Pengolahan Sinyal Digital

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System Representation in the z-Domain

The System Function

- The system function $H(z)$ is given by

$$H(z) \triangleq \mathcal{Z}\left[h(n)\right] = \sum_{-\infty}^{\infty} h(n) z^{-n}; \quad R_{h-} < |z| < R_{h+}$$

- Using the convolution property of the z-transform, the output transform $Y(z)$ is given by

$$Y(z) = H(z)X(z) \quad : \quad \mathrm{ROC}_y = \mathrm{ROC}_h \cap \mathrm{ROC}_x$$

The System Function

- Therefore a linear and timeinvariant system can be represented in the *z*-domain by

$$X(z) \;\rightarrow\; \boxed{H(z)} \;\rightarrow\; Y(z) = H(z)X(z)$$

System Function From The Difference Equation Representation

- LTI systems are described by a difference equation

$$y(n)+\sum_{k=1}^{N}a_k y(n-k)=\sum_{\ell=1}^{M}b_\ell x(n-\ell)$$

- System Function Representation

$$Y(z)+\sum_{k=1}^{N}a_k z^{-k}Y(z)=\sum_{\ell=0}^{M}b_\ell z^{-\ell}X(z)$$

$$H(z)\triangleq\frac{Y(z)}{X(z)}=\frac{\sum_{\ell=0}^{M}b_\ell z^{-\ell}}{1+\sum_{k=1}^{N}a_k z^{-k}}=\frac{B(z)}{A(z)}=\frac{b_o z^{-M}\left(z^{M}+\ldots+\frac{b_M}{b_o}\right)}{z^{-N}\left(z^{N}+\ldots+a_N\right)}$$

System Function From The Difference Equation Representation

- After factorization

$$H(z) = b_o z^{N-M} \frac{\prod_{\ell=1}^{N}(z - z_\ell)}{\prod_{k=1}^{N}(z - p_k)}$$

  - z_ℓ are the system zeros
  - p_k are the system poles
- $H(z)$ (an LTI system) can also be represented in the z-domain using a pole-zero plot

Transfer Function Representation

- A frequency response function or transfer function $H(z)$ on the unit circle $z = e^{j\omega}$

$$H\left(e^{j\omega}\right) = b_o z e^{j(N-M)\omega} \frac{\prod_{\ell=1}^{N}\left(e^{j\omega} - z_\ell\right)}{\prod_{k=1}^{N}\left(e^{j\omega} - p_k\right)}$$

- The factor $(e^{j\omega} - z_\ell)$ can be interpreted as a *vector* in the complex *z*-plane from a zero z to the unit circle at $z = e^{j\omega}$
- The factor $(e^{j\omega} - p_k)$ can be interpreted as a vector from a pole p_k to the unit circle at $z = e^{j\omega}$.

Transfer Function Representation

- The magnitude response function

$$\left|H\left(e^{j\omega}\right)\right| = |b_o| \frac{\left|e^{j\omega} - z_1\right| \ldots \left|e^{j\omega} - z_M\right|}{\left|e^{j\omega} - p_1\right| \ldots \left|e^{j\omega} - p_N\right|}$$

  - a product of the lengths of vectors from zeros to the unit circle *divided* by the lengths of vectors from poles to the unit circle and *scaled* by $|b_o|$.

Transfer Function Representation

- The phase response function

$$\angle H\left(e^{j\omega}\right)=\left[0 \text{ or } \pi\right]+\left[\left(N-M\right)\omega\right]+\sum_{1}^{M}\angle\left(e^{j\omega}-z_k\right)-\sum_{1}^{N}\angle\left(e^{j\omega}-p_k\right)$$

  - a sum of a constant factor, a linear-phase factor, and a nonlinear-phase factor (angles from the "zero vectors" *minus* the sum of angles from the "pole vectors").

Example

- Given a causal system

$$y(n) = 0.9y(n-1) + x(n)$$

  - Determine $H(z)$ and sketch its pole-zero plot.
  - Plot $|H(e^{j\omega})|$ and $\angle H(e^{j\omega})$.
  - Determine the impulse response $h(n)$.

Example

- The difference equation can be put in the form

$$y(n) - 0.9y(n-1) = x(n)$$

$$H(z) = b_o z^{N-M} \frac{\prod_{\ell=1}^{N}(z - z_\ell)}{\prod_{k=1}^{N}(z - p_k)}$$

$$H(z) = \frac{1}{1 - 0,9z^{-1}}; \quad |z| > 0,9$$

- since the system is causal. There is one pole at 0.9 and one zero at the origin

Example

- illustrate using the zplane function

```
>> b = [1, 0]; a = [1, -0.9]; zplane(b,a)
```

Example

- determine the magnitude and phase of $H(ej\omega)$ using freqz function.
- take 100 points along the upper half of the unit circle

```
>> [H,w] = freqz(b,a,100); magH = abs(H); phaH = angle(H);
>> subplot(2,1,1);plot(w/pi,magH);grid
>> xlabel('frequency in pi units'); ylabel('Magnitude');
>> title('Magnitude Response')
>> subplot(2,1,2);plot(w/pi,phaH/pi);grid
>> xlabel('frequency in pi units'); ylabel('Phase in pi units');
>> title('Phase Response')
```

Example

- From the *z*-transform in Table

$$h(n) = Z^{-1}\left[\frac{1}{1-0,9z^{-1}}; \quad |z| > 0,9\right]$$

$$= (0,9)^n u(n)$$

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