/ 16.11.18 dl GAN's

$x_1 \dots x_N \sim p(x)$, estimate $p(x)$

$$\arg\max_\theta \frac{1}{N} \sum_n \ln q_\theta(x_n) = \arg\min_\theta KL(p\|q_\theta)$$

$$x_1 \dots x_N \sim p(x)$$

$x_i \in \{1, 2, \dots C\}$, $x_1 \dots x_N \sim q_\theta(x)$

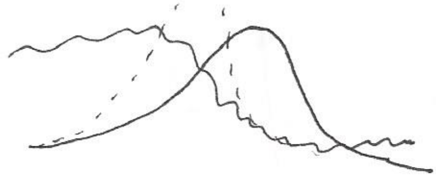
$$p(x) = \prod_{n=1}^{N} q_\theta(x_n) = \prod_c [q_\theta(c)]^{n_c}, \quad n_i := \#\{x_n = i\}, \quad \sum_{i=1}^{C} n_i = N$$

$$\frac{1}{N} \ln p(x) = \sum_c \underbrace{\frac{n_c}{N}}_{\hat{p}(c)} \ln q_\theta(c) \to \max_\theta$$

$$\sum_c \hat{p}(c) \ln q_\theta(c) = \sum_c \hat{p}(c) \ln \frac{q_\theta(c)}{\hat{p}(c)} = -KL(p\|q_\theta(c)) \to \max_\theta$$

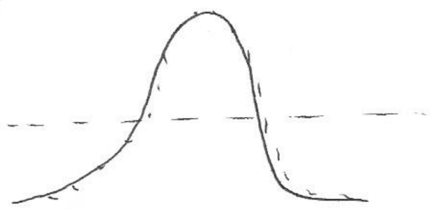
$$\cancel{\sum_c \hat{p}(c) \ln q_\theta(c)}$$

$$\sum_c \hat{p}(c) \ln \frac{\hat{p}(c)}{q_\theta(c)} \to \max_\theta$$



Generator : $G(z) : Z \to X$

Discriminator : $D(x) : X \to [0, 1]$



Game : $\begin{cases} D(x), & x \text{ is real} \\ 1 - D(x), & x \text{ is fake} \end{cases}$

$p(\text{real}) = p(\text{fake}) = \frac{1}{2}$

$$V(D, G) = \mathbb{E}_{x \sim p(x)} \log D(x) + \mathbb{E}_{z \sim p(z)} \log(1 - D(G(z)))$$

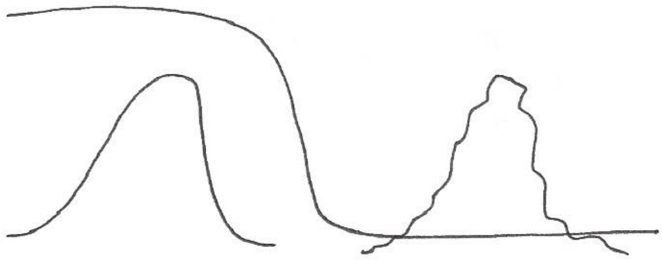
$$\min_G \max_D V(D, G)$$

D task (for fixed G)

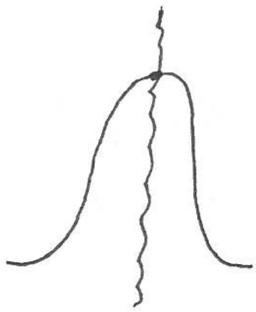
$$\mathbb{E}_{x\sim p(x)} \log D(x) + \mathbb{E}_{z\sim p(z)} \log(1 - D(G(z))) \to \max_{D}$$

G task (for fixed D)

$$\mathbb{E}_{z\sim p(z)} \log(1 - D(G(z))) \to \min_{G}$$

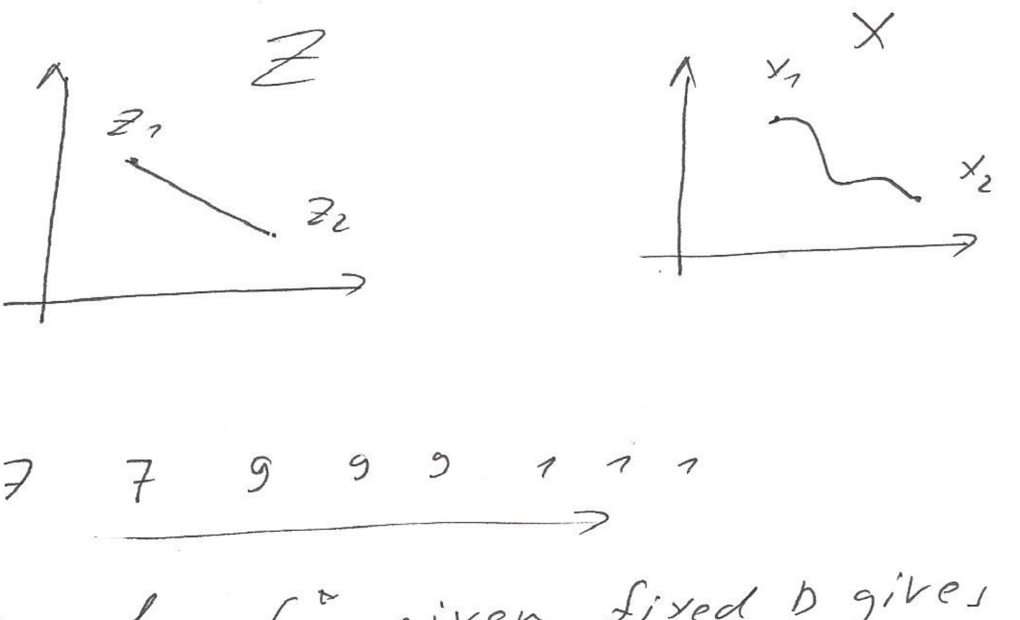


переобучение дискриминатора



переобучение генератора

дискриминатор должен быть обучен лучше генератора

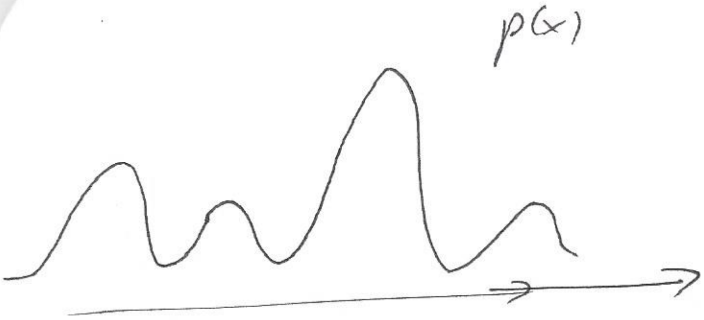


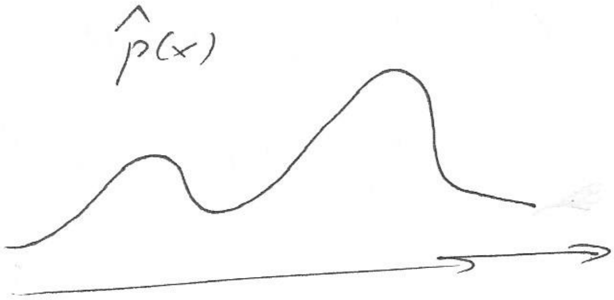
7 7 9 9 9 1 1 1 $\longrightarrow$

optimal G^* given fixed D gives delta:

$G^*(x|D) = \delta(x - \arg\max_{x} D(x))$ mode collapse

no diversity of samples





Optimal D^* given fixed G

$$D^*(x \mid G) = \frac{p(x)}{p(x) + q(x)} \qquad \underline{\text{Theorem 1}}$$

$$\int p(x) \ln D(x)\, dx + \int q(x) \ln (1 - D(x))\, dx =$$

$$= \int \left(p(x) \ln D(x) + q(x) \ln(1 - D(x))\right) dx$$

$$p \ln d + q \ln (1-d) \to \max_d \,, \quad p, q \geq 0$$

$$\frac{p}{d} + \frac{q}{1-d}(-1) = 0, \quad p(1-d) = q d$$

$$-pd + p = qd$$

$$d(p+q) = p, \quad d = \frac{p}{p+q}$$

<u>Theorem 2</u>

Given optimal D^*, optimal G^* should yield $p(x) = q(x)$

$$V(G, D^*) = \int p(x) \ln \frac{p(x)}{p(x) + q(x)}\, dx + \int q(x) \ln \frac{q(x)}{p(x) + q(x)}\, dx =$$

$$= KL(p \,\|\, p+q) + KL(q \,\|\, p+q) \to \min_q$$

$G^*(x): \ q(x) = p(x)$

$V(G^*, D^*) = -2\ln 2$

$$V(G, D^*) - V(G^*, D^*) = \int p(x) \ln \frac{p(x)}{p(x)+q(x)} \, dx$$

$$\begin{cases} p = \alpha(p+q) \\ q = \alpha(p+q) \end{cases}, \quad p+q = 2\alpha(p+q), \quad \alpha = \frac{1}{2}, \quad \frac{1}{2}p = \frac{1}{2}q, \quad p = q$$

improve G from another criterion:

$$\mathbb{E}_{z\sim p(z)} \log D(G(z)) \to \max_G$$

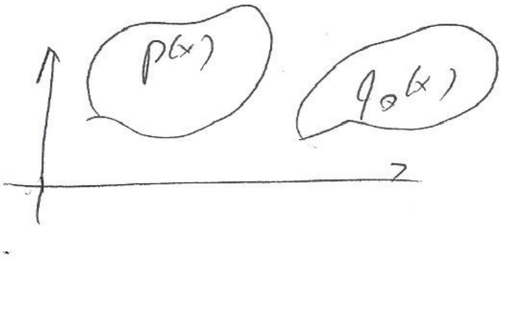
$$\{ \mathbb{E}_{z\sim p(z)} \log(1 - D(G(z))) \to \min_G \}$$

don't mix labels!

автоматически настраиваемая ф-ия потерь

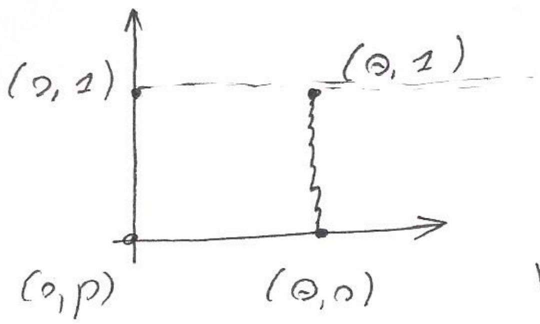
Wasserstein GAN

$\rho(p(x), q_\theta(x)) \to \min_\theta$



$\rho(p(x), q_\theta(x)) = const$

расстояние Вассерштейна



$$W(p,q) = |\theta|$$

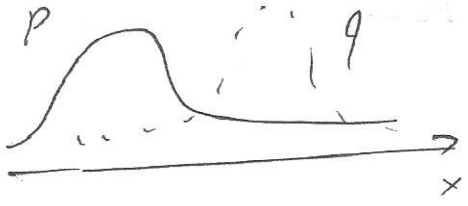
$$W(p,q) = \inf_{\gamma \in \Pi(p,q)} \mathbb{E}_{(x,y)\sim\gamma}[\|x-y\|]$$

парное распределение γ

Jensen-Shannon (JS) divergence

$$J(p,q) = \frac{1}{2}(KL(p\|a) + KL(q\|a))$$

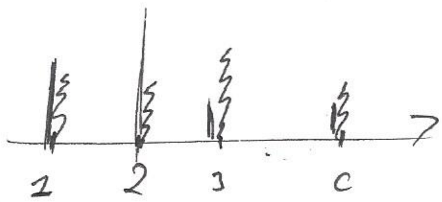
$$a = \frac{p+q}{2}$$



$\gamma(x,y)$

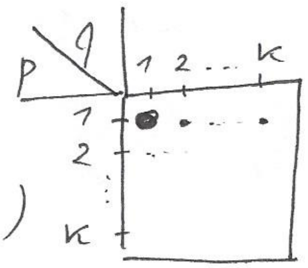
$\int \gamma(x,y)\,dx = q(y)$ кучи

$\int \gamma(x,y)\,dy = p(x)$ с песком!)



$\|x-y\|$

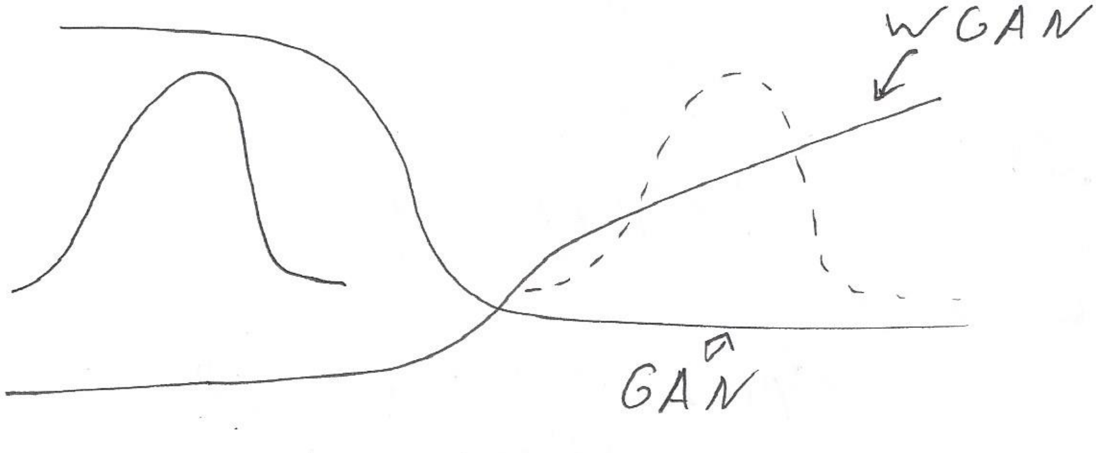
$x \sim p(x)$, $y \sim p(y)$



Kantorovich - Rubinstein duality

$$W(p,q_\theta) = \sup_{\|f\|_2 < 1} \mathbb{E}_{x\sim p}[f(x)] - \mathbb{E}_{x\sim q_\theta}[f(x)]$$

$F^{-1}(x)$



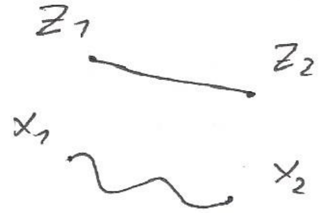
Improved WGAN

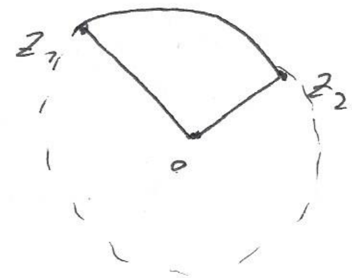
* remove weights clipping
* add $\lambda(\|\nabla_x f(x)\| - 1)^2$

семинар

$p(z)$, $G_\theta(z)$

RL





$\mathbb{E}_{p_d(x)} \log D(x) + \mathbb{E}_{p_d(x)} \log(1-1$

$\mathbb{E}_{x\sim p_d(x)} \mathbb{E}_{q_\varphi(z|x)} \log p_\theta(x|z) - KL(\mathbb{E}_{x\sim p_d(x)} q_\varphi(z|x) \| p(z))$

VAE / GAN