Institut des sciences actuariales et financieres

Master 1 Internship

On the range of Admissible Term Structure

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1 Introduction

1.1 SAF Lab

The laboratory of actuarial and financial sciences (SAF) EA2429 was established in 1997 in the Institute of financial sciences and insurance (ISFA) [1], formalizing the intern research activities.

The SAF Lab currently has 20 faculty members (witch are in the section 05, 06 and 26 of the CNU[2]) and 20 PhD.

The multidisciplinary SAF Lab research activities, revolve around risk finance and insurance:

- Modeling and risk measurement (probability and statistics)
- Risk management
- Medico-economic analysis and economic risk

In partnership with the IUT University Lyon 1, SAF lab also plans to develop a research on the marketing of banking and insurance services.

The research themes of the lab evolve to incorporate new risks, recent accounting standards (IFRS)[3] and the new prudential regulations (Basel 3, Solvency 2) to a global consideration of all risks to a company or institution, including the environmental dimension.

The main projects of the laboratory involve the Business Risk Management, risks related to the extension of human life and natural hazards. This is to model these risks and dependence, to study their economic, financial and insurance impact, aggregate them in the context of Solvency 2 and Basel 3, and propose a comprehensive debate on the impact of modeling management companies in the financial sector and insurance.

[1] Internal school of the university Lyon 1

[2] CNU(Conseil national des universities) : it is a national authority governed by Decree No. 92-70 of 16 January 1992. It makes decisions on individual measures relating to qualification, recruitment and careers of university professors and lecturers governed by Decree No. 84-431 of 6 June 1984 establishing the statutory provisions applicable to common teacher-researchers and the special status of the body of university professors and lecturers body

[3] International Financial Reporting Standards (IFRS) are designed as a common global language for business affairs so that company accounts are understandable and comparable across international boundaries. They are a consequence of growing international shareholding and trade and are particularly important for companies that have dealings in several countries. They are progressively replacing the many different national accounting standards. The rules to be followed by accountants to maintain books of accounts which is comparable, understandable, reliable and relevant as per the users internal or external.

1.2 Position of the problem

In the heart of risk management and modern asset pricing, the yield curve construction takes his big interest. A term-structure curve describe the evolution of a particular variable (such as interest rate, yield-to-maturity, credit spread, volatility) as a function of time-to-maturity.

Unfortunately, the market quotes are given for a small number of points of the variable. Therefore, the financial industry have to found a method in order to interpolate the credit curve. In other word, it has to supply the rest of the missing information of the curve, and this is the heart of several projects that have been done in the Contrepartie Credit Risk; specially the yield curve construction which constitute the main subject of my internship.

1.3 Mission

A.Cousin & I.Niang presented some interesting results about the term-structure construction in there article [AC14]. My mission was to :

- Develop the bounds presented in the article [AC14] and research some applications of this results.
- refine this bounds specially for CDS.
- Research the impact of market on CDS contracts of the company AIG.
- Research the impact of CVA on the CDS contracts of the company AIG.

I will first present some general notions, that was new for me, present briefly the results of A.Cousin & I.Niang and finally present the applications of this work.

2 Credit default swap (CDS)

In the credit derivative market, the credit default swaps takes a big place. A CDS contains one only underlying, it allowed the market investors to manage dynamically the underlying default risk.

A CDS looks like an interest rate swap, because there is an exchanging floating payment and fixed payment between the buyer and the seller. A CDS carry the advantage of an insurance product witch give to his owner a protection over the underlying default risk .

the buyer of the protection pays a fixed amount s called the *spread*, at a regular and pre-fixed dates ($t_1, t_2, \ldots, T$)[4], until the default date τ if the default occurs before the maturity T. Else, he will pays the previous amount until T.

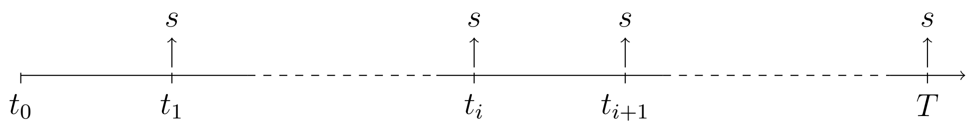

Figure 1: In the case of no default

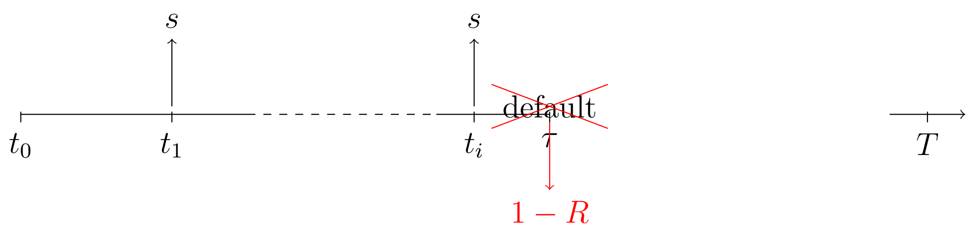

Figure 2: In the case of occuring default ($\tau < T$)

The floating part payed by the protection seller depends on the default condition of the underlying before the maturity. In the case of default, the seller will refund to the buyer a part R of the nominal, depending on the recovery rate R of the underlying. In the case of no defect, the seller will pay nothing.

The recovery rate will remain unknown until the maturity date. Not easy to estimate, he varies depending on the company.

The CDS's price or the spread is determined at the initial date (t_0) by equalizing the expected value of the two previous cash flows.

[4]In the market, the spreads are issued evry 3 months ($t_{k+1} - t_k$=3 months)

Let's specify first some notations :

τ the underlying default date.

R his recovery rate wich is a predictable process of $[0, 1]$

T_0 =0 the CDS signature date

T the maturity of the CDS

t_i The payment dates of the buyer where $\Delta t = t_i - t_{i-1}$ are equal $\forall i \in 1, \ldots, n$

$\beta(t)$ an index in which $t \in [t_{\beta(t)-1}, t_{\beta(t)}]$.

$\mathbf{r}$ the short rate and $D(t,T) = \exp(-\int_t^T r_s ds)$

For the seller, the future cash flow that he will **receive** at $t < T \wedge \tau$:

$$s \left\{ (T_{\beta(t)} - t) P^D(t, T_{\beta(t)}) \mathbb{1}_{\tau > T_{\beta(t)}} + \sum_{i=\beta(t)+1}^{n} \Delta t P^D(t, t_i) \mathbb{1}_{\tau > t_i} + (\tau - T_{\beta(\tau)-1} P^D(t, \tau) \mathbb{1}_{\tau \leq T}) \right\}$$

this formula can be approximated by the continuous flow

$$\int_t^T s P^D(t, u) du$$

Instead the seller will pay

$$\mathbb{1}_{\tau \leq T} P^D(t, \tau)(1 - R)$$

We have then at $t = t_0 = 0$ the following result :

$$s\mathbb{E}_{\mathbb{Q}} \left[\mathbb{1}_{\tau \leq T}(1 - R) P^D(t_0, \tau) \right] = \mathbb{E}_{\mathbb{Q}} \left[\int_t^T \mathbb{1}_{\tau > u} P^D(t, u) du \right]$$

(1)

where $\mathbb{Q}$ is a free-risk probability.

3 Admissible Term Structures

Term-structure construction consist of finding a function $T \longrightarrow P(t_0, T)$ given a small number market quotes $S_1, ..., S_n$. For a CDS, the AIG market quotes are given for maturities 3, 5 ,7 and 10 years :

maturity (year)	3	5	7	10
CDS spread (bp)	58	54	52	49

Table 1: AIG CDS spread at Dec. 17, 2007

Indeed, We have to rely on interpolation/calibration schemes to construct the curve for the missing maturities. In previous case the curve would be the CDS implied survival probability.
This will lead us to define what will be understood by a good yield curve construction.

3.1 Market fit condition

The Term structure function $T \longrightarrow P(t_0, T)$ is built from market quotes of standard product. Let's define some notations :

n The number of product

S $= (S_1, S_2, \ldots, S_n)$ The set of market quotes at t_0

T $= (T_1, \ldots, T_n)$ the corresponding set of increasing maturities

t $= (t_1, \ldots, t_m)$ payment time grid

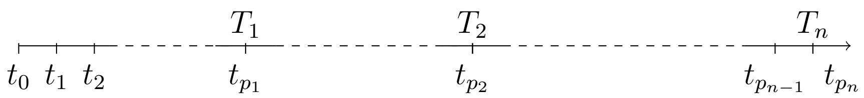

Figure 3: Time grid

Notice that $\forall i,\ T_i = t_{p_i}$.

Let $P = (P^D(t_0, t_1), \ldots, P^D(t_0, t_m))$ be the vector of the values of the curve at the payment dates $t_1, \ldots, t_m$:

The market fit condition can be restated as a rectangular system of linear equations :

$$A \cdot P = B$$

where :

P $=(P^D(t_0, t_1), \ldots, P^D(t_0, t_m))'$

A is a $n \times m$ matrix

B is a $n \times 1$ matrix with positive coefficients

A and B only depend on current market quotes S, on standard maturities T, on payment dates t and on products characteristics.

Example : credit curve based on CDS for the CDS we can conclude from 2 that under the market fit condition can be expressed as :

$$s_i \sum_{k=1}^{p_i} \delta_k P^D(t_0, t_k) Q(t_0, t_k) - (1-R) P^D(t_0, T) Q(t_0, T)$$
$$+(1-R) \int_{t_0}^{T_i} f^D(t_0, t) P^D(t_0, t) Q(t_0, t) dt \quad = 1 - R, \ i = 1, \dots, n.$$

where $f^D(t0, u)$ is the instantaneous forward (discount) rate associated with maturity date u.

In order to get an admissible curve $T \longrightarrow P(t_0, T)$ have to fit some others conditions.

3.2 Arbitrage-free conditions

Definition 3.1 (*arbitrage-free condition*)

A credit curve is said to be arbitrage-free if the curve corresponds to a well-defined default distribution function. In other words P had to verify the following conditions :

- $P(t_0, t_0) = 1$
- $T \longmapsto P(t_0, T)$ *is non increasing function (i.e* $\exists x, y, P(t_0, x) < P(t_0, y) \& x > y$*)*

Therefore we can have the following inequalities, called *Arbitrage-free inequalities* :

$$\begin{array}{lr} P(t_0, T_1) \leq (P(t_0, t_k)) \leq 1 & \forall k \in 1, \dots, p_1 \\ P(t_0, T_i) \leq (P(t_0, t_k)) \leq P(t_0, T_{i-1}) & \forall k \in [p_{i-1}+1, p_i - 1] \end{array}$$

A.Cousin had demonstrate the following proposition :

Proposition 3.1

Assume that, at time t_0, quoted fair spreads $S_1, \dots, S_n$ are reliable for standard CDS maturities $T_1 < \dots < T_n$. For any $i = 1, \dots, n$ the survival probability $Q(t_0, T_i)$ associated with a market-compatible and arbitrage-free credit curve is such that:

$$Q_{min}(t_0, T_i) \leq Q(t_0, T_i) \leq Q_{max}(t_0, T_i)$$

where :

$$\begin{aligned} Q_{max}(t_0, T_i) &= \frac{1 - R - \sum_{k=1}^{i-1}((1-R)M_k + S_i N_k)Q(t_0, T_k)}{P^D(t_0, T_{i-1})(1-R) + S_i(N_i + \delta_{p_i} P^D(t_0, T_i))} \\ Q_{min}(t_0, T_i) &= \frac{1 - R - \sum_{k=1}^{i}((1-R)M_k + S_i N_k)Q(t_0, T_{k-1})}{P^D(t_0, T_i)(1 - R + S_i \delta_{p_i})} \end{aligned}$$

with :

- $p_0 = 1$, $T_0 = t_0$ and $P^D(t_0, T_0) = Q(t_0, t_0) = 1$
- $\forall i \in 1, \dots, n$, $M_i = P^D(t_0, T_{i-1}) - P^D(t_0, T_i)$ *and* $N_i = \displaystyle\sum_{k=p_{i-1}}^{p_i - 1} \delta_k P^D(t_0, t_k)$

this bounds can be computed recursively. Will have a series of decreasing rectangles in wich every cds credit curve should cross.

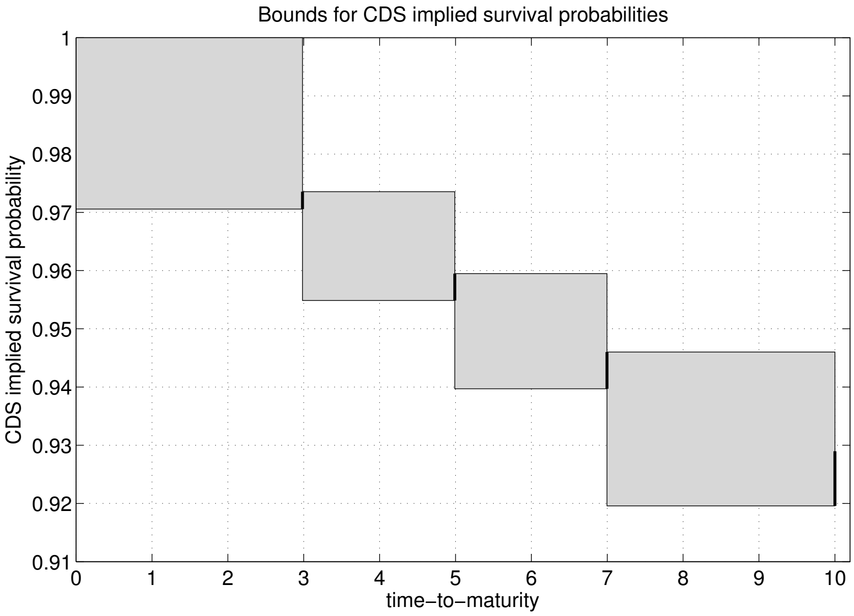

Figure 4: *Union of decreasing rectangles for CDS Spreads : R=40% and* $P^D(t_0, t) = \exp(-3\%(t - t_0))$

3.3 Application on AIG CDS data spreads

If the assumption arbitrage-free is not verified the previous results will not be relevant. Indeed we see this phenomenon when we tried to apply the previous bounds over the data provided by *AIG (France)*. We note that the arbitrage-free inequalities where no longer verified.

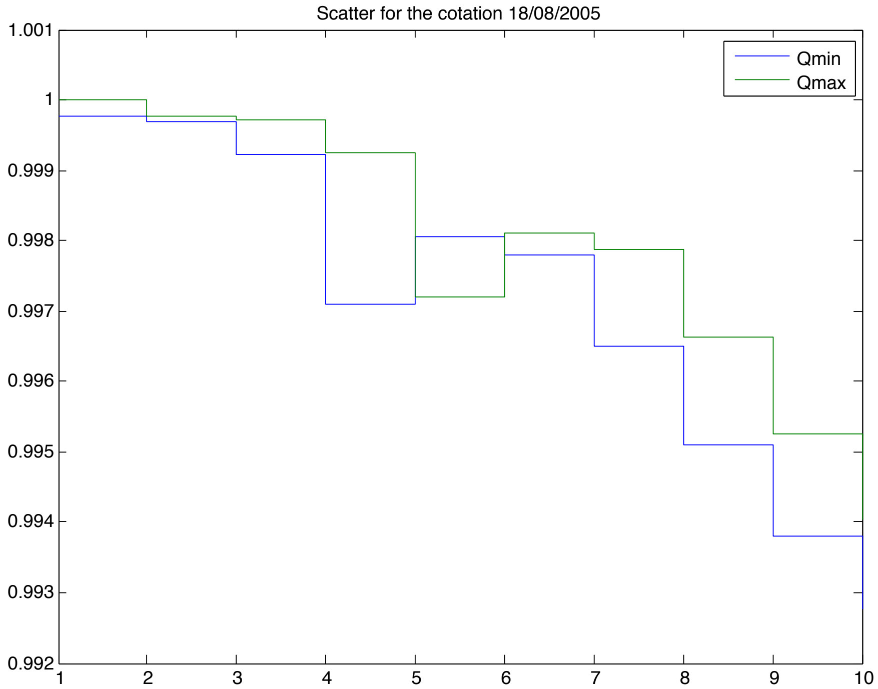

Figure 5: Credit curve among 18/08/2005 - 28/09/2007

It means that the market since 18/08/2005 contained arbitrage opportunities. Indeed between 18/08/2005 - 28/09/2007 the market wasn't complete.

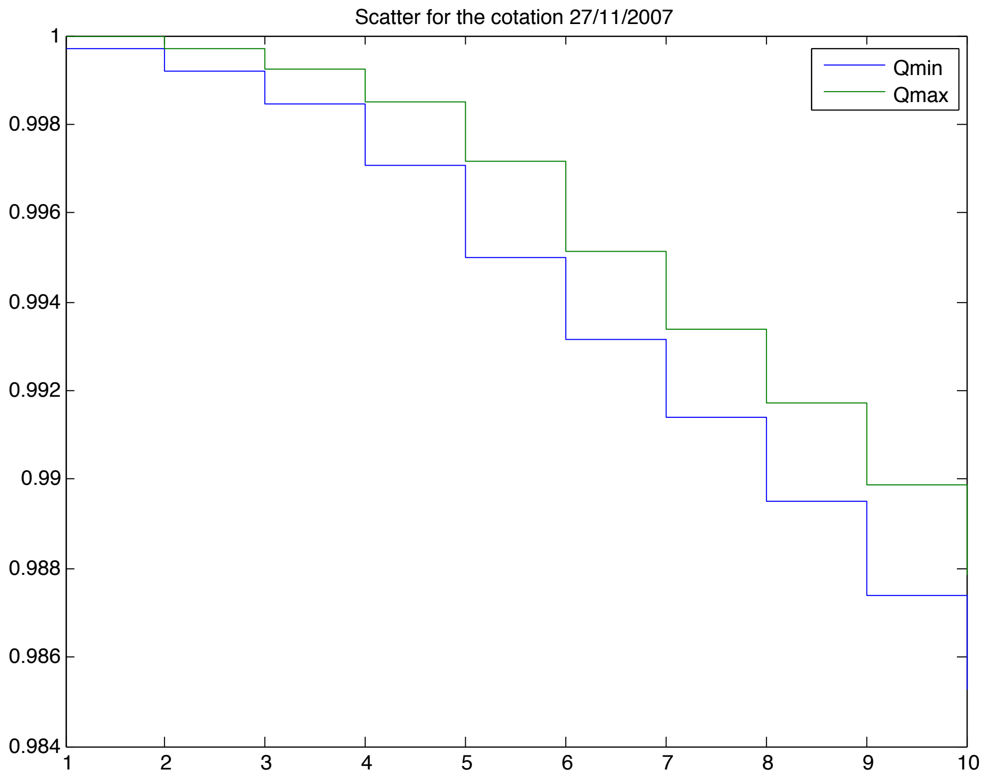

Figure 6: Credit curve since 28/09/2007 : here in 27/11/2007

4 My following work

The applications of the previous notion seems to be numerous and quite interesting. Two other applications that I'm still working on them :

- The impact of 2008 crisis on the CDS contracts
- The impact of CDS contracts on CVA
- Refine the bounds presented on the previous section

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References

[AC14] Ibrahima Niang Areski Cousin. On the range of admissible term structures. *Hal*, 2014.