DELFT UNIVERSITY OF TECHNOLOGY

Introduction to High Performance Computing WI4049TU

Lab Report

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General Remarks

This final Lab report includes the answers for the exercises (base grad denoted in paranthesis):

- 0. Introductory exercise (0.5)
- 1. Poisson solver (1.75)
- 2. Finite elements simulation (1.0)
- 3. Eigenvalue solution by Power Method on GPU (1.75)

The optional **shining points** (e.g., performance analysis, optimization, discussion, and clarifying figures) which yield further points are usually marked by a small blue heading in the text or an additional note is added under a figure or table. For example:

This is a shining point.

0 Introductory exercise

In the introductory lab session, we are taking a look at some basic features of MPI. We start out very simple with a hello world program on two nodes.

Hello World

```
#include "mpi.h"
#include <stdio.h>

int np, rank;

int main(int argc, char **argv)

{
    MPI_Init(&argc, &argv);
    MPI_Comm_size(MPI_COMM_WORLD, &np);
    MPI_Comm_rank(MPI_COMM_WORLD, &rank);

printf("Node %d of %d says: Hello world!\n", rank, np);

MPI_Finalize();
    return 0;
}
```

This program can be compiled with the following command:

```
mpicc -o helloworld1.out helloworld1.c
```

And run with:

```
srun -n 2 -c 4 --mem-per-cpu=1GB ./helloworld1.out
```

We get the following output:

```
Node 0 of 2 says: Hello world!
Node 1 of 2 says: Hello world!
```

From now on I'll skip the compilation and only mention on how many nodes the program is run and what the output is / interpretation of the output.

0.a) Ping Pong

I used the template to check how long MPI_Send and MPI_Recv take. The code can be found in the appendix for this section.

I've modified the printing a bit to make it easier to gather the information. Then I piped the program output into a textfile for further processing in python. I ran it first on one and then on two nodes as specified in the

assignment sheet. Opposed to the averaging over 5 send / receive pairs, I've done 1000 pairs. Furthmore I reran the whole programm 5 times to gather more data. All this data is shown in the following graph:



Figure 1: Ping Pong: Number of bytes sent vs. average time taken from 1000 pairs of send / receive. 5 runs shown for each size as scatter plot. Mean of these 5 runs shown as line. Blue small fit includes all data points up to 131072 bytes, blue large from there. Red small fit includes all data points up to 32768 bytes, red large from there.

As can be seen in the data and the fits, there are outliers especially for the larger data sizes. For our runs we get the following fits and Rš values:

Run Type	Data Size	Fit Equation	Rš Value
Single Node	Small (<=131072)	$5.95 \times 10^{-7} \cdot x + 7.97 \times 10^{-4}$	0.92
Single Node	Large ($>= 131072$)	$4.61 \times 10^{-7} \cdot x + 1.23 \times 10^{-2}$	0.89
Two Node	Small (<=32768)	$1.07 \times 10^{-6} \cdot x + 2.60 \times 10^{-3}$	0.97
Two Node	Large (>=32768)	$4.41 \times 10^{-7} \cdot x + 3.42 \times 10^{-3}$	0.97

Table 1: Fit Equations and Rš Values for Single Node and Two Node Runs

Note: Each run was performed 5 times (for 1 and 2 nodes) to get a fit on the data and calculate a Rš value. TODO: **Further analysis needed?**

Extra: Ping Pong with MPI_SendRecv

We do the same analysis for the changed program utilizing MPI_SendRecv. The code can be found in the appendix for this section.

We get the following graph from the measurements which were performed in the same way as for the previous program:



Figure 2: Ping Pong with MPI_SendRecv: Number of bytes sent vs. average time taken from 1000 pairs of send / receive. 5 runs shown for each size as scatter plot. Mean of these 5 runs shown as line. Blue small fit includes all data points up to 32768 bytes, blue large from there. Red small fit includes all data points up to 32768 bytes, red large from there.

We get the following fits and Rš values for the runs:

Run Type	Data Size	Fit Equation	Rš Value
Single Node	Small (<=32768)	$3.51 \times 10^{-7} \cdot x + 1.19 \times 10^{-3}$	0.98
Single Node	Large (>= 32768)	$1.69 \times 10^{-7} \cdot x + 4.00 \times 10^{-3}$	1.00
Two Node	Small (<=32768)	$6.65 \times 10^{-7} \cdot x + 9.47 \times 10^{-4}$	0.98
Two Node	Large (>= 32768)	$1.66 \times 10^{-7} \cdot x + 4.93 \times 10^{-3}$	1.00

Table 2: Fit Equations and Rš Values for Single Node and Two Node Runs

TODO: Further analysis needed?

0.b) MM-product

After an introduction of the matrix-matrix multiplication code in the next section, the measured speedups are discussed in the subsequent section.

Explanation of the code

For this excercise I've used the template provided in the assignment sheet as a base to develop my parallel implementation for a matrix-matrix multiplication. The code can be found in the appendix for this section.

The porgam can be run either in sequential (default) or parallel mode (parallel as a command line argument). For the sequential version, the code is practically unchanged and just refactored into a function for timing purposes. The parallel version is more complex and works as explained bellow:

First, rank 0 computes a sequential reference solution. Then rank 0 distributes the matrices in the following way in splitwork:

- Matrix A is split row-wise by dividing the number of rows by the number of nodes.
- The first worker (=rank 1) gets the most rows starting from row 0: total_rows (nr_workers 1) $\cdot floor(\frac{total_rows}{nr_workers})$.
- All other workers and the master (= rank 0) get the same number of rows: $floor(\frac{\text{total_rows}}{\text{nr workers}})$.
- The master copies the corresponding rows of matrix A and the whole transposed matrix B* into a buffer (for details on MM_input buffer see bellow) for each worker and sends them off using MPI_ISend.
- The workers receive the data using MPI_Recv and then compute their part of the matrix product and send only the rows of the result matrix back to the master using MPI_Send.
- In the meanwhile the master computes its part of the matrix product.
- Using MPI_Waitall the master waits for all data to be sent to the workers and only afterwards calls MPI_Recv to gather the results from the workers.
- Finally all results are gathered by the master in the result matrix.

Assume we have a 5x5 matrix A and 2 workers (rank 1 and rank 2) and master (rank 0). The partitioning is done row-wise as follows:

Partitioning Example

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{pmatrix} \rightarrow \begin{pmatrix} \text{Worker 1} \\ \text{Worker 1} \\ \text{Worker 1} \\ \text{Master} \\ \text{Master} \end{pmatrix}$$

- Rank 0 (Master): Rows 4 and 5 (last two rows)
- Rank 1 (Worker 1): Rows 1 to 3 (first three rows) Worker 1 always gets the most rows

This partitioning can be visually represented as:

Master (rank 0):
$$\begin{pmatrix} a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{pmatrix}$$
Worker 1 (rank 1):
$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \end{pmatrix}$$

Each worker computes its part of the matrix product, and the master gathers the results at the end and compiles them into the final matrix.

The MM_input buffer is used to store the rows of matrix A and the whole matrix B for each worker. It is implemented using a simple struct:

```
typedef struct MM_input {
    size_t rows;
double *a;
double *b;
} MM_input;
```

*[Optimization] Note on transposed matrix B: It is usually beneficial from a cache perspective to index arrays sequentially or in a row-major order. However, in the matrix-matrix multiplication, we access the elements of matrix B in a column-wise order. This leads to cache misses and is not optimal. To mitigate this, we can transpose matrix B and then access it in a row-wise order. This is done in the code by the master before sending the data to the workers.

Discussion of the speedups

The code was run on Delft's cluster with 1, 2, 4, 8, 16, 24, 32, 48, and 64 nodes. For the experiments the matrix size of A and B was set to 2000×2000 . This means that the program has to evaluate 2000 multiplications and 1999 additions for each element of the resulting matrix C. In total this results in $\approx 2000^3 = 8 \times 10^9$ operations. The command looked similar to the following for the different node counts:

srun -n 48 --mem-per-cpu=4GB --time=00:02:00 ./MM.out parallel

For this experiment, the execution time was measured and the speedup was calculated. The results are shown in Table 3 and Figure 3.

CPU Count	Execution Time / s	Approx. Speedup
1	47.11	1.0
2	10.26	4.6
4	10.30	4.6
8	5.20	9.1
16	2.97	15.9
24	2.54	18.5
32	2.29	20.6
48	2.98	15.8
64	1.72	27.4

Table 3: Execution Time vs CPU Count



Figure 3: Speedup vs CPU Count Black \times marks the average of the rerun for n=48.

Note: The speedup is calculated as $S = \frac{T_1}{T_p}$, where T_1 is the execution time on 1 node and T_p is the execution time on p nodes.

Discussion:

As one can cleary discern from the data in Table 3 and Figure 3, the speedup increases with the number of nodes (with the exception of n = 48). This is expected as the more nodes we have, the more work can be done in

parallel. However, the speedup is not linear. This is due to the overhead of communication between the nodes. The more nodes we have, the more communication is needed, and this overhead increases. This is especially visible in the data for n = 48. Here the speedup is lower than for n = 32. For this run the communication didn't went as smooth as for the other runs. This can potentially be attributed to the fact that one (or more) of the nodes or the network was under heavy load during this task.

[Further investigation] After observing this slower speed for the n=48, I reran the tests multiple times and got a runtime of around 1.9s which was to be expected initially. Therefore, this one run is an odd one out, most likely due to the reasons mentioned above! I've also added the averaged data of the reruns as a datapoint in Figure 3.

Another interesting fact can be seen when comparing the time taken for n = 1 and n = 2. They don't at all scale with the expected factor of 2. This is could be due to the fact, that the resource management system prefers runs with multiple nodes instead of a single node (= sequential).

Additional notes: The flag <code>-mem-per-cpu=<#>GB</code> was set depending on the number of nodes used. For 1-24 nodes 8GB was used, for 32-48 nodes 4GB, and for 64 nodes 3GB. This had to be done to comply with QOS policy on the cluster.

TODO: Data locality?

1 Poisson solver

In this section of the lab report, we will dicuss a prallel implementation of the Poisson solver. The Poisson solver is a numerical method used to solve the Poisson equation, which is a partial differential equation that is useful in many areas of physics.

Note: For local testing and development I'll run the code with mpirun instead of the srun command on the cluster.

1.1 Building a parallel Poisson solver

For the first part of the exercise we follow the steps lined out in the assignment sheet. I'll comment on the steps 1 through 10 and related questions bellow. The finished implementation can be found in the appendix for this section.

1. **Step:** After adding MPI_Init and MPI_Finalize, we can run the program with multiple processes. We can see that the program runs with 4 processes in Figure 4 via the quadrupeled output.

```
etschgi1@Deep-Thought:~/REPOS/HPC/01_lab1/src$ mpirun -np 4 ./mpi.out
Number of iterations : 2355
Number of iterations : 2355
Elapsed processortime: 0.133189 s
Number of iterations : 2355
Elapsed processortime: 0.134150 s
Elapsed processortime: 0.134474 s
Elapsed processortime: 0.135356 s
```

Figure 4: MPI Poisson after Step 1 - Running with 4 processes

2. **Step:** To see which process is doing what, I included the rank of the process for the print statements as shown in Figure 5.

Figure 5: MPI_Poisson after Step 2 - Running with 4 processes

3. **Step:** Next we define wtime as a global double and replace the four utility timing functions with the ones given on Brightspace. A quick verification as shown in Figure 6 shows that the program still runs as expected.

Figure 6: MPI Poisson after Step 3 - Running with 4 processes

- 4. **Step:** Next we check if two processes indeed give the same output. Both need 2355 iterations to converge and the diff command returned no output, which means that the files content is identical.
- 5. **Step:** Now only the process with rank 0 will read data from files and subsequently broadcast it to the others. Testing this again with 2 processes, we see an empty diff of the output files and the same number of iterations needed to converge.

6. **Step:** We create a cartesian grid of processes using MPI_Cart_create and use MPI_Cart_shift to find the neighbors of each process. We can see that the neighbors are correctly identified in Figure 7.

```
(0) (x,y)=(0,0)

(0) top 1, right -2, bottom -2, left 2

(1) (x,y)=(0,1)

(1) top -2, right -2, bottom 0, left 3

(2) (x,y)=(1,0)

(2) top 3, right 0, bottom -2, left -2

(3) (x,y)=(1,1)

(3) top -2, right 1, bottom 2, left -2
```

Figure 7: MPI_Poisson after Step 6 - Running with 4 processes on a 2x2 grid

When there is no neighbor in a certain direction, -2 (or MPI_PROC_NULL) is returned.

7. **Step:** We overhaul the setup to get a proper local grid for each process. Furthermore, we only save the relevant source fields in the local grid for each process.

With for instance 3 processes you should see that 1 or 2 processes do not do any iteration. Do you understand why?

If we have a look at the input file we see that there are only 3 source fields in the grid. This means that the process that does not have a source field in its local grid will not do any iterations (or only 1). Therefore, if we have 3 processes and the distribution of source fields as given in the input file only 1 process will do iterations if processes are ordered in x-direction and 2 if ordered in y-direction. From this we can conclude that indeed all processes have different local grids and perform different calculations.

```
• etschgi1@Deep-Thought:~/REPOS/HPC/02_lab1/src$ mpirun -np 3 ./mpi.out 3 1
(0) (x,y)=(0,0)
(0) top -2, right -2, bottom -2, left 1
(1) (x,y)=(1,0)
(2) (x,y)=(2,0)
(3) Number of iterations: 1
(2) Elapsed Wtime 0.000618 s (95.3% CPU)
(3) Elapsed Wtime 0.000477 s (95.2% CPU)
(4) Number of iterations: 695
(5) Elapsed Wtime 0.017636 s (95.3% CPU)
(6) Elapsed Wtime 0.017636 s (95.3% CPU)
(7) Elapsed Wtime 0.017636 s (95.3% CPU)
(8) Elapsed Wtime 0.017636 s (95.3% CPU)
(9) Elapsed Wtime 0.017636 s (95.3% CPU)
(10) Elapsed Wtime 0.017636 s (95.3% CPU)
(21) Elapsed Wtime 0.017636 s (95.3% CPU)
(22) Elapsed Wtime 0.017636 s (95.3% CPU)
(23) Elapsed Wtime 0.017636 s (95.3% CPU)
(24) Elapsed Wtime 0.017636 s (95.3% CPU)
(25) Elapsed Wtime 0.017636 s (95.3% CPU)
```

Figure 8: MPI_Poisson after Step 7 - Running with 3 processes on a 3x1 (left) vs. 1x3 (right) grid For the 3x1 grid, only rank 1 does iterations (> 1), for the 1x3 grid, ranks 0 and 2 do iterations (> 1).

- 8. **Step:** After defining and committing two special datatypes for vertical and horizontal communication, we setup the communication logic to exchange the boundary values between the processes. We call our Exchange_Borders function after each iteration (for both red / black grid points). Now we face the problem in which some processes may stop instantly (no source in their local grid). They will not supply any data to their neighbors, which will cause the program to hang. We shall fix this in the next step.
- 9. **Step:** Finally we need to implement the logic to check for convergence (in a global sense). We do this by using a MPI_Allreduce call with the MPI_MAX operation. This way we aggregate all deltas and choose the biggest one for the global delta which we use in the while-loop-condition to check for convergence. We can see that the program now runs as expected in Figure 9.

```
(0) (x,y)=(0,0)

(0) top -1, right 2, bottom 1, left -1

(1) (x,y)=(0,1)

(1) top 0, right 3, bottom -1, left -1

(2) (x,y)=(1,0)

(2) top -1, right -1, bottom 3, left 0

(3) (x,y)=(1,1)

(3) top 2, right -1, bottom -1, left 1

(0) Number of iterations : 2355

(1) Number of iterations : 2355

(2) Number of iterations : 2355

(3) Number of iterations : 2355

(1) Elapsed Wtime 0.287549 s (99.9% CPU)

(2) Elapsed Wtime 0.287537 s (100.0% CPU)

(3) Elapsed Wtime 0.287537 s (100.0% CPU)

(0) Elapsed Wtime 0.295957 s (99.9% CPU)
```

Figure 9: MPI_Poisson after Step 9 - Running with 4 processes on a 2x2 grid

Note that this run in Figure 9 was done with another pc and another MPI implementation. Therefore, we see -1 for cells without a neighbor! However, other than that cosmetic difference it has no impact on the programm.

10. **Step:** Now we only have to fix two remaining things. First we have to make sure that each process uses the right global coordinates for the output file in the end. Therefore, we change the function a bit to include the specific x-/y-offset for each processor. The second thing is the potential problem, that different processors might start with different (red/black) parities. In order to accomplish a global parity we simply have to change the calculation in the if in Do_Step from

```
if ((x + y) % 2 == parity && source[x][y] != 1)

to

if ((x + offset[X_DIR] + y + offset[Y_DIR]) % 2 == parity && source[x][y] != 1)
```

this guarantees that during a given iteration all processors are using the same parity.

This just leaves one question open: Are the results acutally the same?

Checking the output files of the MPI-implementation with the sequential reference indeed shows identical numerical values for the calculated points. Furthermore, the needed iteration count is also identical which isn't a big surprise, given that the two programms perform the exact same calculation steps.

1.2 Exercises, modifications, and performance aspects

For this subsection we'll define the following shorthand notation:

n:	the number of iterations
<i>g</i> :	gridsize
t:	time needed in seconds
pt:	processor topology in form pxy , where:
p:	number of processors used
x:	number of processors in x-direction
y:	number of processors in y-direction

Table 4: Notation for this section

pt = 414 means 4 processors in a 1×4 topology.

1.2.1 Over-relaxation (SOR)

We start of by rewriting the Do_Step routine to facilitate SOR updates. Furthermore, we need h^2 , the grid spacing (which is 1 in our case) and the relaxation parameter ω to calculate the updated values. A quick test shows a speedup of roughly a factor of 10. More systematic tests will be done in the next section.

1.2.2 Optimal ω for 4 proc. on a 4x1 grid

With the power of a little python scripting we can easily test different values for ω and plot the results as seen in Figure 10.

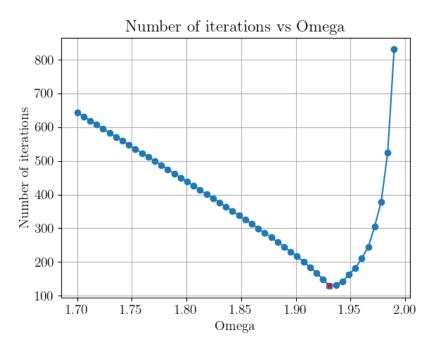


Figure 10: Optimal ω for 4 processors on a 4x1 grid

We find that the optimal ω is at about 1.93 for this setup with only 129 iterations. This constitutes a speedup of about 1825% compared to the sequential implementation.

2 Finite elements simulation

 ${f 3}$ Eigenvalue solution by Power Method on GPU

Appendix - Introductory exercise

The following code was used for the ping pong task:

```
#include <stdio.h>
#include <stdlib.h>
3 #include <mpi.h>
5 // Maximum array size 2^20= 1048576 elements
6 #define MAX_EXPONENT 20
7 #define MAX_ARRAY_SIZE (1<<MAX_EXPONENT)</pre>
8 #define SAMPLE_COUNT 1000
int main(int argc, char **argv)
11
      // Variables for the process rank and number of processes
12
13
      int myRank, numProcs, i;
      MPI_Status status;
14
15
      // Initialize MPI, find out MPI communicator size and process rank
16
      MPI Init(&argc, &argv);
17
      MPI_Comm_size(MPI_COMM_WORLD, &numProcs);
18
19
      MPI_Comm_rank(MPI_COMM_WORLD, &myRank);
20
21
      int *myArray = (int *)malloc(sizeof(int)*MAX_ARRAY_SIZE);
22
      if (myArray == NULL)
24
          printf("Not enough memory\n");
25
26
          exit(1);
27
      // Initialize myArray
28
      for (i=0; i<MAX_ARRAY_SIZE; i++)</pre>
29
          myArray[i]=1;
30
31
      int number_of_elements_to_send;
      int number_of_elements_received;
33
34
35
      // PART C
      if (numProcs < 2)
36
37
          printf("Error: Run the program with at least 2 MPI tasks!\n");
38
          MPI_Abort(MPI_COMM_WORLD, 1);
39
40
      double startTime, endTime;
41
42
43
      // TODO: Use a loop to vary the message size
      for (size_t j = 0; j <= MAX_EXPONENT; j++)</pre>
44
45
          number_of_elements_to_send = 1<<j;</pre>
46
          if (myRank == 0)
47
          {
              myArray[0]=myArray[1]+1; // activate in cache (avoids possible delay when sending
49
      the 1st element)
              startTime = MPI_Wtime();
              for (i=0; i<SAMPLE_COUNT; i++)</pre>
51
                  MPI_Send(myArray, number_of_elements_to_send, MPI_INT, 1, 0,
53
                       MPI_COMM_WORLD);
54
                  MPI_Probe(MPI_ANY_SOURCE, MPI_ANY_TAG, MPI_COMM_WORLD, &status);
55
                  MPI_Get_count(&status, MPI_INT, &number_of_elements_received);
56
57
58
                  MPI_Recv(myArray, number_of_elements_received, MPI_INT, 1, 0,
                      MPI_COMM_WORLD, MPI_STATUS_IGNORE);
59
              } // end of for-loop
60
61
              endTime = MPI_Wtime();
62
              number_of_elements_received,(endTime - startTime)/(2*SAMPLE_COUNT));
64
          }
65
          else if (myRank == 1)
66
67
               // Probe message in order to obtain the amount of data
              MPI_Probe(MPI_ANY_SOURCE, MPI_ANY_TAG, MPI_COMM_WORLD, &status);
68
```

```
MPI_Get_count(&status, MPI_INT, &number_of_elements_received);
69
70
                   for (i=0; i<SAMPLE_COUNT; i++)</pre>
71
                   {
                         MPI_Recv(myArray, number_of_elements_received, MPI_INT, 0, 0,
73
                         MPI_COMM_WORLD, MPI_STATUS_IGNORE);
74
                         {\tt MPI\_Send} \, ({\tt myArray} \, , \, \, {\tt number\_of\_elements\_to\_send} \, , \, \, {\tt MPI\_INT} \, , \, \, {\tt 0} \, , \, \, {\tt 0} \, , \, \,
75
                        MPI_COMM_WORLD);
76
77
                   } // end of for-loop
              }
78
79
80
        // Finalize MPI
81
        MPI_Finalize();
83
84
        return 0;
```

For the bonus task, the following code was used:

```
#include <stdio.h>
#include <stdlib.h>
3 #include <mpi.h>
5 // Maximum array size 2^20= 1048576 elements
6 #define MAX_EXPONENT 20
7 #define MAX_ARRAY_SIZE (1<<MAX_EXPONENT)</pre>
8 #define SAMPLE_COUNT 1000
int main(int argc, char **argv)
11 {
12
       // Variables for the process rank and number of processes
       int myRank, numProcs, i;
       MPI_Status status;
14
15
       // Initialize MPI, find out MPI communicator size and process rank
16
       MPI_Init(&argc, &argv);
17
       MPI_Comm_size(MPI_COMM_WORLD, &numProcs);
18
       MPI_Comm_rank(MPI_COMM_WORLD, &myRank);
19
20
21
22
       int *myArray = (int *)malloc(sizeof(int)*MAX_ARRAY_SIZE);
      if (myArray == NULL)
23
24
       {
25
           printf("Not enough memory\n");
           exit(1);
26
27
       // Initialize myArray
28
       for (i=0; i<MAX_ARRAY_SIZE; i++)</pre>
29
30
           myArray[i]=1;
31
       int number_of_elements_to_send;
32
       int number_of_elements_received;
33
34
       // PART C
35
       if (numProcs < 2)</pre>
36
       {
37
38
           printf("Error: Run the program with at least 2 MPI tasks!\n");
           MPI_Abort(MPI_COMM_WORLD, 1);
39
40
41
       double startTime, endTime;
42
       // TODO: Use a loop to vary the message size \,
43
       for (size_t j = 0; j <= MAX_EXPONENT; j++)</pre>
44
45
46
           number_of_elements_to_send = 1<<j;</pre>
47
           if (myRank == 0)
           {
48
               myArray[0]=myArray[1]+1; // activate in cache (avoids possible delay when sending
49
       the 1st element)
               startTime = MPI Wtime();
50
51
               for (i=0; i<SAMPLE_COUNT; i++)</pre>
               {
52
53
                    MPI_Sendrecv(myArray, number_of_elements_to_send, MPI_INT, 1,0,myArray,
```

```
number_of_elements_to_send, MPI_INT, 1, 0, MPI_COMM_WORLD, &status);
54
55
                endTime = MPI_Wtime();
56
                \label{lem:printf("Rank %2.1i: Received %i elements: Ping Pong took %f seconds \\ \normalfont{n", myRank,}
57
       number_of_elements_to_send,(endTime - startTime)/(2*SAMPLE_COUNT));
           }
58
59
           else if (myRank == 1)
60
            {
                for (i=0; i<SAMPLE_COUNT; i++)</pre>
61
62
                    MPI_Sendrecv(myArray, number_of_elements_to_send, MPI_INT, 0,0,myArray,
63
       number_of_elements_to_send , MPI_INT , 0 , 0 , MPI_COMM_WORLD , &status);
           }
65
66
67
       // Finalize MPI
68
69
       MPI_Finalize();
70
       return 0:
71
```

The matrix multiplication used the following code:

```
* FILE: mm.c
   * DESCRIPTION:
3
       This program calculates the product of matrix a[nra][nca] and b[nca][ncb],
       the result is stored in matrix c[nra][ncb].
5
      The max dimension of the matrix is constraint with static array
   *declaration, for a larger matrix you may consider dynamic allocation of the
   *arrays, but it makes a parallel code much more complicated (think of
   *communication), so this is only optional.
10
11
13 #include <math.h>
#include <mpi.h>
#include <stdbool.h>
16 #include <stdio.h>
#include <stdlib.h>
18 #include <string.h>
19
20 #define NRA 2000 /* number of rows in matrix A */
#define NCA 2000 /* number of columns in matrix A */
_{\rm 22} #define NCB 2000 /* number of columns in matrix B */
23 // #define N 1000
#define EPS 1e-9
#define SIZE_OF_B NCA*NCB*sizeof(double)
26
27 bool eps_equal(double a, double b) { return fabs(a - b) < EPS; }</pre>
28
void print_flattened_matrix(double *matrix, size_t rows, size_t cols, int rank) {
      printf("[%d]\n", rank);
30
      for (size_t i = 0; i < rows; i++) {</pre>
31
          for (size_t j = 0; j < cols; j++) {
32
              printf("%10.2f ", matrix[i * cols + j]); // Accessing element in the 1D array
33
34
          printf("\n"); // Newline after each row
35
36
37 }
38
  int checkResult(double *truth, double *test, size_t Nr_col, size_t Nr_rows) {
39
      for (size_t i = 0; i < Nr_rows; ++i) {</pre>
40
41
          for (size_t j = 0; j < Nr_col; ++j) {</pre>
              size_t index = i * Nr_col + j;
42
              if (!eps_equal(truth[index], test[index])) {
43
                  return 1;
44
              }
45
          }
46
      }
      return 0;
48
49 }
```

```
51 typedef struct {
       size_t rows;
       double *a;
       double *b:
54
55 } MM_input;
56
57 char* getbuffer(MM_input *in, size_t size_of_buffer){
58
       char* buffer = (char*)malloc(size_of_buffer * sizeof(char));
       if (buffer == 0)
59
60
            printf("Buffer couldn't be allocated.");
61
            return NULL;
62
63
       size_t offset = 0;
64
       memcpy(buffer + offset, &in->rows, sizeof(size_t));
65
66
       offset += sizeof(size_t);
       size_t matrix_size = in->rows * NCA * sizeof(double);
67
       memcpy(buffer + offset, in->a, matrix_size);
68
69
       offset += matrix_size;
       memcpy(buffer + offset, in->b, NCA*NCB*sizeof(double));
70
71
       return buffer;
72 }
73
   MM_input* readbuffer(char* buffer, size_t size_of_buffer){
74
       MM_input *mm = (MM_input*)malloc(sizeof(MM_input));
75
76
77
       mm->rows = ((size_t*)buffer)[0];
       size_t offset = sizeof(size_t);
78
       size_t matrix_size = mm->rows * NCA;
79
       mm->a = (double*)malloc(sizeof(double)*matrix_size);
80
       mm->b = (double*)malloc(sizeof(double)*matrix_size);
81
       memcpy(mm->a, &(buffer[offset]), matrix_size);
82
       offset += matrix_size;
83
84
       memcpy(mm->b, &(buffer[offset]), NCA*NCB*sizeof(double));
       free(buffer);
85
86
       return mm;
87 }
88
89
90
   void setupMatrices(double (*a)[NCA], double (*b)[NCB], double (*c)[NCB]){
       for (size_t i = 0; i < NRA; i++) {</pre>
91
92
            for (size_t j = 0; j < NCA; j++) {</pre>
93
                a[i][j] = i + j;
94
96
       for (size_t i = 0; i < NCA; i++) {</pre>
97
            for (size_t j = 0; j < NCB; j++) {</pre>
                b[i][j] = i * j;
99
            }
100
101
102
        for (size_t i = 0; i < NRA; i++) {</pre>
            for (size_t j = 0; j < NCB; j++) {
   c[i][j] = 0;</pre>
104
105
106
108 }
109
double multsum(double* a,double* b_transposed, size_t size){
       double acc = 0;
       for (size_t i = 0; i < size; i++)</pre>
            acc += a[i]*b_transposed[i];
114
116
       return acc;
117 }
118
double productSequential(double *res) {
       // dynamically allocate to not run into stack overflow - usually stacks are
120
        // 8192 bytes big \rightarrow 1024 doubles but we have 1 Mio. per matrix
       double(*a)[NCA] = malloc(sizeof(double) * NRA * NCA);
```

```
double(*b)[NCB] = malloc(sizeof(double) * NCA * NCB);
123
       double(*c)[NCB] = malloc(sizeof(double) * NRA * NCB);
124
125
       /*** Initialize matrices ***/
126
       setupMatrices(a,b,c);
128
       /* Parallelize the computation of the following matrix-matrix
129
130
      multiplication. How to partition and distribute the initial matrices, the
      work, and collecting final results.
132
       // multiply
133
       double start = MPI_Wtime();
for (size_t i = 0; i < NRA; i++) {</pre>
134
135
            for (size_t j = 0; j < NCB; j++) {</pre>
136
                for (size_t k = 0; k < NCA; k++) {</pre>
137
                    res[i * NCB + j] += a[i][k] * b[k][j];
138
139
           }
140
141
       /* perform time measurement. Always check the correctness of the parallel
142
          results by printing a few values of c[i][j] and compare with the
143
           sequential output.
145
       double time = MPI_Wtime()-start;
146
147
       free(a);
       free(b):
148
       free(c);
149
       return time;
150
151 }
   double splitwork(double* res, size_t num_workers){
153
154
       if (num_workers == 0) // sadly noone will help me :((
            printf("Run sequential!\n");
156
            return productSequential(res);
158
159
       double(*a)[NCA] = malloc(sizeof(double) * NRA * NCA);
       double(*b)[NCB] = malloc(sizeof(double) * NCA * NCB);
161
       double(*c)[NCB] = malloc(sizeof(double) * NRA * NCB);
162
       // Transpose matrix b to make accessing columns easier - in row major way - better cache
       performance
       setupMatrices(a,b,c);
164
       double start_time = MPI_Wtime();
166
       double (*b_transposed)[NCA] = malloc(sizeof(double) * NCA * NCB);
       for (size_t i = 0; i < NCA; i++) {</pre>
168
            for (size_t j = 0; j < NCB; j++) {</pre>
169
                b_transposed[j][i] = b[i][j];
170
172
       /*** Initialize matrices ***/
174
       // given number of workers I'll split
       size_t rows_per_worker = NRA / (num_workers+1); //takes corresponding columns from other
176
       matrix
       printf("rows per worker: %zu\n", rows_per_worker);
       size_t row_end_first = NRA - rows_per_worker*num_workers;
178
       printf("first gets most: %zu\n", row_end_first);
179
180
       //setup requests
181
       MPI_Request requests[num_workers];
       MM_input *data_first = (MM_input*)malloc(sizeof(MM_input));
183
184
       data_first->rows = row_end_first;
       data_first->a = (double*)a; //they both start of with no offset!
       data_first->b = (double*)b_transposed;
186
187
       size_t total_size = sizeof(size_t) + (data_first->rows * NCA)*sizeof(double)+SIZE_OF_B;
       char* buffer = getbuffer(data_first, total_size);
                                                             //first one
188
189
        // Tag is just nr-cpu -1
       MPI_Isend(buffer, total_size, MPI_CHAR, 1, 0, MPI_COMM_WORLD, &requests[0]);
191
192
       free(data_first);
       total_size = sizeof(size_t) + (rows_per_worker * NCA)*sizeof(double) + SIZE_OF_B; //size
```

```
is the same for all other - just compute once!
194
       size t i;
       for (i = 0; i < (num_workers-1); ++i)</pre>
195
196
            MM input *data = (MM input*)malloc(sizeof(MM input)):
197
198
            data->rows = rows_per_worker;
            data->a = (double*)(a + (row_end_first + rows_per_worker*i));
199
            data->b = (double*)(b_transposed); // send everyting - all needed
200
            buffer = getbuffer(data, total_size);
201
            printf("nr_worker - %zu\n", i);
202
            MPI_Isend(buffer, total_size, MPI_CHAR, i+2, i+1, MPI_COMM_WORLD, &requests[i+1]);
203
204
            free(data);
205
       double* my_a = (double*)(a + (row_end_first + rows_per_worker*i));
206
207
208
       //I multiply the rest
       size_t offset = 0;
209
       for (size_t row = (NRA-rows_per_worker); row < NRA; row++)</pre>
210
211
            for (size_t col = 0; col < NCB; col++)</pre>
212
213
214
                res[row * NCB + col] = multsum(my_a+offset, (((double*)b_transposed)+col*NCA), NCA
       );
215
           }
216
            offset += NCA;
217
       printf("My c: \n");
218
        //wait for rest
219
       MPI_Status stats[num_workers];
220
       if(MPI_Waitall(num_workers, requests, stats) == MPI_ERR_IN_STATUS){
221
            printf("Communication failed!!! - abort\n");
222
223
       printf(">>>Everything sent and recieved\n");
224
225
226
       // reviece rest
       size_t buf_size = sizeof(double)*row_end_first*NCB;
227
228
       double* revbuf;
       offset = 0;
       for (size_t worker = 0; worker < num_workers; worker++)</pre>
230
231
232
            revbuf = (double*)malloc(buf_size); //first gets largest buffer
            MPI_Recv(revbuf, buf_size/sizeof(double), MPI_DOUBLE, worker+1, worker, MPI_COMM_WORLD
233
        ,&stats[worker]);
            memcpy(&res[offset/sizeof(double)], revbuf, buf_size);
234
235
            free(revbuf):
            offset += buf_size;
236
            buf_size = sizeof(double)*rows_per_worker*NCB;
237
238
       double time = MPI_Wtime()-start_time;
239
       //free all pointers!
240
241
       free(a):
       free(b);
242
       free(b_transposed);
243
       free(c);
       return time;
245
246 }
247
248
249
double work(int rank, size_t num_workers){
       size_t rows_per_worker = NRA / (num_workers+1);
251
252
       char* buffer;
       MPI_Status status;
253
       if (rank == 1) // first always get's most work
254
255
            rows_per_worker = NRA - rows_per_worker*num_workers;
256
257
258
       size_t size_of_meta = sizeof(size_t);
       size_t size_of_a = sizeof(double)*rows_per_worker*NCA;
259
       size_t buffersize = size_of_meta+size_of_a + SIZE_OF_B;
260
       buffer = (char*)malloc(buffersize);
261
262
       MPI_Recv(buffer, buffersize, MPI_CHAR, 0, rank-1, MPI_COMM_WORLD, &status);
```

```
264
       double start = MPI_Wtime();
265
       int count;
       MPI_Get_count(&status, MPI_CHAR, &count);
       printf("I'm rank %d and I got %d bytes (%ld doubles) of data from %d with tag %d.\n", rank
267
        , count , (count - size of (size _t)) / size of (double) , status . MPI_SOURCE , status . MPI_TAG);
268
       MM_input *mm = (MM_input*)malloc(sizeof(MM_input));
269
270
       mm->a = (double*)&buffer[size_of_meta];
271
       mm->b = (double*)&buffer[size_of_meta+size_of_a];
272
       double *res =(double*)malloc(sizeof(double)*rows_per_worker*NCB);
273
274
       size t offset = 0:
275
       for (size_t row = 0; row < rows_per_worker; row++)</pre>
276
277
            for (size_t col = 0; col < NCB; col++)</pre>
278
279
            {
                res[row * NCB + col] = multsum(mm->a+offset, (((double*)mm->b)+col*NCA), NCA);
280
281
            offset += NCA;
282
283
284
       MPI_Send(res, rows_per_worker*NCB, MPI_DOUBLE, 0,rank-1, MPI_COMM_WORLD);
       printf("[%d] sent res home\n",rank);
285
286
       free (res);
287
       return MPI_Wtime() - start;
288 }
289
   int main(int argc, char *argv[]) {
290
       int tid, nthreads;
291
       /* for simplicity, set NRA=NCA=NCB=N */
292
       // Initialize MPI, find out MPI communicator size and process rank
293
294
       int myRank, numProcs;
       MPI_Status status;
295
       MPI_Init(&argc, &argv);
296
       MPI_Comm_size(MPI_COMM_WORLD, &numProcs);
297
       MPI_Comm_rank(MPI_COMM_WORLD, &myRank);
298
       int num_Workers = numProcs-1;
299
        if (argc > 1 && strcmp(argv[1], "parallel") == 0) {
            // Variables for the process rank and number of processes
301
           if (myRank == 0) {
302
303
                printf("Run parallel!\n");
                double *truth = malloc(sizeof(double) * NRA * NCB);
304
                double time = productSequential(truth);
305
                printf("Computed reference results in %.6f s\n", time);
306
                printf("Hello from master! - I have %d workers!\n", num_Workers);
307
                // send out work
308
                double *res = malloc(sizeof(double)*NRA*NCB);
309
310
                time = splitwork(res, num_Workers);
                if (checkResult(res, truth, NCB, NRA)) {
311
                    printf("Matrices do not match!!!\n");
312
                    return 1;
313
314
                printf("Matrices match (parallel [eps \%.10f])! - took: \%.6f s\n", EPS, time);
315
                free(truth);
                free(res);
317
318
            } else {
319
                double time = work(myRank, num_Workers);
                printf("Worker bee %d took %.6f s (after recv) for my work\n", myRank, time);
320
321
322
       } else // run sequantial
323
            printf("Run sequantial!\n");
325
            double *res = malloc(sizeof(double) * NRA * NCB);
326
            double time = productSequential(res);
327
            if (checkResult(res, res, NCB, NRA)) {
328
                printf("Matrices do not match!!!\n");
329
                return 1;
330
331
            printf("Matrices match (sequantial-trivial)! - took: %.6f s\n", time);
332
            free(res);
333
       }
334
```

```
336     MPI_Finalize();
337     return 0;
338 }
```

Appendix - Poisson solver

The parallel Poisson solver used the following code:

```
1 /*
   * MPI_Poisson.c
   * 2D Poison equation solver (parallel version)
6 #include <stdio.h>
7 #include <stdlib.h>
8 #include <math.h>
9 #include <time.h>
10 #include <mpi.h>
#include <assert.h>
#define DEBUG 0
14
#define max(a,b) ((a)>(b)?a:b)
17
18 // defines for Exercises!
19
20 #define SOR 1
#define DEFINES_ON (SOR || 0)
23 //defines end
24
25 enum
26 {
      X_DIR, Y_DIR
27
28 };
30 // only needed for certain configs!
31 #ifdef SOR
32 double sor_omega = 1.9;
33 #endif
35 /* global variables */
36 int gridsize[2];
37 double precision_goal;
                              /* precision_goal of solution */
38 int max_iter; /* maximum number of iterations alowed */
39 int P; //total number of processes
40 int P_grid[2]; // process grid dimensions
41 MPI_Comm grid_comm; //grid communicator
42 MPI_Status status;
43 double h;
45 /* process specific globals*/
46 int proc_rank;
47 double wtime;
48 int proc_coord[2]; // coords of current process in processgrid
49 int proc_top, proc_right, proc_bottom, proc_left; // ranks of neighboring procs
50 // step 7
int offset[2] = {0,0};
52 // step 8
53 MPI_Datatype border_type[2];
/* benchmark related variables */
clock_t ticks; /* number of systemticks */
int timer_on = 0; /* is timer running? */
/* local grid related variables */
double **phi; /* grid */
int **source; /* TRUE if subgrid element is a source */
                   /* grid dimensions */
62 int dim[2];
64 void Setup_Grid();
```

```
65 double Do_Step(int parity);
66 void Solve();
67 void Write_Grid();
68 void Clean_Up();
void Debug(char *mesg, int terminate);
70 void start_timer();
void resume_timer();
72 void stop_timer();
73 void print_timer();
74
75 void start_timer()
76 {
       if (!timer_on){
77
            MPI_Barrier(grid_comm);
78
           ticks = clock();
wtime = MPI_Wtime();
79
80
81
            timer_on = 1;
       }
82
83 }
84
85 void resume_timer()
86
       if (!timer_on){
87
88
            ticks = clock() - ticks;
            wtime = MPI_Wtime() - wtime;
89
            timer_on = 1;
90
91
92 }
93
94 void stop_timer()
95 {
       if (timer_on){
96
           ticks = clock() - ticks;
97
            wtime = MPI_Wtime() - wtime;
98
99
            timer_on = 0;
100
101 }
void print_timer()
104 {
105
       if (timer_on){
           stop_timer();
106
           printf("(%i) Elapsed Wtime \%14.6f s (\%5.1f\%\% CPU)\n", proc_rank, wtime, 100.0 * ticks
107
       * (1.0 / CLOCKS_PER_SEC) / wtime);
           resume_timer();
108
110
       else{
            printf("(%i) Elapsed Wtime %14.6f s (%5.1f%% CPU)\n", proc_rank, wtime, 100.0 * ticks
111
       * (1.0 / CLOCKS_PER_SEC) / wtime);
112
113 }
114
void Debug(char *mesg, int terminate)
116 {
       if (DEBUG || terminate){
117
118
           printf("%s\n", mesg);
119
       if (terminate){
120
121
            exit(1);
122
123 }
void Setup_Proc_Grid(int argc, char **argv){
       int wrap_around[2];
126
       int reorder;
127
128
       Debug("My_MPI_Init",0);
129
130
       // num of processes
131
132
       MPI_Comm_size(MPI_COMM_WORLD, &P);
133
       //calculate the number of processes per column and per row for the grid
134
       if (argc > 2) {
```

```
P_grid[X_DIR] = atoi(argv[1]);
136
            P_grid[Y_DIR] = atoi(argv[2]);
            if(P_grid[X_DIR] * P_grid[Y_DIR] != P){
                Debug("ERROR Proces grid dimensions do not match with P ", 1);
            }
140
141
            if (argc>3)
            {
142
                // get sor from args
143
                sor_omega = atof(argv[3]);
144
145
                printf("Set sor_omega over argv to %5.f\n", sor_omega);
147
148
       elsef
            Debug("ERROR Wrong parameter input",1);
150
       // Create process topology (2D grid)
       wrap_around[X_DIR] = 0;
        wrap_around[Y_DIR] = 0;
154
       reorder = 1; //reorder process ranks
156
157
        // create grid_comm
       int ret = MPI_Cart_create(MPI_COMM_WORLD, 2, P_grid, wrap_around, reorder, &grid_comm);
158
       if (ret != MPI_SUCCESS){
159
160
            Debug("ERROR: MPI_Cart_create failed",1);
161
        //get new rank and cartesian coords of this proc
        MPI_Comm_rank(grid_comm, &proc_rank);
       MPI_Cart_coords(grid_comm, proc_rank, 2, proc_coord);
164
       printf("(\%i) (x,y)=(\%i,\%i)\n", proc_rank, proc_coord[X_DIR], proc_coord[Y_DIR]);
        //calc neighbours
166
167
        // MPI_Cart_shift(grid_comm, Y_DIR, 1, &proc_bottom, &proc_top);
        MPI_Cart_shift(grid_comm, Y_DIR, 1, &proc_top, &proc_bottom);
168
        MPI_Cart_shift(grid_comm, X_DIR, 1, &proc_left, &proc_right);
169
        printf("(%i) top %i, right %i, bottom %i, left %i\n", proc_rank, proc_top,
       proc_right, proc_bottom, proc_left);
171 }
173 void Setup_Grid()
174 {
        int x, y, s;
        double source_x, source_y, source_val;
176
       FILE *f:
177
178
       Debug("Setup_Subgrid", 0);
179
       if (proc_rank == 0) {
181
            f = fopen("input.dat", "r");
182
            if (f == NULL){
183
184
                Debug("Error opening input.dat", 1);
185
            fscanf(f, "nx: %i\n", \&gridsize[X_DIR]);
186
           fscanf(f, "ny: %i\n", &gridsize[Y_DIR]);
fscanf(f, "precision goal: %lf\n", &precision_goal);
fscanf(f, "max iterations: %i\n", &max_iter);
187
189
190
        MPI_Bcast(&gridsize, 2, MPI_INT, 0, grid_comm);
191
       MPI_Bcast(&precision_goal, 1, MPI_DOUBLE, 0, grid_comm);
192
       MPI_Bcast(&max_iter, 1, MPI_INT, 0, grid_comm);
193
194
       h = 1;
       /* Calculate dimensions of local subgrid */ //! We do that later now!
195
        // dim[X_DIR] = gridsize[X_DIR] + 2;
        // dim[Y_DIR] = gridsize[Y_DIR] + 2;
198
        //! Step 7
       int upper_offset[2] = {0,0};
200
        // Calculate top left corner cordinates of local grid
201
        offset[X_DIR] = gridsize[X_DIR] * proc_coord[X_DIR] / P_grid[X_DIR];
202
        offset[Y_DIR] = gridsize[Y_DIR] * proc_coord[Y_DIR] / P_grid[Y_DIR];
203
        upper_offset[X_DIR] = gridsize[X_DIR] * (proc_coord[X_DIR] + 1) / P_grid[X_DIR];
204
        upper_offset[Y_DIR] = gridsize[Y_DIR] * (proc_coord[Y_DIR] + 1) / P_grid[Y_DIR];
205
206
       // dimensions of local grid
```

```
dim[X_DIR] = upper_offset[X_DIR] - offset[X_DIR];
208
        dim[Y_DIR] = upper_offset[Y_DIR] - offset[Y_DIR];
209
        // Add space for rows/columns of neighboring grid
       dim[X_DIR] += 2;
211
       dim[Y_DIR] += 2;
212
        //! Step 7 end
213
214
215
        /* allocate memory */
       if ((phi = malloc(dim[X_DIR] * sizeof(*phi))) == NULL){
216
            Debug("Setup_Subgrid : malloc(phi) failed", 1);
217
218
       if ((source = malloc(dim[X_DIR] * sizeof(*source))) == NULL){
219
            Debug("Setup_Subgrid : malloc(source) failed", 1);
220
221
       if ((phi[0] = malloc(dim[Y_DIR] * dim[X_DIR] * sizeof(**phi))) == NULL){
222
223
            Debug("Setup_Subgrid : malloc(*phi) failed", 1);
224
       if ((source[0] = malloc(dim[Y_DIR] * dim[X_DIR] * sizeof(**source))) == NULL){
225
            Debug("Setup_Subgrid : malloc(*source) failed", 1);
227
       for (x = 1; x < dim[X_DIR]; x++)</pre>
228
229
            phi[x] = phi[0] + x * dim[Y_DIR];
230
231
            source[x] = source[0] + x * dim[Y_DIR];
232
233
        /* set all values to '0' */
234
       for (x = 0; x < dim[X_DIR]; x++){</pre>
235
            for (y = 0; y < dim[Y_DIR]; y++)</pre>
236
237
                phi[x][y] = 0.0;
238
                source[x][y] = 0;
            }
240
       }
241
242
        /* put sources in field */
243
            if (proc_rank==0)
244
                s = fscanf(f, "source: %lf %lf %lf\n", &source_x, &source_y, &source_val);
246
            }
247
            MPI_Bcast(&s, 1, MPI_INT, 0, grid_comm);
248
            if (s==3){
249
                MPI_Bcast(&source_x, 1, MPI_DOUBLE, 0, grid_comm);
250
                MPI_Bcast(&source_y, 1, MPI_DOUBLE, 0, grid_comm);
MPI_Bcast(&source_val, 1, MPI_DOUBLE, 0, grid_comm);
251
252
                x = source_x * gridsize[X_DIR];
                y = source_y * gridsize[Y_DIR];
254
                x = x + 1 - offset[X_DIR]; // Step 7 --> local grid transform
255
                y = y + 1 - offset[Y_DIR]; // Step 7 --> local grid transform
256
                if(x > 0 \&\& x < dim[X_DIR] -1 \&\& y > 0 \&\& y < dim[Y_DIR] -1){ // check if in local}
257
       grid
                     phi[x][y] = source_val;
                     source[x][y] = 1;
259
            }
261
262
        while (s==3);
263
264
       if (proc_rank == 0) {
265
266
            fclose(f);
267
268 }
269
void Setup_MPI_Datatypes()
271
       Debug("Setup_MPI_Datatypes",0);
272
273
        // vertical data exchange (Y_Dir)
274
       MPI_Type_vector(dim[X_DIR] - 2, 1, dim[Y_DIR], MPI_DOUBLE, &border_type[Y_DIR]);
275
        // horizontal data exchange (X_Dir)
276
       MPI_Type_vector(dim[Y_DIR] - 2, 1, 1, MPI_DOUBLE, &border_type[X_DIR]);
277
278
       MPI_Type_commit(&border_type[Y_DIR]);
```

```
280
        MPI_Type_commit(&border_type[X_DIR]);
281 }
283 void Exchange_Borders()
284 €
285
        Debug("Exchange_Borders",0);
        // top direction
286
         \texttt{MPI\_Sendrecv(\&phi[1][1], 1, border\_type[Y\_DIR], proc\_top, 0, \&phi[1][dim[Y\_DIR] - 1], 1, } 
287
        border_type[Y_DIR], proc_bottom, 0, grid_comm, &status);
288
        // bottom direction
        MPI_Sendrecv(&phi[1][dim[Y_DIR] - 2], 1, border_type[Y_DIR], proc_bottom, 0, &phi[1][0],
        1, border_type[Y_DIR], proc_top, 0, grid_comm, &status);
        // left direction
290
        MPI_Sendrecv(&phi[1][1], 1, border_type[X_DIR], proc_left, 0, &phi[dim[X_DIR]-1][1], 1,
291
        border_type[X_DIR], proc_right, 0, grid_comm, &status);
        // right direction
        MPI_Sendrecv(&phi[dim[X_DIR]-2][1], 1, border_type[X_DIR], proc_right, 0, &phi[0][1], 1,
293
        border_type[X_DIR], proc_left, 0, grid_comm, &status);
294 }
295
   double Do_Step(int parity)
296
297
298
      int x, y;
299
     double old_phi, c_ij;
     double max_err = 0.0;
300
301
      /* calculate interior of grid */
302
        for (x = 1; x < dim[X_DIR] - 1; x++){</pre>
303
            for (y = 1; y < dim[Y_DIR] - 1; y++){</pre>
304
                if ((x + offset[X_DIR] + y + offset[Y_DIR]) % 2 == parity && source[x][y] != 1){
305
                     old_phi = phi[x][y];
306
307
                     #ifndef SOR
                     phi[x][y] = (phi[x + 1][y] + phi[x - 1][y] + phi[x][y + 1] + phi[x][y - 1]) *
308
        0.25:
                     #endif
                     #ifdef SOR //! I'm not quite sure about the h and source parts here
310
311
                     c_{ij} = (phi[x + 1][y] + phi[x - 1][y] + phi[x][y + 1] + phi[x][y - 1] + h*h*
        source[x][y]) * 0.25 - phi[x][y];
                     phi[x][y] += sor_omega*c_ij;
312
313
                     #endif
                     if (max_err < fabs(old_phi - phi[x][y])){
  max_err = fabs(old_phi - phi[x][y]);</pre>
314
315
316
                }
317
            }
318
319
320
321
     return max_err;
322 }
323
   void Solve()
324
   {
325
        int count = 0;
326
        double delta;
327
        double global delta;
328
329
        double delta1, delta2;
330
        Debug("Solve", 0);
331
332
333
        /* give global_delta a higher value then precision_goal */
        global_delta = 2 * precision_goal;
334
335
        while (global_delta > precision_goal && count < max_iter)</pre>
336
337
            Debug("Do_Step 0", 0);
338
            delta1 = Do_Step(0);
339
            Exchange_Borders();
340
            Debug("Do_Step 1", 0);
341
            delta2 = Do_Step(1);
342
            Exchange_Borders();
            delta = max(delta1, delta2);
344
            MPI_Allreduce(&delta, &global_delta, 1, MPI_DOUBLE, MPI_MAX, grid_comm);
345
```

```
347
348
        printf("(%i) Number of iterations : %i\n", proc_rank, count);
350 }
351
352 double* get_Global_Grid()
353 {
354
        Debug("get_Global_Grid", 0);
355
        //!! DEBUG only
        for (size_t i = 0; i < dim[X_DIR]; i++)</pre>
356
357
            for (size_t j = 0; j < dim[Y_DIR]; j++)</pre>
358
359
                phi[i][j] = proc_rank;
360
361
362
363
364
        // only process 0 needs to store all data!
365
       double* global_phi = NULL;
366
        if (proc_rank == 0) {
367
            global_phi = malloc(gridsize[X_DIR] * gridsize[Y_DIR] * sizeof(double));
368
            if (global_phi == NULL) {
369
370
                Debug("get_Global_Grid : malloc(global_phi) failed", 1);
            }
371
       }
372
373
       // copy own part into buffer - flatten!
size_t buf_size = (dim[X_DIR] - 2) * (dim[Y_DIR] - 2) * sizeof(double);
374
375
        double* local_phi = malloc(buf_size);
376
        int idx = 0;
377
378
        for (int x = 1; x < dim[X_DIR] - 1; x++) {</pre>
            for (int y = 1; y < dim[Y_DIR] - 1; y++) {</pre>
379
                local_phi[idx++] = phi[x][y];
380
381
382
       printf("I'm proc %d and i have a buffer of size %zu\n", proc_rank, buf_size);
383
385
        // only proc 0 needs sendcounts and displacements for the gatherv operation
386
387
        int* sendcounts = NULL;
        int* displs = NULL;
388
        if (proc_rank == 0) {
389
            sendcounts = malloc(P * sizeof(int));
390
            displs = malloc(P * sizeof(int));
391
392
            // size and offset of different subgrids
393
            //! Note that this only works if every process has the same subgrid
394
            if (gridsize[X_DIR] % P_grid[X_DIR] != 0 || gridsize[Y_DIR] % P_grid[Y_DIR] != 0)
395
396
            {
                Debug("!!!A grid dimension is not a multiple of the P_grid in this direction!", 1)
397
            }
398
            int subgrid_width = gridsize[X_DIR] / P_grid[X_DIR];
400
            int subgrid_height = gridsize[Y_DIR] / P_grid[Y_DIR];
401
            for (int px = 0; px < P_grid[X_DIR]; px++) {</pre>
402
                for (int py = 0; py < P_grid[Y_DIR]; py++) {</pre>
403
                     int rank = px * P_grid[Y_DIR] + py;
404
                     sendcounts[rank] = subgrid_width * subgrid_height;
405
                     displs[rank] = (px * subgrid_width * gridsize[Y_DIR]) + (py * subgrid_height);
406
407
                }
            }
408
409
        Debug("get_Global_Grid : MPI_Gatherv", 0);
410
        //! TODO this Gatherv does something wrong - all local grids are alright!!!
411
       MPI_Gatherv(local_phi, (dim[X_DIR] - 2) * (dim[Y_DIR] - 2), MPI_DOUBLE, global_phi,
412
       sendcounts, displs, MPI_DOUBLE, 0, MPI_COMM_WORLD);
413
        free(local_phi);
        if (proc_rank == 0) {
415
            free(sendcounts):
416
            free(displs);
```

```
418
419
        return global_phi;
420
421 }
422
   void Write_Grid_global(){
423
        int x, y;
424
425
        FILE *f;
        char filename[40]; //seems danagerous to use a static buffer but let's go with the steps
426
        sprintf(filename, "output_MPI_global_%i.dat", proc_rank);
427
        if ((f = fopen(filename, "w")) == NULL){
428
            Debug("Write_Grid : fopen failed", 1);
429
430
431
        Debug("Write_Grid", 0);
432
433
434
        for (x = 1; x < dim[X_DIR]-1; x++){</pre>
            for (y = 1; y < dim[Y_DIR]-1; y++){
435
436
                 int x_glob = x + offset[X_DIR];
                 int y_glob = y + offset[Y_DIR];
437
                 fprintf(f, "%i %i %f\n", x_glob, y_glob, phi[x][y]);
438
439
440
441
        fclose(f);
442
443
   void Write_Grid()
444
445 {
        double* global_phi = get_Global_Grid();
446
        if(proc_rank != 0){
            assert (global_phi == NULL);
448
449
            return;
450
451
        int x, y;
452
        FILE *f;
        char filename[40]; //seems danagerous to use a static buffer but let's go with the steps
453
        sprintf(filename, "output_MPI%i.dat", proc_rank);
454
        if ((f = fopen(filename, "w")) == NULL){
            Debug("Write_Grid : fopen failed", 1);
456
457
458
        Debug("Write_Grid", 0);
459
460
        for (x = 0; x < gridsize[X_DIR]; x++){</pre>
461
            for (y = 0; y < gridsize[Y_DIR]; y++){
    fprintf(f, "%i %i %f\n", x+1, y+1, global_phi[x*gridsize[Y_DIR] + y]);</pre>
462
464
465
        fclose(f);
466
467
        free(global_phi);
468
469
470 void Clean_Up()
471
        Debug("Clean_Up", 0);
472
473
        free(phi[0]);
474
        free(phi);
475
476
        free(source[0]);
477
        free(source);
478 }
479
480 int main(int argc, char **argv)
481
   {
        MPI_Init(&argc, &argv);
482
        Setup_Proc_Grid(argc,argv); // was earlier MPI_Comm_rank(MPI_COMM_WORLD, &proc_rank);
483
484
        start_timer();
485
        Setup_Grid();
486
        Setup_MPI_Datatypes();
488
        #ifdef SOR
489
        if (proc_rank == 0)
```

```
491
               printf("SOR using omega: \%.5f\n", sor_omega);
492
493
         #endif
494
495
         Solve();
496
497
         // Write_Grid();
Write_Grid_global();
print_timer();
498
499
500
501
         Clean_Up();
MPI_Finalize();
502
503
         return 0;
504
505 }
```