PENGOLAHAN SINYAL DIGITAL

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FIR FILTER DESIGN

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FIR FILTER DESIGN

- The first type of systems perform signal filtering in the time domain and hence are called *digital filters*
- The second type of systems provide signal representation in the frequency domain and are called *spectrum analyzers*.

FIR AND IIR FILTERS

- The designs are mostly of the *frequency selective* type multiband lowpass, highpass, bandpass, and bandstop filters.

- FIR filter design
  - Differentiators or Hilbert transformers, which, although not frequency-selective filters, nevertheless follow the design techniques being considered.
  - More sophisticated filter designs are based on arbitrary frequency-domain specifications

DESIGN OF A DIGITAL FILTER

- **Specifications:** Before we can design a filter, we must have some specifications. These specifications are determined by the applications.

- **Approximations:** Once the specifications are defined, we use various concepts and mathematics that we studied so far to come up with a filter description that approximates the given set of specifications. This step is the topic of filter design.

- **Implementation:** The product of the above step is a filter description in the form of either a difference equation, or a system function $H(z)$, or an impulse response $h(n)$.

DESIGN OF A DIGITAL FILTER

- Specifications are required in the frequency-domain in terms of the desired magnitude and phase response of the filter
- A linear phase response in the passband is desirable.
- The magnitude specifications are given in one of two ways:
  - *Absolute specifications*, which provide a set of requirements on the magnitude response function $|H(e^{j\omega})|$
  - *Relative specifications*, which provide requirements in *decibels* (dB),

$$dB\ scale = -20\log_{10}\frac{\left|H\left(e^{j\omega}\right)\right|}{\left|H\left(e^{j\omega}\right)\right|_{\max}} \geq 0$$

ABSOLUTE SPECIFICATIONS

- A typical absolute specification of a lowpass filter

- band $[0, \omega_p]$ is called the *passband*, and δ_1 is the tolerance (or ripple)
- band $[\omega_s, \pi]$ is called the *stopband*, and δ_2 is the corresponding tolerance (or ripple)
- band $[\omega_p, \omega_s]$ is called the *transition band*, and there are no restrictions on the magnitude response in this band

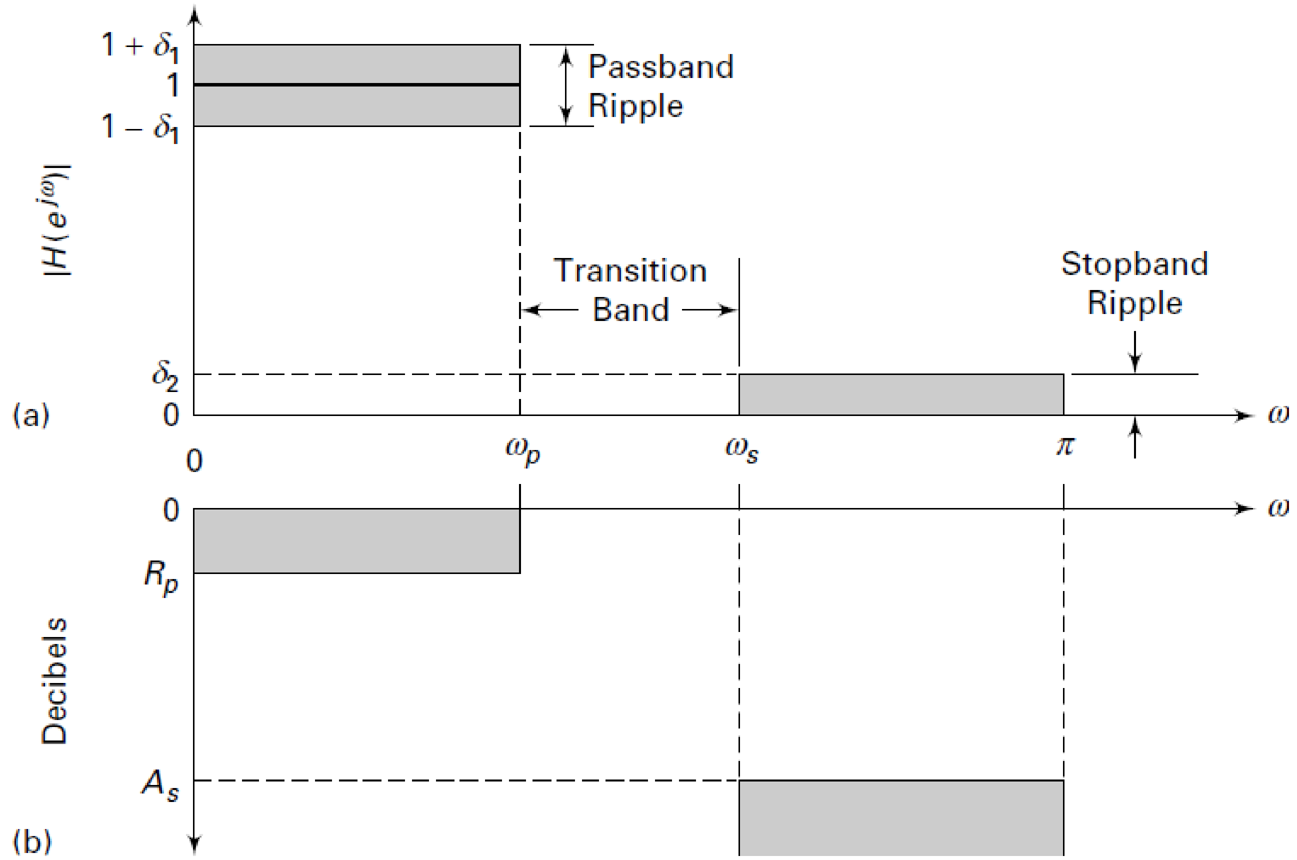

FIGURE 7.1 *FIR filter specifications: (a) absolute (b) relative*

RELATIVE (DB) SPECIFICATIONS

- R_p is the passband ripple in dB
- A_s is the stopband attenuation in dB
- Since $|H(ej\omega)|_{max}$ in absolute specifications is equal to $(1 + \delta_1)$

$$R_p = -20\log_{10}\frac{1-\delta_1}{1+\delta_1} > 0(\approx 0)$$

$$A_s = -20\log_{10}\frac{\delta_2}{1+\delta_1} > 0(\gg 1)$$

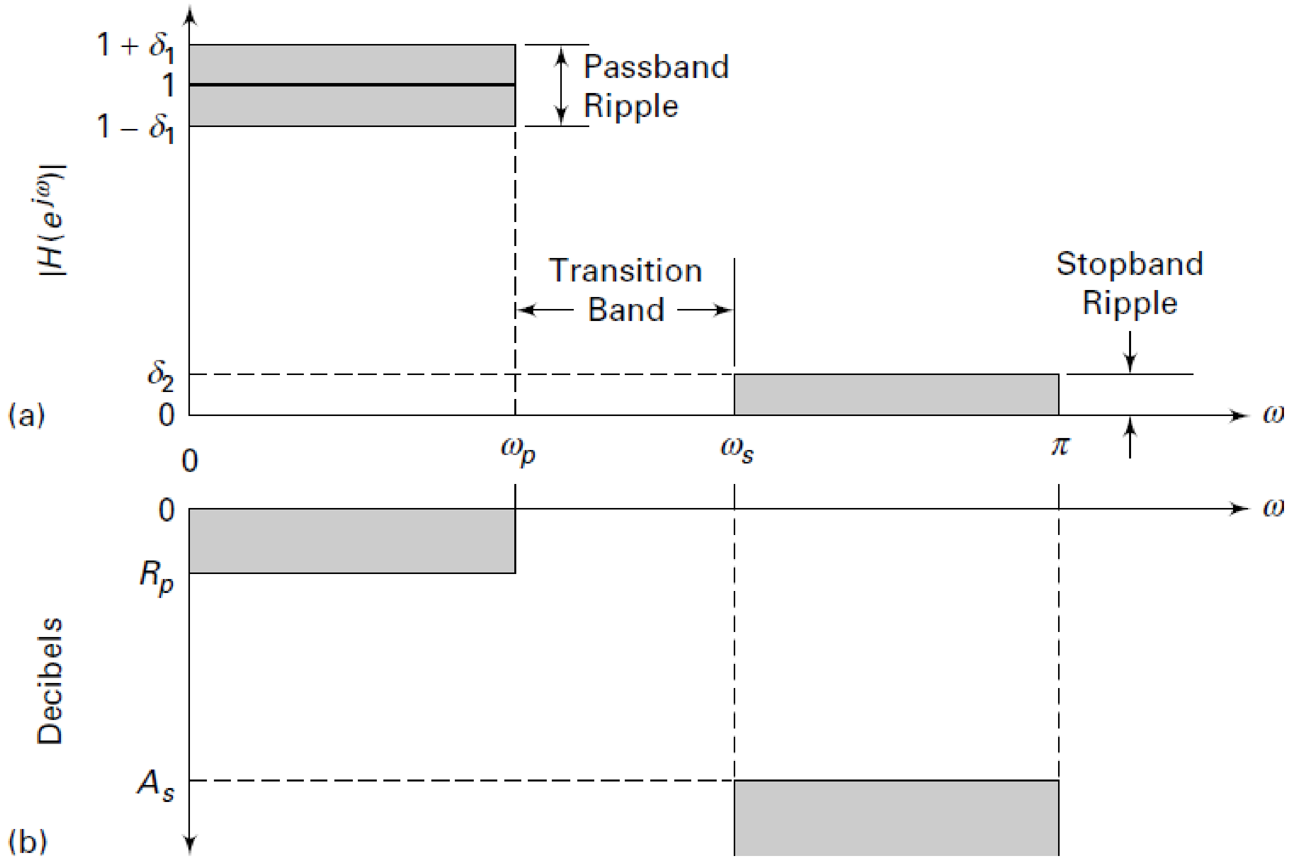

FIGURE 7.1 *FIR filter specifications: (a) absolute (b) relative*

EXAMPLE

- In a certain filter’s specifications the passband ripple is 0.25 dB, and the stopband attenuation is 50 dB.

- Determine δ_1 and δ_2.

EXAMPLE

- In a certain filter's specifications the passband ripple is 0.25 dB, and the stopband attenuation is 50 dB.
- Determine δ_1 and δ_2.
- Solution:

$$R_p = 0.25 = -20\log_{10}\frac{1-\delta_1}{1+\delta_1} \Rightarrow \delta_1 = 0.0144$$

$$A_s = 50 = -20\log_{10}\frac{\delta_2}{1+\delta_1} \Rightarrow \delta_2 = 0.0032$$

DESIGN OF A DIGITAL FILTER

- The most important design parameters are *frequency-band tolerances* (or ripples) and *band-edge frequencies*.
- The given band is a passband or a stopband is a relatively minor issue.

- ***Problem statement :***
  Design a lowpass filter (i.e., obtain its system function $H(z)$ or its difference equation) that has a passband $[0, \omega_p]$ with tolerance δ_1 (or R_p in dB) and a stopband $[\omega_s, \pi]$ with tolerance δ_2 (or A_s in dB).

DESIGN OF A DIGITAL FILTER

- The design and approximation of FIR digital filters that have several design and implementational advantages:
  - The phase response can be exactly linear.
  - They are relatively easy to design since there are no stability problems.
  - They are efficient to implement.
  - The DFT can be used in their implementation.

DESIGN OF A DIGITAL FILTER

- Usinh linear phase frequency-selective FIR filters
- Advantages of a linear-phase response are:
  - design problem contains only real arithmetic and not complex arithmetic
  - linear-phase filters provide no delay distortion and only a fixed amount of delay
  - for the filter of length M (or order $M - 1$) the number of operations are of the order of $M/2$ as we discussed in the linear-phase filter implementation

PROPERTIES OF LINEAR-PHASE FIR FILTERS

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PROPERTIES OF LINEAR-PHASE FIR FILTERS

- Discuss shapes of impulse and frequency responses and locations of system function zeros of linear-phase FIR filters
- Let $h(n)$, $0 \le n \le M - 1$ be the impulse response of length (or duration) M.
- The system function is

$$H(z) = \sum_{n=0}^{M-1} h(n) z^{-n} = z^{-(M-1)} \sum_{n=0}^{M-1} h(n) z^{M-1-n}$$

  - which has $(M - 1)$ poles at the origin $z = 0$ (trivial poles) and $(M - 1)$ zeros located anywhere in the z-plane.
- The frequency response function is

$$H(e^{j\omega}) = \sum_{n=0}^{M-1} h(n) e^{-j\omega}, \quad -\pi < \omega \le \pi$$

IMPULSE RESPONSE $h(n)$

- Impose a linear-phase constraint

$$\angle H\left(e^{j\omega}\right) = -\alpha\omega, \quad -\pi < \omega \leq \pi$$

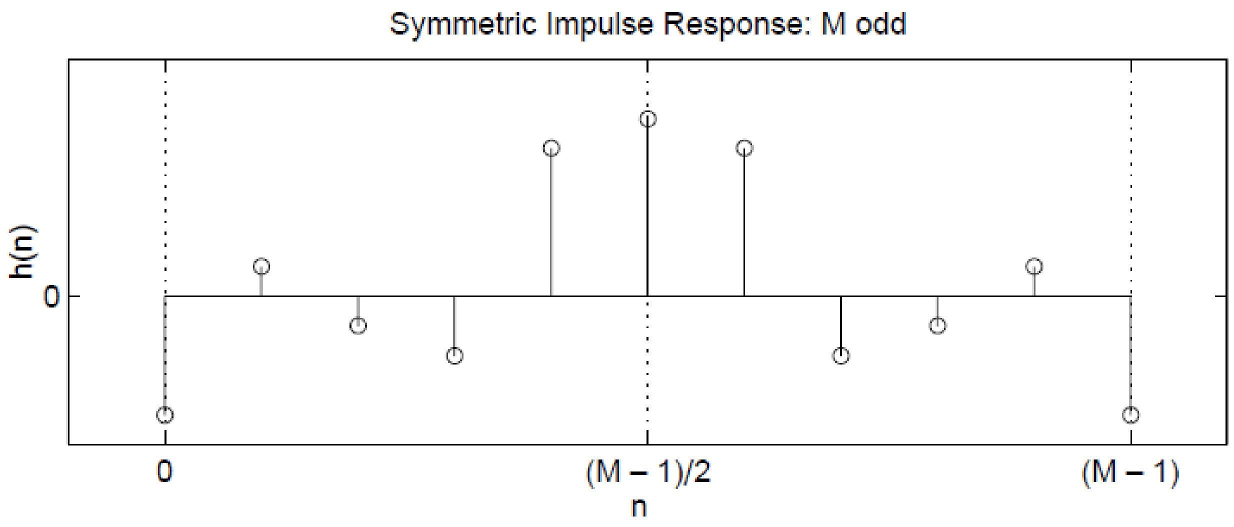
  - where α is a *constant phase delay*.
- *h*(*n*) must be symmetric

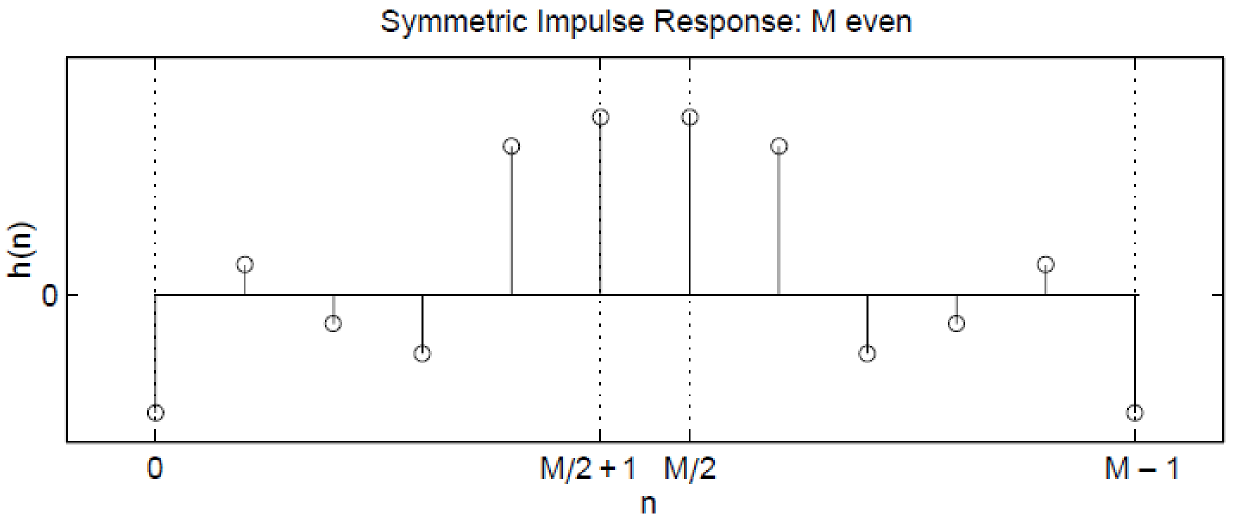
$$h(n) = h(M-1-n), \quad 0 \leq n \leq (M-1), \quad \alpha = \frac{M-1}{2}$$

- Hence *h*(*n*) is symmetric about α, which is the index of symmetry

IMPULSE RESPONSE $h(n)$

- There are two possible types of symmetry:
  - *M odd:* In this case $\alpha = (M-1)/2$ is an integer.
  - *M even:* In this case $\alpha = (M-1)/2$ is not an integer.





SECOND TYPE OF "LINEAR-PHASE"

- The phase response $H(e^{j\omega})$ satisfy the condition

$$\angle H\left(e^{j\omega}\right) = \beta - \alpha\omega$$

  - which is a straight line but not through the origin.
- In this case α is not a constant phase delay, but

$$\frac{d\angle H\left(e^{j\omega}\right)}{d\omega} = -\alpha$$

  - is constant, which is the group delay.
- α is called a *constant group delay*, frequencies are delayed at a constant rate.

SECOND TYPE OF "LINEAR-PHASE"

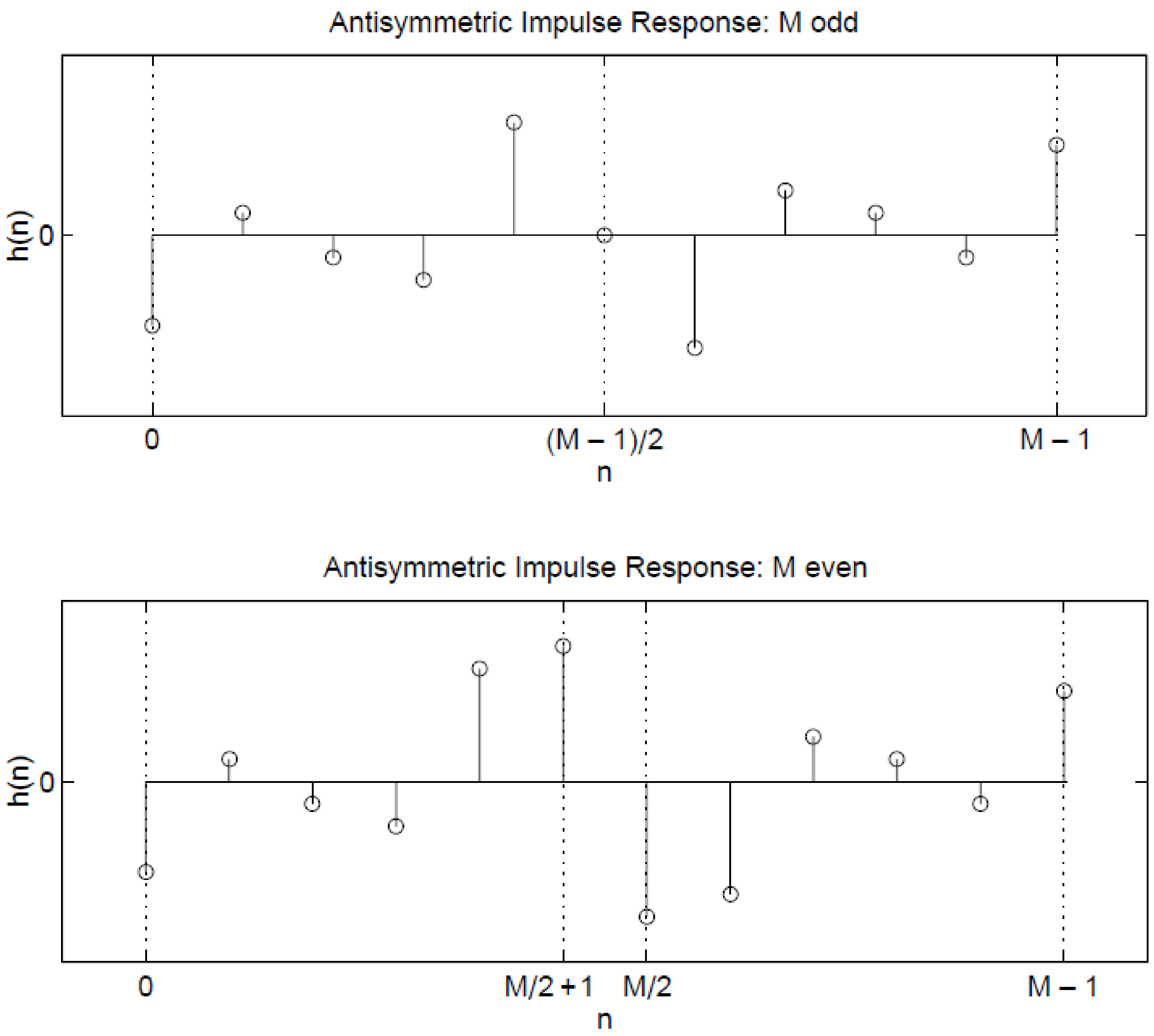
- the impulse response $h(n)$ is *antisymmetric*

$$h(n) = -h(M-1-n), \quad 0 \le n \le (M-1), \quad \alpha = \frac{M-1}{2}, \quad \beta = \pm\frac{\pi}{2}$$

- The index of symmetry is still $\alpha = (M - 1)/2$.

SECOND TYPE OF "LINEAR-PHASE"

- Two possible types
- *M odd:*
  In this case $\alpha = (M - 1)/2$ is an integer
  The sample $h(\alpha)$ at $\alpha = (M - 1)/2$ must necessarily be equal to zero, i.e., $h((M - 1)/2) = 0$.
- *M even:*
  In this case $\alpha = (M - 1)/2$ is not an integer



FREQUENCY RESPONSE $H(e^{j\omega})$

- When the cases of symmetry and antisymmetry are combined with odd and even *M*, there are four types of linear-phase FIR filters.
- Frequency response functions for each of these types have some peculiar expressions and shapes.

$$H\left(e^{j\omega}\right)=H_r\left(\omega\right)e^{j(\beta-\alpha\omega)};\quad \beta=\pm\frac{\pi}{2},\alpha=\frac{M-1}{2}$$

  - $H_r(\omega)$ is an *amplitude response* function and not a magnitude response function
- The amplitude response is a real function, but unlike the magnitude response, which is always positive, the amplitude response may be both positive and negative.
- The phase response associated with the magnitude response is a *discontinuous* function, while that associated with the amplitude response is a *continuous linear* function.

EXAMPLE

- Let the impulse response be

$$h(n) = \left\{ \underset{\uparrow}{1}, 1, 1 \right\}$$

- Determine and draw frequency responses.

EXAMPLE: SOLUTION

$$h(n) = \{\underset{\uparrow}{1}, 1, 1\}$$

- The frequency response function is

$$H\left(e^{j\omega}\right) = \sum_{0}^{2} h(n) e^{-j\omega n} = 1 + e^{-j\omega} + e^{-2j\omega} = \left\{e^{j\omega} + 1 + e^{-j\omega}\right\} e^{j\omega}$$

$$= \{1 + 2\cos\omega\} e^{j\omega}$$

- From this the magnitude and the phase responses are

$$\left|H\left(e^{j\omega}\right)\right| = \left|1 + 2\cos\omega\right|, \quad 0 < \omega \leq \pi$$

$$\angle H\left(e^{j\omega}\right) = \begin{cases} -\omega, & 0 < \omega < 2\pi/3 \\ \pi - \omega, & 2\pi/3 < \omega < \pi \end{cases}$$

- since cos ω can be both positive and negative
- the phase response is *piecewise linear*

EXAMPLE: SOLUTION

$$h(n) = \{\underset{\uparrow}{1}, 1, 1\}$$

- amplitude and the corresponding phase responses are

$$H_r\left(e^{j\omega}\right) = 1 + 2\cos\omega$$

$$\angle H\left(e^{j\omega}\right) = -\omega$$

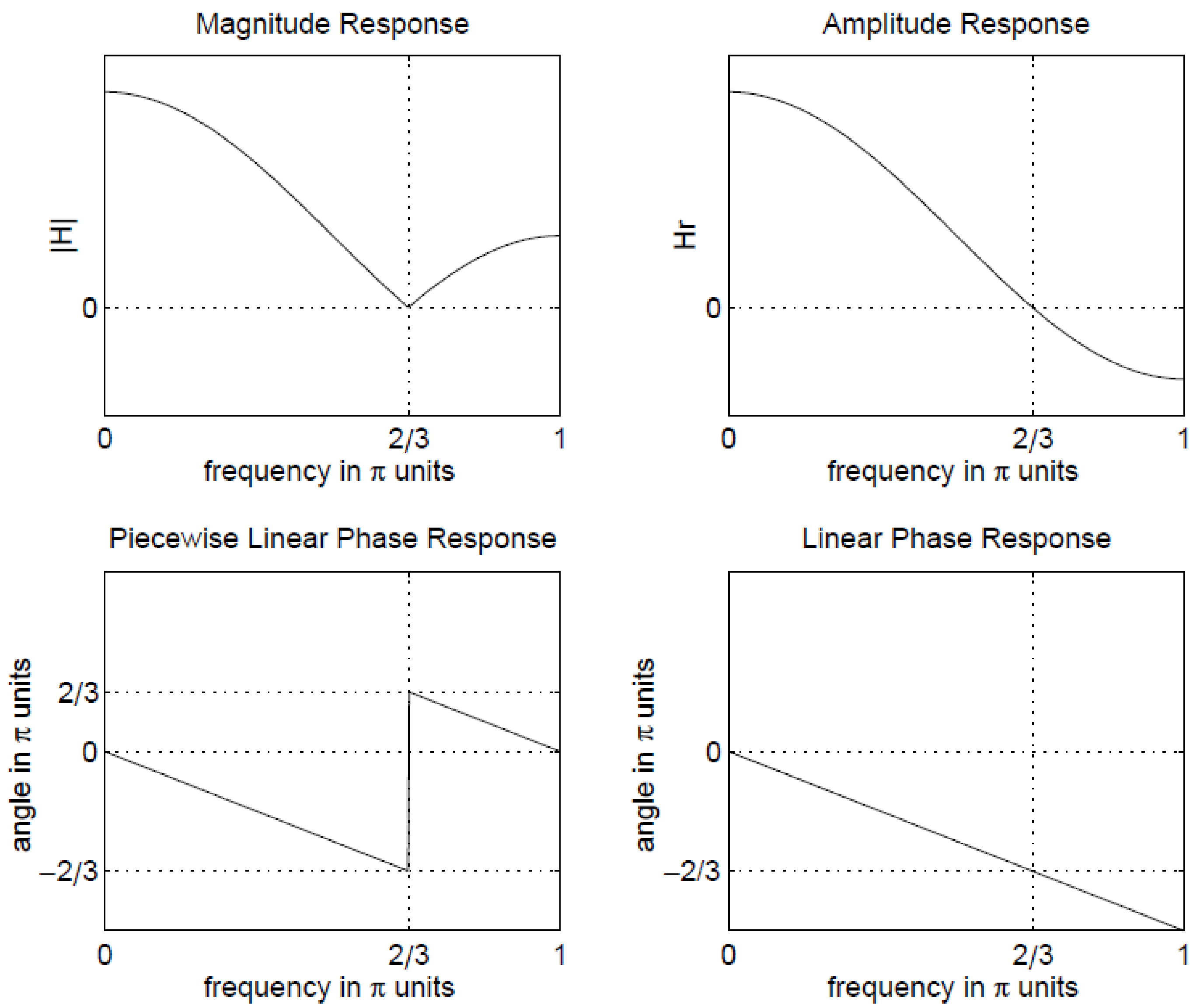
$$-\pi < \omega < \pi$$

- the phase response is *truly linear*

EXAMPLE: SOLUTION

$$h(n) = \{\underset{\uparrow}{1}, 1, 1\}$$

- The difference between the magnitude and the amplitude (or between the piecewise linear and the linear-phase) responses should be clear.



TYPE-1 LINEAR-PHASE FIR FILTER

- ***Symmetrical impulse response, M odd***
- In this case $\beta = 0$, $\alpha = (M - 1)/2$ is an integer, and $h(n) = h(M - 1 - n)$, $0 \le n \le M - 1$.

$$H\left(e^{j\omega}\right) = \left[\sum_{n=0}^{(M-1)/2} a(n)\cos\omega n\right] e^{-j\omega(M-1)/2}$$

- sequence $a(n)$ is obtained from $h(n)$

$$a(0) = h\left(\frac{M-1}{2}\right) : \text{the middle sample}$$

$$a(n) = 2h\left(\frac{M-1}{2} - n\right), 1 \le n \le \frac{M-3}{2}$$

$$H_r(\omega) = \sum_{n=0}^{(M-1)/2} a(n)\cos\omega n$$

TYPE-2 LINEAR-PHASE FIR FILTER

- ***Symmetrical impulse response, M even***
- In this case $\beta = 0$, $h(n) = h(M-1-n)$, $0 \le n \le M-1$, but $\alpha = (M-1)/2$ is not an integer.

$$H\left(e^{j\omega}\right) = \left[\sum_{n=1}^{\mathrm{M}/2} b(n)\cos\left\{\omega\left(n-\frac{1}{2}\right)\right\}\right] e^{-j\omega(M-1)/2}$$

$$b(n) = 2h\left(\frac{M}{2}-n\right),\ n = 1, 2, \ldots, \frac{M}{2}$$

$$H_r(\omega) = \sum_{n=1}^{M/2} b(n)\cos\left\{\omega\left(n-\frac{1}{2}\right)\right\}$$

- At $\omega = \pi$

$$H_r(\pi) = \sum_{n=1}^{M/2} b(n)\cos\left\{\pi\left(n-\frac{1}{2}\right)\right\} = 0$$

TYPE-2 LINEAR-PHASE FIR FILTER

- cannot use this type (i.e., symmetric $h(n)$, M even) for highpass or bandstop filters

TYPE-3 LINEAR-PHASE FIR FILTER

- ***Antisymmetric impulse response,***
  M ***odd*** In this case $\beta = \pi/2$, $\alpha = (M - 1)/2$ is an integer, $h(n) = -h(M - 1 - n)$, $0 \le n \le M - 1$, and $h((M - 1)/2) = 0$.

$$H\left(e^{j\omega}\right) = \left[\sum_{n=1}^{(M-1)/2} c(n)\sin\omega n\right] e^{j\left[\frac{\pi}{2}-\left(\frac{M-1}{2}\right)\omega\right]}$$

$$c(n) = 2h\left(\frac{M-1}{2} - n\right),\ n = 1, 2, \ldots, \frac{M-1}{2}$$

$$H_r(\omega) = \sum_{n=1}^{(M-1)/2} c(n)\sin\omega n$$

TYPE-3 LINEAR-PHASE FIR FILTER

- At $\omega = 0$ and $\omega = \pi$ we have $Hr(\omega) = 0$, regardless of $c(n)$ or $h(n)$.
- $e^{j\pi/2} = j$, which means that $jHr(\omega)$ is purely imaginary.
- This type of filter is not suitable for designing a lowpass filter or a highpass filter.
- This behavior is suitable for approximating ideal digital Hilbert transformers and differentiators.
- An ideal Hilbert transformer is an all-pass filter that imparts a 90° phase shift on the input signal
- It is frequently used in communication systems for modulation purposes.
- Differentiators are used in many analog and digital systems to take the derivative of a signal.

TYPE-4 LINEAR-PHASE FIR FILTER

- ***Antisymmetric impulse response, M even***
  This case is similar to Type-2.

$$H\left(e^{j\omega}\right)=\left[\sum_{n=1}^{M/2} d(n)\sin\left\{\omega\left(n-\frac{1}{2}\right)\right\}\right]e^{-j\left[\frac{\pi}{2}-\omega(M-1)/2\right]}$$

$$d(n)=2h\left(\frac{M}{2}-n\right),\ n=1,2,\ldots,\frac{M}{2}$$

$$H_r(\omega)=\sum_{n=1}^{M/2} d(n)\sin\left\{\omega\left(n-\frac{1}{2}\right)\right\}$$

- At ω = 0, $H_r(0)$ = 0 and $e^{j\pi/2} = j$. Hence this type is also suitable for designing digital Hilbert transformers and differentiators.

MATLAB IMPLEMENTATION

- The MATLAB function freqz computes the frequency response from which we can determine the magnitude response but not the amplitude response.
- The function zerophase that can compute the amplitude response.
- The invocation [Hr,w, phi] = zerophase(b,a) returns the amplitude response in Hr, evaluated at 512 values around the top half of the unit circle in the array w and the continuous phase response in phi.

HR TYPE 1

```
function [Hr,w,a,L] = Hr_Type1(h);
% Computes Amplitude response Hr(w) of a Type-1 LP FIR filter
% -------------------------------------------------------------
% [Hr,w,a,L] = Hr_Type1(h)
% Hr = Amplitude Response
% w = 500 frequencies between [0 pi] over which Hr is computed
% a = Type-1 LP filter coefficients
% L = Order of Hr
% h = Type-1 LP filter impulse response
%
M = length(h); L = (M-1)/2;
a = [h(L+1) 2*h(L:-1:1)]; % 1x(L+1) row vector
n = [0:1:L]; % (L+1)x1 column vector
w = [0:1:500]'*pi/500; Hr = cos(w*n)*a';
```

HR TYPE2

```
function [Hr,w,b,L] = Hr_Type2(h);
% Computes Amplitude response of a Type-2 LP FIR filter
% -----------------------------------------------------
% [Hr,w,b,L] = Hr_Type2(h)
% Hr = Amplitude Response
% w = frequencies between [0 pi] over which Hr is computed
% b = Type-2 LP filter coefficients
% L = Order of Hr
% h = Type-2 LP impulse response
%
M = length(h); L = M/2;
b = 2*[h(L:-1:1)]; n = [1:1:L]; n = n-0.5;
w = [0:1:500]'*pi/500; Hr = cos(w*n)*b';
```

HR TYPE3

```
function [Hr,w,c,L] = Hr_Type3(h);
% Computes Amplitude response Hr(w) of a Type-3 LP FIR filter
% -----------------------------------------------------------
% [Hr,w,c,L] = Hr_Type3(h)
% Hr = Amplitude Response
% w = frequencies between [0 pi] over which Hr is computed
% c = Type-3 LP filter coefficients
% L = Order of Hr
% h = Type-3 LP impulse response
%
M = length(h); L = (M-1)/2;
c = [2*h(L+1:-1:1)]; n = [0:1:L];
w = [0:1:500]'*pi/500; Hr = sin(w*n)*c';
```

HR TYPE4

```
function [Hr,w,d,L] = Hr_Type4(h);
% Computes Amplitude response of a Type-4 LP FIR filter
% ---------------------------------------------------
% [Hr,w,d,L] = Hr_Type4(h)
% Hr = Amplitude Response
% w = frequencies between [0 pi] over which Hr is computed
% d = Type-4 LP filter coefficients
% L = Order of d
% h = Type-4 LP impulse response
%
M = length(h); L = M/2;
d = 2*[h(L:-1:1)]; n = [1:1:L]; n = n-0.5;
w = [0:1:500]'*pi/500; Hr = sin(w*n)*d';
```

ZERO LOCATIONS

- Recall that for an FIR filter there are $(M - 1)$ (trivial) poles at the origin and $(M - 1)$ zeros located somewhere in the z-plane.

- For linear-phase FIR filters, these zeros possess certain symmetries that are due to the symmetry constraints on $h(n)$.

ZERO LOCATIONS

- A general *zero constellation* is a quadruplet

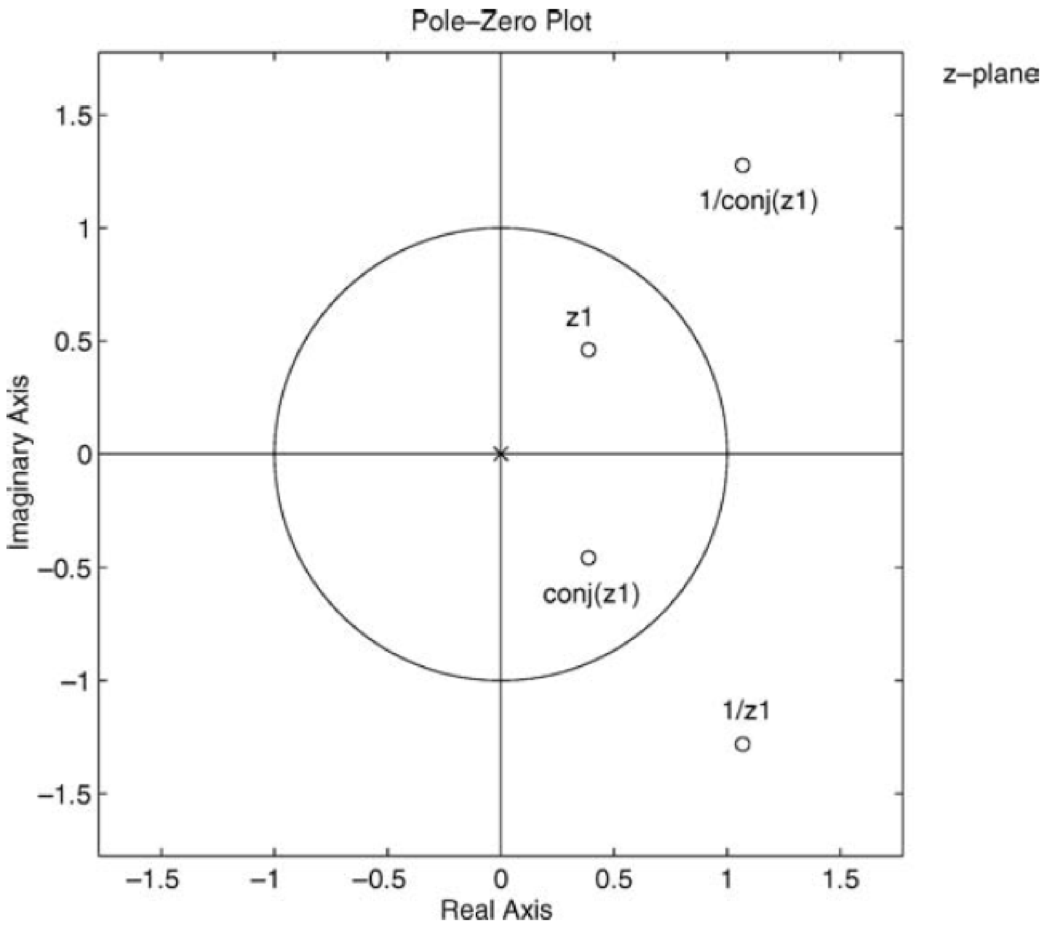
$$re^{j\theta} \qquad \frac{1}{r}e^{j\theta} \qquad re^{-j\theta} \qquad \frac{1}{r}e^{-j\theta}$$

- If r = 1, then 1/r = 1, and hence the zeros are on the unit circle and occur in pairs

$$e^{j\theta} \qquad e^{-j\theta}$$

- If θ = 0 or θ = π, then the zeros are on the real line and occur in pairs

$$r \qquad \frac{1}{r}$$



EXAMPLE

- Let

$$h(n) = \left\{ \underset{\uparrow}{-4}, 1, -1, -2, 5, 6, 5, -2, -1, 1, -4 \right\}$$

- Determine the amplitude response *Hr* (*ω*) and the locations of the zeros of *H* (*z*).

EXAMPLE: SOLUTION

- Since M = 11, which is odd, and since $h(n)$ is symmetric about $\alpha = (11-1)/2 = 5$, this is a Type-1 linear-phase FIR filter.

$$a(0) = h(\alpha) = h(5) = 6,\ a(1) = 2h(5-1) = 10,\ a(2) = 2h(5-2) = -4$$

$$a(3) = 2h(5-3) = -2,\ a(4) = 2h(5-4) = 2,\ a(5) = 2h(5-5) = -8$$

- An *amplitude response* function

$$Hr(\omega) = a(0)+a(1)\cos\omega + a(2)\cos 2\omega + a(3)\cos 3\omega + a(4)\cos 4\omega + a(5)\cos 5\omega$$

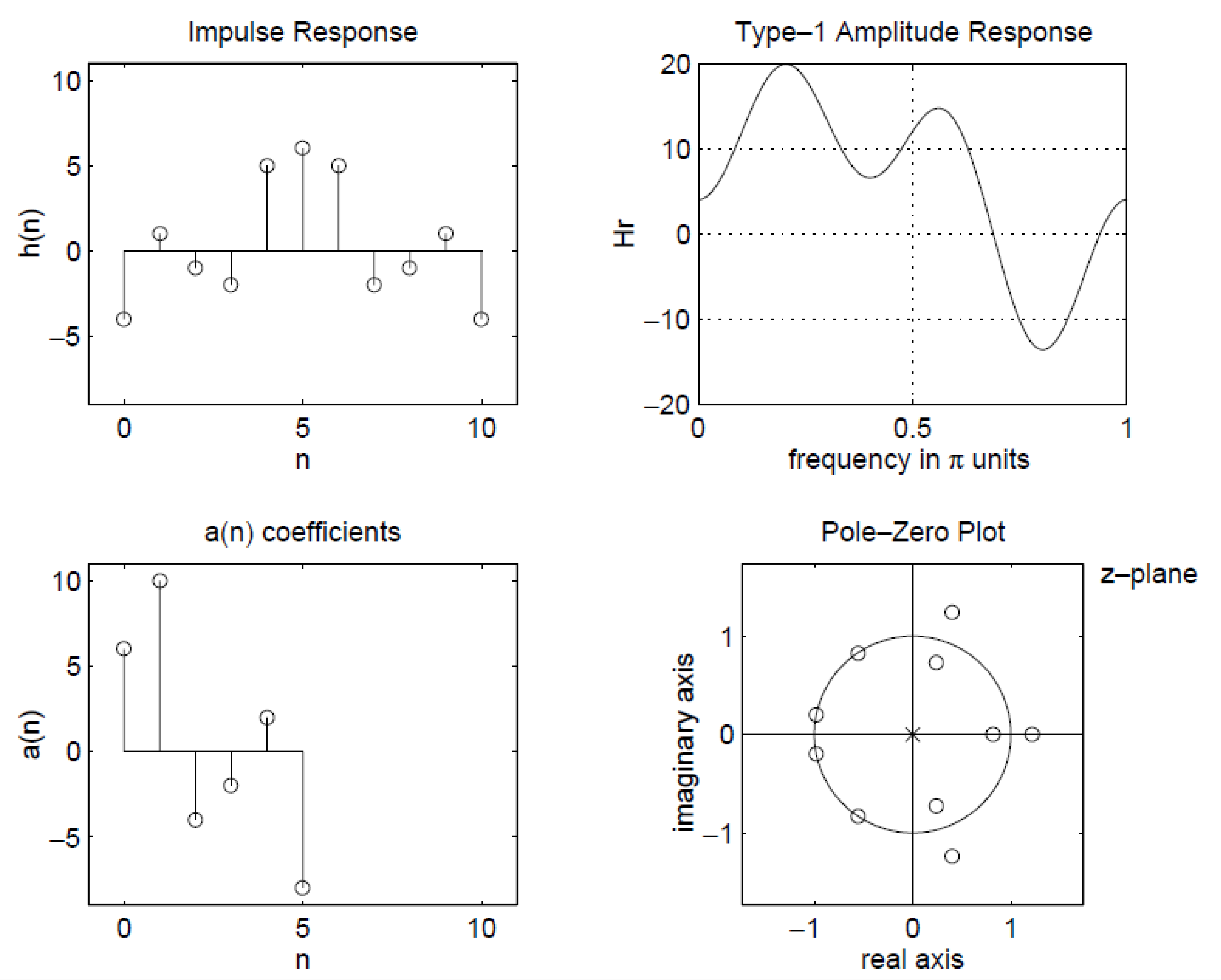
$$= 6 + 10\cos\omega - 4\cos 2\omega - 2\cos 3\omega + 2\cos 4\omega - 8\cos 5\omega$$

EXAMPLE: SOLUTION

```
h = [-4,1,-1,-2,5,6,5,-2,-1,1,-4];
M = length(h); n = 0:M-1;
[Hr,w,a,L] = Hr_Type1(h);
amax = max(a)+1; amin = min(a)-1;
subplot(2,2,1); stem(n,h); axis([-1 2*L+1 amin amax])
xlabel('n'); ylabel('h(n)'); title('Impulse Response')
subplot(2,2,3); stem(0:L,a); axis([-1 2*L+1 amin amax])
xlabel('n'); ylabel('a(n)'); title('a(n) coefficients')
subplot(2,2,2); plot(w/pi,Hr);grid
xlabel('frequency in pi units'); ylabel('Hr')
title('Type-1 Amplitude Response')
subplot(2,2,4); % pzplot(h,1);
```

EXAMPLE: SOLUTION

- There are no restrictions on $H_r(\omega)$ either at $\omega = 0$ or at $\omega = \pi$.
- There is one zero-quadruplet constellation and three zero pairs



EXAMPLE

- Let

$$h(n) = \left\{ \underset{\uparrow}{-4}, 1, -1, -2, 5, 6, 6, 5, -2, -1, 1, -4 \right\}$$

- Determine the amplitude response *Hr* (*ω*) and the locations of the zeros of *H* (*z*).

EXAMPLE: SOLUTION

- This is a Type-2 linear-phase FIR filter since $M = 12$ and since $h(n)$ is symmetric with respect to $\alpha = (12 - 1)/2 = 5.5$.

$$b(1) = 2h\left(\frac{12}{2}-1\right)=12, \quad b(2) = 2h\left(\frac{12}{2}-2\right)=10, \quad b(3) = 2h\left(\frac{12}{2}-3\right)=-4$$

$$b(4) = 2h\left(\frac{12}{2}-4\right)=-2, \quad b(5) = 2h\left(\frac{12}{2}-5\right)=2, \quad b(6) = 2h\left(\frac{12}{2}-6\right)=-8$$

- An *amplitude response* function

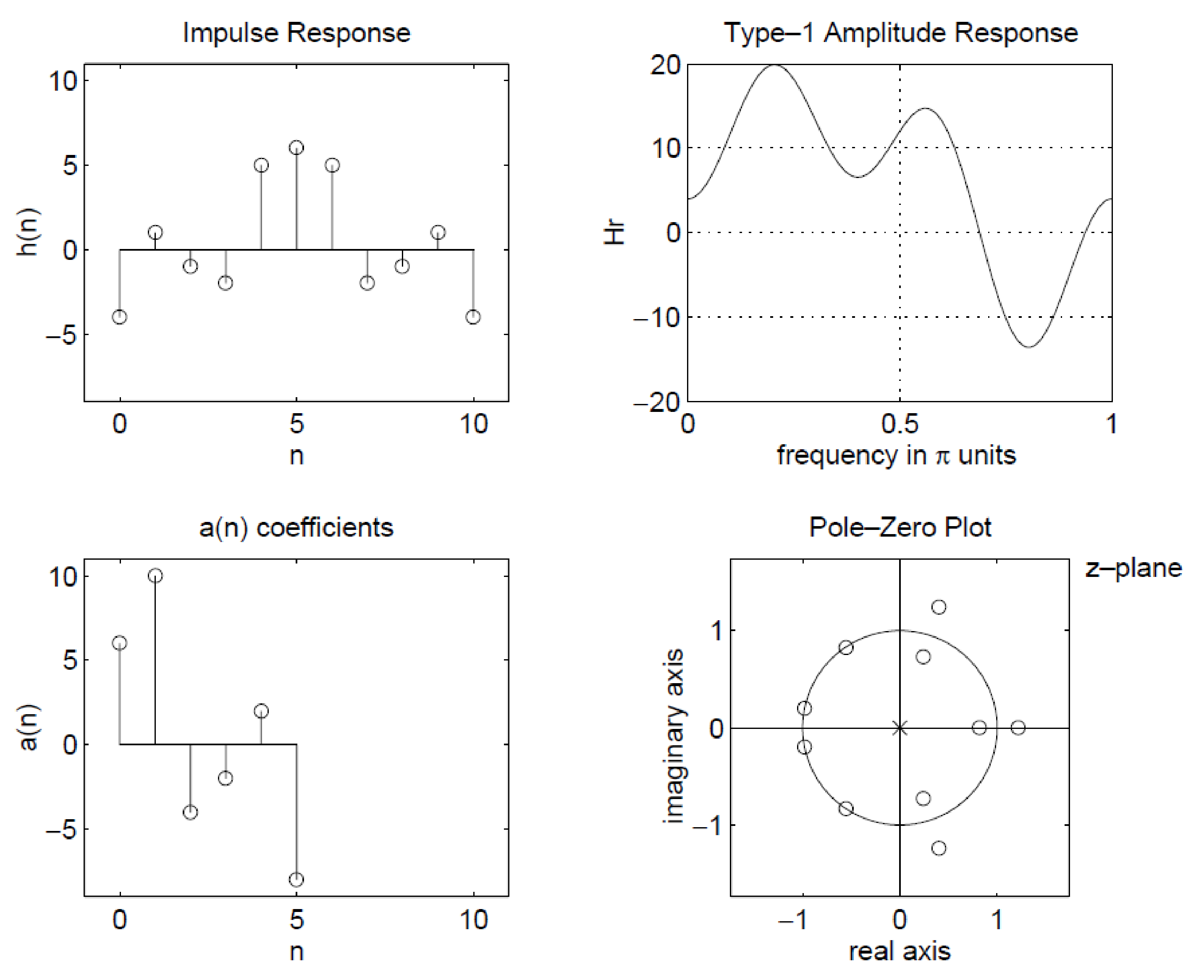
$$\begin{aligned} H_r(\omega) &= b(1)\cos\left[\omega\left(1-\tfrac{1}{2}\right)\right] + b(2)\cos\left[\omega\left(2-\tfrac{1}{2}\right)\right] + b(3)\cos\left[\omega\left(3-\tfrac{1}{2}\right)\right] \\ &\quad + b(4)\cos\left[\omega\left(4-\tfrac{1}{2}\right)\right] + b(5)\cos\left[\omega\left(5-\tfrac{1}{2}\right)\right] + b(6)\cos\left[\omega\left(6-\tfrac{1}{2}\right)\right] \\ &= 12\cos\left(\frac{\omega}{2}\right) + 10\cos\left(\frac{3\omega}{2}\right) - 4\cos\left(\frac{5\omega}{2}\right) - 2\cos\left(\frac{7\omega}{2}\right) \\ &\quad + 2\cos\left(\frac{9\omega}{2}\right) - 8\cos\left(\frac{11\omega}{2}\right) \end{aligned}$$

EXAMPLE: SOLUTION

```
h = [-4,1,-1,-2,5,6,6,5,-2,-1,1,-4];
M = length(h); n = 0:M-1;
[Hr,w,b,L] = Hr_Type2(h);
bmax = max(b)+1; bmin = min(b)-1;
subplot(2,2,1); stem(n,h); axis([-1 2*L+1 bmin bmax])
xlabel('n'); ylabel('h(n)'); title('Impulse Response')
subplot(2,2,3); stem(1:L,b); axis([-1 2*L+1 bmin bmax])
xlabel('n'); ylabel('b(n)'); title('b(n) coefficients')
subplot(2,2,2); plot(w/pi,Hr);grid
xlabel('frequency in pi units'); ylabel('Hr')
title('Type-1 Amplitude Response')
subplot(2,2,4); % pzplotz(h,1)
```

EXAMPLE: SOLUTION

- observe that there are no restrictions on $Hr(\omega)$ either at $\omega = 0$ or at $\omega = \pi$.
- There is one zero-quadruplet constellation and three zero pairs.



EXAMPLE

- Let

$$h(n) = \{\underset{\uparrow}{-4}, 1, -1, -2, 5, 0, -5, 2, 1, -1, 4\}$$

- Determine the amplitude response *Hr* (*ω*) and the locations of the zeros of *H* (*z*).

EXAMPLE: SOLUTION

- Since M = 11, which is odd, and since $h(n)$ is antisymmetric about α = (11 – 1)/2 = 5, this is a Type-3 linear-phase FIR filter.

$$c(0) = h(\alpha) = h(5) = 0,\ c(1) = 2h(5-1) = 10,\ c(2) = 2h(2-2) = -4$$

$$c(3) = 2h(5-3) = -2,\ c(4) = 2h(5-4) = 2,\ c(5) = 2h(5-5) = -8$$

- An *amplitude response* function

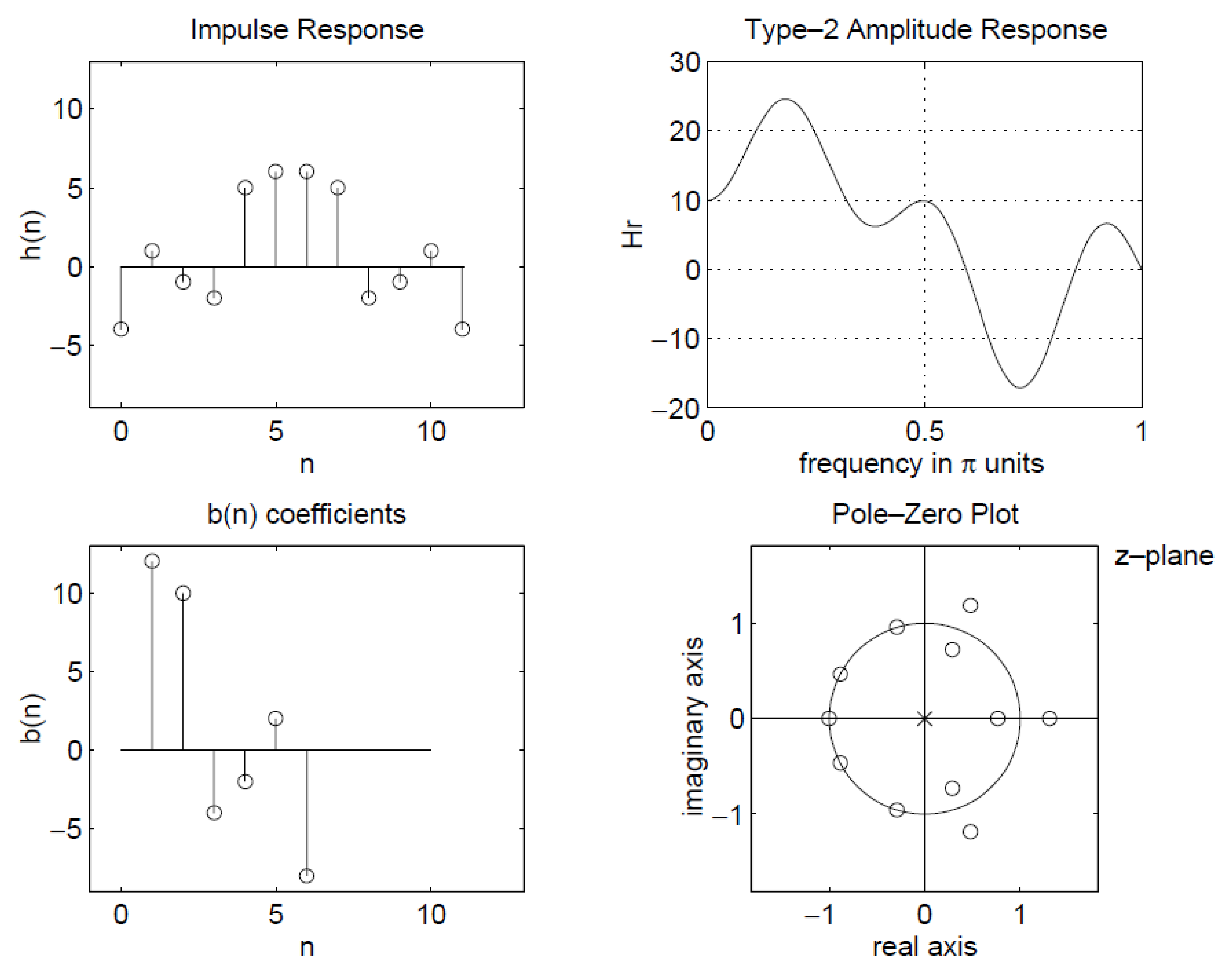
$$\begin{aligned} H_r(\omega) &= c(0) + c(1)\sin\omega + c(2)\sin 2\omega + c(3)\sin 3\omega + c(4)\sin 4\omega + c(5)\sin 5\omega \\ &= 0 + 10\sin\omega - 4\sin 2\omega - 2\sin 3\omega + 2\sin 4\omega - 8\sin 5\omega \end{aligned}$$

EXAMPLE: SOLUTION

```
h = [-4,1,-1,-2,5,0,-5,2,1,-1,4];
M = length(h); n = 0:M-1; [Hr,w,c,L] = Hr_Type3(h);
cmax = max(c)+1; cmin = min(c)-1;
subplot(2,2,1); stem(n,h); axis([-1 2*L+1 cmin cmax])
xlabel('n'); ylabel('h(n)'); title('Impulse Response')
subplot(2,2,3); stem(0:L,c); axis([-1 2*L+1 cmin cmax])
xlabel('n'); ylabel('c(n)'); title('c(n) coefficients')
subplot(2,2,2); plot(w/pi,Hr);grid
xlabel('frequency in pi units'); ylabel('Hr')
title('Type-1 Amplitude Response')
subplot(2,2,4); % pzplotz(h,1)
```

EXAMPLE: SOLUTION

- observe that $Hr(\omega)$ is zero at $\omega = \pi$.
- There is one zero-quadruplet constellation, three zero pairs, and one zero at $\omega = \pi$ as expected.



EXAMPLE

- Let

$$h(n) = \{\underset{\uparrow}{-4}, 1, -1, -2, 5, 6, -6, -5, 2, 1, -1, 4\}$$

- Determine the amplitude response $H_r(\omega)$ and the locations of the zeros of $H(z)$.

EXAMPLE: SOLUTION

- This is a Type-4 linear-phase FIR filter since $M = 12$ and since $h(n)$ is antisymmetric with respect to $\alpha = (12 - 1)/2 = 5.5$.

$$d(1) = 2h\left(\tfrac{12}{2} - 1\right) = 12, \;\; d(2) = 2h\left(\tfrac{12}{2} - 2\right) = 10, \;\; d(3) = 2h\left(\tfrac{12}{2} - 3\right) = -4$$

$$d(4) = 2h\left(\tfrac{12}{2} - 4\right) = -2, \;\; d(5) = 2h\left(\tfrac{12}{2} - 5\right) = 2, \;\; d(6) = 2h\left(\tfrac{12}{2} - 6\right) = -8$$

- An *amplitude response* function

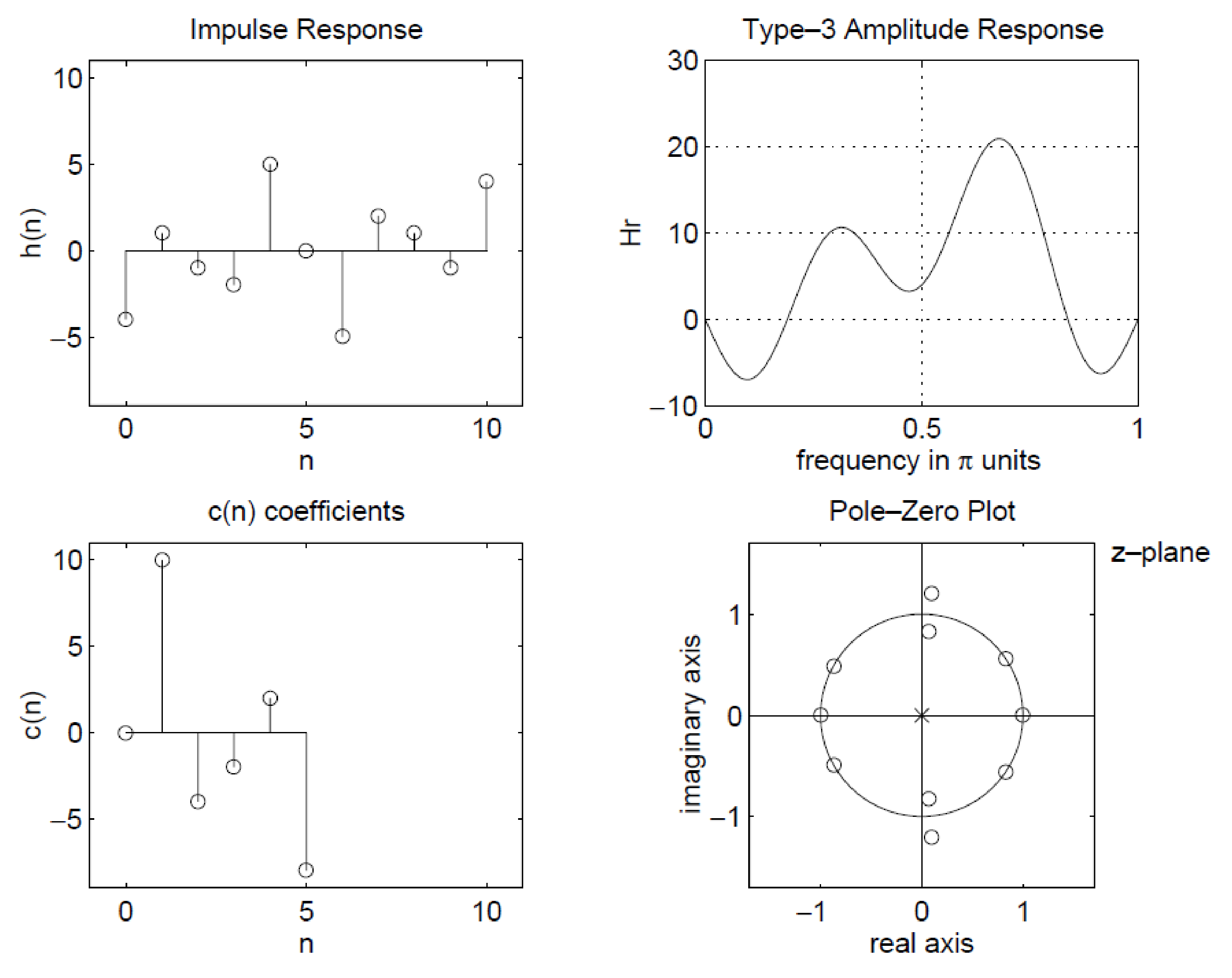
$$\begin{aligned} H_r(\omega) &= d(1)\sin\left[\omega\left(1 - \frac{1}{2}\right)\right] + d(2)\sin\left[\omega\left(2 - \frac{1}{2}\right)\right] + d(3)\sin\left[\omega\left(3 - \frac{1}{2}\right)\right] \\ &\quad + d(4)\sin\left[\omega\left(4 - \frac{1}{2}\right)\right] + d(5)\sin\left[\omega\left(5 - \frac{1}{2}\right)\right] + d(6)\sin\left[\omega\left(6 - \frac{1}{2}\right)\right] \\ &= 12\sin\left(\frac{\omega}{2}\right) + 10\sin\left(\frac{3\omega}{2}\right) - 4\sin\left(\frac{5\omega}{2}\right) - 2\sin\left(\frac{7\omega}{2}\right) \\ &\quad + 2\sin\left(\frac{9\omega}{2}\right) - 8\sin\left(\frac{11\omega}{2}\right) \end{aligned}$$

EXAMPLE: SOLUTION

```
h = [-4,1,-1,-2,5,6,-6,-5,2,1,-1,4];
M = length(h); n = 0:M-1; [Hr,w,d,L] = Hr_Type4(h);
dmax = max(d)+1; dmin = min(d)-1;
subplot(2,2,1); stem(n,h); axis([-1 2*L+1 dmin dmax])
xlabel('n'); ylabel('h(n)'); title('Impulse Response')
subplot(2,2,3); stem(1:L,d); axis([-1 2*L+1 dmin dmax])
xlabel('n'); ylabel('d(n)'); title('d(n) coefficients')
subplot(2,2,2); plot(w/pi,Hr);grid
xlabel('frequency in pi units'); ylabel('Hr')
title('Type-1 Amplitude Response')
subplot(2,2,4); pzplotz(h,1)
```

EXAMPLE: SOLUTION

- observe that $Hr(\omega) = 0$ at $\omega = 0$ and at $\omega = \pi$.
- There is one zero-quadruplet constellation, two zero pairs, and zeros at $\omega = 0$ and $\omega = \pi$ as expect



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