2nd August, 2020

To: Professor Sury,
cc: Mr. Wu

From:
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Following our recent discussions, our team assessed your current portfolio and prepared the following analysis. To restate our intentions, our goal is to review a series of broad asset classes to clearly identify any potential opportunities to improve your current allocation, either via enhanced returns or reduced risk. The broad indices and respective publicly traded versions are listed below.

ASSET CLASSES	ACCEPTABLE PROXY/TICKER
S&P 500	SPY
DJIA	DIA
Nasdaq 100	QQQ
Barclays Aggregate	AGG
Gold	GLD
Russell 2000	IWM
High Yield Bonds	YLD

The team evaluated the trailing 5-years monthly return data for each proxy, adjusted for dividends and corporate actions, to provide a suitable measure of expected return. Using a set of optimally designed portfolios, solved for specified levels of return and deviation, we are able to visualize your current portfolio and provide a series of preferable alternatives. As the findings included below will support, our team strongly suggests a reallocation to one of these portfolios along the Efficient Frontier.

Step 1: Efficient Frontier

As we agreed, our output is included in both a Matrix and a long-hand approach for your evaluation. Both approaches lead to the same results. In the following section, we will walk through the process we took for each approach and how we arrived at our conclusion.

Under the long-hand approach:

To perform the long-hand approach, we performed matrix multiplication on the A-inverse matrix and b-matrix. By multiplying these two matrices together with varying μ_0 values, we were able to determine different weights per asset class, along with restraints λ_1 and λ_2. The objective of the long-hand approach was to minimize portfolio variance, such that the sum of the asset weights total 1 and the expected portfolio return equals the specified μ_0 or return value. In the place of Excel functions such as MMULT, TRANSPOSE or MINVERSE, our group utilized simple multiplication by referencing individual cells on our worksheet.

Though the Excel function ‘SUMPRODUCT’ is traditionally used in this scenario, the function was not working so we had to perform the “very long-hand approach”. The process for each individual μ_0 was to multiply each row in the A-inverse matrix with the entire b-matrix and then sum the products.

A^(-1)

11845.34	-5047.77	-2852.95	424.56	-287.47	-1659.07	-2422.64	0.46	-138.13
-5047.77	4032.47	454.83	-67.20	-31.74	137.76	521.65	-0.15	97.33
-2852.95	454.83	1152.00	-65.05	-105.21	363.30	1053.08	-0.35	83.72
424.56	-67.20	-65.05	982.33	-283.20	190.95	-1182.38	1.18	-24.33
-287.47	-31.74	-105.21	-283.20	445.78	332.95	-71.11	-0.21	27.87
-1659.07	137.76	363.30	190.95	332.95	1227.80	-593.69	0.00	0.64
-2422.64	521.65	1053.08	-1182.38	-71.11	-593.69	2695.08	0.06	-47.11
0.46	-0.15	-0.35	1.18	-0.21	0.00	0.06	0.00	0.03
-138.13	97.33	83.72	-24.33	27.87	0.64	-47.11	0.03	-7.83

b

0
0
0
0
0
0
0
1
0.004

For example, we multiplied 11,845.34 by 0 and then added it to the sum of the product of -5,047.77 and 0 and so on. Once this process had been completed for the entire first row of the A-inverse matrix, the result would indicate the weight for SPY in a portfolio with an overall return of 0.004 (μ_0) in the x-matrix. After that, the same process was carried out for the second row, then the third, and so on until all rows had gone through this process. We did this for the following 13 μ_0, which led us to 13 x-matrices:

μ_0 = 0.004, 0.005, 0.006, 0.007, 0.008, 0.009, 0.010, 0.015, 0.020, 0.025, 0.030, 0.035, 0.040.

The following image shows the resulting x-matrices after performing the long-hand approach.

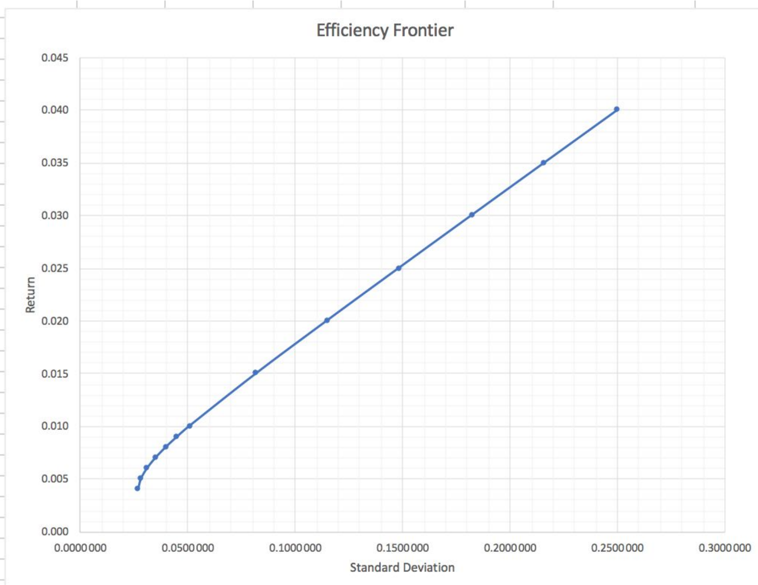
μ_0	w_SPY	w_DIA	w_QQQ	w_AGG	w_GLD	w_IWM	w_YLD	λ_1	λ_2
0.004	-0.088848	0.2394057	-0.010971	1.0820275	-0.100501	0.0036972	-0.12481	-0.00011285	-0.002322
0.005	-0.226978	0.3367387	0.0727515	1.0576981	-0.072633	0.0043405	-0.171918	-8.3871E-05	-0.010148
0.006	-0.365108	0.4340716	0.1564742	1.0333686	-0.044764	0.0049838	-0.219027	-5.4891E-05	-0.017973
0.007	-0.503237	0.5314045	0.2401968	1.0090392	-0.016895	0.005627	-0.266135	-2.5912E-05	-0.025799
0.008	-0.641367	0.6287374	0.3239194	0.9847097	0.0109735	0.0062703	-0.313243	3.0674E-06	-0.033624
0.009	-0.779497	0.7260703	0.4076421	0.9603802	0.0388423	0.0069136	-0.360352	3.2047E-05	-0.04145
0.010	-0.917627	0.8234032	0.4913647	0.9360508	0.066711	0.0075569	-0.40746	6.1026E-05	-0.049275
0.015	-1.608275	1.3100678	0.9099779	0.8144035	0.2060546	0.0107732	-0.643002	0.00020592	-0.088402
0.020	-2.298924	1.7967324	1.328591	0.6927561	0.3453982	0.0139895	-0.878543	0.00035082	-0.12753
0.025	-2.989573	2.283397	1.7472042	0.5711088	0.4847419	0.0172059	-1.114085	0.00049572	-0.166657
0.030	-3.680222	2.7700616	2.1658174	0.4494615	0.6240855	0.0204222	-1.349626	0.00064061	-0.205784
0.035	-4.37087	3.2567262	2.5844305	0.3278142	0.7634291	0.0236385	-1.585168	0.00078551	-0.244911
0.040	-5.061519	3.7433908	3.0030437	0.2061669	0.9027727	0.0268549	-1.82071	0.00093041	-0.284039

<u>Under the matrix approach:</u>

Performing the matrix approach is fairly similar to the long-hand approach, as it features very similar multiplication formulas within Excel. As previously mentioned, we needed to find the x-matrix by multiplying the A-inverse matrix with the b-matrix. This was done by using MMULT, an Excel function for matrix multiplication.

When using these functions with matrices, it is necessary to consider dimensionality. Strictly speaking, a matrix can only be multiplied if certain terms match up. As an example, an (mXn) matrix will multiply with a (nXp), as the n's match up, resulting in a matrix with (mXp) dimensions. Alternatively, if the matrix (mXn) was multiplied with (pXn), this would output an error as n ≠ p. To make sure we didn't encounter any errors during our matrix multiplication, we also utilized the TRANSPOSE function which changed the dimensionality of a matrix from (mXn) to (nXm).

Additionally, we used Excel's 'Solver', to determine the Global Minimum Variance (GMV). The GMV portfolio is simply the efficient portfolio in which the standard deviation is minimized. This was done by setting the Solver objective to minimize the variance, manipulating the weights of the asset class, and following the constraint that the total weights should equal 1. After all this had been completed, we then were able to plot our Efficient Frontier starting with GMV and ending with our greatest μ_0.



Step 2: Optimal Portfolio Selection

Our next step was to introduce your current portfolio into the Capital Allocation Diagram. The strategy produced annual returns of 12.85% with a standard deviation of 9.09%. In a traditional exercise, when evaluating an allocation is to identify the equivalent strategy from either a risk or return perspective.

It's important to recall the restraints and allowances that you requested we place on your portfolio. Namely:

- You would like to utilize all available cash, leaving no liquidity buffer
- You are comfortable shorting any of these positions, which are well traded, though be aware that here we are ignoring any potential dividend liability
- You are comfortable taking considerable leverage on any of these positions

Original Portfolio								
	SPY	DIA	QQQ	AGG	GLD	IWM	YLD	**Sum of Weights**
Weights	0.2	0.2	0.2	0.2	0.2	-0.1	0.1	**1**
Expected returns	0.002141	0.002162	0.003453	0.000741	0.001906	-0.000702	0.000421	
Monthly returns	0.010122							
Annual returns	0.128465							
Monthly Std	0.02624875							
Annual Std	0.09092834							

When we plot the strategy into our diagram, we can clearly see that it's not situated along the Efficient Frontier. Though this is both good and bad news, it means that we will have ways to optimize your current allocation.

Out of many possibilities, there are two simple ways to maximize this portofolio: The first one is to hold fixed the current annualized risk of the portofolio, 9.09%, and identify the portfolio weights that maximize the return at that level.

Maximize Return								
	SPY	DIA	QQQ	AGG	GLD	IWM	YLD	**Sum of Weights**
Weights	-1.8029148	1.41830838	1.03410287	0.78407466	0.24912887	0.03150758	-0.7142075	1
Expected returns	-0.019304	0.015329	0.017854	0.002906	0.002374	0.000221	-0.003008	
Monthly returns	0.016373							
Annual returns	0.215166							
Monthly Std	0.02624975							
Annual Std	0.09093179							

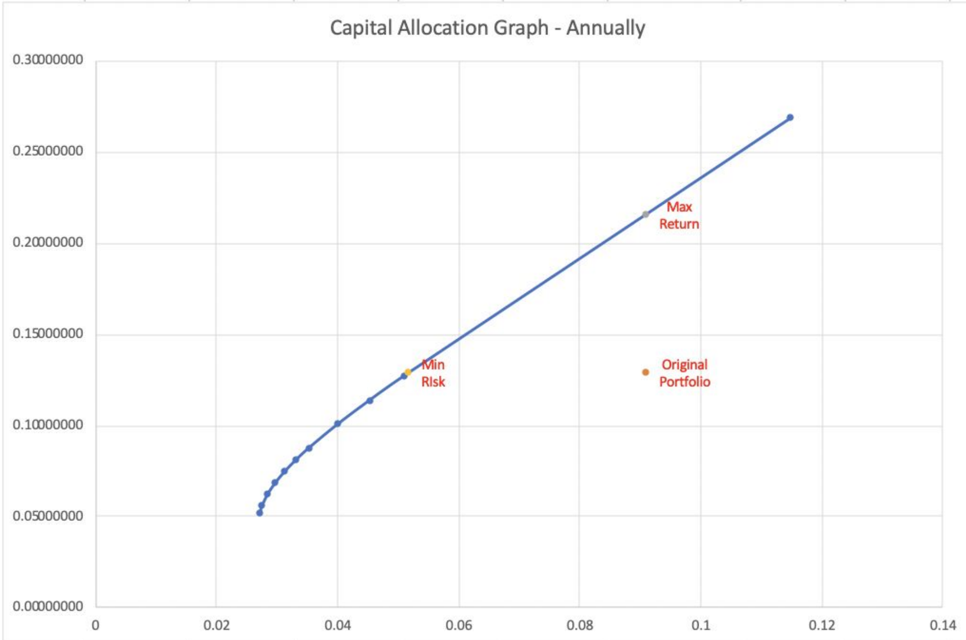
Holding fixed the annual standard deviation, we can see that the annual return increases significantly from 12.8% to 21.5%.

The second approach is to hold fixed your current level of annual return, 12.85%, and minimize the level of risk for that given return.

Minimize Risk								
	SPY	DIA	QQQ	AGG	GLD	IWM	YLD	**Sum of Weights**
Weights	-0.9346363	0.83537072	0.50163625	0.93308527	0.07012346	0.00765199	-0.4132313	1
Expected returns	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	
Monthly returns	0.010122							
Annual returns	0.128465							
Monthly Std	0.01489815							
Annual Std	0.05160871							

Holding fixed the annual expected return, we can see that the annual standard deviation (risk) decreases from 9.1% to 5.2%.

From the graph below, you will see that your current allocation, the Original Portfolio, is far from an "efficient" strategy. Our team used Solver to identify portfolios above, which each sit on the Efficiency Frontier. These are the exact allocations that maximize return for your current level of risk, or minimize risk for today's return.



These two optimized portfolios are equally efficient, with key differences in return and risk characteristics.

Finally, we introduced the concept of a risk-free rate, in order to identify an “optimal portfolio” along our Efficient Frontier. The stand-in for the risk-free rate is the 5-years average return of the U.S. Treasury 3-month t-bill. The bill is quoted in annual percentage, regardless of its short duration.

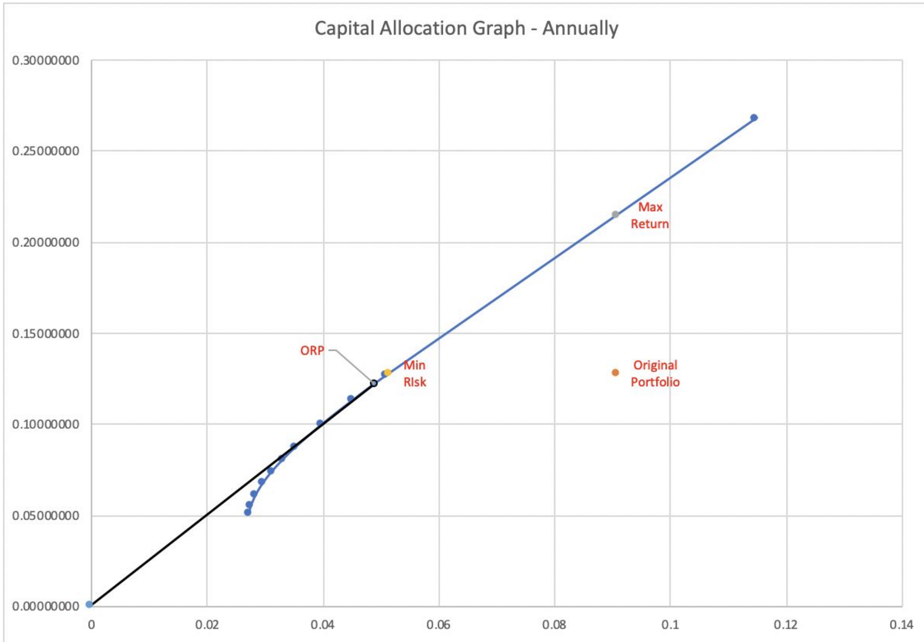
As this proxy is riskless, its standard deviation is zero. Therefore it sits on the y-axis of our graph and serves as an intercept for the Capital Allocation Line (CAL). The CAL can be any line between this intercept and a single portfolio in our universe; however, the greater the slope of the CAL, the higher the risk-adjusted-return, or sharpe ratio, of the portfolio it connects with is. Drawing a CAL tangent to the Efficient Frontier maximizes this measure. This is done using Solver to maximize the sharpe ratio, the return of the portfolio minus the risk-free rate, divided by the standard deviation of the risky portfolio, subject to spending all of the available capital.

Our optimal portfolio is the set of weights that sits on the Efficient Frontier is tangent to this line.

OPR Portfolio Weights and Characteristics:

weights	
SPY	-0.870537672
DIA	0.7902221
QQQ	0.462823345
AGG	0.944344774
GLD	0.057210436
IWM	0.007337558
YLD	-0.391400542

Std. dev	Expected Return	Sharpe Ratio
0.014125129	0.009659096	0.174975309
Annual Std. Dev	Annual Return	
0.048930883	0.122269469	



We can see from the graph that introducing the risk-free rate, the UST 3-month bill, we can identify an Optimal Risky Portfolio (ORP). This portfolio has a standard deviation of 4.89% and an annualized return of 12.23%. While this allocation is an improvement over your current portfolio, it is equally efficient as the two alternatives we've already suggested.

However, a major component of the ORP is its relationship to the risk-free rate. By buying the 3-month t-bill, you are able to create a portfolio that in fact, lies above the Efficient Frontier. This allocation to the risk-free rate reduces your weighting to the ORP, thereby reducing your strategy's capacity for return, but improving its risk characteristics.

On the other hand, you can increase your level of potential returns, again to a point that lies above the Frontier, by borrowing funds at that same rate. This allows you to increase the amount of funds allocated to the risky portfolio, while taking on only a marginal cost.

Conclusion and Recommendation:

The methods listed above provide just a handful of opportunities to improve upon your current allocation, both in terms of risk and return. Just 3 portfolios are specified above, including the efficient strategies that would improve your current return holding your risk constant, or reduce your level of risk for your same average return. Furthermore, a shift to the Optimal Risky Portfolio presents a nearly infinite number of available portfolios at a set level of risk or return that you specify.

Our team looks forward to continuing this conversation with you and evaluating your options further.

"I attest that this assignment has been completed in accordance with the academic integrity protocols and honor codes of the McCombs School of Business and the University of Texas. I relied on no unauthorized resources (including material submitted or produced by other students—past or present, individuals not listed below, internet message boards, solution manuals, or other forms of outside assistance). This work product is the sole result of my efforts and is being submitted as such. (Emmanuelle Eguche (ee8324), Jing Fang (jf36536), Sitong Li (sl43736), Joe Niehaus (jfn258), Matthew Streichler(mrs4732))."