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Facultat de Matemàtiques i Estadística

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Bachelor's Degree Thesis

A learning approach to the FOM problem

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Abstract

The family of modes (FOM) approach to solving model predictive control (MPC) problems is a novel heuristic technique developed at MIT to solve the time complexity of traditional MPC solving techniques. This study addresses some of the issues associated with the previous formulations of this technique by increasing the sequential robustness of the FOM and providing methodologies to choose the parameters required for the heuristic. A general simulation interface is developed together with techniques to score and compare the obtained trajectories. Then an statistics and machine learning based methodology is developed to tune the parameters is proposed and the results are compared with the original ones. Finally, experimental procedures are developed to validate the results.

Keywords

Manipulation, Online Control, MPC, FOM, Underactuation, Hybridness

1. Introduction

Recently, there has been a lot of progress in the worlds of manipulation and locomotion. ADD SOME EXAMPLES HERE.

Complementarity constraints led to hybrid state formulations. MPC allowed to convexify those formulations. MPC grows exponential with the number of lookahead time steps.

Feedback is hard/requires effort. All the teams in the ARC used open loop systems (as of the 2nd edition of the competition) [add reference]

Feedback is important. Robots fall because control in hybrid systems still has a long way to go.

This allowed to tackle problems as state hybridness (ADD COMPLEMENTARITY CONSTRAINTS AND HYBRID PROBLEMS). This allowed further progress into the planning branch of these problems. Unluckily, the complexity of the traditional formulations for these problems requires discretization of the time space of the problem and the algorithmic cost of the used solvers usually grows exponentially with the number of considered time steps. This has been translated into a relatively lower progress of the correspondent online control branch.

1.1 Prior Work

1.2 Complementarity Constraints

The Complementarity constraint formulation [TODO: INSERT REFERENCES] presented a systematic way of formulating hybrid system problems and the work of Anitescu and Potra [1] guaranteed a solution for this formulation on multi-rigid-body Coulomb friction contact problems and impact problems with friction and nonzero restitution coefficient. However, the complementarity constraints are generally non convex and using them in practical problems without further treatment makes the problem non-computable.

1.3 Model Predictive Control

To tackle the computing cost of complementarity constraints, model predictive control(MPC) is usually used. This formulation defines a discrete set of positions (x_1, \dots, x_n) and control inputs (u_0, \dots, u_{n-1}) and assumes that the hybrid state the system is in time t_i cannot change until time t_{i+1} . Given convex dynamics and cost function and fixing the state the system is during each time transition, the optimization problem on the control inputs is convex [TODO: Citation]. Thus, the global problem can be solved by addressing each of the convex fixed hybrid state sub-problems. As the number of sub-problems grows exponentially with the number of time steps, a mixed integer programming (MIP) strategy is usually adopted. The drawbacks of MIP is that it doesn't escalate well with the number of possible hybrid states and, even with a small number of states, the optimizers aren't generally fast enough to solve the problem online, as required in control problems.

[TODO: Add this: Es pot representar el conjunt de possibles problemes de programacio lineal que s'han de resoldre a traves d'un arbre. Cada node representa un dels tres estats possibles. L'arrel de l'arbre es un node buit i a cada pas de la trajectoria l'arbre es ramifica 3 nodes (un per cada possible estat). Cada cami des de l'arrel fins a una de les fulles defineix de manera unica un problema a solucionar.]

1.4 Family of Modes

Recently, Hogan and Rodriguez [3] proposed an heuristic based procedure to solve MPC problems online called Family of Modes (FOM). In their work, they assume a finite horizon MPC problem. Periodically, the heuristic method explores the optimization problem over only a small set of all the possible combinations of hybrid states to decide the instantaneous controller policy. This policy is then recomputed frequently to take into account new information as the finite horizon advances in time as well as to tackle external interference with the system and errors in the dynamic model.

FOM is applied to solve a planar system formed by a point pusher and a square slider supported on a board. The contact between the pusher and the slider is modelled as a single point Coulomb contact. Three possible hybrid states are considered with regards to this contact point: pushing without sliding and pushing while sliding in each of the two possible directions. The contact between the slider and the board is modelled by deriving the limit surface of the slider [TODO: Citation]. The motion and cone constraints are obtained by assuming a quasi-static formulation and linearising the motion equations and the motion cone [TODO: explain more and explain why we use full dynamics on the first step]. By exploring only three pre-chosen modes at every step their controller is able to successfully track a straight trajectory or pass through a set of waypoints even under external perturbations or perturbations on the dynamical model used. [TODO: add this:(per exemple, podriem estar esperant que en un pas el dit robotic es mantingues adherit, pero el coeficient de fricció pot ser diferent a la zona on esta en aquell moment i lliscar en comptes de mantenir-se adherit)]

However, the method depends on the chosen modes to explore. In this simple case a good set of modes could be derived intuitively or by trial and error, but for more complex and higher dimensional problems, this may not be possible. Furthermore, adding more modes to consider into this heuristic doesn't generally translate into better results and, in some cases, it may lead to results far worse than the original one. So even guessing a good set of modes doesn't provide an easy or systematic way of improving the obtained result.

In this study the FOM heuristic is reinforced to provide non-worsening local solutions to the optimization problem as new modes are added into the family and a systematic way of choosing the initial modes. Finally, a generalized method is proposed to track complex trajectories. [TODO: Explain better].

1.5 Nomenclature

From here on:

- (1) `hybrid state` or `state`: Each of the possible dynamic states the system can be in. [TODO: further define]
- (2) `hybrid mode` or `mode`: Ordered set of states. It defines a finite horizon convex optimization program.
- (3) `family of modes` or `family`: Unordered set of modes. Corresponds to each of the convex problems explored at each iteration of the FOM heuristic. [TODO: Formally define to avoid any kind of ambiguities]

1.6 Sequential Properties

Let's cite [2]

2. My Work. TODO: Change name

2.1 Chameleon Mode

Un dels defectes era el fet que en cada iteració només s'executes el primer pas de la trajectòria obtinguda. Això feia que la immensa majoria de famílies de camins fossin, a efectes pràctics, inútils. Per mostrar un exemple senzill, es pot considerar el cas en que la família conte dos camins de tres iteracions, per exemple:

$$\text{cami1} = \text{adherencia}, \text{lliscar}_a \text{munt}, \text{adherencia} \quad \text{cami2} = \text{adherencia}, \text{lliscar}_a \text{vall}, \text{adherencia}$$

. Encara que entre els dos camins es tingui en compte la totalitat dels estats possibles, el dit robòtic mai lliscara, ja que el primer pas dels dos camins és adherència, i el controlador només executa el primer pas. Vaig estar mirant maneres de solucionar aquest problema i, si es volien mantenir camins estàtics (és a dir, executar sempre els mateixos camins en cada iteració), el nombre de camins necessaris també creixia molt ràpidament, ja que per cada camí, hauries d'afegir totes les possibles translacions periòdiques del camí.

Vaig considerar camins dinàmics i vaig obtenir dues possibles solucions. La primera era principalment imaginar els camins com a camins periòdics i traslladar-los una posició al final de cada iteració. La segona era afegir un mode camaleó, que en cada iteració simulés la continuació del camí seleccionat com a solució de la iteració anterior.

2.2 Simulation Interface

De moment he decidit utilitzar mateixos costos que s'utilitzen localment per solucionar els problemes de programació lineal en l'horitzó reduït per avaluar la solució total, ja que això em permet garantir alguns resultats teòrics, com ara que, a part d'errors numèrics al solucionar el MIP, la solució del MIP ha de ser millor que la de la família de modes.

Aquests últims mesos he estat programant un codi en MATLAB per simular els dos mètodes (família de modes i MIP) basant-me en el codi anterior del meu company de laboratori. Ell tenia un codi prototip per solucionar un cas molt concret: família de modes amb tres camins de 35 iteracions que segueix una línia recta. Ho he reestructurat tot per tal que es puguin generar camins arbitraris que combinin els estats que jo defineixi i seguir trajectòries arbitràries. Això ha de permetre, per una banda, estendre el problema a casos més complexos (trajectòries més complexes, un dit que pugui canviar la cara del cub que empeny, múltiples dits que empenyin diferents cares, etc).

2.3 Systematic Protocol to Choose the Modes of the Family

Initially, a single trajectory tracking learning protocol was devised. A single euler integration step of the controlling policy was executed with a lot of initial conditions homogeneously distributed in a four dimensional ball centered around the expected initial conditions. This guaranteed enough generality to represent any possible situation in a static trajectory (either a line or a circle). The cost for every initial

condition and hybrid mode was recorded. The cost was normalized for every experiment across all the hybrid states and then the k modes with highest normalized mean were chosen.

After testing it, the same procedure was applied to a wide range of circular trajectories as well as a line trajectory and the best modes for each curvature were chosen.

Finally, at every iteration of the controlling policy, the local curvature is computed and the adequate family is used.

3. Experiments

4. TODO stuff

I have to talk about close loops and about manipulation. then first approach to the family of modes. Then sequential consistency, improvement.

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6. Environments

In this section you can see examples of different environments.

6.1 Theorem-like environments

Theorem 6.1. *This is an example of a theorem, numbered with the section number.*

Proposition 6.2. *This is a proposition, numbered the same way. You can reference theorems and propositions using the labels, see for instance Theorem [6.1](#).*

Lemma 6.3. *This is a lemma, numbered the same way.*

Corollary 6.4. *This is a corollary, numbered the same way.*

Conjecture 6.5. *This is a conjecture, numbered the same way.*

If you need any other environment, you can add it to the preamble following the examples.

6.2 Definition-like environments

Definition 6.6. This is a definition. Notice that the style is different than in theorems.

Example 6.7. This is an example. Same style as definitions.

6.3 Remark-like environments

Remark 6.8. This is a remark.

Remark. This is a remark with no numbered label. You may create other environments with no numbered label by copying from this example.

7. Figures

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<http://www.kwasan.kyoto-u.ac.jp/solarb6/usinggraphicx.pdf>

8. Mathematics and packages

By default, the following packages are uploaded:

- (1) `enumerate`: It allows you to make list with specific somehow arbitrary labels, like this one.
- (2) `amsthm`: To make environments with different styles.

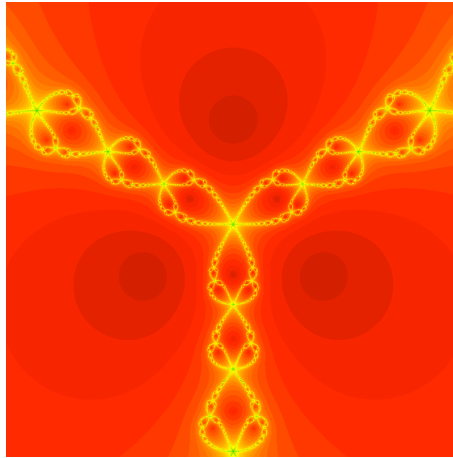


Figure 1: The caption of this figure is "Newton's method of a cubic polynomial".

- (3) `amsmath`, `amssymb`, `amsfonts`: Multiple mathematics symbols and fonts.
- (4) `graphicx`: To include figures in a simple and intuitive way.
- (5) `amscd`: To make commutative diagram with horizontal and vertical arrows. See below.
- (6) `xy`: To make really fancy commutative arrows. See below.
- (7) `booktabs`: To make fancy tables.

You may add other standard packages if you need them but try to avoid it if at all possible.

If you need to use them, you will find information about these packages in the usual internet places.

Here are examples of two commutative diagrams, one made with the package `amscd` and the other one with `xy`.

$$\begin{array}{ccc} A & \xrightarrow{g} & B \\ \downarrow \pi & & \downarrow \pi \\ X & \xrightarrow{f} & Y \end{array}$$

9. Bibliography

You may include your references by hand using the `bibliography` (see an example below) or, alternatively, you may use a `.bib` file and use BibTeX. In any case, we ask you to use a reasonable **consistent** format for all your references. Our recommendation is using BibTeX with the style "plain" or "amsalpha".

References

- [1] M. Anitescu and F. Potra. Formulating dynamic multi-rigid-body contact problems with friction as solvable linear complementarity problems. 14(93):231–247, 1997.
- [2] Dimitri P. Bertsekas, John N. Tsitsiklis, and Cynara. Wu. Rollout algorithms for combinatorial optimization. 3(3):245–262, 1997.
- [3] Franois Hogan and Alberto Rodriguez. Feedback control of the pusher-slider system: A story of hybrid and underactuated contact dynamics. 2016.

A. Title of the appendix

You can include here an appendix with details that can not be included in the core of the document. You should reference the sections in this appendix in the core document.

B. Title of the appendix

Second appendix.