r,t,n,s)	The delta variables for the voltage phase angles.
VAR_GRIDIO(r,t,c,n,s,io)	The variable representing the injection/withdrawal of grid-connected commodity **c** to/from grid node **n** in timeslice s.
VAR_GNETINJ(r,t,n,s)	The artificial net injection variables in the PTDF formulation, for node **n** and timeslice **s**.
VAR_GVIRT(r,t,p,n,s)	Variable for the virtual flows over candidate grid lines **p** for node **n** and timeslice **s**.
VAR_XCAP(r,t,p)	Virtual capacity variable for the PTDF formulation with transmission expansion planning
VAR_GG_PR(r,t,n,s)	Variable for the gas pressure at node **n** and timeslice **s**.
VAR_GG_MF(r,t,p,m,r2,u,ts)	Variable for the mass flow of gas from node **m** to node **u**.
VAR_GG_PDIF(r,t,p,m,r2,u,ts)	Variable for the pressure difference in pipeline **p** between node **m** and node **u** timeslice **s**.
VAR_GG_PADD(r,t,p,m,ts)	Variable for the pressure boost in pipeline **p** at node **m**.
VAR_GG_Y(r,t,p,m,r2,u,ts)	Binary variable for the gas flow direction.
VAR_GG_STEP(r,t,p,c,ts,j,j)	Step variables for alternate Weymouth linearization

3.2.2 Reporting parameters

The energy flows in the grid are reported normally in the reporting parameters for process flows, and the grid node balances in the reporting parameters for commodity balances.

The annual costs related to the new parameter COM_CSTBAL are reported in the CST_COMC reporting parameter and in the variable cost component of the REG_ACOST reporting parameter. The corresponding objective component is reported in the variable cost component of the REG_WOBJ reporting parameter.

3.3 Variables

There are only a few sets of new variables introduced in the implementation of the grid modeling features, which are shown in Table 2.

The variables VAR_COMAUX represent the voltage phase angle variables for each grid node, timeslice and period. The new variables VAR_GRIDIO represent the amounts of each electricity generation or other supply commodity injected into each of the grid nodes, and the corresponding amounts of these commodities withdrawn from each of the grid nodes. Also these injection/load variables are timeslice and period specific.

As there are thus essentially no other changes to the model variables, the user is referred to Chapter 4 of the TIMES Reference Manual for details on the variables of the model.

3.4 Equations

Below in Table 3 the few new equations related to the grid modeling features are listed and briefly described. The equations include the linear DC power flow equations (section 2.1) and gas pipeline grids equations (section 2.2), equations for the allocation of the generation and demands to the grid nodes of an optional add-on grid (section 2.3), and the simplified N–1 security constraints described earlier in section 2.4.

Note that the equation for the augmented final objective function, **EQ_OBJ**, has the same name as in standard TIMES, only the definition of this equation is different when costs have been defined on the flows in the grid by using the **COM_CSTBAL** parameter. Similarly, also the objective variable ObjZ has the same name as in standard TIMES.

Table 3. Equations for the grid modeling features in TIMES.

Equation	Description
EQ_GR_POWFLO(r,t,n,s,reg,com)	Linear DC power load flow equations for each grid line – phase-angle formulation.
EQ_GR_PTDFLO(r,t,p,n,s)	Linear DC power load flow equations for each grid pipeline – PTDF formulation.
EQ_GR_VIRTCP(r,t,p,n,s,bd)	Implied capacity requirement of virtual powerflow.
EQ_GR_VIRTBD(r,t,p)	Bound on the virtual powerflow capacity.
EQ_GR_GENALL(r,t,n,s,cg)	The equations for allocating electricity generation (or other grid-connected supply) to grid nodes.
EQ_GR_DEMALL(r,t,n,s)	The equations for allocating electricity demands (or other grid-connected demands) to grid nodes.
EQ_GR_XBND(r,t,s,ie)	The simplified N-1 security constraints for the total amount of imported / exported electricity into / from region r.
EQ_OBJBAL(r,cur)	The equations for the costs on the grid flows, which are summed into a new component of VAR_OBJ.
EQ_GG_MFLO(r,t,p,m,r2,u,ts)	Equations linking the IRE variables to mass flows
EQ_GG_GAMA(r,t,p,m,r2,u,ts)	Pressure boost defining equation
EQ_GG_MBND(r,t,p,m,r2,u,ts)	Mass flow bounds in bi-directional case
EQ_GG_PDIF1(r,t,p,m,r2,u,ts)	Pressure difference bounds in bi-directional case
EQ_GG_PDIF2(r,t,p,m,r2,u,ts)	Pressure difference defining equation
EQ_GG_HLIP(r,t,p,m,r2,u,ts)	Line-pack storage balance equation
EQ_GG_HLEV(r,t,p,m,r2,u,ts)	Defining equations for line-pack levels
EQ_GG_WEYMTX(r,t,p,m,r2,u,ts,o)	Weymouth equation with Taylor expansion
EQ_GG_WEYMST(r,t,p,m,r2,u,ts)	Weymouth equation with stepped linearization

3.5 Changes in Model Generator Code

The implementation required only small modifications to the existing code and only one new component in the model generator source code. The new and modified code components are listed in Table 4.

The new source file **powerflo.vda** contains most of the new code for the grid modeling features. This file is automatically called from the file equ_ext.vda, if either the parameter PRC_REACT or the parameter COM_CSTBAL has been defined. Also the new variables and equations as well as the additional term in the objective function have been conditionally defined in this file. Finally, the reporting file cost_ann.rpt also conditionally calls the file powerflo.vda whenever the parameter COM_CSTBAL has been defined.

The second new file **gasgrids.vda** includes the new code for the features specific to gas grids.

Table 4. New and modified files in the TIMES model generator code.

Added file	Description
powerflo.vda	Declarations, preprocessing and equations for the elc grid features
gasgrids.vda	Declarations, preprocessing and equations for the gas grid features
Modified file	**Description of changes made**
cal_ire.mod	Conditional addition of grid variable terms to balance equations
mod_vars.mod	Conditional additions to variable declarations
initmty.vda	Declarations for the grid modeling features
init_ext.vda	Initializantions for the grid modeling features
prep_ext.vda	Conditional calling of powerflo.vda for parameter interpolation
pp_prelv.vda	Conditional calling of powerflo.vda for pre-processing
coef_ext.vda	Conditional calling of powerflo.vda for pre-processing
equ_ext.vda	Conditional calling of powerflo.vda for equation definitions
mod_ext.vda	Conditional additions for the model statement
cost_ann.rpt	Conditional calling of powerflo.vda for cost reporting purposes

4. USER'S REFERENCE

4.1 Activating the Grid Equations

The linearized DC power flow equations are automatically activated in TIMES whenever the PRC_REACT, GR_PTDF or the GG_KGF parameter is specified for some transmission lines. However, the activation of the feature can be done also manually by using the following settings in the run file (the trailing comment part starting from '!' should not be included):

```
$  SET POWERFLO YES   ! Activate power flow analysis feature
```

Nonetheless, in any case, the power flow equations are generated only for those lines that have a non-zero value specified either for the PRC_REACT or the GR_PTDF parameter.

The automated allocation facilities for the add-on-grid approach are also activated by the corresponding input parameters, but for electricity grids, to ensure that the grid topology is fully defined they may require that the PRC_REACT or GR_PTDF parameters are appropriately defined for all grid connections. Zero values for PRC_REACT can be used for including lines in the grid but excluding them from power flow equations.

4.2 Defining Electricity Grids

- In general, all transmission lines in electricity grids should be bi-directional trade processes (unless only one direction is technically possible);
- All inter-regional transmission line processes should have unique names, but intra-regional processes may utilize the same process names in all regions;
- Grid nodes are represented in TIMES by specific electricity commodities;
- The generation and demand of electricity can be either explicitly modeled for each of the grid nodes, or they can also be automatically allocated to the nodes;
- If the mechanism for allocating generation and demands to add-on grid nodes is to be used, the nodes in each region should be defined using completely distinct new commodity names (of type NRG+ELC, at DAYNITE level), e.g. GN1, …, GN4;
- All lines should, in general, have either investment costs defined or PRC_RESID / NCAP_PASTI specified, or both (to avoid arbitrary flows in both directions);
- For any inter-regional line between regions R1 and R2, one should preferably define the PRC_RESID/NCAP_PASTI and investment costs only on either side of the link (either in region R1 or in R2), to have a *single* consistent line capacity;
- For all bi-directional transmission lines, availability factors can also be defined to be direction-dependent by using the NCAP_AFC parameter instead of NCAP_AF:
  - For inter-regional lines, by defining NCAP_AFC both on the traded commodity (for imports) and on NRG (for exports) in the region that has the line capacity;
  - For intra-regional lines, by defining NCAP_AFC on both of the two grid node commodities involved.

4.3 Transmission Losses

- Optional transmission losses can be defined for each transmission line, and should be specified by using a negative IRE_FLOSUM(reg,y,line,n,ts,'IMP',n,'OUT') parameter on the imports into node ***n***. In other words, one should not use the IRE_FLO parameter for defining transmission efficiencies, but instead using the IRE_FLOSUM parameter is advised.
- If the allocation methods are used for the generation and demands (see section 4.5), the intra-regional transmission losses have usually been already accounted in the original model. In that case, transmission losses should probably not be added to the intra-regional grid-lines, to avoid double counting of the losses.

4.4 Linear DC Power Flow Equations

- The reactances (inverse of susceptance) for the transmission lines are defined by PRC_REACT(r,y,p). For inter-regional lines, the parameter can be defined on either side of the link (if defined on both sides, the maximum value is taken);
- Alternatively, PTDF factors can be defined by GR_PTDF(r,y,p,n,r2,k);
- The reactance / PTDF parameters also define the lines included in the grid topology;
- If zero reactance is specified, the line is included in the grid but is excluded from the power flow equations; a zero PTDF on the other hand disables the flow in a line.
- Transmission expansion planning is genuinely only supported with the PTDF formulation. When using the phase-angle formulation, any new flows through new lines installed are basically treated like additional net injection or withdrawal.
- Under the PDTF formulation, candidate new lines can be consistently included by including them in the PTDF factors, as if those lines had been already installed. The candidate lines must also have capacity bounds, either via NCAP_DISC (MIP option) or by specifying CAP_BND (LP relaxation).

4.5 Specification of Gas Grids with Weymouth equations

- All grids for gaseous commodities must have **nodes** represented by commodities that have one of the NRG_GRID types (ELC, GAS, or LTHEAT). Under VEDA, the grid type can be defined in the **FI_Comm** table (regardless of the commodity type).
- All pipelines processes **p** in the grid must defined between a single pair of regions only, and between a single pair of grid node commodities. However, the pipelines can also be defined between additional pairs of commodities, which are not grid nodes (multi-carrier pipelines). Pipelines can be unidirectional or bi-directional.
- All pipelines in the gas grid must have the Weymouth constant (GG_KGF) specified (for the input node or at least either of the input nodes in bi-directional case).
- All the grid nodes must have pressure bounds (LO/UP) defined by GG_PRBD.
- At least one of the grid nodes must have the GG_DENS parameter specified, for defining the conversion from the mass flows to the standard TIMES IRE flows.
- A pressure boost by a compressor can be modeled at the beginning of each pipeline by specifying GG_GAMMA, giving the maximum proportional pressure increase.
- Binary variables for bidirectional flows can be activated with any data for GG_PP.

- All pipelines of gas grids must have **one** output grid node in the PCG in the region of the output node (if any). Bi-directional intra-regional lines require making a choice.
- If a pipeline is a **multi-carrier pipeline**, i.e. defined between **several pairs of energy or material carriers** distributed through the same pipeline, only **one output grid node** should be in the PCG in each region where it has an output node (as mentioned above), and any other commodities associated with that same node should also be included in the PCG. For example, a pipeline distributing GASNGA1 and GASBIO1 (input) to GASNGA2 and GASBIO2 (output) should have both GASNGA2 and GASBIO2 in the PCG, but the associated grid nodes should be chosen to be either GASNGA1 and GASNGA2, or GASBIO1 and GASBIO2. The choice of the grid node carrier will only affect the linepack equation. All the other gas flow equations will apply to the total pipeline flow (in this case the flow of GASNGA + GASBIO).

4.6 Allocating Generation and Demands to Add-on Grid Nodes

- All electricity generation (or other supply) commodities ***com*** to be included in the allocation of generation must have the GR_GENLEV(reg,com) parameter defined:
  - GR_GENLEV(reg,com)=1 signifies that the commodity can be withdrawn from and injected to the grid in a fully flexible way;
  - GR_GENLEV(reg,com)=2 signifies that the commodity can only be injected to, but not withdrawn from the grid in net terms;
  - GR_GENLEV(reg,com)=3 signifies that the commodity can neither be withdrawn from nor injected to the grid in net terms;
- All generation or supply technologies ***prc*** should be mapped to unit types ***u*** by using the GR_GENMAP(reg, prc, u) parameter (normally with the value 1)
- All unit types should be allocated to the grid nodes ***n*** by using the GR_GENFR(reg, y, n, u) parameter, representing the fraction of the generation of type ***u*** to be allocated to the node ***n*** in region ***reg***;
- The regional demand for electricity (or other commodity) ***elcs*** can be allocated to the grid nodes in two alternative ways A and B, as follows:
  - A. By allocating the total demand in each timeslice to the grid nodes; or
  - B. By allocating each sectoral electricity demand to the grid nodes.
- If using alternative A, one should use the GR_DEMFR(reg, y, n, ts) parameter for defining the fraction of the total demand in timeslice ***ts*** to be allocated to the grid node ***n*** in region ***reg***.
- If using alternative B, where the demand for each sectoral commodity ***elcs*** is to be allocated to each grid node ***n***, one should use the following approach:
  - Use the GR_ENDFR(reg, y, n, elcs) parameter for specifying the fraction of the sectoral demand for ***elcs*** to be allocated to the node ***n*** in region ***reg***;
  - As the demands usually represent amounts after endogenous transmission losses, one can additionally specify approximate transmission losses for each sectoral demand ***elcs*** by using the GR_ENDFR(reg, y, elcs, elcs) parameter, representing the amount of losses in proportion to the demand; otherwise transmission losses are allocated to an (arbitrary) single node;
- All the fractional allocation parameters are automatically normalized by TIMES, such that the sum of them over the grid nodes (or unit types) equals to 1.

4.7 Notes on other types of grids

- The add-on grid facility, where the supply and demand can be allocated to grid nodes, is now available irrespective of modeling also with one of the DC power flow or Weymouth gas flow formulations.
- If found useful, the PTDF formulation can also be applied to other than electricity grids, for example to gas grids, instead of using the Weymouth formulation.

4.8 Additional Parameters

- Costs on the total generation into grid node ***n***, or on the total or net transmission to/from node ***n***, can be specified by using the COM_CSTBAL(reg,y,n,s,type,cur) parameter, either differentiated by timeslice ***s***, or for all timeslices (***s***=ANNUAL). The ***type*** can be any of the following:
  - PRD – production (generation into the node)
  - CON – consumption (demand from the node)
  - IMP – transmission into the node from other grid nodes
  - EXP – transmission from the node to other grid nodes
  - NTX – net imports to the node (IMP – EXP)
  - NPG – net positive generation injected into the node (Max(0,$G_{r,n} – L_{r,n}$))
- The total net imports to and exports from region ***reg***, in proportion to the total transmission line capacity between ***reg*** and other model regions, can be bounded by using the GR_XBND(reg,y) parameter. The bound is an upper bound. As the net exports are equal to the total net injection $G_r – L_r$, the constraints generated are equivalent to the equations $|G(\text{reg,y,s}) – L(\text{reg,y,s})| \leq \text{GR\_XBND(reg,y)} \bullet \sum P_{max}$ for each year ***y*** and timeslice ***s***.

4.9 Specification of Input Parameters

The following Table 5 lists the available user-input parameters. The **PRC_REACT** parameter is required to be provided by the user to define the grid topology for all of the grid modeling features described here. The following indexes are used in the index domain of the parameters:

Index	Meaning	Index	Meaning
r	Region	c	Commodity (other than grid node)
datayear	Period / Milestone year	s	Timeslice
p	Process	u	Unit type
n	Grid node commodity	cur	Currency
bd	Bound type	omg	Omega (pressure points)

Table 5: Input parameters for TIMES Grid Features

Input parameter (Indexes)[1]	**Related parameters**[2]	**Units / Ranges & Default values & Default inter-/extrapolation**[3]	**Instances**[4] (Required / Omit / Special conditions)	**Description**	**Affected equations or variables**[5]
COM_CSTBAL (r,datayear,n,s,type, cur)	COM_CSTNET, COM_CSTPRD	• TIMES cost unit • [0, INF); default value: none • Default i/e[6]: standard	• Required for each commodity for which grid balance costs are to be accounted.	Cost on generation or other supply, imports, exports, net imports, consumption, or net positive supply at grid node n.	• EQ_OBJBAL
PRC_REACT (r,datayear,p)		• Relative value • [0, INF); default value: none • Default i/e: standard	• Required for each transmission line to be included in the grid system	Reactance of grid transmission line p.	• EQ_GR_POWFLO
GR_GENLEV (r,c)	GR_GENLEV, GR_GENMAP, GR_GENFR	• Value of 1, 2 or 3. • default value: none	• Required for all electricity generation or other supply commodities related to grid	Characterization of grid connectivity of generation or other supply commodity	• EQ_GENALL • EQ_DEMALL
GR_GENMAP (r,p,u)	See above	• Dimensionless • [0, 1] (usually 1); default value: none	• Required for all generation or supply processes and exogenous imports / exports (if any)	Mapping of generation or other supply technologies to generation or other supply unit types.	• EQ_GENALL

[1] The first row contains the parameter name, the second row contains in brackets the index domain over which the parameter is defined.
[2] This column gives references to related input parameters or sets being used in the context of this parameter as well as internal parameters/sets or result parameters being derived from the input parameter.
[3] This column lists the unit of the parameter, the possible range of its numeric value [in square brackets] and the inter-/extrapolation rules that apply.
[4] An indication of circumstances for which the parameter is to be provided or omitted, as well as description of inheritance/aggregation rules applied to parameters having the timeslice (**s)** index.
[5] Equations or variables that are directly affected by the parameter.
[6] Abbreviation i/e = inter-/extrapolation

Input parameter (Indexes)[1]	**Related parameters[2]**	**Units / Ranges & Default values & Default inter-/extrapolation[3]**	**Instances[4]** (Required / Omit / Special conditions)	**Description**	**Affected equations or variables[5]**
GR_GENFR (r,datayear,n,u)	See above	• Dimensionless • [0, 1], default value: none • Default i/e: standard	• Automatically normalized	Fraction of generation or supply from unit type u to be allocated to grid node n.	• EQ_GENALL
GR_DEMFR (r,datayear,n,s)	GR_ENDFR GR_GENFR	• Dimensionless • [0, 1]; default value: none • Default i/e: standard	• Only taken into account in region r if GR_ENDFR has <u>not</u> been specified in r • Automatically normalized	Fraction of total demand to be allocated to grid node n in timeslice s.	• EQ_GR_DEMALL
GR_ENDFR (r,datayear,n,c)	GR_DEMFR GR_GENFR	• Dimensionless • [0, 1]; default value: none • Default i/e: standard	• Automatically normalized	Fraction of sectoral demand *c* to be allocated to grid node *n*.	• EQ_GR_DEMALL
GR_PTDF (r,datayear,p,n,r2,k)	PRC_REACT	• Dimensionless • [0, ∞); default value: none • Default i/e: standard	• Required for each transmission line to be included in the grid system (on either side)	PTDF factor for transmission line **p**, out of node **n**, with respect to reference node **k** in region r2	• EQ_GR_PTDF
GR_XBND (r,t)	IRE_XBND	• Dimensionless • [0, 1]; default value: none • Default i/e: none	• Optional bounding parameter	Line overloading coefficient α defining the maximum power level of imports/exports	• EQ_GR_XBND
GG_DENS (r,n)		• Flow units per gas flow mass units • (0, ∞); default value: none	• Mandatory gas parameter (energy or mass density) for at least one node in each disjoint gas grid network	Conversion factor from the mass unit of the gas flow to the TIMES IRE process flow unit	• EQ_GG_MFLO

Input parameter (Indexes)[1]	Related parameters[2]	Units / Ranges & Default values & Default inter-/extrapolation[3]	Instances[4] (Required / Omit / Special conditions)	Description	Affected equations or variables[5]
GG_GAMMA (r,datayear,p,n)	GG_PRBD	• Dimensionless • [1, 10]; default value: 1 • Default i/e: standard	• Required for each pipeline with a compressor, at the input node	Factor describing the pressure increase from the nodal pressure of the input node	• EQ_GG_GAMA • EQ_GG_WEYMTX • EQ_GG_WEYMST
GG_KGF (r,datayear,p,n)	GG_KLP	• Mass units per pressure units per hour • [0, ∞); default value: none • Default i/e: standard	• Required for all gas grid pipeliness, at the input node	Weymouth constant for gas flow in pipeline p, out of node n, expressed in mass units per pressure units per hour	• EQ_GG_WEYMTX • EQ_GG_WEYMST
GG_KLP (r,datayear,p,n)	GG_KGF	• Mass units per pressure • [0, ∞); default value: none • Default i/e: standard	• Optional line-pack constant when line-pack is modeled	Constant for the gas flow in pipeline p out of node n, expressed in mass units per pressure units	• EQ_GG_HLIP • EQ_GG_HLEV
GG_PP (r,year,p,n,bd,omg)	GG_PRBD	• Pressure units • (0, ∞); default value: none • Default i/e: standard	• Required only when using the linearized Taylor expansion formulation of the Weymouth equation • Each point omg required for both input and output node, and all pairs must always have input pr. > output pr.	Sample pressure points omg for node n of line p, where bd=UP/LO indicates direction of flow such that pr(UP) > pr(LO)	• EQ_GG_WEYMTX
GG_PRBD (r,datayear,n,bd)	GG_GAMMA	• Pressure units • (0, ∞); default value: none • Default i/e: standard	• Required for all nodes of the gas grid • If either bound is missing, set equal to other bound	Minimum (bd=LO) and maximum (bd=UP) bounds for nodal gas pressure of node n in region r.	• EQ_GG_WEYMTX • EQ_GG_WEYMST

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