UNIVERSITAS INDONESIA
Veritas, Probitas, Iustitia

Pengolahan Sinyal Digital

Adhi Harmoko Saputro

FIR Filter Structures

FIR Filter Structures

- A finite-duration impulse response filter has a system function of the form

$$H(z) = b_o + b_1 z^{-1} + \ldots + b_{M-1} z^{1-M} = \sum_{n=0}^{M-1} b_n z^{-n}$$

- The impulse response *h*(*n*) is

$$h(n) = \begin{cases} b_n, & 0 \le n \le M-1 \\ 0, & \text{else} \end{cases}$$

- The difference equation representation is

$$y(n) = b_o x(n) + b_1 x(n-1) + \ldots + b_{M-1} x(n-M+1)$$

- The order of the filter is $M - 1$, and the *length* of the filter (which is equal to the number of coefficients) is M

FIR Filter Structures

- The FIR filter structures are
  - a linear convolution of finite support
  - always stable
  - relatively simple compared to IIR structures
- FIR filters can be designed to have a linear-phase response, which is desirable in some applications.

FIR Filter Structures

1. **Direct form:**

$$y(n) = b_o x(n) + b_1 x(n-1) + \ldots + b_{M-1} x(n-M+1)$$

2. **Cascade form:**

$$H(z) = b_o + b_1 z^{-1} + \ldots + b_{M-1} z^{1-M} = \sum_{n=0}^{M-1} b_n z^{-n}$$

this form the system function *H*(*z*) is factored into 2nd-order factors, which are then implemented in a cascade connection.

FIR Filter Structures

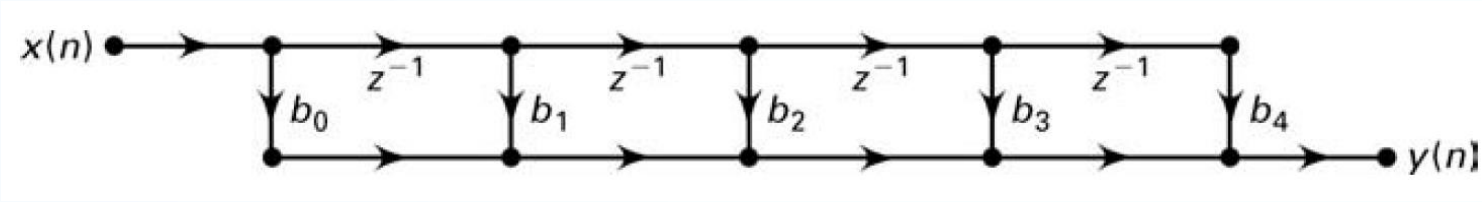
3. **Linear-phase form:**
its impulse response exhibits certain symmetry conditions
to reduce multiplications by about half.

4. **Frequency-sampling form:**
This structure is based on the DFT of the impulse response $h(n)$ and leads to a parallel structure
suitable for a design technique based on the sampling of frequency response $H(e^{j\omega})$.

Direct form

- The difference equation is implemented as a tapped delay line since there are no feedback paths.
- Let $M = 5$ (i.e., a 4th-order FIR filter)

$$y(n) = b_o x(n) + b_1 x(n-1) + b_2 x(n-2) + b_3 x(n-3) + b_4 x(n-4)$$



Direct form
Matlab Implementation

- The direct form FIR structure is described by the row vector b containing the *{bn}* coefficients.
- The structure is implemented by the filter function, in which the vector a is set to the scalar value 1

Cascade form

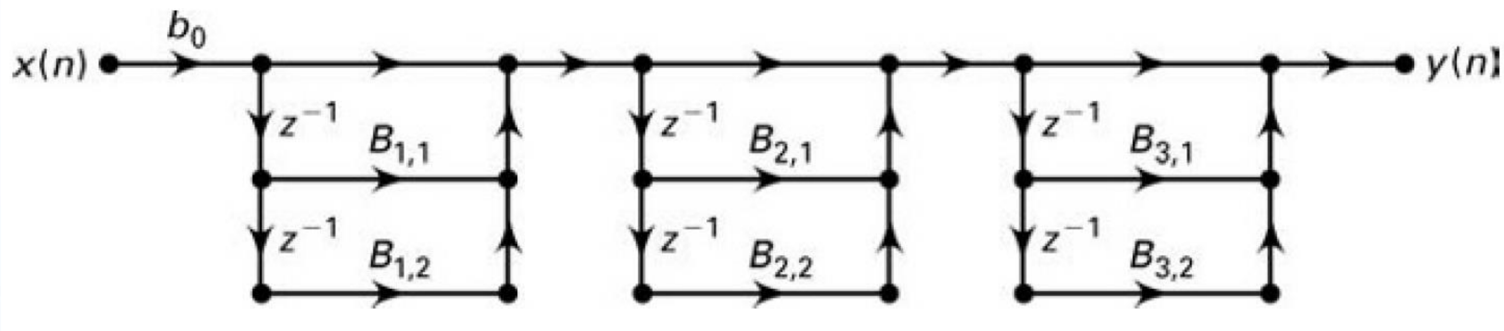
- The system function $H(z)$ is converted into products of 2nd-order sections with real coefficients

$$
\begin{aligned}
H(z) &= b_o + b_1 z^{-1} + \ldots + b_{M-1} z^{-M+1} \\
&= b_o \left(1 + \frac{b_1}{b_o} z^{-1} + \ldots + \frac{b_{M-1}}{b_o} z^{-M+1} \right) \\
&= b_o \prod_{k=1}^{K} \left(1 + B_{k,1} z^{-1} + B_{k,2} z^{-2} \right)
\end{aligned}
$$

- K is equal to $M/2$, and $B_{k,1}$ and $B_{k,2}$ are real numbers representing the coefficients of 2nd-order sections.

Cascade form

- For M = 7 the cascade form is shown as



Cascade form
Matlab Implementation

- Use our dir2cas function by setting the denominator vector a equal to 1
- Use cas2dir to obtain the direct form from the cascade form.

Linear-phase form

- For frequency-selective filters (e.g., lowpass filters) it is generally desirable to have a phase response that is a linear function of frequency

$$\angle H\left(e^{j\omega}\right) = \beta - \alpha\omega, \quad -\pi < \omega \leq \pi$$

- where $\beta = 0$ or $\pm\pi/2$ and α is a constant.

Linear-phase form

- The linear-phase condition imposes the following symmetry conditions on the impulse response *h*(*n*)
  - a symmetric impulse response

$$h(n) = h(M-1-n); \quad \beta = 0, \alpha = \frac{M-1}{2}, \quad 0 \le n \le M-1$$

  - an antisymmetric impulse response

$$h(n) = -h(M-1-n); \quad \beta = \pm\pi/2, \alpha = \frac{M-1}{2}, \quad 0 \le n \le M-1$$

Linear-phase form

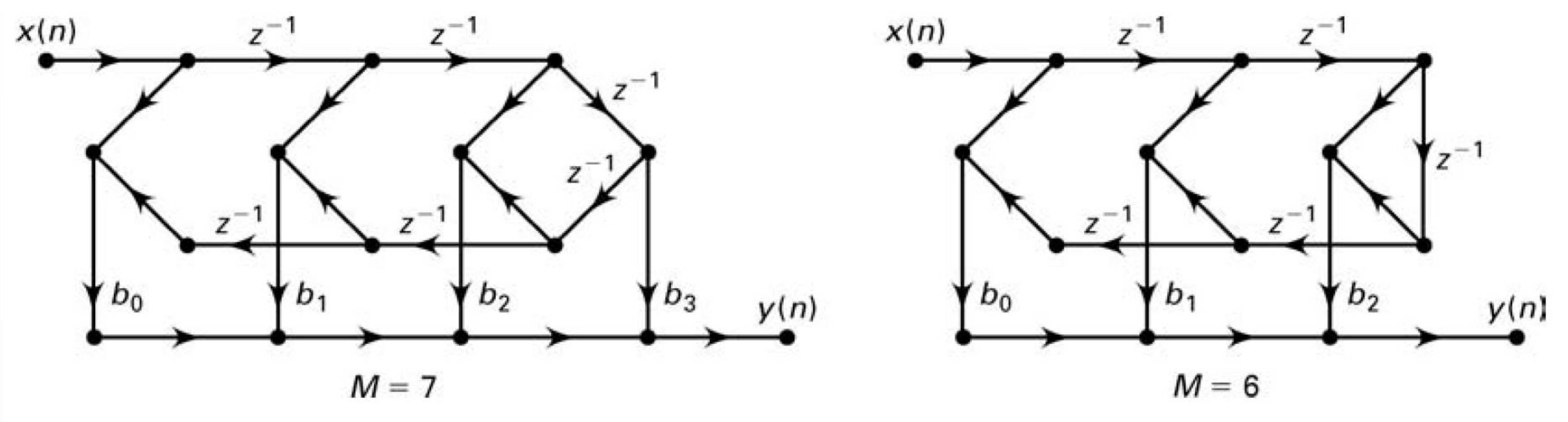
- Consider the difference equation with a symmetric impulse response in a symmetric impulse response

$$y(n) = b_o x(n) + b_1 x(n-1) + \ldots + b_1 x(n-M+2) + b_o x(n-M+1)$$

$$= b_o\left[x(n) + x(n-M+1)\right] + b_1\left[x(n-1) + x(n-M+2)\right] + \ldots$$

Linear-phase form

- The block diagram implementation of the previous difference equation for both odd and even *M*.



Linear-phase form
Matlab Implementation

- The linear-phase structure is essentially a direct form drawn differently to save on multiplications.
- Hence in a MATLAB representation of the linear-phase structure is equivalent to the direct form.

Example

- An FIR filter is given by the system function

$$H(z) = 1 + 16\frac{1}{16}z^{-4} + z^{-8}$$

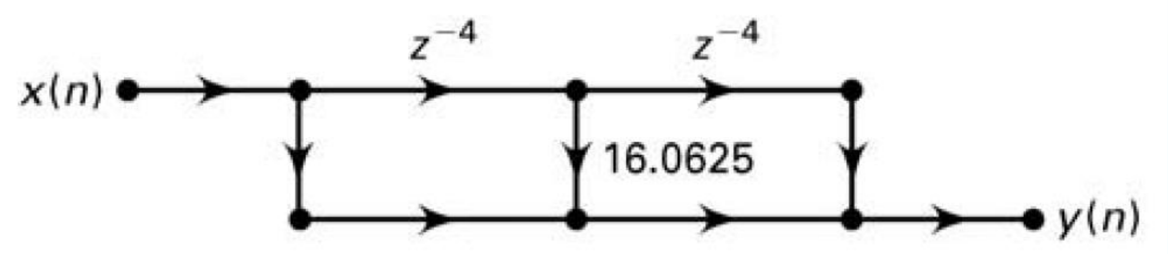
- Determine and draw the direct, linear-phase, and cascade form structures

Example

$$H(z) = 1 + 16\frac{1}{16}z^{-4} + z^{-8}$$

- **Direct form:** The difference equation is given by

$$y(n) = x(n) + 16.0625x(n-4) + x(n-8)$$

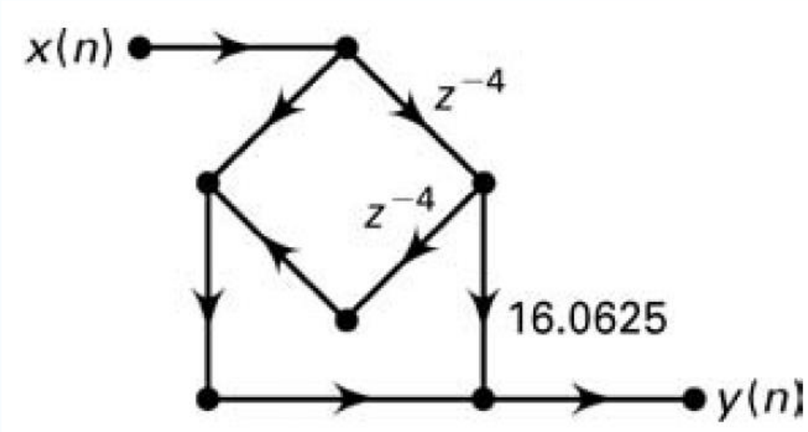


Example

$$H(z) = 1 + 16\frac{1}{16}z^{-4} + z^{-8}$$

- **Linear-phase form:** The difference equation can be written in the form

$$y(n) = [x(n) + x(n-8)] + 16.0625x(n-4)$$

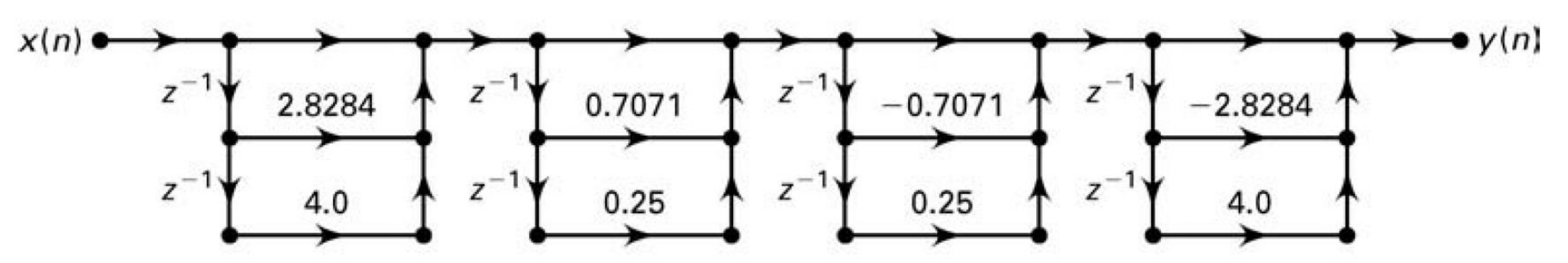


Example

$$H(z) = 1 + 16\frac{1}{16}z^{-4} + z^{-8}$$

- **Cascade form:**

>> b=[1,0,0,0,16+1/16,0,0,0,1]; [b0,B,A] = dir2cas(b,1)



Proyek

1. Ricky: Pitch Extraction
2. Bhakti: Speech End Pointing
3. Ibrohim: Compression
4. Yousa: DTMF
5. Sulkhan: Speech Distraction
6. Wahyu: Sound Visualisation

Presentasi dan demonstrasi dilaksanakan pada Rabu 16 Desember 2015 Pukul 13.00 – 17.00 di Lab Kom

UAS dilaksanakan pada Jumat 18 Desember 2015 Pukul 08.00 – 10.00

Terima Kasih