Fusion Physics

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A. Chapter 0: Energy and Global Income Distribution

A.1. How is income distributed globally, and how does it relate to energy consumption?

Solution

If we divide the world into 4 groups $1/7$ would earn under 2\$, another $3/7$ between 2\$ and 8\$, another $2/7$ between 8\$ and 32\$ and the last $1/7$ would earn more than 32\$ a day. The energy consumption is distributed in a similar way.
In richer countries people eat more, drive more, fly more, and use more utilities, all leading to higher energy consumption.

A.2. Compute the primary energy consumption in a fully developed country per capita and day from:

A.2.1. a) Estimating a person's individual consumption (heating, electricity, car, etc.)

Solution

I roughly pay for about 1000 Kwh of electricity per year. That is about 3 Kwh == 3*(3600*1000) = 10.8 MJ per day.
Heating is roughly double to triple that, so about 25 MJ per day.
1 L of gasoline has about 32 MJ of energy and I drive about 10.000 km a year: 10.000 km / 5 L per 100 km = 2000 L of gasoline per year. That is 64 GJ per year or 175 MJ per day.
Clearly I'm driving a lot. With production of goods + transportation and other stuff I'd say I'm at about 300 MJ per day roughly 85 Kwh/day
Note: 1 J = 1 Ws; 1 Kwh = 3600 * 1000 J = 3,6 MJ; 1 MJ = 0,277 Kwh

A.2.2. b) From the macroeconomic perspective of a whole country

Solution

Primary energy consumption of Austria is about $1{,}4 * 10^18$ J/year which is (divided by 365* 10 Mio. [people]) about 380 MJ per person per day or about 105 Kwh per person per day.

A.3. Explain energy intensity

Solution

Energy intensity is a measure of the energy inefficiency of an economy. It is calculated as units of energy per unit of GDP (Gross Domestic Product) or some other measure of economic output. High energy intensities indicate a high price or cost of converting energy into GDP. (Wikipedia)
Depends on multiple factors: climate, energy mix and sectors in the economy of the given country (e.g. industry vs. services vs. agriculture, etc.)

A.4. How do primary energy consumption and consumer electricity differ?

Solution

1) Primary energy consumption is the energy contained in the fuel (e.g. coal, oil, gas, uranium, etc.)
2) Consumer electricity is the energy that is delivered to the consumer (e.g. electricity from the wall socket).
As seen in the question above, the actual electricity consumption (bill) I gave was about 10% of the primary energy consumption per person.
Of the primary energy only about 20% are actually converted to electricity and half of that is used by consumers (rest in industry, loss etc.)

A.5. What's the energy mix in Austria?

Solution

See this [LINK]

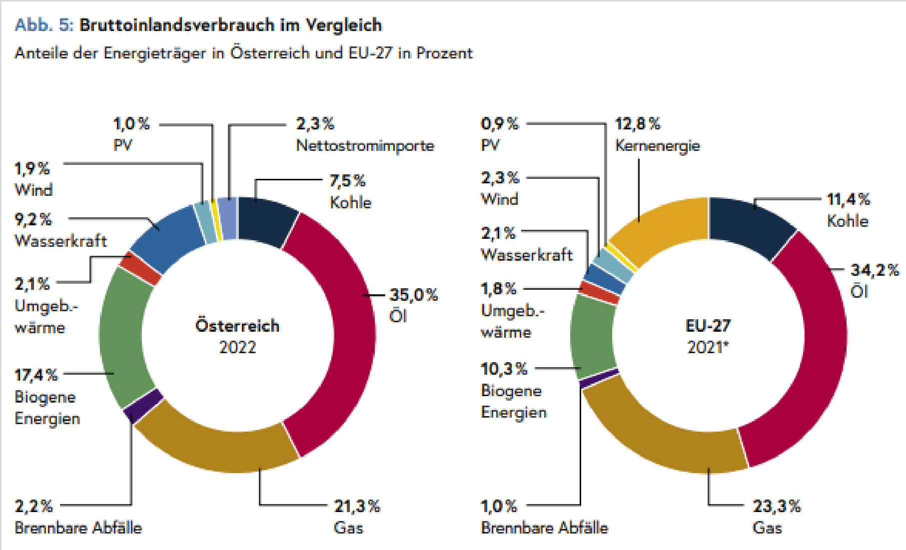

Abbildung 1: Energy mix in Austria

So we have mostly oil and gas (55%) and then renewable energy sources (35%) and coal and waste + import (10%).

A.6. Given a number for reserves of a single fossil resource, compute:

Let's take coal as an example: $1 * 10^{15}$ kg of coal reserves.

A.6.1. a) What part of the energy mix it can contribute sustainably (1000 years)

Solution

If we take roughly 100 Kwh per day and person we have roughly 10^{12} Kwh per day for the world. 1 kg of coal has about 30 MJ of energy or about 8,3 Kwh. So we need (roughly) about 10^{11} kg of coal per day, this lasts us 10^5 days or roughly 27 years

A.6.2. b) How long would it last at current consumption levels

Solution

See a)

A.7. How much W/m^2 can various energy sources produce? How would you compute it?

Solution

1. Solar: peak: 1000 W/m^2 (depends on location, time of day, weather, etc.) - avg. yield (electricity 10-20%): 10-20 W/m^2
2. Biofuels:
   a) Wood: $<$ 0.5 W/m^2
   b) rape to biodiesel: $<$ 0.2 W/m^2
   c) sugarcane: 1-1.5 W/m^2
3. Wind:
   a) onshore: 1-2 W/m^2
   b) offshore: 2-4 W/m^2
4. Nuclear (fission): 1000 W/m^2

A.8. Discuss whether renewables compete with arable land for food production. What about biofuels?

Solution

Yes they do compete. Recently I read that about 2/3 of the arable land is used for animal feed. So if we would stop eating meat we could free up a lot of land for biofuels.
But than again it would be more efficient to use the land for solar panels or wind turbines.

A.9. Discuss the ongoing price-drop in solar, and what it means for other alternatives, such as future fusion energy

Solution

Solar hits a middle ground between biofuels and nuclear energy. World solar capacity has increased by a factor of 50 in 13 years. And price has dropped by over 99% since 1976. Dropped fourfold in the last 10 years.
With cheaper solar energy, nuclear and fusion becomes less attractive.

A.10. Discuss the question of perceived and quantitative risk for the environment from various aspects of civilization (e.g., birds vs cats/wind turbines)

Solution

Birds aren't real. They are government surveillance drones.

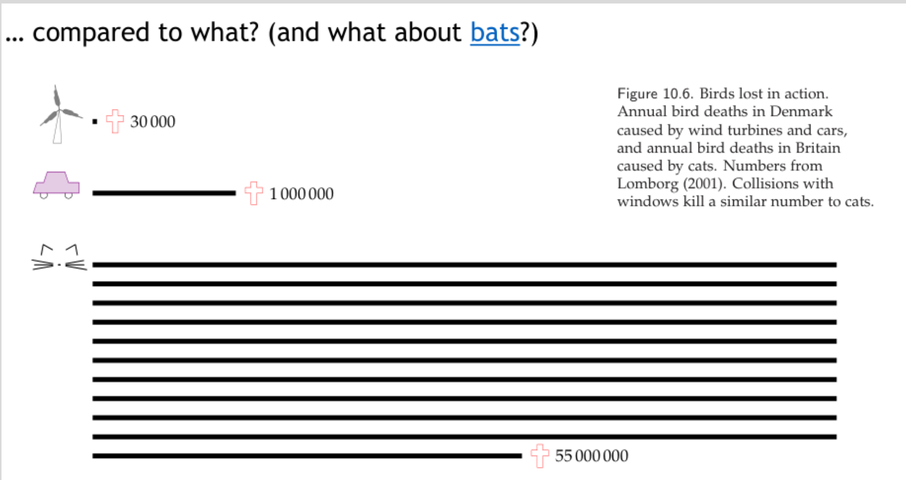

Abbildung 2: Birds aren't real

Way more birds are killed by cats than by wind turbines.
There are certainly drawbacks with all kinds of energy production sites. Some destroy the landscape, some kill birds, some are noisy, some emit CO_2, some are dangerous, etc. In the end we have to decide which drawbacks we can live with.

A.11. Are there CO_2-free energy sources? Why/why not?

Solution

No there aren't basically. I mean photosynthesis is CO_2-free (or negative I'd say) but it is not an energy source which we are able to use.
While some energy sources are quite CO_2 intensive, such as Coal 1000 g CO_2/kWh, others are less so, such as natural gas 200 g CO_2/kWh, Photovoltaics 50 g CO_2/kWh, Hydro 17 g CO_2/kWh, Nuclear 20 g CO_2/kWh, ITER-like 44 g CO_2/kWh.

B. Chapter 1: Nuclear Energy and Fusion

B.1. Give a historical perspective of the controlled use of nuclear energy in general and fusion in particular.

Solution

In the early days scientists dreamed about changing Lead into Gold. With better knowledge of chemical reactions they at least tried but were doomed to fail from the beginning.
At the end of the 19th century scientists discovered that atoms are not the smallest particles and that they can be split. Especially the discovery of radiation and radioactive decay was a big step forward.
Looking at the sun, they wondered how it could burn for so long. The answer was nuclear fusion and Einstein gave us the world famous formula $E = mc^2$ in 1905, which relates energy and mass. Yet it was Francis Aston's discovery or rather precise measurement of the mass of Helium and Hydrogen that gave us the final piece of the puzzle. He measured that the mass of Helium was less than the mass of 4 Hydrogen atoms and postulated that the difference in mass was converted to energy by a nuclear reaction.
Only in the late 20s a complete understanding of the nuclear fusion process was achieved by also taking quantum mechanics into account (otherwise the sun would have been to cold for any reaction to take place).
In the late 30s Otto Hahn and Fritz Strassmann discovered nuclear fission of uranium by bombarding it with neutrons.
Fission: After the second world war first attempts were made to build a fusion reactor. The idea was to fuse deuterium and tritium together to form Helium. Deuterium is a Hydrogen Isotope naturally found in water and tritium was made using Lithium in so called breeder reactors. The US, UK and Soviet Union all started their classified fusion programs in the 50s. First the confinement was done using magnetic fields, this turned out to be very difficult. Later the idea of inertial confinement was born with the invention of the laser (1960). There a laser is used to insert energy into a pellet of fuel which then is only confined by its own inertia.
Both inertial and magnetic confinement fusion are still being researched today and are the most promising ways to achieve fusion in a viable way.

B.2. Draw and explain a schematic fusion power plant.

Solution

A note on Li: To get tritium for the fusion reaction Lithium is used as a blanket around the plasma. The neutrons from the fusion reaction hit the Lithium and produce tritium.

Like a nuclear fission plant a fusion plant is also a thermal power plant using the heat from the fusion reaction to produce steam and drive a turbine.

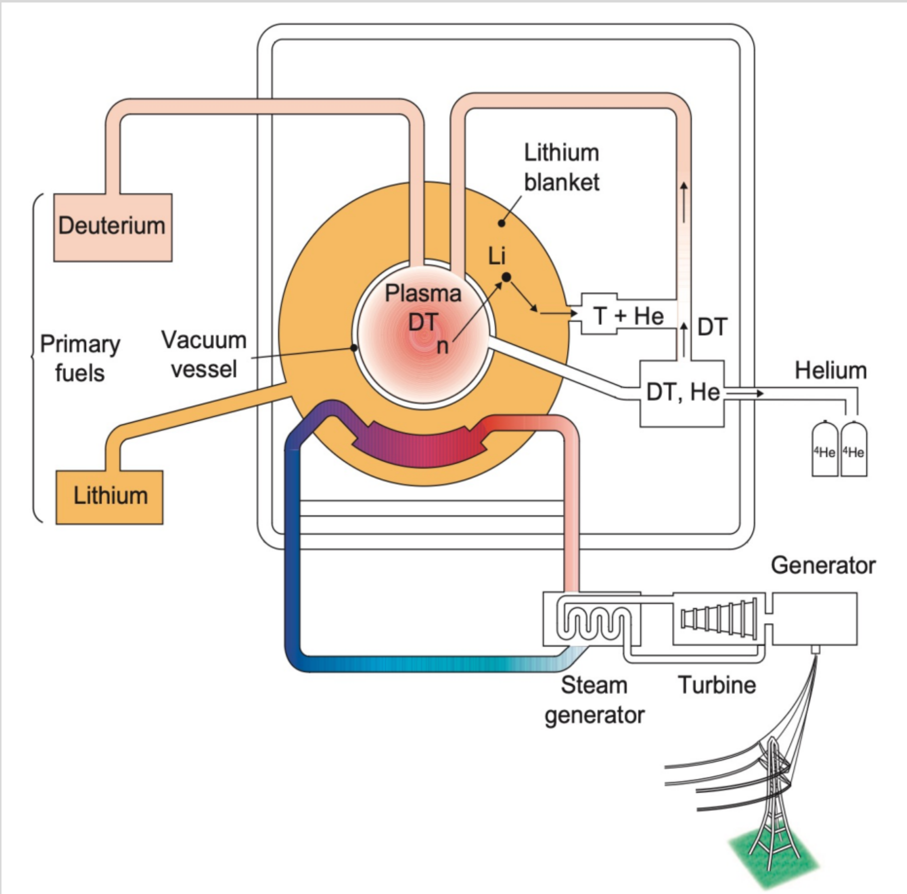

Abbildung 3: Schematic diagram of nuclear fusion power plant

B.3. Explain the difference between magnetic and inertial confinement fusion.

Solution

Magnetic confinement fusion uses magnetic fields (e.g. tokamak) to confine the plasma and inertial confinement fusion uses lasers to heat up a pellet of fuel and then uses the inertia of the pellet to confine the plasma.

C. Chapter 2: Discoveries Leading to Fusion

C.1. Give a historical perspective on the scientific discoveries that led to the discovery of fusion.

Solution

In Keywords:
1) $E = m \cdot c^2$: Probably one of the most important formulas in physics.
2) Einsteins equivalence of mass and energy led to the concept of fusion energy.
3) Francis Aston invented mass spectroscopy and measured that Helium is lighter than 4 Hydrogen atoms. (also explained Isotopes). Eddington realized that Francis measurements mean, that stars convert mass to energy.
4) James Chadwick discovered the neutron which could explain isotopes in 1932.
5) Bonding force in nucleus is given by the mass defect - the difference between the mass of the nucleus and the sum of the masses of the protons and neutrons.
6) Otto Hahn and Fritz Strassmann discovered nuclear fission of uranium by bombarding it with neutrons.
7) In 1939 Lise Meitner and Otto Frisch discovered that the nucleus of Uranium splits into two lighter nuclei and that the mass defect is converted to energy.
8) In 1942 Enrico Fermi built the first nuclear reactor.
9) In 1945 the first atomic bomb was detonated.

C.2. Explain how the mass defect of various isotopes can be used to extract energy in fission and fusion.

Solution

Free neutrons and protons have slightly more mass than neutrons and protons in a nucleus. The difference in mass is called the mass defect. The mass defect is converted to energy when the nucleus is formed - this energy is called binding energy.
In fission the mass defect is converted to energy when the nucleus splits into two lighter nuclei. Fission releases energy for nuclei heavier than Iron and fusion releases energy for nuclei lighter than Iron.

D. Chapter 3: Stellar Energy and Fusion

D.1. Where does the energy in stars come from, and how was it conjectured?

Solution

Was conjectured by Sir Arthur Eddington in 1920s. He realized that the sun gains energy from the mass defect of Hydrogen to Helium.
If the sun was a lump of coal it would burn out in a few thousand years.
The sun must be older than the earth they argued and the earth is (according to Lord Kelvin) at least 20 million years old [Spoiler: he was was way off].

D.2. Write the reaction of the proton-proton cycle. Why is it important for stars?

Solution

1) Two protons combine to form deuterium (D) and a positron (e+) and a neutrino (ν)
2) The deuterium combines with another proton to form Helium-3 (He-3) and a gamma ray (γ)
3) Two Helium-3 nuclei combine to form Helium-4 (He-4) and two protons.
This process releases about 10 million times more energy than the chemical bonding of Hydrogen with Oxygen to form Water. $\rightarrow$ from that we conclude that the sun must live not for thousands, but for billions of years.

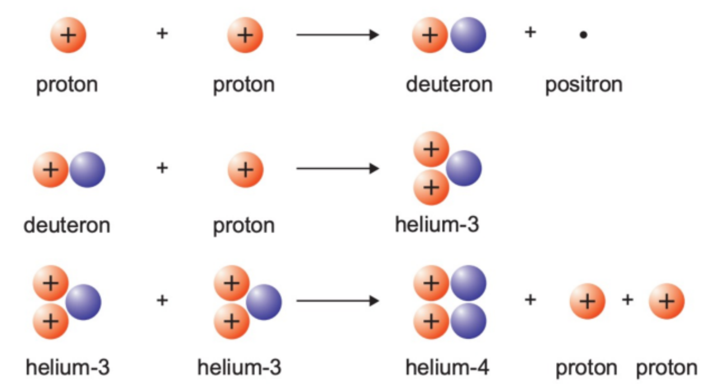

FIGURE 3.2 The three reactions that convert hydrogen to helium in the Sun. In the first stage two protons combine together. One proton converts to a neutron and the overall charge balance is conserved through emission of a *positron*, a particle with the same mass and properties as an electron but with positive electric charge.

Abbildung 4: Proton-Proton Cycle

D.3. How are stars able to generate conditions for fusion, and why do we need other ways on earth?

Solution

They are massive, so gravity compresses them and heats them up. They have to constantly balance the gravitational force with the pressure from the fusion reaction.
In the sun fusion only happens near the core - practically within a sphere 1/10 of the sun's radius. About 270 watts are produced per cubic meter in the core which is then transported through radiation and further out through convection.
In the core the sun has a temperature of about 14 million Kelvin while on the surface it is only about 6000 Kelvin.
On Earth we need other confinement methods because we need much higher energy densities than the sun.

D.4. What's the meaning of different star stages and their composition? What stage is the Sun and elements can it produce?

Solution

One might wonder how heavier elements than Helium are produced.
For bigger stars (than our sun) the fusion process continues after Helium-4 is produced. The star burns at a higher temperature and fuses Helium-4 to Carbon-12 and Carbon-12 to Oxygen-16, ... and so on Neon-20, Magnesium-24,...
Our sun can only produce Helium-4 and is subsequently doomed to cool down and shrink into a white dwarf.
Bigger stars explode in a supernova and produce all the heavier elements which are the building blocks for so called secondary stars (like our sun) and planets and *us*.

D.5. Explain primordial nucleosynthesis and how it led to the current universe.

Solution

Also known as Big Bang Nucleosynthesis this refers to the production of nuclei other than Hydrogen during the early phases of the universe.
The universe was extremely hot and dense a few seconds after the big bang. As it expanded and cooled down the protons and neutrons combined to form Hydrogen, Helium-4 (pp-cycle) and a small amount of Lithium.
By pure change certain regions of the universe had a slightly higher density than others. These regions attracted more matter and eventually formed stars.

E. Chapter 4: Fusion on Earth

E.1. Write equations and explain similarities and differences between fusion reactions that are realistically possible to do on earth.

Solution

Different reaction types:

1. D-T: D + T $\rightarrow$ ^{4}He + n + 17.6 MeV
2. D-D: D + D $\rightarrow$ ^{3}He + n + 3.3 MeV
3. D-D: D + D $\rightarrow$ T + p + 4.0 MeV
4. D-Helium-3: D + ^{3}He $\rightarrow$ ^{4}He + p + 18.3 MeV

The D-T reaction has the highest cross-section and can be accomplished with the lowest temperatures. The D-D reaction is the more difficult to achieve but doesn't require Tritium. Nevertheless, the most realistic to achieve is the D-T reaction.

E.2. Why is tritium a scarce resource and how to produce it?

Solution

Because tritium is a radioactive isotope of Hydrogen with a half-life of 12.3 years. It is produced in the upper atmosphere by cosmic rays. Other than that there are no significant natural sources of tritium.
Tritium is therefore produced by irradiating Lithium-6 or Lithium-7 with neutrons:

^{6}Li + n $\rightarrow$ T + ^{4}He + 4.8 MeV

^{7}Li + n $\rightarrow$ T + ^{4}He + n - 2.5 MeV

Some would even be produced inside a fusion reactor by the D-D reaction.

E.3. Draw the overall fuel cycle of a D-T fusion plant.

Solution

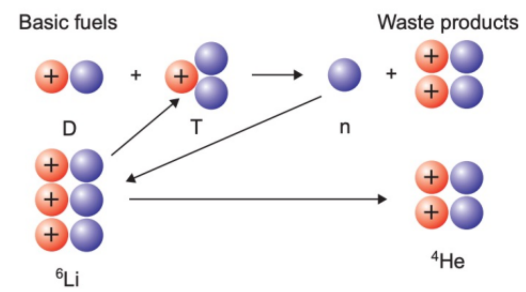

FIGURE 4.3 The overall fusion reaction. The basic fuels are deuterium and lithium; the waste product is helium.

Abbildung 5: Fuel Cycle of a D-T Fusion Plant

E.4. Explain the difficulty to achieve fusion regarding the Coulomb barrier. What makes it easier than an estimation via classical physics?

Solution

For fusion to occur the Coulomb barrier has to be overcome. This is the repulsive force between the two nuclei due to their positive charge. However, if two nuclei get exeptionally close to each other the strong nuclear force takes over and binds them together releasing lots of energy in the process.
In classical physics the Coulomb barrier is so high that the probability of two nuclei getting close enough to fuse is practically zero. However, in quantum mechanics there is a small probability that the nuclei can tunnel through the barrier. This probability is dependent on the energy of the nuclei and thus on the temperature of the plasma.

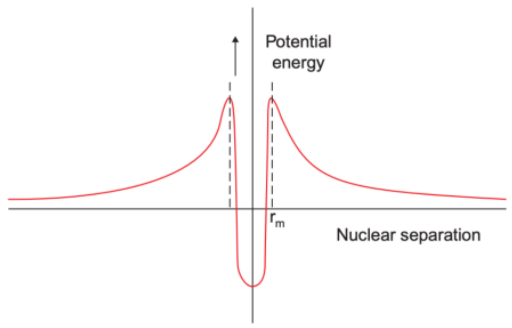

FIGURE 4.4 A diagram illustrating the potential energy of two nuclei as their distance apart is varied. When far apart, they repel each other—their electrostatic repulsion increases as they get closer together. When they become very close, a nuclear attraction becomes effective and the potential energy drops.

Abbildung 6: Coulomb Barrier

E.5. What temperatures are needed for thermonuclear fusion? Explain with regard to the reaction cross-section.

Solution

The D-T reaction has the highest cross section at about 100 keV. The cross section gives the probability of a reaction to occur. The higher the cross section the higher the probability. Each reaction has a characteristic cross section curve which peaks at a certain energy. The D-T reaction requires the lowest temperature to achieve fusion.

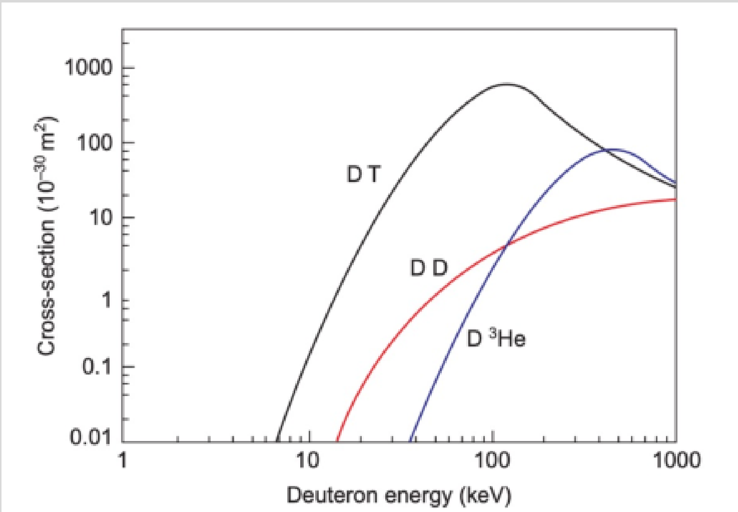

Abbildung 7: Cross Section for different reactions

E.6. Explain the power amplification factor, break-even point, and ignition criterion.

Solution

Power amplification factor: A fusion energy gain factor, usually expressed with the symbol Q, is the ratio of fusion power produced in a nuclear fusion reactor to the power required to maintain the plasma in steady state. There are two break-even points: **engineering break-even** which is the point where the fusion power equals the heating power and **economic break-even** which is the point where the fusion power equals the heating power plus the power needed to produce the plasma i.e. operating costs of the plant.
Self heating of the plasma is only achieved when the gain factor is $Q \approx 5$.
Ignition criterion: Ignition is the point where the fusion power produced is higher than the heating power. This is the point where the plasma is self-heating (menaing $Q \approx 5$).

E.7. What is the fusion triple-product? Explain all three terms and the ways to get a high value in magnetic and inertial confinement fusion.

Solution

The triple product is a figure of merit used in fusion research. It is the product of the plasma density n, the plasma temperature T and the energy confinement time τ_E.

$$nT\tau_E > 3 \cdot 10^{21} m^{-3} keV\ s \tag{1}$$

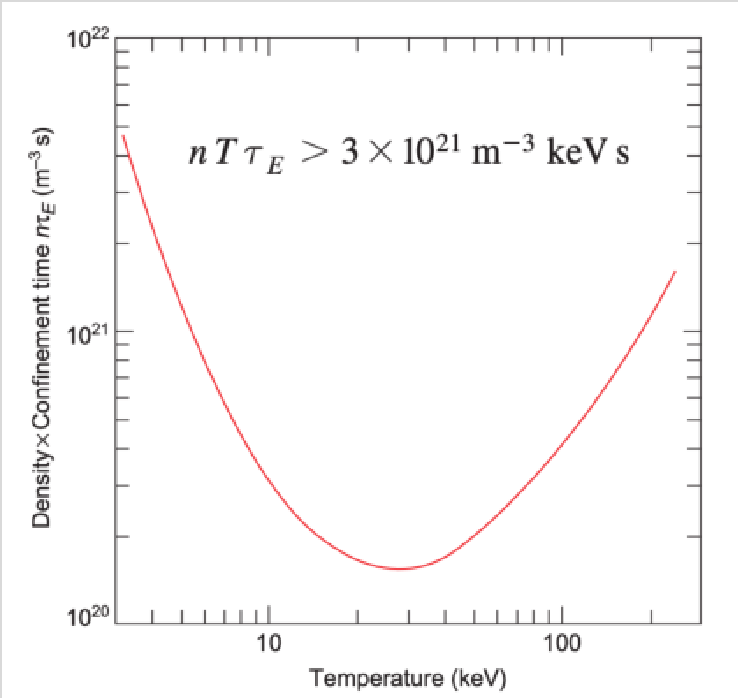

Abbildung 8: The ignition criterion: the value of the product of density and confinement time $n\tau_E$, necessary to obtain plasma ignition, plotted as a function of plasma temperature T (on x-axis). The curve has a minimum at about $T = 30\,keV$ (roughly 300 million K).

For the two different types of confinement we can achieve a high triple product in different ways. For magnetic confinement we can achieve a somewhat high confinement time at a lower preasure. For inertial confinement we can achieve a high pressure at a lower confinement time.
For both cases a temperature T of about 20-30 keV is required. A Q of about 5, which is needed for ignition may be achieved using magnetic confinement at 5 bars for 1 second or inertial confinement at 5 billion bars for 1 nanosecond.

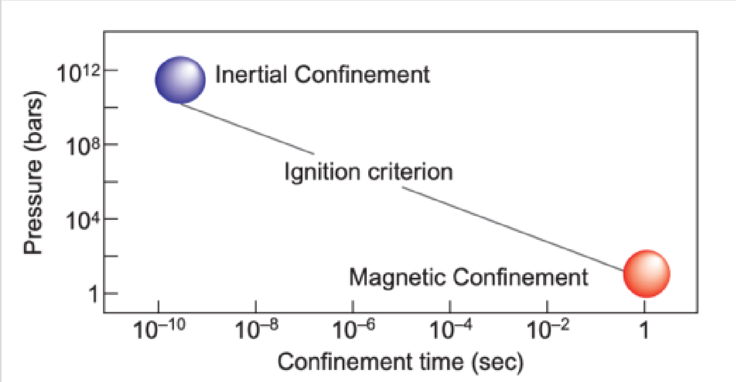

Abbildung 9: The conditions required for fusion plotted in terms of plasma pressure (bars) against confinement time (in seconds).

E.8. Discuss from a historical perspective how close we are to reach scientific and technical break-even and ignition for various fusion technologies.

Solution

From an engineering standpoint we are just now able to produce magnetic confinement devices that can achieve a high enough triple product to reach ignition. However, we are still far of when it comes to ignition an thus self heating. The constant of fusion research is that we are always 30 years away from a working fusion reactor.
Actually we are getting closer: recent experiments at the National Ignition Facility (NIF) in the US have achieved a triple product of $Q = 1.54$ which is a factor of about 3 shy of the ignition criterion.
For magnetic confinement the record is held by the Joint European Torus (JET) in the UK with a triple product of $Q = 0.67$. For Q_{ext} (only measuring the energy input from external sources) the record stands at $Q_{ext} = 1.25$ slightly besting JET's Q_{ext} of 1.14.

F. Chapter 5: MISSING

G. Chapter 6: The Hydrogen Bomb

G.1. Give a historical international perspective on the development of the H-bomb.

Solution

G.2. Explain the requirement for a H-bomb and advantages over pure fission.

Solution

G.3. Discuss limited resources and processing for Uranium and Tritium.

Solution

G.4. How does an H-bomb in the Teller-Ulam design work? Draw and explain.

Solution

G.5. Discuss ideas for civil uses of nuclear bombs and why they failed.

Solution

H. Chapter 7: MISSING