

Fusion Physics

Newest version (link)

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A. Chapter 0: Energy and Global Income Distribution

A.1. How is income distributed globally, and how does it relate to energy consumption?

Solution

If we divide the world into 4 groups $1/7$ would earn under 2\$, another $3/7$ between 2\$ and 8\$, another $2/7$ between 8\$ and 32\$ and the last $1/7$ would earn more than 32\$ a day. The energy consumption is distributed in a similar way.

In richer countries people eat more, drive more, fly more, and use more utilities, all leading to higher energy consumption.

A.2. Compute the primary energy consumption in a fully developed country per capita and day from:

A.2.1. a) Estimating a person's individual consumption (heating, electricity, car, etc.)

Solution

I roughly pay for about 1000 Kwh of electricity per year. That is about 3 Kwh $\Rightarrow 3 \cdot (3600 \cdot 1000) = 10.8$ MJ per day.

Heating is roughly double to triple that, so about 25 MJ per day.

1 L of gasoline has about 32 MJ of energy and I drive about 10.000 km a year: $10.000 \text{ km} / 5 \text{ L per } 100 \text{ km} = 2000 \text{ L of gasoline per year}$. That is 64 GJ per year or 175 MJ per day.

Clearly I'm driving a lot. With production of goods + transportation and other stuff I'd say I'm at about 300 MJ per day roughly 85 Kwh/day

Note: $1 \text{ J} = 1 \text{ Ws}$; $1 \text{ Kwh} = 3600 \cdot 1000 \text{ J} = 3,6 \text{ MJ}$; $1 \text{ MJ} = 0,277 \text{ Kwh}$

A.2.2. b) From the macroeconomic perspective of a whole country

Solution

Primary energy consumption of Austria is about $1,4 \cdot 10^{18} \text{ J/year}$ which is (divided by $365 \cdot 10 \text{ Mio. [people]}$) about 380 MJ per person per day or about 105 Kwh per person per day.

A.3. Explain energy intensity

Solution

Energy intensity is a measure of the energy inefficiency of an economy. It is calculated as units of energy per unit of GDP (Gross Domestic Product) or some other measure of economic output. High energy intensities indicate a high price or cost of converting energy into GDP. (Wikipedia)

Depends on multiple factors: climate, energy mix and sectors in the economy of the given country (e.g. industry vs. services vs. agriculture, etc.)

A.4. How do primary energy consumption and consumer electricity differ?

Solution

1) Primary energy consumption is the energy contained in the fuel (e.g. coal, oil, gas, uranium, etc.)

2) Consumer electricity is the energy that is delivered to the consumer (e.g. electricity from the wall socket).

As seen in the question above, the actual electricity consumption (bill) I gave was about 10% of the primary energy consumption per person.

Of the primary energy only about 20% are actually converted to electricity and half of that is used by consumers (rest in industry, loss etc.)

A.5. What's the energy mix in Austria?

Solution

See this [\[LINK\]](#)

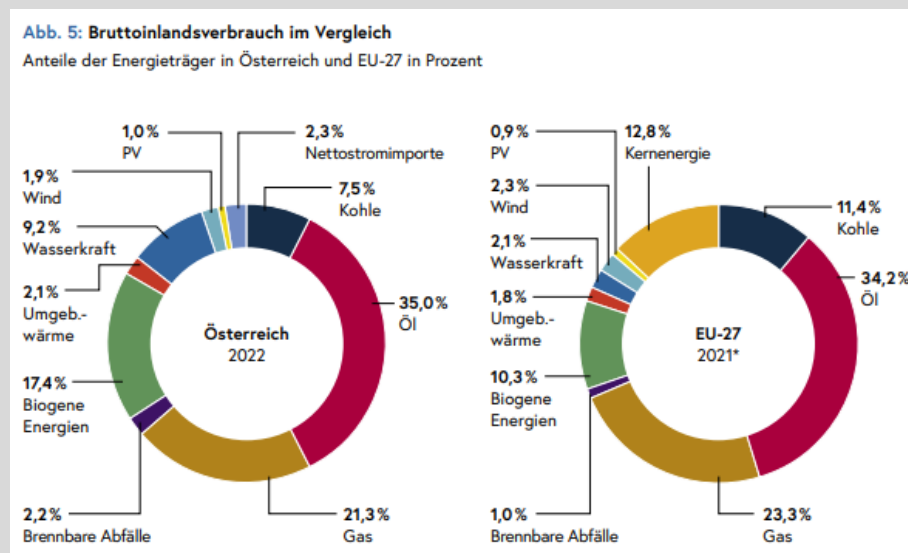


Abbildung 1: Energy mix in Austria

So we have mostly oil and gas (55%) and then renewable energy sources (35%) and coal and waste + import (10%).

A.6. Given a number for reserves of a single fossil resource, compute:

Let's take coal as an example: $1 * 10^{15}$ kg of coal reserves.

A.6.1. a) What part of the energy mix it can contribute sustainably (1000 years)

Solution

If we take roughly 100 Kwh per day and person we have roughly 10^{12} Kwh per day for the world. 1 kg of coal has about 30 MJ of energy or about 8,3 Kwh. So we need (roughly) about 10^{11} kg of coal per day, this lasts us 10^5 days or roughly 27 years

A.6.2. b) How long would it last at current consumption levels**Solution**

See a)

A.7. How much W/m^2 can various energy sources produce? How would you compute it?**Solution**

1. Solar: peak: 1000 W/m^2 (depends on location, time of day, weather, etc.) - avg. yield (electricity 10-20%): $10\text{-}20 \text{ W/m}^2$
2. Biofuels:
 - a) Wood: $< 0.5 \text{ W/m}^2$
 - b) rape to biodiesel: $< 0.2 \text{ W/m}^2$
 - c) sugarcane: $1\text{-}1.5 \text{ W/m}^2$
3. Wind:
 - a) onshore: $1\text{-}2 \text{ W/m}^2$
 - b) offshore: $2\text{-}4 \text{ W/m}^2$
4. Nuclear (fission): 1000 W/m^2

A.8. Discuss whether renewables compete with arable land for food production. What about biofuels?**Solution**

Yes they do compete. Recently I read that about $2/3$ of the arable land is used for animal feed. So if we would stop eating meat we could free up a lot of land for biofuels. But than again it would be more efficient to use the land for solar panels or wind turbines.

A.9. Discuss the ongoing price-drop in solar, and what it means for other alternatives, such as future fusion energy

Solution

Solar hits a middle ground between biofuels and nuclear energy. World solar capacity has increased by a factor of 50 in 13 years. And price has dropped by over 99% since 1976. Dropped fourfold in the last 10 years.

With cheaper solar energy, nuclear and fusion becomes less attractive.

A.10. Discuss the question of perceived and quantitative risk for the environment from various aspects of civilization (e.g., birds vs cats/wind turbines)

Solution

Birds aren't real. They are government surveillance drones.

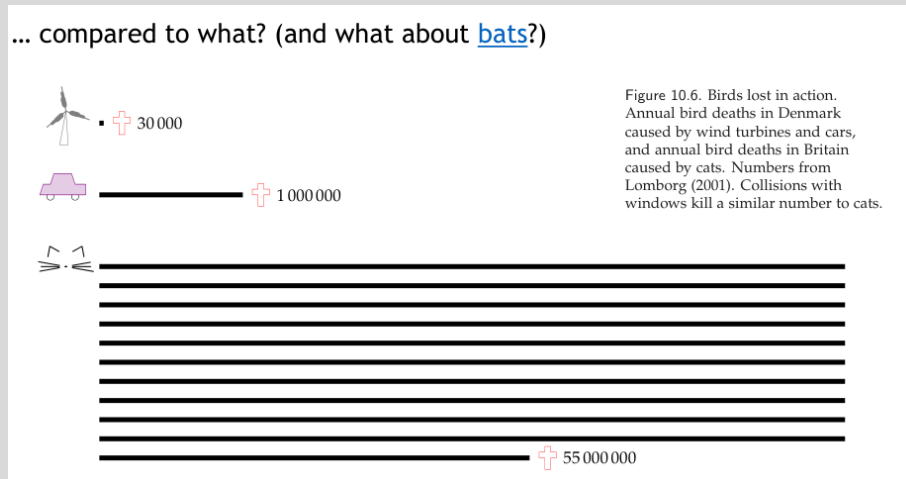


Abbildung 2: Birds aren't real

Way more birds are killed by cats than by wind turbines.

There are certainly drawbacks with all kinds of energy production sites. Some destroy the landscape, some kill birds, some are noisy, some emit CO₂, some are dangerous, etc.

In the end we have to decide which drawbacks we can live with.

A.11. Are there CO₂-free energy sources? Why/why not?**Solution**

No there aren't basically. I mean photosynthesis is CO₂-free (or negative I'd say) but it is not an energy source which we are able to use.

While some energy sources are quite CO₂ intensive, such as Coal 1000 g CO₂/kWh, others are less so, such as natural gas 200 g CO₂/kWh, Photovoltaics 50 g CO₂/kWh, Hydro 17 g CO₂/kWh, Nuclear 20 g CO₂/kWh, ITER-like 44 g CO₂/kWh.

B. Chapter 1: Nuclear Energy and Fusion

B.1. Give a historical perspective of the controlled use of nuclear energy in general and fusion in particular.

Solution

In the early days scientists dreamed about changing Lead into Gold. With better knowledge of chemical reactions they at least tried but were doomed to fail from the beginning.

At the end of the 19th century scientists discovered that atoms are not the smallest particles and that they can be split. Especially the discovery of radiation and radioactive decay was a big step forward.

Looking at the sun, they wondered how it could burn for so long. The answer was nuclear fusion and Einstein gave us the world famous formula $E = mc^2$ in 1905, which relates energy and mass. Yet it was Francis Aston's discovery or rather precise measurement of the mass of Helium and Hydrogen that gave us the final piece of the puzzle. He measured that the mass of Helium was less than the mass of 4 Hydrogen atoms and postulated that the difference in mass was converted to energy by a nuclear reaction.

Only in the late 20s a complete understanding of the nuclear fusion process was achieved by also taking quantum mechanics into account (otherwise the sun would have been too cold for any reaction to take place).

In the late 30s Otto Hahn and Fritz Strassmann discovered nuclear fission of uranium by bombarding it with neutrons.

Fission: After the second world war first attempts were made to build a fusion reactor. The idea was to fuse deuterium and tritium together to form Helium. Deuterium is a Hydrogen Isotope naturally found in water and tritium was made using Lithium in so called breeder reactors. The US, UK and Soviet Union all started their classified fusion programs in the 50s. First the confinement was done using magnetic fields, this turned out to be very difficult. Later the idea of inertial confinement was born with the invention of the laser (1960). There a laser is used to insert energy into a pellet of fuel which then is only confined by its own inertia.

Both inertial and magnetic confinement fusion are still being researched today and are the most promising ways to achieve fusion in a viable way.

B.2. Draw and explain a schematic fusion power plant.

Solution

A note on Li: To get tritium for the fusion reaction Lithium is used as a blanket around the plasma. The neutrons from the fusion reaction hit the Lithium and produce tritium.

Like a nuclear fission plant a fusion plant is also a thermal power plant using the heat from the fusion reaction to produce steam and drive a turbine.

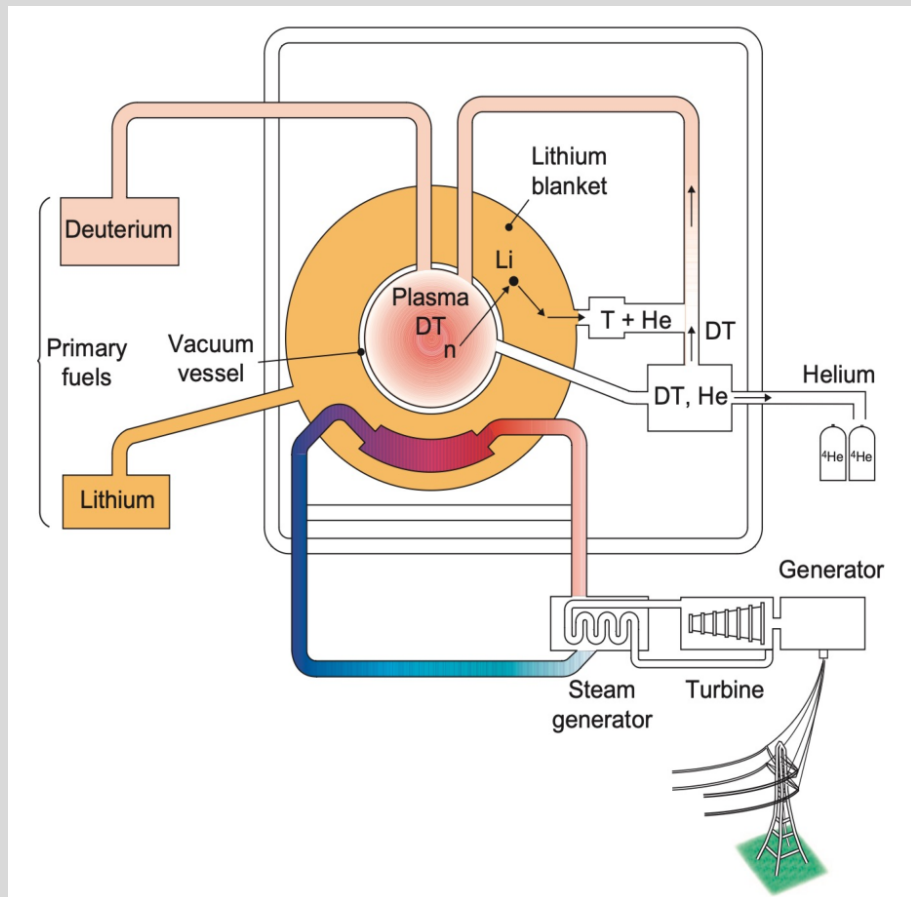


Abbildung 3: Schematic diagram of nuclear fusion power plant

B.3. Explain the difference between magnetic and inertial confinement fusion.

Solution

Magnetic confinement fusion uses magnetic fields (e.g. tokamak) to confine the plasma and inertial confinement fusion uses lasers to heat up a pellet of fuel and then uses the inertia of the pellet to confine the plasma.

C. Chapter 2: Discoveries Leading to Fusion

C.1. Give a historical perspective on the scientific discoveries that led to the discovery of fusion.

Solution

In Keywords:

- 1) $E = m \cdot c^2$: Probably one of the most important formulas in physics.
- 2) Einsteins equivalence of mass and energy led to the concept of fusion energy.
- 3) Francis Aston invented mass spectroscopy and measured that Helium is lighter than 4 Hydrogen atoms. (also explained Isotopes). Eddington realized that Francis measurements mean, that stars convert mass to energy.
- 4) James Chadwick discovered the neutron which could explain isotopes in 1932.
- 5) Bonding force in nucleus is given by the mass defect - the difference between the mass of the nucleus and the sum of the masses of the protons and neutrons.
- 6) Otto Hahn and Fritz Strassmann discovered nuclear fission of uranium by bombarding it with neutrons.
- 7) In 1939 Lise Meitner and Otto Frisch discovered that the nucleus of Uranium splits into two lighter nuclei and that the mass defect is converted to energy.
- 8) In 1942 Enrico Fermi built the first nuclear reactor.
- 9) In 1945 the first atomic bomb was detonated.

C.2. Explain how the mass defect of various isotopes can be used to extract energy in fission and fusion.

Solution

Free neutrons and protons have slightly more mass than neutrons and protons in a nucleus. The difference in mass is called the mass defect. The mass defect is converted to energy when the nucleus is formed - this energy is called binding energy.

In fission the mass defect is converted to energy when the nucleus splits into two lighter nuclei. Fission releases energy for nuclei heavier than Iron and fusion releases energy for nuclei lighter than Iron.

D. Chapter 3: Stellar Energy and Fusion

D.1. Where does the energy in stars come from, and how was it conjectured?

Solution

Was conjectured by Sir Arthur Eddington in 1920s. He realized that the sun gains energy from the mass defect of Hydrogen to Helium.

If the sun was a lump of coal it would burn out in a few thousand years.

The sun must be older than the earth they argued and the earth is (according to Lord Kelvin) at least 20 million years old [Spoiler: he was way off].

D.2. Write the reaction of the proton-proton cycle. Why is it important for stars?

Solution

- 1) Two protons combine to form deuterium (D) and a positron (e^+) and a neutrino (ν)
- 2) The deuterium combines with another proton to form Helium-3 (He-3) and a gamma ray (γ)
- 3) Two Helium-3 nuclei combine to form Helium-4 (He-4) and two protons.

This process releases about 10 million times more energy than the chemical bonding of Hydrogen with Oxygen to form Water. \rightarrow from that we conclude that the sun must live not for thousands, but for billions of years.

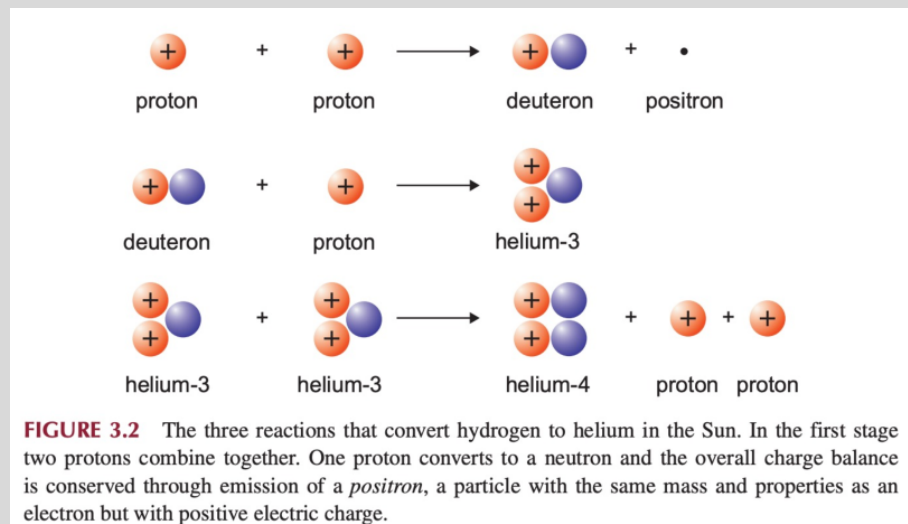


Abbildung 4: Proton-Proton Cycle

D.3. How are stars able to generate conditions for fusion, and why do we need other ways on earth?**Solution**

They are massive, so gravity compresses them and heats them up. They have to constantly balance the gravitational force with the pressure from the fusion reaction.

In the sun fusion only happens near the core - practically within a sphere 1/10 of the sun's radius. About 270 watts are produced per cubic meter in the core which is then transported through radiation and further out through convection.

In the core the sun has a temperature of about 14 million Kelvin while on the surface it is only about 6000 Kelvin.

On Earth we need other confinement methods because we need much higher energy densities than the sun.

D.4. What's the meaning of different star stages and their composition? What stage is the Sun and elements can it produce?**Solution**

One might wonder how heavier elements than Helium are produced.

For bigger stars (than our sun) the fusion process continues after Helium-4 is produced. The star burns at a higher temperature and fuses Helium-4 to Carbon-12 and Carbon-12 to Oxygen-16, ... and so on Neon-20, Magnesium-24,...

Our sun can only produce Helium-4 and is subsequently doomed to cool down and shrink into a white dwarf.

Bigger stars explode in a supernova and produce all the heavier elements which are the building blocks for so called secondary stars (like our sun) and planets and *us*.

D.5. Explain primordial nucleosynthesis and how it led to the current universe.**Solution**

Also known as Big Bang Nucleosynthesis this refers to the production of nuclei other than Hydrogen during the early phases of the universe.

The universe was extremely hot and dense a few seconds after the big bang. As it expanded and cooled down the protons and neutrons combined to form Hydrogen, Helium-4 (pp-cycle) and a small amount of Lithium.

By pure chance certain regions of the universe had a slightly higher density than others. These regions attracted more matter and eventually formed stars.

E. Chapter 4: Fusion on Earth

E.1. Write equations and explain similarities and differences between fusion reactions that are realistically possible to do on earth.

Solution

Different reaction types:

1. D-T: $D + T \rightarrow {}^4\text{He} + n + 17.6 \text{ MeV}$
2. D-D: $D + D \rightarrow {}^3\text{He} + n + 3.3 \text{ MeV}$
3. D-D: $D + D \rightarrow T + p + 4.0 \text{ MeV}$
4. D-Helium-3: $D + {}^3\text{He} \rightarrow {}^4\text{He} + p + 18.3 \text{ MeV}$

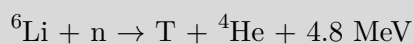
The D-T reaction has the highest cross-section and can be accomplished with the lowest temperatures. The D-D reaction is the more difficult to achieve but doesn't require Tritium. Nevertheless, the most realistic to achieve is the D-T reaction.

E.2. Why is tritium a scarce resource and how to produce it?

Solution

Because tritium is a radioactive isotope of Hydrogen with a half-life of 12.3 years. It is produced in the upper atmosphere by cosmic rays. Other than that there are no significant natural sources of tritium.

Tritium is therefore produced by irradiating Lithium-6 or Lithium-7 with neutrons:



Some would even be produced inside a fusion reactor by the D-D reaction.

E.3. Draw the overall fuel cycle of a D-T fusion plant.

Solution

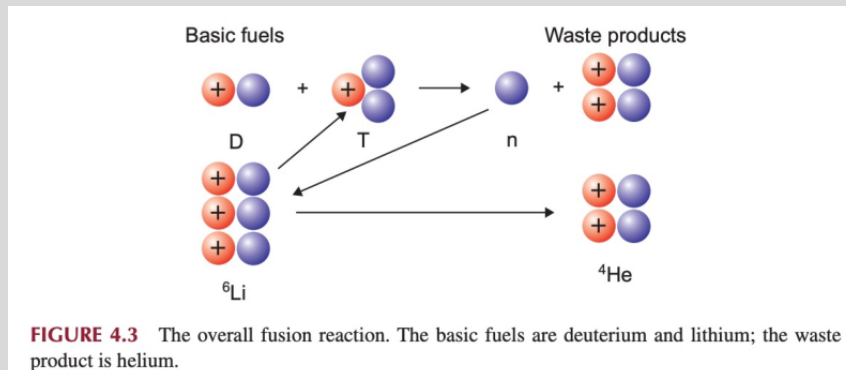


Abbildung 5: Fuel Cycle of a D-T Fusion Plant

E.4. Explain the difficulty to achieve fusion regarding the Coulomb barrier. What makes it easier than an estimation via classical physics?

Solution

For fusion to occur the Coulomb barrier has to be overcome. This is the repulsive force between the two nuclei due to their positive charge. However, if two nuclei get exceptionally close to each other the strong nuclear force takes over and binds them together releasing lots of energy in the process.

In classical physics the Coulomb barrier is so high that the probability of two nuclei getting close enough to fuse is practically zero. However, in quantum mechanics there is a small probability that the nuclei can tunnel through the barrier. This probability is dependent on the energy of the nuclei and thus on the temperature of the plasma.

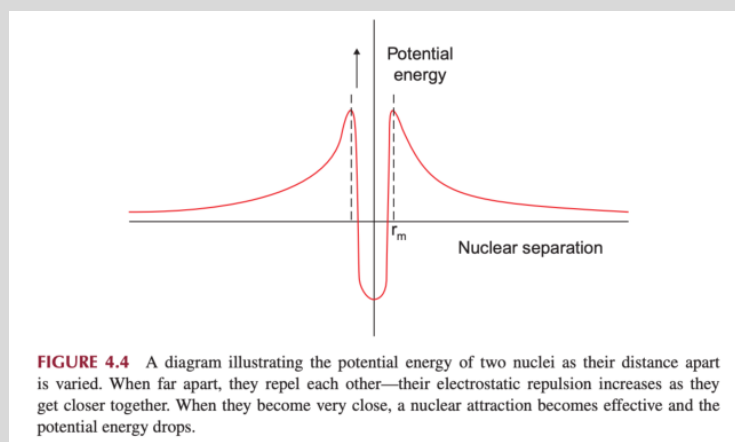


Abbildung 6: Coulomb Barrier

E.5. What temperatures are needed for thermonuclear fusion? Explain with regard to the reaction cross-section.

Solution

The D-T reaction has the highest cross section at about 100 keV. The cross section gives the probability of a reaction to occur. The higher the cross section the higher the probability. Each reaction has a characteristic cross section curve which peaks at a certain energy. The D-T reaction requires the lowest temperature to achieve fusion.

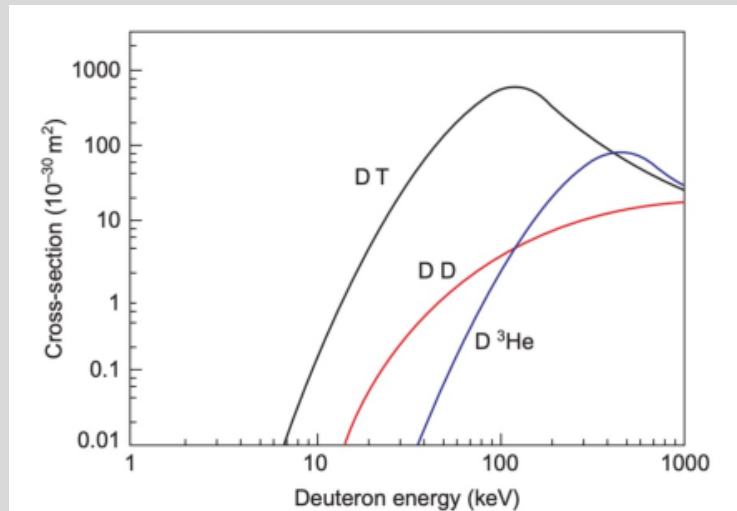


Abbildung 7: Cross Section for different reactions

E.6. Explain the power amplification factor, break-even point, and ignition criterion.

Solution

Power amplification factor: A fusion energy gain factor, usually expressed with the symbol Q , is the ratio of fusion power produced in a nuclear fusion reactor to the power required to maintain the plasma in steady state. There are two break-even points: **engineering break-even** which is the point where the fusion power equals the heating power and **economic break-even** which is the point where the fusion power equals the heating power plus the power needed to produce the plasma i.e. operating costs of the plant.

Self heating of the plasma is only achieved when the gain factor is $Q \approx 5$.

Ignition criterion: Ignition is the point where the fusion power produced is higher than the heating power. This is the point where the plasma is self-heating (meaning $Q \approx 5$).

E.7. What is the fusion triple-product? Explain all three terms and the ways to get a high value in magnetic and inertial confinement fusion.

Solution

The triple product is a figure of merit used in fusion research. It is the product of the plasma density n , the plasma temperature T and the energy confinement time τ_E .

$$nT\tau_E > 3 \cdot 10^{21} \text{ m}^{-3} \text{ keV s} \quad (1)$$

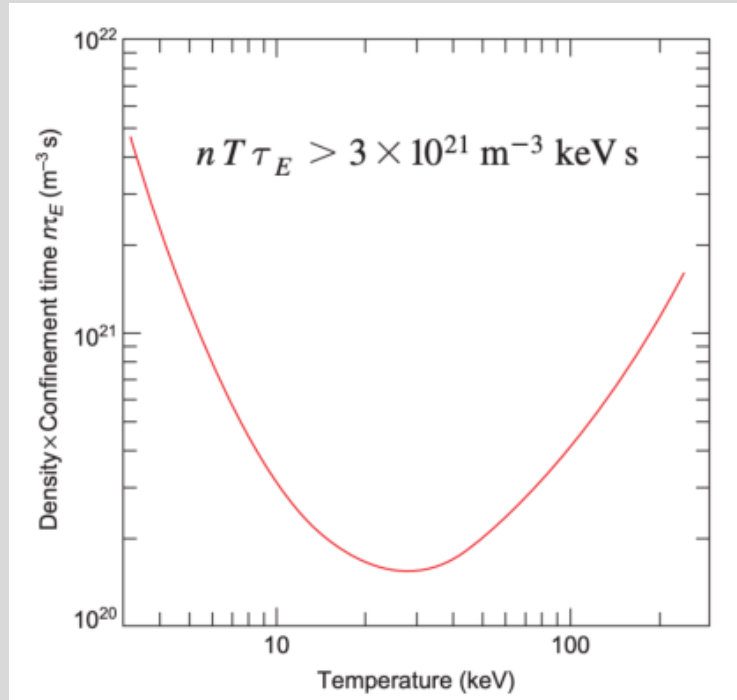


Abbildung 8: The ignition criterion: the value of the product of density and confinement time $n\tau_E$, necessary to obtain plasma ignition, plotted as a function of plasma temperature T (on x-axis). The curve has a minimum at about $T = 30 \text{ keV}$ (roughly 300 million K).

For the two different types of confinement we can achieve a high triple product in different ways. For magnetic confinement we can achieve a somewhat high confinement time at a lower pressure. For inertial confinement we can achieve a high pressure at a lower confinement time.

For both cases a temperature T of about 20-30 keV is required. A Q of about 5, which is needed for ignition may be achieved using magnetic confinement at 5 bars for 1 second or inertial confinement at 5 billion bars for 1 nanosecond.

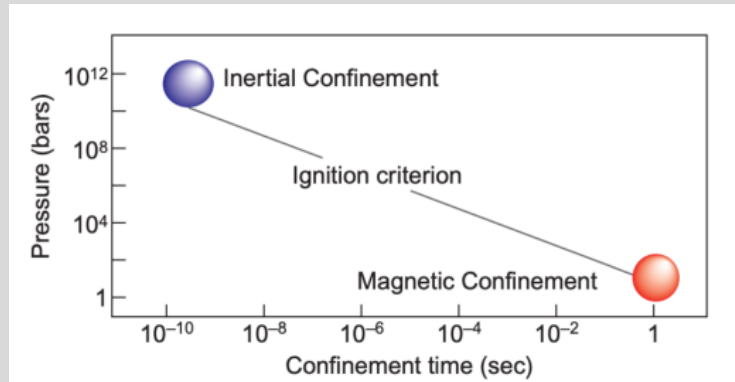


Abbildung 9: The conditions required for fusion plotted in terms of plasma pressure (bars) against confinement time (in seconds).

E.8. Discuss from a historical perspective how close we are to reach scientific and technical break-even and ignition for various fusion technologies.

Solution

From an engineering standpoint we are just now able to produce magnetic confinement devices that can achieve a high enough triple product to reach ignition. However, we are still far of when it comes to ignition and thus self heating. The constant of fusion research is that we are always 30 years away from a working fusion reactor.

Actually we are getting closer: recent experiments at the National Ignition Facility (NIF) in the US have achieved a triple product of $Q = 1.54$ which is a factor of about 3 shy of the ignition criterion.

For magnetic confinement the record is held by the Joint European Torus (JET) in the UK with a triple product of $Q = 0.67$. For Q_{ext} (only measuring the energy input from external sources) the record stands at $Q_{ext} = 1.25$ slightly besting JET's Q_{ext} of 1.14.

F. Chapter 5: Magnetic Confinement

F.1. Explain the main ideas and principles of magnetic plasma confinement.

Solution

The main idea of magnetic plasma confinement is to use magnetic fields to confine a plasma. Plasma is charged and will therefore gyrate in an applied magnetic field.

F.2. Draw and explain the mechanism of the pinch effect in a plasma with a current flowing inside.

Solution

It was discovered that a high current flowing through a conductor will cause the conductor to contract (pinch). This is due to the magnetic field generated by the current, which will cause a force on the conductor.

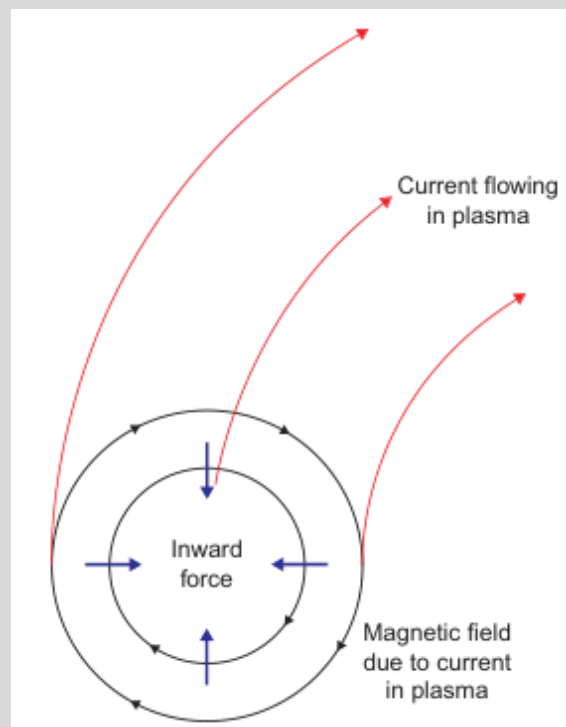


Abbildung 10: Pinch effect in a plasma with a current flowing inside.

The inward force on the plasma is explained by the Lorentz force acting on the plasma.

F.3. Draw and explain the gyro-orbits of ions and electrons in a magnetic field. What happens if particles collide?

Solution

Charged particles (ions and electrons) will gyrate in a magnetic field. Since the plasma is moving in circular orbits and the magnetic field produced in a Tokamak follows these orbits, the particles will gyrate along their paths.

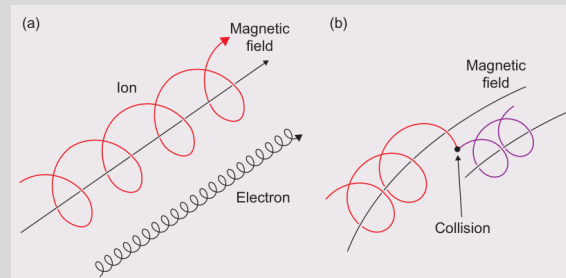


Abbildung 11: Gyro-orbits of ions and electrons in a magnetic field.

Collisions between particles will cause the particles to move to a new orbit.

F.4. Draw and explain Z-pinch, theta-pinch, and magnetic mirror. How are they related?

Solution

All three are different ways to confine a plasma using magnetic fields in a linear fashion.

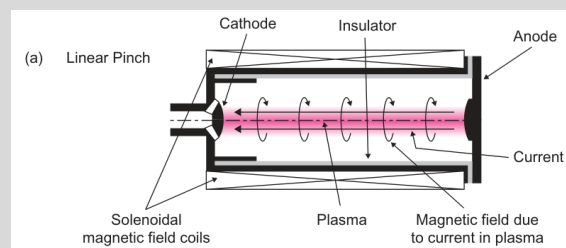


Abbildung 12: Z-pinch or linear pinch.

The Z-pinch uses a current flowing through the plasma to generate a magnetic field that will cause the plasma to contract. It uses an Anode and a Cathode to generate the current in the plasma.

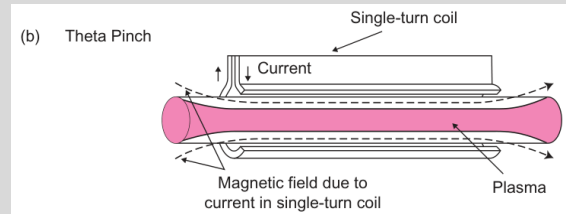


Abbildung 13: Theta-pinch.

The theta-pinch uses a current flowing through the plasma to generate a magnetic field that will cause the plasma to contract. It uses a current running around the plasma in a single turn coil - along the theta axis in cylindrical coordinates (hence the name).

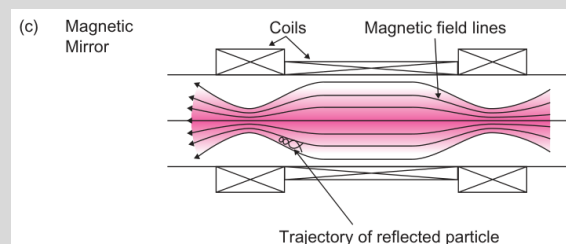


Abbildung 14: Magnetic mirror.

Magnetic mirrors use a magnetic field that is stronger at the ends of the plasma than in the middle. This will cause the plasma to be reflected back and forth between the ends of the plasma. Using this method less plasma escapes the magnetic field on each end.

F.5. What is the plasma beta? Sketch its derivation. How is it related to stability?

Solution

Plasma beta β is the ratio of plasma pressure p to magnetic pressure p_{mag} :

$$\beta = \frac{p}{p_{mag}} \quad (2)$$

Plasma pressure is given by the density n and the temperature T of the plasma:

$$p = nk_B T \quad (3)$$

and the magnetic pressure is given by the magnetic field B :

$$p_{mag} = \frac{B^2}{2\mu_0} \quad (4)$$

Combining these equations gives:

$$\beta = \frac{nk_B T}{B^2/2\mu_0} \quad (5)$$

β is related to stability in the sense that at a higher β the plasma is more prone to instabilities. Since for higher plasma pressure the plasma will expand and the magnetic field will be less able to confine the plasma in the device.

F.6. Draw and explain instabilities in a Z-pinch. How to avoid them?

Solution

When an attempt is made to confine the plasma in a Z-pinch, instabilities will occur. Even a small deviation of the magnetic field lines will rapidly lead to a drastic deformation of the plasma.

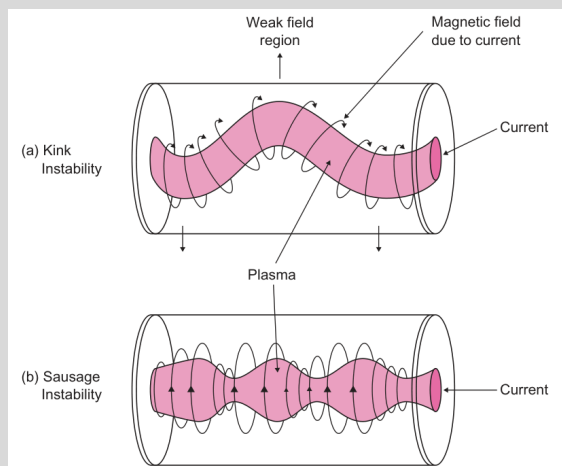


Abbildung 15: Instabilities in Z-pinch.

To avoid instabilities, the plasma could be confined in a torus. This is the idea behind the Tokamak.

F.7. Draw and explain a screw pinch and a tokamak. What are similarities and differences?

Solution

A screw pinch is a combination of a Z-pinch and a theta-pinch much like the graphic bellow:

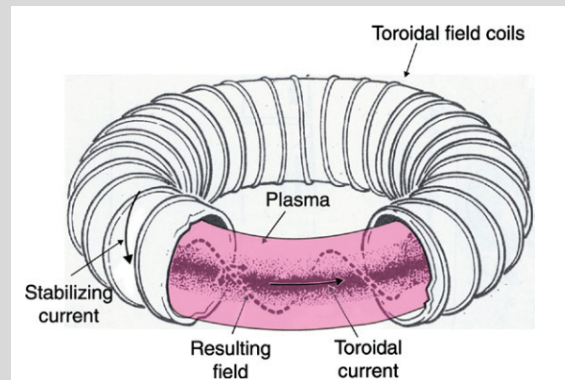


Abbildung 16: NOTE: This is a toroidal pinch -> Screw Pinch is linear!!!

A Tokamak is a toroidal pinch. It uses a current flowing through the plasma to generate a magnetic field that will cause the plasma to contract.

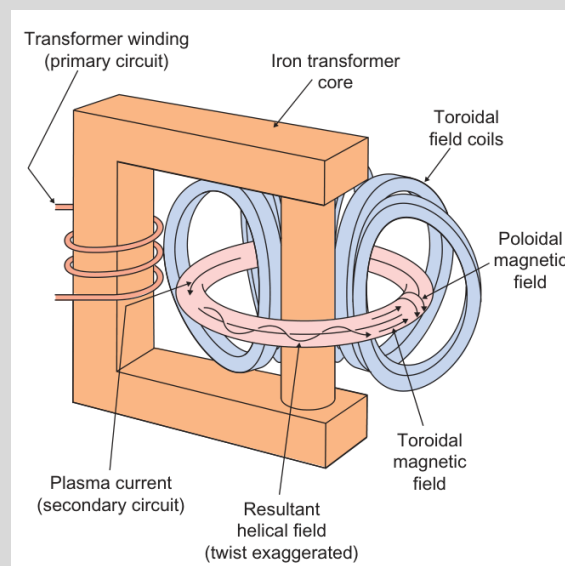


Abbildung 17: Tokamak

Similarities: Both use a current flowing through the plasma to generate a magnetic field that will cause the plasma to contract.

Differences: The Tokamak is toroidal and the Screw Pinch is linear. Tokamaks are also more stable than Screw Pinches and don't lose their plasma as easily.

F.8. Draw and explain a stellarator and explain similarities and differences to a tokamak.**Solution**

A stellarator is a toroidal device that uses external coils to generate the magnetic field. This means that the current in the plasma is not needed to generate the magnetic field. Compared to a tokamak which is can only be operated in pulsed mode, a stellarator can be operated in steady-state mode.

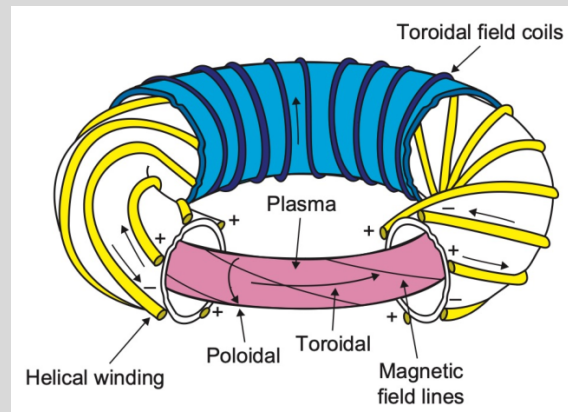


Abbildung 18: Stellarator

Similarities: Both are toroidal devices.

Differences: The Tokamak uses a current flowing through the plasma to generate a magnetic field that will cause the plasma to contract. The Stellarator uses external coils to generate the magnetic field.

G. Chapter 6: The Hydrogen Bomb

G.1. Give a historical international perspective on the development of the H-bomb.

Solution

Some isotopes of uranium and plutonium have nuclei that are so close to being unstable that they fragment and release energy when bombarded with neutrons. A fission chain reaction builds up because each fragmenting nucleus produces several neutrons that can initiate further reactions. An explosion occurs if the piece of uranium or plutonium exceeds a certain critical mass — thought to be a few kilograms (i.e., smaller than a grapefruit). In order to bring about the explosion, this critical mass has to be assembled very quickly, either by firing together two subcritical pieces or by compressing a subcritical sphere using conventional explosives. The US developed the first atom bombs in great secrecy during World War II at Los Alamos, New Mexico. The first test weapon, exploded in New Mexico in July 1945, had a force equivalent to 21 kilotons of high explosive. A few days later, bombs of similar size devastated the Japanese cities of Hiroshima and Nagasaki. **Producing the fissile materials for such weapons was difficult and expensive and required an enormous industrial complex. Less than 1 % of natural uranium is the “explosive” isotope ^{235}U , and separating it from the more abundant ^{238}U is very difficult.** Plutonium does not occur naturally at all and must be manufactured in a fission reactor and then extracted from the intensely radioactive waste. Moreover, the size of a **pure fission bomb was limited by the requirement that the component parts be below the critical mass. Fusion does not suffer from this size limitation and might allow bigger bombs to be built. The fusion fuel, deuterium, is much more abundant than ^{235}U and is easier to separate.** Even as early as 1941, before he had built the very first nuclear (fission) reactor in Chicago, the physicist Enrico Fermi speculated to Edward Teller that **a fission bomb might be able to ignite the fusion reaction in deuterium in order to produce an even more powerful weapon**—this became known as the **hydrogen bomb**, or **H-bomb**. These ideas were not pursued seriously until after the war ended. Many of the scientists working at Los Alamos then left to go back to their academic pursuits. Robert Oppenheimer, who had led the development of the fission bomb, resigned as director of the weapons laboratory at Los Alamos to become director of the Princeton Institute of Advanced Study and was replaced by Norris Bradbury. Edward Teller, after a brief period in academic life, returned to Los Alamos and became the main driving force behind the development of the H-bomb, with a concept that was called the **Classical Super**.

There was, however, much soul-searching in the US as to whether it was justified to try and build a fusion bomb at all. In 1949, Enrico Fermi and Isidor Rabi, both distinguished physicists and Nobel Prize winners, wrote a report for the Atomic Energy Commission in which they said:

Necessarily such a weapon goes far beyond any military objective and enters the range of very great natural catastrophes. . . . It is clear that the use of such a weapon cannot be justified on any ethical ground which gives a human being a certain individuality and dignity even if he happens to be a resident of an enemy country. . . . The fact that no limits exist to the destructiveness of this weapon makes its existence and the knowledge of its construction a danger to humanity as a whole. It is an evil thing considered in any light.

The debate was cut short in early 1950 by the unexpected detonation of the first Soviet fission bomb. Prompted by the suspicion that East German spy Klaus Fuchs had supplied information about US hydrogen-bomb research to the Soviet Union, President Truman ordered that the Super be developed as quickly as possible. However, no one really knew how to do this, so new fears were raised that Truman's statement might simply encourage the Soviet Union to speed up its own efforts to build a fusion bomb and, more seriously, that the Soviet scientists might already know how to do it.

G.2. Explain the requirement for a H-bomb and advantages over pure fission.

Solution

see above

G.3. Discuss limited resources and processing for Uranium and Tritium.

Solution

There were further serious problems in terms of providing the fusion fuel. A mixture of deuterium and tritium would ignite most easily, but **tritium does not occur naturally and must be manufactured in a nuclear reactor**. A DT fusion bomb with an explosive yield equal to that of 10 million tons of TNT (10 megatons) would require hundreds of kilograms of tritium. The rough size of the bomb can be estimated from the density of solid deuterium; it would be equivalent to a sphere about 1 meter in diameter. Smaller quantities of deuterium and tritium could be used to boost the explosive force of fission bombs or to initiate the ignition of a fusion weapon. But to manufacture tritium in the quantities needed for a significant number of pure DT bombs would require a massive production effort—dwarfing even the substantial program already under way to manufacture plutonium—and would be prohibitively expensive.

For Uranium, the problem is that **less than 1 % of natural uranium is the “explosive” isotope ^{235}U , and separating it from the more abundant ^{238}U is very difficult** (see above).

G.4. How does an H-bomb in the Teller-Ulam design work? Draw and explain.

Solution

Due to the reasons mentioned above, Teller created a new idea, which didn't work. With the help of Ulam they created the Teller-Ulam design:

Early in 1951, Ulam made an important conceptual breakthrough, and Teller quickly refined the idea. This followed an idea known as radiation implosion that had first been broached in 1946 by Klaus Fuchs before he was arrested for giving atomic secrets to the

Soviet Union. Most of the energy leaves the fission trigger as X-rays. Traveling at the speed of light, the X-rays can reach the nearby fusion fuel almost instantaneously and be used to compress and ignite it before it is blown apart by the blast wave from the fission explosion, which travels only at the speed of sound. This is analogous to the delay between seeing a flash of lightning and hearing the sound of the thunder, though the much smaller distances in the bomb reduce the delay to less than a millionth of a second. A second important requirement is that the radiation from the fission bomb needs to compress the fusion fuel before it is heated to the high temperature at which it will ignite. It is much easier to compress a gas when it is cold than when it is hot.

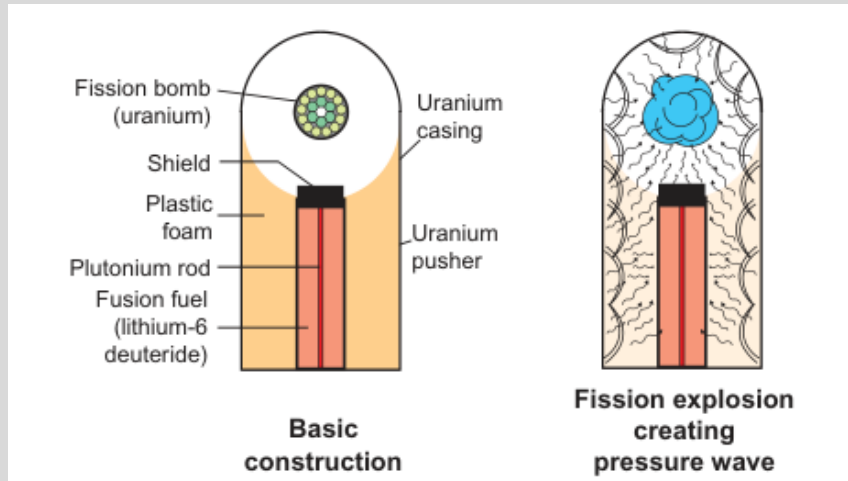


Abbildung 19: Schematic diagram of the elements of an H-bomb. A fission explosion is first triggered by high explosive. This explosion is contained inside a heavy metal case. The radiation from the fission bomb causes the implosion and heating of the fusion fuel and sets off the fusion bomb.

The fission bomb trigger is set off at one end of a cylindrical casing in which the fusion fuel is also contained. The fusion fuel is thought to be in the form of a cylinder surrounding a rod of plutonium, and a layer of very dense material—usually natural uranium or tungsten—surrounds the fuel itself. The X-ray radiation from the fission bomb is channeled down the radial gap between the outer casing and the fusion fuel cylinder. The gap is filled with plastic foam that is immediately vaporized and turned into hot plasma. The plasma is transparent to the X-rays, allowing the inner surface of the cylindrical casing and the outer surface of the dense layer surrounding the fusion fuel to be heated quickly to very high temperatures. As the outer surface of the layer surrounding the fuel vaporizes, it exerts a high inward pressure—rather like an inverted rocket engine.

Enormous pressures are generated instantaneously—several billion times atmospheric pressure—and the fuel is compressed to typically 300 times its normal density. It is salutary to realize that the explosive force released by the fission trigger, enough to destroy an entire city, is being used briefly to squeeze a few kilograms of fuel! The compression and the neutrons from the fission bomb cause the plutonium rod down the middle of the fusion fuel to become critical and to explode—in effect a second fission bomb goes off. This explosion rapidly heats the already compressed fusion fuel to the temperature required for the fusion reactions to start. Once ignited, the fusion fuel burns outward and extremely high temperatures—up to 300 million degrees—are reached as almost the whole of the fuel is consumed. Some reports suggest that the design has been refined so much that the plutonium “spark plug” is no longer needed.

G.5. Discuss ideas for civil uses of nuclear bombs and why they failed.

Solution

Project Plowshare (USA): The project considered civil engineering applications, such as using a series of nuclear explosions to either widen the existing Panama Canal or to construct a new sea-level waterway connecting the Atlantic and Pacific oceans, to construct new dams and harbors and to cut railroads and highways through mountainous areas. Other ideas involved blasting underground caverns for storing water, natural gas, or petroleum and using underground explosions to improve the flow from difficult oil or natural gas fields by fragmenting rock formations that have low natural permeability. The US carried out nearly thirty underground explosions during the 1960s and '70s to study some of these ideas, but **Project Plowshare was ended in 1977 due to growing concerns about radioactive fallout and about the ethics of using nuclear weapons under any circumstances.**

Energy conversion: One peaceful application studied in the United States was the possibility of converting the energy released by a nuclear explosion into electricity. The simplest idea was to set off a small nuclear bomb deep underground inside a natural formation of rock salt. The energy of the explosion would melt the salt—and then water would be pumped into the cavity and the steam extracted to generate electricity. This concept was tested in December 1961 in the first explosion of the Plowshare series—a test known as Gnome using a small fission bomb with a yield of about 3 kilotons.

Project Pacer: The concept was followed up in the mid-1970s at Los Alamos National Laboratory with a study (known as the Pacer Project) of a more sophisticated scheme to **generate a steady supply of electricity from nuclear explosions.** A typical design envisaged exploding the bombs inside an underground blast-chamber—a steel cylinder 30 m in diameter and 100 m tall with 4 m thick walls filled with molten fluoride salt. The molten salt would absorb the energy of the explosions (and also the neutrons, to reduce damage to the blast-chamber) and would then be used to heat water to drive a steam turbine. A 1 kiloton bomb releases about 4000 GJ (the same amount of energy that is obtained by burning about 175 tons of coal) and, at the planned rate of one bomb every 45 min, the power output (if converted to electricity with the 30% efficiency typical of a coal-fired steam plant) would be about 500 MW. However, a large supply of nuclear bombs, roughly 10,000 bombs per year, would be required for the plant to operate continuously.

Leaving aside the **environmental and political issues** involved in producing nuclear bombs on such a massive scale, the economics of such a system are very doubtful given the **enormous costs and difficulties of producing fissile material.** In principle, it would be more economical to use fusion bombs, which deliver much more energy per unit of fissile material, but **in practice it would be impossible to contain explosions larger than a few kilotons within a realistic engineered structure.** It is not surprising that Project Pacer never progressed beyond the conceptual stage.

H. Chapter 7: Inertial Confinement

H.1. What are typical orders of magnitude for terms in the triple product for magnetic and inertial confinement? What does this mean for steady-state operation?

Solution

See Figure 9:

Magnetic Confinement:

$$n \approx 10^{20} \text{ particles / m}^3, \quad T \text{ in range } 10 - 20 \text{ keV} \approx 100 - 200 \text{ million K}, \quad \tau_E \approx 10^{0.5} \text{ s}$$

Inertial Confinement:

$$n > 10^{30} \text{ particles / m}^3, \quad T \text{ in range } 10 - 20 \text{ keV} \approx 100 - 200 \text{ million K}, \quad \tau_E \approx 10^{-9} \text{ s}$$

A typical value would be $nT\tau_E \approx 10^{21} \text{ m}^{-3} \text{ keV s}$.

Practically this means for the inertial confinement the energy gained by one pulse must be enough to heat the next fuel pellet.

H.2. Draw and explain an inertial fusion reaction by direct and indirect drive. How is it related to the hydrogen bomb?

Solution

In the direct drive the laser beams are directly focused on the fuel pellet. In the indirect drive the laser beams are focused on a hohlraum which then heats the fuel pellet:

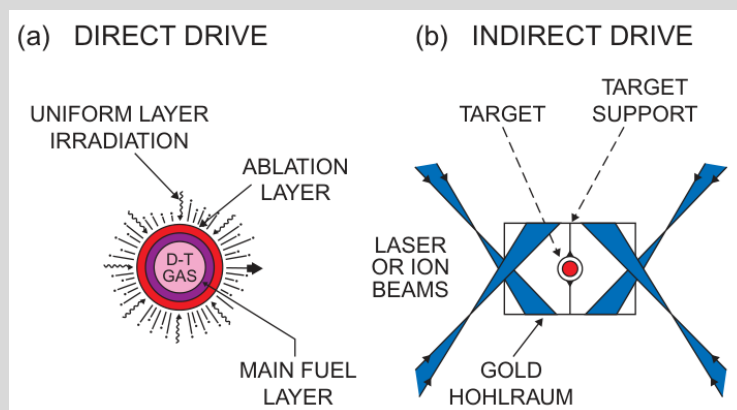


Abbildung 20: Comparison of the principles of direct and indirect drive. Uniform irradiation is produced by many laser beams in the direct-drive case and by the production of X-rays at the wall of the hohlraum in the indirect case. The targets are similar in size in the two cases, but the direct drive has been shown enlarged to illustrate the typical structure.

The hydrogen bomb uses a fission bomb to create the conditions for fusion. Both the bomb and inertial confinement creates the conditions for fusion by compressing the fuel pellet. While the bomb uses a fission bomb to compress the fuel pellet, inertial confinement uses (mainly) lasers to heat the outer layers which then compress the fuel pellet.

H.3. How does a laser work? Discuss efficiency and future possibilities for technical break-even.

Solution

A laser is a device that emits light through a process of optical amplification based on the stimulated emission of electromagnetic radiation. The term "laser" originated as an acronym for "light amplification by stimulated emission of radiation".

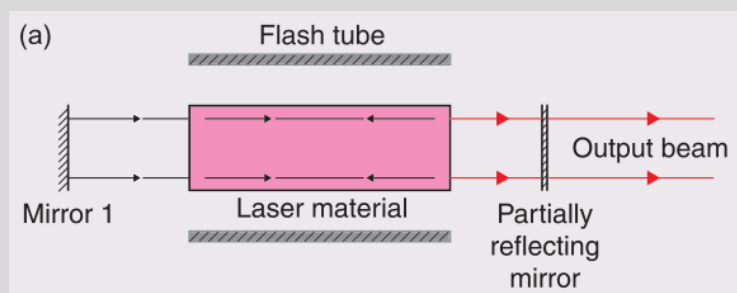


Abbildung 21: Schematic diagram of a typical laser.

In principle a laser pumps energy into a medium (gas, liquid or solid) which raises the electrons to a higher energy level. These electrons then fall back to a lower energy level if they are stimulated by a photon of the same energy as the energy drop between the two levels. This will cause the electron to emit a photon of the same energy as the stimulating photon. This process is called stimulated emission. These photons are then reflected back and forth between two mirrors stimulating more emissions on their way. One of the mirrors is partially transparent and lets some of the photons out.

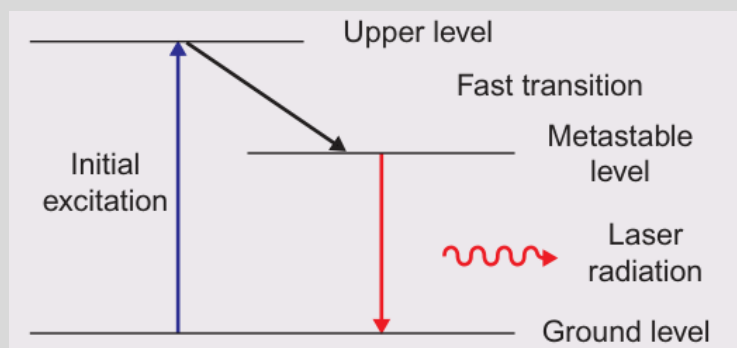


Abbildung 22: Principle of stimulated emission.