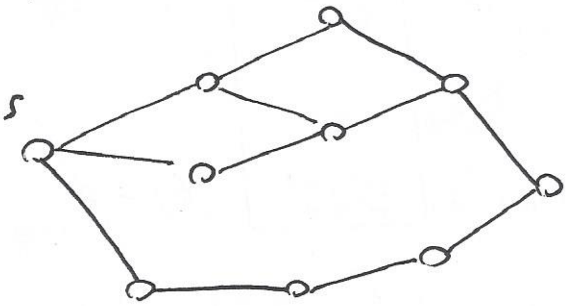
Разрезы графов.



$f(i,j)$ – поток

$c(i,j) \geq 0$
$\ast\ c(j,i)$, $0 \leq f(i,j) \leq c(i,j)$

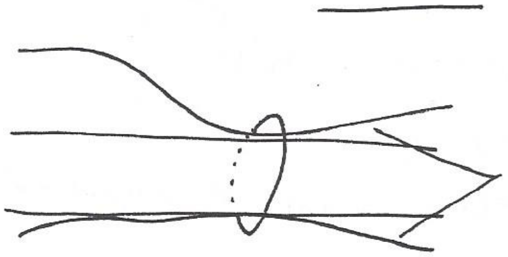
поиск максимального потока в сети

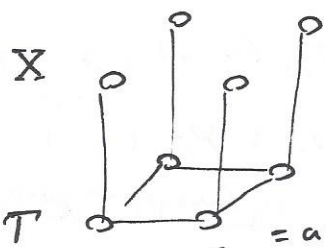
$$\begin{cases} \sum\limits_{i:(s,i)\in E} f(s,i) = \Phi \to \max & \text{величина потока} \\ \sum\limits_{j:(i,j)\in E} f(i,j) = \sum\limits_{j:(j,i)\in E} f(j,i) & \forall i \neq s,t \\ 0 \leq f(i,j) \leq c(i,j) & \forall (i,j)\in E \end{cases}$$

(s,t) – cut, $V = V_s \cup V_t$; $V_s \cap V_t = \varnothing$
$s \in V_s$, $t \in V_t$

$$A = \sum_{\substack{(i,j)\in E \\ i\in V_s,\, j\in V_t}} c(i,j) \to \min_{V_s, V_t} \quad \text{величина разреза}$$

$\underline{\Phi \leq A}$ Т Ф-М: $\max \Phi = \min A$





$T^* = \arg\max\limits_T p(T \mid X) = \arg\max\limits_T p(T,X) = \arg\min E$

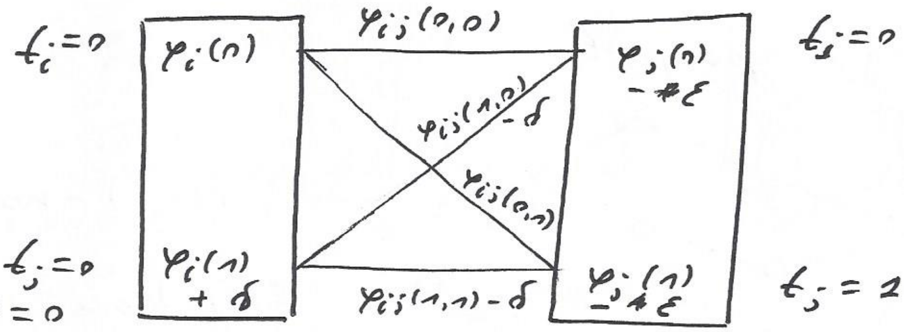
Бинарная сегментация изображений

$X, T \in \{0,1\}^n$, $p(X,T) = \prod\limits_{i\in V} \psi_i(t_i, x_i) \prod\limits_{(i,j)\in E} \psi_{ij}(t_i, t_j)$

$$T^* = \arg\min_T \Big(\sum_{i\in V} \varphi_i(t_i, x_i) + \sum_{(i,j)\in E} \varphi_{ij}(t_i, t_j) \Big)$$

$$\varphi_{ij}(t_i, t_j) = \exp(-\|x_i - x_j\|^2)[t_i \neq t_j]$$

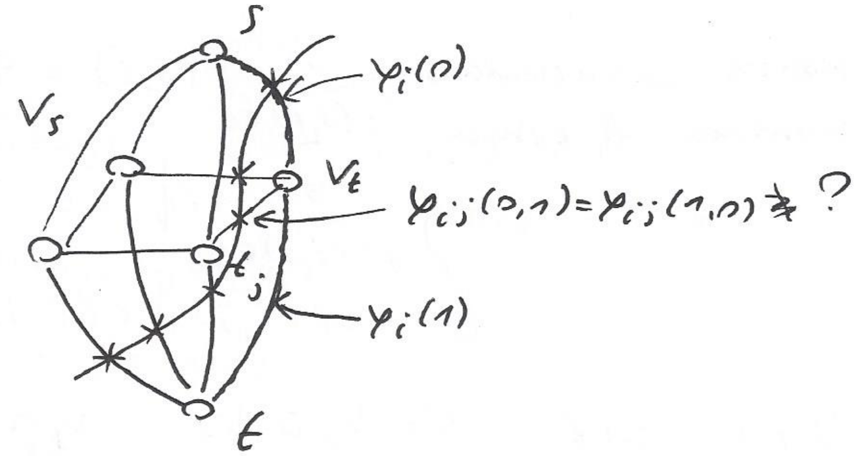
$$E(T) = \sum_{i\in V} \varphi_i(t_i) + \sum_{(i,j)\in E} \varphi_{ij}(t_i, t_j) \to \min_{T\in\{0,1\}^n}$$



1) $\varphi_i(t_i) \geq 0$

2) $\varphi_{ij}(0,0)=\varphi_{ij}(1,1)=0$

3) $\varphi_{ij}(0,1)=\varphi_{ij}(1,0)$



$$A = \sum_{i\in V_s} \varphi_i(1) + \sum_{i\in V_t} \varphi_i(0) + \sum_{\substack{(i,j)\in E\\ i\in V_s,\ j\in V_t}} \varphi_{ij}(0,1) =$$

$$= \left\{ \begin{array}{l} t_i=1,\ i\in V_s \\ t_i=0,\ i\in V_t \end{array} \right\} = \sum_{i\in V} \varphi_i(t_i) + \sum_{\substack{(i,j)\in E\\ \cancel{i\in V_s,\ j\in V_t}\\ t_i\neq t_j}} \varphi_{ij}(t_i,t_j) +$$

$$+ \sum_{\substack{(i,j)\in E\\ t_i=t_j}} \overset{0}{\varphi_{ij}(t_i,t_j)} = E(T)$$

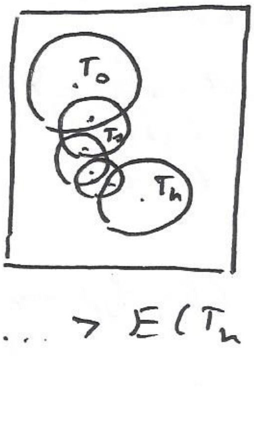
условие $\varphi_{ij}(1,0)=\varphi_{ij}(0,1) \geq 0$ субмодулярности:

$$\varphi_{ij}(1,0)+\varphi_{ij}(0,1) \geq \varphi_{ij}(0,0)+\varphi_{ij}(1,1)$$

$f: A\subset U \to \mathbb{R}$, f субмодулярная, если для $\forall$ подмн-в A,B

$f(A\cup B)+f(A\cap B) \leq f(A)+f(B)$ {аналог выпуклых ф-ий в задачах д.о.}

$E \to \min\limits_{T\in\{1..k\}^n}$



мн-во решений

○ подмн-во реш.

глобальные min в ○

T_n - приближённое решение

$$E(T_0) > E(T_1) > \ldots > E(T_n) = E(T_{n+1})$$

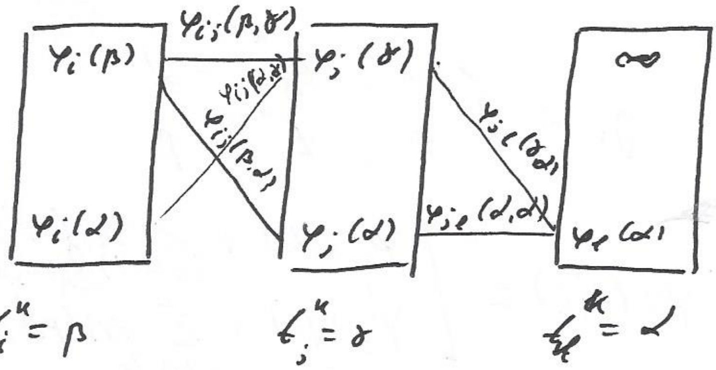
α - расширение

α-расш.

$T^k \quad \alpha \in \{1 \dots k\}$

$$y_i = \begin{cases} 0, & t_i^{k+1} = t_i^k \neq \alpha \\ 1, & t_i^{k+1} = \alpha \end{cases}$$

$E(Y) \to \min$



Пусть $\varphi_{ij}(\alpha,\alpha) = 0 \quad \forall (i,j) \in \mathcal{E}, \forall \alpha$

$\varphi_{ij}(\beta,\alpha) + \varphi_{ij}(\alpha,\gamma) \geq \varphi_{ij}(\beta,\gamma) \quad \forall \alpha,\beta,\gamma$

нер-во Δ.

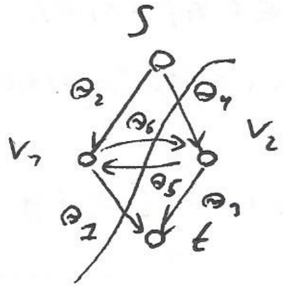
Тогда φ_{ij} должна быть метрикой

$\varphi_{ij}(t_i,t_j) = C[t_i \neq t_j]$ тоже метрика

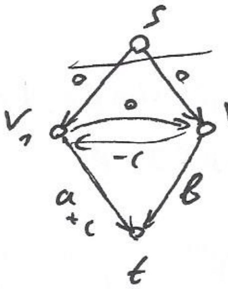
24.03.17 2м сем

$$E(x_1,x_2) = \theta_1 x_1 + \theta_2(1-x_1) + \theta_3 x_2 + \theta_4(1-x_2) + \theta_5 x_1(1-x_2) + \theta_6 x_2(1-x_2)$$

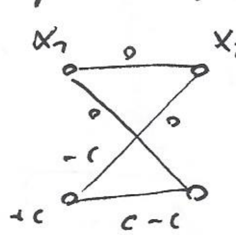
$x_1, x_2 \in \{0,1\}$

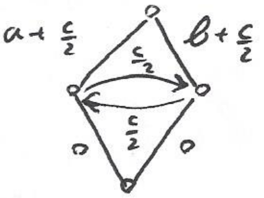


$E(x_1,x_2) = a x_1 + b x_2 + c x_1 x_2$



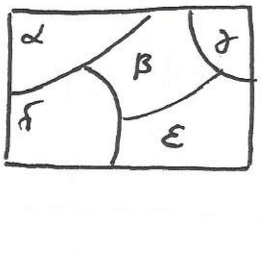
репараметризация

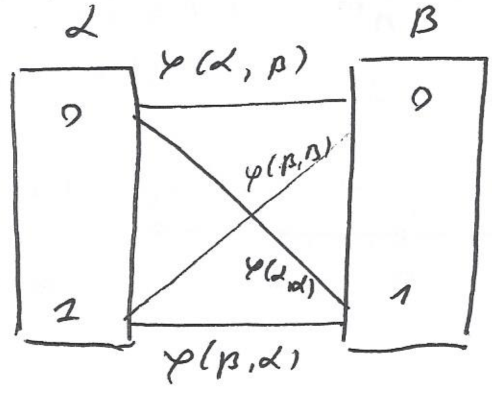




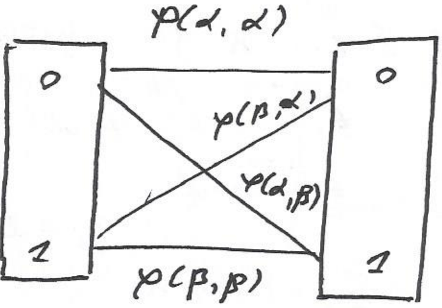
$$E(T) = \sum_{i \in V} \varphi_i(t_i) + \sum_{(i,j) \in \mathcal{E}} \varphi_{ij}(t_i,t_j)$$

α-β — swap





$$y_i = \begin{cases} 0, & t_i^{n+1} = \alpha \\ 1, & t_i^{n+1} = \beta \end{cases}$$



$$\varphi_i(y_i) = \left[\begin{array}{l} \varphi_i(\alpha) + \sum\limits_{j\in N(i),\, \hat{t}_j \neq \alpha,\beta} \varphi_{ij}(\alpha, \hat{t}_j) \\ \varphi_i(\beta) + \sum\limits_{\substack{j \neq \alpha,\beta \\ j \in N(i)}} \varphi_{ij}(\beta, \hat{t}_j) \end{array} \right.$$

$$\varphi_{ij}(\alpha,\alpha) + \varphi_{ij}(\beta,\beta) \le \varphi_{ij}(\alpha,\beta) + \varphi_{ij}(\beta,\alpha)$$

$X = \mathbb{R}$, $\{1 \dots k\}$

(1) $|x-y|$, (2) $\cancel{(x-y)^2 = x^2 - 2xy + y^2 > x^2 + y^2}$

$\cancel{x;y > 0}$

$$(x-z)^2 \le (x-y)^2 + (y-z)^2$$

$$|x-z| \le |x-y| + |y-z|$$

(3) $[x \neq y] \le [x \neq z] + [z \neq y]$

(4) $\min(c, |x-y|) \le \min(c, |x-z|) + \min(c, |z-y|)$

$\rho\ \rho\ \rho\ +$

$\rho\ \rho\ c\ +$

$\rho\ c\ \rho$

$$\rho(x,y) + \rho(y,z) \le c \le \rho(x,z)$$

$$\rho(x,y) + c \le c + c \le \rho(x,z) + c$$

$$\min(c, |x-z|) \le \min(c, |x-y|) + \min(c, |y-z|)$$

$$\rho(x,y) + \rho(y,z)$$

$\cancel{N}$ n

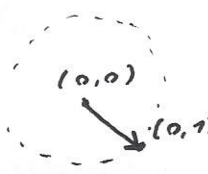
$\cancel{C_n^1 (\tfrac{1}{N})^m}$ $C_n^1 \cdot \left(\frac{1}{n}\right)^m$ $1 - \frac{1}{n^m}$

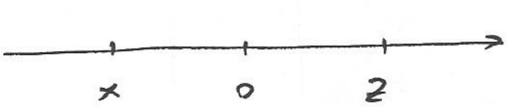
$\cancel{M}$ m

$C_n^2 (\cancel{\tfrac{1}{n}^m}$ $\cancel{2} (\frac{1}{n}$

$p(x_1 > 0 \mid x_2 = 0, \dots, x_n = 0) =$

$= \frac{1}{n^m}$, $p(x_1 > 0, x_2 > 0 \mid x_3 = 0, \dots, x_n = 0)$





$$\min(c, |x|) + \min(c, |z|) \ge \min(c, |x-z|)$$