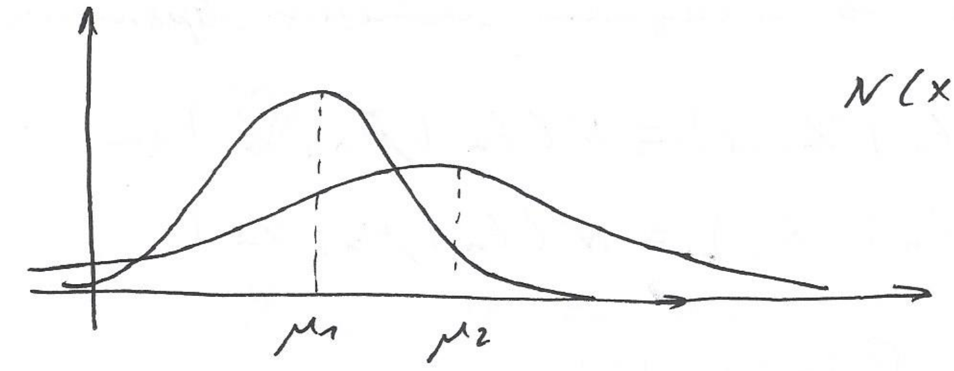
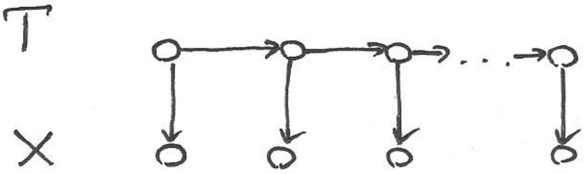
17.03.17 гм V

Линейные динамические системы.



$N(x|\mu_1, \sigma_1^2)$
$*$
$N(x|\mu_2, \sigma_2^2)$

$$\mu = \frac{\mu_1\sigma_1^2 + \mu_2\sigma_2^2}{\sigma_1^2 + \sigma_2^2}, \quad \sigma^2 = \frac{\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2}$$



$x_n \in \mathbb{R}^d, \ t_n \in \mathbb{R}^D$

$d = 2, \ D = 6$

$$p(X, T|\Theta) = p(t_1) \prod_n p(t_n|t_{n-1}) \prod_n p(x_n|t_n)$$

$$p(t_1) = N(t_1|\mu_0, V_0), \quad p(t_n|t_{n-1}) = N(t_n|At_{n-1}, \Gamma)$$

$$p(x_n|t_n) = N(x_n|Ct_n, \Sigma), \quad \Theta = \{\mu_0, V_0, A, \Gamma, C, \Sigma\}$$

$$z(t_{n+1}) = z(t_n) + v(t_n)\Delta t + a(t_n)\frac{\Delta t^2}{2}$$

$A \in \mathbb{R}^{6\times 6}$, $A = \begin{bmatrix} 1 & 1 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

$C \in \mathbb{R}^{2\times 6}$

$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$

$\Sigma \in \mathbb{R}^{2\times 2}$

$\Gamma \in \mathbb{R}^{6\times 6}$, $\Gamma = \begin{bmatrix} \sigma^{\xi} & & & & & 0 \\ & \sigma^{\xi} & & & & \\ & & \sigma_1 & & & \\ & & & \sigma^{\xi} & & \\ & & & & \sigma^{\xi} & \\ 0 & & & & & \sigma_2 \end{bmatrix}$

$\Sigma = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}$

1) $p(X_{t2}, T_{t2} | \Theta) \to \max\limits_{\Theta}$

2) $p(t_n | X_n, \Theta)$, $X_n = (x_1 \dots x_n)$ фильтрация сигнала

параметры объекта в текущий момент времени

1) Predictor $p(t_n | X_{n-1}) = \mathcal{N}(t_n | \tilde{\mu}_n, \tilde{V}_n)$

2) Corrector $p(t_n | X_n) = \mathcal{N}(t_n | \mu_n, V_n)$

$n = \cancel{0}\, 1$, $p(t_1)$, $(\tilde{\mu}_1, \tilde{V}_1) = (\mu_0, V_0)$

$$p(t_1 | x_1) = \frac{p(x_1 | t_1) p(t_1)}{\int p(x_1 | t_1) p(t_1) dt_1} = \mathcal{N}(t_1 | \mu_1, V_1)$$

Пусть известно $p(t_{n-1} | X_{n-1})$

$$p(t_n | X_{n-1}) = \int p(t_n, t_{n-1} | X_{n-1}) d t_{n-1} =$$

$$= \int p(t_n | \underbrace{t_{n-1}, X_{n-1}}_{\text{условная нез-ть}}) p(t_{n-1} | X_{n-1}) d t_{n-1} =$$

$$= \int p(t_n | t_{n-1}) p(t_{n-1} | X_{n-1}) d t_{n-1} = \mathcal{N}(t_n | \tilde{\mu}_n, \tilde{V}_n)$$

$\cancel{\square}$ $t_n = A t_{n-1} + \varepsilon$, $\varepsilon \sim \mathcal{N}(\varepsilon | 0, \Gamma)$

$t_{n-1} = \mu_{n-1} + \delta$, $\delta \sim \mathcal{N}(\delta | 0, V_{n-1})$

$\tilde{\mu}_n = A \mu_{n-1}$, $\tilde{V}_n = \Gamma + A V_{n-1} A^T$

$$\mathbb{E}[(A\delta)(A\delta)^T] = \mathbb{E}[(A\delta)(\delta^T A^T)] = A(\mathbb{E}\, \delta\delta^T) A^T = A V_{n-1} A^T$$

$$p(t_n | X_n) = \frac{p(X_n, t_n)}{p(X_n)} = \frac{p(x_n | X_{n-1}, t_n) p(t_n | X_{n-1}) p(X_{n-1})}{p(X_n)}$$

$$= \frac{p(x_n | t_n) p(t_n | X_{n-1}) p(X_{n-1})}{p(X_n)} \propto p(x_n | t_n) p(t_n | X_{n-1}) =$$

$$= \mathcal{N}(t_n | \mu_n, V_n)$$

$$\mu_n = \tilde{\mu}_n + K_n (x_n - C\tilde{\mu}_n)$$

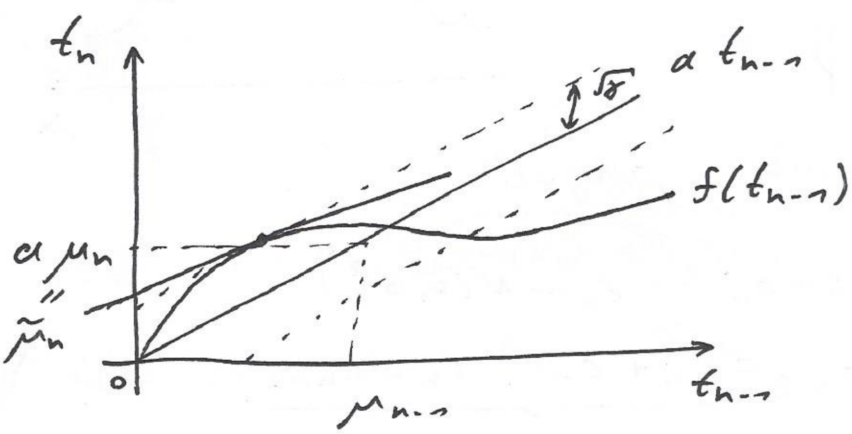
$$V_n = (I - K_n C)\tilde{V}_n$$

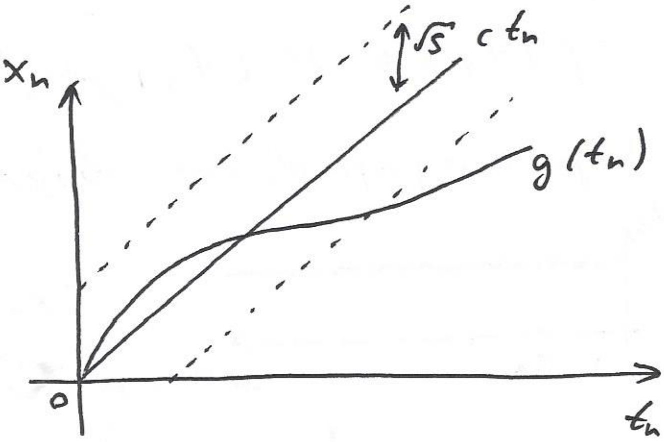
$$K_n = \tilde{V}_n C^T (C\tilde{V}_n C^T + \Sigma)^{-1}$$

Пусть $d = D = 1$, $t_n = a\,t_{n-1} + \varepsilon$, $\varepsilon \sim N(\varepsilon | 0, \sigma)$

$x_n = c\,t_n + \delta$, $\delta \sim N(\delta | 0, s)$

$$p(t_{n-1} | X_{n-1}) = N(t_n | \mu_n, \vartheta_n)$$





$t_n \sim N(t_n | \tilde{\mu}_n, \tilde{\vartheta}_n)$, $t_n = \frac{x_n}{c} - \frac{\delta}{c} \sim N(t_n | \frac{x_n}{c}, \frac{s}{c^2})$

$$\mu_n = \frac{\tilde{\mu}_n \frac{s}{c^2} + \frac{x_n}{c}\tilde{\vartheta}_n \pm \tilde{\mu}_n \tilde{\vartheta}_n}{\frac{s}{c^2} + \tilde{\vartheta}_n} = \frac{\tilde{\mu}_n (\frac{s}{c^2} + \tilde{\vartheta}_n) + \frac{x_n}{c}\tilde{\vartheta}_n - \tilde{\mu}_n \tilde{\vartheta}_n}{\frac{s}{c^2} + \tilde{\vartheta}_n} =$$

$$= \tilde{\mu}_n + \underbrace{\frac{c\tilde{\vartheta}_n}{s + c^2\tilde{\vartheta}_n}}_{k_n} (x_n - c\tilde{\mu}_n)$$

$$\vartheta_n = \frac{\tilde{\vartheta}_n \cdot \frac{s}{c^2}}{\tilde{\vartheta}_n + \frac{s}{c^2}} = \frac{s\tilde{\vartheta}_n \pm c^2\tilde{\vartheta}_n}{c^2\tilde{\vartheta}_n + \frac{s}{\cancel{c^2}}} = \frac{\tilde{\vartheta}_n (s + c^2\tilde{\vartheta}_n - c^2\tilde{\vartheta}_n)}{s + c^2\tilde{\vartheta}_n} =$$

$$= \tilde{\vartheta}_n \left(1 - \frac{c^2\tilde{\vartheta}_n}{s + c^2\tilde{\vartheta}_n}\right) = \tilde{\vartheta}_n (1 - k_n \cdot c)$$

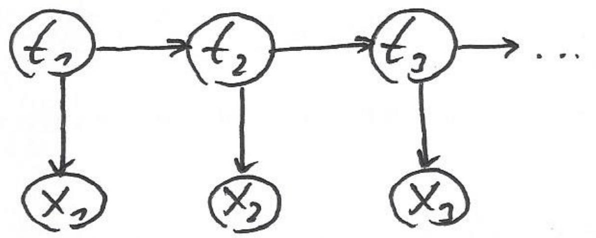
$$A_n = \frac{\partial f}{\partial t_{n-1}}\Big|_{\mu_{n-1}}$$

$$C_n = \frac{\partial g}{\partial t_n}\Big|_{\tilde{\mu}_n}$$

линеаризация: $a_n = \frac{\partial f}{\partial t_{n-1}}\Big|_{\mu_{n-1}}$, $c_n = \frac{\partial g}{\partial t_n}\Big|_{\tilde{\mu}_n}$

$c_n = \frac{\partial g^{-1}}{\partial x_n}\Big|_{x_n}$ {если $\exists\, g^{-1}$}

17.03.17 гм сем



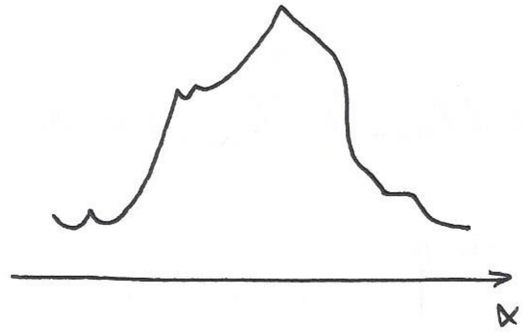
$x_n \in \mathbb{R}^D$

$t_n \in \mathbb{R}^d$

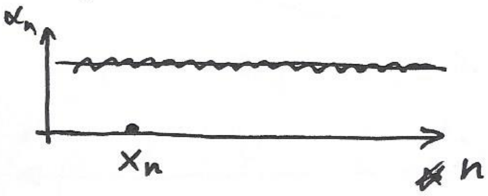
$$p(t_n \mid t_{n-1}) = N(t_n \mid A t_{n-1}, \Gamma)$$

$$p(x_n \mid t_n) = N(x_n \mid C t_n, \Sigma)$$

$$p(t_1) = N(t_1 \mid \mu_0, V_0)$$



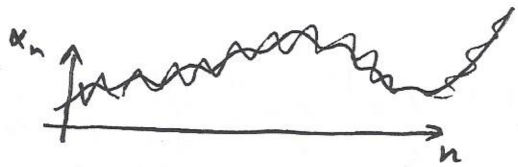
прогнозирование временных рядов



$x_n = c + \varepsilon_n$, $\varepsilon_n \sim N(0, \sigma_x^2)$

$\Rightarrow \underbrace{x_n = c_n + \varepsilon_n, \quad c_n = c_{n-1}}$

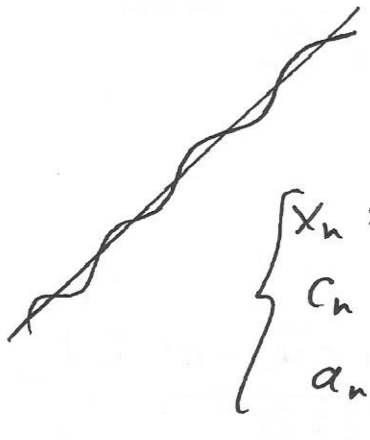
$$\begin{cases} x_n = c_n + \varepsilon_n \\ c_n = c_{n-1} + \delta_n \end{cases}$$



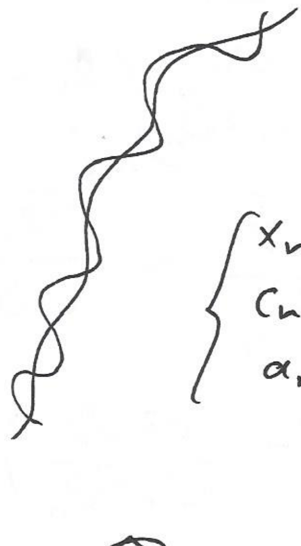
может уходить в ∞

$\delta_n \sim N(0, \sigma^2)$

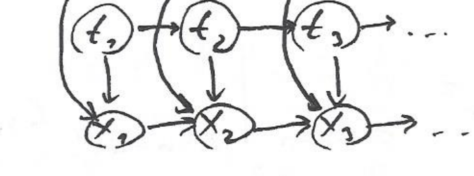
$$\begin{cases} x_n = c_n + \varepsilon_n \\ c_n = \alpha c_{n-1} + \delta_n \end{cases}, \quad 0 < \alpha < 1$$

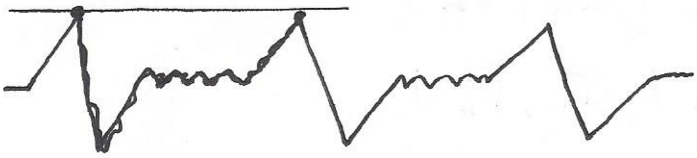


$$\begin{cases} x_n = c_n + \varepsilon_n \\ c_n = c_{n-1} + a_{n-1} \\ a_n = a_{n-1} \end{cases}$$



$$\begin{cases} x_n = c_n + \varepsilon_n \\ c_n = c_{n-1} + a_{n-1} + \delta_n \\ a_n = a_{n-1} \end{cases}$$





$$x_n = c_n + \varepsilon_n$$

$$c_n = -\sum_{i=1}^{m} c_{n-i} + \delta_n$$

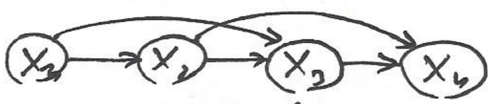
$$\tilde{c}_n = A\tilde{c}_{n-1} \qquad \tilde{c}_n = [c_n \ c_{n-1} \dots c_{n-m}]^T$$

$$A = \begin{bmatrix} -1 & -1 & \dots & -1 \\ 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ & & \ddots & \\ 0 & & 1 & 0 \end{bmatrix}$$

AR(m) авторегрессия

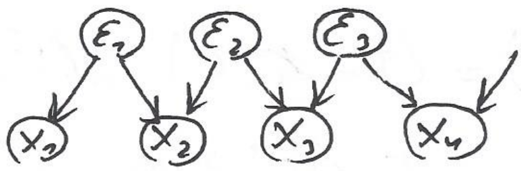
$$x_n = \sum_{i=1}^{m} w_i x_{n-i} + \varepsilon_n , \quad \varepsilon_n \sim N(0, \sigma_x^2)$$

AR(2):



MA(q): $x_n = \varepsilon_n + \sum_{j=1}^{q} v_j \varepsilon_{n-j}$, $\varepsilon_n \sim N$ {по ε}

$q = 1$

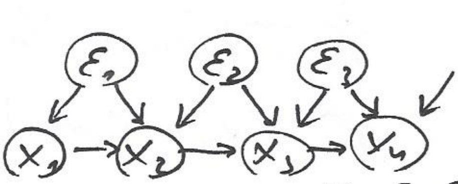


скользящее среднее

ARMA(m, q): $x_n = \sum_{i=1}^{m} w_i x_{n-i} + \sum_{j=1}^{q} v_j \varepsilon_{n-j} + \varepsilon_n$

$$\hat{\varepsilon}_n = \begin{bmatrix} \varepsilon_n \\ \varepsilon_{n-1} \end{bmatrix}$$

ARMA(1, 1):



скрытые: $t_n = \begin{bmatrix} \varepsilon_n \\ \varepsilon_{n-1} \\ x_{n-1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & v_1 & w_1 \end{bmatrix} \begin{bmatrix} \varepsilon_{n-1} \\ \varepsilon_{n-2} \\ x_{n-2} \end{bmatrix}$

$$x_n = \begin{bmatrix} 1 & v_1 & w_1 \end{bmatrix} \begin{bmatrix} \varepsilon_n \\ \varepsilon_{n-1} \\ x_{n-1} \end{bmatrix}$$

$$p(t_n \mid x_1 \ldots x_n) = \mathcal{N}(t_n \mid \mu_n, V_n)$$

$$p(t_n \mid x_1 \ldots x_{n-1}) = \mathcal{N}(\tilde{t}_n \mid \tilde{\mu}_n, \tilde{V}_n)$$

$\cancel{\text{ugb. } \mu_{n-1}, V_{n-1}}$

$$p(t_n \mid x_1 \ldots x_n) \propto p(t_n, x_n \mid x_1 \ldots x_{n-1}) =$$

$$= p(X_n \mid t_n)\, p(t_n \mid x_1 \ldots x_{n-1})$$

$$\text{''}\ \mathcal{N}(x_n \mid C t_n, \Sigma) \qquad \text{''}\ \mathcal{N}(t_n \mid \tilde{\mu}_n, \tilde{V}_n)$$

$$p(x) = \mathcal{N}(x \mid \mu, \Sigma), \quad p(y \mid x) = \mathcal{N}(y \mid Ax, \Gamma)$$

$$p(x \mid y) = \mathcal{N}(x \mid P(\Sigma^{-1}\mu + A^T \Gamma^{-1} y), P = (\Sigma^{-1} + A^T \Gamma^{-1} A)^{-1})$$

$$V_n = (\tilde{V}_n^{-1} + C^T \Sigma^{-1} C)^{-1} = \tilde{V}_n - \underbrace{\tilde{V}_n C^T (\Sigma + C \tilde{V}_n C^T)^{-1}}_{K_n} C \tilde{V}_n =$$

$$= (I - K_n C)\tilde{V}_n$$

$$\mu_n = V_n(\tilde{V}_n^{-1}\tilde{\mu}_n + C^T \Sigma^{-1} x_n) = (I - K_n C)\tilde{V}_n(\tilde{V}_n^{-1}\tilde{\mu}_n + C^T \Sigma^{-1} x_n)$$

$$= (I - K_n C)\tilde{\mu}_n + \tilde{V}_n C^T \Sigma^{-1} x_n - K_n C \tilde{V}_n C^T \Sigma^{-1} x_n =$$

$$= (I - K_n C)\tilde{\mu}_n + \tilde{V}_n C^T \Sigma^{-1} x_n - \tilde{V}_n C^T (\Sigma + C\tilde{V}_n C^T)^{-1} C \tilde{V}_n C^T \Sigma^{-1} x_n =$$

$$= (I - K_n C)\tilde{\mu}_n + \tilde{V}_n C^T (\Sigma + C\tilde{V}_n C^T)^{-1}(\cancel{\Sigma} + \cancel{C\tilde{V}_n C^T} - \cancel{C\tilde{V}_n C^T})\cancel{\Sigma^{-1}} x_n =$$

$$= \tilde{\mu}_n + K_n(x_n - C\tilde{\mu}_n)$$

$$p(t_n \mid x_1 \ldots x_N) = \mathcal{N}(t_n \mid \hat{\mu}_n, \hat{V}_n)$$

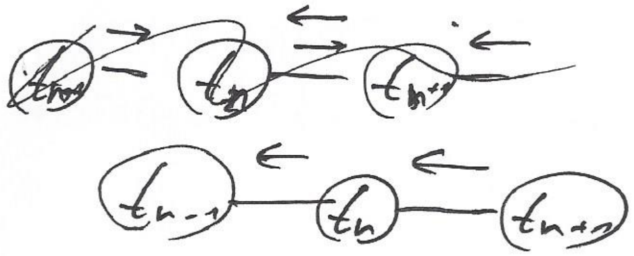
$$p(T \mid X) \qquad t_1 - t_2 - t_3 -$$

$$\text{''}\ \frac{1}{Z} \prod_n \psi_n(t_n) \prod_n \psi_{n,n+1}(t_n, t_{n+1})$$

$$Z = p(X), \quad \psi_n(t_n) = p(x_n \mid t_n), \quad \psi_{n,n+1}(t_n, t_{n+1}) = p(t_{n+1} \mid t_n)$$

$$\mu_{t_{n-1} \to t_n}(t_n), \quad \mu_{t_n \to t_{n+1}}(t_{n+1}), \quad b_n(t_n)\ ?$$

$$b_n(t_n) \propto \psi_n(t_n)\, \mu_{t_{n-1} \to t_n}(t_n)\, \mu_{t_{n+1} \leftarrow t_n}(t_n)$$



$$b_n(t_n) \propto \psi_n(t_n)\,\mu_{t_{n-1}\to t_n}(t_n)\,\mu_{t_{n+1}\leftarrow t_n}(t_n) \propto$$

$$\propto \psi_n(t_n)\,\mu_{t_{n-1}\to t_n}(t_n)\sum_{t_{n+1}} \psi_{n,n+1}(t_n, t_{n+1}) \times$$

$$\times \underbrace{\psi_{n+1}(t_{n+1})\,\mu_{t_{n+2}\to t_{n+1}}(t_{n+1})}_{\dfrac{b_{n+1}(t_{n+1})}{\mu_{t_n\to t_{n+1}}(t_{n+1})}}$$