Delft University of Technology

Introduction to High Performance Computing
WI4049TU

Lab Report

Author:
Elias Wachmann (6300421)

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General Remarks

This final Lab report includes the answers for the exercises (base grad denoted in paranthesis):

0. Introductory exercise (0.5)
1. Poisson solver (1.75)
2. Finite elements simulation (1.0)
3. Eigenvalue solution by Power Method on GPU (1.75)

The optional **shining points** (e.g., performance analysis, optimization, discussion, and clarifying figures) which yield further points are usually marked by a small blue heading in the text or an additional note is added under a figure or table. For example:
This is a shining point.

0 Introductory exercise

In the introductory lab session, we are taking a look at some basic features of MPI.
We start out very simple with a hello world program on two nodes.

Hello World

```c
#include "mpi.h"
#include <stdio.h>

int np, rank;

int main(int argc, char **argv)
{
  MPI_Init(&argc, &argv);
  MPI_Comm_size(MPI_COMM_WORLD, &np);
  MPI_Comm_rank(MPI_COMM_WORLD, &rank);

  printf("Node %d of %d says: Hello world!\n", rank, np);

  MPI_Finalize();
  return 0;
}
```

This program can be compiled with the following command:

```
mpicc -o helloworld1.out helloworld1.c
```

And run with:

```
srun -n 2 -c 4 --mem-per-cpu=1GB  ./helloworld1.out
```

We get the following output:

```
Node 0 of 2 says: Hello world!
Node 1 of 2 says: Hello world!
```

From now on I'll skip the compilation and only mention on how many nodes the program is run and what the output is / interpretation of the output.

0.a) Ping Pong

I used the template to check how long `MPI_Send` and `MPI_Recv` take. The code can be found in the appendix for this section.

I've modified the printing a bit to make it easier to gather the information. Then I piped the program output into a textfile for further processing in python. I ran it first on one and then on two nodes as specified in the

assignment sheet. Opposed to the averaging over 5 send / receive pairs, I've done 1000 pairs. Furthmore I reran the whole programm 5 times to gather more data. All this data is shown in the following graph:

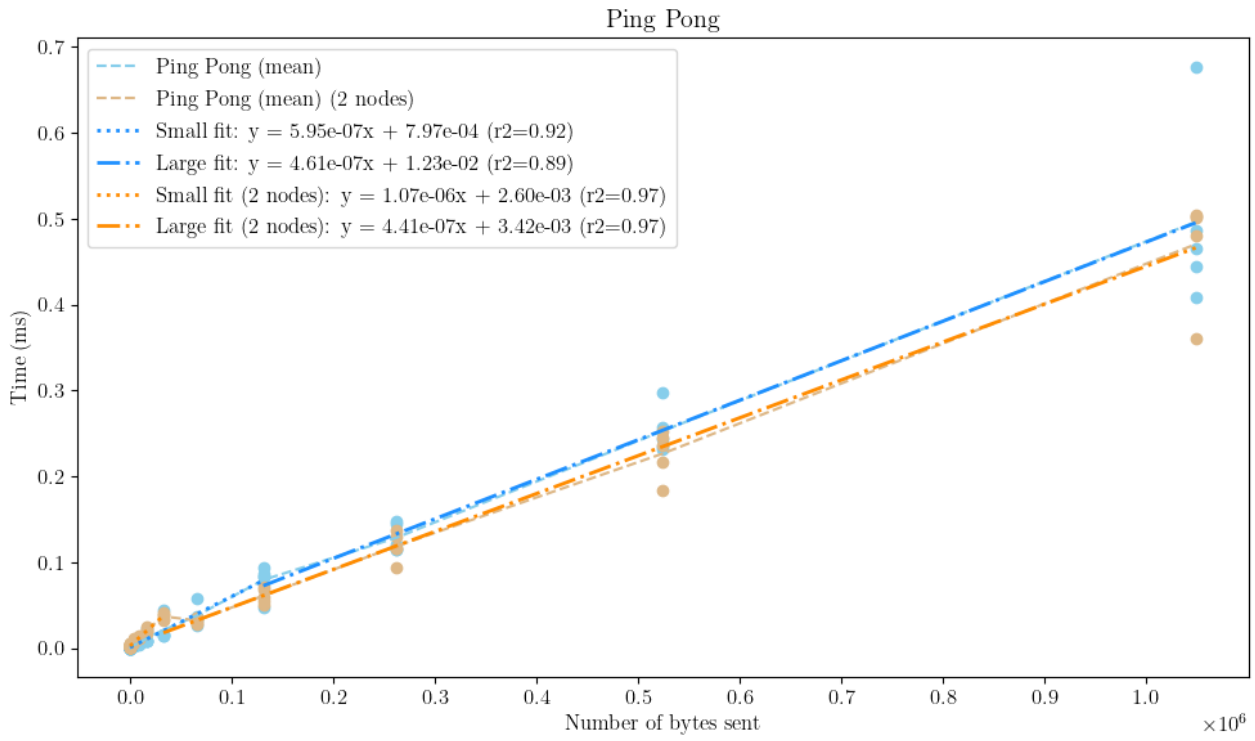

Figure 1: Ping Pong: Number of bytes sent vs. average time taken from 1000 pairs of send / receive. 5 runs shown for each size as scatter plot. Mean of these 5 runs shown as line. Blue small fit includes all data points up to 131072 bytes, blue large from there. Red small fit includes all data points up to 32768 bytes, red large from there.

As can be seen in the data and the fits, there are outliers especially for the larger data sizes.
For our runs we get the following fits and R^2 values:

Run Type	**Data Size**	**Fit Equation**	**R^2 Value**
Single Node	Small (<=131072)	$5.95 \times 10^{-7} \cdot x + 7.97 \times 10^{-4}$	0.92
Single Node	Large (>= 131072)	$4.61 \times 10^{-7} \cdot x + 1.23 \times 10^{-2}$	0.89
Two Node	Small (<=32768)	$1.07 \times 10^{-6} \cdot x + 2.60 \times 10^{-3}$	0.97
Two Node	Large (>=32768)	$4.41 \times 10^{-7} \cdot x + 3.42 \times 10^{-3}$	0.97

Table 1: Fit Equations and R^2 Values for Single Node and Two Node Runs

Note: Each run was performed 5 times (for 1 and 2 nodes) to get a fit on the data and calculate a R^2 value.
TODO: **Further analysis needed?**

Extra: Ping Pong with MPI_SendRecv

We do the same analysis for the changed program utilizing `MPI_SendRecv`. The code can be found in the appendix for this section.
We get the following graph from the measurements which were performed in the same way as for the previous program:

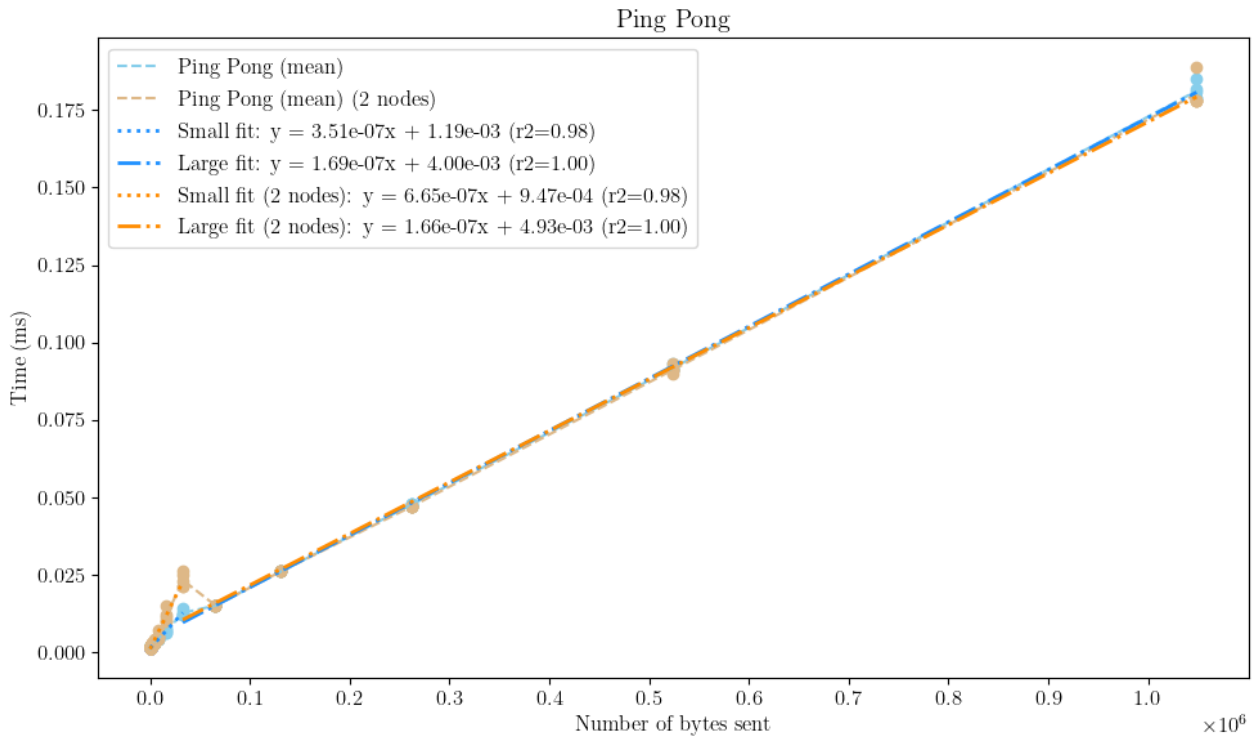

Figure 2: Ping Pong with MPI_SendRecv: Number of bytes sent vs. average time taken from 1000 pairs of send / receive. 5 runs shown for each size as scatter plot. Mean of these 5 runs shown as line. Blue small fit includes all data points up to 32768 bytes, blue large from there. Red small fit includes all data points up to 32768 bytes, red large from there.

We get the following fits and R^2 values for the runs:

Run Type	**Data Size**	**Fit Equation**	**R^2 Value**
Single Node	Small (<=32768)	$3.51 \times 10^{-7} \cdot x + 1.19 \times 10^{-3}$	0.98
Single Node	Large (>=32768)	$1.69 \times 10^{-7} \cdot x + 4.00 \times 10^{-3}$	1.00
Two Node	Small (<=32768)	$6.65 \times 10^{-7} \cdot x + 9.47 \times 10^{-4}$	0.98
Two Node	Large (>=32768)	$1.66 \times 10^{-7} \cdot x + 4.93 \times 10^{-3}$	1.00

Table 2: Fit Equations and R^2 Values for Single Node and Two Node Runs

TODO: **Further analysis needed?**

0.b) MM-product

After an introduction of the matrix-matrix multiplication code in the next section, the measured speedups are discussed in the subsequent section.

Explanation of the code

For this excercise I've used the template provided in the assignment sheet as a base to develop my parallel implementation for a matrix-matrix multiplication. The code can be found in the appendix for this section.

The porgam can be run either in sequential (default) or parallel mode (parallel as a command line argument). For the sequential version, the code is practically unchanged and just refactored into a function for timing purposes. The parallel version is more complex and works as explained bellow:
First, rank 0 computes a sequential reference solution. Then rank 0 distributes the matrices in the following way in `splitwork`:

- Matrix A is split row-wise by dividing the number of rows by the number of nodes.
- The first worker (=rank 1) gets the most rows starting from row 0:
  $\texttt{total\_rows} - (\texttt{nr\_workers} - 1) \cdot \mathit{floor}(\frac{\texttt{total\_rows}}{\texttt{nr\_workers}})$.
- All other workers and the master (= rank 0) get the same number of rows: $\mathit{floor}(\frac{\texttt{total\_rows}}{\texttt{nr\_workers}})$.
- The master copies the corresponding rows of matrix A and the whole transposed matrix B* into a buffer (for details on `MM_input` buffer see bellow) for each worker and sends them off using `MPI_ISend`.
- The workers receive the data using `MPI_Recv` and then compute their part of the matrix product and send only the rows of the result matrix back to the master using `MPI_Send`.
- In the meanwhile the master computes its part of the matrix product.
- Using `MPI_Waitall` the master waits for all data to be sent to the workers and only afterwards calls `MPI_Recv` to gather the results from the workers.
- Finally all results are gathered by the master in the result matrix.

Assume we have a 5x5 matrix A and 2 workers (rank 1 and rank 2) and master (rank 0). The partitioning is done row-wise as follows:

Partitioning Example

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{pmatrix} \rightarrow \begin{pmatrix} \text{Worker 1} \\ \text{Worker 1} \\ \text{Worker 1} \\ \text{Master} \\ \text{Master} \end{pmatrix}$$

- **Rank 0 (Master)**: Rows 4 and 5 (last two rows)
- **Rank 1 (Worker 1)**: Rows 1 to 3 (first three rows) - Worker 1 always gets the most rows

This partitioning can be visually represented as:

$$\text{Master (rank 0):} \begin{pmatrix} a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{pmatrix}$$

$$\text{Worker 1 (rank 1):} \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \end{pmatrix}$$

Each worker computes its part of the matrix product, and the master gathers the results at the end and compiles them into the final matrix.
The `MM_input` buffer is used to store the rows of matrix A and the whole matrix B for each worker. It is implemented using a simple struct:

```
typedef struct MM_input {
    size_t rows;
    double *a;
    double *b;
} MM_input;
```

***[Optimization] Note on transposed matrix B:** It is usually beneficial from a cache perspective to index arrays sequentially or in a row-major order. However, in the matrix-matrix multiplication, we access the elements of matrix B in a column-wise order. This leads to cache misses and is not optimal. To mitigate this, we can transpose matrix B and then access it in a row-wise order. This is done in the code by the master before sending the data to the workers.

Discussion of the speedups

The code was run on Delft's cluster with 1, 2, 4, 8, 16, 24, 32, 48, and 64 nodes. For the experiments the matrix size of A and B was set to 2000×2000. This means that the program has to evaluate 2000 multiplications and 1999 additions for each element of the resulting matrix C. In total this results in $\approx 2000^3 = 8 \times 10^9$ operations. The command looked similar to the following for the different node counts:

```
srun -n 48 --mem-per-cpu=4GB --time=00:02:00 ./MM.out parallel
```

For this experiment, the execution time was measured and the speedup was calculated. The results are shown in Table 3 and Figure 3.

CPU Count	Execution Time / s	Approx. Speedup
1	47.11	1.0
2	10.26	4.6
4	10.30	4.6
8	5.20	9.1
16	2.97	15.9
24	2.54	18.5
32	2.29	20.6
48	2.98	15.8
64	1.72	27.4

Table 3: Execution Time vs CPU Count

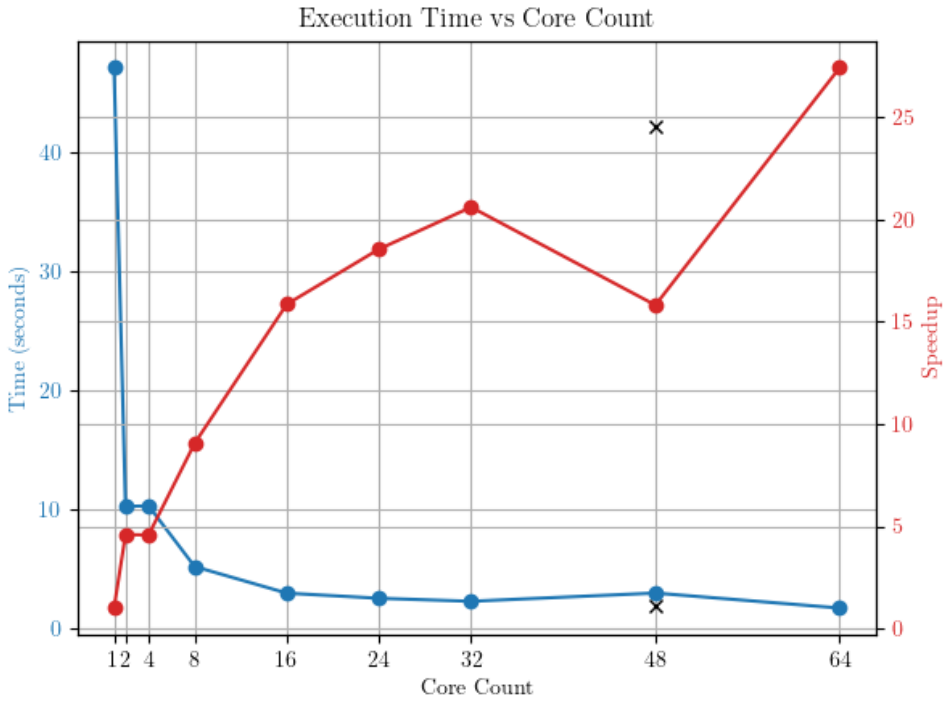

Figure 3: Speedup vs CPU Count
Black × marks the average of the rerun for $n = 48$.

Note: The speedup is calculated as $S = \frac{T_1}{T_p}$, where T_1 is the execution time on 1 node and T_p is the execution time on p nodes.

Discussion:
As one can cleary discern from the data in Table 3 and Figure 3, the speedup increases with the number of nodes (with the exception of $n = 48$). This is expected as the more nodes we have, the more work can be done in

parallel. However, the speedup is not linear. This is due to the overhead of communication between the nodes. The more nodes we have, the more communication is needed, and this overhead increases. This is especially visible in the data for $n = 48$. Here the speedup is lower than for $n = 32$. For this run the communication didn't went as smooth as for the other runs. This can potentially be attributed to the fact that one (or more) of the nodes or the network was under heavy load during this task.
[Further investigation] After observing this slower speed for the $n = 48$, I reran the tests multiple times and got a runtime of around $1.9s$ which was to be expected initially. Therefore, this one run is an odd one out, most likely due to the reasons mentioned above! I've also added the averaged data of the reruns as a datapoint in Figure 3.
Another interesting fact can be seen when comparing the time taken for $n = 1$ and $n = 2$. They don't at all scale with the expected factor of 2. This is could be due to the fact, that the resource management system prefers runs with multiple nodes instead of a single node (= sequential).

Additional notes: The flag `-mem-per-cpu=<#>GB` was set depending on the number of nodes used. For 1-24 nodes 8GB was used, for 32-48 nodes 4GB, and for 64 nodes 3GB. This had to be done to comply with QOS policy on the cluster.

1 Poisson solver

2 Finite elements simulation

3 Eigenvalue solution by Power Method on GPU

Appendix - Introductory exercise

The following code was used for the ping pong task:

```c
#include <stdio.h>
#include <stdlib.h>
#include <mpi.h>

// Maximum array size 2^20= 1048576 elements
#define MAX_EXPONENT 20
#define MAX_ARRAY_SIZE (1<<MAX_EXPONENT)
#define SAMPLE_COUNT 1000

int main(int argc, char **argv)
{
    // Variables for the process rank and number of processes
    int myRank, numProcs, i;
    MPI_Status status;

    // Initialize MPI, find out MPI communicator size and process rank
    MPI_Init(&argc, &argv);
    MPI_Comm_size(MPI_COMM_WORLD, &numProcs);
    MPI_Comm_rank(MPI_COMM_WORLD, &myRank);


    int *myArray = (int *)malloc(sizeof(int)*MAX_ARRAY_SIZE);
    if (myArray == NULL)
    {
        printf("Not enough memory\n");
        exit(1);
    }
    // Initialize myArray
    for (i=0; i<MAX_ARRAY_SIZE; i++)
        myArray[i]=1;

    int number_of_elements_to_send;
    int number_of_elements_received;

    // PART C
    if (numProcs < 2)
    {
        printf("Error: Run the program with at least 2 MPI tasks!\n");
```

```
        MPI_Abort(MPI_COMM_WORLD, 1);
    }
    double startTime, endTime;

    // TODO: Use a loop to vary the message size
    for (size_t j = 0; j <= MAX_EXPONENT; j++)
    {
        number_of_elements_to_send = 1<<j;
        if (myRank == 0)
        {
            myArray[0]=myArray[1]+1; // activate in cache (avoids possible delay when sending the 1st element)
            startTime = MPI_Wtime();
            for (i=0; i<SAMPLE_COUNT; i++)
            {
                MPI_Send(myArray, number_of_elements_to_send, MPI_INT, 1, 0,
                    MPI_COMM_WORLD);
                MPI_Probe(MPI_ANY_SOURCE, MPI_ANY_TAG, MPI_COMM_WORLD, &status);
                MPI_Get_count(&status, MPI_INT, &number_of_elements_received);

                MPI_Recv(myArray, number_of_elements_received, MPI_INT, 1, 0,
                    MPI_COMM_WORLD, MPI_STATUS_IGNORE);
            } // end of for-loop

            endTime = MPI_Wtime();
            printf("Rank %2.1i: Received %i elements: Ping Pong took %f seconds\n", myRank, number_of_elements_received,(endTime - startTime)/(2*SAMPLE_COUNT));
        }
        else if (myRank == 1)
        {
            // Probe message in order to obtain the amount of data
            MPI_Probe(MPI_ANY_SOURCE, MPI_ANY_TAG, MPI_COMM_WORLD, &status);
            MPI_Get_count(&status, MPI_INT, &number_of_elements_received);

            for (i=0; i<SAMPLE_COUNT; i++)
            {
                MPI_Recv(myArray, number_of_elements_received, MPI_INT, 0, 0,
                MPI_COMM_WORLD, MPI_STATUS_IGNORE);
                MPI_Send(myArray, number_of_elements_to_send, MPI_INT, 0, 0,
                MPI_COMM_WORLD);
            } // end of for-loop
        }
    }

    // Finalize MPI
    MPI_Finalize();

    return 0;
}
```

For the bonus task, the following code was used:

```
#include <stdio.h>
#include <stdlib.h>
#include <mpi.h>

// Maximum array size 2^20= 1048576 elements
#define MAX_EXPONENT 20
#define MAX_ARRAY_SIZE (1<<MAX_EXPONENT)
#define SAMPLE_COUNT 1000

int main(int argc, char **argv)
{
    // Variables for the process rank and number of processes
    int myRank, numProcs, i;
    MPI_Status status;

    // Initialize MPI, find out MPI communicator size and process rank
    MPI_Init(&argc, &argv);
    MPI_Comm_size(MPI_COMM_WORLD, &numProcs);
    MPI_Comm_rank(MPI_COMM_WORLD, &myRank);


    int *myArray = (int *)malloc(sizeof(int)*MAX_ARRAY_SIZE);
```

```
    if (myArray == NULL)
    {
        printf("Not enough memory\n");
        exit(1);
    }
    // Initialize myArray
    for (i=0; i<MAX_ARRAY_SIZE; i++)
        myArray[i]=1;

    int number_of_elements_to_send;
    int number_of_elements_received;

    // PART C
    if (numProcs < 2)
    {
        printf("Error: Run the program with at least 2 MPI tasks!\n");
        MPI_Abort(MPI_COMM_WORLD, 1);
    }
    double startTime, endTime;

    // TODO: Use a loop to vary the message size
    for (size_t j = 0; j <= MAX_EXPONENT; j++)
    {
        number_of_elements_to_send = 1<<j;
        if (myRank == 0)
        {
            myArray[0]=myArray[1]+1; // activate in cache (avoids possible delay when sending the 1st element)
            startTime = MPI_Wtime();
            for (i=0; i<SAMPLE_COUNT; i++)
            {
                MPI_Sendrecv(myArray, number_of_elements_to_send, MPI_INT, 1,0,myArray, number_of_elements_to_send, MPI_INT, 1, 0, MPI_COMM_WORLD, &status);
            }

            endTime = MPI_Wtime();
            printf("Rank %2.1i: Received %i elements: Ping Pong took %f seconds\n", myRank, number_of_elements_to_send,(endTime - startTime)/(2*SAMPLE_COUNT));
        }
        else if (myRank == 1)
        {
            for (i=0; i<SAMPLE_COUNT; i++)
            {
                MPI_Sendrecv(myArray, number_of_elements_to_send, MPI_INT, 0,0,myArray, number_of_elements_to_send, MPI_INT, 0, 0, MPI_COMM_WORLD, &status);
            }
        }
    }

    // Finalize MPI
    MPI_Finalize();

    return 0;
}
```

The matrix multiplication used the following code:

```
/******************************************************************************
 * FILE: mm.c
 * DESCRIPTION:
 *   This program calculates the product of matrix a[nra][nca] and b[nca][ncb],
 *   the result is stored in matrix c[nra][ncb].
 *   The max dimension of the matrix is constraint with static array
 *declaration, for a larger matrix you may consider dynamic allocation of the
 *arrays, but it makes a parallel code much more complicated (think of
 *communication), so this is only optional.
 *
 ******************************************************************************/

#include <math.h>
#include <mpi.h>
#include <stdbool.h>
#include <stdio.h>
#include <stdlib.h>
```

```c
#include <string.h>

#define NRA 2000 /* number of rows in matrix A */
#define NCA 2000 /* number of columns in matrix A */
#define NCB 2000 /* number of columns in matrix B */
// #define N 1000
#define EPS 1e-9
#define SIZE_OF_B NCA*NCB*sizeof(double)

bool eps_equal(double a, double b) { return fabs(a - b) < EPS; }

void print_flattened_matrix(double *matrix, size_t rows, size_t cols, int rank) {
    printf("[%d]\n", rank);
    for (size_t i = 0; i < rows; i++) {
        for (size_t j = 0; j < cols; j++) {
            printf("%10.2f ", matrix[i * cols + j]);  // Accessing element in the 1D array
        }
        printf("\n");  // Newline after each row
    }
}

int checkResult(double *truth, double *test, size_t Nr_col, size_t Nr_rows) {
    for (size_t i = 0; i < Nr_rows; ++i) {
        for (size_t j = 0; j < Nr_col; ++j) {
            size_t index = i * Nr_col + j;
            if (!eps_equal(truth[index], test[index])) {
                return 1;
            }
        }
    }
    return 0;
}

typedef struct {
    size_t rows;
    double *a;
    double *b;
} MM_input;

char* getbuffer(MM_input *in, size_t size_of_buffer){
    char* buffer = (char*)malloc(size_of_buffer * sizeof(char));
    if (buffer == 0)
    {
        printf("Buffer couldn't be allocated.");
        return NULL;
    }
    size_t offset = 0;
    memcpy(buffer + offset, &in->rows, sizeof(size_t));
    offset += sizeof(size_t);
    size_t matrix_size = in->rows * NCA * sizeof(double);
    memcpy(buffer + offset, in->a, matrix_size);
    offset += matrix_size;
    memcpy(buffer + offset, in->b, NCA*NCB*sizeof(double));
    return buffer;
}

MM_input* readbuffer(char* buffer, size_t size_of_buffer){
    MM_input *mm = (MM_input*)malloc(sizeof(MM_input));

    mm->rows = ((size_t*)buffer)[0];
    size_t offset = sizeof(size_t);
    size_t matrix_size =  mm->rows * NCA;
    mm->a = (double*)malloc(sizeof(double)*matrix_size);
    mm->b = (double*)malloc(sizeof(double)*matrix_size);
    memcpy(mm->a, &(buffer[offset]), matrix_size);
    offset += matrix_size;
    memcpy(mm->b, &(buffer[offset]), NCA*NCB*sizeof(double));
    free(buffer);
    return mm;
}


void setupMatrices(double (*a)[NCA], double (*b)[NCB], double (*c)[NCB]){
```

```
    for (size_t i = 0; i < NRA; i++) {
        for (size_t j = 0; j < NCA; j++) {
            a[i][j] = i + j;
        }
    }

    for (size_t i = 0; i < NCA; i++) {
        for (size_t j = 0; j < NCB; j++) {
            b[i][j] = i * j;
        }
    }

    for (size_t i = 0; i < NRA; i++) {
        for (size_t j = 0; j < NCB; j++) {
            c[i][j] = 0;
        }
    }
}

double multsum(double* a,double* b_transposed, size_t size){
    double acc = 0;
    for (size_t i = 0; i < size; i++)
    {
        acc += a[i]*b_transposed[i];
    }
    return acc;
}

double productSequential(double *res) {
    // dynamically allocate to not run into stack overflow - usually stacks are
    // 8192 bytes big -> 1024 doubles but we have 1 Mio. per matrix
    double(*a)[NCA] = malloc(sizeof(double) * NRA * NCA);
    double(*b)[NCB] = malloc(sizeof(double) * NCA * NCB);
    double(*c)[NCB] = malloc(sizeof(double) * NRA * NCB);

    /*** Initialize matrices ***/
    setupMatrices(a,b,c);

    /* Parallelize the computation of the following matrix-matrix
   multiplication. How to partition and distribute the initial matrices, the
   work, and collecting final results.
    */
    // multiply
    double start = MPI_Wtime();
    for (size_t i = 0; i < NRA; i++) {
        for (size_t j = 0; j < NCB; j++) {
            for (size_t k = 0; k < NCA; k++) {
                res[i * NCB + j] += a[i][k] * b[k][j];
            }
        }
    }
    /*  perform time measurement. Always check the correctness of the parallel
       results by printing a few values of c[i][j] and compare with the
       sequential output.
    */
    double time = MPI_Wtime()-start;
    free(a);
    free(b);
    free(c);
    return time;
}

double splitwork(double* res, size_t num_workers){
    if (num_workers == 0) // sadly noone will help me :((
    {
        printf("Run sequential!\n");
        return productSequential(res);
    }

    double(*a)[NCA] = malloc(sizeof(double) * NRA * NCA);
    double(*b)[NCB] = malloc(sizeof(double) * NCA * NCB);
    double(*c)[NCB] = malloc(sizeof(double) * NRA * NCB);
    // Transpose matrix b to make accessing columns easier - in row major way - better cache
```

```
    performance
    setupMatrices(a,b,c);

    double start_time = MPI_Wtime();
    double (*b_transposed)[NCA] = malloc(sizeof(double) * NCA * NCB);
    for (size_t i = 0; i < NCA; i++) {
        for (size_t j = 0; j < NCB; j++) {
            b_transposed[j][i] = b[i][j];
        }
    }

    /*** Initialize matrices ***/
    // given number of workers I'll split
    size_t rows_per_worker = NRA / (num_workers+1); //takes corresponding columns from other
    matrix
    printf("rows per worker: %zu\n", rows_per_worker);
    size_t row_end_first = NRA - rows_per_worker*num_workers;
    printf("first gets most: %zu\n", row_end_first);

    //setup requests
    MPI_Request requests[num_workers];
    MM_input *data_first = (MM_input*)malloc(sizeof(MM_input));
    data_first->rows = row_end_first;
    data_first->a = (double*)a; //they both start of with no offset!
    data_first->b = (double*)b_transposed;
    size_t total_size = sizeof(size_t) + (data_first->rows * NCA)*sizeof(double)+SIZE_OF_B;
    char* buffer = getbuffer(data_first, total_size);    //first one

    // Tag is just nr-cpu -1
    MPI_Isend(buffer, total_size, MPI_CHAR, 1, 0,MPI_COMM_WORLD, &requests[0]);
    free(data_first);
    total_size = sizeof(size_t) + (rows_per_worker * NCA)*sizeof(double) + SIZE_OF_B; //size
    is the same for all other - just compute once!
    size_t i;
    for (i = 0; i < (num_workers-1); ++i)
    {
        MM_input *data = (MM_input*)malloc(sizeof(MM_input));
        data->rows = rows_per_worker;
        data->a = (double*)(a + (row_end_first + rows_per_worker*i));
        data->b = (double*)(b_transposed); // send everyting - all needed
        buffer = getbuffer(data, total_size);
        printf("nr_worker - %zu\n", i);
        MPI_Isend(buffer, total_size, MPI_CHAR, i+2, i+1,MPI_COMM_WORLD, &requests[i+1]);
        free(data);
    }
    double* my_a = (double*)(a + (row_end_first + rows_per_worker*i));

    //I multiply the rest
    size_t offset = 0;
    for (size_t row = (NRA-rows_per_worker); row < NRA; row++)
    {
        for (size_t col = 0; col < NCB; col++)
        {
            res[row * NCB + col] = multsum(my_a+offset, (((double*)b_transposed)+col*NCA), NCA
    );
        }
        offset += NCA;
    }
    printf("My c: \n");
    //wait for rest
    MPI_Status stats[num_workers];
    if(MPI_Waitall(num_workers, requests, stats) == MPI_ERR_IN_STATUS){
        printf("Communication failed!!! - abort\n");
    }
    printf(">>>Everything sent and recieved\n");

    // reviece rest
    size_t buf_size = sizeof(double)*row_end_first*NCB;
    double* revbuf;
    offset = 0;
    for (size_t worker = 0; worker < num_workers; worker++)
    {
        revbuf = (double*)malloc(buf_size); //first gets largest buffer
```

```
233         MPI_Recv(revbuf, buf_size/sizeof(double), MPI_DOUBLE, worker+1, worker, MPI_COMM_WORLD
    ,&stats[worker]);
234         memcpy(&res[offset/sizeof(double)], revbuf, buf_size);
235         free(revbuf);
236         offset += buf_size;
237         buf_size = sizeof(double)*rows_per_worker*NCB;
238     }
239     double time = MPI_Wtime()-start_time;
240     //free all pointers!
241     free(a);
242     free(b);
243     free(b_transposed);
244     free(c);
245     return time;
246 }
247
248
249
250 double work(int rank, size_t num_workers){
251     size_t rows_per_worker = NRA / (num_workers+1);
252     char* buffer;
253     MPI_Status status;
254     if (rank == 1) // first always get's most work
255     {
256         rows_per_worker = NRA - rows_per_worker*num_workers;
257     }
258     size_t size_of_meta = sizeof(size_t);
259     size_t size_of_a = sizeof(double)*rows_per_worker*NCA;
260     size_t buffersize = size_of_meta+size_of_a + SIZE_OF_B;
261     buffer = (char*)malloc(buffersize);
262
263     MPI_Recv(buffer, buffersize, MPI_CHAR, 0, rank-1, MPI_COMM_WORLD, &status);
264     double start = MPI_Wtime();
265     int count;
266     MPI_Get_count(&status, MPI_CHAR, &count);
267     printf("I'm rank %d and I got %d bytes (%ld doubles) of data from %d with tag %d.\n", rank
    , count, (count-sizeof(size_t))/sizeof(double), status.MPI_SOURCE, status.MPI_TAG);
268
269     MM_input *mm = (MM_input*)malloc(sizeof(MM_input));
270     mm->a = (double*)&buffer[size_of_meta];
271     mm->b = (double*)&buffer[size_of_meta+size_of_a];
272
273     double *res =(double*)malloc(sizeof(double)*rows_per_worker*NCB);
274
275     size_t offset = 0;
276     for (size_t row = 0; row < rows_per_worker; row++)
277     {
278         for (size_t col = 0; col < NCB; col++)
279         {
280             res[row * NCB + col] = multsum(mm->a+offset, (((double*)mm->b)+col*NCA), NCA);
281         }
282         offset += NCA;
283     }
284     MPI_Send(res, rows_per_worker*NCB, MPI_DOUBLE, 0,rank-1, MPI_COMM_WORLD);
285     printf("[%d] sent res home\n",rank);
286     free(res);
287     return MPI_Wtime() - start;
288 }
289
290 int main(int argc, char *argv[]) {
291     int tid, nthreads;
292     /* for simplicity, set NRA=NCA=NCB=N  */
293     // Initialize MPI, find out MPI communicator size and process rank
294     int myRank, numProcs;
295     MPI_Status status;
296     MPI_Init(&argc, &argv);
297     MPI_Comm_size(MPI_COMM_WORLD, &numProcs);
298     MPI_Comm_rank(MPI_COMM_WORLD, &myRank);
299     int num_Workers = numProcs-1;
300     if (argc > 1 && strcmp(argv[1], "parallel") == 0) {
301         // Variables for the process rank and number of processes
302        if (myRank == 0) {
303             printf("Run parallel!\n");
```

```
            double *truth = malloc(sizeof(double) * NRA * NCB);
            double time = productSequential(truth);
            printf("Computed reference results in %.6f s\n", time);
            printf("Hello from master! - I have %d workers!\n", num_Workers);
            // send out work
            double *res = malloc(sizeof(double)*NRA*NCB);
            time = splitwork(res, num_Workers);
            if (checkResult(res, truth, NCB, NRA)) {
                printf("Matrices do not match!!!\n");
                return 1;
            }
            printf("Matrices match (parallel [eps %.10f])! - took: %.6f s\n", EPS, time);
            free(truth);
            free(res);
        } else {
            double time = work(myRank, num_Workers);
            printf("Worker bee %d took %.6f s (after recv) for my work\n", myRank, time);
        }

    } else  // run sequantial
    {
        printf("Run sequantial!\n");
        double *res = malloc(sizeof(double) * NRA * NCB);
        double time = productSequential(res);
        if (checkResult(res, res, NCB, NRA)) {
            printf("Matrices do not match!!!\n");
            return 1;
        }
        printf("Matrices match (sequantial-trivial)! - took: %.6f s\n", time);
        free(res);
    }

    MPI_Finalize();
    return 0;
}
```

Appendix - Poisson solver