Institut des sciences actuariales et financieres

Master 1 Internship

On the range of Admissible Term Structure

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1 Credit default swap (CDS)

In the credit derivative market, the credit default swaps takes a big place. A CDS contains one only underlying, it allowed the market investors to manage dynamically the underlying default risk.

A CDS looks like an interest rate swap, because there is an exchanging floating payment and fixed payment between the buyer and the seller. A CDS carry the advantage of an insurance product witch give to his owner a protection over the underlying default risk .

The CDS contract involve 3 entities : the *buyer*, the *seller* and the *reference entity*. The seller of a CDS will garanty, a recovery $1 - R$ of the nominal, in a period of time T called the *maturity*, if the reference entity fails.

Instead, the buyer of the protection pays a fixed amount s called the *spread*, at a regular and prefixed dates ($t_1, t_2, \ldots, T$)[1], until the default date τ if the default occurs before the maturity T. Else, he will pays the previous amount until T.

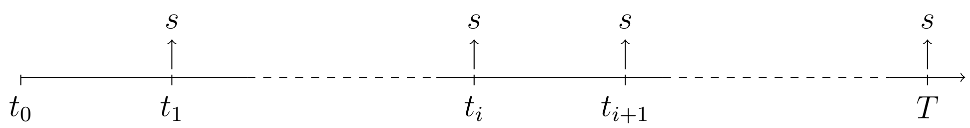


Figure 1: In the case of no default

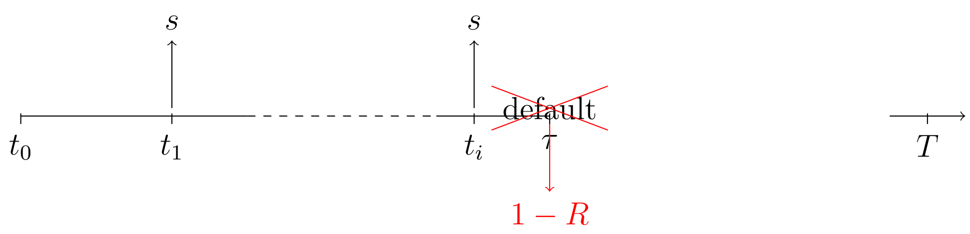


Figure 2: In the case of occuring default ($\tau < T$)

[1]In the market, the spreads are issued evry 3 months ($t_{k+1} - t_k$=3 months)

The floating part payed by the protection seller depends on the default condition of the underlying before the maturity. In the case of default, the seller will refund to the buyer a part R of the nominal, depending on the recovery rate R of the underlying. In the case of no defect, the seller will pay nothing.

The recovery rate will remain unknown until the maturity date. Not easy to estimate, he varies depending on the company.

The CDS's price or the spread is determined at the initial date (t_0) by equalizing the expected value of the two previous cash flows.

Let's specify first some notations :

τ the underlying default date.

R his recovery rate wich is a predictable process of $[0,1]$

T_0 =0 the CDS signature date

T the maturity of the CDS

t_i The payment dates of the buyer where $\Delta t = t_i - t_{i-1}$ are equal $\forall i \in 1, \dots, n$

$\beta(t)$ an index in which $t \in [t_{\beta(t)-1}, t_{\beta(t)}]$.

r the short rate and $D(t,T) = \exp(-\int_t^T r_s ds)$

For the seller, the future cash flow that he will **receive** at $t < T \wedge \tau$:

$$s\left\{(T_{\beta(t)} - t)P^D(t, T_{\beta(t)})\mathbb{1}_{\tau > T_{\beta(t)}} + \sum_{i=\beta(t)+1}^{n} \Delta t P^D(t,t_i)\mathbb{1}_{\tau > t_i} + (\tau - T_{\beta(\tau)-1}P^D(t,\tau)\mathbb{1}_{\tau \le T})\right\}$$

this formula can be approximated by the continuous flow

$$\int_t^T sP^D(t,u)du$$

Instead the seller will pay

$$\mathbb{1}_{\tau \le T} P^D(t,\tau)(1-R)$$

We have then at $t = t_0 = 0$ the following result :

$$s\mathbb{E}_{\mathbb{Q}}\left[\mathbb{1}_{\tau \le T}(1-R)P^D(t_0,\tau)\right] = \mathbb{E}_{\mathbb{Q}}\left[\int_t^T \mathbb{1}_{\tau > u} P^D(t,u)du\right]$$

(1)

where $\mathbb{Q}$ is a free-risk probability.

2 Admissible Term Structures

Term-structure construction consist of finding a function $T \longrightarrow P(t_0, T)$ given a small number market quotes $S_1, ..., S_n$. For a CDS, the AIG market quotes are given for maturities 3, 5 ,7 and 10 years :

maturity (year)	3	5	7	10
CDS spread (bp)	58	54	52	49

Table 1: AIG CDS spread at Dec. 17, 2007

Indeed, We have to rely on interpolation/calibration schemes to construct the curve for the missing maturities. In previous case the curve would be the CDS implied survival probability.
This will lead us to define what will be understood by a good yield curve construction.

2.1 Market fit condition

The Term structure function $T \longrightarrow P(t_0, T)$ is built from market quotes of standard product. Let's define some notations :

n The number of product

S $= (S_1, S_2, \ldots, S_n)$ The set of market quotes at t_0

T $= (T_1, \ldots, T_n)$ the corresponding set of increasing maturities

t $= (t_1, \ldots, t_m)$ payment time grid

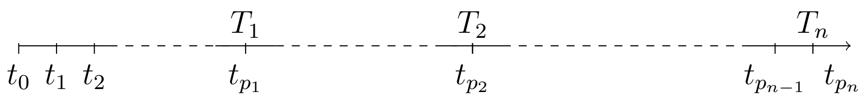


Figure 3: Time grid

Notice that $\forall i,\ T_i = t_{p_i}$.

Let $P = (P^D(t_0, t_1), \ldots, P^D(t_0, t_m))$ be the vector of the values of the curve at the payment dates $t_1, \ldots, t_m$:

The market fit condition can be restated as a rectangular system of linear equations :

$$A \cdot P = B$$

where :

P $=(P^D(t_0, t_1), \ldots, P^D(t_0, t_m))'$

A is a $n \times m$ matrix

B is a $n \times 1$ matrix with positive coefficients

A and B only depend on current market quotes S, on standard maturities T, on payment dates t and on products characteristics.

Example : Credit curve based on CDS for the CDS we can conclude from 1 that under the market fit condition can be expressed as :

$$s_i \sum_{k=1}^{p_i} \delta_k P^D(t_0, t_k) Q(t_0, t_k) - (1-R) P^D(t_0, T) Q(t_0, T)$$
$$+(1-R) \int_{t_0}^{T_i} f^D(t_0, t) P^D(t_0, t) Q(t_0, t) dt \quad = 1 - R, \ i = 1, \ldots, n.$$

where $f^D(t0, u)$ is the instantaneous forward (discount) rate associated with maturity date u.

In order to get an admissible curve $T \longrightarrow P(t_0, T)$ have to fit some others conditions.

2.2 Arbitrage-free conditions

Definition 2.1 (*arbitrage-free condition*)

A credit curve is said to be arbitrage-free if the curve corresponds to a well-defined default distribution function. In other words P had to verify the following conditions :

- $P(t_0, t_0) = 1$
- $T \longmapsto P(t_0, T)$ *is non increasing function (i.e* $\exists x, y, P(t_0, x) < P(t_0, y) \& x > y$*)*

Therefore we can have the following inequalities, called *Arbitrage-free inequalities* :

$$\begin{array}{lr} P(t_0, T_1) \leq (P(t_0, t_k)) \leq 1 & \forall k \in 1, \ldots, p_1 \\ P(t_0, T_i) \leq (P(t_0, t_k)) \leq P(t_0, T_{i-1}) & \forall k \in [p_{i-1}+1, p_i - 1] \end{array}$$

A.Cousin had demonstrate the following proposition :

Proposition 2.1

Assume that, at time t_0, quoted fair spreads $S_1, \dots, S_n$ are reliable for standard CDS maturities $T_1 < \dots < T_n$. For any $i = 1, \dots, n$ the survival probability $Q(t_0, T_i)$ associated with a market-compatible and arbitrage-free credit curve is such that:

$$Q_{min}(t_0, T_i) \leq Q(t_0, T_i) \leq Q_{max}(t_0, T_i)$$

where :

$$\begin{aligned} Q_{max}(t_0, T_i) &= \frac{1 - R - \sum_{k=1}^{i-1}((1-R)M_k + S_i N_k)Q(t_0, T_k)}{P^D(t_0, T_{i-1})(1-R) + S_i(N_i + \delta_{p_i} P^D(t_0, T_i))} \\ Q_{min}(t_0, T_i) &= \frac{1 - R - \sum_{k=1}^{i}((1-R)M_k + S_i N_k)Q(t_0, T_{k-1})}{P^D(t_0, T_i)(1 - R + S_i \delta_{p_i})} \end{aligned}$$

with :

- $p_0 = 1$, $T_0 = t_0$ and $P^D(t_0, T_0) = Q(t_0, t_0) = 1$
- $\forall i \in 1, \dots, n, \;\; M_i = P^D(t_0, T_{i-1}) - P^D(t_0, T_i)$ *and* $N_i = \sum_{k=p_{i-1}}^{p_i - 1} \delta_k P^D(t_0, t_k)$

this bounds can be computed recursively. Knowing that :
Will have a series of decreasing rectangles in wich every cds credit curve should cross.

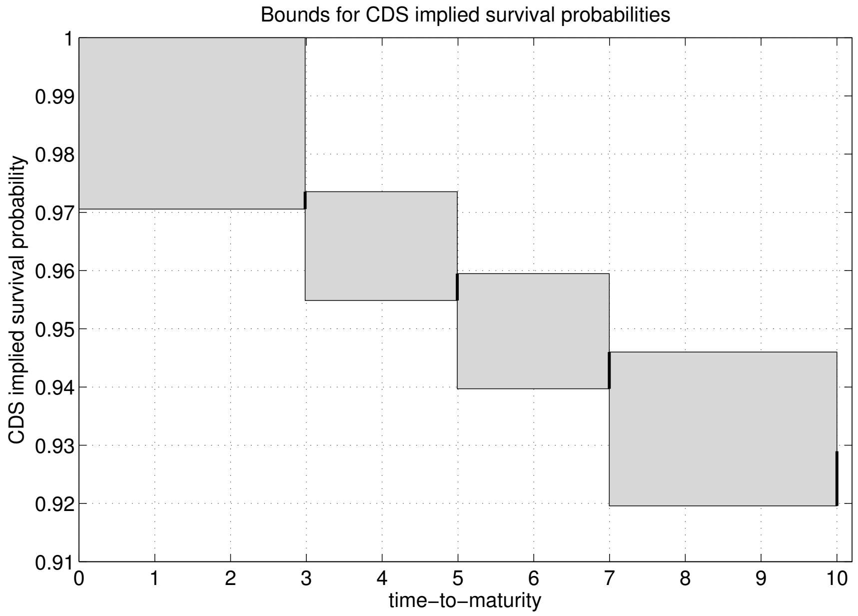


Figure 4: *Union of decreasing rectangles for CDS Spreads : R=40% and* $P^D(t_0, t) = \exp(-3\%(t - t_0))$

2.3 Application on AIG CDS data spreads

If the assumption arbitrage-free is not verified the previous results will not be relevant. Indeed we see this phenomenon when we tried to apply the previous bounds over the data provided by *AIG (France)*. We note that the arbitrage-free inequalities where no longer verified.

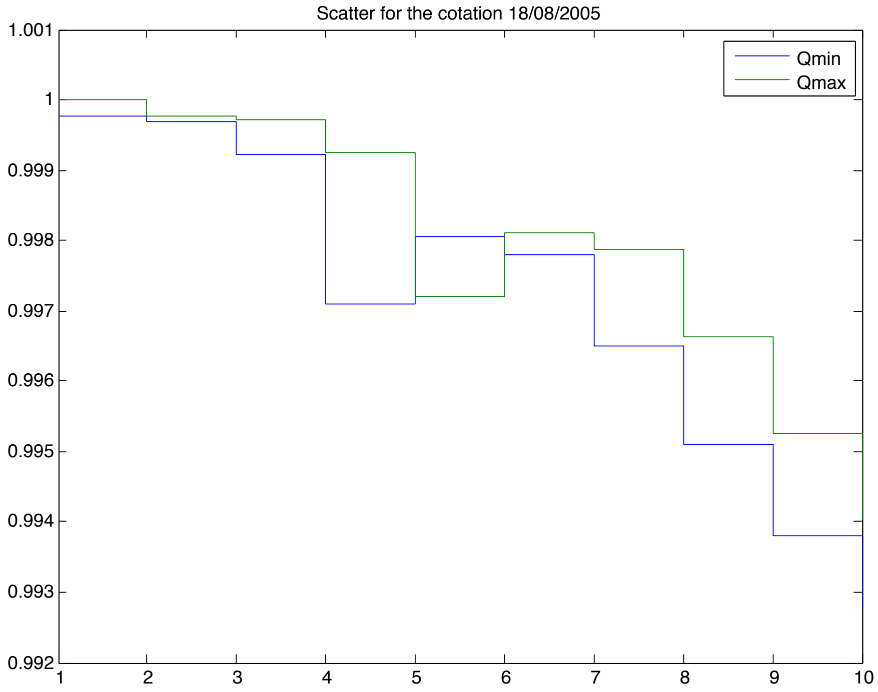


Figure 5: Credit curve among 18/08/2005 - 28/09/2007

It means that the market since 18/08/2005 contained arbitrage opportunities. Indeed between 18/08/2005 - 28/09/2007 the market wasn't complete.

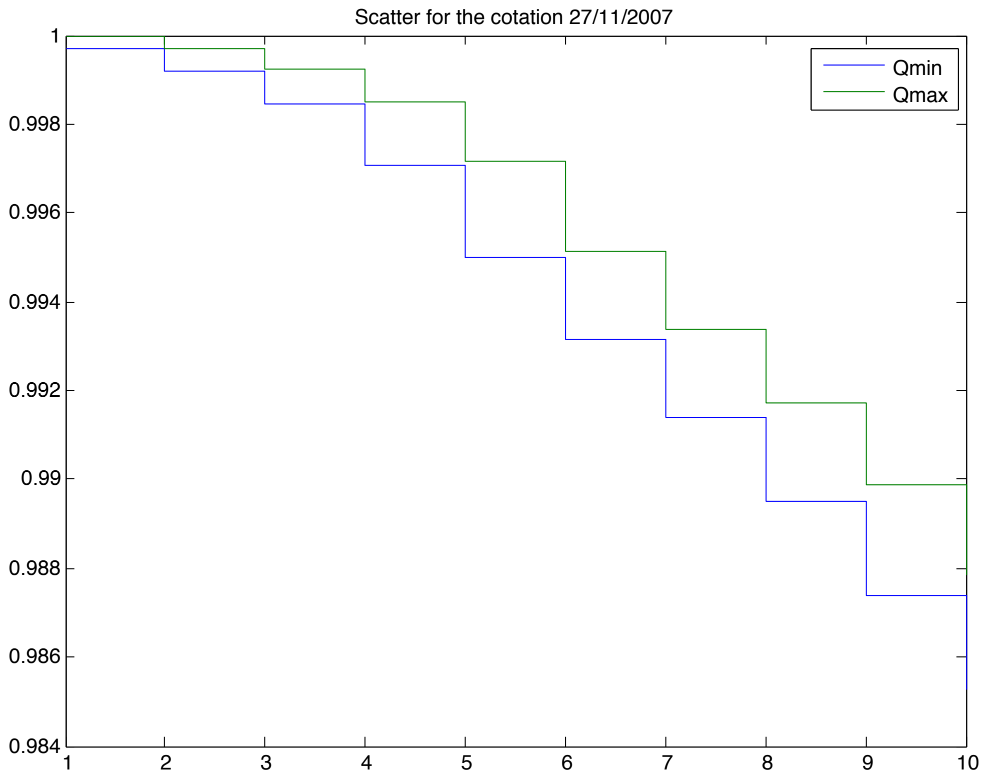


Figure 6: Credit curve since 28/09/2007 : here in 27/11/2007

3 AIG CDS data over 2005 - 2010

3.1 The data dynamic

In this section we will apply the previous results on datas provided by AIG.

In the paper [AC14], A.cousin & I.Niang showed that the CDS market contains no arbitration opportunity if credit curve is a default distribution function witch implied that this function must be a decreasing function.
Among the period 16/08/2005 - 28/09/2007 we can see that the credit curve appears to be non decreasing :

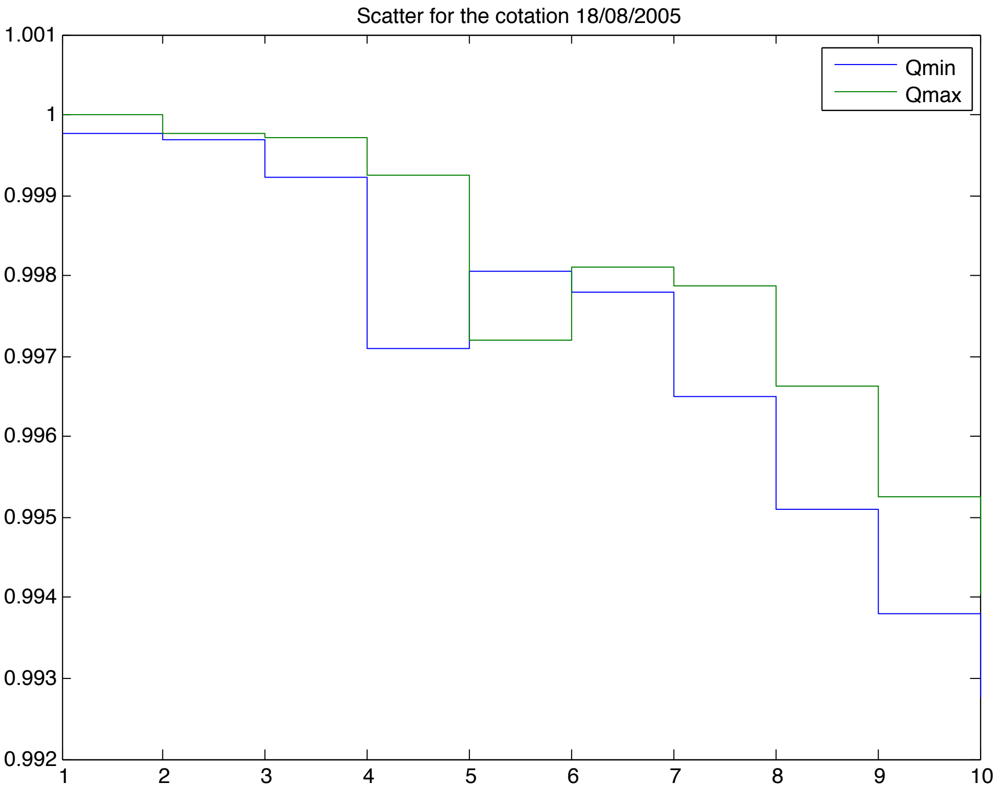


Figure 7: Credit curve among 18/08/2005 - 28/09/2007

Since 28/09/2007 we can see that the credit curve fit well the market condition :

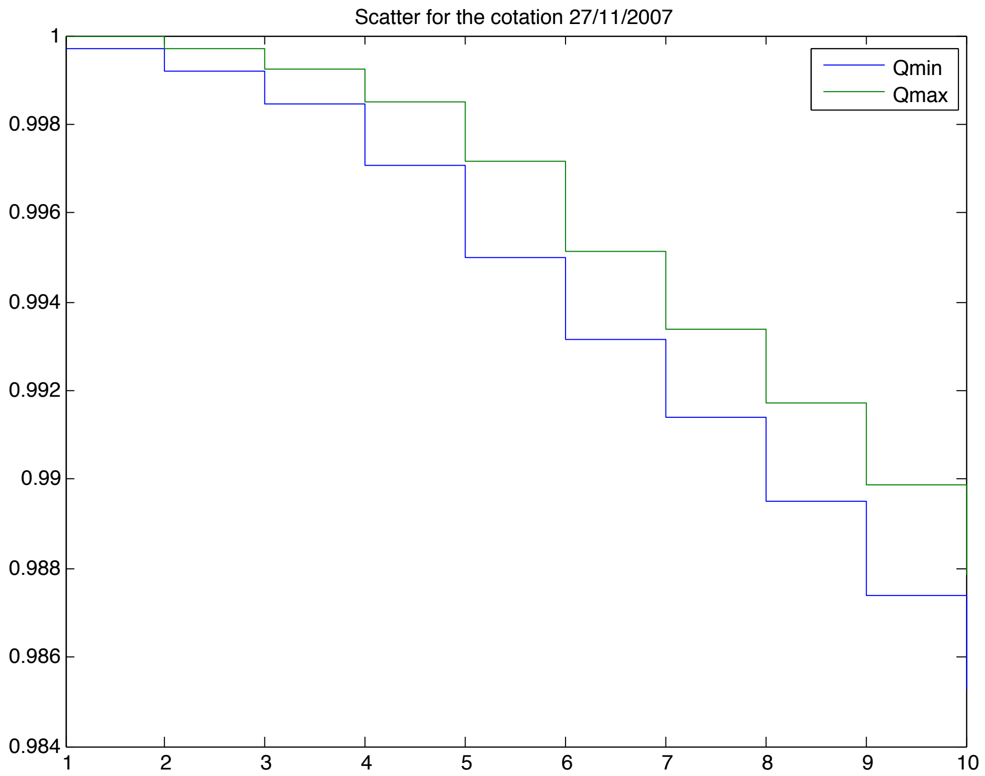
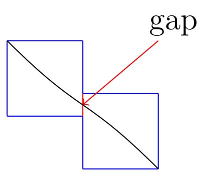


Figure 8: Credit curve since 28/09/2007 : here in 27/11/2007

This is due to the lake of liquidity.
Let's remark that all curves must go through the gap defined at T_i by $Qmax(T_i) - Qmin(T_{i-1})$:



In a crisis period, the company will naturally increase her spreads values, because the risk of illequidity will increase. The company will be more mefiant that of the entities that insure them and will impose a higher spreads value.

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References

[AC14] Ibrahima Niang Areski Cousin. On the range of admissible term structures. *Hal*, 2014.