Pengolahan Sinyal Digital

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Sampling of Continuous-Time Signals

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Signal Types

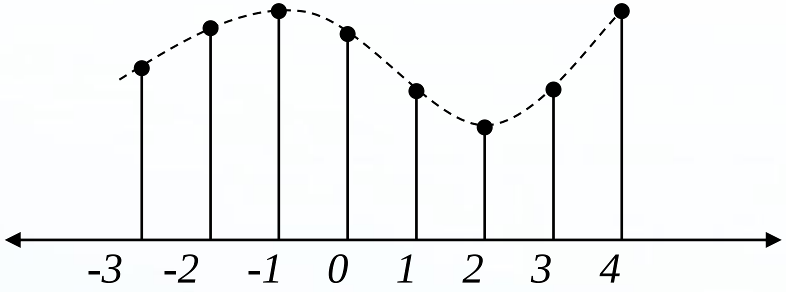
- Analog signals: continuous in time and amplitude
  - Example: voltage, current, temperature,…
- Digital signals: discrete both in time and amplitude
  - Example: attendance of this class, digitizes analog signals,…
- Discrete-time signal: discrete in time, continuous in amplitude
  - Example:hourly change of temperature in Austin

Signal Types

- Theory for digital signals would be too complicated
  - Requires inclusion of nonlinearities into theory
- Theory is based on discrete-time continuous-amplitude signals
  - Most convenient to develop theory
  - Good enough approximation to practice with some care
- In practice we mostly process digital signals on processors
  - Need to take into account finite precision effects
- Our text book is about the theory hence its title
  - Discrete-Time Signal Processing

Periodic (Uniform) Sampling

- Sampling is a continuous to discrete-time conversion



- Most common sampling is periodic

$$x[n] = x_c(nT) \quad -\infty < n < \infty$$

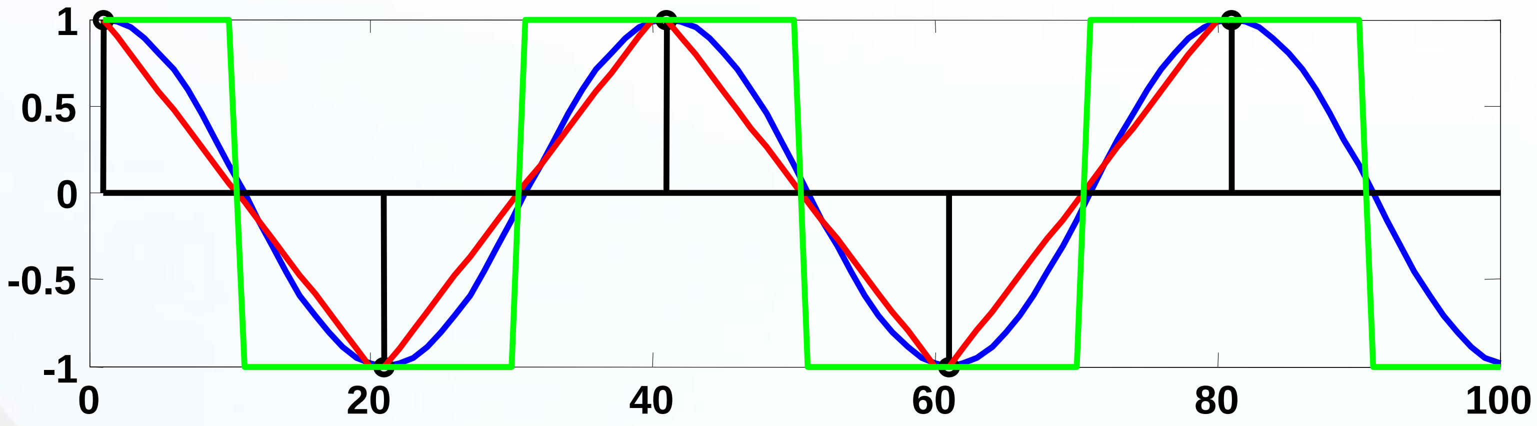
- T is the sampling period in second
- $f_s = 1/T$ is the sampling frequency in Hz

Periodic (Uniform) Sampling

- Sampling frequency in radian-per-second $\Omega_s = 2\pi f_s$ rad/sec
- Use [.] for discrete-time and (.) for continuous time signals
- This is the ideal case not the practical but close enough
  - In practice it is implement with an analog-to-digital converters
  - We get digital signals that are quantized in amplitude and time

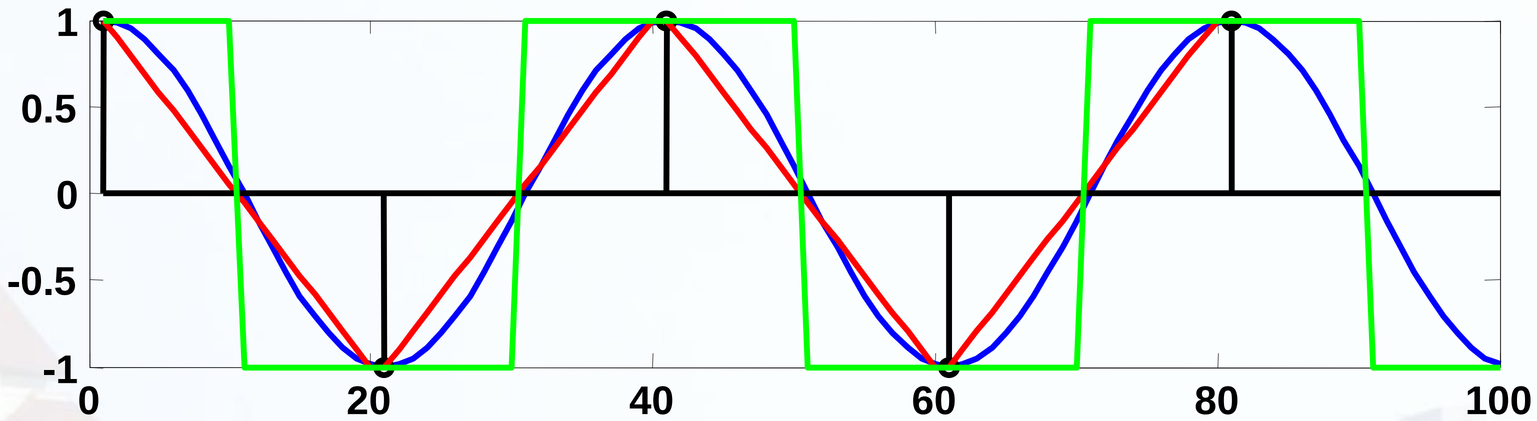
Periodic Sampling

- Sampling is, in general, not reversible
- Given a sampled signal one could fit infinite continuous signals through the samples



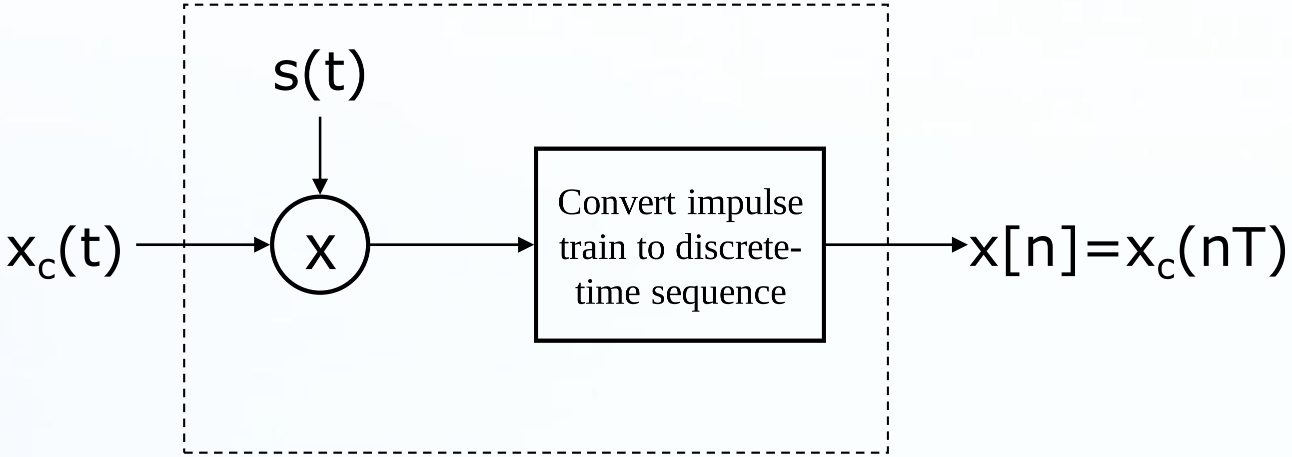
Periodic Sampling

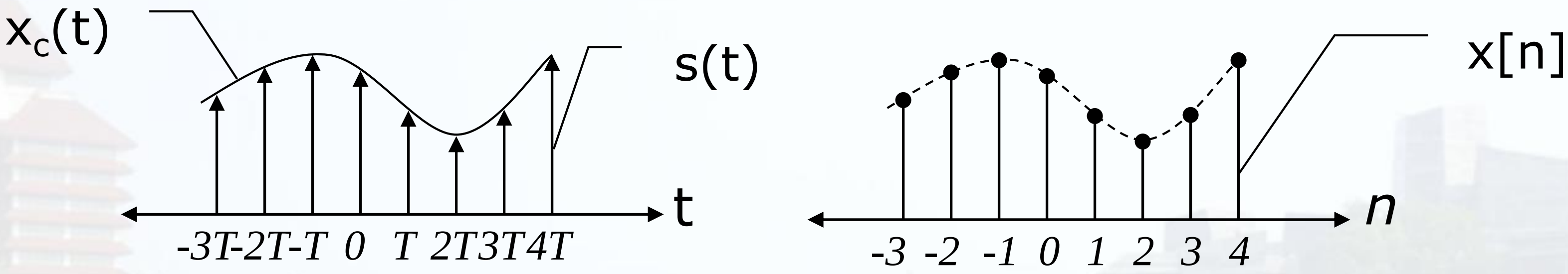
- Fundamental issue in digital signal processing
  - If we loose information during sampling we cannot recover it
- Under certain conditions an analog signal can be sampled without loss so that it can be reconstructed perfectly



Representation of Sampling

- Mathematically convenient to represent in two stages
  - Impulse train modulator
  - Conversion of impulse train to a sequence



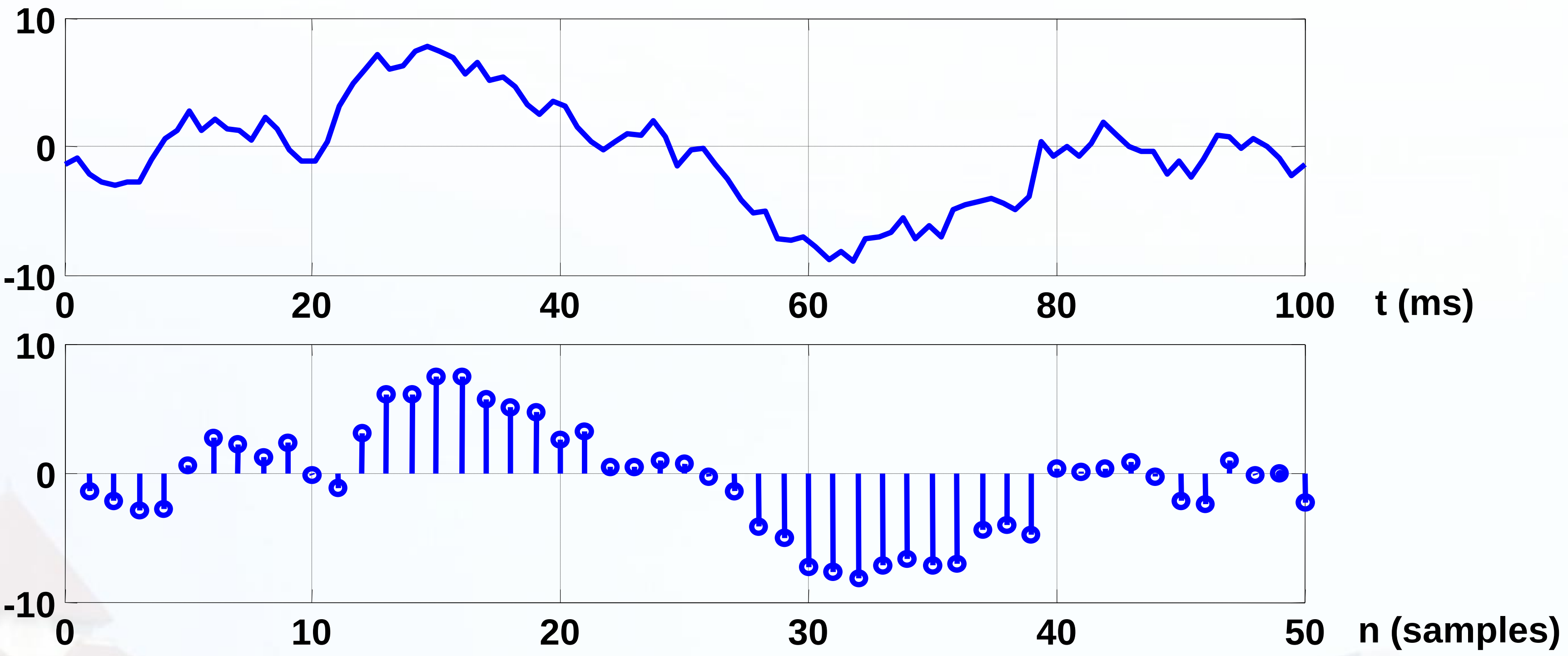


Discrete-Time Signals and Systems

Discrete-Time Signals: Sequences

- Discrete-time signals are represented by sequence of numbers
  - The n^{th} number in the sequence is represented with $x[n]$
- Often times sequences are obtained by sampling of continuous-time signals
  - In this case $x[n]$ is value of the analog signal at $x_c(nT)$
  - Where T is the sampling period

Discrete-Time Signals: Sequences



Basic Sequences and Operations

- Delaying (Shifting) a sequence

$$y[n] = x[n - n_o]$$

- Unit sample (impulse) sequence

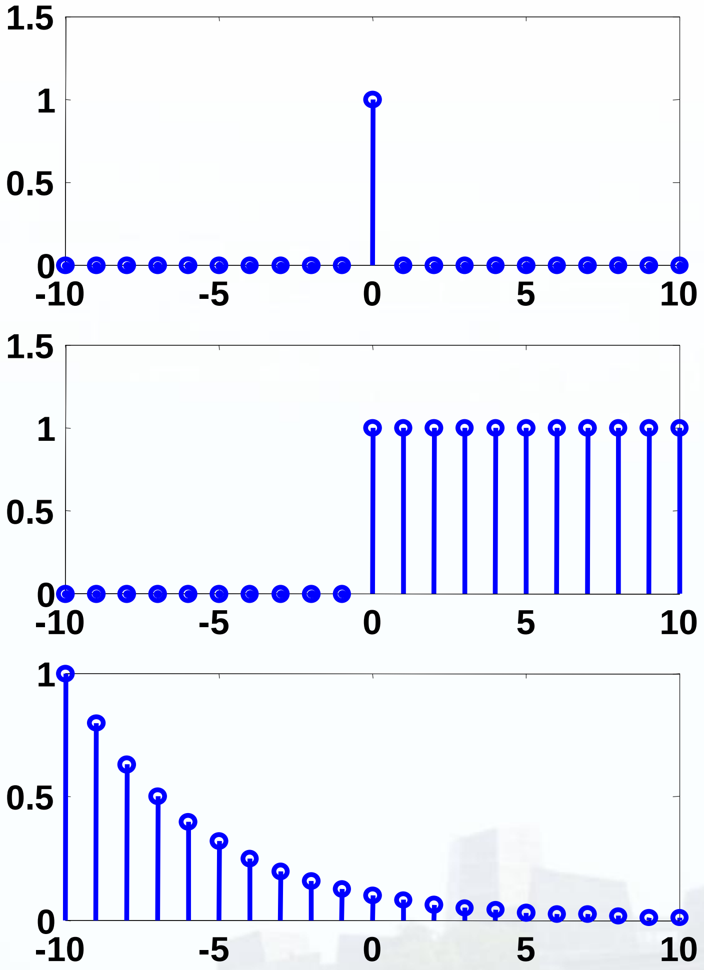
$$\delta[n] = \begin{cases} 0 & n \neq 0 \\ 1 & n = 0 \end{cases}$$

- Unit step sequence

$$u[n] = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$$

- Exponential sequences

$$x[n] = A\alpha^n$$



Sinusoidal Sequences

- Important class of sequences

$$x[n] = \cos(\omega_o n + \varphi)$$

- An exponential sequence with complex

$$\alpha = |\alpha| e^{j\omega_o} \text{ and } A = |A| e^{j\varphi}$$

$$x[n] = A\alpha^n = |A| e^{j\varphi} |\alpha|^n e^{j\omega_o n} = |A||\alpha|^n e^{j(\omega_o n + \varphi)}$$

$$x[n] = |A||\alpha|^n \cos(\omega_o n + \varphi) + j|A||\alpha|^n \sin(\omega_o n + \varphi)$$

- x[n] is a sum of weighted sinusoids

Sinusoidal Sequences

- Different from continuous-time, discrete-time sinusoids
  - Have ambiguity of $2\pi k$ in frequency

$$\cos\left(\left(\omega_o + 2\pi k\right)n + \varphi\right) = \cos\left(\omega_o n + \varphi\right)$$

  - Are not necessary periodic with $2\pi/\omega_o$

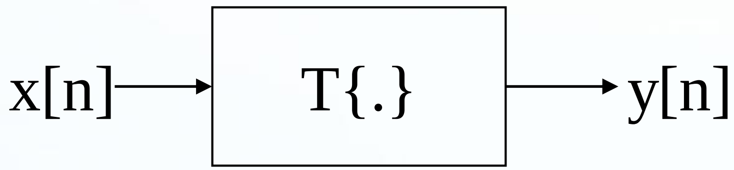
$$\cos\left(\omega_o n + \varphi\right) = \cos\left(\omega_o n + \omega_o N + \varphi\right) \text{ only if } N = \frac{2\pi k}{\omega_o} \text{ is an integer}$$

Discrete-Time Systems

Discrete-Time Systems

- Discrete-Time Sequence is a mathematical operation that maps a given input sequence x[n] into an output sequence y[n]

$$y[n] = T\{x[n]\}$$



Discrete-Time Systems

- Example Discrete-Time Systems
  - Moving (Running) Average

$$y[n] = x[n] + x[n-1] + x[n-2] + x[n-3]$$

  - Maximum

$$y[n] = \max\left\{x[n], x[n-1], x[n-2]\right\}$$

  - Ideal Delay System

$$y[n] = x[n-n_o]$$

Memoryless System

- Memoryless System
  - A system is memoryless if the output y[n] at every value of n depends only on the input x[n] at the same value of n
- Example Memoryless Systems
  - Square

$$y[n] = \left(x[n]\right)^2$$

  - Sign

$$y[n] = sign\left\{x[n]\right\}$$

Memoryless System

- Counter Example
  - Ideal Delay System

$$y[n] = x[n - n_o]$$

Linear Systems

- Linear System: A system is linear if and only if

$$T\{x_1[n]+x_2[n]\}=T\left\{x_1[n]\right\}+T\left\{x_2[n]\right\} \quad \text{(additivity)}$$

$$\textit{and}$$

$$T\left\{ax[n]\right\}=aT\left\{x[n]\right\} \quad \text{(scaling)}$$

Linear Systems

- Examples
  - Ideal Delay System

$$y[n] = x[n - n_o]$$

$$\begin{aligned}
T\{x_1[n] + x_2[n]\} &= x_1[n - n_o] + x_2[n - n_o] \\
T\{x_2[n]\} + T\{x_1[n]\} &= x_1[n - n_o] + x_2[n - n_o] \\
T\{ax[n]\} &= ax_1[n - n_o] \\
aT\{x[n]\} &= ax_1[n - n_o]
\end{aligned}$$

Time-Invariant Systems

- Time-Invariant (shift-invariant) Systems
  - A time shift at the input causes corresponding time-shift at output

$$y[n] = T\{x[n]\} \Rightarrow y[n-n_o] = T\{x[n-n_o]\}$$

- Example
  - Square

$$y[n] = \left(x[n]\right)^2$$

Delay the input the output is $\mathrm{y}_1[n] = \left(x[n-n_o]\right)^2$

Delay the output gives $\mathrm{y}[\mathrm{n\text{-}n_o}] = \left(x[n-n_o]\right)^2$

Time-Invariant Systems

- Time-Invariant (shift-invariant) Systems
  - A time shift at the input causes corresponding time-shift at output

$$y[n] = T\{x[n]\} \Rightarrow y[n - n_o] = T\left\{x[n - n_o]\right\}$$

- Counter Example
  - Compressor System

$$y[n] = x[Mn]$$

Delay the input the output is $\quad \mathrm{y}_1\left[n\right] = x[Mn - n_o]$

Delay the output gives $\quad \mathrm{y}\left[\mathrm{n\text{-}n_o}\right] = x\left[M\left(n - n_o\right)\right]$

Causal System

- Causality
  - A system is causal it's output is a function of only the current and previous samples

- Examples
  - Backward Difference

$$y[n] = x[n] - x[n-1]$$

- Counter Example
  - Forward Difference

$$y[n] = x[n+1] + x[n]$$

Stable System

- Stability (in the sense of bounded-input bounded-output BIBO)
  - A system is stable if and only if every bounded input produces a bounded output

$$|x[n]| \le B_x < \infty \Rightarrow \ |y[n]| \le B_y < \infty$$

- Example
  - Square

$$y[n] = \left(x[n]\right)^2$$

if input is bounded by $|x[n]| \le B_x < \infty$

output is bounded by $|y[n]| \le B_x^2 < \infty$

Stable System

- Counter Example
  - Log

$$y[n] = \log_{10}\left(\left|x[n]\right|\right)$$

even if input is bounded by $\left|x[n]\right| \le B_x < \infty$

output not bounded for $\mathrm{x}\left[\mathrm{n}\right] = 0 \ \Rightarrow \mathrm{y}\left[0\right] = \log_{10}\left(\left|x\left[n\right]\right|\right) = -\infty$

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