14.04.17 2м IX

$$p(x) \simeq q(x) = \frac{f(x)}{g(\theta)} \exp(\theta^T u(x))$$

Expectation Propagation

$KL(p\|q) \to \min_\theta$

$$\int p(x) \ln \frac{p(x) g(\theta)}{f(x) \exp(\theta^T u(x))} \to \min_\theta$$

$$\parallel$$

$$\int p(x) \ln \frac{p(x)}{f(x)} dx + \int p(x) \ln g(\theta) dx - \int p(x) \theta^T u(x) \to \min_\theta$$

$$C + \ln g(\theta) - \int p(x) \theta^T u(x) dx \to \min_\theta \quad \Big| \frac{\partial}{\partial \theta_j}$$

$$\frac{\partial}{\partial \theta_j} \ln g(\theta) - \int p(x) u_j(x) = 0$$

$$\mathbb{E}_q u_j(x) = \mathbb{E}_p u_j(x)$$ moment matching

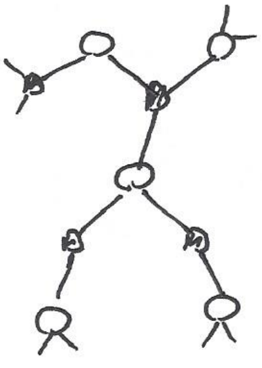
$$p(x) \simeq \prod_j q_j(x_j) : \quad q_j(x_j) = \frac{f_j(x_j)}{g_j(\theta)} \exp(\theta^T u(x_j))$$

$$\int q_i(x_i) u_j(x_i) dx_i = \int p_i(x_i) u_j(x_i) dx_i$$

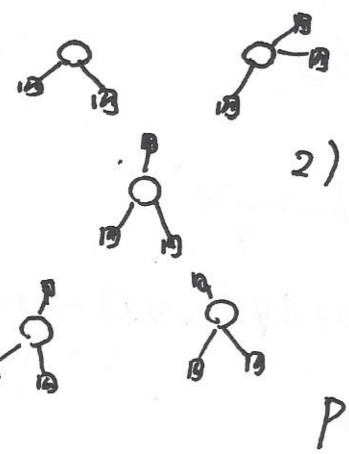
$$p(x) = \frac{1}{Z} \prod_f \psi_f(x_f)$$

1) $q(x) = \prod_i q_i(x_i) = \arg\min KL(p\|q)$

$q(x) = \arg\min \sum KL(p(x_i) \| q(x_i))$



$\approx$

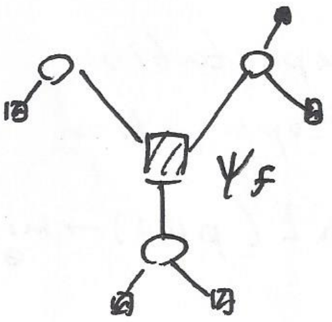


2) $\psi_f(x_f) \simeq \prod_{i \in f} \mu_{f\to i}(x_i) =$

$= \arg\min_\mu KL\left(\frac{\psi_f(x_f)}{Z_f} \Big\| \prod_{i \in f} \mu_{f \to i}(x_i) \right)$

$p_i(x_i) \simeq \frac{1}{Z_i} \prod_{g: i \in g} \mu_{g \to i}(x_i)$

приближение в контексте
текущего приближения



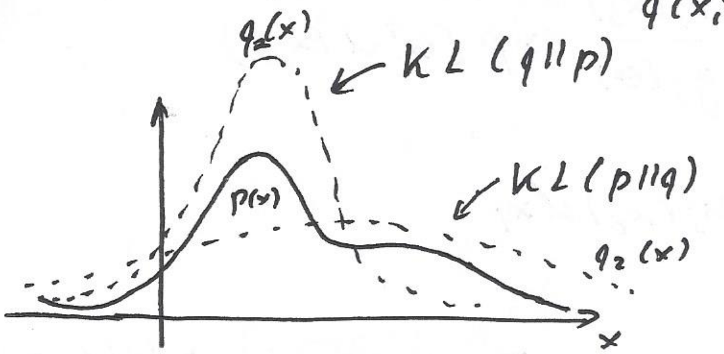
$$KL\left(\frac{1}{Z_f}\cdot \Psi_f(x_f)\prod_{g\neq f}\prod_{i\in g}\mu_{g\to i}(x_i)\,\Big\|\,\prod_{j\in f}\mu_{f\to j}(x_j)\prod_{g\neq f}\prod_{i\in g}\mu_{g\to i}(x_i)\right)\to\min_{\mu_{f\to j}} \quad (*)$$

$$p(x)\simeq q(x)=\prod_i q_i(x_i)$$

$$KL(p\|q)=\int p(x)\ln p(x)\,dx\,-$$

$$-\sum_i\left(\int p(x_i)\ln q_i(x_i)\,dx_i \mp \int p(x_i)\ln p(x_i)\,dx_i\right)=$$

$$=C+\sum_i\int p(x_i)\ln\frac{p(x_i)}{q(x_i)}\,dx_i\to\min_{q_i} \qquad \{q_i=p_i\}$$



$$p(x,y,z)=p(x)p(y,z)$$
$$q(x,y,z)=p(x)q(y,z)$$

$$KL(q\|p)=\int \cancel{p(x)}\,p(y,z)\ln\frac{\cancel{p(x)}\,p(y,z)}{\cancel{p(x)}\,q(y,z)}\,\cancel{dx}\,dy\,dz=$$

$$=\int p(y,z)\ln\frac{p(y,z)}{q(y,z)}\,dy\,dz$$

$$KL(p(x)p(y,z)\|p(x)q(y,z))\to\min_q$$

$$\Leftrightarrow KL(p(y,z)\|q(y,z))\to\min_q$$

$$(*)\simeq KL\left(\frac{1}{Z_f}\Psi_f(x_f)\prod_{i\in f}\prod_{\substack{g\neq f\\ i\in g}}\mu_{g\to i}(x_i)\,\Big\|\,\prod_{i\in f}\Big(\mu_{f\to i}(x_i)\prod_{\substack{g\neq f\\ i\in g}}\mu_{g\to i}(x_i)\Big)\right)$$

$$\min\{\mu_{f\to i}\}$$

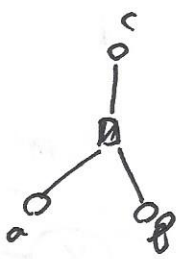
$$\frac{1}{Z_f}\int \Psi_f(X_f)\prod_{j\in f}\prod_{\substack{g\neq f\\ j\in g}}\mu_{g\to j}(x_j)\,dX_{f\setminus i} =$$

$$= \mu_{f\to i}(x_i)\cdot\prod_{\substack{g\neq f\\ i\in g}}\mu_{g\to i}(x_i) \qquad \mathrm{Proj}$$

$$\mu_{f\to i}(x_i) = \frac{\frac{1}{Z_f}\int \Psi_f(X_f)\prod_{j\in f}\prod_{\substack{g\neq f\\ j\in g}}\mu_{g\to j}(x_j)\,dX_{f\setminus i}}{\prod_{\substack{g\neq f\\ i\in g}}\mu_{g\to i}(x_i)}$$

1) Z_f , 2) $\mathbb{E}_{\sim 1/Z_f} x_i$, 3) $\mathbb{E}_{\sim 1/Z_f} x_i^2$

$$\mu_{f\to i}(x_i) = \frac{\mathrm{Proj}\,(-''-)}{-''-}$$



$\Psi_f(a,b,c) =$
$= [c=a+b]$
$p(c)$?

для дискретних:

$$\mu_{f\to i}(x_i) \propto \sum_{X_{f\setminus i}} \Psi_f(X_f)\prod_{g\neq f}\prod_{\substack{j\in g\\ j\neq i}}\mu_{g\to j}(x_j) =$$

$$= \sum_{X_{f\setminus i}} \Psi_f(X_f)\prod_{j\neq i}\prod_{\substack{g: j\in g\\ g\neq f}}\mu_{g\to j}(x_j) =$$

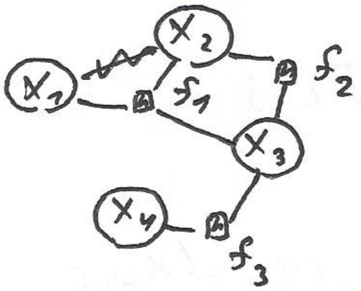
$$= \sum_{X_{f\setminus i}} \Psi_f(X_f)\prod_{j\neq i}\mu_{j\to f}(x_j) \qquad \max(St+\chi^2, \ln G)\ ?$$

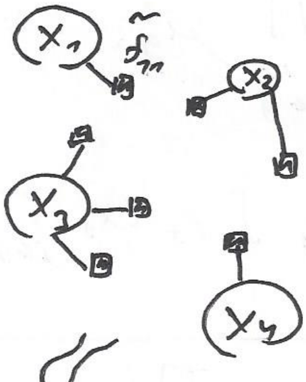
14.04.17 гм сем

EP $\quad p(x)\propto\prod_i f_i(X_i)\approx q(x)=\prod_i \tilde{f}_i(X_i) = \prod_i\prod_{x_j\in X_i}\tilde{f}_{ij}(x_j)=$

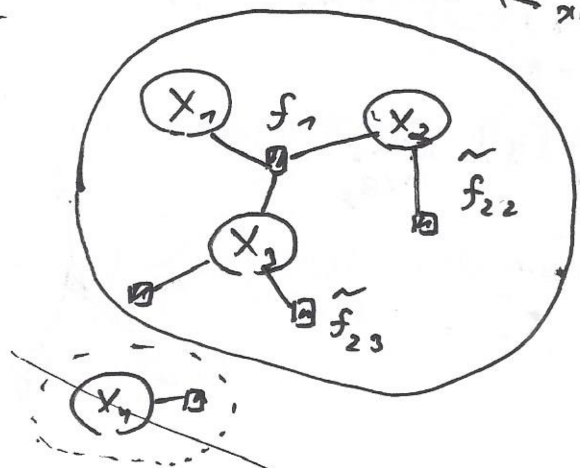
$$= \prod_{x_j}\underbrace{\prod_{i:x_j\in X_i}\tilde{f}_{ij}(x_j)}_{q(x_j)} \qquad \mathrm{proj}(p(x)) = \arg\min_{q\in Q} KL(p\|q)$$

$p(x)$: $\approx$ невозможно приближение напрямую $q(x)$:





контекстное приближение $\cancel{\approx}$ $\parallel$ ← хорошо



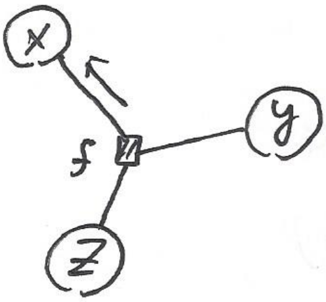
$$q^{\setminus i}(x) = \frac{q(x)}{\tilde f_i(x_i)}, \qquad q^{new}(x) = proj\left(f_i(x_i)\, q^{\setminus i}(x)\right)$$

$$\tilde f_i^{new}(x_i) = \frac{q^{new}(x)}{q^{\setminus i}(x)}$$

$$\mu_{f_i \to x_j}(x_j) = \tilde f_{ij}(x_j)$$

$$\mu_{x_j \to f_i}(x_j) = \prod_{\substack{k:\, x_j \in f_k \\ k \neq i}} \mu_{f_k \to x_j}(x_j)$$

$$\mu_{f_i \to x_j}(x_j) = \frac{proj\left(\int f_i(x_i) \prod_{k:\, x_k \in f_j} \mu_{x_k \to f_i}(x_k)\, dx^{\setminus j}\right)}{\mu_{x_j \to f_i}(x_j)}$$



EP

$$f(x,y,z) = [x = y+z] \quad , \; x,y,z \in \mathbb{R}$$

$$\mu_{x \to f}(x) = \mathcal{N}(x \mid \mu_x, \sigma_x^2)$$

$$\mu_{y \to f}(y) = \mathcal{N}(y \mid \mu_y, \sigma_y^2)$$

$$\mu_{z \to f}(z) = \mathcal{N}(z \mid \mu_z, \sigma_z^2)$$

$$\mu_{f \to x}(x) = \frac{\operatorname{proj} \int f(x)\, \mu_{y \to f}(y)\, \mu_{z \to f}(z)\, dy\, dz}{\mu_{x \to f}} =$$

$$= \frac{\operatorname{proj} \int [x = y+z]\, \mathcal{N}(y \mid \mu_y, \sigma_y^2)\, \mathcal{N}(z \mid \mu_z, \sigma_z^2)\, \overset{\mathcal{N}(x \mid \mu_x, \sigma_x^2)}{\cancel{dy\,dz}}\, dy\,dz}{\mathcal{N}(x \mid \mu_x, \sigma_x^2)} =$$

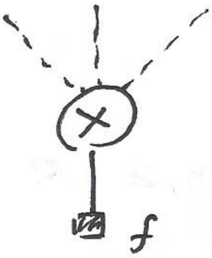
$$= \frac{\operatorname{proj} \mathcal{N}(x \mid \mu_x, \sigma_x^2) \int [x = y+z]\, \mathcal{N}(y \mid \mu_y, \sigma_y^2)\, \mathcal{N}(z \mid \mu_z, \sigma_z^2)\, dy\,dz}{\mathcal{N}(x \mid \mu_x, \sigma_x^2)} =$$

$$= \frac{\operatorname{proj} \mathcal{N}(x \mid \mu_x, \sigma_x^2) \int \mathcal{N}(y \mid \mu_y, \sigma_y^2)\, \mathcal{N}(x-y \mid \mu_{z}, \sigma_{z}^2)\, dy}{\mathcal{N}(x \mid \mu_x, \sigma_x^2)} =$$

$$= \frac{\operatorname{proj} \mathcal{N}(x \mid \mu_x, \sigma_x^2)\, \mathcal{N}(x \mid \overset{\mu_y+\mu_z}{\cancel{\mu_{x+y}}}, \overset{\sigma_y^2+\sigma_z^2}{\cancel{\sigma^2_{x+y}}})}{\mathcal{N}(x \mid \mu_x, \sigma_x^2)} = \mathcal{N}(x \mid \overset{\mu_y+\mu_z}{\cancel{\mu_{x+y}}}, \overset{\sigma_y^2+\sigma_z^2}{\cancel{\sigma^2_{x+y}}})$$

$$\mathcal{N}(x \mid \mu, \Sigma) \qquad \mathcal{N}(y \mid Ax, \Gamma)$$

$$\int \mathcal{N}(x \mid \mu, \Sigma)\, \mathcal{N}(y \mid Ax, \Gamma)\, dx = \mathcal{N}(y \mid A\mu, \Gamma + A\Sigma A^T)$$



f

$$f(x) = [x > \varepsilon], \quad x \in \mathbb{R}$$

$$\mu_{x\to f}(x) = \mathcal{N}(x \mid \mu, \sigma^2)$$

$$\mu_{f\to x}(x) = \frac{p2\left(f(x)\,\mu_{x\to f}(x)\right)}{\mu_{x\to f}(x)} =$$

$$= \frac{p2\big(\overbrace{[x>\varepsilon]\,\mathcal{N}(x \mid \mu, \sigma^2)}^{\tau(x)}\big)}{\mathcal{N}(x \mid \mu, \sigma^2)}$$

$$\tau(x) = \frac{1}{Z}[x>\varepsilon]\,\mathcal{N}(x \mid \mu, \sigma^2)$$

$$\int_{-\infty}^{\infty} [x>\varepsilon]\,\mathcal{N}(x \mid \mu, \sigma^2)\,dx = \int_{\varepsilon}^{\infty} \mathcal{N}(x \mid \mu, \sigma^2)\,dx =$$

$$= \left[y = \frac{x-\mu}{\sigma},\ dy = \frac{dx}{\sigma} \right] = \int_{\frac{\varepsilon-\mu}{\sigma}}^{\infty} \mathcal{N}(y \mid 0, 1)\,dy =$$

$$= 1 - F\left(\frac{\varepsilon-\mu}{\sigma}\right) = Z$$

$$\tau(x) = \frac{1}{Z}[x>\varepsilon]\frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} = \frac{1}{Z}\frac{[x>\varepsilon]}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2\sigma^2}(x^2-2x\mu+\mu^2)} =$$

$$= \frac{1}{Z}\frac{[x>\varepsilon]}{\sqrt{2\pi}\,\sigma} e^{\left(-\frac{x^2}{2\sigma^2} + \frac{x\mu}{\sigma^2} - \frac{\mu^2}{2\sigma^2}\right)} =$$

$$u(x) = (x^2, x)$$

$$\frac{f(x)}{g(\theta)} \exp\left(\theta^T u(x)\right) \qquad \theta = \left(-\frac{1}{2\sigma^2}, \frac{\mu}{\sigma^2}\right)$$

$$g(\theta) = Z\sigma e^{\frac{\mu^2}{2\sigma^2}} = \left(1 - F\left(\frac{\varepsilon-\mu}{\sigma}\right)\right)\sigma e^{\frac{\mu^2}{2\sigma^2}}$$

$$\theta_1 = -\frac{1}{2\sigma^2}, \quad \sigma^2 = -\frac{1}{2\theta_1}, \quad \theta_2 = \frac{\mu}{\sigma^2}, \quad \mu = \sigma^2\theta_2 = -\frac{\theta_2}{2\theta_1}$$

$$\frac{\varepsilon-\mu}{\sigma} = \frac{\varepsilon + \frac{\theta_2}{2\theta_1}}{\sqrt{-\frac{1}{2\theta_1}}} = \left(\varepsilon + \frac{\theta_2}{2\theta_1}\right)\sqrt{-2\theta_1}\ \cancel{\sqrt{-\frac{1}{2\theta_1}}\, e^{\frac{\theta_2^2}{2\theta_1}}}$$

$$\frac{\mu^2}{2\sigma^2} = \frac{\frac{\theta_2^2}{4\theta_1^2}}{\frac{1}{2\theta_1}} = \frac{\theta_2^2}{2\theta_2}$$

$$\cancel{= \frac{(2\theta_1\varepsilon + \theta_2)\, e^{\frac{\theta_2^2}{2\theta_1}}}{2\theta_2}}$$

$g(\theta) =$

$$\left(1 - F\left(\frac{\varepsilon-\mu}{\sigma}\right)\right)\sigma\, e^{\frac{\mu^2}{2\sigma^2}} = \left(1 - F\left(\left(\varepsilon + \frac{\theta_2}{2\theta_1}\right)\sqrt{-2\theta_1}\right)\right)\cdot$$

$$\cdot \sqrt{-\frac{1}{2\theta_1}}\ e^{-\frac{\theta_2^2}{2\theta_1}}$$

$$\ln g(\theta) = \ln(1-F) \mp \frac{1}{2}\ln 2\theta_1 + \frac{\theta_2^2}{2\theta_1} =$$

$$= \ln(1-F) - \frac{1}{2}\ln\theta_1 \mp \frac{\theta_2^2}{2\theta_2}$$

$$\frac{\partial}{\partial\theta_2}\ln g(\theta) = \frac{\sigma^2}{1 - F\left(\frac{\varepsilon-\mu}{\sigma}\right)} \quad N$$

<u>True Skill</u>

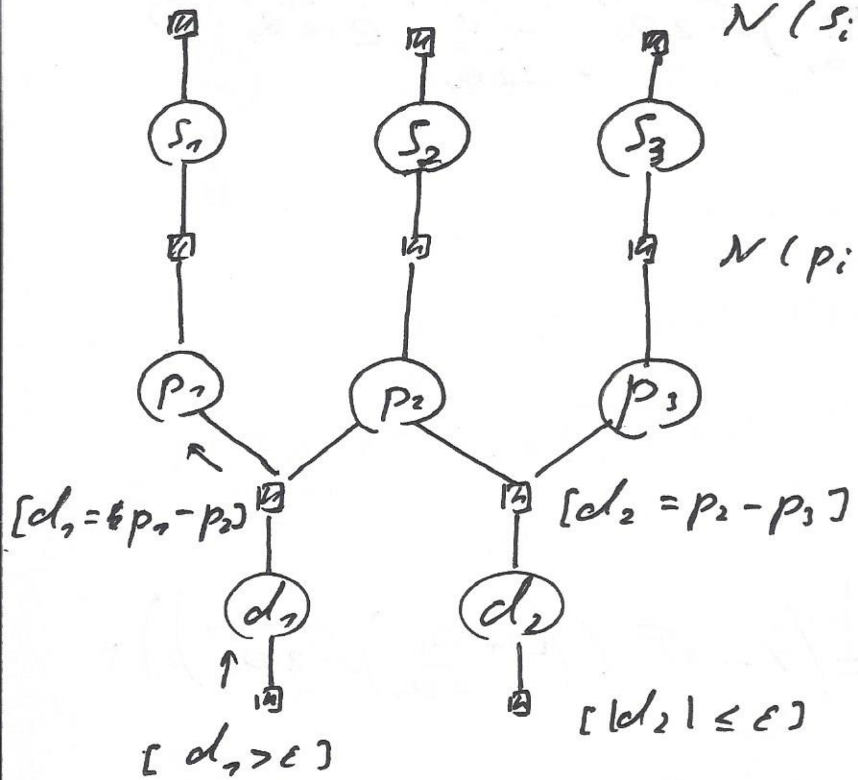
Игра: 1 место Игрок 1

2–3 место Игрок 2, Игрок 3

$s_i \in \mathbb{R},\ i = 1, 2, 3$, $N(s_i \mid \mu_i, \sigma_i^2)$ – априорное распр-е

$p_i \sim N(p_i \mid s_i, \beta^2)$, $d_1 = p_1 - p_2$, $d_2 = p_2 - p_3$

$d_1 > \varepsilon$, $|d_2| \le \varepsilon$



$f = \mathcal{N}(s_1 \mid \mu_1, \sigma_1^2)$

$$\mu_{f \to s_1}(s_1) = \frac{\mathrm{proj}\left(\mathcal{N}(s_1 \mid \mu_1, \sigma_1^2) \cdot \mathcal{N}(s_1 \mid \mu, \sigma^2)\right)}{\mathcal{N}(s_1 \mid \mu_1, \sigma_1^2)} =$$

$$= \mathcal{N}(s_1 \mid \mu, \sigma^2)$$

$f = \mathcal{N}(p_1 \mid s_1, \beta^2)$

$$\cancel{\mu_{f \to p_1}(p_1) = \mathrm{proj}\left(\mathcal{N}(p_1 \mid s_1, \beta^2)\, \mathcal{N}(p_1 \mid s, \beta^2)\, \mu_{s_1 \to f}(s_1)\right)}$$

$$\mu_{f \to p_1}(p_1) = \frac{\mathrm{proj}\left(\int \mathcal{N}(p_1 \mid s_1, \beta^2)\, \mu_{p_1 \to f}(p_1)\, \mathcal{N}(s_1 \mid \mu, \sigma^2)\, ds_1\right)}{\mu_{p_1 \to f}(p_1)} =$$

$$= \mathcal{N}(p_1 \mid \mu, \beta^2 + \sigma^2)$$