Politecnico di Milano

Final project: Introduction to space missions' analysis
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1. Introduction

The main purpose of the assignment is to analyse different strategies of manoeuvring to transfer a satellite from a starting position given the velocity (position and velocity vector characterize the initial orbit) to a final position on a different orbit given the Keplerian parameters.
Working autonomously and in group we researched different strategies, each one of them with specific objectives and we compared and verified the results we obtained through calculations and charts.
Starting from the theoretical knowledge we learned during the course we decided to present three of the possible sequences of manoeuvring that we believe are of particular interest: a standard manoeuvre (aiming to find a compromise between the costs in terms of energy and time required) and two alternative manoeuvres to further decrease Δt and ΔV.

The spacecraft moves subjected to the gravitational influence of the Earth: all the considerations and calculations made are based on the two-body model, the gravitational parameter utilised is $\mu = 398600\ km^3/s^2$ and the manoeuvres are considered impulsive. Calculations and charts presented in the report have been realized with the software MATLAB®.

Table of the symbols used and units of measure.

a	Semi-major axis	km
e	Eccentricity	-
i	Inclination	rad
Ω	Right Ascension of the Ascending Node (RAAN)	rad
ω	Argument of pericentre	rad
θ	True anomaly	rad
Δt	Requested time	s
ΔV	Cost of manoeuvre	km/s
E	Specific energy	km^2/s^2
T	Orbital period	s
I, **J**, **K**	ECI coordinate system axis	-

2. Initial orbit characterisation

2.1 Initial orbit's parameters

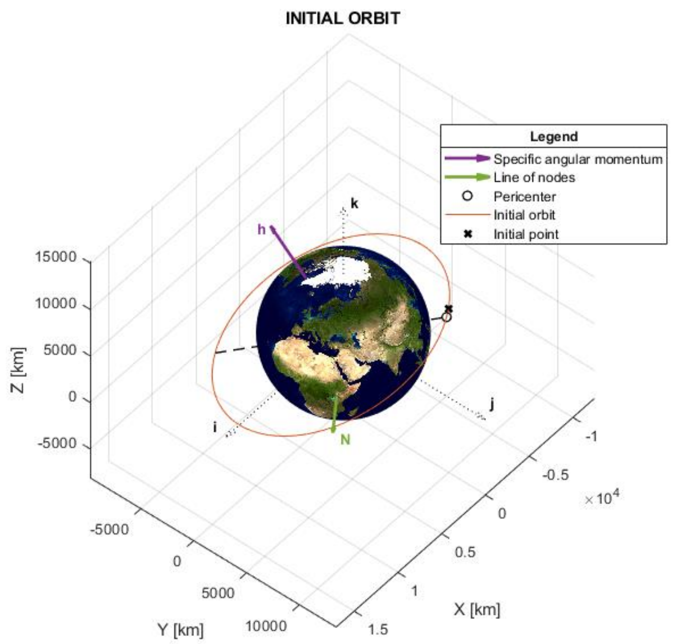
Given the position and velocity vectors of the starting point in the initial orbit in the ECI coordinate system (Earth-centred inertial), using the equation of the specific energy and the orbital parameters definitions, we determined the Keplerian orbital parameters that describe the initial orbit.

x (km)	y (km)	z (km)	v_x (km/s)	v_y (km/s)	v_z (km/s)
-4091.2866	6563.7099	3713.1316	-6.0730	-3.8110	0.1904

a (Km)	e (-)	i (rad)	Ω (rad)	ω (rad)	θ (rad)
9610.0091	0.1076	0.4482	0.6157	1.4280	0.0903

2.2 Initial orbit's features

The orbit has pericentre altitude above sea level of about 2205 km and altitude of apocentre of about 4273 km, so it can be defined as MEO (Medium Earth Orbit) included between the Van Allen belts and the geostationary orbit. It also has a low eccentricity, specific energy $E = -20.7388\ km^2/s^2$ and orbital period $T = 9376\ s = 156.27\ min$.
The true anomaly of the given initial position is close to 0 rad, so it is near the pericentre.



3. Final orbit characterisation

3.1 Final orbit's parameters

Given the Keplerian orbital parameters it is possible to obtain the position and velocity vectors in the perifocal coordinate system. Then, using the correct rotation matrixes we shifted the vectors into the ECI coordinate system.

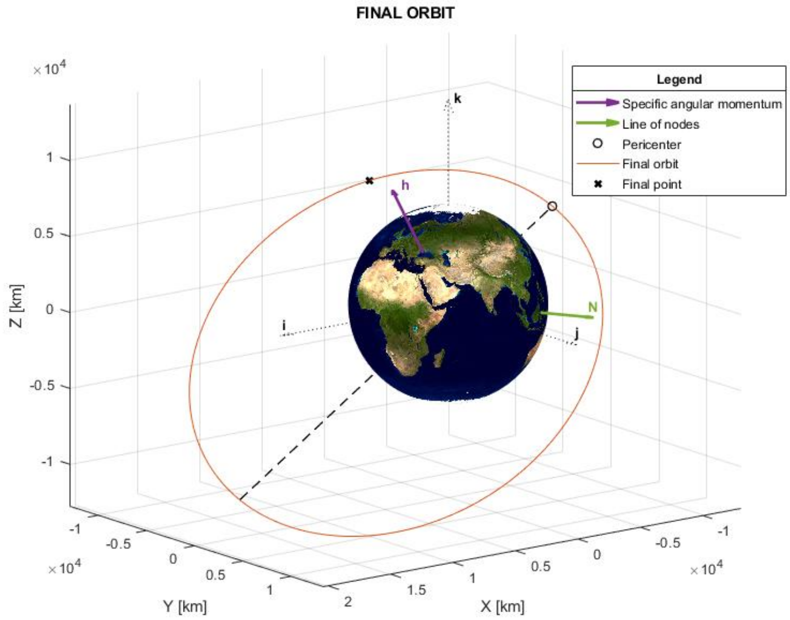
a (km)	e (-)	i (rad)	Ω (rad)	ω (rad)	θ (rad)
13960.0000	0.3339	0.7923	2.1220	1.0030	1.3000

x (km)	y (km)	z (km)	v_x (km/s)	v_y (km/s)	v_z (km/s)
-1079.2453	-9598.9116	6029.1334	4.7018	-3.9317	-1.9734

3.2 Final orbit's features

The orbit has pericentre altitude above sea level of about 2928 km and altitude of apocentre of 12250 km, so it can also be classified as MEO. It has eccentricity, specific energy, and orbital period greater than the initial orbit, in particular $E = -14.2765\ km^2/s^2$ and $T = 16415\ s = 273.58\ min$.

The final orbit has a different shape than the initial one and, with the values of inclination (i) and RAAN (Ω), it is possible to calculate $\Delta i = 0.3441$ rad and $\Delta\Omega = 1.5063$ rad. Therefore, the two orbits lie on different planes, and they do not have any intersection points.



4. Transfer trajectories definition and analysis

4.1 General considerations

In our report we focused on both the parameters Δt e ΔV necessary for the orbital transfer and we tried to find manoeuvres that would reduce one or the other or that gave a good compromise between the two parameters. Mainly starting from observations regarding the geometry of the problem, we realised different sequences of manoeuvring to be able to change the orbital plane (i, Ω), orbital shape (a, e) and the orientation of the orbit on the plane (ω) to arrive at the final point.

4.2 Standard strategy

The standard strategy for orbital transfer is composed of three distinct manoeuvres: variation of orbital plane, variation of orbital shape using the bitangent transfer, variation of pericentre anomaly.

In our standard strategy we focused on reducing the overall energy cost (ΔV), trying at the same time to contain the Δt.

Between the three main manoeuvres the variation of orbital plane is more expensive because it is directly proportional to the tangent velocity in the manoeuvring point.

Hence, it is convenient to do it after the change of orbital shape in order to have a lower tangent velocity. With these considerations in mind, we decided to execute the variation of orbital shape as the first manoeuvre of the sequence.

Between the possible bitangent transfers (pericentre to apocentre, apocentre to pericentre, pericentre to pericentre and apocentre to apocentre) the most convenient is the pericentre to apocentre one with ΔV = 0.9047 km/s.

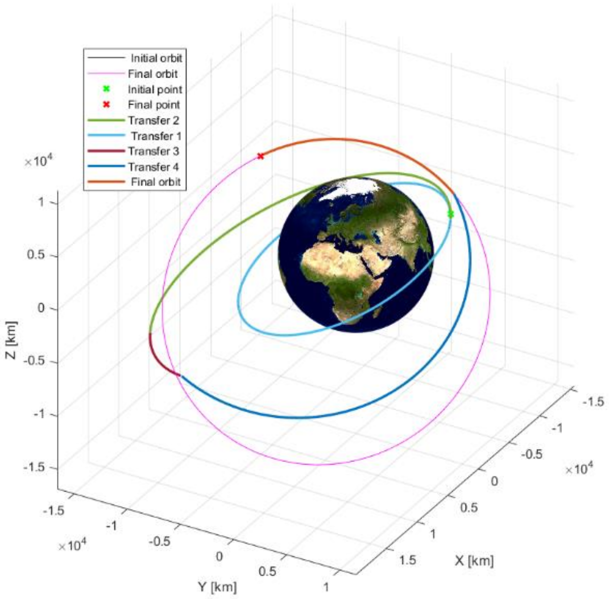
This manoeuvre is the most convenient in terms of fuel consumption and, even though it is not the best for Δt it allows to spare time in the following manoeuvres because the arrival point at the apocentre is closer to the point of orbital plane change.

In fact, the possible points of manoeuvring are at the intersection between the two planes containing the first and second orbit, in our case the two possible points are $\theta_1 = 0.4973$ rad and $\theta_2 = (0.4973 + \pi)$ rad. Since $\cos(\theta_2) < 0$ we will use the second point.

After these manoeuvres the satellite is on an intermediate orbit with orbital shape and plane equal to the final orbit and orientation as the initial orbit plus the parasitic Δω caused by the change of plane.

The last necessary manoeuvre is the variation of pericentre anomaly, this can be done in two points: θ_A = 0.4464 rad that is further from the apocentre but closer to the final point (on the new orbit θ = 5.8368 rad); θ_B = 3.5880 rad that is closer to the apocentre but further from the final point (on the new orbit θ = 2.6952 rad). Since the energetic cost for the manoeuvre is the same, we decided to execute it in the first point (θ_A) because it is more convenient for Δt.

4.2.1 Standard strategy sequence



Coasting phase from initial point θ = 0.0903 rad until pericentre θ = 0 rad for shape change (Δt = 9267 s)

Orbital shape change from pericentre to apocentre (coasting on the transfer orbit from pericentre to apocentre) (Δt = 7890 s) (ΔV = 0.9047 Km/s)

Coasting phase from apocentre until θ = 3.6389 rad for plane change (Δt = 2357s)

Plane change at θ = 3.6389 rad (ΔV = 3.3381 km/s)

Coasting phase from θ = 3.6389 rad to θ = 0.4464 rad for pericentre anomaly change (Δt = 6408 s)

Pericentre anomaly change in θ = 0.4464 rad (ΔV = 1.6343 km/s)

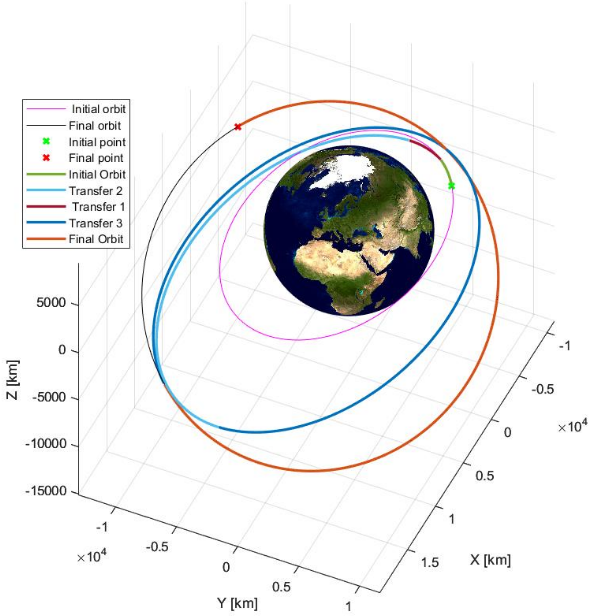
Coasting phase from θ = 5.8368 rad to the final point θ = 1.3000 rad (Δt = 2406 s)

Total Cost: 5.8771 km/s
Total Time: 28331 s (approximately 7.8 hours)

4.3 First alternative strategy

In the first alternative strategy our objective is to reduce the energetic cost, so decreasing the overall ΔV for the transfer despite it requires a higher Δt.
In our case the semi-major axis ratio $a_{final} / a_{initial}$ = 1.4527 could suggest that it would be not convenient to use bielliptic manoeuvres to change the orbital shape. Besides, although the plane change angle is α = 49.25°, we verified that a bielliptic strategy for the plane change manoeuvre is not convenient either.
Even if we circularise the initial orbit in order to try a bielliptic strategy on a bigger circular orbit, using a Hohmann manoeuvre for the shape change, it is not really useful.
So we focused on the pericentre anomaly change manoeuvre and we verified that executing this manoeuvre as the first of the sequence allows us to obtain an improvement in the overall efficiency of the transfer. Since the parasitic Δω caused by the plane change manoeuvre only depends on the inclination and RAAN of the initial and final orbit, thanks to the spherical geometry it is possible to calculate the parasitic $\Delta\omega = u_2 - u_1$ = -1.3178 rad. The main feature of this strategy consists in the execution of the first manoeuvre to reach the pericentre anomaly equal to $\omega_{final} - \Delta\omega$: in this way after the change of plane the satellite will have the orientation of the final orbit.
After the change of pericentre anomaly in the first available point (θ = 0.4464 rad), we executed a bitangent change of shape from pericentre to apocentre (a possible secant manoeuvre it is not energetically convenient). Lastly, we executed the change of plane in θ = 2.7461 rad arriving directly on the final orbit without necessity of any ulterior orientation adjustments.

4.3.1 First alternative strategy sequence



- Coasting phase from initial point θ = 0.0903 rad to θ = 0.4464 rad for pericentre anomaly change (Δt = 429 s)

  Pericentre anomaly change at θ = 0.4464 rad (ΔV = 0.6017 Km/s)
- Coasting phase from θ = 5.8368 rad to pericentre for orbital shape change (Δt = 537 s)
- Orbital shape change from pericentre to apocentre (coasting the transfer orbit from pericentre to apocentre) (Δt = 7890 s) (ΔV = 0.9047 Km/s)
- Coasting phase from the apocentre to θ = 2.7461 rad for plane change (Δt = 14514 s)

  Plane change in θ = 2.7461 rad (ΔV= 3.2687 km/s)
- Coasting phase from θ = 2.7461 rad to the final point θ = 1.3000 rad (Δt = 11957 s)

Total Cost: 4.7752 km/s
Total Time: 35328 s (approximately 9.8 hours)

4.4 Second alternative strategy

For this strategy we focused on finding the quickest transfer possible directly from the initial to the final point at the expense of a much higher energetic cost than the previous sequences. We first obtained the necessary parameters to define the transfer orbit plane using the initial and final position vectors rr_i and rr_f.

Versor perpendicular to the plane	Inclination	Ascending node	RAAN
h_n	i_t	N_t	Ω_t
$\dfrac{rr_i \,\Lambda\, rr_f}{\lVert rr_i \,\Lambda\, rr_f \rVert}$	$\mathrm{acos}\,(h_n \cdot \boldsymbol{K})$	$\dfrac{\boldsymbol{K} \,\Lambda\, h_n}{\lVert \boldsymbol{K} \,\Lambda\, h_n \rVert}$	$\mathrm{acos}\,(N_t \cdot \boldsymbol{I})$

Once we found these values, to define all the possible transfer orbits we solved the following system:

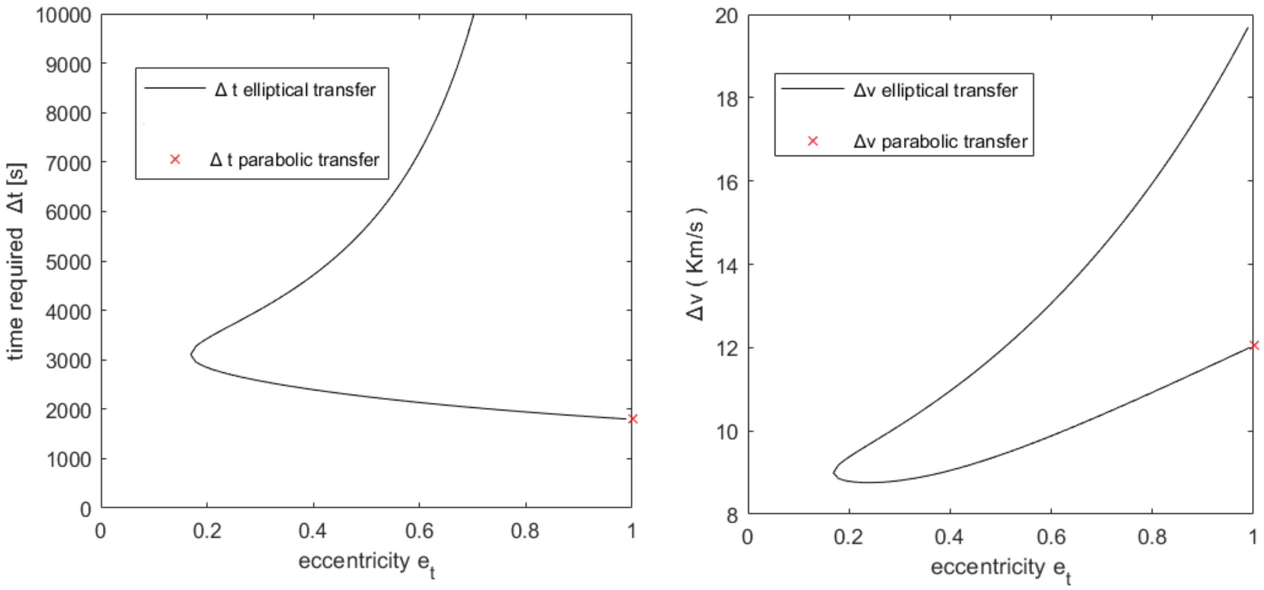
$$\begin{cases} r_i = \dfrac{p_t}{1 + e_t \cos(\theta_{t1})} \\ r_f = \dfrac{p_t}{1 + e_t \cos(\theta_{t1} + \Delta\theta)} \end{cases}$$

$$r_i = \frac{a_i \cdot (1 - e_i^2)}{1 + e_i \cos(\theta_i)}$$ Magnitude of rr_i

$$r_f = \frac{a_f \cdot (1 - e_f^2)}{1 + e_f cos(\theta_f)}$$ Magnitude of rr_f

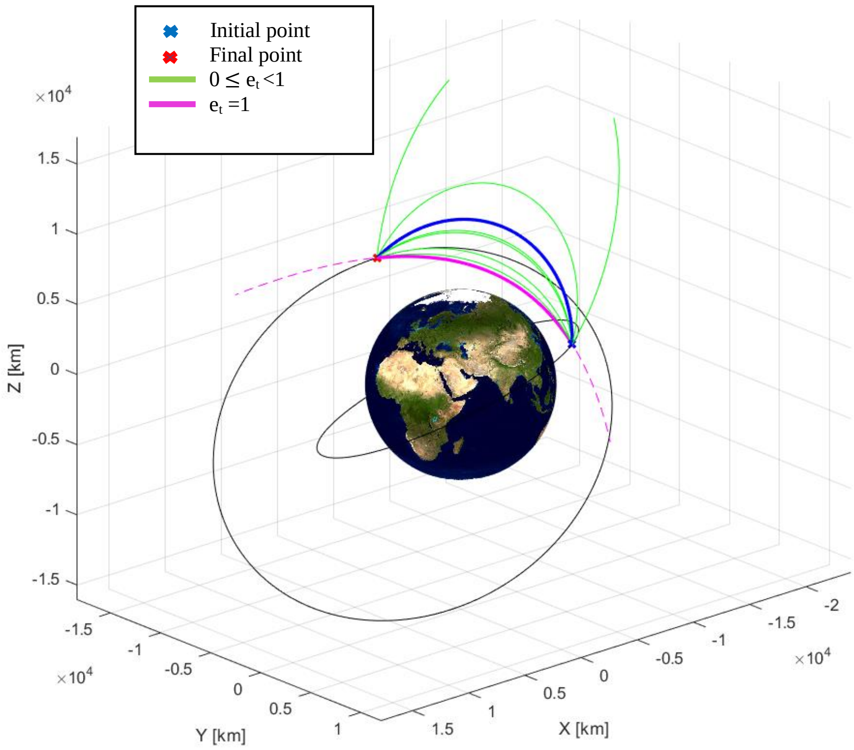
$p_t = a_t \cdot (1 - e_t{}^2)$	Semi-latus rectum elliptic transfer orbit ($0 \le e_t < 1$)
$p_t = 2 \cdot r_{pt}$	Semi-latus rectum parabolic transfer orbit ($e_t = 1$)
r_{pt}	Pericentre radius of the parabolic transfer orbit
θ_{t1}	True anomaly of the initial point on the transfer orbit
$\Delta\theta = acos\,(\frac{rr_i \cdot rr_f}{r_i \cdot r_f})$	Difference of anomaly between initial and final point on the transfer orbit

For each transfer orbit is also possible to obtain $\beta = \text{acos}\,(\frac{rr_i \cdot N_t}{r_i})$ that is the angle between the ascending line of nodes and the initial radius. Since $rr_i \cdot \boldsymbol{K} > 0$ we can calculate the pericentre anomaly as $\omega_t = \beta - \theta_{t1}$.
By varying the eccentricity as a parameter, we obtained the energetic and temporal costs for all the possible elliptic and parabolic orbits.



From observing the obtained plots, it is clear that by increasing the eccentricity the transfer time decreases at the expense of a significant increase in ΔV and we have the minimum Δt for $e_t = 1$ (we decided to not take into consideration iperbolic orbits with $e_t > 1$).
However, the energetic cost for the parabolic manoeuvre is very high especially due to the difference of inclination between the orbits. Therefore, we believe it is significant to consider two cases: the direct transfer with minimum Δt (which was the objective of this strategy) and the case with lower ΔV between the direct transfers (which we verified to be for $e_t = 0.2391$).

4.4.1 Second alternative strategy sequences



Transfer 2a (parabolic orbit)

- Immediate change of plane and acceleration at the initial point (ΔV = 8.2850 km/s) (θ_i = 0.0903 rad → θ_{t1} = -0.7103 rad)
- Coasting phase from θ = -0.7103 rad to θ = 1.2401 rad (Δt = 1795 s)
- Change of plane and deceleration at the final point (ΔV = 3.7602 km/s) (θ_{tf} = 1.2401 rad → θ_f = 1.3000 rad)

Total Cost: 12.0452 km/s
Total Time: 1795 s (approximately 30 min)

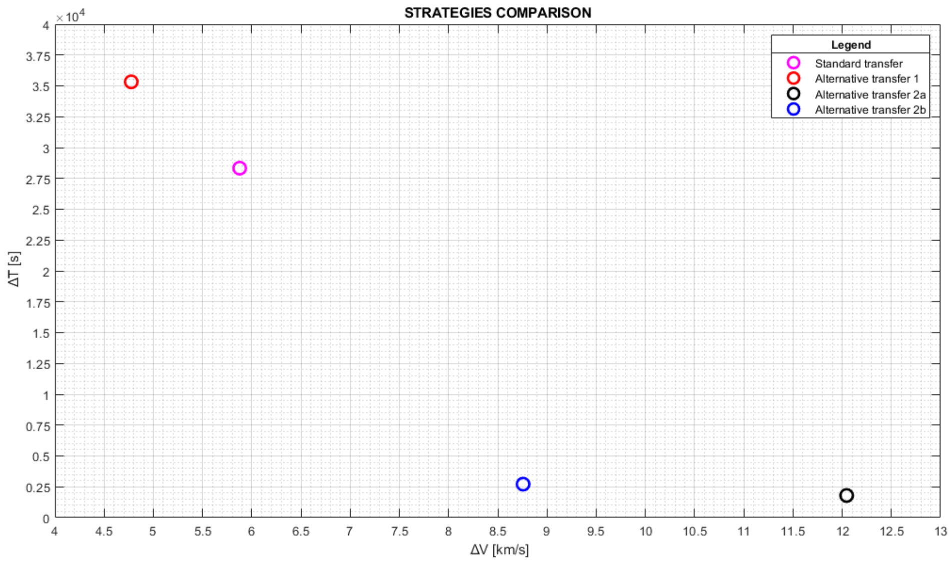
Transfer 2b (elliptic orbit)

- Immediate change of plane and acceleration at the initial point (ΔV = 6.7159 km/s) (θ_i = 0.0903 rad → θ_{t1} = -0.0949 rad)
- Coasting phase from θ = -0.0949 rad to θ = 1.8556 rad (Δt = 2715 s)
- Change of plane and deceleration at the final point (ΔV = 2.0417 km/s) (θ_{tf} = 1.8556 rad → θ_f = 1.3000 rad)

Total Cost: 8.7576 km/s
Total Time: 2715 s (approximately 45 min)

5. Conclusions

Analysing the results, it is evident that the standard strategy is not best in terms of time spent and neither in terms of energetic cost.
Considering this manoeuvre as reference we analysed the alternatives we proposed. The first alternative strategy reduces the energetic cost by 19% against the increase in time required by 25%.
Since the transfer is overall expensive mainly due to the elevated Δi between initial and final orbit, using a strategy as this one that can reduce the consumption of propellant can be very important, in particular for long-term space missions.
The strategy 2a, on the contrary, minimizes the required time by reducing it by 94% at the expense of a more than doubled energetic consumption: the total ΔV is too elevated. For example, using a range value of specific impulse $I_{sp} = 200 \div 400s$, it would require a $m_{propellant}/m_0 = 0.9978 \div 0.9537$.
Lastly, the strategy 2b reduces the time by 90% and increases by 49% the energetic cost: the required fuel mass is still very elevated (using the previous I_{sp} hypothetical range value $m_{propellant}/m_0 = 0.9885 \div 0.8929$) but it could be taken into consideration for specific missions that require a very quick transfer, without worrying about other factors such as fuel consumption or payload.



6. Appendix

Standard strategy

t (s)	a (km)	e (-)	i (rad)	Ω (rad)	ω (rad)	θ (rad)	Δv (km/s)
0	9610.0091	0.1076	0.4482	0.6157	1.4280	0.0903	-
9267	9610.0091	0.1076	0.4482	0.6157	1.4280	0	0.8028
	13598	0.3693	0.4482	0.6157	1.4280	0	
17159	13598	0.3693	0.4482	0.6157	1.4280	3.1416	0.1018
	13960.0000	0.3339	0.4482	0.6157	1.4280	3.1416	
19516	13960.0000	0.3339	0.4482	0.6157	1.4280	3.6389	3.3381
	13960.0000	0.3339	0.7923	2.1220	0.1102	3.6389	
25924	13960.0000	0.3339	0.7923	2.1220	0.1102	0.4464	1.6343
	13960.0000	0.3339	0.7923	2.1220	1.0030	5.8368	
28331	13960.0000	0.3339	0.7923	2.1220	1.0030	1.3000	-

First alternative strategy

t (s)	a (km)	e (-)	i (rad)	Ω (rad)	ω (rad)	θ (rad)	Δv (km/s)
0	9610.0091	0.1076	0.4482	0.6157	1.4280	0.0903	-
429	9610.0091	0.1076	0.4482	0.6157	1.4280	0.4464	0.6017
	9610.0091	0.1076	0.4482	0.6157	2.3208	5.8368	
966	9610.0091	0.1076	0.4482	0.6157	2.3208	0	0.8029
	13598	0.3693	0.4482	0.6157	2.3208	0	
8857	13598	0.3693	0.4482	0.6157	2.3208	3.1416	0.1018
	13960.0000	0.3339	0.4482	0.6157	2.3208	3.1416	
23371	13960.0000	0.3339	0.4482	0.6157	2.3208	2.7461	3.2687
	13960.0000	0.3339	0.7923	2.1220	1.0030	2.7461	
35328	13960.0000	0.3339	0.7923	2.1220	1.0030	1.3000	-

Second alternative strategy (transfer 2a)

t (s)	a (km)	e (-)	i (rad)	Ω (rad)	ω (rad)	θ (rad)	Δv (km/s)
0	9610.0091	0.1076	0.4482	0.6157	1.4280	0.0903	8.2850
	-	1	1.0346	1.8389	1.2379	5.5729	
1795	-	1	1.0346	1.8389	1.2379	1.2401	3.7602
	13960.0000	0.3339	0.7923	2.1220	1.0030	1.3000	

Second alternative strategy (transfer 2b)

t (s)	a (km)	e (-)	i (rad)	Ω (rad)	ω (rad)	θ (rad)	Δv (km/s)
0	9610.0091	0.1076	0.4482	0.6157	1.4280	0.0903	6.7157
	11264	0.2390	1.0346	1.8389	1.2379	6.1888	
2715	11264	0.2390	1.0346	1.8389	1.2379	1.8560	2.0418
	13960.0000	0.3339	0.7923	2.1220	1.0030	1.3000	