Barcelona Graduate School of Economics

Maximum Likelihood Estimation of APARCH(1,1) Model

Laura Roman, Angus McKay, Veronika Kyuchukova, Euan Dowers

March 26, 2017

APARCH(1,1) Model

Consider a time series

$$r_t = \mu + \epsilon_t$$

with

$$\epsilon_t = \sigma_t z_t$$

z_t standard Gaussian and

$$\sigma_t^\delta = \omega + \alpha(|\epsilon_{t-1}| - \gamma\epsilon_{t-1})^\delta + \beta\sigma_{t-1}^\delta.$$

We want to estimate $\mu, \omega, \alpha, \beta, \delta$. We will do this by maximising the conditional likelihood of ϵ_t, which is distributed as

$$\mathcal{N}(0, \sigma_t^2).$$

Given data $\hat{r}_t$ and estimates $\hat{\mu}, \hat{\omega}, \hat{\alpha}, \hat{\beta}, \hat{\delta}$ we can choose an initial $\hat{\sigma}_0^\delta$ and calculate subsequent $\hat{\sigma}_t^\delta$ by the equation above.

Algorithm

We initialise $\hat{\sigma}_0^\delta$ by calculating the mean of ϵ_t^2 using $t \in \{1, \ldots, 10\}$, which gives an estimate of the unconditional variance σ^2, which we then take to the power $\frac{\delta}{2}$ to obtain $\hat{\sigma}_0^\delta$.

We then compute subsequent σ_t^δ and use these to compute a conditional log-likelihood of our observations. The function that computes this log-likelihood is then simultaneously maximised over our parameters.

The final step involves calculating a numerical Hessian matrix by finite difference approximations, which is used to calculate standard errors, which are used for inference on our parameter estimates.