/ 10.02.17 гм I

многомерные распр-я

$$p(x_1\dots x_n)=p(x_n|x_1\dots x_{n-1})p(x_{n-1}|x_1\dots x_{n-2})\dots p(x_1)$$

2^n памяти

n памяти: $p(x_1\dots x_n)=\prod_{j=1}^{n} p(x_j)$

условная независимоть

$$x\perp y\,|\,z \Leftrightarrow p(x,y|z)=p(x|z)p(y|z)$$

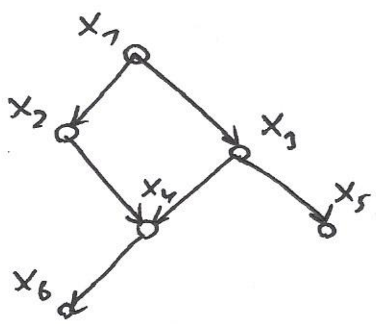
$$p(x,y)=\int p(x,y,z)dz=\int p(x|y,z)p(y|z)p(z)dz =$$

$$=\int p(x|z)p(y|z)p(z)dz \neq p(x)\cdot p(y)$$

$$\| \int p(x,y,z)dz=\int \cancel{p(x|y,z)}\, p(x,y|z)p(z)dz$$

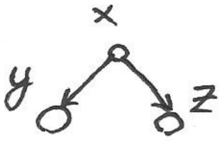
$$p(x,y,z)=p(x|z)p(y|z)p(z)$$

байесовская сеть



$$p(x_1\dots x_6)=p(x_6|x_4)p(x_5|x_3)p(x_4|x_2,x_3)\cdot$$
$$\cdot p(x_3|x_1)p(x_2|x_1)p(x_1)$$

$$p(x_1\dots x_n)=\prod_{j=1}^{n} p(x_j|Pa_j)$$

① 

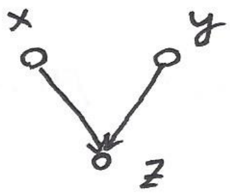
$$p(x,y,z)=p(y|x)p(z|x)p(x)$$

$$y\perp z\,|\,x$$

(2) 

$$p(x,y,z)=p(z|y)\,p(y|x)\,p(x)$$

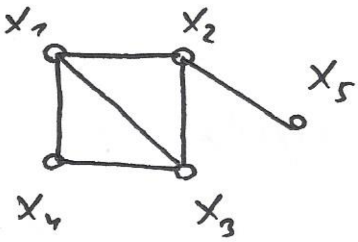
$$x \perp z \mid y$$

(3) 

$$p(x,y,z)=p(z|x,y)\,p(x)\,p(y)$$

$x \perp y$, но $x \perp y \mid z$ неверно

марковская сеть



$$p(x_1 \dots x_5)=\psi_1(x_1,x_2,x_3)\,\psi_2(x_1,x_3,x_4)\,\psi_3(x_2,x_5)\cdot\frac{1}{Z}$$

$$\psi_j \geqslant 0$$

$$Z=\sum_{x_1 \dots x_5}\psi_1\cdot\psi_2\cdot\psi_3$$

$$p(x_1 \dots x_5)=\frac{1}{Z}\exp(-E(x_1 \dots x_5))$$

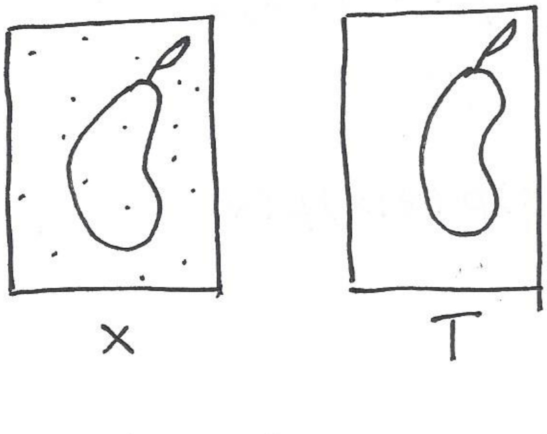
$$E(x_1 \dots x_5)=\varphi_1(x_1,x_2,x_3)+\varphi_2(x_1,x_3,x_4)+\varphi_3(x_2,x_5)$$

$\varphi_j=-\ln\psi_j$, ψ_j – факторы

φ_j – потенциалы

E – энергия

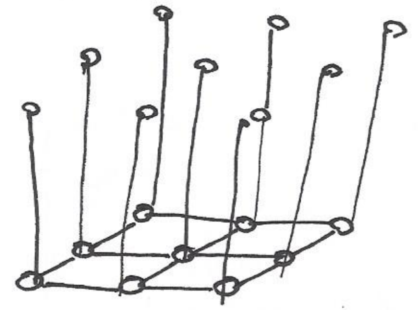
удаление шума



$p(X,T)$

$p(T|X)\to\max\limits_{T}$

$x_i \in \{-1,+1\}$



$t_i \in \{-1,+1\}$

$$p(x,T) = \frac{1}{Z}\prod_{(i,j)\in\mathcal{E}} \psi_{ij}(t_i,t_j)\prod_i \psi_i(x_i,t_i) =$$

$\mathcal{E}$ — отношение соседства

$$= \frac{1}{Z}\exp\Big(-\sum_{(i,j)\in\mathcal{E}} \varphi_{ij}(t_i,t_j) - \sum_i \varphi_i(x_i,t_i)\Big) =$$

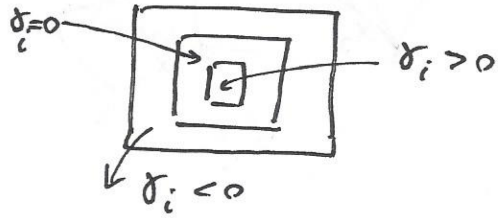
хотим поощрять одинаковые знаки

$$= \frac{1}{Z}\exp\Big(+\alpha\sum_{(i,j)\in\mathcal{E}} t_i t_j + \beta\sum_i x_i t_i\Big)$$

$$p(T|X) \to \max_T$$

$$\arg\max_T p(T|X) = \arg\max_T p(T,X) = \arg\min_T E(X,T) =$$

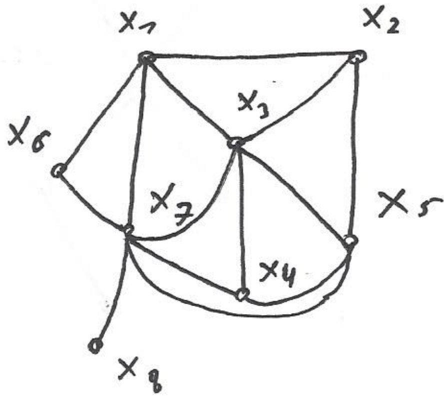
$$= \arg\min_T \Big(-\alpha\sum_{i,j\in\mathcal{E}} t_i t_j - \beta\sum_i x_i t_i\Big) \Big\{+\sum_i \gamma_i t_i\Big\}$$



$$p(x,T) = \frac{1}{Z}\prod_{i,j\in\mathcal{E}} \psi_{ij}(t_i,t_j)\underbrace{\prod_i \psi_i(x_i,t_i)\prod_i \psi_i'(t_i)}_{\prod_i \psi_i''(x_i,t_i)}$$

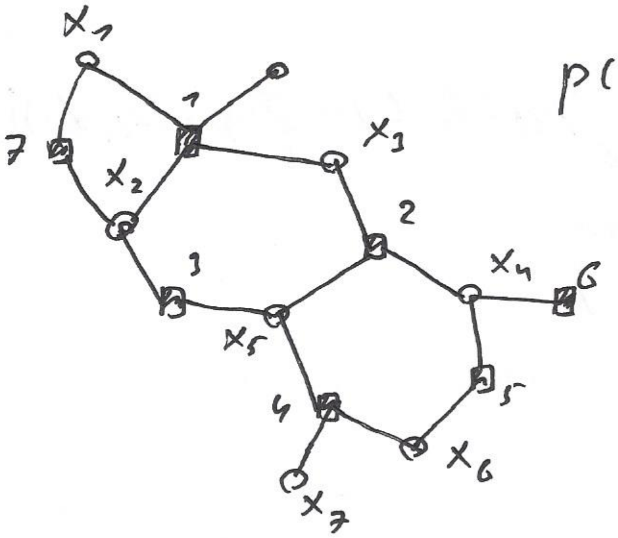
раскраска атласа

с, к, з, ж

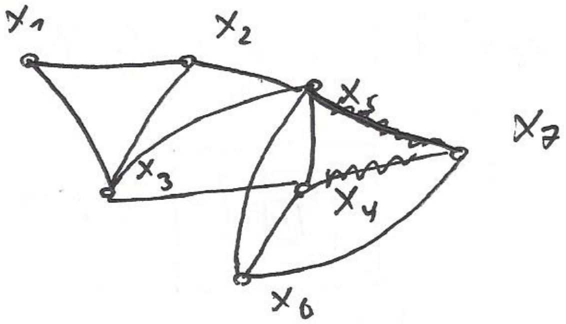


$$p(x_1 \dots x_8) = \frac{1}{Z} \psi_1(x_1, x_2, x_3) \dots \psi_5(x_3, x_4, x_5, x_7) \psi_6(x_7, x_8) \dots$$

фактор-графы

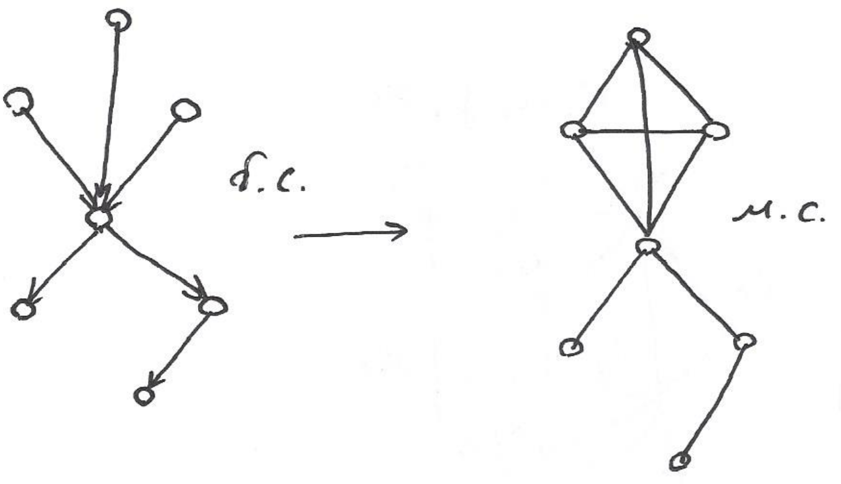


$$p(x_1 \dots x_7) = \frac{1}{Z} \psi_1(x_1, x_2, x_3) \cdot$$
$$\cdot \psi_2(x_3, x_4, x_5) \psi_3(x_2, x_5) \cdot$$
$$\cdot \psi_4(x_5, x_6, x_7) \psi_5(x_4, x_6) \psi_6(x_4) \cdot$$
$$\cdot \psi_7(x_1, x_2)$$

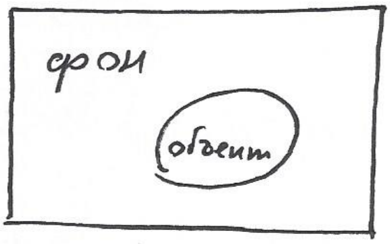


$p(x_i, x_j \mid X_{\setminus i,j}) \neq$ условная независимость

$$p(X) = \frac{1}{Z} \prod_{k: i \in k} \psi_k(x_i, \dots) \prod_{k: j \in k} \psi_k(x_j, \dots) \prod_{\substack{k: i \notin k \\ j \notin k}} \psi_k(\dots)$$



Сегментация изображений



$y_i \in \{0, 1\}$

фон ↗ ↖ объект

$$\Theta_i(y_i; x_i) = -\log p(y_i | x_i)$$

$$\Theta_{ij}(y_i, y_j) = \alpha [y_i \neq y_j]$$

$$\Theta'_{ij}(y_i, y_j; x_i, x_j) = \alpha [y_i \neq y_j] \frac{1}{\|x_i - x_j\|^2 + 1}$$

семена, seeds

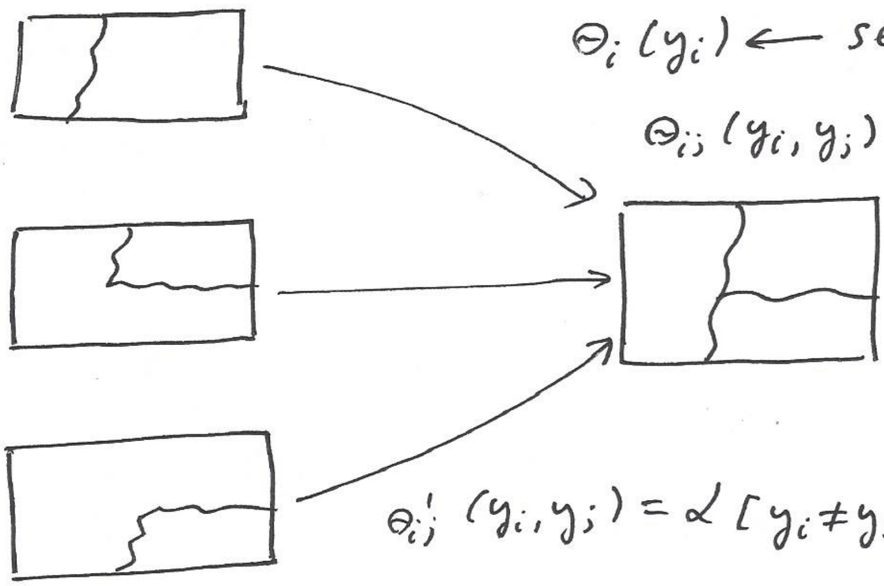
$$\Theta_i(y_i = 1) = \begin{cases} +\infty, & i \in seeds \\ 0, & \text{иначе} \end{cases}$$

Коллаж

$y_i \in \{1, \dots, k\}$

$\Theta_i(y_i) \leftarrow$ seeds

$\Theta_{ij}(y_i, y_j) = \alpha [y_i \neq y_j]$



$$\Theta'_{ij}(y_i, y_j) = \alpha [y_i \neq y_j] (|I_i(y_i) - I_i(y_j)| + |I_j(y_i) - I_j(y_j)|)$$

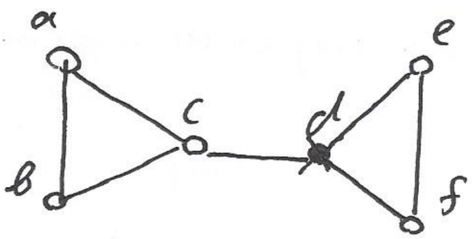
10.02.17 2м сем

условная нез-ть

$a \perp b \mid c \Leftrightarrow p(a,b \mid c) = p(a \mid c)\, p(b \mid c)$

$a \perp b \mid c \;\not\Rightarrow\; a \perp b$
$\;\not\Leftarrow$

~~$p(a,b|c)$~~ $\quad p(a,b,c) = p(a,b \mid c)\,p(c) = p(a \mid c)\,p(b \mid c)\,p(c)$



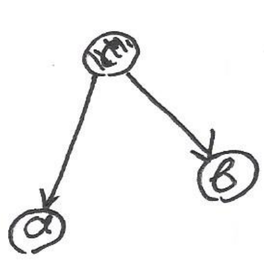
$p(a \ldots f) = \frac{1}{Z}\psi_1(a,b,c)\,\psi_2(c,d)\,\psi_3(d,e,f)$

$e \perp c \mid d$

$p(e,c,d) = p(e \mid d) \cdot p(c \mid d)$? Да

$p(e,c,d) = \int p(a \ldots f)\,da\,db\,df = \frac{1}{Z}\psi_2(c,d)\int \psi_1(a,b,c)\,\psi_3(d,e,f) \cdot da\,db\,df = \frac{1}{Z}\psi_2(c,d)\int \psi_1(a,b,c)\,da\,db \cdot \int \psi_3(d,e,f)\,d\cancel{d}f =$

$= \frac{1}{Z}\psi_2(c,d)\,\psi_1'(c)\,\psi_3'(d,e)$ => условная независимость

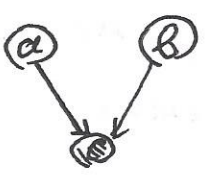
① 

$a \perp b \mid c$? Да

,c' блокирует путь 

② 

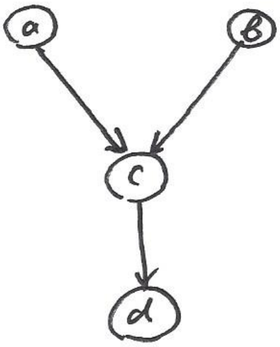
 тоже блокирует

③ 

$a \perp b$

но нет $a \perp b \mid c$

 'c' не блокирует путь



d блокирует a и в

d не блокирует a и в

d - separation

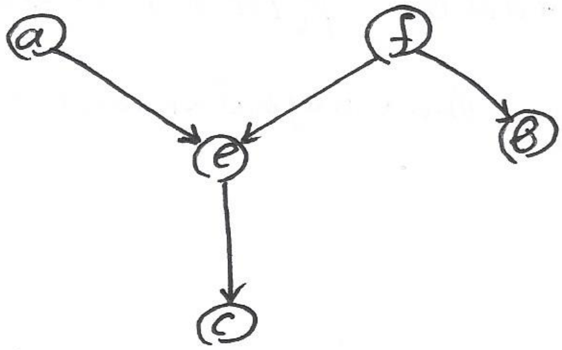
н — переменная наблюдаема

Путь блокирован, если

1)  (тогда условная независимость)

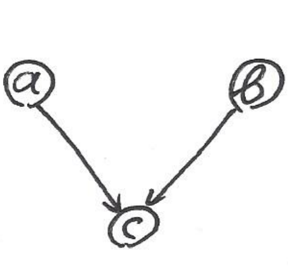
2) 

3)  и нет прямых наблюдаемых потомков

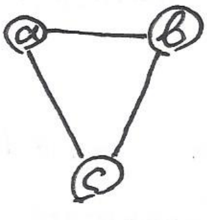


$a \perp\!\!\!\perp b \mid c$? нет

$a \perp\!\!\!\perp b \mid f$ да

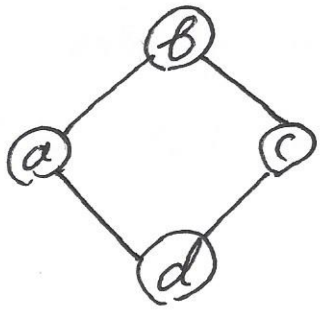


$a \perp\!\!\!\perp b \mid \emptyset$



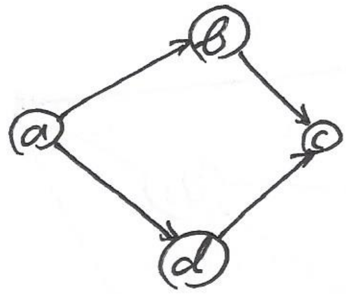
$a \not\!\perp\!\!\!\perp b$

марковские сети не покрывают все распределения!



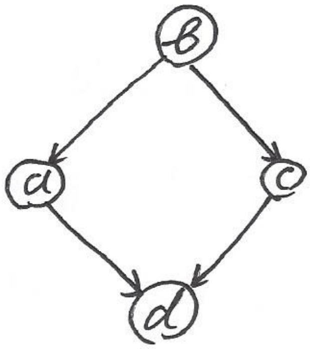
$a \perp\!\!\!\perp c \mid b, d$ +

$b \perp\!\!\!\perp d \mid a, c$ +



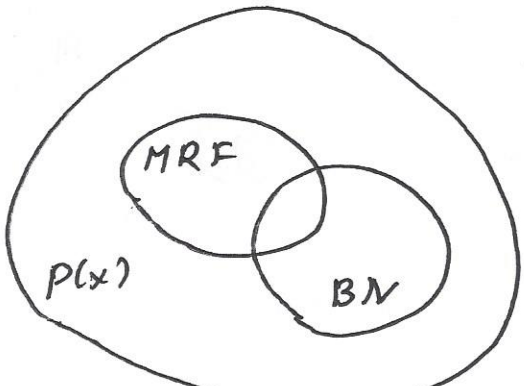
$a \perp\!\!\!\perp c \mid b, d$ +

$b \perp\!\!\!\perp d \mid a, c$ −



$a \perp\!\!\!\perp c \mid b, d$ −

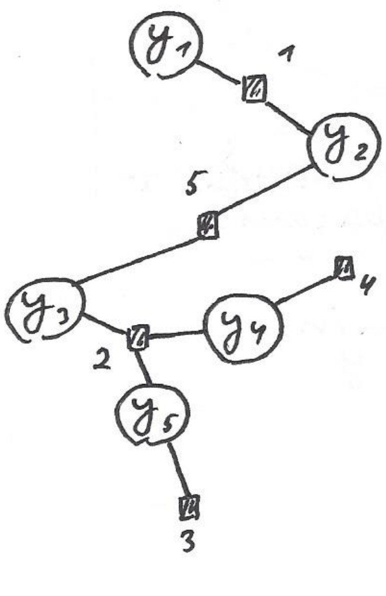
$b \perp\!\!\!\perp d \mid a, c$ +

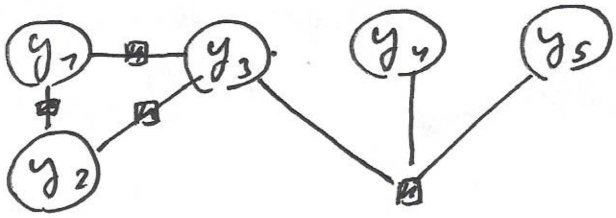


условные независимости тройки групп переменных

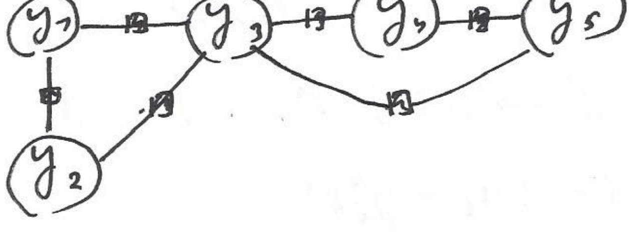
фактор-граф

$$E(y) = y_1 y_2 - 10 y_1 y_2 - y_3 y_4 y_5 + 3 y_2 y_3 + y_5 - y_4$$



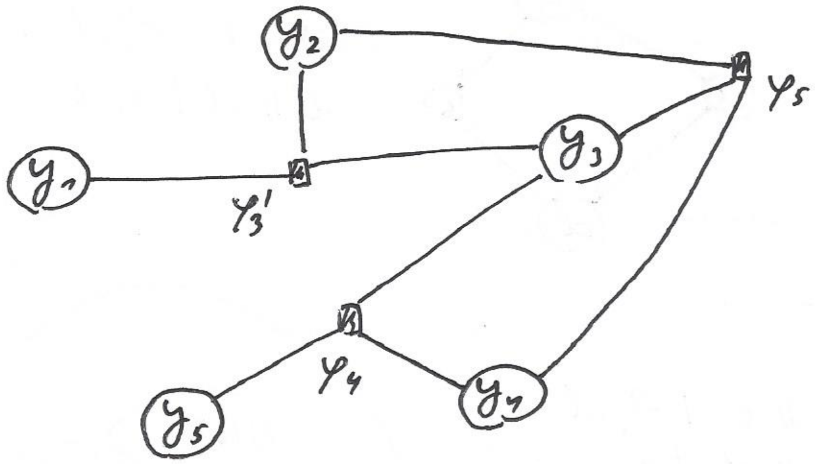


+



−

$$p(y_1 \dots y_5) = p(y_1)\,p(y_2|y_1)\,p(y_3|y_1,y_2)\,p(y_4|y_2,y_3)\,p(y_5|y_3,y_4)$$

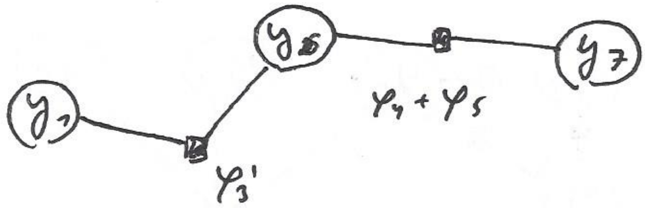


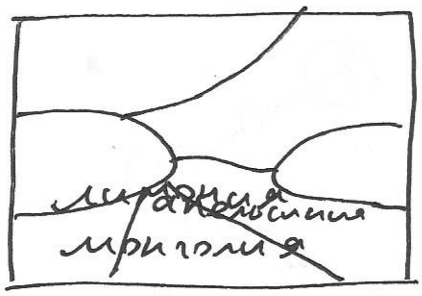
$$E(y) = \varphi_1(y_1) + \varphi_2(y_1, y_2) + \varphi_3(y_1, y_2, y_3) +$$

$$+ \varphi_4(y_4, y_2, y_3) + \varphi_5(y_5, y_3, y_4) =$$

$$= \varphi_3'(y_1, y_2, y_3) + \varphi_4(y_2, y_3, y_4) + \varphi_5(y_3, y_4, y_5)$$

$$y_6 = (y_2, y_3), \quad y_7 = (y_4, y_5)$$





$$E(y) = \sum_i \Theta_i(y_i) + \sum_{(i,j)\in E} \Theta_{ij}(y_i, y_j) \to \min_y$$

$$y_i \in \{\to, \leftarrow, \nearrow, \swarrow, \dots\}$$

$$\Theta_i(y_i) = \|y_i - y_i^0\|^2$$