01.03.19 нейробайес III

$x \in \mathbb{R}^D$, $z \in \mathbb{R}^d$

$$p(x, z \mid \Theta) = \prod_{i=1}^{n} p(x_i, z_i \mid \Theta) = \prod_{i=1}^{n} p(x_i, z_i \mid \Theta) =$$

$$= \prod_{i=1}^{n} p(x_i \mid z_i, \Theta)\, p(z_i) = \prod_{i=1}^{n} \mathcal{N}(x_i \mid W z_i + \mu, \sigma^2 I)\, \mathcal{N}(z_i \mid 0, I)$$

$X = (x_1 \dots x_n)$, $\Theta = \{W, \mu, \sigma\}$, $p(X \mid \Theta) \to \max_{\Theta}$

$$p(X \mid \Theta) = \prod_{i=1}^{n} p(x_i \mid \Theta) = \prod_{i=1}^{n} \int p(x_i \mid z, \Theta)\, p(z)\, dz$$

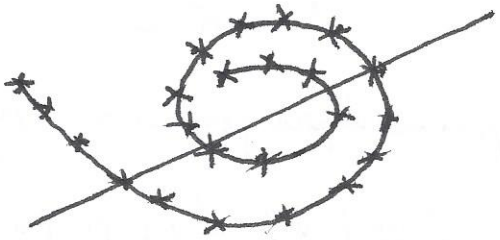
E-step: $q(z) = p(z \mid x, \Theta) = \prod_{i=1}^{n} p(z_i \mid x_i, \Theta)$

M-step: $\mathbb{E}_{q(z)} \ln p(x, z \mid \Theta) \to \max_{\Theta}$

Стохастический EM

E'-step: $i \sim U\{1 \dots n\}$

$q(z_i) = p(z_i \mid x_i, \Theta)$



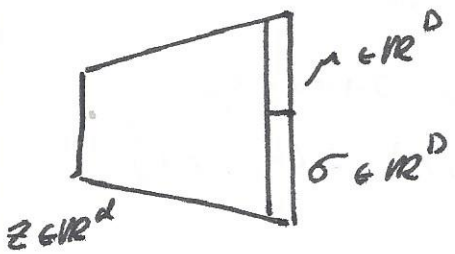
нелинейное моделирование

M'-step:

$\Theta_{t+1} = \Theta_t + \alpha_t \left(\nabla_{\Theta} \log p(x_i, \hat{z} \mid \Theta) \right)$, $\hat{z} \sim q(z_i)$

$$p(x, z \mid \Theta) = \prod_{i=1}^{n} \mathcal{N}(x_i \mid \mu(z_i, \Theta), \Sigma(z_i, \Theta))\, \mathcal{N}(z_i \mid 0, I)$$

$\Sigma = \mathrm{diag}(\sigma_1^2 \dots \sigma_d^2)$



$$\log p(x \mid \Theta) = \sum_{i=1}^{n} \log \int p(z)\, p(x_i \mid z, \Theta)\, dz$$

$$\neq \sum_{i=1}^{n} \log p(x_i \mid \hat{z}_i, \Theta)$$

потому что $\log \mathbb{E} \geq \mathbb{E} \log$ $\quad \hat{z}_i \sim p(z)$

$$\log p(x|\Theta) \geq \mathcal{L}(\Theta, \varphi) = \int q(z|\varphi) \log \frac{p(x,z|\Theta)}{q(z|\varphi)} dz \Leftarrow$$

хорошая оценка

$$\log \int p(z) p(x|z,\Theta) dz \geq \int p(z) \log p(x|z,\Theta) dz \to \max_{\Theta}$$

плохая оценка

$$q(z|\varphi) = \prod_{i=1}^{n} q_i(z_i|\varphi) = \prod_{i=1}^{n} \prod_{j=1}^{d} \mathcal{N}(z_{ij} | m_{ij}, s_{ij}^2)$$

$$\varphi = \{m_i, s_i\}_{i=1}^{n}$$

$$\Leftarrow \sum_{i=1}^{n} \Big[\int q_i(z_i|\varphi) \log p(x_i|z_i,\Theta) dz_i - \underbrace{\int q(z_i|\varphi) \log \frac{q(z_i|\varphi)}{p(z_i)} dz_i}_{KL} \Big]$$

Репараметризация на φ

$$\nabla_\varphi \int q_i(z_i|\varphi) \log p(x_i|z_i,\Theta) dz_i \Leftarrow, \quad z_i = g(\varepsilon, \varphi)$$

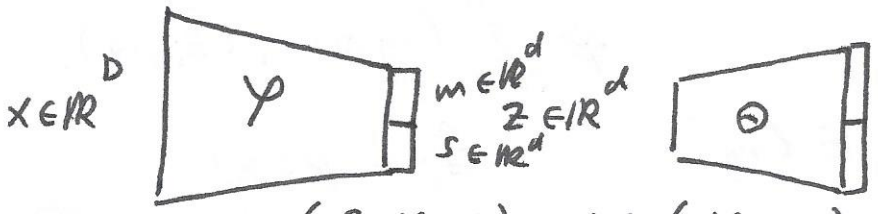
и $z_{ij} = m_{ij} + \hat{\varepsilon} s_{ij}$, $\hat{\varepsilon} \sim \mathcal{N}(0,1)$

$$\Leftarrow \nabla_\varphi \int r(\varepsilon) \log p(x_i | g(\varepsilon, \varphi), \Theta) d\varepsilon \approx$$

$$\approx \nabla_\varphi \log p(x_i | g(\hat{\varepsilon}, \varphi), \Theta), \quad \hat{\varepsilon} \sim \mathcal{N}(0,1)$$

$q_i(z_i | \varphi, \underline{x_i})$ — одни параметры φ для всех z => энкодер

$$KL = f(m(x_i,\varphi), s(x_i,\varphi))$$



$$z = g(\varepsilon, \varphi, x) = m(\varphi, x) + \hat{\varepsilon} s(\varphi, x)$$