Neural Network and Deep Learning

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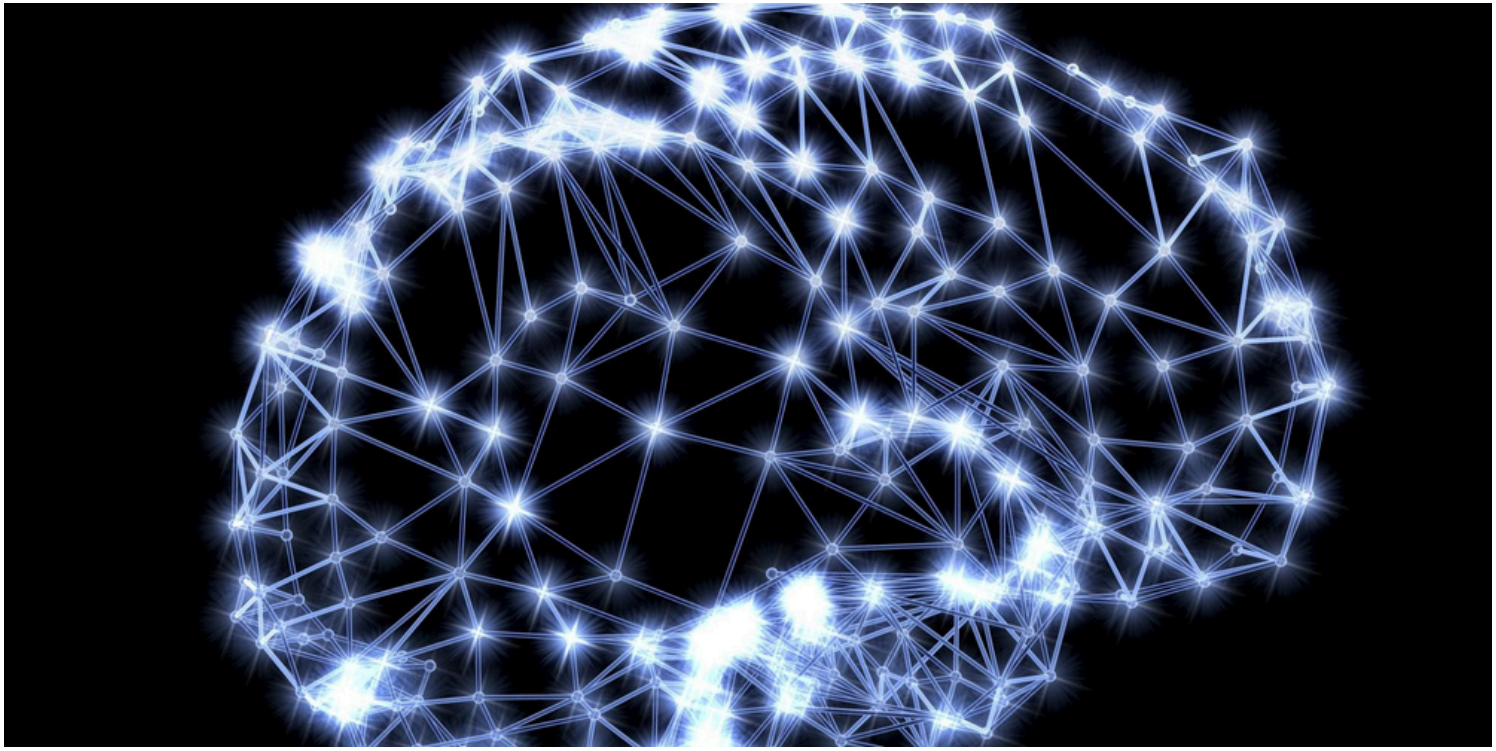
Naxcen Quantum Society

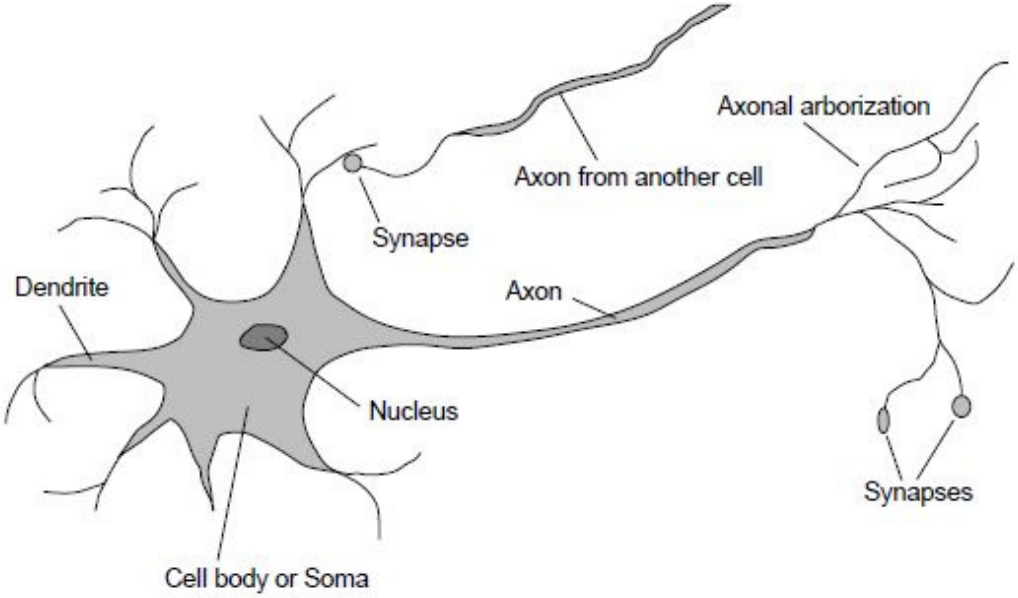
December 2024 **Research Community**

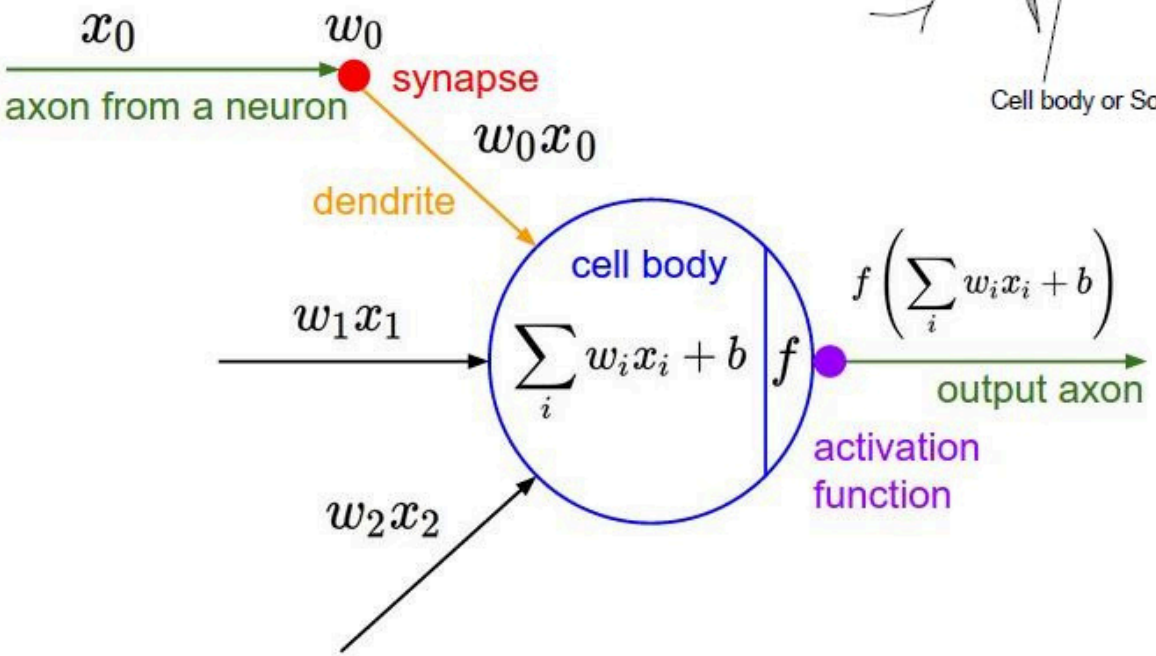
Neural Network

- *Mimics the functionality of a brain.*
- *A neural network is a graph with neurons (nodes, units etc.) connected by links.*



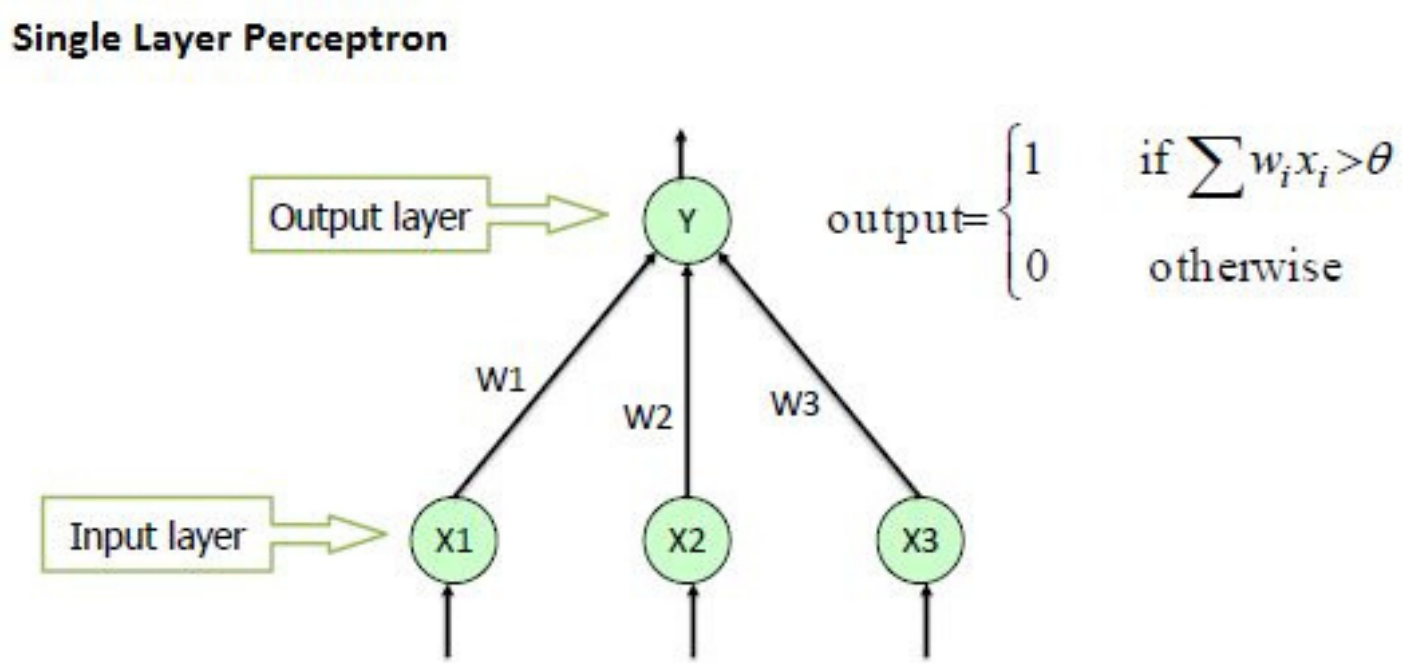
Neural Network: Neuron



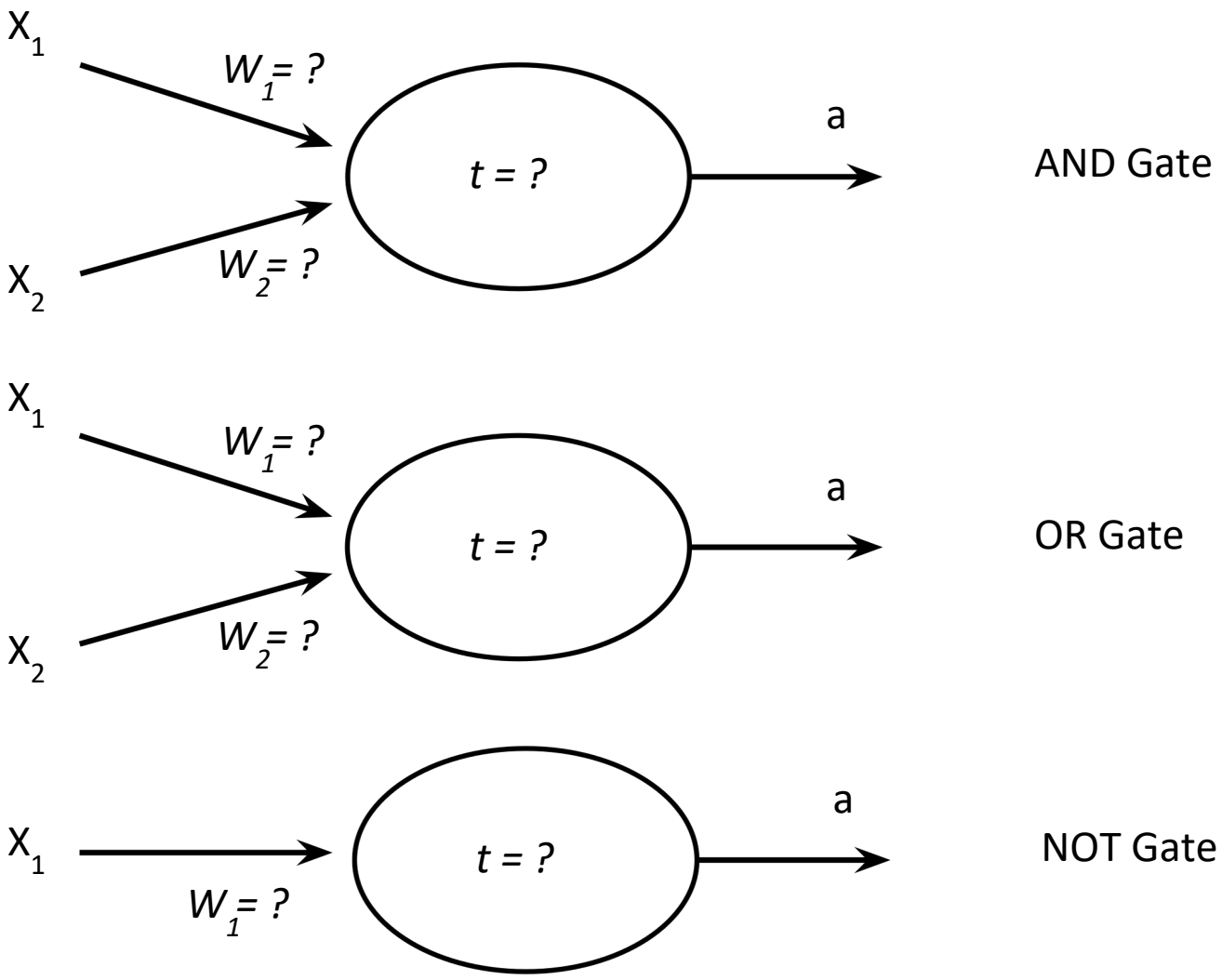


Neural Network: Perceptron

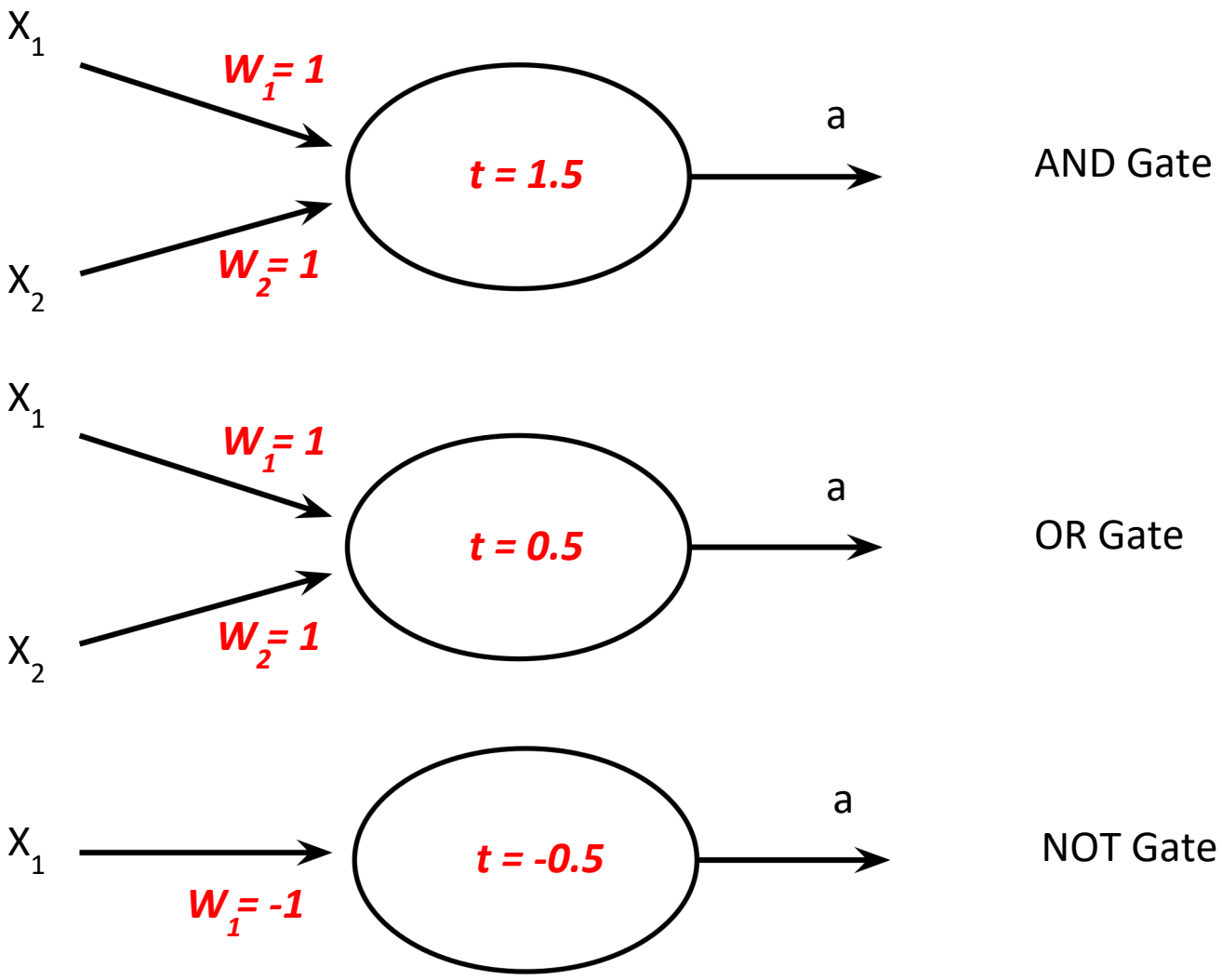
- Network with only single layer.
- No hidden layers



Neural Network: Perceptron

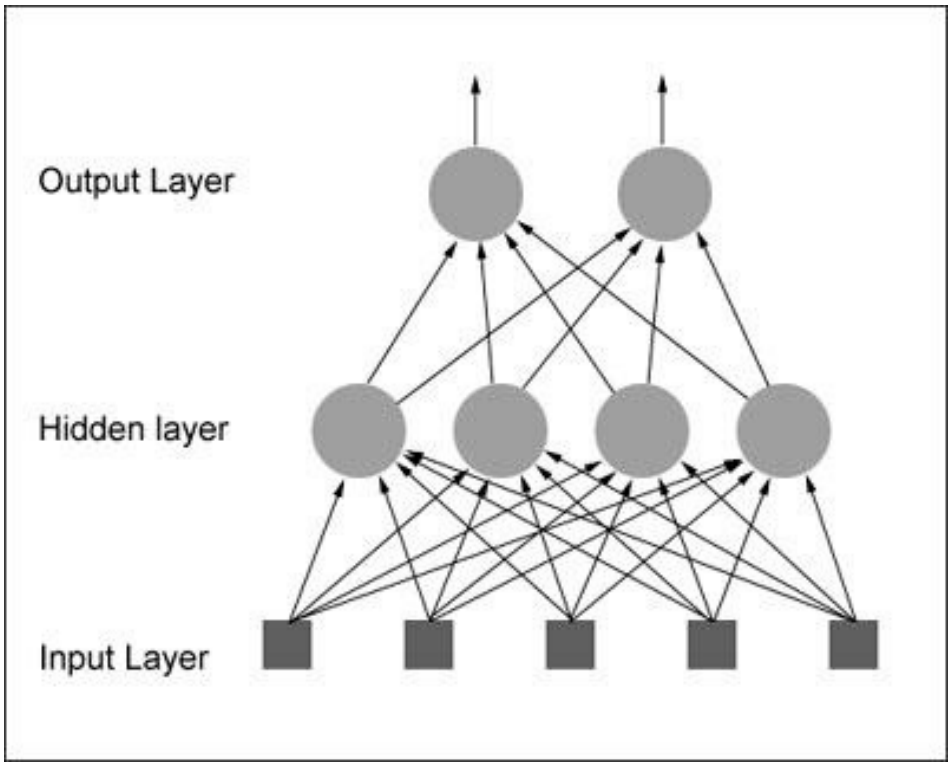


Neural Network: Perceptron



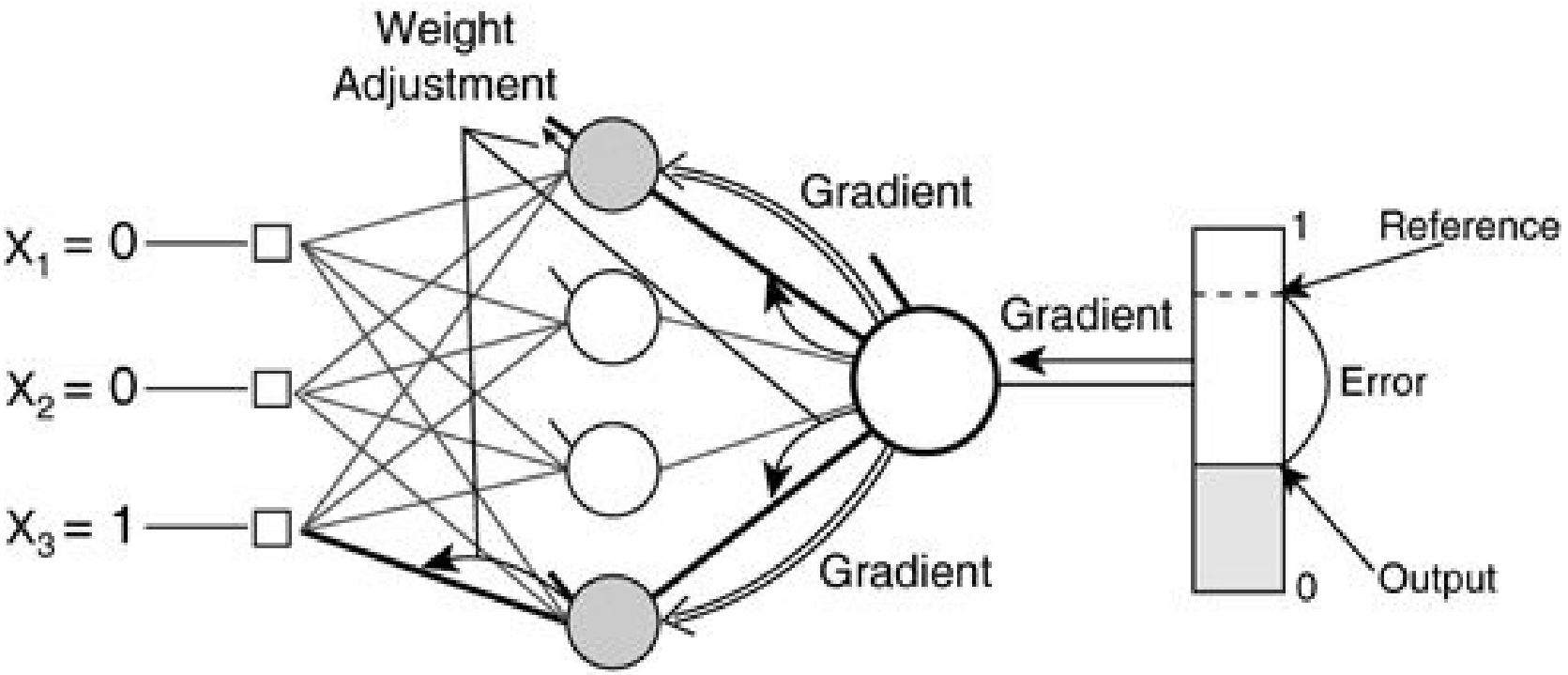
Neural Network: Multi Layer Perceptron (MLP) or Feed-Forward Network (FNN)

- Network with *n*+1 layers
- One output and *n* hidden layers.

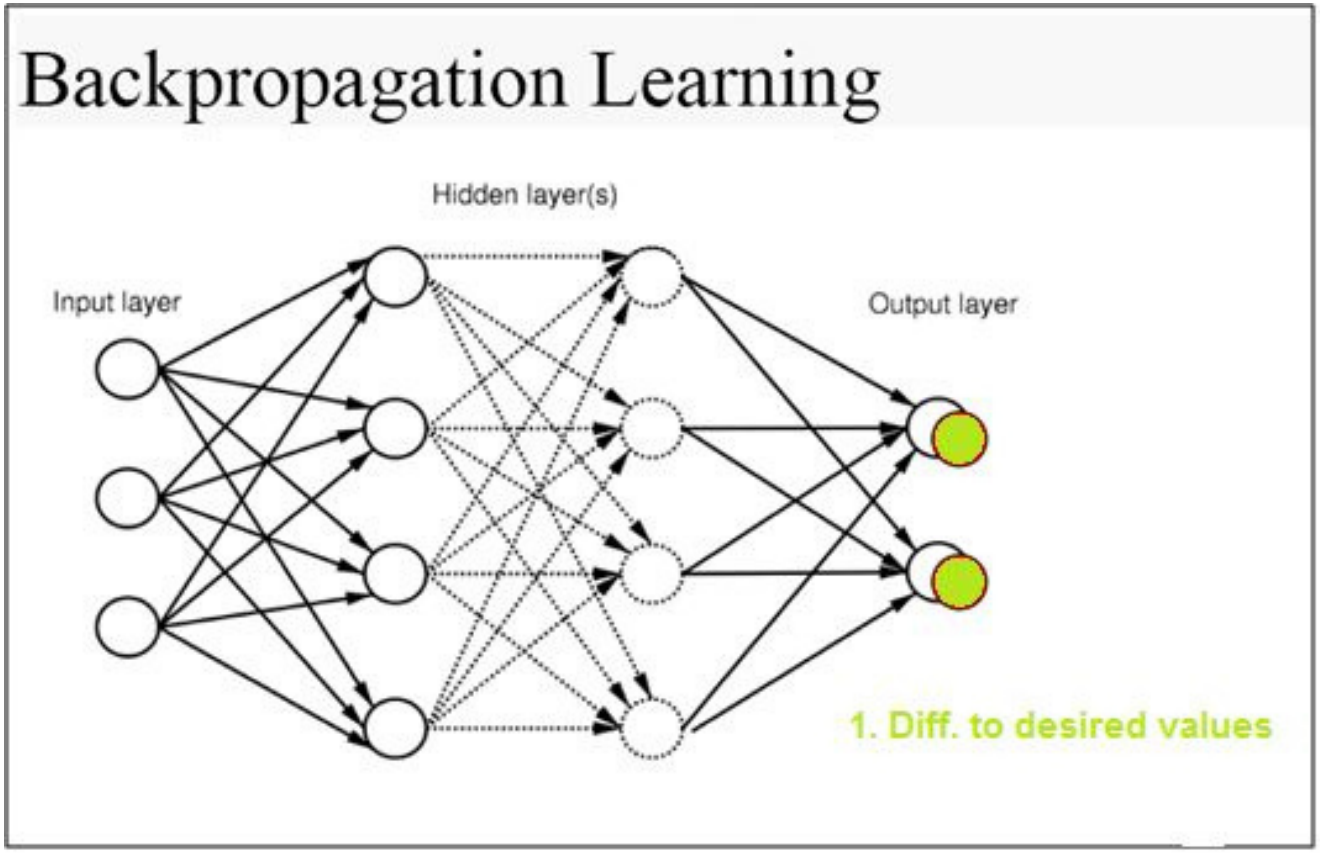


Training: Back propagation algorithm

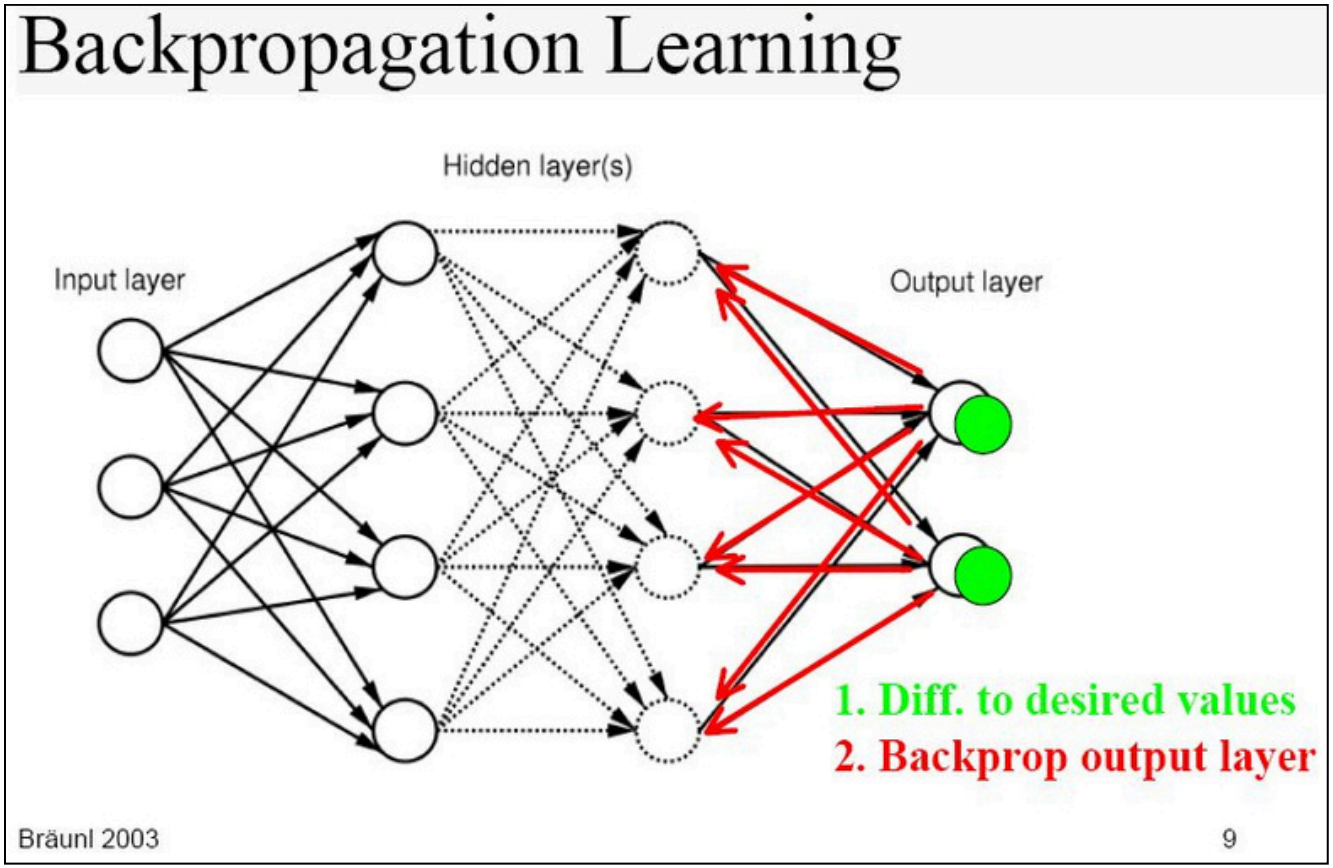
- Gradient decent algorithm



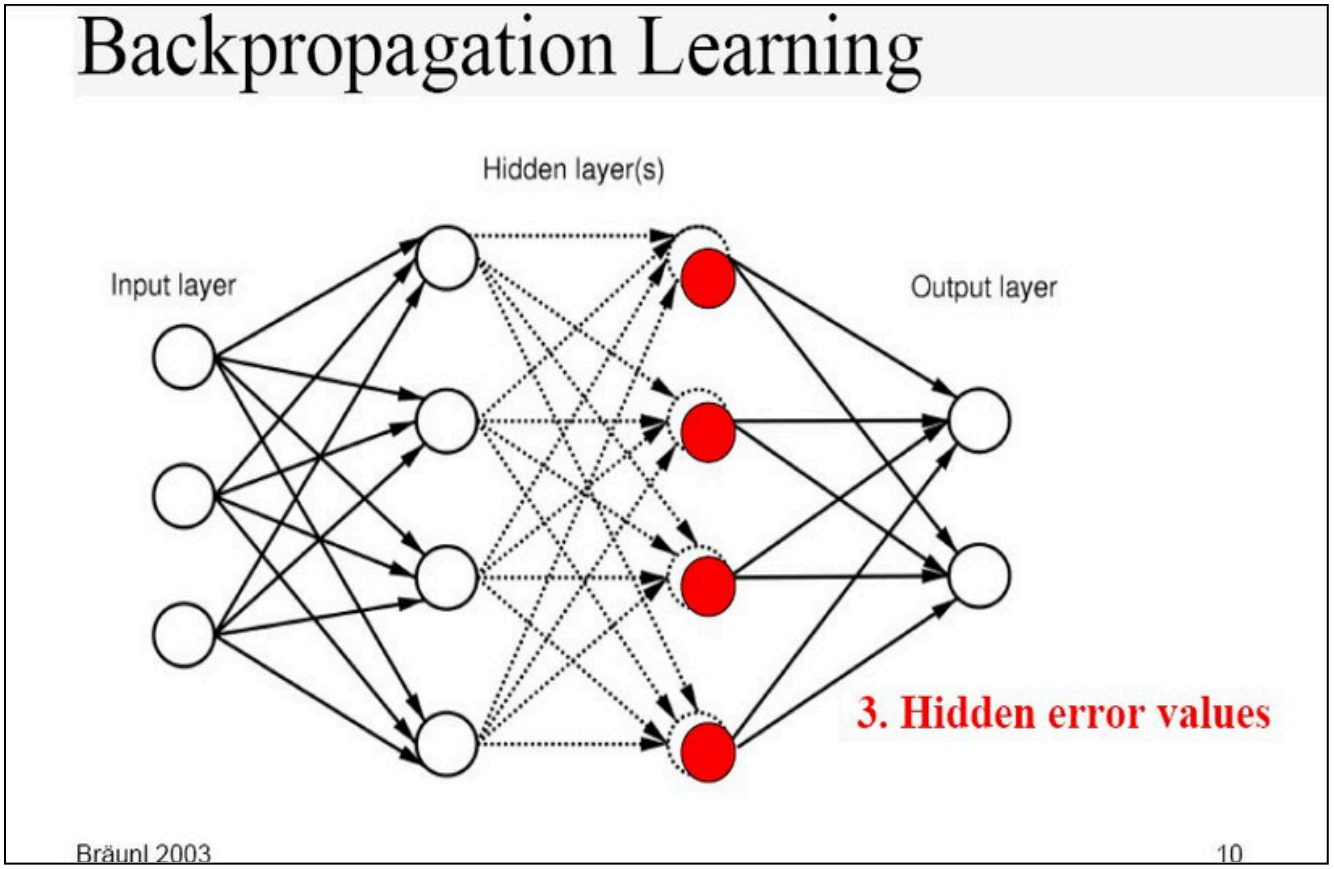
Training: Back propagation algorithm



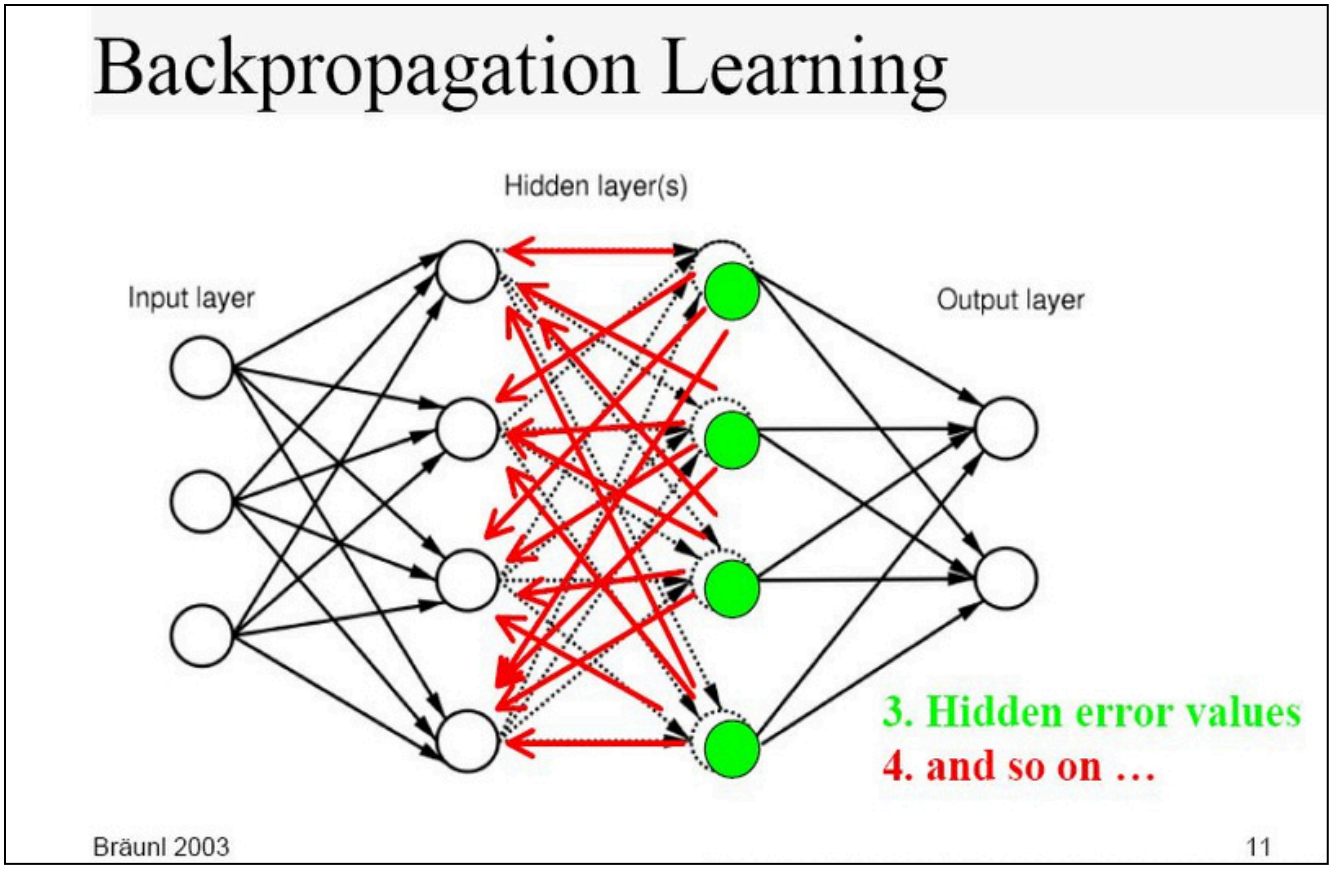
Training: Back propagation algorithm



Training: Back propagation algorithm



Training: Back propagation algorithm



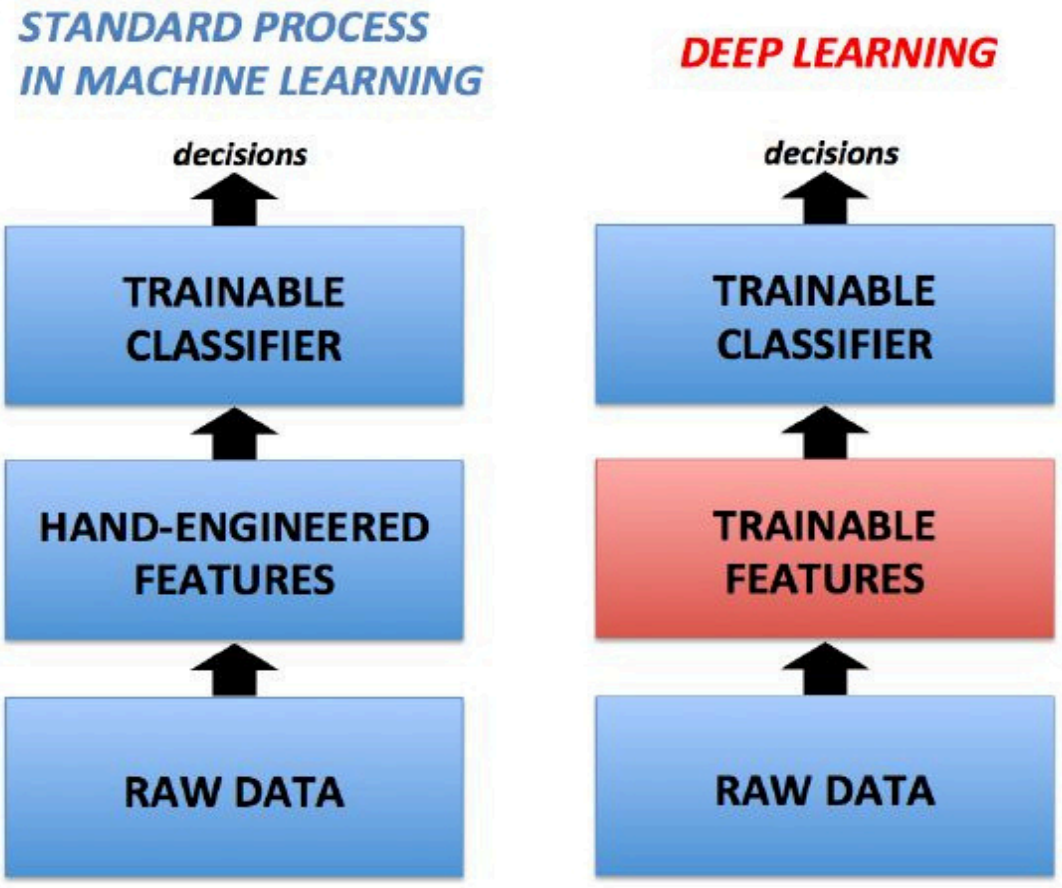
Training: Back propagation algorithm

1. Initialize network with random weights
2. For all training cases (called examples):
   a. Present training inputs to network and calculate output
   b. For all layers (starting with output layer, back to input layer):
      i. Compare network output with correct output (error function)
      ii. Adapt weights in current layer

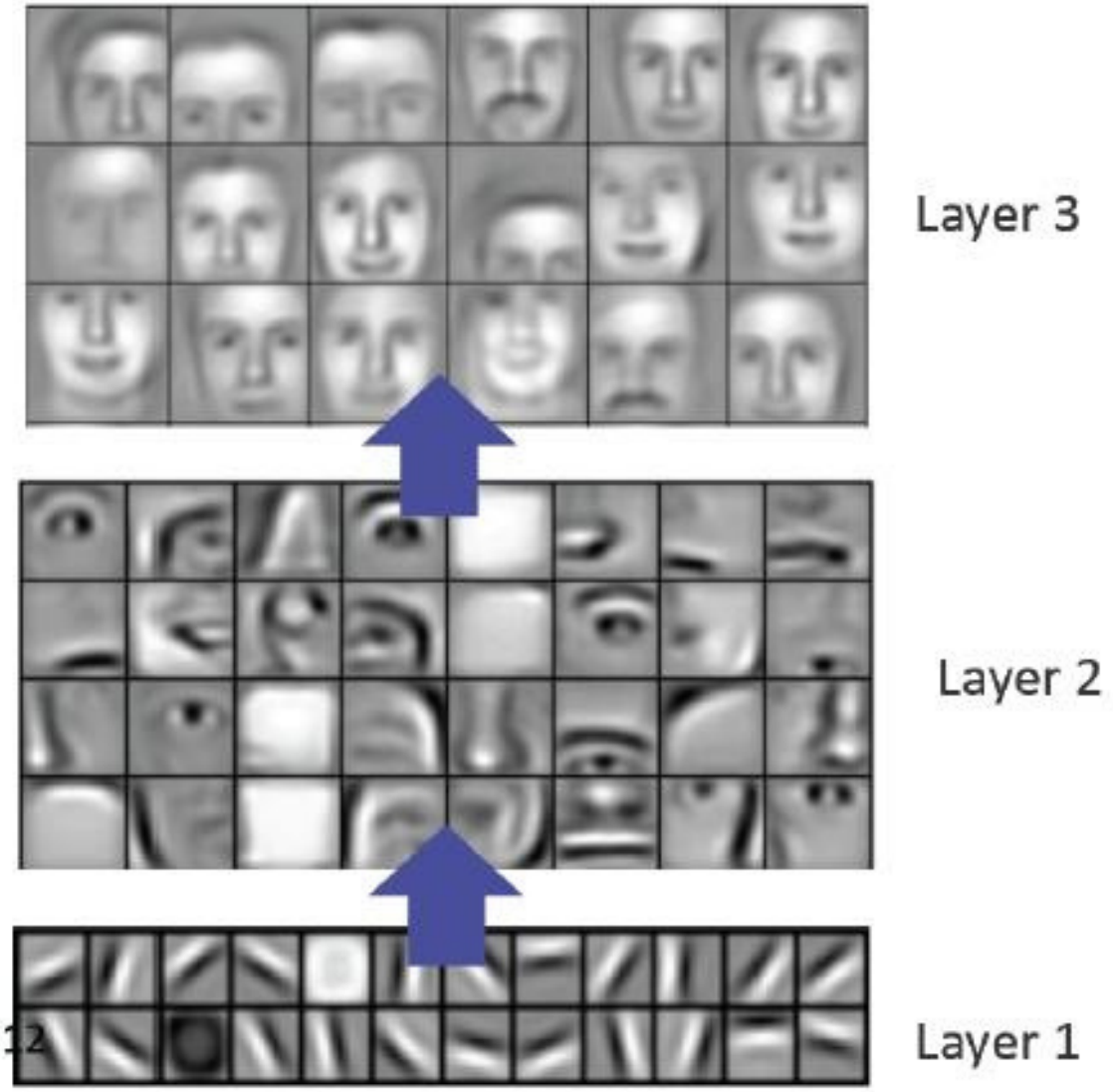
Deep Learning

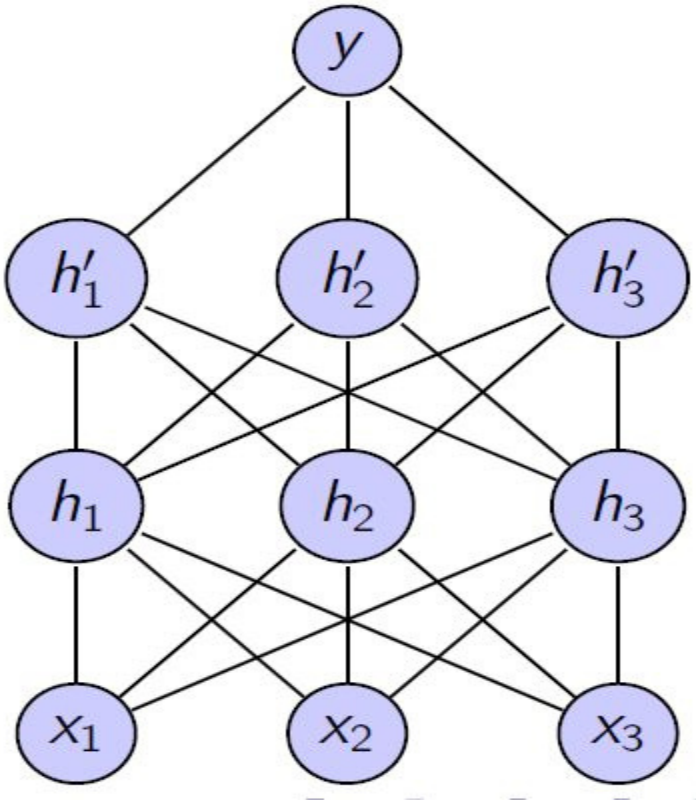
What is Deep Learning?

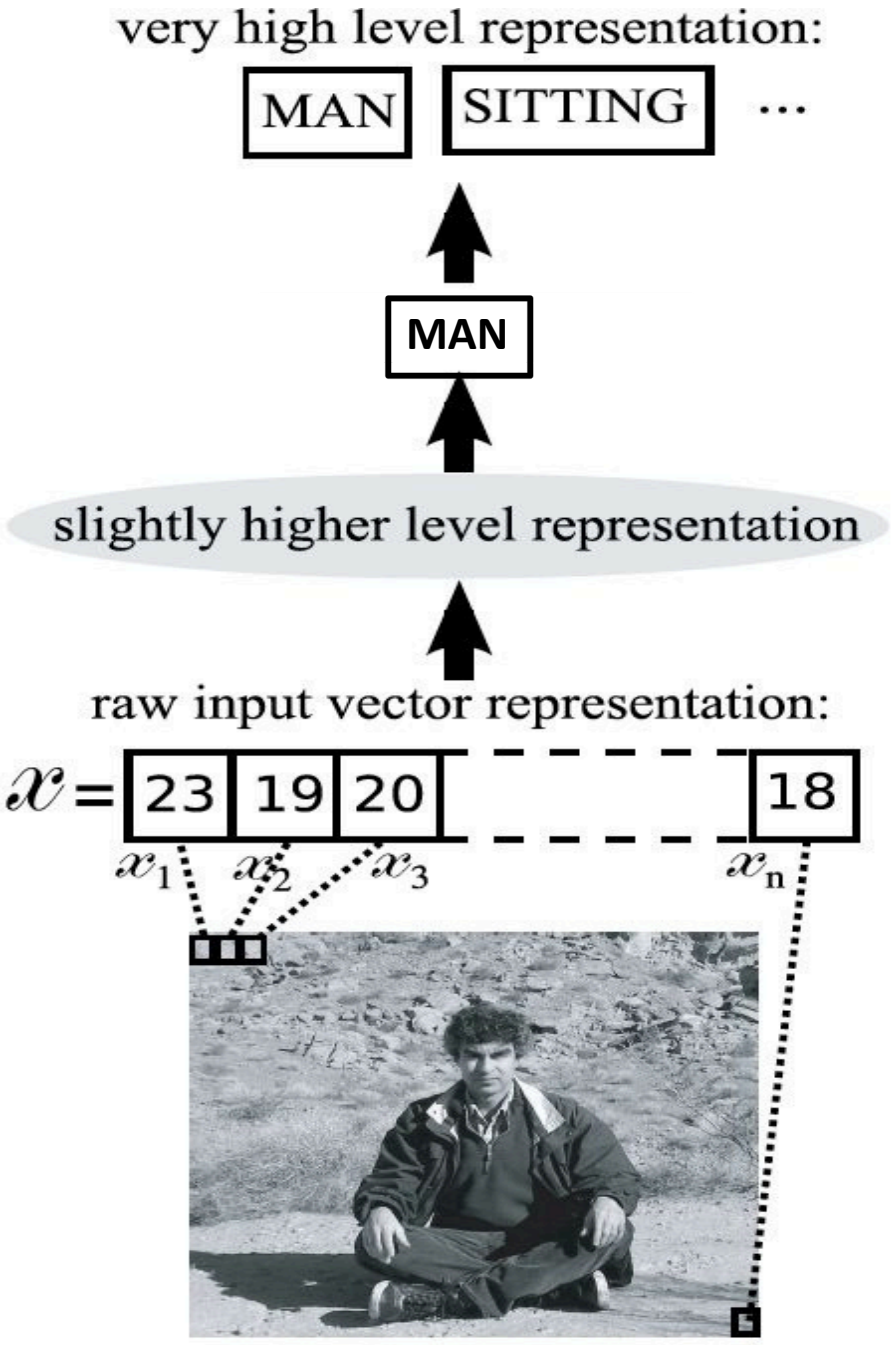
- A family of methods that uses deep architectures to learn high-level feature representations



Example 1



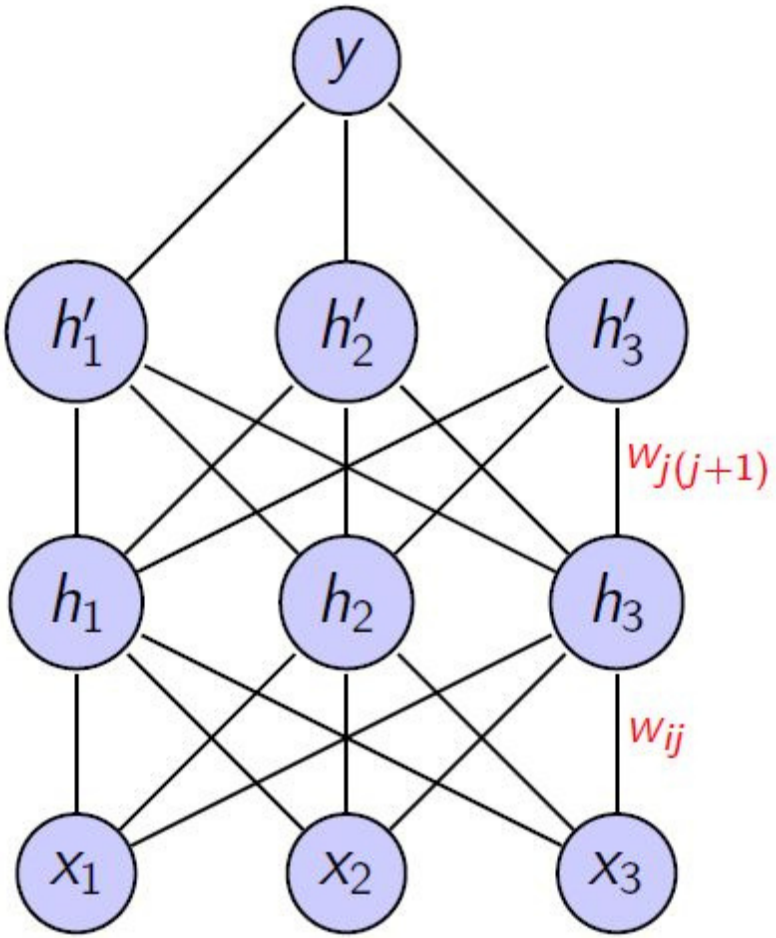

y
h'_1
h'_2
h'_3
h_1
h_2
h_3
x_1
x_2
x_3


very high level representation:
MAN
SITTING
...
MAN
slightly higher level representation
raw input vector representation:
x = 23 19 20 18
x_1 x_2 x_3 x_n

Example 2

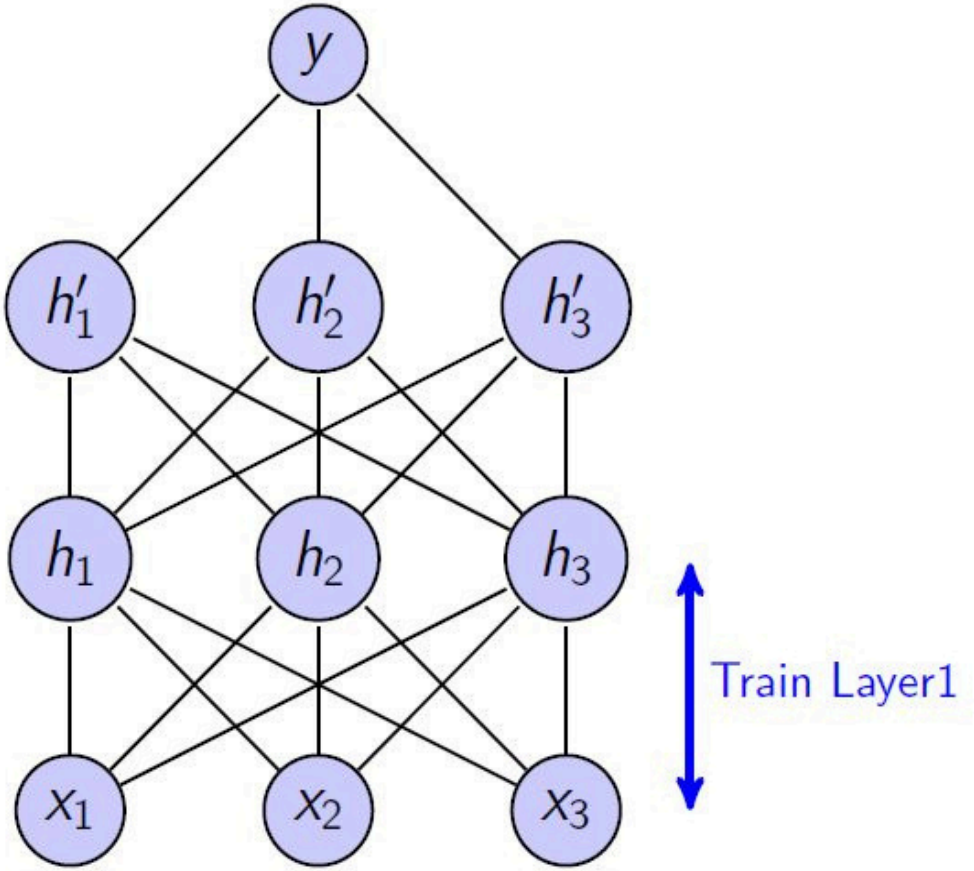
Why are Deep Architectures hard to train?

- Vanishing/Exploding gradient problem in Back Propagation
  - $\frac{\partial Loss}{\partial w_{ij}} = \frac{\partial Loss}{\partial in_j}\frac{\partial in_j}{\partial w_{ij}} = \delta_j x_i$
  - $\delta_j = \left[\sum_{j+1} \delta_{j+1} w_{j(j+1)}\right] \sigma'(in_j)$
  - δ_j may vanish after repeated multiplication



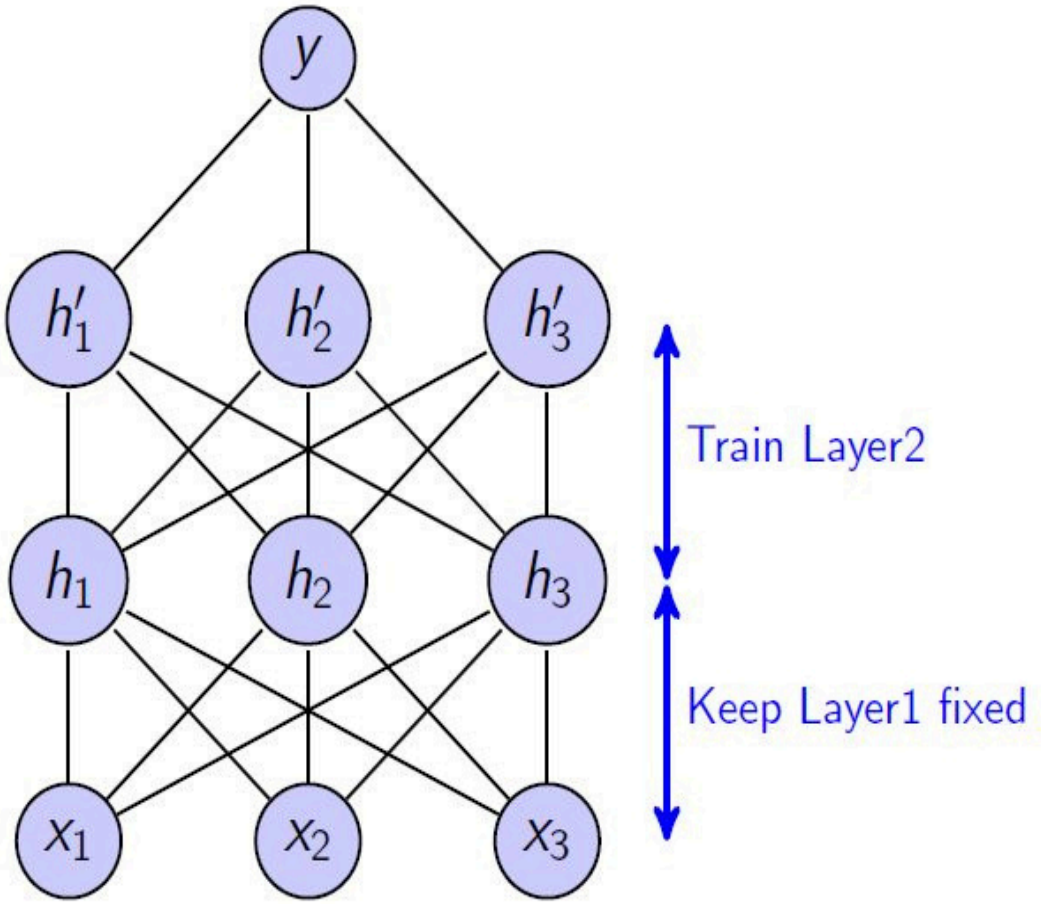
Layer-wise Pre-training

- First, train one layer at a time, optimizing data-likelihood objective P(x)



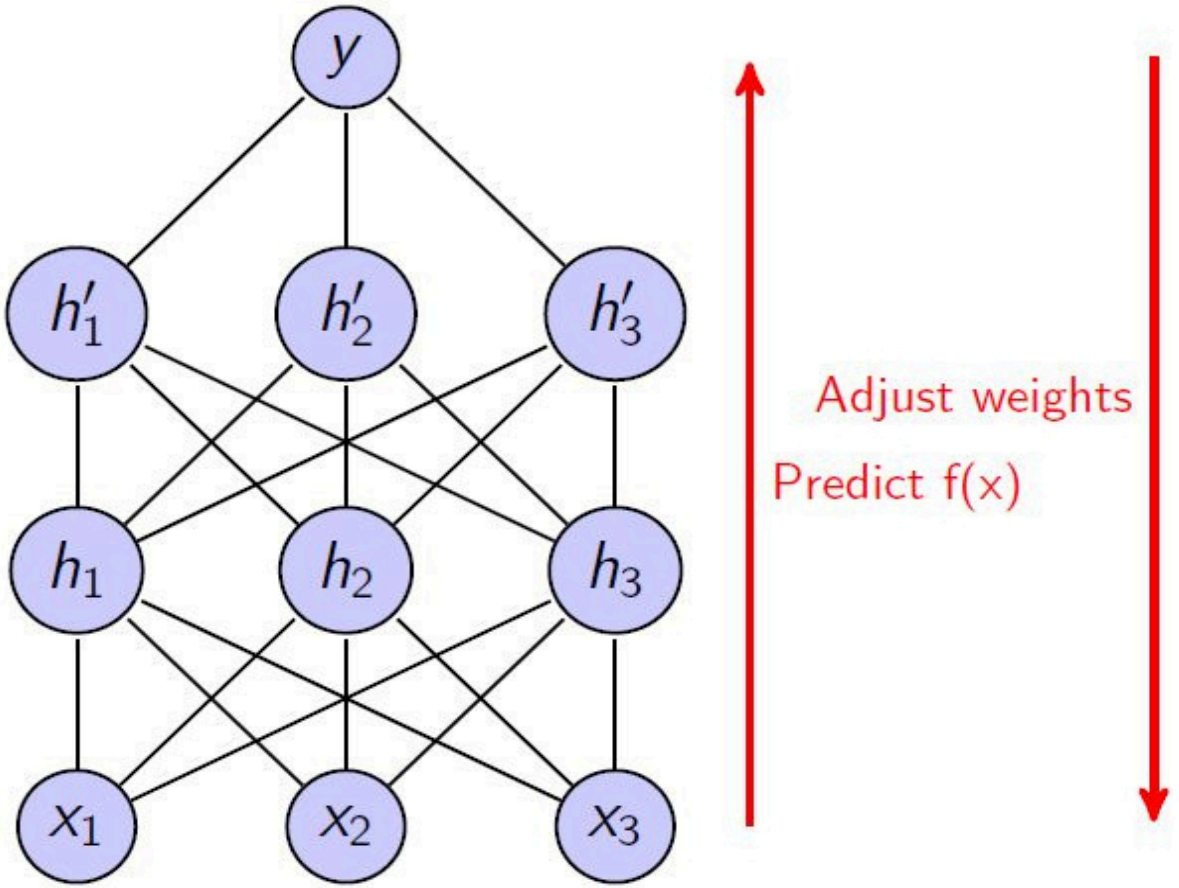
Layer-wise Pre-training

- Then, train second layer next, optimizing data-likelihood objective P(h)



Layer-wise Pre-training

- Finally, fine-tune labelled objective P(y|x) by Backpropagation



Deep Belief Nets

- Uses Restricted Boltzmann Machines (RBMs)
- Hinton et al. (2006), *A fast learning algorithm for deep belief nets*.

Restricted Boltzmann Machine (RBM)

- RBM is a simple energy-based model:

$$p(x,h) = \frac{1}{Z_\theta} \exp\left(-E_\theta(x,h)\right)$$

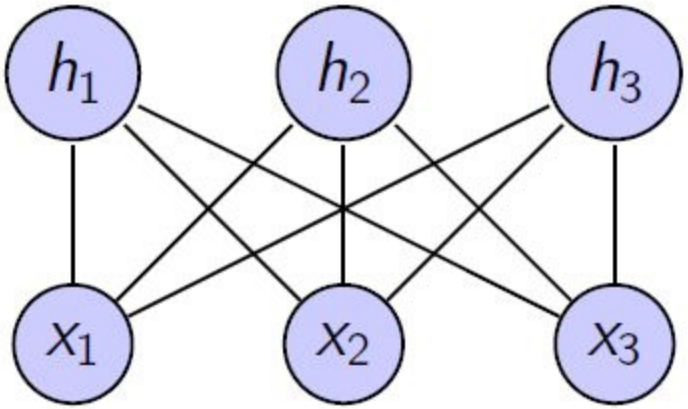
where

$$E_\theta(x,h) = -x^T W h - b^T x - d^T h$$

$$Z_\theta = \sum_{(x,h)} \exp(-E_\theta(x,h))$$

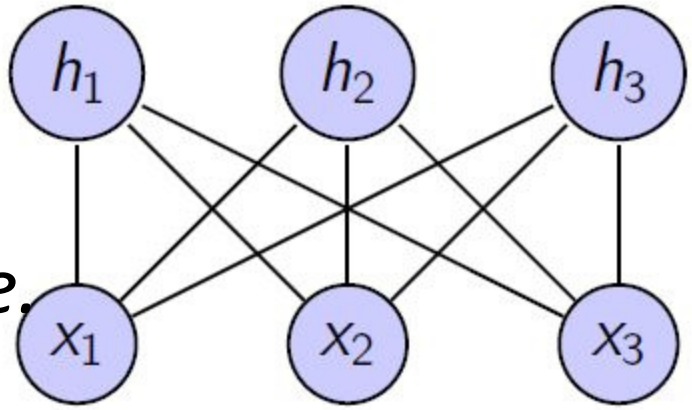
Example:

- Let weights $(h_1; x)_1$, $(h; x_1)$ be positive, others be zero, b = d = 0.
- Calculate *p(x,h)* ?
- Ans: p(x1 = 1; x2 = 0; x3 = 1; h1 = 1; h2 = 0; h3 = 0)



Restricted Boltzmann Machine (RBM)

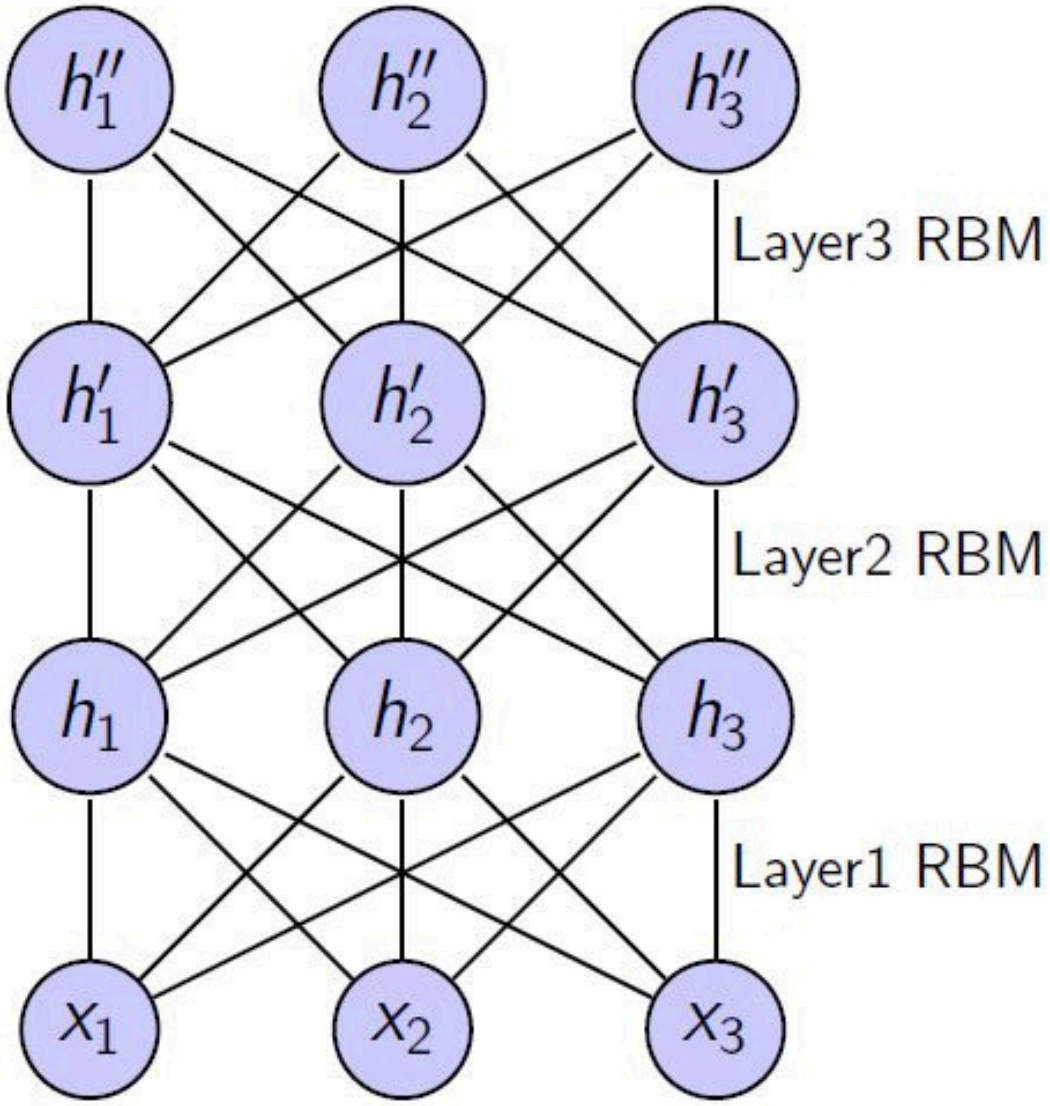
- *P(x, h) = P(h|x) P(x)*
- *P(h|x): easy to compute*
- *P(x): hard if datasets are large.*



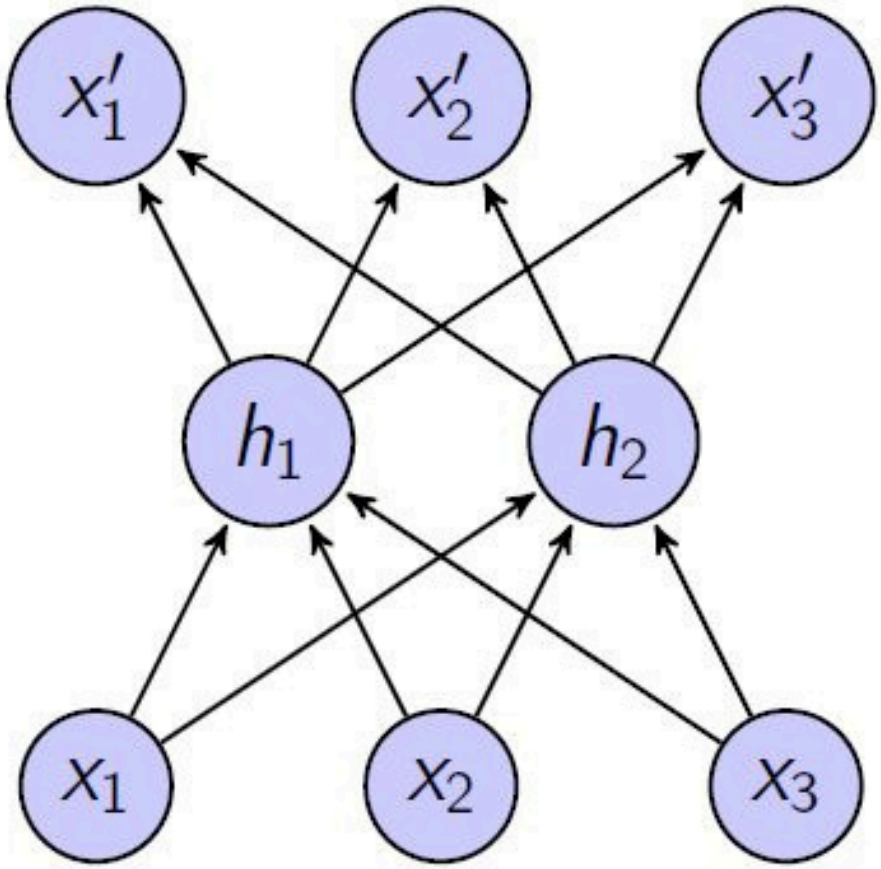
Contrastive Divergence:

1. Let $x^{(m)}$ be training point, $W = [w_{ij}]$ be current model weights
2. Sample $\hat{h}_j \in \{0, 1\}$ from $p(h_j | x = x^{(m)}) = \sigma(\sum_i w_{ij} x_i^{(m)} + d_j)\ \forall j.$
3. Sample $\tilde{x}_i \in \{0, 1\}$ from $p(x_i | h = \hat{h}) = \sigma(\sum_j w_{ij} \hat{h}_j + b_i)\ \forall i.$
4. Sample $\tilde{h}_j \in \{0, 1\}$ from $p(h_j | x = \tilde{x}) = \sigma(\sum_i w_{ij} \tilde{x}_i + d_j)\ \forall j.$
5. $w_{ij} \leftarrow w_{ij} + \gamma(x_i^{(m)} \cdot \hat{h}_j - \tilde{x}_i \cdot \tilde{h}_j)$

Deep Belief Nets (DBN) = Stacked RBM



Auto-Encoders: Simpler alternative to RBMs



Decoder: $x' = \sigma(W'h + d)$

Encoder: $h = \sigma(Wx + b)$

Deep Learning - Architecture

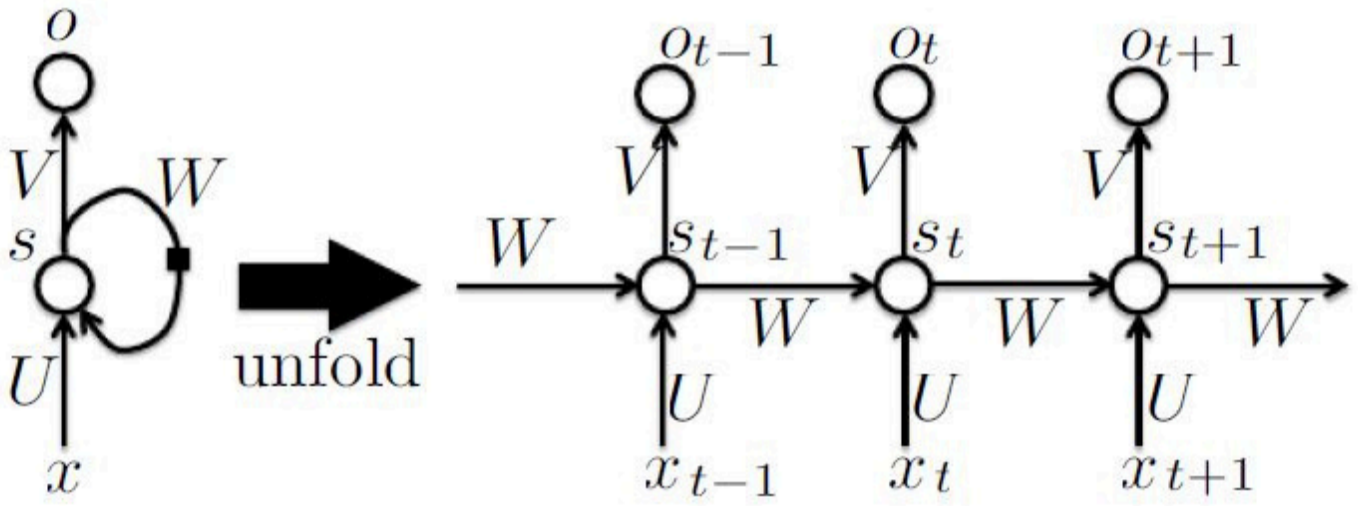
- Recurrent Neural Network (RNN)
- Convolution Neural Network (CNN)

Recurrent Neural Network (RNN)

Recurrent Neural Network (RNN)

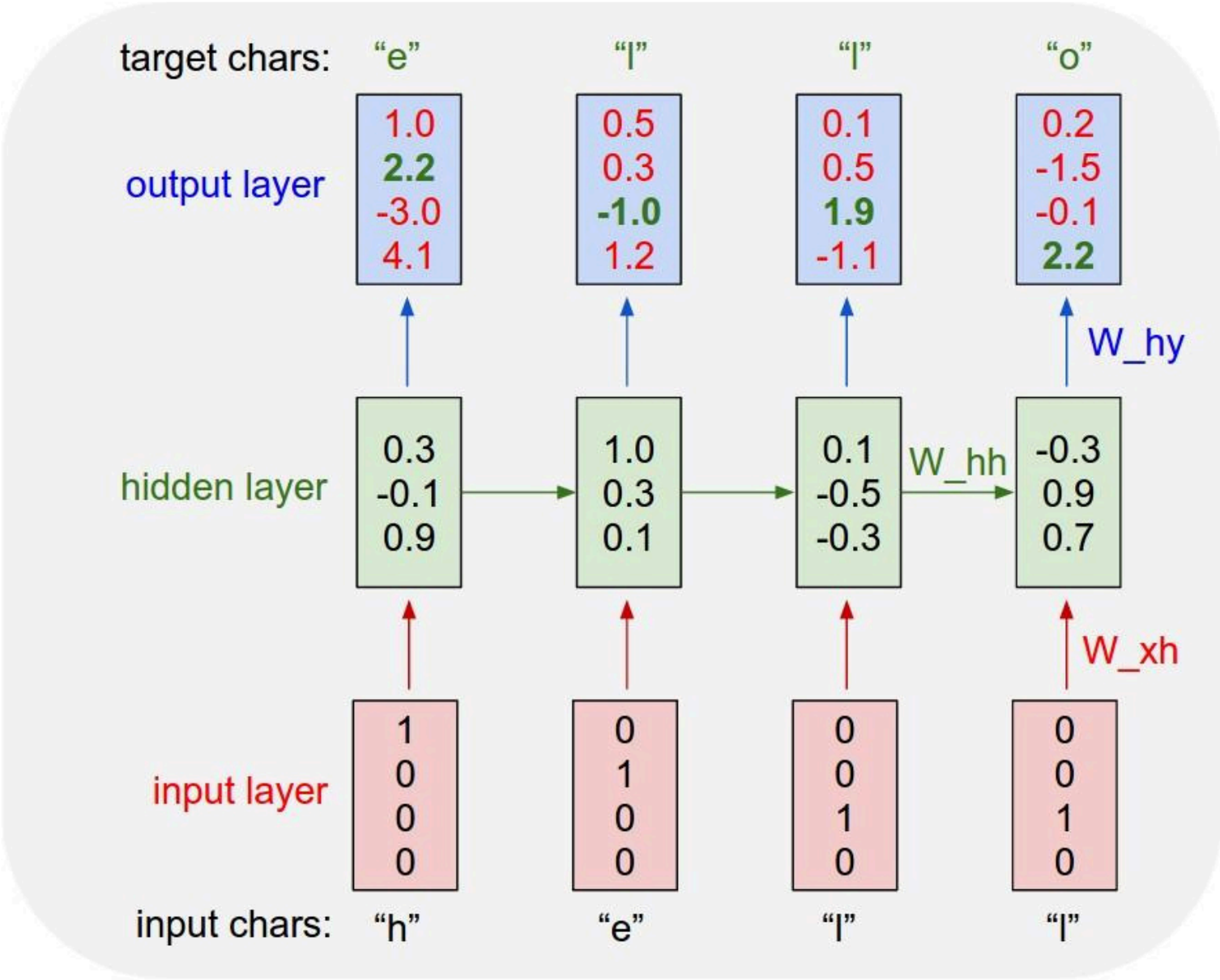
- Enable networks to do temporal processing

  and learn sequences

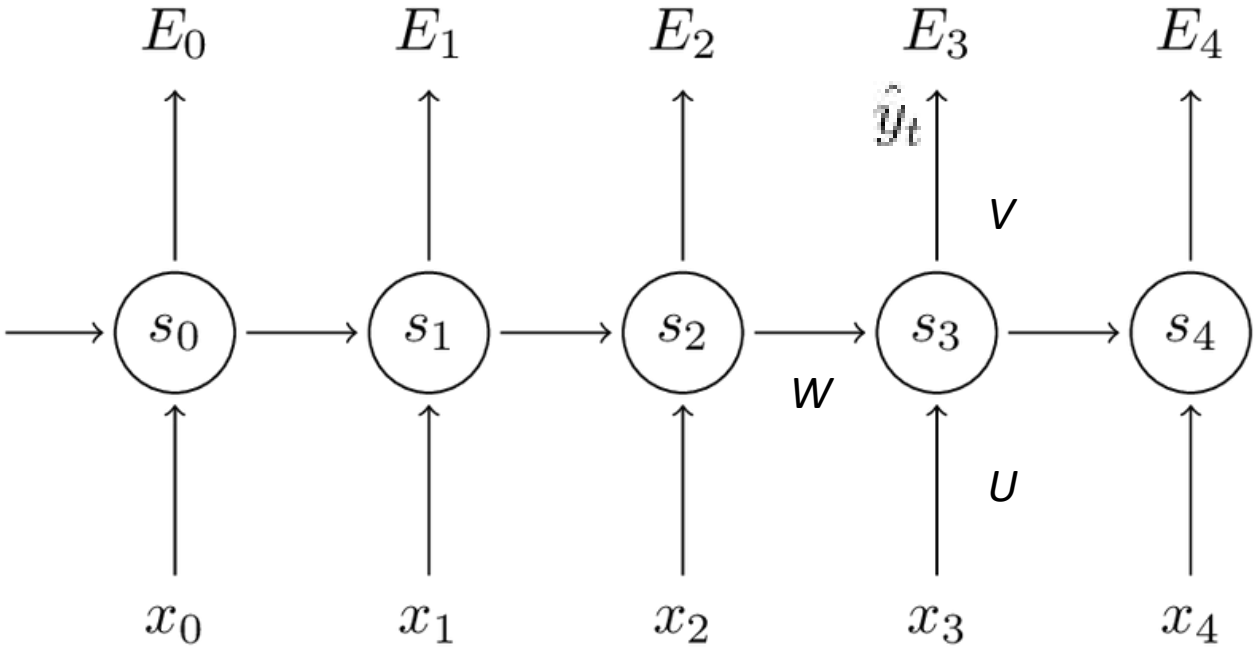


$$
\begin{aligned}
\boldsymbol{a}_t &= \boldsymbol{b} + \boldsymbol{W}\boldsymbol{s}_{t-1} + \boldsymbol{U}\boldsymbol{x}_t \\
\boldsymbol{s}_t &= \tanh(\boldsymbol{a}_t) \\
\boldsymbol{o}_t &= \boldsymbol{c} + \boldsymbol{V}\boldsymbol{s}_t \\
\boldsymbol{p}_t &= \mathrm{softmax}(\boldsymbol{o}_t)
\end{aligned}
$$

Character level language model **Vocabulary: [h,e,l,o]**



Training of RNN: BPTT



$\hat{y}_t$: Predicted

y_t : Actual

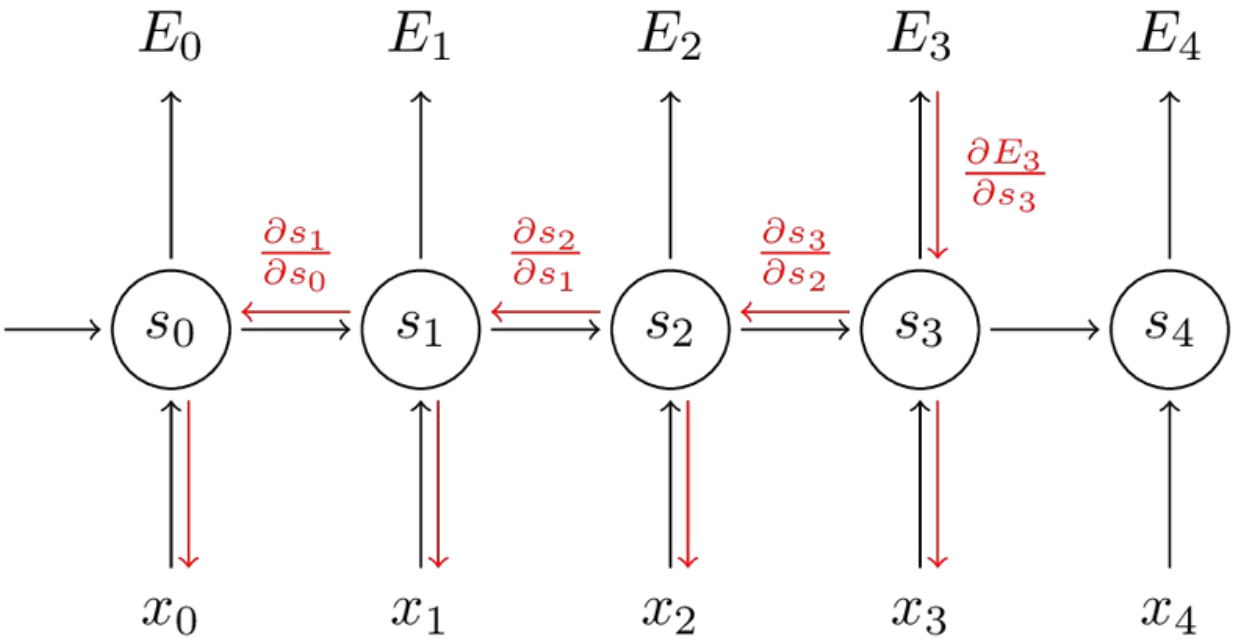
$$\frac{\partial E}{\partial W} = \sum_t \frac{\partial E_t}{\partial W}$$

$$\begin{aligned}\frac{\partial E_3}{\partial V} &= \frac{\partial E_3}{\partial \hat{y}_3}\frac{\partial \hat{y}_3}{\partial V} \\ &= \frac{\partial E_3}{\partial \hat{y}_3}\frac{\partial \hat{y}_3}{\partial z_3}\frac{\partial z_3}{\partial V} \\ &= (\hat{y}_3 - y_3) \otimes s_3\end{aligned}$$

$$\frac{\partial E_3}{\partial W} = \frac{\partial E_3}{\partial \hat{y}_3}\frac{\partial \hat{y}_3}{\partial s_3}\frac{\partial s_3}{\partial W}$$

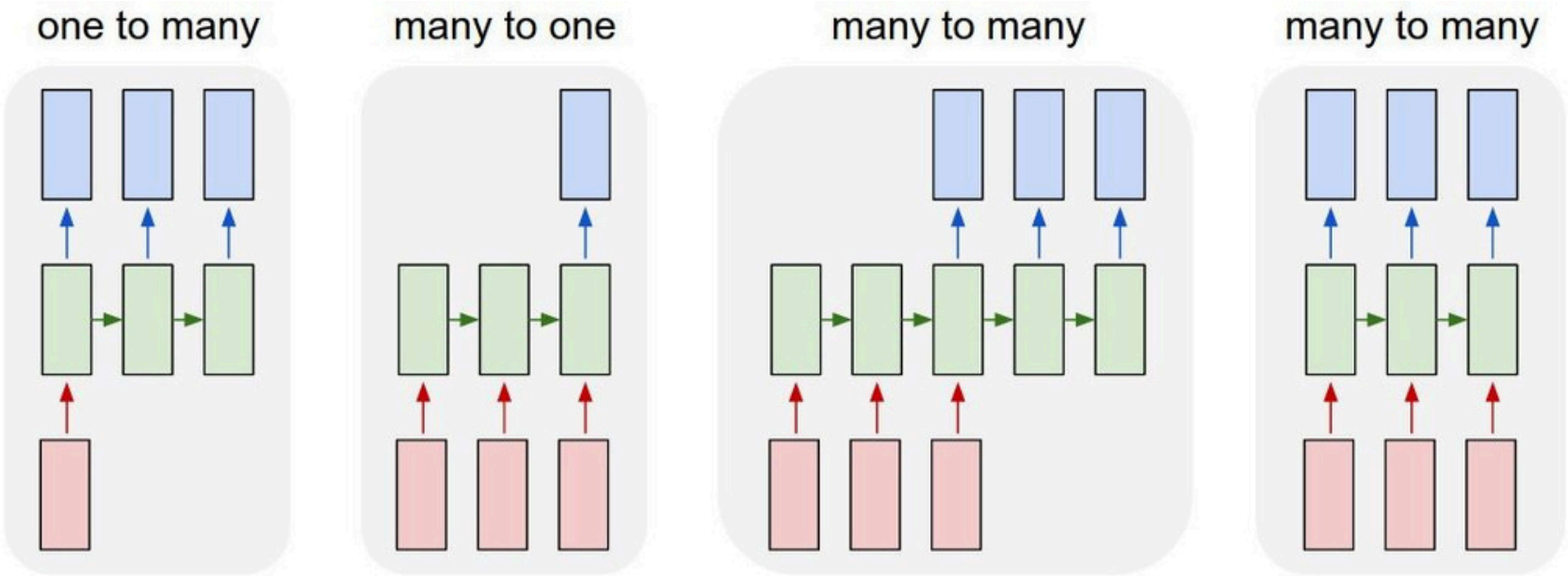
$$\frac{\partial E_3}{\partial W} = \sum_{k=0}^{3} \frac{\partial E_3}{\partial \hat{y}_3}\frac{\partial \hat{y}_3}{\partial s_3}\frac{\partial s_3}{\partial s_k}\frac{\partial s_k}{\partial W}$$

Training of RNN: BPTT



$$\frac{\partial E_3}{\partial W} = \frac{\partial E_3}{\partial \hat{y}_3}\frac{\partial \hat{y}_3}{\partial s_3}\frac{\partial s_3}{\partial W}$$

$$\frac{\partial E_3}{\partial W} = \sum_{k=0}^{3} \frac{\partial E_3}{\partial \hat{y}_3}\frac{\partial \hat{y}_3}{\partial s_3}\frac{\partial s_3}{\partial s_k}\frac{\partial s_k}{\partial W}$$



One to many:

Sequence output (e.g. image captioning takes an image and outputs a sentence of words)

Many to one:

Sequence input (e.g. sentiment analysis where a given sentence is classified as expressing positive or negative sentiment)

Many to many:

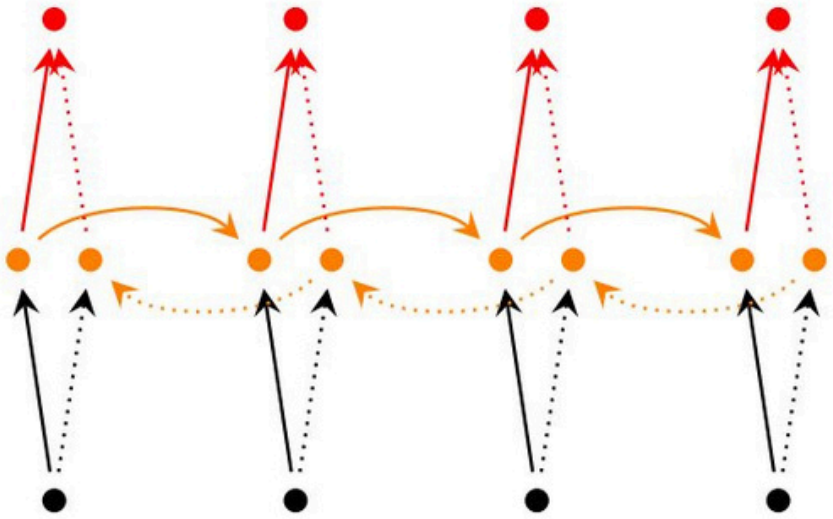
Sequence input and sequence output (e.g. Machine Translation: an RNN reads a sentence in English and then outputs a sentence in French)

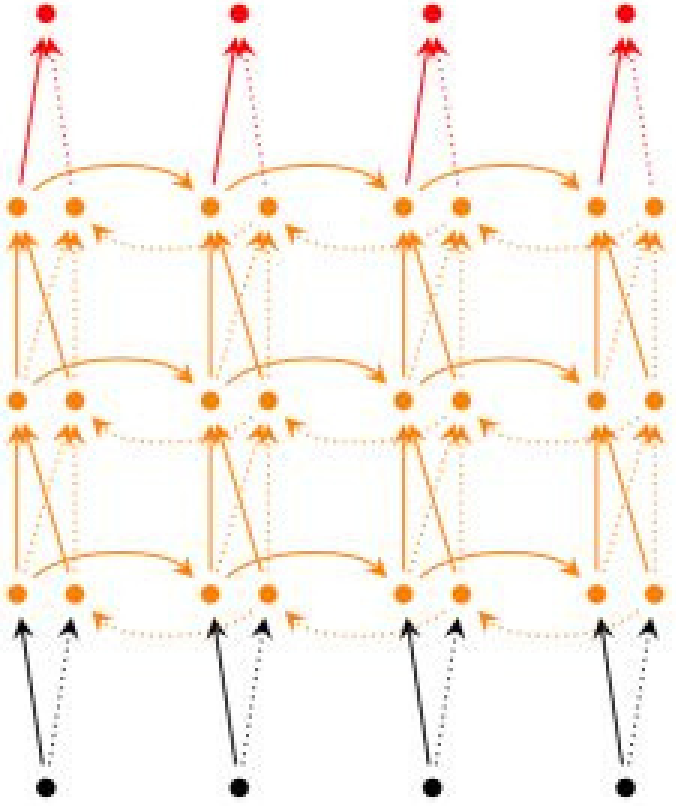
Many to many:

Synced sequence input and output (e.g. Language modelling where we wish to predict next words.

RNN Extensions

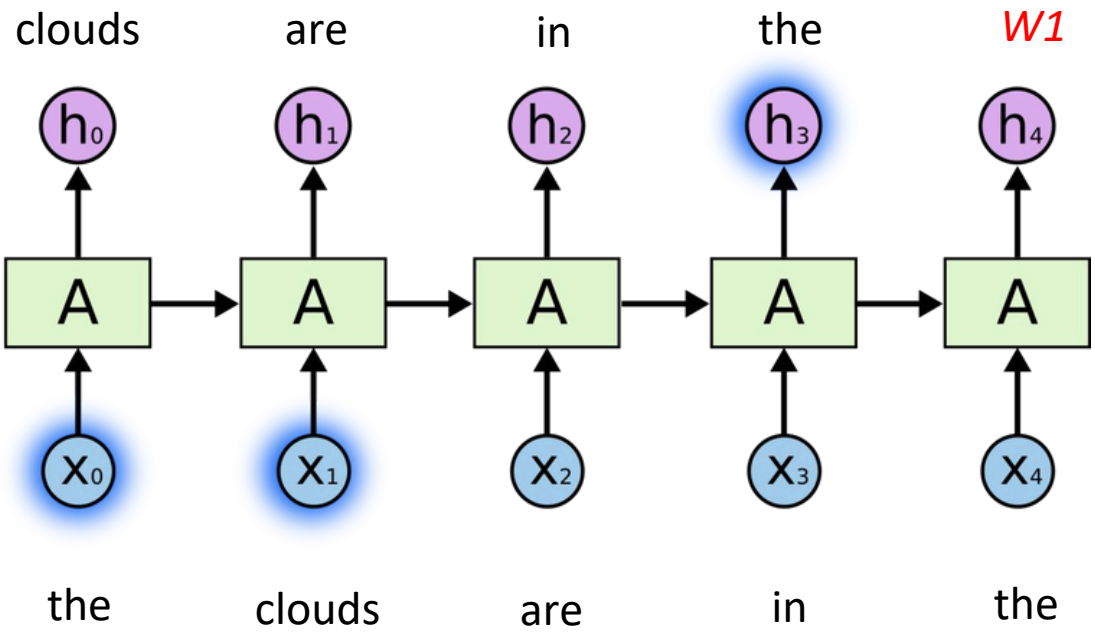
- Bidirectional RNN
- Deep (Bidirectional) RNNs





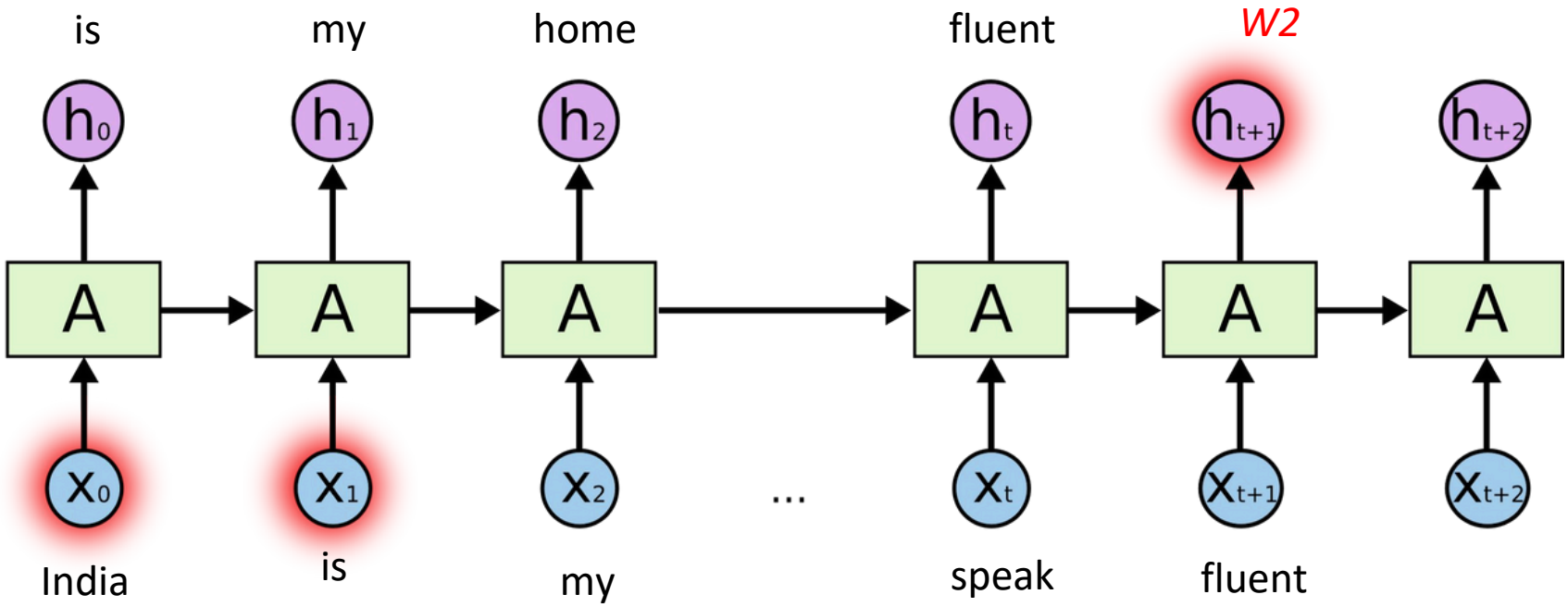
RNN (Cont..)

- “the clouds are in the *sky*”



RNN (Cont..)

- “India is my home country. I can speak fluent *Hindi*.”

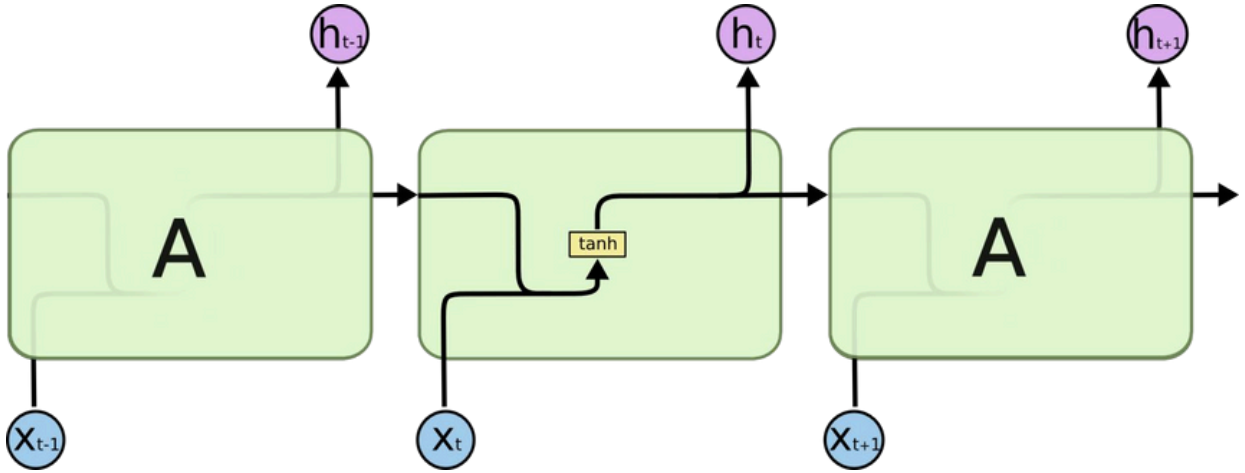


It is very hard for RNN to learn “Long Term Dependency”.

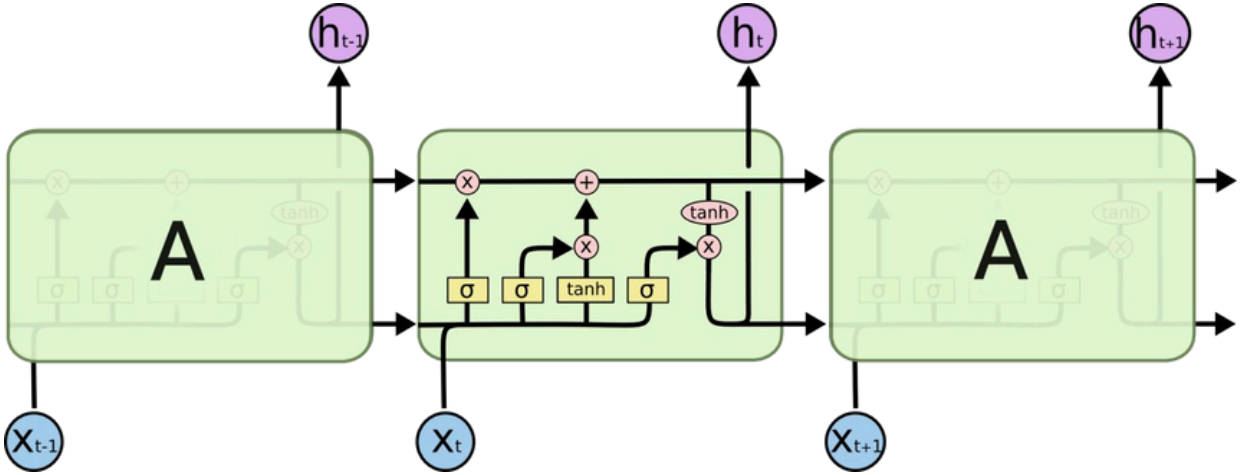
LSTM

- Capable of learning long-term dependencies.

Simple RNN



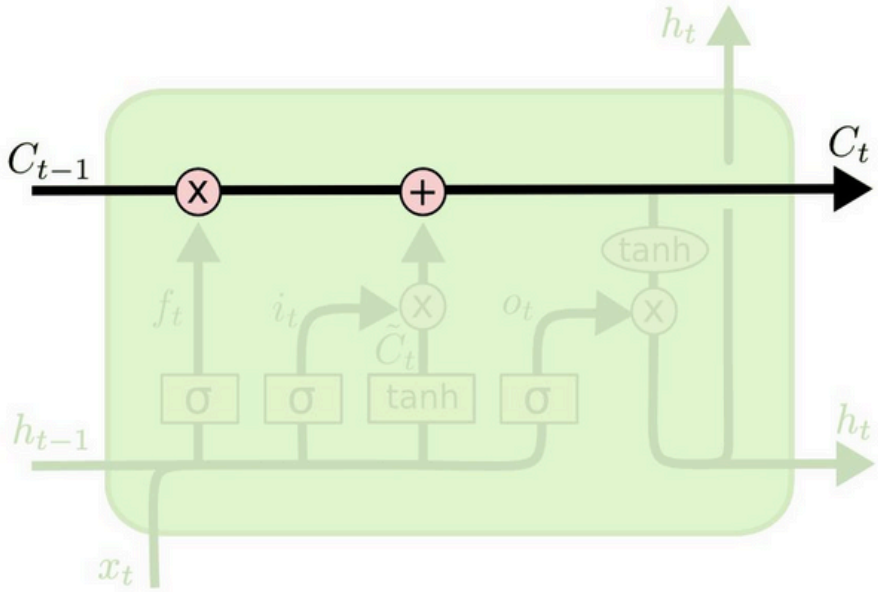
LSTM



LSTM

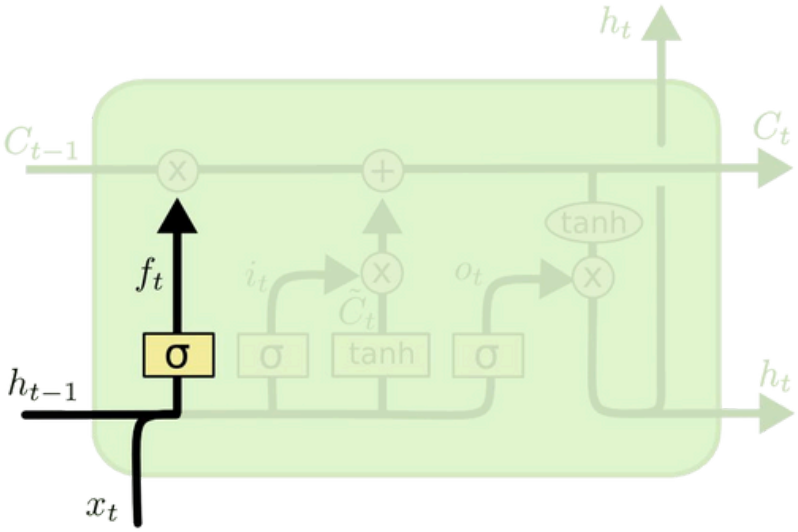
- LSTM remove or add information to the cell state, carefully regulated by structures called gates.

- Cell state: Conveyer belt of the cell



LSTM

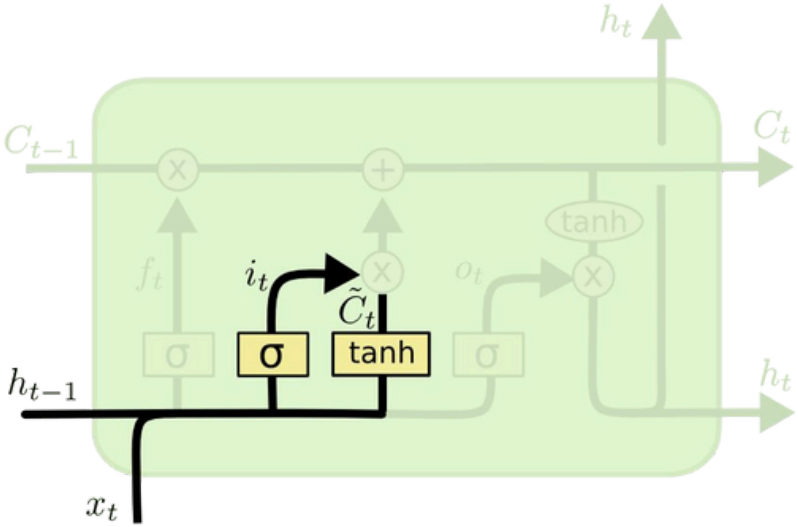
- Gates
  - Forget Gate
  - Input Gate
  - Output Gate



$$f_t = \sigma\left(W_f \cdot [h_{t-1}, x_t] \; + \; b_f\right)$$

LSTM

- Gates
  - Forget Gate
  - Input Gate
  - Output Gate

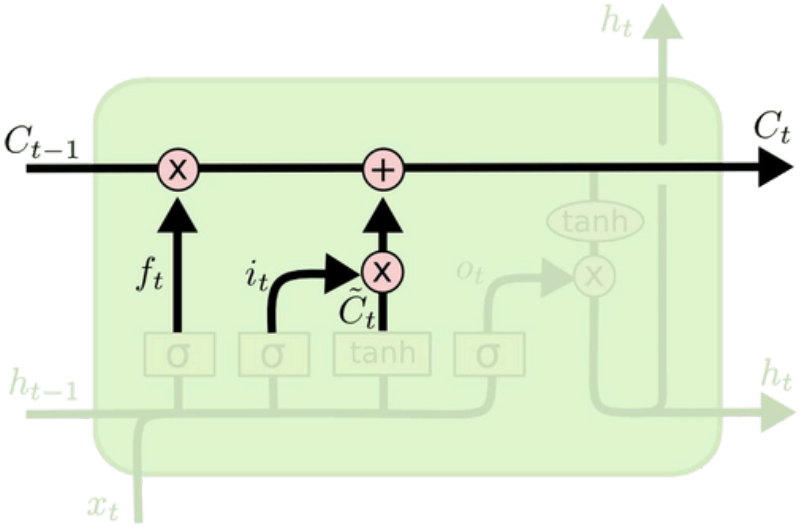


$$i_t = \sigma\left(W_i \cdot [h_{t-1}, x_t] \; + \; b_i\right)$$

$$\tilde{C}_t = \tanh(W_C \cdot [h_{t-1}, x_t] \; + \; b_C)$$

LSTM

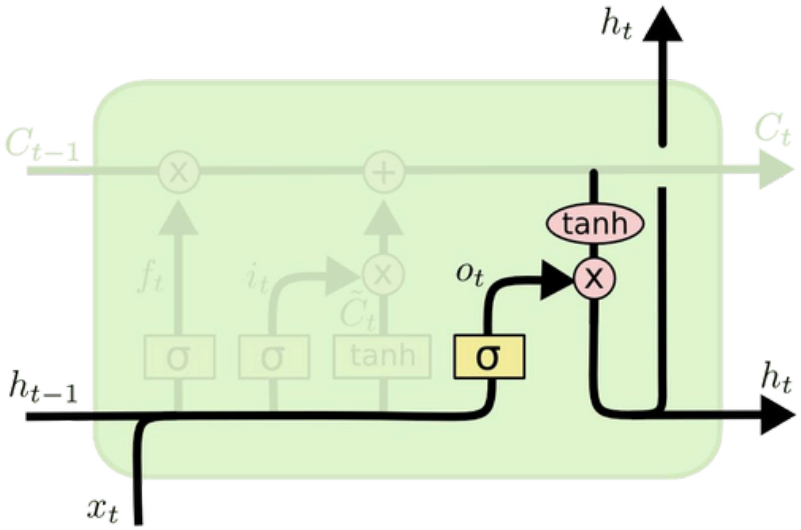
- Gates
  - Forget Gate
  - Input Gate
  - Output Gate



$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

LSTM

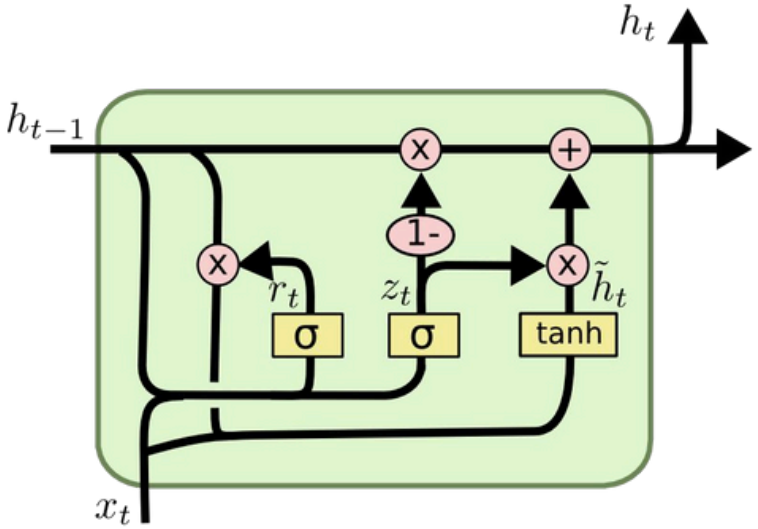
- Gates
  - Forget Gate
  - Input Gate
  - Output Gate



$$o_t = \sigma\left(W_o\left[h_{t-1}, x_t\right] + b_o\right)$$

$$h_t = o_t * \tanh\left(C_t\right)$$

LSTM- Variants

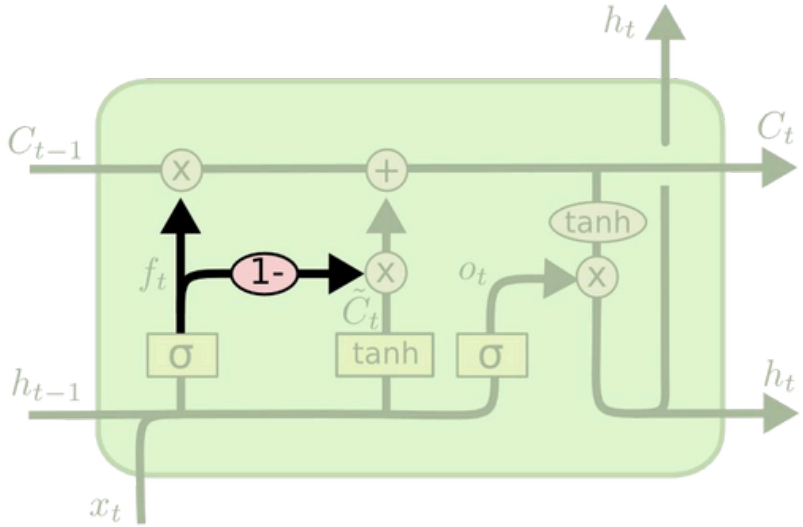


$$z_t = \sigma\left(W_z \cdot [h_{t-1}, x_t]\right)$$

$$r_t = \sigma\left(W_r \cdot [h_{t-1}, x_t]\right)$$

$$\tilde{h}_t = \tanh\left(W \cdot [r_t * h_{t-1}, x_t]\right)$$

$$h_t = (1 - z_t) * h_{t-1} + z_t * \tilde{h}_t$$



$$C_t = f_t * C_{t-1} + (1 - \boldsymbol{f_t}) * \tilde{C}_t$$

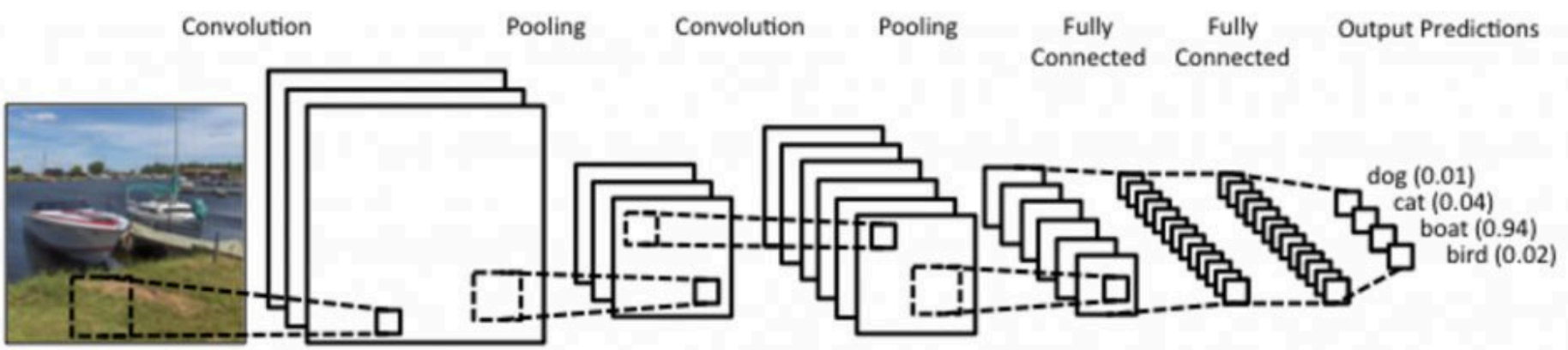
Convolutional Neural Network (CNN)

Convolutional Neural Network (CNN)

- A special kind of multi-layer neural networks.
- Implicitly extract relevant features.
- Fully-connected network architecture does not take into account the spatial structure.
- In contrast, CNN tries to take advantage of the spatial structure.

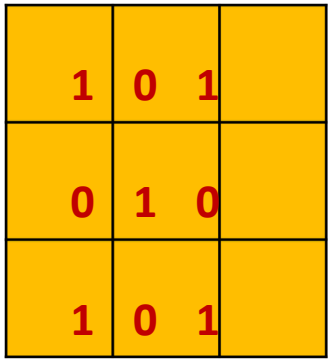
Convolutional Neural Network (CNN)

1. Convolutional layer
2. Pooling layer
3. Fully connected layer



Convolutional Neural Network (CNN)

1. Convolutional layer



Convolution Filter

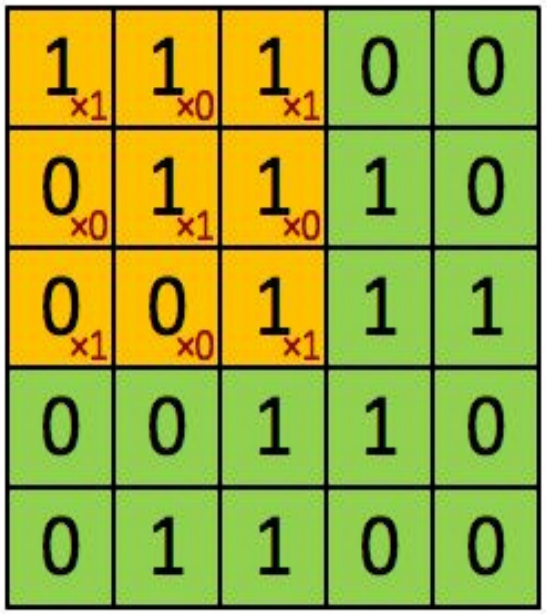
1	1	1	0	0
0	1	1	1	0
0	0	1	1	1
0	0	1	1	0
0	1	1	0	0

Image

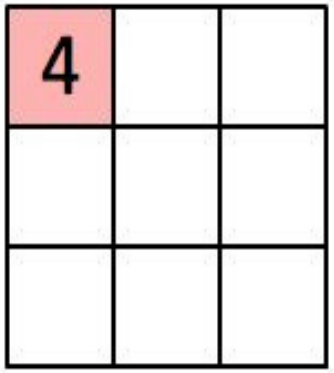
Convolutional Neural Network (CNN)

1. Convolutional layer

1	0	1
0	1	0
1	0	1



Image

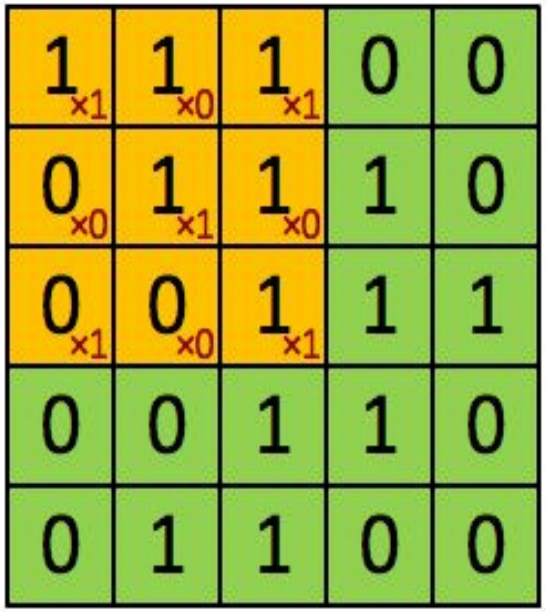


Convolved Feature

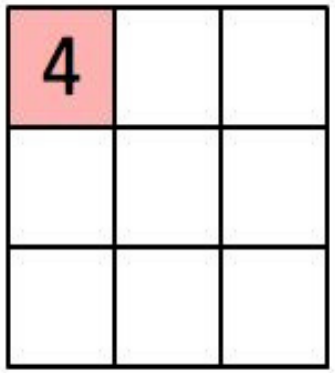
Convolutional Neural Network (CNN)

1. Convolutional layer
   - Local receptive field
   - Shared weights

1	0	1
0	1	0
1	0	1



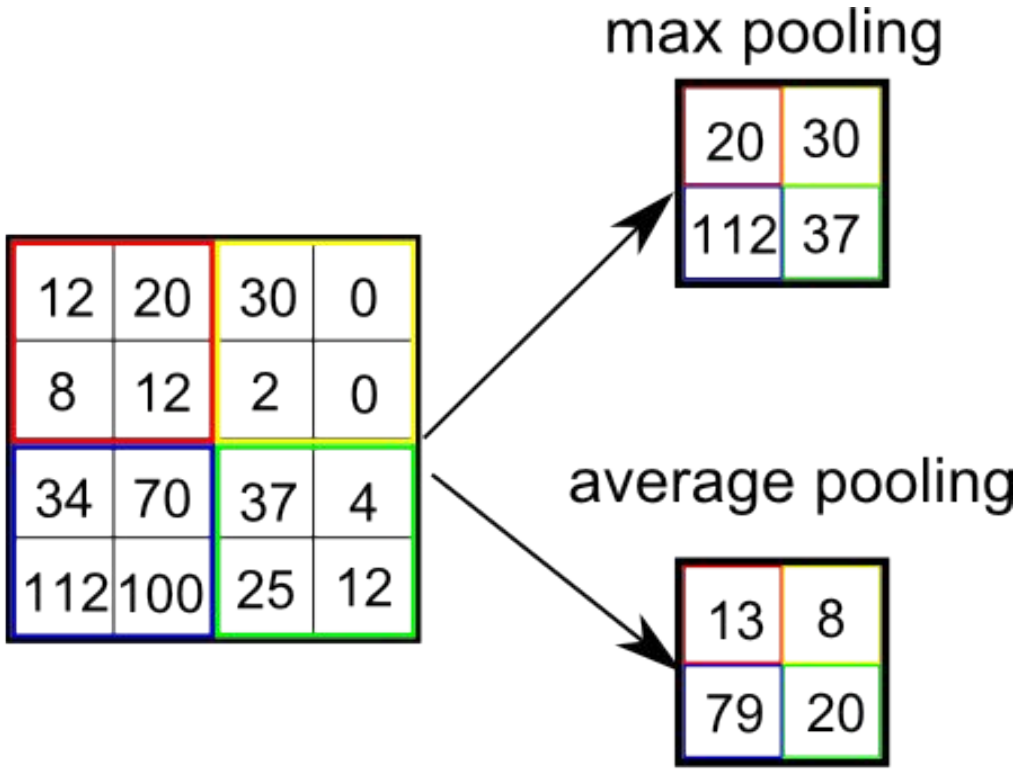
Image



Convolved Feature

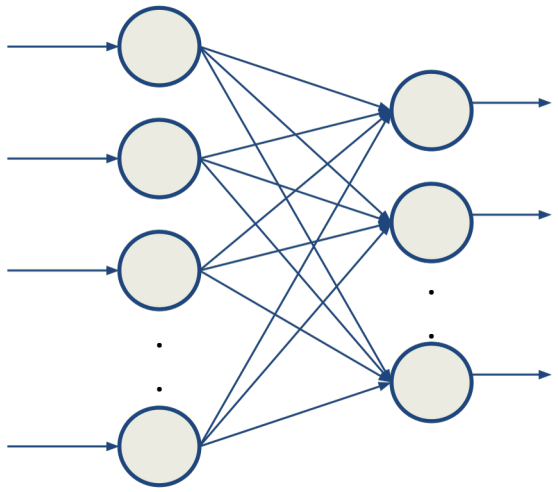
Convolutional Neural Network (CNN)

2. Pooling layer



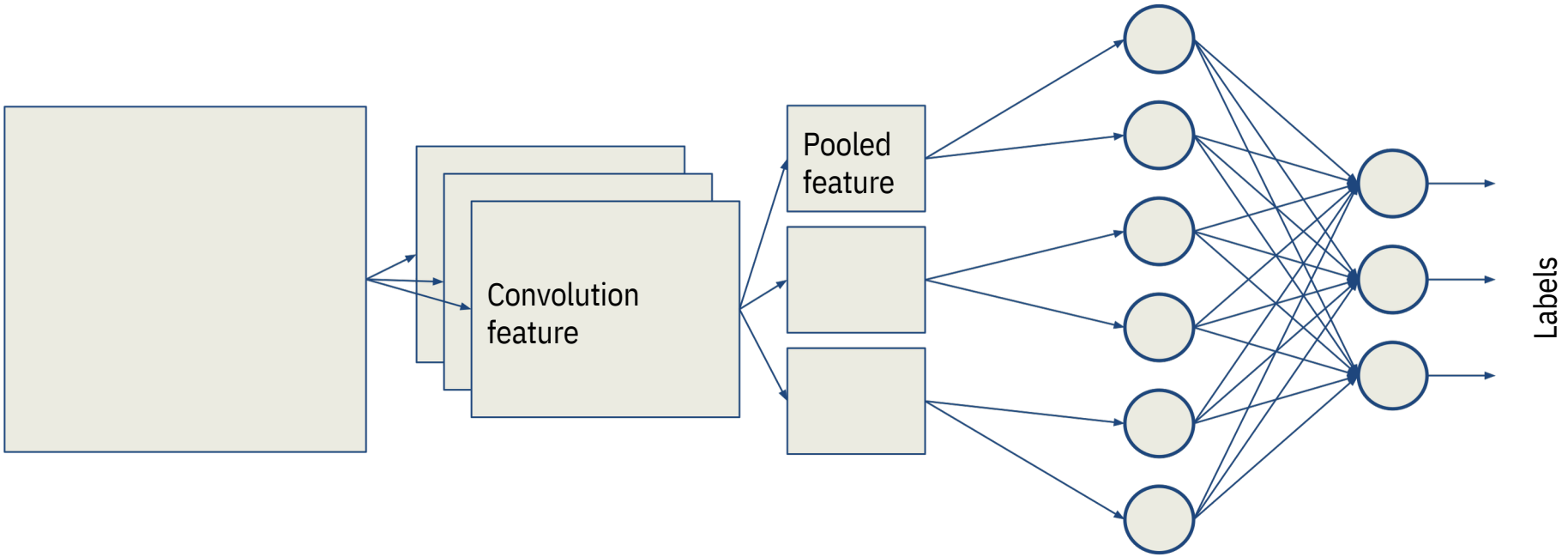
Convolutional Neural Network (CNN)

3. Fully connected layer

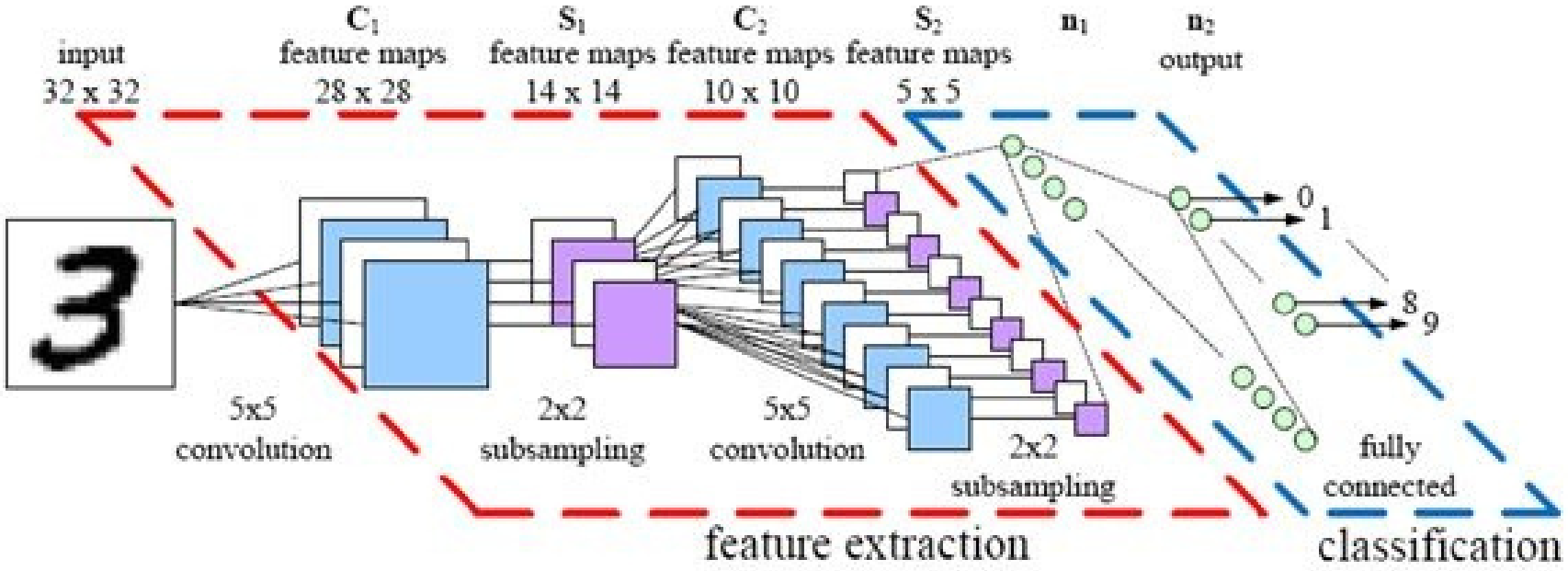


Convolutional Neural Network (CNN)

Putting it all together



Example 1: CNN for Image



Example 2: CNN for Text