11.11.16 Бимо лек

X

Распределение Дирихле

$(q_1 \dots q_m)$, $q_j \geq 0$ $\sum_j q_j = 1$

$$Dir(q_1 \dots q_m | \alpha_1 \dots \alpha_m) = \frac{\Gamma(\sum_{j=1}^m \alpha_j)}{\prod_{j=1}^m \Gamma(\alpha_j)} \prod_{j=1}^m q_j^{\alpha_j - 1}$$

1) $(q_1 + q_2, q_3, \dots, q_m) \sim Dir(q_1+q_2, q_3, \dots, q_m | \alpha_1 + \alpha_2, \alpha_3, \dots, \alpha_m)$

2) $$p(q_2, \dots, q_m | q_1) = \frac{p(q_1 \dots q_m)}{p(q_1)} = \frac{Dir(q_1 \dots q_m | \alpha_1 \dots \alpha_m)}{Beta(q_1 | \alpha_1, \sum_{j=2}^m \alpha_j)} =$$

$$= Dir\left(\frac{q_2}{1-q_1} \dots \frac{q_m}{1-q_1} \middle| \alpha_2 \dots \alpha_m\right)$$

$$p(q_1, \sum_{j=2}^m q_j) \sim Dir(q_1, \sum_{j=2}^m q_j | \alpha_1, \sum_{j=2}^m \alpha_j)$$

$$p(q_1) \sim Beta(q_1 | \alpha_1, \sum_{j=2}^m q_j)$$

$x \sim \{0,1\}^m$ Полиномиальное

$$p(x,q) = p(x|q)p(q|\alpha) = \prod_{i=1}^m q_i^{x_i} Dir(q|\alpha)$$

$$p(X, q) = \prod_{k=1}^n \prod_{i=1}^m q_i^{x_{ki}} Dir(q|\alpha)$$

$$\int p(X,q)\,dq = \int p(X|q)p(q|\alpha)\,dq = p(X) \overset{?}{=} \prod_k p(x_k)$$

$$p(x_{ki} | x_1 \dots x_{k-1}) = \frac{\alpha_i + \nu_i}{\sum_{j=1}^m \alpha_j + k - 1}$$

ν_i – число объектов, принявших значение i

Stick-Breaking

$\theta_1 \dots \theta_n \dots$

$\pi_1 \dots \pi_n \dots$

for $i = 1, \dots, n, \dots$

$\theta_i \sim G_0(\dots)$

$v_i \sim Beta(v_i | 1, \alpha)$

$\pi_i = \prod_{j=1} (1 - v_j) v_i$

Процесс китайского ресторана

$$p(z_m \mid z_1 \dots z_{m-1}) = \begin{cases} k & ; \frac{\nu_k}{\alpha+m-1} ; \ ++\nu_k \\ k_* & ; \frac{\alpha}{\alpha+m-1} ; \ \nu_{k_*} = 1 ; ++k_* \end{cases}$$

α - параметр энтропии

$\pi_1 \dots \pi_n \dots \quad z \in \{1 \dots n \dots\}$

$$p(z ; \pi) = p(z \mid \pi) p(\pi)$$

$$p(z) = \int p(z \mid \pi) p(\pi) d\pi = p(z_1) p(z_2 \mid z_1) \dots p(z_n \mid z_{m-1} \dots z_1)$$

$$p(X, z \mid \pi, \theta) = \prod_n p(x_n \mid z_n, \theta) p(z_n \mid \pi)$$

$$p(X \mid \pi, \theta) \to \max_{\pi, \theta}$$

$$p(\pi, \theta)\, p(x, z \mid \pi, \theta) = \prod_n p(x_n \mid z_n, \theta_n) p(z_n \mid \pi) \underbrace{p(\pi, \theta)}_{p(\pi) \cdot p(\theta)}$$

Collapsed Gibbs Sampling

$$p(x, z, \pi, \theta) = \prod_n p(x_n \mid z_n, \theta) p(z_n \mid \pi) p(\pi) \cdot p(\theta)$$

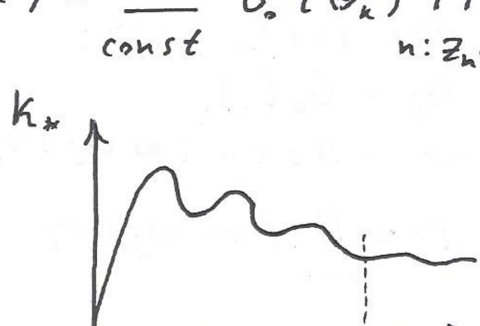
$$p(x, z, \theta) = \int p(x, z, \pi, \theta) d\pi = \prod_n p(x_n \mid z_n, \theta) \cdot \prod_k p(\theta_k) \cdot p(z)$$

1) $$p(z_n \mid z_{\backslash n}, \theta, X) = \begin{cases} k, & \sim n_k^{\backslash n} \cdot p(x_n \mid \theta_k) ; \ n_k = n_k^{\backslash n} + 1 \\ k_*, & \sim \alpha \int p(x_n \mid \theta) G_0(\theta) d\theta ; \end{cases}$$

$$\theta_{k_*+1} \sim p(x_n \mid \theta) G_0(\theta), \ n_{k_*} = 1, \ k_*{+}{+}$$

2) $$p(\theta_k \mid z_n, \theta_{\backslash k}, X) = \frac{1}{const} G_0(\theta_k) \prod_{n: z_n = k} p(x_n \mid \theta_k)$$

3) clear garbage



11.11.16 было сем

$$X: p_x(x) \quad : \quad p_y(y) = p_x(F^{-1}(y)) \left| \frac{\partial F^{-1}(y)}{\partial x} \right|$$

$$(c_1 \dots c_k) \to (q_1 \dots q_k)$$

$$(q_1 \dots q_k) = \left(\frac{c_1}{\sum_k c_k}, \dots, \frac{c_k}{\sum_k c_k} \right)$$

$$c_k \sim \mathrm{Gamma}(c_k \mid \alpha_k, 1)$$

$$F(c_1 \dots c_k) \to (q_1 \dots q_{k-1}, z)$$

$$\{1, 2, \dots, k\} = I, \quad I = A_1 \sqcup A_2 \sqcup \dots \sqcup A_2$$

$$(q_1 \dots q_k) \sim \mathrm{Dir}(\alpha_1 \dots \alpha_k)$$

$$\left(\sum_{i \in A_1} q_i, \dots, \sum_{i \in A_2} q_i \right) \sim \mathrm{Dir}\left(\sum_{i \in A_1} \alpha_i, \dots, \sum_{i \in A_2} \alpha_i \right)$$

$$x \sim G(x \mid a, 1), \quad y \sim G(y \mid b, 1)$$

$$B(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}, \quad x + y \sim G(x+y \mid a+b, 1)$$

$$(q_1 \dots q_k) \sim \mathrm{Dir}(\alpha_1 \dots \alpha_k)$$

$$(q_1, 1 - q_1) \sim \mathrm{Dir}\left(\alpha_1, \sum_{i=2}^{k} \alpha_i\right); \quad q_i \text{ и } q_{\setminus i} \text{ зависимы}$$

$$\frac{q_{\setminus i}}{1 - q_i} \sim \mathrm{Dir}(\alpha_1 \dots \alpha_{i-1} \alpha_{i+1} \dots \alpha_k)$$

$$p(q_1 \dots q_k) = p(q_k \mid q_1 \dots q_{k-1}) \, p(q_{k-1} \mid q_1 \dots q_{k-2}) \dots p(q_1)$$

q_1 сгенерировали, $\underset{(v_2 \dots v_k)}{\dfrac{q_{\setminus 1}}{1 - q_1}} \sim \mathrm{Dir}(\underset{(v_2 \dots v_k)}{\alpha_2 \dots \alpha_k})$

$$q_2 = (1 - q_1) \cdot v_2$$

$$v_2 \sim B\left(\alpha_2, \sum_{i=3}^{k} \alpha_i\right)$$

$$\left(\frac{V_3}{1-V_2}, \ldots, \frac{V_k}{1-V_2}\right) \sim \mathcal{D}ir(\alpha_3, \ldots, \alpha_k)$$

$$V_3 (u_3 \ldots u_k)$$

$$V_3 = (1-V_2)\, u_3$$

$$q_i = V_i \prod_{j=1}^{i-1} (1-V_j), \quad V_i \sim B\left(\alpha_i, \sum_{j=i+1}^{K} \alpha_j\right)$$

$\mathcal{D}P(\alpha, H)$

$\hat{\theta}_1 \ldots \hat{\theta}_N$

$\mathcal{D}P(\alpha, G_0)$ $\quad \theta_k \sim P_{G_0}(\theta)$

$\theta_1 \ldots \theta_N \sim G_0 \Leftrightarrow V_k \sim Beta(v \mid 1, \alpha)$

$x_i \sim p(x \mid \theta_i)$ $\quad P_G(\theta) = \sum_{k=1}^{\infty} \pi_k \delta_{\theta_k}(\theta)$

$\pi_k = V_k \prod_{i<k} (1-V_i)$

$z_1 \ldots z_n \sim Discrete(\pi_1 \ldots \pi_n)$

$x_i \sim p(x \mid \theta_{z_i})$

$$p(X, V, Z, \Theta \mid \alpha, G_0) = \prod_{n=1}^{N} \prod_{k=1}^{\infty} [p(x_n \mid \theta_k) \cdot \pi_k]^{z_{nk}} \cdot \prod_{k=1}^{\infty} Beta(V_k \mid 1, \alpha) \prod_{k=1}^{\infty} P_{G_0}(\theta_k)$$

$$G_0 = N(\theta_k \mid 0, \sigma^2 I)$$

$$p(z, \theta, V \mid X, \alpha, G_0) \approx q(V) \cdot q(\theta) \cdot q(z)$$

$$\ln q(V) \overset{const}{=} \mathbb{E}_{\theta, Z} \ln p(z, \theta, V, X \mid \alpha, G_0) =$$

$$= \sum_{n,k} \mathbb{E}_{\theta, Z} [z_{nk} \cancel{\ln p(x_n \mid \theta_k)} + z_{nk} \ln \pi_k] + (\alpha - 1) \sum_k \mathbb{E}_{\theta, Z} \ln(1 - V_k) \cdot B(1, \alpha) +$$

$$+ \sum_k \cancel{\ln P_{G_0}(\theta_k)} = \sum_{n,k} \mathbb{E}_{\theta, Z}\, z_{nk} \ln V_k \prod_{i<k} (1 - V_i) +$$

$$+ (\alpha - 1) \sum_k \mathbb{E}_{\theta, Z} (1 - V_k) = (\alpha - 1) \sum_k \mathbb{E}_{\theta, Z} (1 - V_k) +$$

$$+ \sum_{n,k} \mathbb{E}_{\theta, Z}\, z_{nk} \left(\sum_{i<k} \ln(1 - V_i) + \ln V_k \right) =$$

$$= \sum_k \ln B\left(V_k \mid \sum_n \mathbb{E}_{\theta, Z} z_{nk} + 1, \sum_n \sum_{j>k} \mathbb{E} z_{nj} + \alpha\right)$$

$$p(X, Z, w, \Theta \mid \alpha, H) = \left[\prod_n \prod_k \left(w_k \, p(x_n \mid \Theta_k)\right)^{[z_n = k]} \right] \cdot$$

$$\cdot \, \mathcal{D}ir(w \mid \alpha) \prod_k p_H(\Theta_k)$$

$$\int_w p(X \mid \Theta, Z) \cdot p(Z \mid w) \, p(w) \, p(\Theta) \, dw =$$

$$= p(\Theta) \, p(X \mid \Theta, Z) \, p(Z)$$

$$p(Z) = \int p(Z \mid w) p(w) \, dw = \int \prod_n \prod_k w_k^{[z_n = k]} \cdot \frac{\Gamma(\sum_k \alpha_k)}{\prod_k \Gamma(\alpha_k)} \cdot$$

$$\cdot \, w_k^{\alpha_k - 1} \, dw_k = \int \prod_k w_k^{\sum_n [z_n = k] + \alpha_k - 1} \, dw_k \cdot \frac{\Gamma(\sum_k \alpha_k)}{\prod_k \Gamma(\alpha_k)} =$$

$$= \frac{\Gamma(\sum_k \alpha_k)}{\prod_k \Gamma(\alpha_k)} \cdot \frac{\prod_k \Gamma(\alpha_k + \sum_n [z_n = k])}{\Gamma(\sum_k \alpha_k + \underbrace{\sum_{k,n} [z_n = k]}_{N})}$$

$$p(z_N = \cancel{k} i \mid z_1 \ldots z_{N-1}) \propto p(z_N = \cancel{k} i, z_1 \ldots z_{N-1}) =$$

$$= \prod_k \Gamma(\alpha_k + \sum_n [z_n = k]) = (\alpha_i + \sum_{n < N} [z_n = i]) =$$

$$\{ a \Gamma(a) = \Gamma(a+1) \}$$

$$= \prod_k \Gamma(\alpha_k + \sum_{n<N} [z_n = k])$$

Процессы

	Гауссовский	Пуассоновский	Дирихле
индексирующий эл-т	$x \in \mathbb{R}^d$	$t \in \mathbb{R}_+$	$A \subset U$
реализация	$f(x)$	$\tau_1 \dots \tau_n \dots$ 	$\xi(A)$ вероятностная мера
одномерная проекция	$f(x_0) \sim$ $N(f(x_0) \mid m(x_0), c(x_0, x_0))$	$n(t_0) \sim \Pi(\lambda t_0)$	$\xi(A_0) \sim Beta(\alpha G_0(A_0), \alpha(1 - G_0(A_0)))$ $\mathbb{E}\,\xi(A_0) = G_0(A_0)$
многомерная проекция	$(f(x_1) \dots f(x_n)) \sim$ $N(\mu, \Sigma)$	$(n(t_1), n(t_2) - n(t_1), \dots, n(t_n) - n(t_{n-1})) \sim$ $(\Pi(\lambda t_1); \Pi(\lambda(t_2 - t_1)); \dots; \Pi(\lambda(t_n - t_{n-1})))$	$A_1 \dots A_k : A_i A_j = \varnothing,\ i \neq j$ $\cup A_i = U$ $(\xi(A_1) \dots \xi(A_k)) \sim$ $Dir(\alpha G_0(A_1) \dots \alpha G_0(A_k))$