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1) $T(n) = aT(n/a) + n^b$; $a, b \in \mathbb{N}_{\geq 2}$ $T(n) = 1 \ \forall n \leq 2$

① $T\left(\frac{n}{a}\right) = aT\left(\frac{n}{a^2}\right) + \left(\frac{n}{a}\right)^b$

$\Rightarrow T(n) = a\left(aT\left(\frac{n}{a^2}\right) + \left(\frac{n}{a}\right)^b\right) + n^b$

$T(n) = a^2T\left(\frac{n}{a^2}\right) + a\left(\frac{n}{a}\right)^b + n^b = a^2T\left(\frac{n}{a^2}\right) + a^1\left(\frac{n}{a^1}\right)^b + a^0\left(\frac{n}{a^0}\right)^b$

② $T\left(\frac{n}{a^2}\right) = \cancel{a}\,aT\left(\frac{n}{a^3}\right) + \left(\frac{n}{a^2}\right)^b$

$\Rightarrow T(n) = a^2\left(aT\left(\frac{n}{a^3}\right) + \left(\frac{n}{a^2}\right)^b\right) + a\left(\frac{n}{a}\right)^b + n^b$

$= a^3T\left(\frac{n}{a^3}\right) + a^2\left(\frac{n}{a^2}\right)^b + a\left(\frac{n}{a}\right)^b + n^b$

$= a^kT\left(\frac{n}{a^k}\right) + \sum_{i=0}^{i=k-1}\left(a^i\cdot\left(\frac{n}{a^i}\right)^b\right)$

Ist ~~...~~ beendet, wenn $T\left(\frac{n}{a^k}\right) = T(1)$, also

wenn $\frac{n}{a^k} = 1 \mid \cdot a^k \Rightarrow n = a^k \Rightarrow \log_a(n) = k$

$\Rightarrow T(n) = a^{\log_a(n)}\,T\left(\frac{n}{a^{\log_a(n)}}\right) + \sum_{i=0}^{\log_a(n)-1}\left(a^i\cdot\left(\frac{n}{a^i}\right)^b\right)$

$\cancel{T(n)} = n\cdot T(1) = n\cdot 1 = n \Rightarrow T(n) = n + \sum_{i=0}^{\log_a(n)-1} a^i\cdot\left(\frac{n}{a^i}\right)^b$

$= n + \sum a^i\left(\frac{n^b}{a^{i\cdot b}}\right) = n + \sum n^b\cdot\left(\frac{a^i}{a^{i\cdot b}}\right) = \cancel{n + n \sum \frac{a^i}{a}} =$

$n + n^b\sum_{i=0}^{\log_a(n)-1}\left(\frac{a^i}{a^{i\cdot b}}\right) = n + n^b\cdot\sum_{i=0}^{\log_a(n)-1}\left(\frac{a}{a^b}\right)^i = n + n^b\cdot\sum\left(\frac{1}{a^{b-1}}\right)^i$

$= n + n^b\cdot\sum_{i=0}^{\log_a(n)-1}\left(\frac{1}{a^{(b-1)\cdot i}}\right) \Rightarrow n + n^b\cdot\frac{a^b}{a^b - a} =$ (geom. Reihe!!)

$= n + n^b\cdot\frac{a^b}{a^b - a}$ → konstant und > 0, auch da $b \geq 2$:

$T(n) = n + n^b\cdot\frac{a^b}{a^b - a} = \Theta\left(n^b\cdot\frac{a^b}{a^b - a}\right) = \boxed{\Theta(n^b)}$

2) $T(n) = T(n-2) + \log(n)$, $T(n) = O(1)\ \forall n \le 2$

[redacted]

(1) $T(n-2) = T(n-2-2) + \log(n-2) = T(n-4) + \log(n-2)$

$T(n) = T(n-4) + \log(n-2) + \log(n)$

(2) $T(n-4) = T(n-4-2) + \log(n-4) = T(n-6) + \log(n-4)$

$T(n) = T(n-6) + \log(n-4) + \log(n-2) + \log(n)$

$T(n) = T(n-3\cdot 2) + \log(n-2\cdot 2) + \log(n-1\cdot 2) + \log(n-0\cdot 2)$

$T(n) = T(n-2k) + \sum_{i=0}^{k-1}(\log(n-2i))$

~~$T(n)$~~ Beendet wenn $T(n-2k) = T(1)$, $\Rightarrow n-2k = 1$

$2k = n-1 \Rightarrow k = \frac{(n-1)}{2} \Rightarrow T(n) = \cancel{T\left(n-2\left(\frac{n-1}{2}\right)\right)}$

$T\left(n-2\left(\frac{(n-1)}{2}\right)\right) = T(n-(n-1)) = T(n-n+1) = T(1)$ ✓

$\Rightarrow T(n) = T(1) + \sum_{i=0}^{\frac{(n-1)}{2}-1} \log(n-2i) = O(1) + \sum_{i=0}^{\frac{(n-3)}{2}}(\log(n-2i))$

$\Rightarrow$ Abschätzen! $\rightarrow$ Obere Schranke: Die Summe ist sicherlich kleiner als $\sum_{i=0}^{\frac{(n-3)}{2}} \log(n-0)$, da $\log(n-0)$ der größte Summand ist! $\Rightarrow$

$O(1) + \sum_{i=0}^{\frac{(n-3)}{2}} \log(n-2i) \le O(1) + \sum_{i=0}^{\frac{(n-3)}{2}} \log(n-0) = O(1) +$

$+ \frac{(n-3)}{2} \cdot \log(n) \overset{O(1)}{=} (1/2) \cdot (n-3) \log(n) = \cancel{(1/2)\, n \log(n)}$

$= O(1) + (1/2) \cdot (n \log(n) - 3 \log(n)) = O(n \log n)$

konstanter Faktor

negativ

obere Schranke

~~Untere Schranke: $O(1) + \sum_{i=0}^{(n-3)} \log(n-2i) \geq O(1)$~~

Untere Schranke:

$$O(1) + \sum_{i=0}^{\frac{(n-3)}{2}} \log(n-2i) \geq O(1) + \sum_{i=0}^{\frac{(n-3)}{2}} \log\left(n-2\left(\frac{(n-3)}{2}\right)\right)$$

$$= O(1) + \sum \log(n-(n-3)) = O(1) + \sum \log(\cancel{n}-\cancel{n}+3)$$

$= O(1) + \sum_{i=0}^{\frac{(n-3)}{2}} \log(3)$ -> Einzelnen Summenglieder sind wieder unabhängig vom Iterator "i" -> auflösen!

$$= O(1) + \left(\frac{(n-3)}{2}\right)\log(3) = O(1) + \frac{\log(3)}{2}\cdot(n-3) =$$

$$O(1) + \frac{\log(3)}{2}\cdot n - \frac{\log(3)}{2}\cdot 3 = \underline{\underline{\Omega(n)}}$$

konstanter Faktor

konstant

Also $T(n) \in O(n \log n)$ und $T(n) \in \Omega(n)$

3) $T(n) = T(\sqrt{n}) + \Theta(\log_2(\log_2(n)))$

~~Sei m = ld ld~~ Sei $m = ld(ld(n))$, $m = ld\, ld\, n \mid 2^{()}$

$2^m = ld\, n \mid 2^{()} \Rightarrow 2^{(2^m)} = n \Rightarrow \sqrt{n} = \sqrt{2^{(2^m)}} = 2^{\frac{(2^m)}{2}}$

$\sqrt{n} = 2^{2^{m-1}}$; $T(2^{2^m}) = T(2^{2^{m-1}}) + \Theta(m)$

Sei $S(m) = T(2^{2^m})$, $\Rightarrow T(2^{2^{m-1}}) = S(m-1)$?

$S(m) = S(m-1) + \Theta(m)$

① $S(m-1) = S(m-2) + \Theta(m-1) \overset{?}{=} S(m-2) + \Theta(m)$

$\Rightarrow S(m) = S(m-2) + \Theta(m) + \Theta(m) = S(m-2) + 2\Theta(m)$

② $S(m-2) = S(m-3) + \Theta(m-2) = S(m-3) + \Theta(m)$

$\Rightarrow S(m) = S(m-3) + \Theta(m) + 2(\Theta(m)) = S(m-3) + 3\Theta(m)$

~~Abbruchbedingung: S(m-3) m-3 = 1 |+3 ⇒ m = 4 also~~

$S(m) = S(m-k) + k\Theta(m) \Rightarrow$ Abbruchbedingung:

$m-k = 1$, also wenn $S(1)$ passiert! $\Rightarrow m-k = 1 \mid +k$

$m = 1+k \mid -1 \Rightarrow k = m-1$; $S(m) = S(m-(m-1)) + (m-1)\Theta(m)$

$= S(m-m+1) + m\Theta(m) - 1\Theta(m) = S(1) + m\Theta(m) - \Theta(m)$

$= S(1) + m\Theta(m) - \Theta(m) = S(1) + \Theta(m^2) - \Theta(m) = \Theta(m^2)$

$T(n) = T(2^{2^m}) = S(m) = \Theta(m^2)$

$\Rightarrow T(n) = \Theta((\log_2 \log_2 n)^2) =$

$= \Theta((\log_2(\log_2(n)))^2)$