Homework 7

Hartwin Peelaers
PHSX521/EPHX521 Fall 2019
Due: Tuesday November 12, beginning of class

For this homework you are allowed to work in groups of two (or three). If you work in a group, you should solve both questions, if working alone answering the second question is sufficient. Grading will be 60% on having a working code, 20% on commenting that code, 20% on generating (useful) figures and explaining them and the physics they contain. You can use any programming or scripting language (Python, C, C++, Fortran, Matlab, etc.). Since this is not a coding class, your code does not have to be the most optimized one. For figures, do label the axes. To hand in, either print out the code+figures, or send everything to me by email. Make sure all names of your group are listed.

(10) 1. (a) Write a code that solves the equation of motion for 3 particles with mass=1, that interact only through gravitational forces (set G=1). Use the Velocity Verlet algorithm as discussed in class, and summarized as:

$$\vec{x}_{new} = \vec{x}_{old} + \vec{v}_{old}\Delta t + \frac{1}{2}(\Delta t)^2\vec{f}(\vec{x}_{old})$$
$$\vec{v}_{new} = \vec{v}_{old} + \frac{1}{2}\Delta t\left[\vec{f}(\vec{x}_{old}) + \vec{f}(\vec{x}_{new})\right]$$

(b) Is the total energy conserved? Why or why not? Check your answer explicitly by plotting the total mechanical energy as function of time.

(c) Use the following special initial conditions (use all digits), and discuss the resulting trajectory.
Positions in Cartesian coordinates: particle 1: (0.97000436, -0.24308753, 0), particle 2: (-0.97000436, 0.24308753, 0), and particle 3: (0, 0, 0).
Velocities of particle 3: $\vec{v_3}$=(-0.93240737, -0.86473146, 0). The velocities of both particle 1 and 2 are given by -$\vec{v_3}$/2.

(10) 2. (a) Write a code to simulate a damped harmonic oscillator with k=m=1. Use the Velocity Verlet algorithm. Note that the damping force is not conservative, as it depends on $\vec{v}$. We therefore have to modify the Verlet algorithm slightly. One

option would be ($\vec{f}$ is the total force):

$$\vec{x}_{new} = \vec{x}_{old} + \vec{v}_{old}\Delta t + \frac{1}{2}(\Delta t)^2 \vec{f}(\vec{x}_{old}, \vec{v}_{old}) \tag{1}$$

$$\vec{v}_{new} = \vec{v}_{old} + \frac{1}{2}\Delta t \left[\vec{f}(\vec{x}_{old}, \vec{v}_{old}) + \vec{f}(\vec{x}_{new}, \vec{v}_{old})\right] \tag{2}$$

$$\vec{v}_{new} = \vec{v}_{old} + \frac{1}{2}\Delta t \left[\vec{f}(\vec{x}_{old}, \vec{v}_{old}) + \vec{f}(\vec{x}_{new}, \vec{v}_{new})\right] \tag{3}$$

In the last term of line 2, we calculate the forces using the new positions ($\vec{x}_{new}$) using our best estimate of the new velocity, namely the old velocity ($\vec{v}_{old}$). This way we obtain a new value for the new velocity ($\vec{v}_{new}$). We then use that updated value to calculate the new velocity $\vec{v}_{new}$ again in line 3. Of course there are many ways to optimize this procedure, but for our purposes this approximation will be sufficient.

(b) Use your code to explore the different damping regimes and compare with what we did in class.

(c) Extend your code to add a driving force ($\cos \omega t$), and try different values of the damping β and the driving frequency ω. Note that $\omega = 1$ is equivalent to the natural frequency ω_0 due to our choice of units. Discuss your results.