Pengolahan Sinyal Digital

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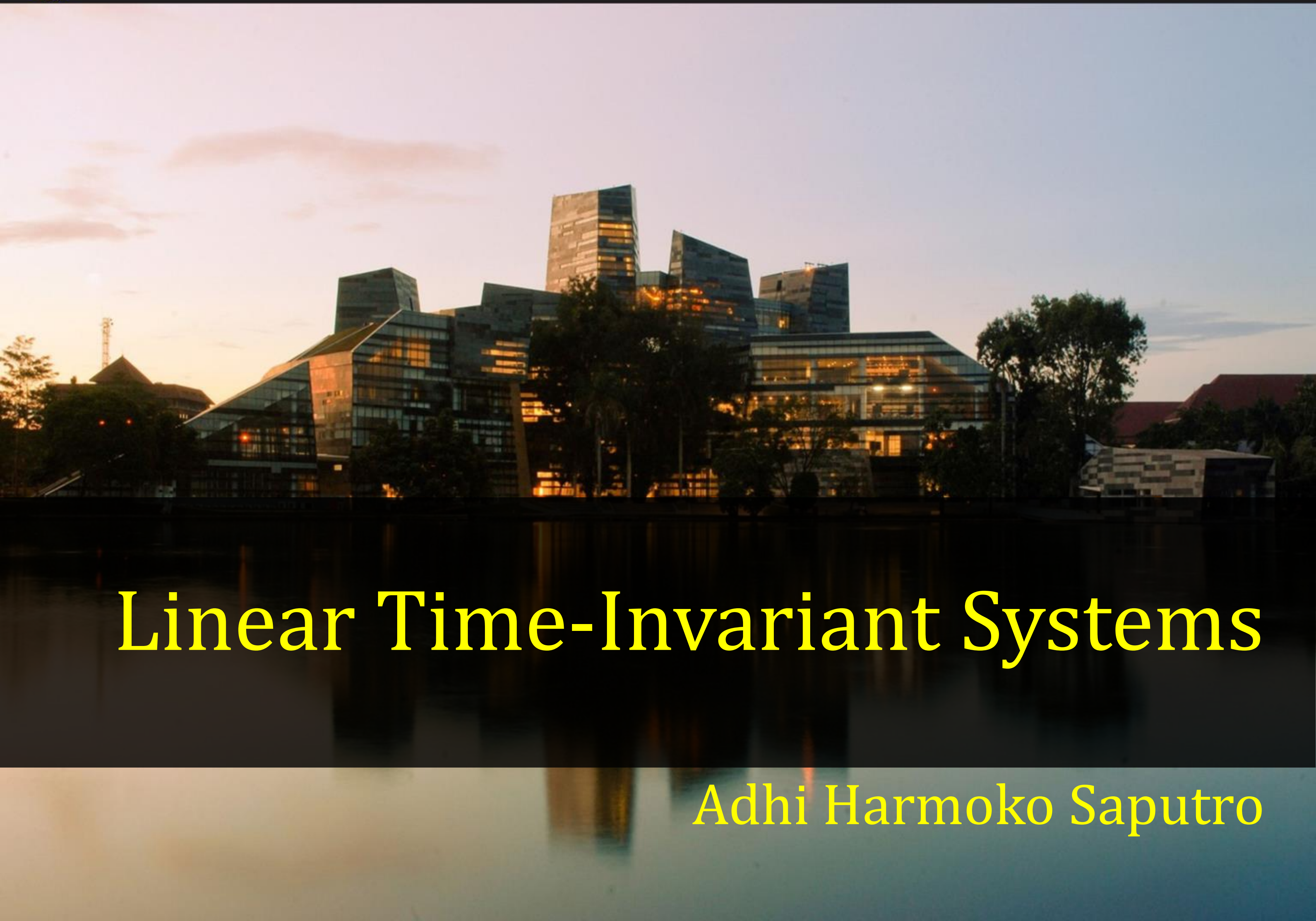

Linear Time-Invariant Systems

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Linear-Time Invariant System

- A linear system in which an input-output pair, $x(n)$ and $y(n)$, is invariant to a shift k in time
- Special importance for their mathematical tractability
- Most signal processing applications involve LTI systems
- LTI system can be completely characterized by their impulse response

$$\delta[n-k] \longrightarrow \boxed{T\{.\}} \longrightarrow h_k[n]$$

Linear-Time Invariant System

- Represent any input $$x[n]=\sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

$$y[n]=T\left\{\sum_{k=-\infty}^{\infty} x[k]\delta[n-k]\right\}=\sum_{k=-\infty}^{\infty} x[k]T\{\delta[n-k]\}=\sum_{k=-\infty}^{\infty} x[k]h_k[n]$$

- From time invariance we arrive at convolution

$$y[n]=\sum_{k=-\infty}^{\infty} x[k]\ h[n-k]=x[k]*h[k]$$

Properties of LTI Systems

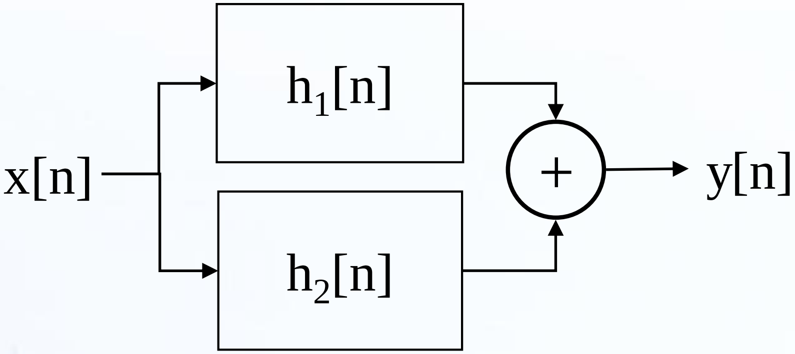
- Convolution is commutative

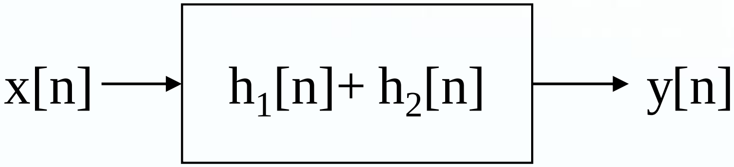
$$x[k]*h[k]=\sum_{k=-\infty}^{\infty} x[k]h[n-k]=\sum_{k=-\infty}^{\infty} h[k]x[n-k]=h[k]*x[k]$$

Properties of LTI Systems

- Convolution is distributive

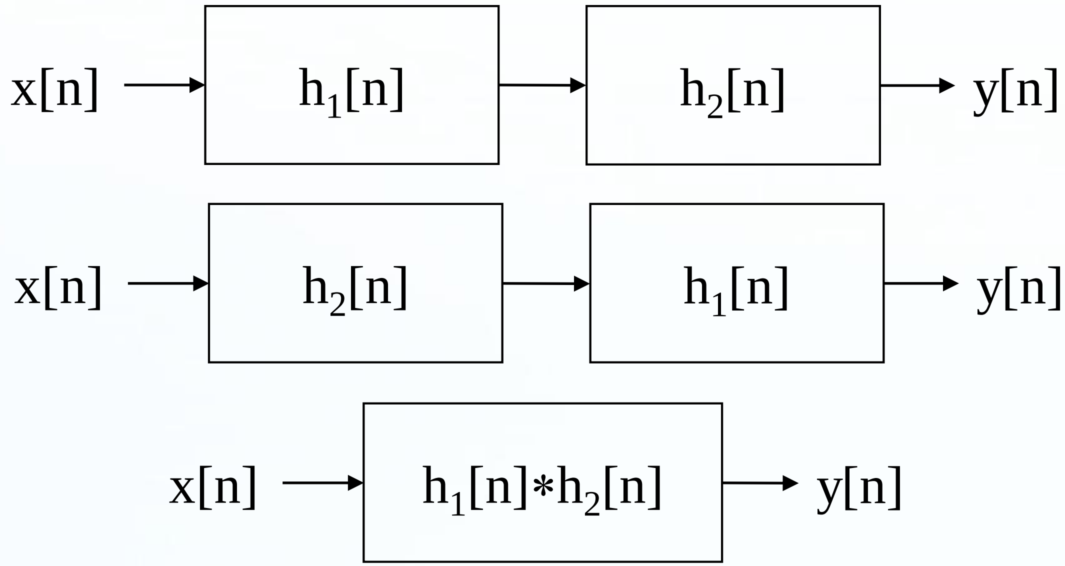
$$x[k]*\left(h_1[k]+h_2[k]\right)=x[k]*h_1[k]+x[k]*h_2[k]$$





Properties of LTI Systems

- Cascade connection of LTI systems



Stable LTI Systems

- An LTI system is (BIBO) stable if and only if
  - Impulse response is absolute summable

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty$$

  - Let’s write the output of the system as

$$|y[n]| = \left|\sum_{k=-\infty}^{\infty} h[k]x[n-k]\right| \le \sum_{k=-\infty}^{\infty} |h[k]||x[n-k]|$$

  - If the input is bounded

$$|x[n]| \le B_x$$

  - Then the output is bounded by

$$|y[n]| \le B_x \sum_{k=-\infty}^{\infty} |h[k]|$$

  - The output is bounded if the absolute sum is finite

Causal LTI Systems

- An LTI system is causal if and only if

$$h[k]=0 \quad \text{for k} < 0$$

Convolution

Example

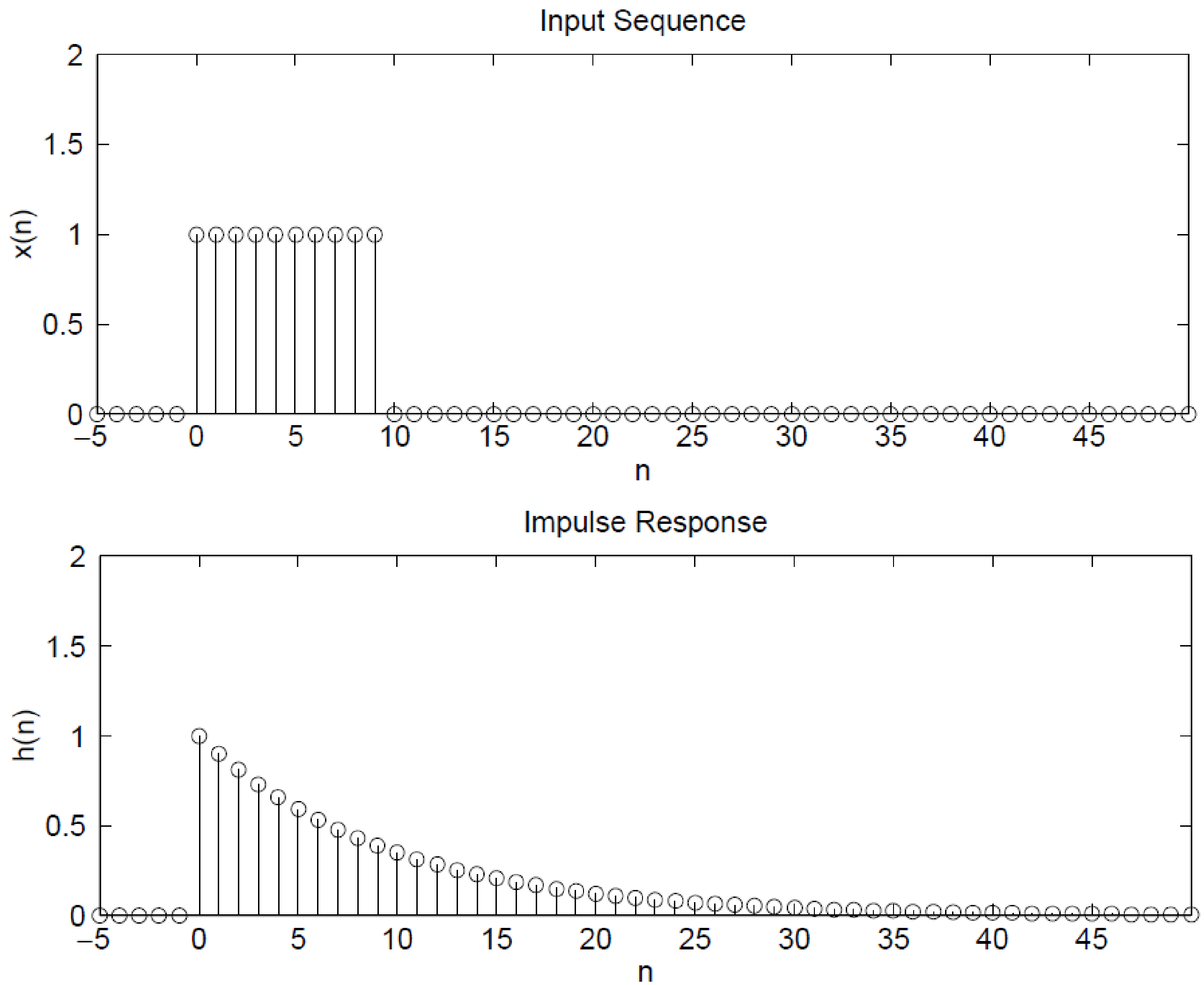
- Let the rectangular pulse $x(n) = u(n) - u(n - 10)$ be an input to an LTI system with impulse response

$$h(n) = (0.9)^n u(n)$$

- Determine the output $y(n)$.

Example

- The input sequence and the impulse response



Example

- From the convolution equation

$$y(n) = x(n) * h(n)$$

$$y(n) = \sum_{k=0}^{9} (1)(0.9)^{(n-k)} u(n-k) = (0.9)^{n} \sum_{k=0}^{9} (0.9)^{-k} u(n-k)$$

- The sum in equation is almost a geometric series sum except that the term $u(n-k)$ takes different values depending on n and k. There are three possible conditions under which $u(n-k)$ can be evaluated.

Example

- Case 1
  - $n < 0$: Then $u(n - k) = 0,\ 0 \leq k \leq 9$

$$y(n) = 0$$

Example

- Case 2
  - The nonzero values of $x(n)$ and $h(n)$ *do not overlap*.
  - $0 \le n < 9$: Then $u(n - k) = 1,\ 0 \le k \le n$.

$$y(n) = (0.9)^n \sum_{k=0}^{n} (0.9)^{-k} = (0.9)^n \sum_{k=0}^{n} \left[(0.9)^{-1}\right]^k$$

$$= (0.9)^n \frac{1-(0.9)^{-(n+1)}}{1-(0.9)^{-1}} = 10\left[1-(0.9)^{(n+1)}\right]$$

Example

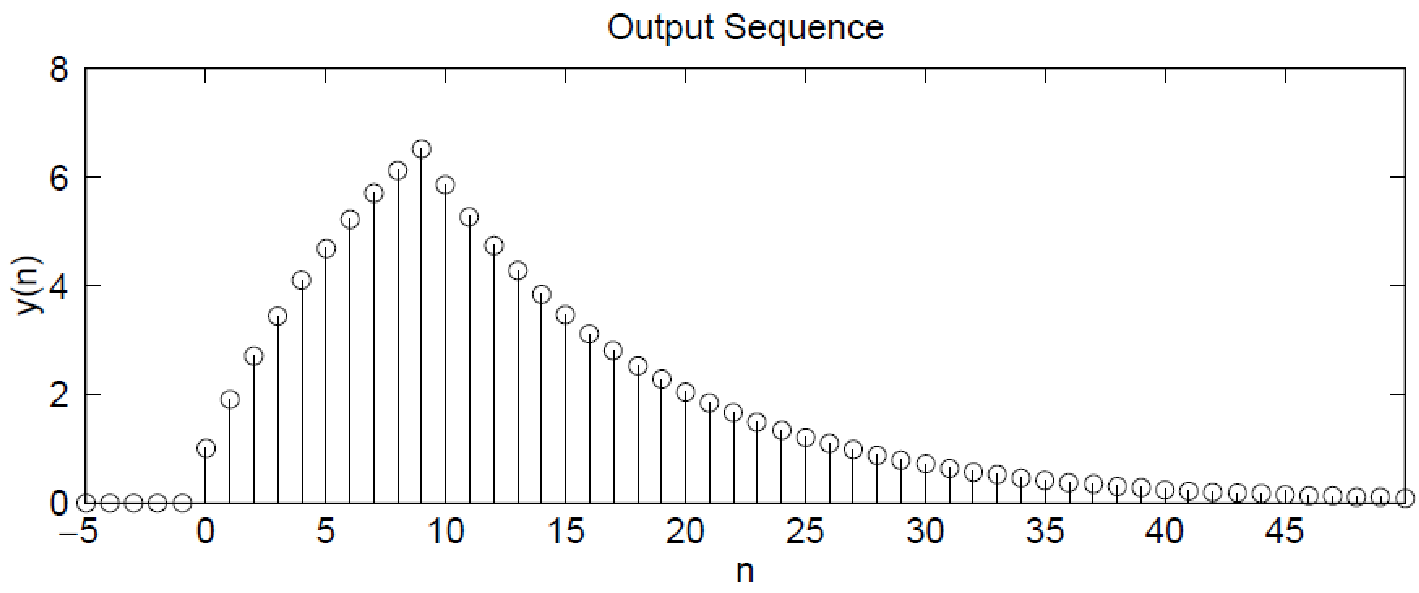
- Case 3
  - The impulse response $h(n)$ *partially overlaps* the input $x(n)$.
  - $n \geq 9$: Then $u(n-k) = 1,\ 0 \leq k \leq 9$

$$y(n) = (0.9)^n \sum_{k=0}^{9} (0.9)^{-k}$$

$$= (0.9)^n \frac{1-(0.9)^{-10}}{1-(0.9)^{-1}} = 10(0.9)^{n-9}\left[1-(0.9)^{10}\right]$$

Example

- The output sequence



Matlab Convolution

- If arbitrary sequences are of infinite duration, then MATLAB cannot be used *directly* to compute the convolution.
- MATLAB does provide a built-in function called conv that computes the convolution between two finite-duration sequences. The conv function assumes that the two sequences begin at $n = 0$ and is invoked by
- `>> y = conv(x,h);`

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