Homework Assignment 1

Data Structures and Algorithms I, WT 2021

Due: 22. 10. 2021

1. (2 Points) We use o-notation to denote an upper bound that is not asymptotically tight. We formally define $o(g(n))$ ("little-oh of g of n") as the set

$$o(g(n)) = \{\, f(n) \mid \text{for any positive constant } c > 0, \text{ there exists a constant } n_0 > 0 \text{ such that } 0 \le f(n) < cg(n) \text{ for all } n \ge n_0 \,\}.$$

Similarly, we use ω-notation to denote a lower bound that is not asymptotically tight. We formally define $\omega(g(n))$ ("little-omega of g of n") as the set

$$\omega(g(n)) = \{\, f(n) \mid \text{for any positive constant } c > 0, \text{ there exists a constant } n_0 > 0 \text{ such that } 0 \le cg(n) < f(n) \text{ for all } n \ge n_0 \,\}.$$

Using the definitions from above, prove that $o(g(n)) \cap \omega(g(n))$ is the empty set.

2. (6 Points) We use the notation $f^{(i)}(n)$ to denote the function $f(n)$ iteratively applied i times to an initial value of n. Formally, let $f(n)$ be a function over the reals. For non-negative integers i, we recursively define

$$f^{(i)}(n) = \begin{cases} n, & \text{if } i = 0, \\ f(f^{(i-1)}(n)), & \text{if } i > 0. \end{cases}$$

Let $\lg^* n = \min\{i \ge 0 \mid \lg^{(i)} n \le 1\}$. Note that $\lg^{(i)} n$ is defined only if $\lg^{(i-1)} n > 0$. Which is asymptotically larger: $\lg(\lg^* n)$ or $\lg^*(\lg n)$? **Hint**: Note that $\lg^*(2^n) = 1 + \lg^*(n)$.

3. (6 Points) Using the basic definition of Θ-notation, prove $\forall a, b \in \mathbb{R}$ with $b > 0$, that n^b is an asymptotically tight bound for $(n + a)^b$.