UNIVERSITAS INDONESIA
Veritas, Probitas, Iustitia

Pengolahan Sinyal Digital

Adhi Harmoko Saputro

Sampling and Reconstruction of Analog Signals

Signal Conversion

- Realworld analog signals are converted into discrete signals using sampling and quantization operations (collectively called analog-to-digital conversion, or ADC).
- These discrete signals are processed by digital signal processors, and the processed signals are converted into analog signals using a reconstruction operation (called digital-to-analog conversion or DAC)

Sampling in Fourier analysis

- Describing the sampling operation from the frequency-domain viewpoint, analyze its effects, and then address the reconstruction operation

- Assuming that the number of quantization levels is sufficiently large that the effect of quantization on discrete signals is negligible

Aliasing Formula

- Continuous-time Fourier transform (CTFT)

$$X_a(j\Omega) \triangleq \int_{-\infty}^{\infty} x_a(t) e^{-j\Omega t} dt$$

  - $x_a(t)$ be an analog (absolutely integrable) signal
  - Ω is an analog frequency in radians/sec

- Inverse continuous-time Fourier transform (ICTFT)

$$x_a(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_a(j\Omega) e^{j\Omega t} d\Omega$$

Aliasing Formula

- $x_a(t)$ is sample at *sampling interval* T_s seconds apart to obtain the discrete-time signal $x(n)$

$$x(n) \triangleq x_a(nT_s)$$

- The discrete-time Fourier transform of $x(n)$: the *aliasing formula*

$$X\left(e^{j\omega}\right) = \frac{1}{T_s} \sum_{\ell=-\infty}^{\infty} X_a\left[j\left(\frac{\omega}{T_s} - \frac{2\pi}{T_s}\ell \right)\right]$$

Aliasing Formula

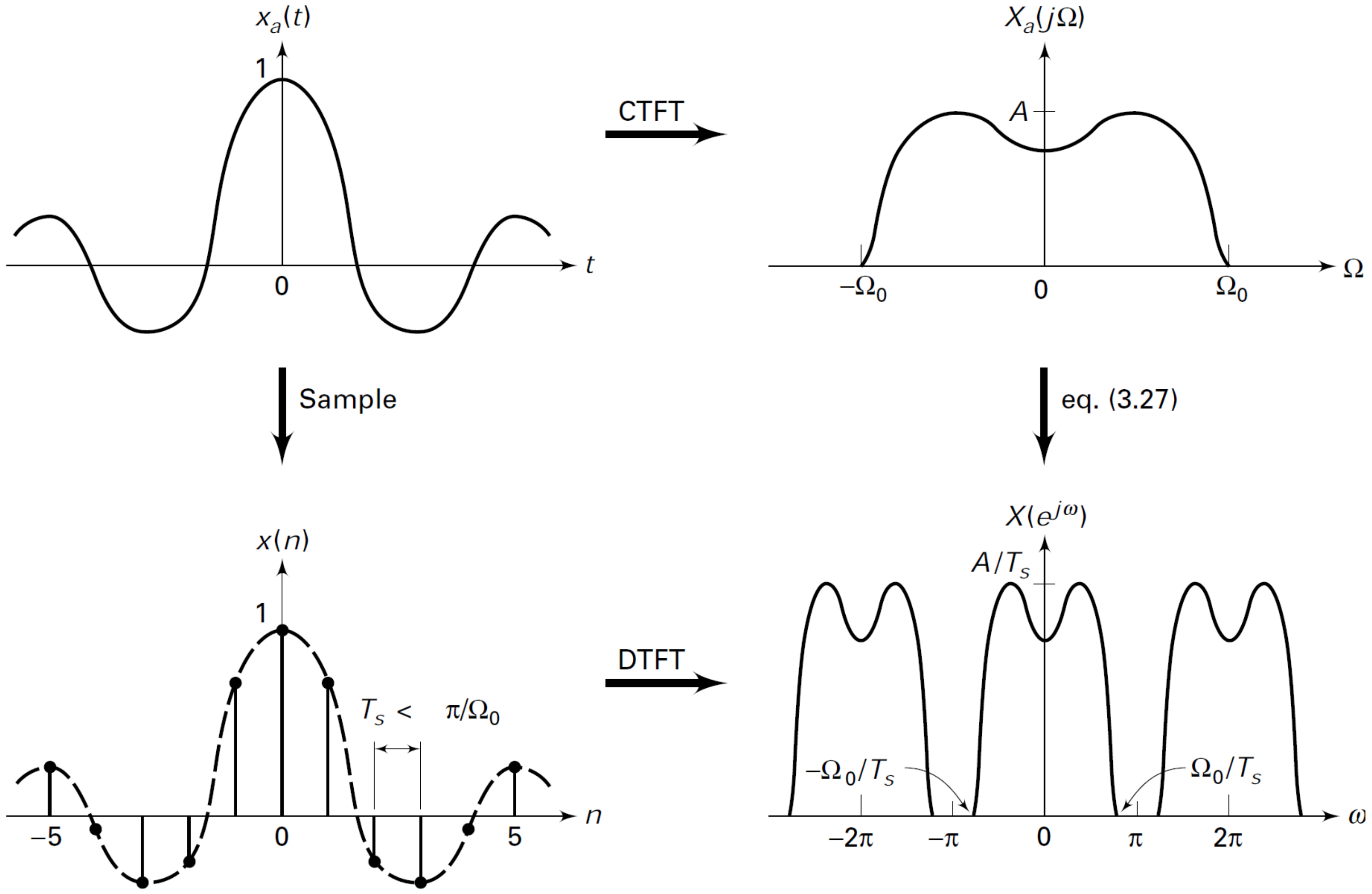
- The analog and digital frequencies are related through T_s

$$\omega = \Omega T_s$$

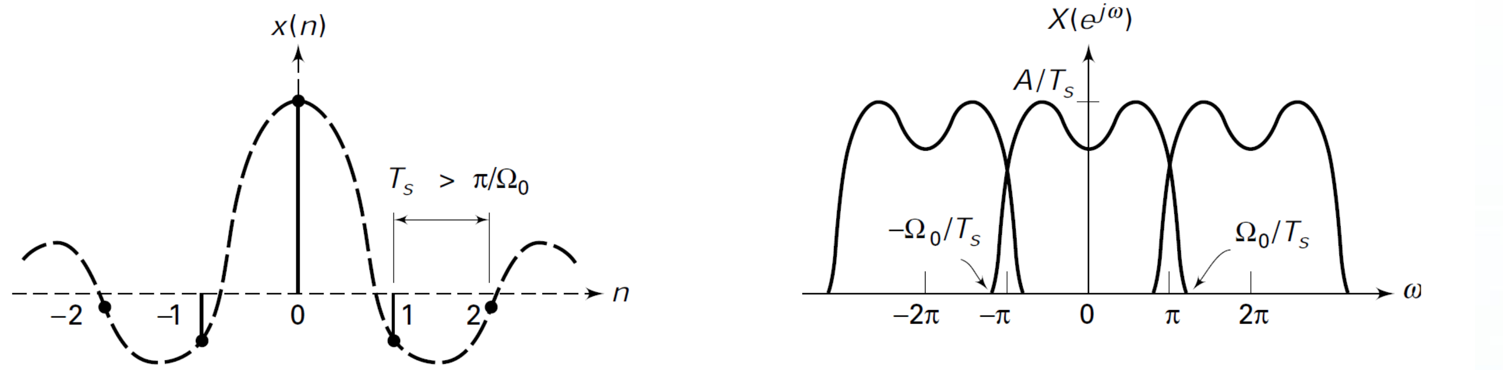
- The sampling frequency F_s is given by

$$F_s \triangleq \frac{1}{T_s} \quad \text{sam/sec}$$

Sampling operation in the time and frequency domains



Sampling operation in the time and frequency domains



Band-limited Signal

- A signal is band-limited if there exists a finite radian frequency Ω_0 such that $X_a(j\Omega)$ is zero for $|\Omega| > \Omega_0$.
- The frequency $F_0=\Omega_0/2\pi$ is called the signal bandwidth in Hz.
- If $\pi > \Omega_0 T_s$ or equivalently, $F_s/2 > F_0$

$$X\left(e^{j\omega}\right) = \frac{1}{T_s} X\left(j\frac{\omega}{T_s} \right); \quad -\frac{\pi}{T_s} < \frac{\omega}{T_s} \leq \frac{\pi}{T_s}$$

Sampling Principle

- A band-limited signal $x_a(t)$ with bandwidth F_0 can be reconstructed from its sample values $x(n) = x_a(nT_s)$ if the sampling frequency $F_s = 1/T_s$ is greater than twice the bandwidth F_0 of $x_a(t)$

$$F_s > 2F_0$$

- Otherwise aliasing would result in $x(n)$.
- The sampling rate of $2F_0$ for an analog band-limited signal is called the Nyquist rate

Signal Reconstruction

- If we sample band-limited $x_a(t)$ above its Nyquist rate, then we can reconstruct $x_a(t)$ from its samples $x(n)$.
  - The samples are converted into a weighted impulse train

$$\sum_{n=-\infty}^{\infty} x(n)\delta(t-nT_s) = \ldots + x(-1)\delta(n+T_s) + x(0)\delta(t) + x(1)\delta(n-T_s) + \ldots$$

  - The impulse train is filtered through an ideal analog lowpass filter band-limited to the $[-F_s/2, F_s/2]$ band.

$$x(n) \longrightarrow \boxed{\begin{array}{c}\text{Impulse train}\\ \text{conversion}\end{array}} \longrightarrow \boxed{\begin{array}{c}\text{Ideal lowpass}\\ \text{filter}\end{array}} \longrightarrow x_a(t)$$

Terima Kasih