SESSION TOPIC: STOCK MARKET PRICE BEHAVIOR

SESSION CHAIRMAN: BURTON G. MALKIEL

EFFICIENT CAPITAL MARKETS: A REVIEW OF THEORY AND EMPIRICAL WORK*

EUGENE F. FAMA**

I. INTRODUCTION

THE PRIMARY ROLE of the capital market is allocation of ownership of the economy's capital stock. In general terms, the ideal is a market in which prices provide accurate signals for resource allocation: that is, a market in which firms can make production-investment decisions, and investors can choose among the securities that represent ownership of firms' activities under the assumption that security prices at any time "fully reflect" all available information. A market in which prices always "fully reflect" available information is called "efficient."

This paper reviews the theoretical and empirical literature on the efficient markets model. After a discussion of the theory, empirical work concerned with the adjustment of security prices to three relevant information subsets is considered. First, *weak form* tests, in which the information set is just historical prices, are discussed. Then *semi-strong form* tests, in which the concern is whether prices efficiently adjust to other information that is obviously publicly available (e.g., announcements of annual earnings, stock splits, etc.) are considered. Finally, *strong form* tests concerned with whether given investors or groups have monopolistic access to any information relevant for price formation are reviewed.[1] We shall conclude that, with but a few exceptions, the efficient markets model stands up well.

Though we proceed from theory to empirical work, to keep the proper historical perspective we should note to a large extent the empirical work in this area preceded the development of the theory. The theory is presented first here in order to more easily judge which of the empirical results are most relevant from the viewpoint of the theory. The empirical work itself, however, will then be reviewed in more or less historical sequence.

Finally, the perceptive reader will surely recognize instances in this paper where relevant studies are not specifically discussed. In such cases my apologies should be taken for granted. The area is so bountiful that some such injustices are unavoidable. But the primary goal here will have been accomplished if a coherent picture of the main lines of the work on efficient markets is presented, along with an accurate picture of the current state of the arts.

* Research on this project was supported by a grant from the National Science Foundation. I am indebted to Arthur Laffer, Robert Aliber, Ray Ball, Michael Jensen, James Lorie, Merton Miller, Charles Nelson, Richard Roll, William Taylor, and Ross Watts for their helpful comments.

** University of Chicago—Joint Session with the Econometric Society.

1. The distinction between weak and strong form tests was first suggested by Harry Roberts.

II. The Theory of Efficient Markets

A. *Expected Return or "Fair Game" Models*

The definitional statement that in an efficient market prices "fully reflect" available information is so general that it has no empirically testable implications. To make the model testable, the process of price formation must be specified in more detail. In essence we must define somewhat more exactly what is meant by the term "fully reflect."

One possibility would be to posit that equilibrium prices (or expected returns) on securities are generated as in the "two parameter" Sharpe [40]-Lintner [24, 25] world. In general, however, the theoretical models and especially the empirical tests of capital market efficiency have not been this specific. Most of the available work is based only on the assumption that the conditions of market equilibrium can (somehow) be stated in terms of expected returns. In general terms, like the two parameter model such theories would posit that conditional on some relevant information set, the equilibrium expected return on a security is a function of its "risk." And different theories would differ primarily in how "risk" is defined.

All members of the class of such "expected return theories" can, however, be described notationally as follows:

$$E(\tilde{p}_{j,t+1}|\Phi_t) = [1 + E(\tilde{r}_{j,t+1}|\Phi_t)]p_{jt}, \tag{1}$$

where E is the expected value operator; p_{jt} is the price of security j at time t; $p_{j,t+1}$ is its price at $t+1$ (with reinvestment of any intermediate cash income from the security); $r_{j,t+1}$ is the one-period percentage return $(p_{j,t+1} - p_{jt})/p_{jt}$; Φ_t is a general symbol for whatever set of information is assumed to be "fully reflected" in the price at t; and the tildes indicate that $p_{j,t+1}$ and $r_{j,t+1}$ are random variables at t.

The value of the equilibrium expected return $E(\tilde{r}_{j,t+1}|\Phi_t)$ projected on the basis of the information Φ_t would be determined from the particular expected return theory at hand. The conditional expectation notation of (1) is meant to imply, however, that whatever expected return model is assumed to apply, the information in Φ_t is fully utilized in determining equilibrium expected returns. And this is the sense in which Φ_t is "fully reflected" in the formation of the price p_{jt}.

But we should note right off that, simple as it is, the assumption that the conditions of market equilibrium can be stated in terms of expected returns elevates the purely mathematical concept of expected value to a status not necessarily implied by the general notion of market efficiency. The expected value is just one of many possible summary measures of a distribution of returns, and market efficiency per se (i.e., the general notion that prices "fully reflect" available information) does not imbue it with any special importance. Thus, the results of tests based on this assumption depend to some extent on its validity as well as on the efficiency of the market. But some such assumption is the unavoidable price one must pay to give the theory of efficient markets empirical content.

The assumptions that the conditions of market equilibrium can be stated in terms of expected returns and that equilibrium expected returns are formed on the basis of (and thus "fully reflect") the information set Φ_t have a major empirical implication—they rule out the possibility of trading systems based only on information in Φ_t that have expected profits or returns in excess of equilibrium expected profits or returns. Thus let

$$x_{j,t+1} = p_{j,t+1} - E(p_{j,t+1}|\Phi_t). \tag{2}$$

Then

$$E(\tilde{x}_{j,t+1}|\Phi_t) = 0 \tag{3}$$

which, *by definition,* says that the sequence $\{x_{jt}\}$ is a "fair game" with respect to the information sequence $\{\phi_t\}$. Or, equivalently, let

$$z_{j,t+1} = r_{j,t+1} - E(\tilde{r}_{j,t+1}|\Phi_t), \tag{4}$$

then

$$E(\tilde{z}_{j,t+1}|\Phi_t) = 0, \tag{5}$$

so that the sequence $\{z_{jt}\}$ is also a "fair game" with respect to the information sequence $\{\Phi\}$.

In economic terms, $x_{j,t+1}$ is the excess market value of security j at time $t+1$: it is the difference between the observed price and the expected value of the price that was projected at t on the basis of the information Φ_t. And similarly, $z_{j,t+1}$ is the return at $t+1$ in excess of the equilibrium expected return projected at t. Let

$$\alpha(\Phi_t) = [\alpha_1(\Phi_t), \alpha_2(\Phi_t), \ldots, \alpha_n(\Phi_t)]$$

be any trading system based on Φ_t which tells the investor the amounts $\alpha_j(\Phi_t)$ of funds available at t that are to be invested in each of the n available securities. The total excess market value at $t+1$ that will be generated by such a system is

$$V_{t+1} = \sum_{j=1}^{n} \alpha_j(\Phi_t)[r_{j,t+1} - E(\tilde{r}_{j,t+1}|\Phi_t)],$$

which, from the "fair game" property of (5) has expectation,

$$E(\tilde{V}_{t+1}|\Phi_t) = \sum_{j=1}^{n} \alpha_j(\Phi_t)E(\tilde{z}_{j,t+1}|\Phi_t) = 0.$$

The expected return or "fair game" efficient markets model[2] has other important testable implications, but these are better saved for the later discussion of the empirical work. Now we turn to two special cases of the model, the submartingale and the random walk, that (as we shall see later) play an important role in the empirical literature.

2. Though we shall sometimes refer to the model summarized by (1) as the "fair game" model, keep in mind that the "fair game" properties of the model are *implications* of the assumptions that (i) the conditions of market equilibrium can be stated in terms of expected returns, and (ii) the information Φ_t is fully utilized by the market in forming equilibrium expected returns and thus current prices.

The role of "fair game" models in the theory of efficient markets was first recognized and studied rigorously by Mandelbrot [27] and Samuelson [38]. Their work will be discussed in more detail later.

B. *The Submartingale Model*

Suppose we assume in (1) that for all t and Φ_t

$$E(\tilde{p}_{j,t+1}|\Phi_t) \geqslant p_{jt}, \quad \text{or equivalently,} \quad E(\tilde{r}_{j,t+1}|\Phi_t) \geqslant 0. \tag{6}$$

This is a statement that the price sequence $\{p_{jt}\}$ for security j follows a submartingale with respect to the information sequence $\{\Phi_t\}$, which is to say nothing more than that the expected value of next period's price, as projected on the basis of the information Φ_t, is equal to or greater than the current price. If (6) holds as an equality (so that expected returns and price changes are zero), then the price sequence follows a martingale.

A submartingale in prices has one important empirical implication. Consider the set of "one security and cash" mechanical trading rules by which we mean systems that concentrate on individual securities and that define the conditions under which the investor would hold a given security, sell it short, or simply hold cash at any time t. Then the assumption of (6) that expected returns conditional on Φ_t are non-negative directly implies that such trading rules based only on the information in Φ_t cannot have greater expected profits than a policy of always buying-and-holding the security during the future period in question. Tests of such rules will be an important part of the empirical evidence on the efficient markets model.[3]

C. *The Random Walk Model*

In the early treatments of the efficient markets model, the statement that the current price of a security "fully reflects" available information was assumed to imply that successive price changes (or more usually, successive one-period returns) are independent. In addition, it was usually assumed that successive changes (or returns) are identically distributed. Together the two hypotheses constitute the random walk model. Formally, the model says

$$f(r_{j,t+1}|\Phi_t) = f(r_{j,t+1}), \tag{7}$$

which is the usual statement that the conditional and marginal probability distributions of an independent random variable are identical. In addition, the density function f must be the same for all t.[4]

3. Note that the expected profitability of "one security and cash" trading systems vis-à-vis buy-and-hold is not ruled out by the general expected return or "fair game" efficient markets model. The latter rules out systems with expected profits in excess of equilibrium expected returns, but since in principle it allows equilibrium expected returns to be negative, holding cash (which always has zero actual and thus expected return) may have higher expected return than holding some security.

And negative equilibrium expected returns for some securities are quite possible. For example, in the Sharpe [40]-Lintner [24, 25] model (which is in turn a natural extension of the portfolio models of Markowitz [30] and Tobin [43]) the equilibrium expected return on a security depends on the extent to which the dispersion in the security's return distribution is related to dispersion in the returns on all other securities. A security whose returns on average move opposite to the general market is particularly valuable in reducing dispersion of portfolio returns, and so its equilibrium expected return may well be negative.

4. The terminology is loose. Prices will only follow a random walk if price changes are independent, identically distributed; and even then we should say "random walk with drift" since expected price changes can be non-zero. If one-period returns are independent, identically distributed, prices will not follow a random walk since the distribution of price changes will depend

Expression (7) of course says much more than the general expected return model summarized by (1). For example, if we restrict (1) by assuming that the expected return on security j is constant over time, then we have

$$E(\tilde{r}_{j,t+1}|\Phi_t) = E(\tilde{r}_{j,t+1}). \tag{8}$$

This says that the mean of the distribution of $r_{j,t+1}$ is independent of the information available at t, Φ_t, whereas the random walk model of (7) in addition says that the entire distribution is independent of Φ_t.[5]

We argue later that it is best to regard the random walk model as an extension of the general expected return or "fair game" efficient markets model in the sense of making a more detailed statement about the economic environment. The "fair game" model just says that the conditions of market equilibrium can be stated in terms of expected returns, and thus it says little about the details of the stochastic process generating returns. A random walk arises within the context of such a model when the environment is (fortuitously) such that the evolution of investor tastes and the process generating new information combine to produce equilibria in which return distributions repeat themselves through time.

Thus it is not surprising that empirical tests of the "random walk" model that are in fact tests of "fair game" properties are more strongly in support of the model than tests of the additional (and, from the viewpoint of expected return market efficiency, superfluous) pure independence assumption. (But it is perhaps equally surprising that, as we shall soon see, the evidence against the independence of returns over time is as weak as it is.)

D. *Market Conditions Consistent with Efficiency*

Before turning to the empirical work, however, a few words about the market conditions that might help or hinder efficient adjustment of prices to information are in order. First, it is easy to determine *sufficient* conditions for capital market efficiency. For example, consider a market in which (i) there are no transactions costs in trading securities, (ii) all available information is costlessly available to all market participants, and (iii) all agree on the implications of current information for the current price and distributions of future prices of each security. In such a market, the current price of a security obviously "fully reflects" all available information.

But a frictionless market in which all information is freely available and investors agree on its implications is, of course, not descriptive of markets met in practice. Fortunately, these conditions are sufficient for market efficiency, but not necessary. For example, as long as transactors take account of all

on the price level. But though rigorous terminology is usually desirable, our loose use of terms should not cause confusion; and our usage follows that of the efficient markets literature.

Note also that in the random walk literature, the information set Φ_t in (7) is usually assumed to include only the past return history, $r_{j,t}, r_{j,t-1}, \ldots$

5. The random walk model does not say, however, that past information is of no value in *assessing* distributions of future returns. Indeed since return distributions are assumed to be stationary through time, past returns are the best source of such information. The random walk model does say, however, that the *sequence* (or the order) of the past returns is of no consequence in assessing distributions of future returns.

available information, even large transactions costs that inhibit the flow of transactions do not in themselves imply that when transactions do take place, prices will not "fully reflect" available information. Similarly (and speaking, as above, somewhat loosely), the market may be efficient if "sufficient numbers" of investors have ready access to available information. And disagreement among investors about the implications of given information does not in itself imply market inefficiency unless there are investors who can consistently make better evaluations of available information than are implicit in market prices.

But though transactions costs, information that is not freely available to all investors, and disagreement among investors about the implications of given information are not necessarily sources of market inefficiency, they are potential sources. And all three exist to some extent in real world markets. Measuring their effects on the process of price formation is, of course, the major goal of empirical work in this area.

III. The Evidence

All the empirical research on the theory of efficient markets has been concerned with whether prices "fully reflect" particular subsets of available information. Historically, the empirical work evolved more or less as follows. The initial studies were concerned with what we call *weak form* tests in which the information subset of interest is just past price (or return) histories. Most of the results here come from the random walk literature. When extensive tests seemed to support the efficiency hypothesis at this level, attention was turned to *semi-strong form* tests in which the concern is the speed of price adjustment to other obviously publicly available information (e.g., announcements of stock splits, annual reports, new security issues, etc.). Finally, *strong form* tests in which the concern is whether any investor or groups (e.g., managements of mutual funds) have monopolistic access to any information relevant for the formation of prices have recently appeared. We review the empirical research in more or less this historical sequence.

First, however, we should note that what we have called *the* efficient markets model in the discussions of earlier sections is the hypothesis that security prices at any point in time "fully reflect" *all* available information. Though we shall argue that the model stands up rather well to the data, it is obviously an extreme null hypothesis. And, like any other extreme null hyposthesis, we do not expect it to be literally true. The categorization of the tests into weak, semi-strong, and strong form will serve the useful purpose of allowing us to pinpoint the level of information at which the hypothesis breaks down. And we shall contend that there is no important evidence against the hypothesis in the weak and semi-strong form tests (i.e., prices seem to efficiently adjust to obviously publicly available information), and only limited evidence against the hypothesis in the strong form tests (i.e., monopolistic access to information about prices does not seem to be a prevalent phenomenon in the investment community).

A. *Weak Form Tests of the Efficient Markets Model*

1. Random Walks and Fair Games: A Little Historical Background

As noted earlier, all of the empirical work on efficient markets can be considered within the context of the general expected return or "fair game" model, and much of the evidence bears directly on the special submartingale expected return model of (6). Indeed, in the early literature, discussions of the efficient markets model were phrased in terms of the even more special random walk model, though we shall argue that most of the early authors were in fact concerned with more general versions of the "fair game" model.

Some of the confusion in the early random walk writings is understandable. Research on security prices did not begin with the development of a theory of price formation which was then subjected to empirical tests. Rather, the impetus for the development of a theory came from the accumulation of evidence in the middle 1950's and early 1960's that the behavior of common stock and other speculative prices could be well approximated by a random walk. Faced with the evidence, economists felt compelled to offer some rationalization. What resulted was a theory of efficient markets stated in terms of random walks, but usually implying some more general "fair game" model.

It was not until the work of Samuelson [38] and Mandelbrot [27] in 1965 and 1966 that the role of "fair game" expected return models in the theory of efficient markets and the relationships between these models and the theory of random walks were rigorously studied.[6] And these papers came somewhat after the major empirical work on random walks. In the earlier work, "theoretical" discussions, though usually intuitively appealing, were always lacking in rigor and often either vague or *ad hoc*. In short, until the Mandelbrot-Samuelson models appeared, there existed a large body of empirical results in search of a rigorous theory.

Thus, though his contributions were ignored for sixty years, the first statement and test of the random walk model was that of Bachelier [3] in 1900. But his "fundamental principle" for the behavior of prices was that speculation should be a "fair game"; in particular, the expected profits to the speculator should be zero. With the benefit of the modern theory of stochastic processes, we know now that the process implied by this fundamental principle is a martingale.

After Bachelier, research on the behavior of security prices lagged until the

6. Basing their analyses on futures contracts in commodity markets, Mandelbrot and Samuelson show that if the price of such a contract at time t is the expected value at t (given information Φ_t) of the spot price at the termination of the contract, then the futures price will follow a martingale with respect to the information sequence $\{\Phi_t\}$; that is, the expected price change from period to period will be zero, and the price changes will be a "fair game." If the equilibrium expected return is not assumed to be zero, our more general "fair game" model, summarized by (1), is obtained.

But though the Mandelbrot-Samuelson approach certainly illuminates the process of price formation in commodity markets, we have seen that "fair game" expected return models can be derived in much simpler fashion. In particular, (1) is just a formalization of the assumptions that the conditions of market equilibrium can be stated in terms of expected returns and that the information Φ_t is used in forming market prices at t.

coming of the computer. In 1953 Kendall [21] examined the behavior of weekly changes in nineteen indices of British industrial share prices and in spot prices for cotton (New York) and wheat (Chicago). After extensive analysis of serial correlations, he suggests, in quite graphic terms:

> The series looks like a wandering one, almost as if once a week the Demon of Chance drew a random number from a symetrical population of fixed dispersion and added it to the current price to determine the next week's price [21, p. 13].

Kendall's conclusion had in fact been suggested earlier by Working [47], though his suggestion lacked the force provided by Kendall's empirical results. And the implications of the conclusion for stock market research and financial analysis were later underlined by Roberts [36].

But the suggestion by Kendall, Working, and Roberts that series of speculative prices may be well described by random walks was based on observation. None of these authors attempted to provide much economic rationale for the hypothesis, and, indeed, Kendall felt that economists would generally reject it. Osborne [33] suggested market conditions, similar to those assumed by Bachelier, that would lead to a random walk. But in his model, independence of successive price changes derives from the assumption that the decisions of investors in an individual security are independent from transaction to transaction—which is little in the way of an economic model.

Whenever economists (prior to Mandelbrot and Samuelson) tried to provide economic justification for the random walk, their arguments usually implied a "fair game." For example, Alexander [8, p. 200] states:

> If one were to start out with the assumption that a stock or commodity speculation is a "fair game" with equal expectation of gain or loss or, more accurately, with an expectation of zero gain, one would be well on the way to picturing the behavior of speculative prices as a random walk.

There is an awareness here that the "fair game" assumption is not sufficient to lead to a random walk, but Alexander never expands on the comment. Similarly, Cootner [8, p. 232] states:

> If any substantial group of buyers thought prices were too low, their buying would force up the prices. The reverse would be true for sellers. Except for appreciation due to earnings retention, the conditional expectation of tomorrow's price, given today's price, is today's price.
>
> In such a world, the only price changes that would occur are those that result from new information. Since there is no reason to expect that information to be non-random in appearance, the period-to-period price changes of a stock should be random movements, statistically independent of one another.

Though somewhat imprecise, the last sentence of the first paragraph seems to point to a "fair game" model rather than a random walk.[7] In this light, the second paragraph can be viewed as an attempt to describe environmental conditions that would reduce a "fair game" to a random walk. But the specification imposed on the information generating process is insufficient for this purpose; one would, for example, also have to say something about investor

7. The appropriate conditioning statement would be "Given the sequence of historical prices."

tastes. Finally, lest I be accused of criticizing others too severely for ambiguity, lack of rigor and incorrect conclusions,

> By contrast, the stock market trader has a much more practical criterion for judging what constitutes important dependence in successive price changes. For his purposes the random walk model is valid as long as knowledge of the past behavior of the series of price changes cannot be used to increase expected gains. More specifically, the independence assumption is an adequate description of reality as long as the actual degree of dependence in the series of price changes is not sufficient to allow the past history of the series to be used to predict the future in a way which makes expected profits greater than they would be under a naive buy-and hold model [10, p 35].

We know now, of course, that this last condition hardly requires a random walk. It will in fact be met by the submartingale model of (6).

But one should not be too hard on the theoretical efforts of the early empirical random walk literature. The arguments were usually appealing; where they fell short was in awareness of developments in the theory of stochastic processes. Moreover, we shall now see that most of the empirical evidence in the random walk literature can easily be interpreted as tests of more general expected return or "fair game" models.[8]

2. Tests of Market Efficiency in the Random Walk Literature

As discussed earlier, "fair game" models imply the "impossibility" of various sorts of trading systems. Some of the random walk literature has been concerned with testing the profitability of such systems. More of the literature has, however, been concerned with tests of serial covariances of returns. We shall now show that, like a random walk, the serial covariances of a "fair game" are zero, so that these tests are also relevant for the expected return models.

If x_t is a "fair game," its unconditional expectation is zero and its serial covariance can be written in general form as:

$$E(\tilde{x}_{t+\tau}\,\tilde{x}_t) = \int_{x_t} x_t E(\tilde{x}_{t+\tau}|x_t) f(x_t) dx_t,$$

where f indicates a density function. But if x_t is a "fair game,"

$$E(\tilde{x}_{t+\tau}|x_t) = 0.^9$$

8. Our brief historical review is meant only to provide perspective, and it is, of course, somewhat incomplete. For example, we have ignored the important contributions to the early random walk literature in studies of warrants and other options by Sprenkle, Kruizenga, Boness, and others. Much of this early work on options is summarized in [8].

9. More generally, if the sequence $\{x_t\}$ is a fair game with respect to the information sequence $\{\Phi_t\}$, (i.e., $E(\tilde{x}_{t+1}|\Phi_t) = 0$ for all Φ_t); then x_t is a fair game with respect to any Φ'_t that is a subset of Φ_t (i.e., $E(\tilde{x}_{t+1}|\Phi'_t) = 0$ for all Φ'_t). To show this, let $\Phi_t = (\Phi'_t, \ \Phi''_t)$. Then, using Stieltjes integrals and the symbol F to denote cumulative distinction functions, the conditional expectation

$$E(\tilde{x}_{t+1}|\Phi'_t) = \int_{\Phi''_t}\int_{x_{t+1}} x_{t+1} dF(x_{t+1}, \Phi''_t|\Phi'_t) = \int_{\Phi''_t}\left[\int_{x_{t+1}} x_{t+1} dF(x_{t+1}|\Phi'_t, \Phi''_t)\right] dF(\Phi''_t|\Phi'_t).$$

From this it follows that for all lags, the serial covariances between lagged values of a "fair game" variable are zero. Thus, observations of a "fair game" variable are linearly independent.[10]

But the "fair game" model does not necessarily imply that the serial covariances of *one-period returns* are zero. In the weak form tests of this model the "fair game" variable is

$$z_{j,t} = r_{j,t} - E(\tilde{r}_{j,t}|r_{j,t-1}, r_{j,t-2}, \ldots). \quad \text{(Cf. fn. 9)} \tag{9}$$

But the covariance between, for example, r_{jt} and $r_{j,t+1}$ is

$$E([\tilde{r}_{j,t+1} - E(\tilde{r}_{j,t+1})]\,[\tilde{r}_{jt} - E(\tilde{r}_{jt})])$$
$$= \int_{r_{jt}} [r_{jt} - E(\tilde{r}_{jt})]\,[E(\tilde{r}_{j,t+1}|r_{jt}) - E(\tilde{r}_{j,t+1})]f(r_{jt})dr_{jt},$$

and (9) does not imply that $E(\tilde{r}_{j,t+1}|r_{jt}) = E(\tilde{r}_{j,t+1})$: In the "fair game" efficient markets model, the deviation of the return for $t+1$ from its conditional expectation is a "fair game" variable, but the conditional expectation itself can depend on the return observed for t.[11]

In the random walk literature, this problem is not recognized, since it is assumed that the expected return (and indeed the entire distribution of returns) is stationary through time. In practice, this implies estimating serial covariances by taking cross products of deviations of observed returns from the overall sample mean return. It is somewhat fortuitous, then, that this procedure, which represents a rather gross approximation from the viewpoint of the general expected return efficient markets model, does not seem to greatly affect the results of the covariance tests, at least for common stocks.[12]

But the integral in brackets is just $E(\tilde{x}_{t+1}|\Phi_t)$ which by the "fair game" assumption is 0, so that

$$E(x_{t+1}|\Phi'_t) = 0 \text{ for all } \Phi'_t \subset \Phi_t.$$

10. But though zero serial covariances are consistent with a "fair game," they do not imply such a process. A "fair game" also rules out many types of non linear dependence. Thus using arguments similar to those above, it can be shown that if x is a "fair game," $E(\tilde{x}_t\tilde{x}_{t+1} \ldots \tilde{x}_{t+\tau}) = 0$ for all τ, which is not implied by $E(\tilde{x}_t\tilde{x}_{t+\tau}) = 0$ for all τ. For example, consider a three-period case where x must be either ± 1. Suppose the process is $x_{t+2} = \text{sign}\,(x_t x_{t+1})$, i.e.,

x_t	x_{t+1}	$\rightarrow$	x_{t+2}
+	+	$\rightarrow$	+
+	−	$\rightarrow$	−
−	+	$\rightarrow$	−
−	−	$\rightarrow$	+.

If probabilities are uniformly distributed across events,

$$E(\tilde{x}_{t+2}|x_{t+1}) = E(\tilde{x}_{t+2}|x_t) = E(\tilde{x}_{t+1}|x_t) = E(\tilde{x}_{t+2}) = E(\tilde{x}_{t+1}) = E(\tilde{x}_t) = 0,$$

so that all pairwise serial covariances are zero. But the process is not a "fair game," since $E(\tilde{x}_{t+2}|x_{t+1}, x_t) \neq 0$, and knowledge of (x_{t+1}, x_t) can be used as the basis of a simple "system" with positive expected profit.

11. For example, suppose the level of one-period returns follows a martingale so that

$$E(\tilde{r}_{j,t+1}|r_{jt}, r_{j,t-1} \ldots) = r_{jt}.$$

Then covariances between successive returns will be nonzero (though in this special case first differences of returns will be uncorrelated).

12. The reason is probably that for stocks, changes in equilibrium expected returns for the

TABLE 1 (from [10])
First-order Serial Correlation Coefficients for One-, Four-, Nine-, and Sixteen-Day Changes in Log_e Price

Stock	Differencing Interval (Days) One	Four	Nine	Sixteen
Allied Chemical	.017	.029	−.091	−.118
Alcoa	.118*	.095	−.112	−.044
American Can	−.087*	−.124*	−.060	.031
A. T. & T.	−.039	−.010	−.009	−.003
American Tobacco	.111*	−.175*	.033	.007
Anaconda	.067*	−.068	−.125	.202
Bethlehem Steel	.013	−.122	−.148	.112
Chrysler	.012	.060	−.026	.040
Du Pont	.013	.069	−.043	−.055
Eastman Kodak	.025	−.006	−.053	−.023
General Electric	.011	.020	−.004	.000
General Foods	.061*	−.005	−.140	−.098
General Motors	−.004	−.128*	.009	−.028
Goodyear	−.123*	.001	−.037	.033
International Harvester	−.017	−.068	−.244*	.116
International Nickel	.096*	.038	.124	.041
International Paper	.046	.060	−.004	−.010
Johns Manville	.006	−.068	−.002	.002
Owens Illinois	−.021	−.006	.003	−.022
Procter & Gamble	.099*	−.006	.098	.076
Sears	.097*	−.070	−.113	.041
Standard Oil (Calif.)	.025	−.143*	−.046	.040
Standard Oil (N.J.)	.008	−.109	−.082	−.121
Swift & Co.	−.004	−.072	.118	−.197
Texaco	.094*	−.053	−.047	−.178
Union Carbide	.107*	.049	−.101	.124
United Aircraft	.014	−.190*	−.192*	−.040
U.S. Steel	.040	−.006	−.056	.236*
Westinghouse	−.027	−.097	−.137	.067
Woolworth	.028	−.033	−.112	.040

* Coefficient is twice its computed standard error.

For example, Table 1 (taken from [10]) shows the serial correlations between successive changes in the natural log of price for each of the thirty stocks of the Dow Jones Industrial Average, for time periods that vary slightly from stock to stock, but usually run from about the end of 1957 to September 26, 1962. The serial correlations of successive changes in $\log_e$ price are shown for differencing intervals of one, four, nine, and sixteen days.[13]

common differencing intervals of a day, a week, or a month, are trivial relative to other sources of variation in returns. Later, when we consider Roll's work [37], we shall see that this is not true for one week returns on U.S. Government Treasury Bills.

13. The use of changes in $\log_e$ price as the measure of return is common in the random walk literature. It can be justified in several ways. But for current purposes, it is sufficient to note that for price changes less than fifteen per cent, the change in $\log_e$ price is approximately the percentage price change or one-period return. And for differencing intervals shorter than one month, returns in excess of fifteen per cent are unusual. Thus [10] reports that for the data of Table 1, tests carried out on percentage or one-period returns yielded results essentially identical to the tests based on changes in $\log_e$ price.

The results in Table 1 are typical of those reported by others for tests based on serial covariances. (Cf. Kendall [21], Moore [31], Alexander [1], and the results of Granger and Morgenstern [17] and Godfrey, Granger and Morgenstern [16] obtained by means of spectral analysis.) Specifically, there is no evidence of substantial linear dependence between lagged price changes or returns. In absolute terms the measured serial correlations are always close to zero.

Looking hard, though, one can probably find evidence of statistically "significant" linear dependence in Table 1 (and again this is true of results reported by others). For the daily returns eleven of the serial correlations are more than twice their computed standard errors, and twenty-two out of thirty are positive. On the other hand, twenty-one and twenty-four of the coefficients for the four and nine day differences are negative. But with samples of the size underlying Table 1 (N = 1200-1700 observations per stock on a daily basis) statistically "significant" deviations from zero covariance are not necessarily a basis for rejecting the efficient markets model. For the results in Table 1, the standard errors of the serial correlations were approximated as $(1/(N-1))^{1/2}$, which for the daily data implies that a correlation as small as .06 is more than twice its standard error. But a coefficient this size implies that a linear relationship with the lagged price change can be used to explain about .36% of the variation in the current price change, which is probably insignificant from an economic viewpoint. In particular, it is unlikely that the small absolute levels of serial correlation that are always observed can be used as the basis of substantially profitable trading systems.[14]

It is, of course, difficult to judge what degree of serial correlation would imply the existence of trading rules with substantial expected profits. (And indeed we shall soon have to be a little more precise about what is implied by "substantial" profits.) Moreover, zero serial covariances are consistent with a "fair game" model, but as noted earlier (fn. 10), there are types of nonlinear dependence that imply the existence of profitable trading systems, and yet do not imply nonzero serial covariances. Thus, for many reasons it is desirable to directly test the profitability of various trading rules.

The first major evidence on trading rules was Alexander's [1, 2]. He tests a variety of systems, but the most thoroughly examined can be decribed as follows: If the price of a security moves up at least y%, buy and hold the security until its price moves down at least y% from a subsequent high, at which time simultaneously sell and go short. The short position is maintained until the price rises at least y% above a subsequent low, at which time one covers the short position and buys. Moves less than y% in either direction are

14. Given the evidence of Kendall [21], Mandelbrot [28], Fama [10] and others that large price changes occur much more frequently than would be expected if the generating process were Gaussian, the expression $(1/(N-1))^{1/2}$ understates the sampling dispersion of the serial correlation coefficient, and thus leads to an overstatement of significance levels. In addition, the fact that sample serial correlations are predominantly of one sign or the other is not in itself evidence of linear dependence. If, as the work of King [23] and Blume [7] indicates, there is a market factor whose behavior affects the returns on all securities, the sample behavior of this market factor may lead to a predominance of signs of one type in the serial correlations for individual securities, even though the population serial correlations for both the market factor and the returns on individual securities are zero. For a more extensive analysis of these issues see [10].

ignored. Such a system is called a y% filter. It is obviously a "one security and cash" trading rule, so that the results it produces are relevant for the submartingale expected return model of (6).

After extensive tests using daily data on price indices from 1897 to 1959 and filters from one to fifty per cent, and after correcting some incorrect presumptions in the initial results of [1] (see fn. 25), in his final paper on the subject, Alexander concludes:

> In fact, at this point I should advise any reader who is interested only in practical results, and who is not a floor trader and so must pay commissions, to turn to other sources on how to beat buy and hold. The rest of this article is devoted principally to a theoretical consideration of whether the observed results are consistent with a random walk hypothesis [8], p. 351).

Later in the paper Alexander concludes that there is some evidence in his results against the independence assumption of the random walk model. But market efficiency does not require a random walk, and from the viewpoint of the submartingale model of (6), the conclusion that the filters cannot beat buy-and-hold is support for the efficient markets hypothesis. Further support is provided by Fama and Blume [13] who compare the profitability of various filters to buy-and-hold for the individual stocks of the Dow-Jones Industrial Average. (The data are those underlying Table 1.)

But again, looking hard one can find evidence in the filter tests of both Alexander and Fama-Blume that is inconsistent with the submartingale efficient markets model, if that model is interpreted in a strict sense. In particular, the results for very small filters (1 per cent in Alexander's tests and .5, 1.0, and 1.5 per cent in the tests of Fama-Blume) indicate that it is possible to devise trading schemes based on very short-term (preferably intra-day but at most daily) price swings that will on average outperform buy-and-hold. The average profits on individual transactions from such schemes are miniscule, but they generate transactions so frequently that over longer periods and ignoring commissions they outperform buy-and-hold by a substantial margin. These results are evidence of persistence or positive dependence in very short-term price movements. And, interestingly, this is consistent with the evidence for slight positive linear dependence in successive daily price changes produced by the serial correlations.[15]

15. Though strictly speaking, such tests of pure independence are not directly relevant for expected return models, it is interesting that the conclusion that very short-term swings in prices persist slightly longer than would be expected under the martingale hypothesis is also supported by the results of non-parametric runs tests applied to the daily data of Table 1. (See [10], Tables 12-15.) For the daily price changes, the actual number of runs of price changes of the same sign is less than the expected number for 26 out of 30 stocks. Moreover, of the eight stocks for which the actual number of runs is more than two standard errors less than the expected number, five of the same stocks have positive daily, first order serial correlations in Table 1 that are more than twice their standard errors. But in both cases the statistical "significance" of the results is largely a reflection of the large sample sizes. Just as the serial correlations are small in absolute terms (the average is .026), the differences between the expected and actual number of runs on average are only three per cent of the total expected number.

On the other hand, it is also interesting that the runs tests do not support the suggestion of slight negative dependence in four and nine day changes that appeared in the serial correlations. In the runs tests such negative dependence would appear as a tendency for the actual number of runs to exceed the expected number. In fact, for the four and nine day price changes, for 17 and

But when one takes account of even the minimum trading costs that would be generated by small filters, their advantage over buy-and-hold disappears. For example, even a floor trader (i.e., a person who owns a seat) on the New York Stock Exchange must pay clearinghouse fees on his trades that amount to about .1 per cent per turnaround transaction (i.e., sales plus purchase). Fama-Blume show that because small filters produce such frequent trades, these minimum trading costs are sufficient to wipe out their advantage over buy-and-hold.

Thus the filter tests, like the serial correlations, produce empirically noticeable departures from the strict implications of the efficient markets model. But, in spite of any statistical significance they might have, from an economic viewpoint the departures are so small that it seems hardly justified to use them to declare the market inefficient.

3. Other Tests of Independence in the Random Walk Literature

It is probably best to regard the random walk model as a special case of the more general expected return model in the sense of making a more detailed specification of the economic environment. That is, the basic model of market equilibrium is the "fair game" expected return model, with a random walk arising when additional environmental conditions are such that distributions of one-period returns repeat themselves through time. From this viewpoint violations of the pure independence assumption of the random walk model are to be expected. But when judged relative to the benchmark provided by the random walk model, these violations can provide insights into the nature of the market environment.

For example, one departure from the pure independence assumption of the random walk model has been noted by Osborne [34], Fama ([10], Table 17 and Figure 8), and others. In particular, large daily price changes tend to be followed by large daily changes. The signs of the successor changes are apparently random, however, which indicates that the phenomenon represents a denial of the random walk model but not of the market efficiency hypothesis. Nevertheless, it is interesting to speculate why the phenomenon might arise. It may be that when important new information comes into the market it cannot always be immediately evaluated precisely. Thus, sometimes the initial price will overadjust to the information, and other times it will underadjust. But since the evidence indicates that the price changes on days following the initial large change are random in sign, the initial large change at least represents an unbiased adjustment to the ultimate price effects of the information, and this is sufficient for the expected return efficient markets model.

Niederhoffer and Osborne [32] document two departures from complete randomness in common stock price changes from transaction to transaction. First, their data indicate that reversals (pairs of consecutive price changes of opposite sign) are from two to three times as likely as continuations (pairs of consecutive price changes of the same sign). Second, a continuation is

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18 of the 30 stocks in Table 1 the actual number of runs is less than the expected number. Indeed, runs tests in general show no consistent evidence of dependence for any differencing interval longer than a day, which seems especially pertinent in light of the comments in footnote 14.

slightly more frequent after a preceding continuation than after a reversal. That is, let (+|++) indicate the occurrence of a positive price change, given two preceding positive changes. Then the events (+|++) and (−|−−) are slightly more frequent than (+|+−) or (−|−+).[16]

Niederhoffer and Osborne offer explanations for these phenomena based on the market structure of the New York Stock Exchange (N.Y.S.E.). In particular, there are three major types of orders that an investor might place in a given stock: (a) buy limit (buy at a specified price or lower), (b) sell limit (sell at a specified price or higher), and (c) buy or sell at market (at the lowest selling or highest buying price of another investor). A book of unexecuted limit orders in a given stock is kept by the specialist in that stock on the floor of the exchange. Unexecuted sell limit orders are, of course, at higher prices than unexecuted buy limit orders. On both exchanges, the smallest non-zero price change allowed is $\frac{1}{8}$ point.

Suppose now that there is more than one unexecuted sell limit order at the lowest price of any such order. A transaction at this price (initiated by an order to buy at market[17]) can only be followed either by a transaction at the same price (if the next market order is to buy) or by a transaction at a lower price (if the next market order is to sell). Consecutive price increases can usually only occur when consecutive market orders to buy exhaust the sell limit orders at a given price.[18] In short, the excessive tendency toward reversal for consecutive non-zero price changes could result from bunching of unexecuted buy and sell limit orders.

The tendency for the events (+|++) and (−|−−) to occur slightly more frequently than (+|+−) and (−|−+) requires a more involved explanation which we shall not attempt to reproduce in full here. In brief, Niederhoffer and Osborne contend that the higher frequency of (+|++) relative to (+|+−) arises from a tendency for limit orders "to be concentrated at integers (26, 43), halves ($26\frac{1}{2}$, $43\frac{1}{2}$), quarters and odd eighths in descending order of preference."[19] The frequency of the event (+|++), which usually requires that sell limit orders be exhausted at at least two consecutively higher prices (the last of which is relatively more frequently at an odd eighth), more heavily reflects the absence of sell limit orders at odd eighths than the event (+|+−), which usually implies that sell limit orders at only one price have been exhausted and so more or less reflects the average bunching of limit orders at all eighths.

But though Niederhoffer and Osborne present convincing evidence of sta-

16. On a transaction to transaction basis, positive and negative price changes are about equally likely. Thus, under the assumption that price changes are random, any pair of non-zero changes should be as likely as any other, and likewise for triplets of consecutive non-zero changes.

17. A buy limit order for a price equal to or greater than the lowest available sell limit price is effectively an order to buy at market, and is treated as such by the broker.

18. The exception is when there is a gap of more than $\frac{1}{8}$ between the highest unexecuted buy limit and the lowest unexecuted sell limit order, so that market orders (and new limit orders) can be crossed at intermediate prices.

19. Their empirical documentation for this claim is a few samples of specialists' books for selected days, plus the observation [34] that actual trading prices, at least for volatile high priced stocks, seem to be concentrated at integers, halves, quarters and odd eighths in descending order.

tistically significant departures from independence in price changes from transaction to transaction, and though their analysis of their findings presents interesting insights into the process of market making on the major exchanges, the types of dependence uncovered do not imply market inefficiency. The best documented source of dependence, the tendency toward excessive reversals in pairs of non-zero price changes, seems to be a direct result of the ability of investors to place limit orders as well as orders at market, and this negative dependence in itself does not imply the existence of profitable trading rules. Similarly, the apparent tendency for observed transactions (and, by implication, limit orders) to be concentrated at integers, halves, even eighths and odd eighths in descending order is an interesting fact about investor behavior, but in itself is not a basis on which to conclude that the market is inefficient.[20]

The Niederhoffer-Osborne analysis of market making does, however, point clearly to the existence of market inefficiency, but with respect to strong form tests of the efficient markets model. In particular, the list of unexecuted buy and sell limit orders in the specialist's book is important information about the likely future behavior of prices, and this information is only available to the specialist. When the specialist is asked for a quote, he gives the prices and can give the quantities of the highest buy limit and lowest sell limit orders on his book, but he is prevented by law from divulging the book's full contents. The interested reader can easily imagine situations where the structure of limit orders in the book could be used as the basis of a profitable trading rule.[21] But the record seems to speak for itself:

> It should not be assumed that these transactions undertaken by the specialist, and in which he is involved as buyer or seller in 24 per cent of all market volume, are necessarily a burden to him. Typically, the specialist sells above his last purchase on 83 per cent of all his sales, and buys below his last sale on 81 per cent of all his purchases ([32], p. 908).

Thus it seems that the specialist has monopoly power over an important block of information, and, not unexpectedly, uses his monopoly to turn a profit. And this, of course, is evidence of market inefficiency in the strong form sense. The important economic question, of course, is whether the market making

20. Niederhoffer and Osborne offer little to refute this conclusion. For example ([32], p. 914):

> Although the specific properties reported in this study have a significance from a statistical point of view, the reader may well ask whether or not they are helpful in a practical sense. Certain trading rules emerge as a result of our analysis. One is that limit and stop orders should be placed at odd eights, preferably at 7/8 for sell orders and at 1/8 for buy orders. Another is to buy when a stock advances through a barrier and to sell when it sinks through a barrier.

The first "trading rule" tells the investor to resist his innate inclination to place orders at integers, but rather to place sell orders 1/8 below an integer and buy orders 1/8 above. Successful execution of the orders is then more likely, since the congestion of orders that occur at integers is avoided. But the cost of this success is apparent. The second "trading rule" seems no more promising, if indeed it can even be translated into a concrete prescription for action.

21. See, for example, ([32], p. 908). But it is unlikely that anyone but the specialist could earn substantial profits from knowledge of the structure of unexecuted limit orders on the book. The specialist makes trading profits by engaging in many transactions, each of which has a small average profit; but for any other trader, including those with seats on the exchange, these profits would be eaten up by commissions to the specialist.

function of the specialist could be fulfilled more economically by some non-monopolistic mechanism.[22]

4. Distributional Evidence

At this date the weight of the empirical evidence is such that economists would generally agree that whatever dependence exists in series of historical returns cannot be used to make profitable predictions of the future. Indeed, for returns that cover periods of a day or longer, there is little in the evidence that would cause rejection of the stronger random walk model, at least as a good first approximation.

Rather, the last burning issue of the random walk literature has centered on the nature of the distribution of price changes (which, we should note immediately, is an important issue for the efficient markets hypothesis since the nature of the distribution affects both the types of statistical tools relevant for testing the hypothesis and the interpretation of any results obtained). A model implying normally distributed price changes was first proposed by Bachelier [3], who assumed that price changes from transaction to transaction are independent, identically distributed random variables with finite variances. If transactions are fairly uniformly spread across time, and if the number of transactions per day, week, or month is very large, then the Central Limit Theorem leads us to expect that these price changes will have normal or Gaussian distributions.

Osborne [33], Moore [31], and Kendall [21] all thought their empirical evidence supported the normality hypothesis, but all observed high tails (i.e., higher proportions of large observations) in their data distributions vis-à-vis what would be expected if the distributions were normal. Drawing on these findings and some empirical work of his own, Mandelbrot [28] then suggested that these departures from normality could be explained by a more general form of the Bachelier model. In particular, if one does not assume that distributions of price changes from transaction to transaction necessarily have finite variances, then the limiting distributions for price changes over longer differencing intervals could be any member of the stable class, which includes the normal as a special case. Non-normal stable distributions have higher tails than the normal, and so can account for this empirically observed feature of distributions of price changes. After extensive testing (involving the data from the stocks in Table 1), Fama [10] concludes that non-normal stable distributions are a better description of distributions of daily returns on common stocks than the normal. This conclusion is also supported by the empirical work of Blume [7] on common stocks, and it has been extended to U.S. Government Treasury Bills by Roll [37].

Economists have, however, been reluctant to accept these results,[23] primar-

22. With modern computers, it is hard to believe that a more competitive and economical system would not be feasible. It does not seem technologically impossible to replace the entire floor of the N.Y.S.E. with a computer, fed by many remote consoles, that kept all the books now kept by the specialists, that could easily make the entire book on any stock available to anybody (so that interested individuals could then compete to "make a market" in a stock) and that carried out transactions automatically.

23. Some have suggested that the long-tailed empirical distributions might result from processes

ily because of the wealth of statistical techniques available for dealing with normal variables and the relative paucity of such techniques for non-normal stable variables. But perhaps the biggest contribution of Mandelbrot's work has been to stimulate research on stable distributions and estimation procedures to be applied to stable variables. (See, for example, Wise [46], Fama and Roll [15], and Blattberg and Sargent [6], among others.) The advance of statistical sophistication (and the importance of examining distributional assumptions in testing the efficient markets model) is well illustrated in Roll [37], as compared, for example, with the early empirical work of Mandelbrot [28] and Fama [10].

5. "Fair Game" Models in the Treasury Bill Market

Roll's work is novel in other respects as well. Coming after the efficient markets models of Mandelbrot [27] and Samuelson [38], it is the first weak form empirical work that is consciously in the "fair game" rather than the random walk tradition.

More important, as we saw earlier, the "fair game" properties of the general expected return models apply to

$$z_{jt} = r_{jt} - E(\tilde{r}_{jt}|\Phi_{t-1}). \tag{10}$$

For data on common stocks, tests of "fair game" (and random walk) properties seem to go well when the conditional expected return is estimated as the average return for the sample of data at hand. Apparently the variation in common stock returns about their expected values is so large relative to any changes in the expected values that the latter can safely be ignored. But, as Roll demonstrates, this result does not hold for Treasury Bills. Thus, to test the "fair game" model on Treasury Bills requires explicit economic theory for the evolution of expected returns through time.

Roll uses three existing theories of the term structure (the pure expectations hypothesis of Lutz [26] and two market segmentation hypotheses, one of which is the familiar "liquidity preference" hypothesis of Hicks [18] and Kessel [22]) for this purpose.[24] In his models r_{jt} is the rate observed from the term structure at period t for one week loans to commence at $t + j - 1$, and can be thought of as a "futures" rate. Thus $r_{j+1,\, t-1}$ is likewise the rate on

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that are mixtures of normal distributions with different variances. Press [35], for example, suggests a Poisson mixture of normals in which the resulting distributions of price changes have long tails but finite variances. On the other hand, Mandelbrot and Taylor [29] show that other mixtures of normals can still lead to non-normal stable distributions of price changes for finite differencing intervals.

If, as Press' model would imply, distributions of price changes are long-tailed but have finite variances, then distributions of price changes over longer and longer differencing intervals should be progressively closer to the normal. No such convergence to normality was observed in [10] (though admittedly the techniques used were somewhat rough). Rather, except for origin and scale, the distributions for longer differencing intervals seem to have the same "high-tailed" characteristics as distributins for shorter differencing intervals, which is as would be expected if the distributions are non-normal stable.

24. As noted early in our discussions, all available tests of market efficiency are implicitly also tests of expected return models of market equilibrium. But Roll formulates explicitly the economic models underlying his estimates of expected returns, and emphasizes that he is simultaneously testing economic models of the term structure as well as market efficiency.

one week loans to commence at $t+j-1$, but observed in this case at $t-1$. Similarly, L_{jt} is the so-called "liquidity premium" in r_{jt}; that is

$$r_{jt} = E(\tilde{r}_{0,t+j-1}|\Phi_t) + L_{jt}.$$

In words, the one-week "futures" rate for period $t+j-1$ observed from the term structure at t is the expectation at t of the "spot" rate for $t+j-1$ plus a "liquidity premium" (which could, however, be positive or negative).

In all three theories of the term structure considered by Roll, the conditional expectation required in (10) is of the form

$$E(\tilde{r}_{j,t}|\Phi_{t-1}) = r_{j+1,t-1} + E(\tilde{L}_{jt}|\Phi_{t-1}) - L_{j+1,t-1}.$$

The three theories differ only in the values assigned to the "liquidity premiums." For example, in the "liquidity preference" hypothesis, investors must always be paid a positive premium for bearing interest rate uncertainty, so that the L_{jt} are always positive. By contrast, in the "pure expectations" hypothesis, all liquidity premiums are assumed to be zero, so that

$$E(\tilde{r}_{jt}|\Phi_{t-1}) = r_{j+1,t-1}.$$

After extensive testing, Roll concludes (i) that the two market segmentation hypotheses fit the data better than the pure expectations hypothesis, with perhaps a slight advantage for the "liquidity preference" hypothesis, and (ii) that as far as his tests are concerned, the market for Treasury Bills is effcient. Indeed, it is interesting that when the best fitting term structure model is used to estimate the conditional expected "futures" rate in (10), the resulting variable z_{jt} seems to be serially independent! It is also interesting that if he simply assumed that his data distributions were normal, Roll's results would not be so strongly in support of the efficient markets model. In this case taking account of the observed high tails of the data distributions substantially affected the interpretation of the results.[25]

6. Tests of a Multiple Security Expected Return Model

Though the weak form tests support the "fair game" efficient markets model, all of the evidence examined so far consists of what we might call "single security tests." That is, the price or return histories of individual securities are examined for evidence of dependence that might be used as the basis of a trading system for *that* security. We have not discussed tests of whether securities are "appropriately priced" vis-à-vis one another.

But to judge whether differences between average returns are "appropriate" an economic theory of equilibrium expected returns is required. At the moment, the only fully developed theory is that of Sharpe [40] and Lintner [24,

25. The importance of distributional assumptions is also illustrated in Alexander's work on trading rules. In his initial tests of filter systems [1], Alexander assumed that purchases could always be executed exactly (rather than at least) y% above lows and sales exactly y% below highs. Mandelbrot [28] pointed out, however, that though this assumption would do little harm with normally distributed price changes (since price series are then essentially continuous), with non-normal stable distributions it would introduce substantial positive bias into the filter profits (since with such distributions price series will show many discontinuities). In his later tests [2], Alexander does indeed find that taking account of the discontinuities (i.e., the presence of large price changes) in his data substantially lowers the profitability of the filters.

25] referred to earlier. In this model (which is a direct outgrowth of the mean-standard deviation portfolio models of investor equilibrium of Markowitz [30] and Tobin [43]), the expected return on security j from time t to t + 1 is

$$E(\tilde{r}_{j,t+1}|\Phi_t) = r_{f,t+1} + \left[\frac{E(\tilde{r}_{m,t+1}|\Phi_t) - r_{f,t+1}}{\sigma(\tilde{r}_{m,t+1}|\Phi_t)}\right] \frac{\text{cov}\,(\tilde{r}_{j,t+1}, \tilde{r}_{m,t+1}|\Phi_t)}{\sigma(\tilde{r}_{m,t+1}|\Phi_t)}, \tag{11}$$

where $r_{f,t+1}$ is the return from t to t + 1 on an asset that is riskless in money terms; $r_{m,t+1}$ is the return on the "market portfolio" m (a portfolio of all investment assets with each weighted in proportion to the total market value of all its outstanding units); $\sigma^2(\tilde{r}_{m,t+1}|\Phi_t)$ is the variance of the return on m; cov $(\tilde{r}_{j,t+1}, \tilde{r}_{m,t+1}|\Phi_t)$ is the covariance between the returns on j and m; and the appearance of Φ_t indicates that the various expected returns, variance and covariance, could in principle depend on Φ_t. Though Sharpe and Lintner derive (11) as a one-period model, the result is given a multiperiod justification and interpretation in [11]. The model has also been extended in (12) to the case where the one-period returns could have stable distributions with infinite variances.

In words, (11) says that the expected one-period return on a security is the one-period riskless rate of interest $r_{f,t+1}$ plus a "risk premium" that is proportional to $\text{cov}(\tilde{r}_{j,t+1}, \tilde{r}_{m,t+1}|\Phi_t)/\sigma(\tilde{r}_{m,t+1}|\Phi_t)$. In the Sharpe-Lintner model each investor holds some combination of the riskless asset and the market portfolio, so that, given a mean-standard deviation framework, the risk of an individual asset can be measured by its contribution to the standard deviation of the return on the market portfolio. This contribution is in fact cov $(\tilde{r}_{j,t+1}, \tilde{r}_{m,t+1}|\Phi_t)/\sigma(\tilde{r}_{m,t+1}|\Phi_t)$.[26] The factor

$$[E(\tilde{r}_{m,t+1}|\Phi_t) - r_{f,t+1}]/\sigma(\tilde{r}_{m,t+1}|\Phi_t),$$

which is the same for all securities, is then regarded as the market price of risk.

Published empirical tests of the Sharpe-Lintner model are not yet available, though much work is in progress. There is some published work, however, which, though not directed at the Sharpe-Lintner model, is at least consistent with some of its implications. The stated goal of this work has been to determine the extent to which the returns on a given security are related to the returns on other securities. It started (again) with Kendall's [21] finding that though common stock price changes do not seem to be serially correlated, there is a high degree of cross-correlation between the *simultaneous* returns of different securities. This line of attack was continued by King [23] who (using factor analysis of a sample of monthly returns on sixty N.Y.S.E. stocks for the period 1926-60) found that on average about 50% of the variance of an individual stock's returns could be accounted for by a "market factor" which affects the returns on all stocks, with "industry factors" accounting for at most an additional 10% of the variance.

26. That is,

$$\sum_j \text{cov}\;(\tilde{r}_{j,t+1}, \tilde{r}_{m,t+1}|\Phi_t)/\sigma(\tilde{r}_{m,t+1}|\Phi_t) = \sigma(\tilde{r}_{m,t+1}|\Phi_t).$$

For our purposes, however, the work of Fama, Fisher, Jensen, and Roll [14] (henceforth FFJR) and the more extensive work of Blume [7] on monthly return data is more relevant. They test the following "market model," originally suggested by Markowitz [30]:

$$\tilde{r}_{j,t+1} = \alpha_j + \beta_j \tilde{r}_{M,t+1} + \tilde{u}_{j,t+1} \tag{12}$$

where $r_{j,t+1}$ is the rate of return on security j for month t, $r_{M,t+1}$ is the corresponding return on a market index M, α_j and β_j are parameters that can vary from security to security, and $u_{j,t+1}$ is a random disturbance. The tests of FFJR and subsequently those of Blume indicate that (12) is well specified as a linear regression model in that (i) the estimated parameters $\hat{\alpha}_j$ and $\hat{\beta}_j$ remain fairly constant over long periods of time (e.g., the entire post-World War II period in the case of Blume), (ii) $r_{M,t+1}$ and the estimated $\hat{u}_{j,t+1}$, are close to serially independent, and (iii) the $\hat{u}_{j,t+1}$ seem to be independent of $r_{M,t+1}$.

Thus the observed properties of the "market model" are consistent with the expected return efficient markets model, and, in addition, the "market model" tells us something about the process generating expected returns from security to security. In particular,

$$E(\tilde{r}_{j,t+1}) = \alpha_j + \beta_j E(\tilde{r}_{M,t+1}). \tag{13}$$

The question now is to what extent (13) is consistent with the Sharpe-Lintner expected return model summarized by (11). Rearranging (11) we obtain

$$E(\tilde{r}_{j,t+1}|\Phi_t) = \alpha_j(\Phi_t) + \beta_j(\Phi_t) E(\tilde{r}_{m,t+1}|\Phi_t), \tag{14}$$

where, noting that the riskless rate $r_{f,t+1}$ is itself part of the information set Φ_t, we have

$$\alpha_j(\Phi_t) = r_{f,t+1}[1 - \beta_j(\Phi_t)], \tag{15}$$

and

$$\beta_j(\Phi_t) = \frac{\text{cov}\,(\tilde{r}_{j,t+1}, \tilde{r}_{m,t+1}|\Phi_t)}{\sigma^2(\tilde{r}_{m,t+1}|\Phi_t)}. \tag{16}$$

With some simplifying assumptions, (14) can be reduced to (13). In particular, if the covariance and variance that determine $\beta_j(\Phi_t)$ in (16) are the same for all t and Φ_t, then $\beta_j(\Phi_t)$ in (16) corresponds to β_j in (12) and (13), and the least squares *estimate* of β_j in (12) is in fact just the ratio of the sample values of the covariance and variance in (16). If we also assume that $r_{f,t+1}$ is the same for all t, and that the behavior of the returns on the market portfolio m are closely approximated by the returns on some representative index M, we will have come a long way toward equating (13) and (11). Indeed, the only missing link is whether in the estimated parameters of (12)

$$\hat{\alpha}_j \cong r_f(1 - \hat{\beta}_j). \tag{17}$$

Neither FFJR nor Blume attack this question directly, though some of Blume's evidence is at least promising. In particular, the magnitudes of the estimated $\hat{\alpha}_j$ are roughly consistent with (17) in the sense that the estimates are always close to zero (as they should be with monthly return data).[27]

In a sense, though, in establishing the apparent empirical validity of the "market model" of (12), both too much and too little have been shown *vis-à-vis* the Sharpe-Lintner expected return model of (11). We know that during the post-World War II period one-month interest rates on riskless assets (e.g., government bills with one month to maturity) have not been constant. Thus, if expected security returns were generated by a version of the "market model" that is fully consistent with the Sharpe-Lintner model, we would, according to (15), expect to observe some non-stationarity in the estimates of α_j. On a monthly basis, however, variation through time in one-period riskless interest rates is probably trivial relative to variation in other factors affecting monthly common stock returns, so that more powerful statistical methods would be necessary to study the effects of changes in the riskless rate.

In any case, since the work of FFJR and Blume on the "market model" was not concerned with relating this model to the Sharpe-Lintner model, we can only say that the results for the former are somewhat consistent with the implications of the latter. But the results for the "market model" are, after all, just a statistical description of the return generating process, and they are probably somewhat consistent with other models of equilibrium expected returns. Thus the only way to generate strong empirical conclusions about the Sharpe-Lintner model is to test it directly. On the other hand, any alternative model of equilibrium expected returns must be somewhat consistent with the "market model," given the evidence in its support.

B. *Tests of Martingale Models of the Semi-strong Form*

In general, semi-strong form tests of efficient markets models are concerned with whether current prices "fully reflect" all obviously publicly available information. Each individual test, however, is concerned with the adjustment of security prices to one kind of information generating event (e.g., stock splits, announcements of financial reports by firms, new security issues, etc.). Thus each test only brings supporting evidence for the model, with the idea that by accumulating such evidence the validity of the model will be "established."

In fact, however, though the available evidence is in support of the efficient markets model, it is limited to a few major types of information generating events. The initial major work is apparently the study of stock splits by Fama,

27. With least squares applied to monthly return data, the estimate of α_j in (12) is

$$\hat{\alpha}_j = \bar{r}_{j,t} - \hat{\beta}_j \bar{r}_{M,t},$$

where the bars indicate sample mean returns. But, in fact, Blume applies the market model to the wealth relatives $R_{jt} = 1 + r_{jt}$ and $R_{Mt} = 1 + r_{Mt}$. This yields precisely the same estimate of β_j as least squares applied to (12), but the intercept is now

$$\hat{\alpha}'_j = \bar{R}_{jt} - \hat{\beta}_j \bar{R}_{Mt} = 1 + \bar{r}_{jt} - \hat{\beta}_j(1 + \bar{r}_{Mt}) = 1 - \hat{\beta}_j + \hat{\alpha}_j.$$

Thus what Blume in fact finds is that for almost all securities, $\hat{\alpha}'_j + \hat{\beta}_j \cong 1$, which implies that $\hat{\alpha}_j$ is close to 0.

Fisher, Jensen, and Roll (FFJR) [14], and all the subsequent studies summarized here are adaptations and extensions of the techniques developed in FFJR. Thus, this paper will first be reviewed in some detail, and then the other studies will be considered.

1. Splits and the Adjustment of Stock Prices to New Information

Since the only apparent result of a stock split is to multiply the number of shares per shareholder without increasing claims to real assets, splits in themselves are not necessarily sources of new information. The presumption of FFJR is that splits may often be associated with the appearance of more fundamentally important information. The idea is to examine security returns around split dates to see first if there is any "unusual" behavior, and, if so, to what extent it can be accounted for by relationships between splits and other more fundamental variables.

The approach of FFJR to the problem relies heavily on the "market model" of (12). In this model if a stock split is associated with abnormal behavior, this would be reflected in the estimated regression residuals for the months surrounding the split. For a given split, define month O as the month in which the effective date of a split occurs, month 1 as the month immediately following the split month, month -1 as the month preceding, etc. Now define the average residual over all split securities for month m (where for each security m is measured relative to the split month) as

$$u_m = \sum_{j=1}^{N} \frac{\hat{u}_{jm}}{N},$$

where $\hat{u}_{jm}$ is the sample regression residual for security j in month m and N is the number of splits. Next, define the cumulative average residual U_m as

$$U_m = \sum_{k=-29}^{m} u_k.$$

The average residual u_m can be interpreted as the average deviation (in month m relative to split months) of the returns of split stocks from their normal relationships with the market. Similarly, U_m can be interpreted as the cumulative deviation (from month -29 to month m). Finally, define u_m^+, u_m^-, U_m^+, and U_m^- as the average and cumulative average residuals for splits followed by "increased" (+) and "decreased" (−) dividends. An "increase" is a case where the percentage change in dividends on the split share in the year after the split is greater than the percentage change for the N.Y.S.E. as a whole, while a "decrease" is a case of relative dividend decline.

The essence of the results of FFJR are then summarized in Figure 1, which shows the cumulative average residuals U_m U_m^+, and U_m^- for $-29 \leq m \leq 30$. The sample includes all 940 stock splits on the N.Y.S.E. from 1927-59, where the exchange was at least five new shares for four old, and where the security was listed for at least twelve months before and after the split.

For all three dividend categories the cumulative average residuals rise in the 29 months prior to the split, and in fact the average residuals (not shown here) are uniformly positive. This cannot be attributed to the splitting process, since in only about ten per cent of the cases is the time between the announcement and effective dates of a split greater than four months. Rather, it seems that firms tend to split their shares during "abnormally" good times—that is, during periods when the prices of their shares have increased more than would

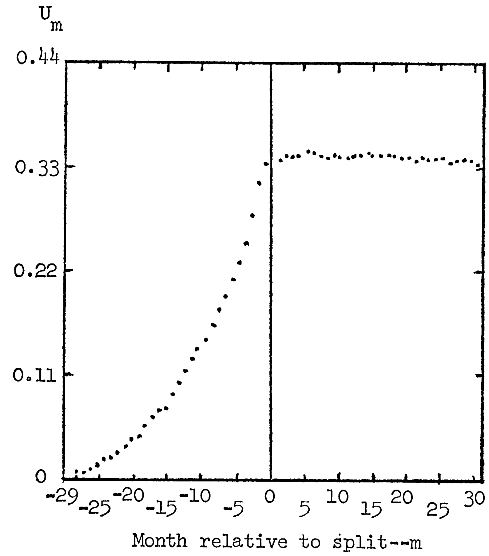

FIGURE 1a
Cumulative average residuals—all splits.

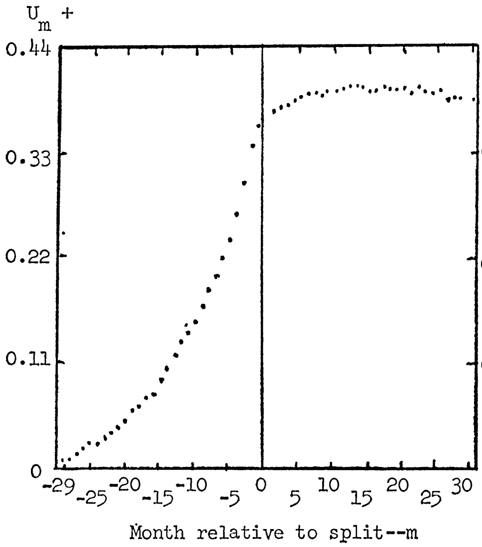

FIGURE 1b
Cumulative average residuals for dividend "increases."

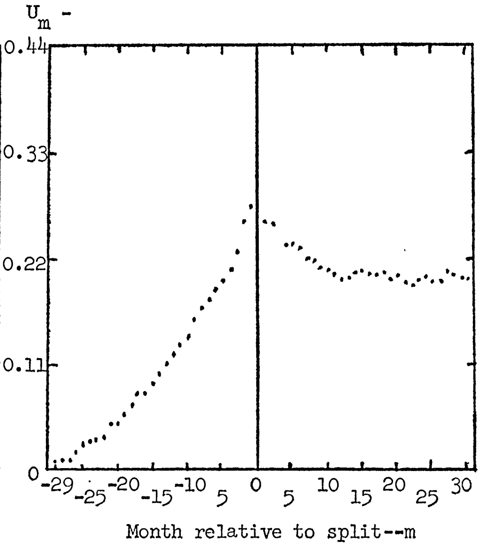

FIGURE 1c
Cumulative average residuals for dividend "decreases."

be implied by their normal relationships with general market prices, which itself probably reflects a sharp improvement, relative to the market, in the earnings prospects of these firms sometime during the years immediately preceding a split.[28]

After the split month there is almost no further movement in U_m, the cumulative average residual for all splits. This is striking, since 71.5 per cent (672 out of 940) of all splits experienced greater percentage dividend increases in the year after the split than the average for all securities on the N.Y.S.E. In light of this, FFJR suggest that when a split is announced the market interprets this (and correctly so) as a signal that the company's directors are probably confident that future earnings will be sufficient to maintain dividend payments at a higher level. Thus the large price increases in the months immediately preceding a split may be due to an alteration in expectations concerning the future earning potential of the firm, rather than to any intrinsic effects of the split itself.

If this hypothesis is correct, return behavior subsequent to splits should be substantially different for the cases where the dividend increase materializes than for the cases where it does not. FFJR argue that in fact the differences are in the directions that would be predicted. The fact that the cumulative average residuals for the "increased" dividends (Figure 1b) drift upward but only slightly in the year *after* the split is consistent with the hypothesis that when the split is *declared*, there is a price adjustment in anticipation of future dividend increases. But the behavior of the residuals for stock splits associated with "decreased" dividends offers even stronger evidence for the split hypothesis. The cumulative average residuals for these stocks (Figure 1c) rise in the few months before the split, but then fall dramatically in the few months after the split when the anticipated dividend increase is not forthcoming. When a year has passed after the split, the cumulative average residual has fallen to about where it was five months prior to the split, which is about the earliest time reliable information about a split is likely to reach the market. Thus by the time it becomes clear that the anticipated dividend increase is not forthcoming, the apparent effects of the split seem to have been wiped away, and the stock's returns have reverted to their normal relationship with market returns.

Finally, and most important, although the behavior of post-split returns will be very different depending on whether or not dividend "increases" occur, and in spite of the fact that a large majority of split securities do experience dividend "increases," when all splits are examined together (Figure 1a), subsequent to the split there is no net movement up or down in the cumulative

28. It is important to note, however, that as FFJR indicate, the persistent upward drift of the cumulative average residuals in the months preceding the split is not a phenomenon that could be used to increase expected trading profits. The reason is that the behavior of the average residuals is not representative of the behavior of the residuals for individual securities. In months prior to the split, successive sample residuals for individual securities seem to be independent. But in most cases, there are a few months in which the residuals are abnormally large and positive. The months of large residuals differ from security to security, however, and these differences in timing explain why the signs of the average residuals are uniformly positive for many months preceding the split.

average residuals. Thus, apparently the market makes unbiased forecasts of the implications of a split for future dividends, and these forecasts are fully reflected in the prices of the security by the end of the split month. After considerably more data analysis than can be summarized here, FFJR conclude that their results lend considerable support to the conclusion that the stock market is efficient, at least with respect to its abiliy to adjust to the information implicit in a split.

2. Other Studies of Public Announcements

Variants of the method of residual analysis developed in [14] have been used by others to study the effects of different kinds of public announcements, and all of these also support the efficient markets hypothesis.

Thus using data on 261 major firms for the period 1946-66, Ball and Brown [4] apply the method to study the effects of annual earnings announcements. They use the residuals from a time series regression of the annual earnings of a firm on the average earnings of all their firms to classify the firm's earnings for a given year as having "increased" or "decreased" relative to the market. Residuals from regressions of monthly common stock returns on an index of returns (i.e., the market model of (12)) are then used to compute cumulative average return residuals separately for the earnings that "increased," and those that "decreased." The cumulative average return residuals rise throughout the year in advance of the announcement for the earnings "increased" category, and fall for the earnings "decreased" category.[29] Ball and Brown [4, p. 175] conclude that in fact no more than about ten to fifteen percent of the information in the annual earnings announcement has not been anticipated by the month of the announcement.

On the macro level, Waud [45] has used the method of residual analysis to examine the effects of announcements of discount rate changes by Federal Reserve Banks. In this case the residuals are essentially just the deviations of the daily returns on the Standard and Poor's 500 Index from the average daily return. He finds evidence of a statistically significant "announcement effect" on stock returns for the first trading day following an announcement, but the magnitude of the adjustment is small, never exceeding .5%. More interesting from the viewpoint of the efficient markets hypothesis is his conclusion that, if anything, the market anticipates the announcements (or information is somehow leaked in advance). This conclusion is based on the non-random patterns of the signs of average return residuals on the days immediately preceding the announcement.

Further evidence in support of the efficient markets hypothesis is provided in the work of Scholes [39] on large secondary offerings of common stock (ie., large underwritten sales of existing common stocks by individuals and institutions) and on new issues of stock. He finds that on average secondary issues are associated with a decline of between one and two per cent in the cumulative average residual returns for the corresponding common stocks. Since the magnitude of the price adjustment is unrelated to the size of the

29. But the comment of footnote 28 is again relevant here.

issue, Scholes concludes that the adjustment is not due to "selling pressure" (as is commonly believed), but rather results from negative information implicit in the fact that somebody is trying to sell a large block of a firm's stock. Moreover, he presents evidence that the value of the information in a secondary depends to some extent on the vendor; somewhat as would be expected, by far the largest negative cumulative average residuals occur where the vendor is the corporation itself or one of its officers, with investment companies a distant second. But the identity of the vendor is not generally known at the time of the secondary, and corporate insiders need only report their transactions in their own company's stock to the S.E.C. within six days after a sale. By this time the market on average has fully adjusted to the information in the secondary, as indicated by the fact that the average residuals behave randomly thereafter.

Note, however, that though this is evidence that prices adjust efficiently to public information, it is also evidence that corporate insiders at least sometimes have important information about their firm that is not yet publicly known. Thus Scholes' evidence for secondary distributions provides support for the efficient markets model in the semi-strong form sense, but also some strong-form evidence against the model.

Though his results here are only preliminary, Scholes also reports on an application of the method of residual analysis to a sample of 696 new issues of common stock during the period 1926-66. As in the FFJR study of splits, the cumulative average residuals rise in the months preceding the new security offering (suggesting that new issues tend to come after favorable recent events)[30] but behave randomly in the months following the offering (indicating that whatever information is contained in the new issue is on average fully reflected in the price of the month of the offering).

In short, the available semi-strong form evidence on the effect of various sorts of public announcements on common stock returns is all consistent with the efficient markets model. The strong point of the evidence, however, is its consistency rather than its quantity; in fact, few different types of public information have been examined, though those treated are among the obviously most important. Moreover, as we shall now see, the amount of semi-strong form evidence is voluminous compared to the strong form tests that are available.

C. *Strong Form Tests of the Efficient Markets Models*

The strong form tests of the efficient markets model are concerned with whether all available information is fully reflected in prices in the sense that no individual has higher expected trading profits than others because he has monopolistic access to some information. We would not, of course, expect this model to be an exact description of reality, and indeed, the preceding discussions have already indicated the existence of contradictory evidence. In particular, Niederhoffer and Osborne [32] have pointed out that specialists on the N.Y.S.E. apparently use their monopolistic access to information concern-

30. Footnote 28 is again relevant here.

ing unfilled limit orders to generate monopoly profits, and Scholes' evidence [39] indicates that officers of corporations sometimes have monopolistic access to information about their firms.

Since we already have enough evidence to determine that the model is not strictly valid, we can now turn to other interesting questions. Specifically, how far down through the investment community do deviations from the model permeate? Does it pay for the average investor (or the average economist) to expend resources searching out little known information? Are such activities even generally profitable for various groups of market "professionals"? More generally, who are the people in the investment community that have access to "special information"?

Though this is a fascinating problem, only one group has been studied in any depth—the managements of open end mutual funds. Several studies are available (e.g., Sharpe [41, 42] and Treynor [44]), but the most thorough are Jensen's [19, 20], and our comments will be limited to his work. We shall first present the theoretical model underlying his tests, and then go on to his empirical results.

1. Theoretical Framework

In studying the performance of mutual funds the major goals are to determine (a) whether in general fund managers seem to have access to special information which allows them to generate "abnormal" expected returns, and (b) whether some funds are better at uncovering such special information than others. Since the criterion will simply be the ability of funds to produce higher returns than some norm with no attempt to determine what is responsible for the high returns, the "special information" that leads to high performance could be either keener insight into the implications of publicly available information than is implicit in market prices or monopolistic access to specific information. Thus the tests of the performance of the mutual fund industry are not strictly strong form tests of the efficient markets model.

The major theoretical (and practical) problem in using the mutual fund industry to test the efficient markets model is developing a "norm" against which performance can be judged. The norm must represent the results of an investment policy based on the assumption that prices fully reflect all available information. And if one believes that investors are generally risk averse and so on average must be compensated for any risks undertaken, then one has the problem of finding appropriate definitions of risk and evaluating each fund relative to a norm with its chosen level of risk.

Jensen uses the Sharpe [40]-Lintner [24, 25] model of equilibrium expected returns discussed above to derive a norm consistent with these goals. From (14)-(16), in this model the expected return on an asset or portfolio j from t to t + 1 is

$$E(\tilde{r}_{j,t+1}|\Phi_t) = r_{f,t+1}\,[1 - \beta_j(\Phi_t)] + E(\tilde{r}_{m,t+1}|\Phi_t)\beta_j(\Phi_t), \tag{18}$$

where the various symbols are as defined in Section III. A. 6. But (18) is an *ex ante* relationship, and to evaluate performance an *ex post* norm is needed.

One way the latter can be obtained is to substitute the realized return on the market portfolio for the expected return in (18) with the result[31]

$$E(\tilde{r}_{j,t+1}|\Phi_t, r_{m,t+1}) = r_{f,t+1}\,[1 - \beta_j(\Phi_t)] + r_{m,t+1}\beta_j(\Phi_t). \qquad (19)$$

Geometrically, (19) says that within the context of the Sharpe-Lintner model, the expected return on j (given information Φ_t and the return $r_{m,t+1}$ on the market portfolio) is a linear function of its risk

$$\beta_j(\Phi_t) = \text{cov}\,(\tilde{r}_{j,t+1}, \tilde{r}_{m,t+1}|\Phi_t)/\sigma^2(\tilde{r}_{m,t+1}|\Phi_t),$$

as indicated in Figure 2. Assuming that the value of $\beta_j(\Phi_t)$ is somehow known, or can be reliably estimated, if j is a mutual fund, its *ex post* performance from t to t + 1 might now be evaluated by plotting its combination of realized return $r_{j,t+1}$ and risk in Figure 2. If (as for the point a) the combination falls above the expected return line (or, as it is more commonly called, the "market line"), it has done better than would be expected given its level of risk, while if (as for the point b) it falls below the line it has done worse.

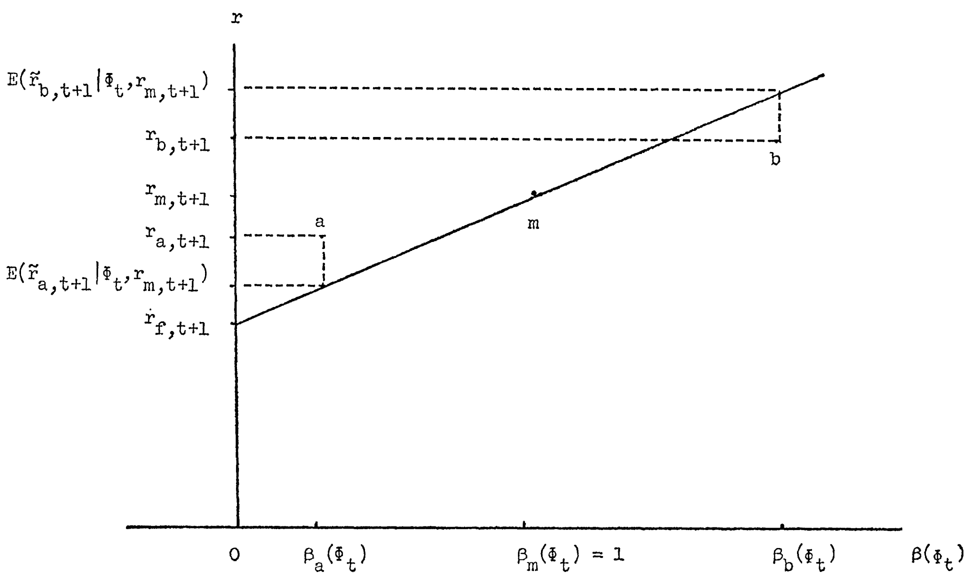

FIGURE 2
Performance Evaluation Graph

Alternatively, the market line shows the combinations of return and risk provided by portfolios that are simple mixtures of the riskless asset and the market portfolio m. The returns and risks for such portfolios (call them c) are

$$r_{c,t+1} = \alpha r_{f,t+1} + (1 - \alpha) r_{m,t+1}$$

$$\beta_c(\Phi_t) = \frac{\text{cov}\,(\tilde{r}_{c,t+1}, \tilde{r}_{m,t+1}|\Phi_t)}{\sigma^2(\tilde{r}_{m,t+1}|\Phi_t)} = \frac{\text{cov}\,((1-\alpha)\tilde{r}_{m,t+1}, \tilde{r}_{m,t+1}|\Phi_t)}{\sigma^2(\tilde{r}_{m,t+1}|\Phi_t)} = 1 - \alpha,$$

31. The assumption here is that the return $\tilde{r}_{j,t+1}$ is generated according to

$$\tilde{r}_{j,t+1} = r_{f,t+1}[1 - \beta_j(\Phi_t)] + r_{m,t+1}\beta_j(\Phi_t) + \tilde{u}_{j,t+1},$$

and

$$E(\tilde{u}_{j,t+1}|r_{m,t+1}) = 0 \text{ for all } r_{m,t+1}.$$

where α is the proportion of portfolio funds invested in the riskless asset. Thus, when $1 \geqslant \alpha \geqslant 0$ we obtain the combinations of return and risk along the market line from $r_{f,t+1}$ to m in Figure 2, while when $\alpha < 0$ (and under the assumption that investors can borrow at the same rate that they lend) we obtain the combinations of return and risk along the extension of the line through m. In this interpretation, the market line represents the results of a naive investment strategy, which the investor who thinks prices reflect all available information might follow. The performance of a mutual fund is then measured relative to this naive strategy.

2. Empirical Results

Jensen uses this risk-return framework to evaluate the performance of 115 mutual funds over the ten year period 1955-64. He argues at length for measuring return as the nominal ten year rate with continuous compounding (i.e., the natural log of the ratio of terminal wealth after ten years to initial wealth) and for using historical data on nominal one-year rates with continuous compounding to estimate risk. The Standard and Poor Index of 500 major common stocks is used as the proxy for the market portfolio.

The general question to be answered is whether mutual fund managements have any special insights or information which allows them to earn returns above the norm. But Jensen attacks the question on several levels. First, can the funds in general do well enough to compensate investors for loading charges, management fees, and other costs that might be avoided by simply choosing the combination of the riskless asset f and the market portfolio m with risk level comparable to that of the fund's actual portfolio? The answer seems to be an emphatic no. As far as net returns to investors are concerned, in 89 out of 115 cases, the fund's risk-return combination for the ten year period is below the market line for the period, and the average over all funds of the deviations of ten year returns from the market time is -14.6%. That is, on average the consumer's wealth after ten years of holding mutual funds is about fifteen per cent less than if he held the corresponding portfolios along the market line.

But the loading charge that an investor pays in buying into a fund is usually a pure salesman's commission that the fund itself never gets to invest. Thus one might ask whether, ignoring loading charges (i.e., assuming no such charges were paid by the investor), in general fund managements can earn returns sufficiently above the norm to cover all other expenses that are presumably more directly related to the management of the fund portfolios. Again, the answer seems to be no. Even when loading charges are ignored in computing returns, the risk-return combinations for 72 out of 115 funds are below the market line, and the average deviation of ten year returns from the market line is -8.9%.

Finally, as a somewhat stronger test of the efficient markets model, one would like to know if, ignoring all expenses, fund managements in general showed any ability to pick securities that outperformed the norm. Unfortunately, this question cannot be answered with precision for individual funds since, curiously, data on brokerage commissions are not published regularly.

But Jensen suggests the available evidence indicates that the answer to the question is again probably negative. Specifically, adding back all other published expenses of funds to their returns, the risk-return combinations for 58 out of 115 funds were below the market line, and the average deviation of ten year return from the line was -2.5%. But part of this result is due to the absence of a correction for brokerage commissions. Estimating these commissions from average portfolio turnover rates for all funds for the period 1953-58, and adding them back to returns for all funds increases the average deviation from the market line from -2.5% to $.09\%$, which still is not indicative of the existence of special information among mutual fund managers.

But though mutual fund managers in general do not seem to have access to information not already fully reflected in prices, perhaps there are individual funds that consistently do better than the norm, and so provide at least some strong form evidence against the efficient markets model. If there are such funds, however, they escape Jensen's search. For example, for individual funds, returns above the norm in one subperiod do not seem to be associated with performance above the norm in other subperiods. And regardless of how returns are measured (i.e., net or gross of loading charges and other expenses), the number of funds with large positive deviations of returns from the market line of Figure 2 is less than the number that would be expected by chance with 115 funds under the assumption that fund managements have no special talents in predicting returns.[32]

Jensen argues that though his results apply to only one segment of the investment community, they are nevertheless striking evidence in favor of the efficient markets model:

> Although these results certainly do not imply that the strong form of the martingale hypothesis holds for all investors and for all time, they provide strong evidence in support of that hypothesis. One must realize that these analysts are extremely well endowed. Moreover, they operate in the securities markets every day and have wide-ranging contacts and associations in both the business and financial communities. Thus, the fact that they are apparently unable to forecast returns accurately enough to recover their research and transactions costs is a striking piece of evidence in favor of the strong form of the martingale hypothesis—at least as far as the extensive subset of information available to these analysts is concerned [20, p. 170].

IV. Summary and Conclusions

The preceding (rather lengthy) analysis can be summarized as follows. In general terms, the theory of efficient markets is concerned with whether prices at any point in time "fully reflect" available information. The theory only has empirical content, however, within the context of a more specific model of

32. On the other hand, there is some suggestion in Scholes' [39] work on secondary issues that mutual funds may occassionally have access to "special information." After corporate insiders, the next largest negative price changes occur when the secondary seller is an investment company (including mutual funds), though on average the price changes are much smaller (i.e., closer to 0) than when the seller is a corporate insider.

Moreover, Jensen's evidence itself, though not indicative of the existence of special information among mutual fund managers, is not sufficiently precise to conclude that such information never exists. This stronger conclusion would require exact data on unavoidable expenses (including brokerage commissions) of portfolio management incurred by funds.

market equilibrium, that is, a model that specifies the nature of market equilibrium when prices "fully reflect" available information. We have seen that all of the available empirical literature is implicitly or explicitly based on the assumption that the conditions of market equilibrium can be stated in terms of expected returns. This assumption is the basis of the expected return or "fair game" efficient markets models.

The empirical work itself can be divided into three categories depending on the nature of the information subset of interest. *Strong-form* tests are concerned with whether individual investors or groups have monopolistic access to any information relevant for price formation. One would not expect such an extreme model to be an exact description of the world, and it is probably best viewed as a benchmark against which the importance of deviations from market efficiency can be judged. In the less restrictive *semi-strong-form* tests the information subset of interest includes all obviously publicly available information, while in the *weak form* tests the information subset is just historical price or return sequences.

Weak form tests of the efficient market model are the most voluminous, and it seems fair to say that the results are strongly in support. Though statistically significant evidence for dependence in successive price changes or returns has been found, some of this is consistent with the "fair game" model and the rest does not appear to be sufficient to declare the market inefficient. Indeed, at least for price changes or returns covering a day or longer, there isn't much evidence against the "fair game" model's more ambitious offspring, the random walk.

Thus, there is consistent evidence of positive dependence in day-to-day price changes and returns on common stocks, and the dependence is of a form that can be used as the basis of marginally profitable trading rules. In Fama's data [10] the dependence shows up as serial correlations that are consistently positive but also consistently close to zero, and as a slight tendency for observed numbers of runs of positive and negative price changes to be less than the numbers that would be expected from a purely random process. More important, the dependence also shows up in the filter tests of Alexander [1, 2] and those of Fama and Blume [13] as a tendency for very small filters to produce profits in excess of buy-and-hold. But any systems (like the filters) that attempt to turn short-term dependence into trading profits of necessity generate so many transactions that their expected profits would be absorbed by even the minimum commissions (security handling fees) that floor traders on major exchanges must pay. Thus, using a less than completely strict interpretation of market efficiency, this positive dependence does not seem of sufficient importance to warrant rejection of the efficient markets model.

Evidence in contradiction of the "fair game" efficient markets model for price changes or returns covering periods longer than a single day is more difficult to find. Cootner [9], and Moore [31] report preponderantly negative (but again small) serial correlations in weekly common stock returns, and this result appears also in the four day returns analyzed by Fama [10]. But it does not appear in runs tests of [10], where, if anything, there is some slight indication of positive dependence, but actually not much evidence of any dependence at all. In any case, there is no indication that whatever dependence exists in weekly returns can be used as the basis of profitable trading rules.

Other existing evidence of dependence in returns provides interesting insights into the process of price formation in the stock market, but it is not relevant for testing the efficient markets model. For example, Fama [10] shows that large daily price changes tend to be folowed by large changes, but of unpredictable sign. This suggests that important information cannot be completely evaluated immediately, but that the initial first day's adjustment of prices to the information is unbiased, which is sufficient for the martingale model. More interesting and important, however, is the Niederhoffer-Osborne [32] finding of a tendency toward excessive reversals in common stock price changes from transaction to transaction. They explain this as a logical result of the mechanism whereby orders to buy and sell at market are matched against existing limit orders on the books of the specialist. Given the way this tendency toward excessive reversals arises, however, there seems to be no way it can be used as the basis of a profitable trading rule. As they rightly claim, their results are a strong refutation of the theory of random walks, at least as applied to price changes from transaction to transaction, but they do not constitute refutation of the economically more relevant "fair game" efficient markets model.

Semi-strong form tests, in which prices are assumed to fully reflect all obviously publicly available information, have also supported the efficient markets hypothesis. Thus Fama, Fisher, Jensen, and Roll [14] find that the information in stock splits concerning the firm's future dividend payments is on average fully reflected in the price of a split share at the time of the split. Ball and Brown [4] and Scholes [39] come to similar conclusions with respect to the information contained in (i) annual earning announcements by firms and (ii) new issues and large block secondary issues of common stock. Though only a few different types of information generating events are represented here, they are among the more important, and the results are probably indicative of what can be expected in future studies.

As noted earlier, the strong-form efficient markets model, in which prices are assumed to fully reflect all available information, is probably best viewed as a benchmark against which deviations from market efficiency (interpreted in its strictest sense) can be judged. Two such deviations have in fact been observed. First, Niederhoffer and Osborne [32] point out that specialists on major security exchanges have monopolistic access to information on unexecuted limit orders and they use this information to generate trading profits. This raises the question of whether the "market making" function of the specialist (if indeed this is a meaningful economic function) could not as effectively be carried out by some other mechanism that did not imply monopolistic access to information. Second, Scholes [39] finds that, not unexpectedly, corporate insiders often have monopolistic access to information about their firms.

At the moment, however, corporate insiders and specialists are the only two groups whose monopolistic access to information has been documented. There is no evidence that deviations from the strong form of the efficient markets model permeate down any further through the investment community. For the purposes of most investors the efficient markets model seems a good first (and second) approximation to reality.

In short, the evidence in support of the efficient markets model is extensive, and (somewhat uniquely in economics) contradictory evidence is sparse. Nevertheless, we certainly do not want to leave the impression that all issues are closed. The old saw, "much remains to be done," is relevant here as elsewhere. Indeed, as is often the case in successful scientific research, now that we know we've been in the past, we are able to pose and (hopefully) to answer an even more interesting set of questions for the future. In this case the most pressing field of future endeavor is the development and testing of models of market equilibrium under uncertainty. When the process generating equilibrium expected returns is better understood (and assuming that some expected return model turns out to be relevant), we will have a more substantial framework for more sophisticated intersecurity tests of market efficiency.

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