Barcelona Graduate School of Economics

Efficient Estimation of Dynamic Conditional Correlation Models

Euan Dowers

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Contents

1 Introduction

The aim of this paper is to allow the DCC model to be efficiently estimated in large dimensions, using two innovations. The first, due to Engle, Ledoit and Wolf [?], is to use the linear shrinkage method of Ledoit and Wolf [?] to estimate the correlation targeting matrix of the DCC model, and the second, which is the original innovation of this paper, is to calculate the log-likelihood of the DCC model using a series of rank-one updates to the cholesky decomposition of the quasi-correlation matrix at each time period, thus avoiding having to refactor this matrix at every time period.

The structure of this report is as follows: Section 2 will describe the Dynamic Conditional Correlation model, including the estimation problem; Section 5 will describe the linear shrinkage estimation for covariance matrices of Ledoit and Wolf.

2 Dynamic Conditional Correlation Model

In order to describe the Dynamic Conditional Correlation Model, it is necessary to build up some notation.

1. Let $r_{i,t}$ denote the return of asset i at time t, and r_t be the N-dimensional vector of all returns at time t.
2. Let $d_{i,t}^2 = V_{t-1}(r_{i,t})$ be the conditional variance of asset i at time t, given information up to time $t-1$.
3. Let D_t be a diagonal matrix whose i-th element is d_{it}.
4. Let H_t be the conditional covariance matrix whose i, j-th element is the conditional covariance between assets i and j at time t.
5. Let $\epsilon_{i,t} = \frac{r_{i,t}}{d_{i,t}}$ be the standardised residuals of asset i at time t.
6. Let R_t be the conditional correlation matrix, whose i,j-th entry is given by the conditional correlation between asset i and asset j at time t.
7. Let σ_i^2 be the unconditional variance of the series r_i, t
8. Let R be the unconditional correlation matrix of the system.

2.1 Dynamic Conditional Correlation Model

The Dynamic Conditional Correlation model of Engle [1] is as follows. Assume we have N financial assets, and we observe for each of these assets a series of returns over a time period $1, \ldots, T$.

Since the conditional correlation and conditional covariance are related by the equation

$$\rho_{i,j,t} = \frac{E_{t-1}((r_{i,t} - E_{t-1}(r_{i,t}))(r_{j,t} - E_{t-1}(r_{j,t})))}{\sqrt{V_{t-1}(r_{i,t})V_{t-1}(y_{j,t})}}, \tag{2.1}$$

these entries are therefore also given by

$$\frac{H_{i,j,t}}{\sqrt{H_{i,i,t}H_{j,j,t}}}. \tag{2.2}$$

Therefore, the conditional correlation matrix and conditional variance matrix are given by

$$R_t = D_t^{-1} H_t D_t^{-1} \quad D_t^2 = diag[H_t] \tag{2.3}$$

To model the quantities $d_{i,t}$ we apply the univariate GARCH(1,1) model to each asset serparately to obtain the dynamics of its conditional variance.

$$H_{i,i,t} = \omega_i + \alpha_i r_{t-1}^2 + \beta_i H_{i,i,t-1}. \tag{2.4}$$

Then, the standardised residuals $\epsilon_{i,t}$ are calculated as described above.

In the mean-reverting DCC model that we use, the dynamics of the quasi-correlation matrix Q_t are governed by the process

$$Q_t = \Omega + \alpha\epsilon_{t-1}\epsilon_{t-1}^T + \beta Q_{t-1}. \tag{2.5}$$

In the correlation targeting version of this model, the intercept matrix Ω is given

$$\Omega = (1 - \alpha - \beta)R \tag{2.6}$$

3 Shrinkage Estimation

4 DCC Model Estimation

5 DCC Model Estimation

References

[1] Engle, R. *Dynamic Conditional Correlation - a simple class of multivariate models.* Journal of Business and Economic Statistics, 2002.

[2] Bollerslev, D *Dynamic Conditional Correlation - a simple class of multivariate models.* Journal of Business and Economic Statistics, 1990.

[3] Engle, R. *Anticipating Correlations: a new paradigm for risk management*