Edge Colouring

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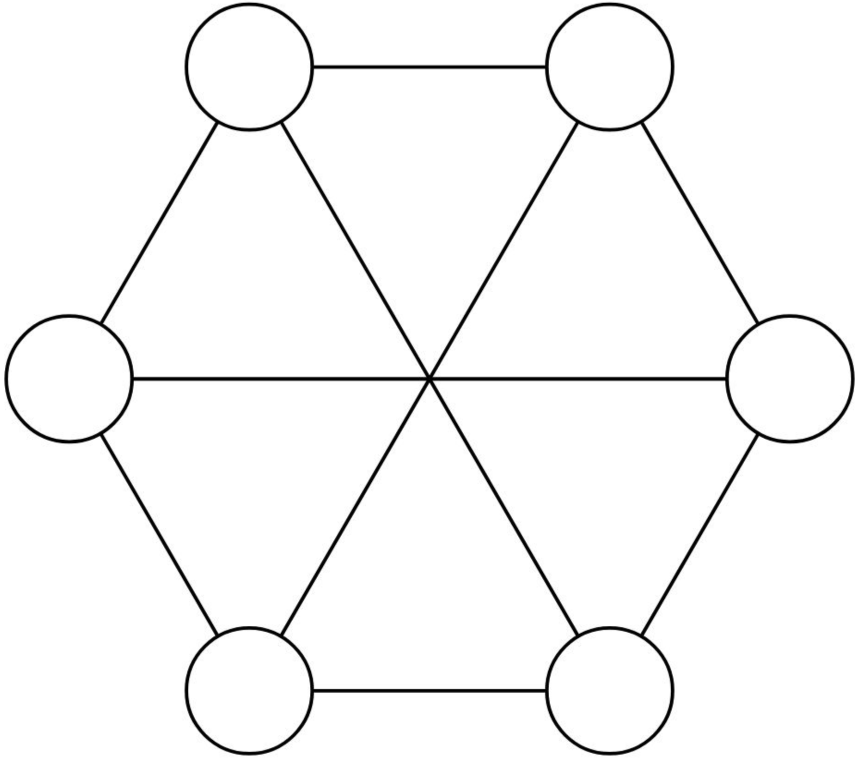
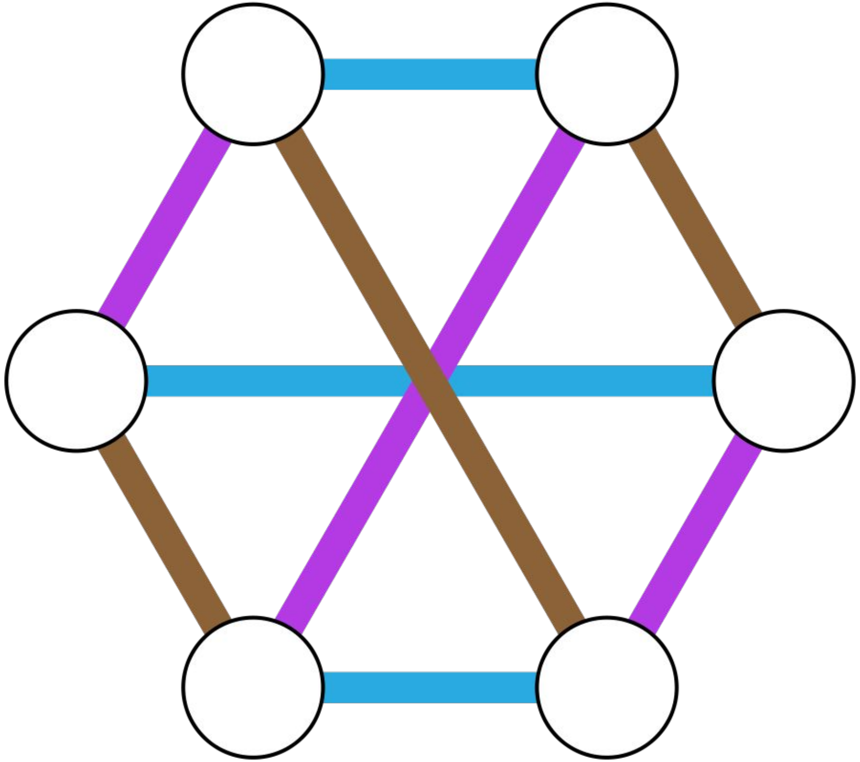


Δ=3

Mapping $c : E \to S$,

$uv, vt \in E \Rightarrow c(uv) \neq c(vt)$



$\Delta=3$

Vizing's Theorem

$$\Delta \leq \chi' \leq \Delta+1$$

Class 1 **Class 2**

Determining if G is Class 1 or 2 is NP-hard in general!

Project Aim

“Do Class 2 graphs possess any common sub-structures?”

Deliverables

Basic:

- **B.1:** Implement Misra-Gries heuristic. (✓)
- **B.2:** Implement Vizing heuristic (by Januario & Urrutia). (✓)
- **B.3:** Duplicate Januario & Urrutia’s results (B1 vs B2). (✓)
- **B.4:** Perform experimental confirmation of Erdos & Wilson’s results. (✓)

Misra-Gries & Vizing Heuristic

- Constructive proofs of Vizing’s Theorem.
- Januario & Urrutia’s idea: try **Δ** colours first, fall back to **Δ+1**.
- Correctness tested on multiple graph families.

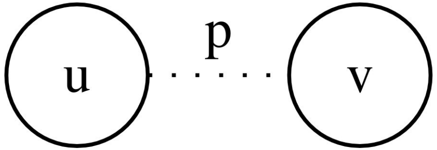
Vizing vs Misra-Gries Heuristic

Instance	**Δ**	**VH colours used**	**MG colours used**
`le450_15a`	99	99	99
`le450_15c`	139	139	140
`myciel3`	5	5	6
`myciel4`	11	11	12
...			

We found the same results!

Random Graphs

Erdos-Renyi model:



G(n, p)

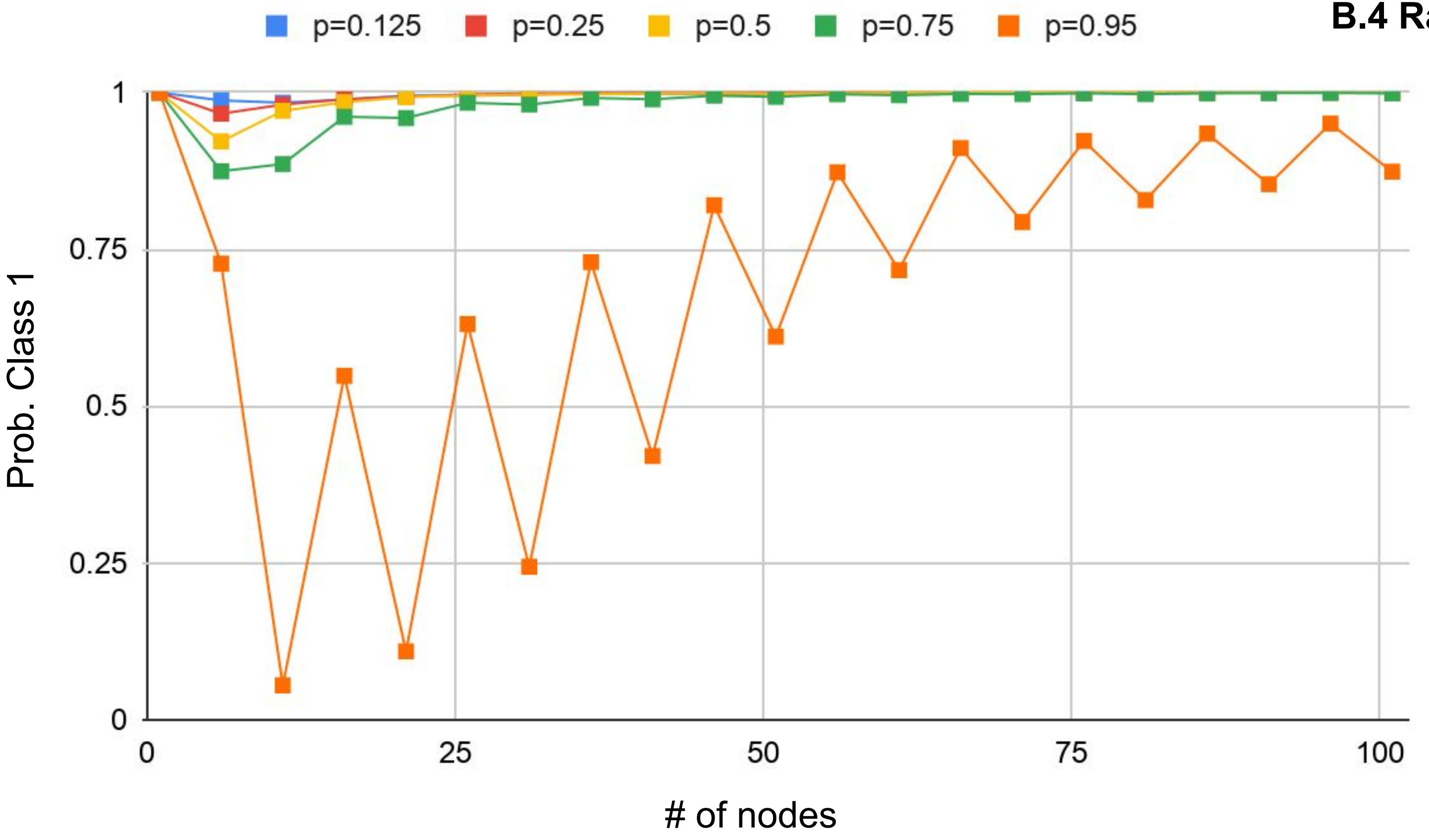
n = # of nodes

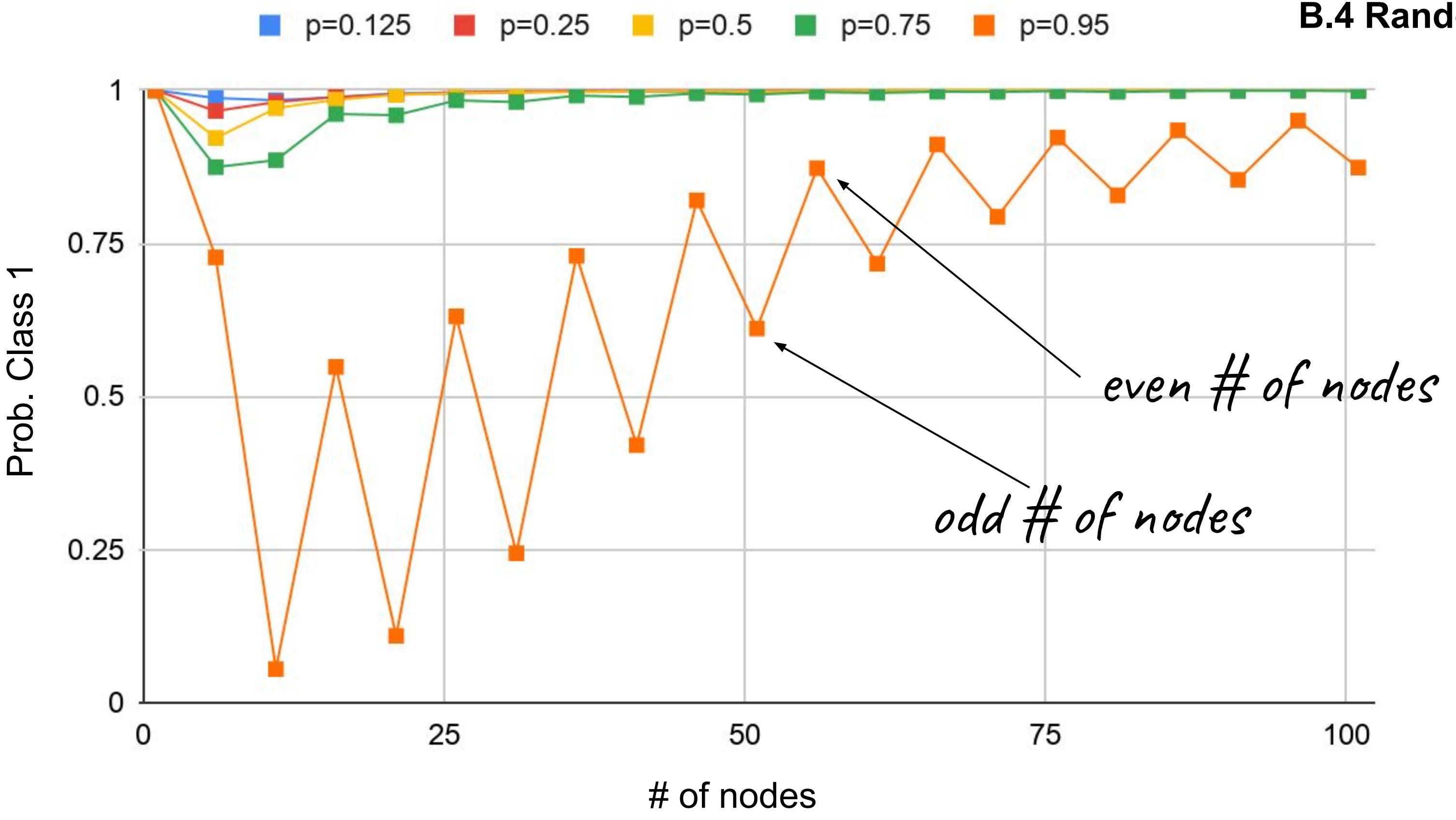
p = probability of an edge between two distinct nodes

Erdos & Wilson:

For G(n, ½): as n→∞, probability that graph is Class 1 →1.

B.1 MG Heuristic
B.2 VH Heuristic
B.3 VH vs MG
B.4 Random graphs



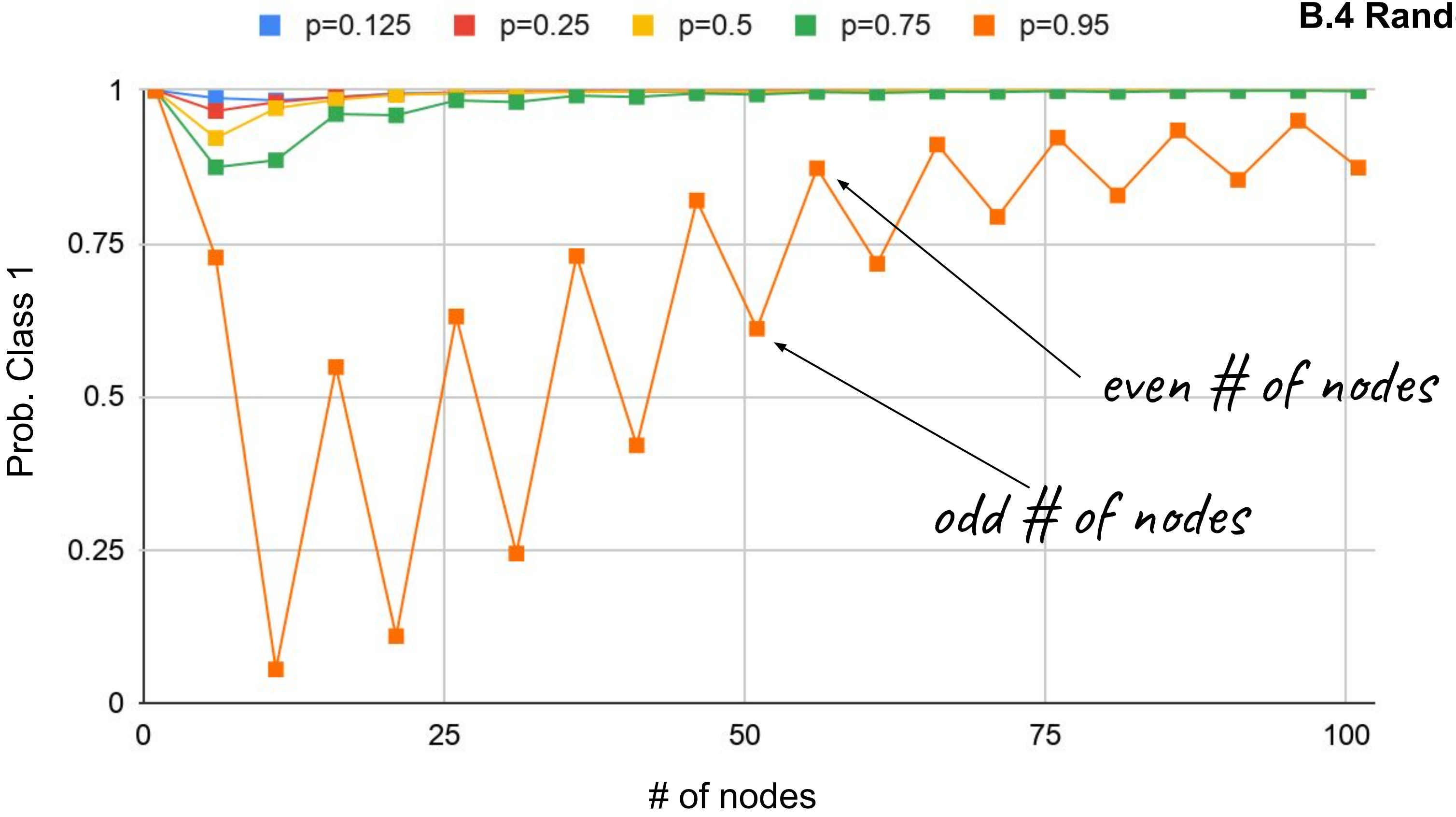

p=0.125
p=0.25
p=0.5
p=0.75
p=0.95
Prob. Class 1
1
0.75
0.5
0.25
0
0
25
50
75
100
# of nodes
even # of nodes
odd # of nodes

Behzad et al.:

$$\chi'(K_n) = \begin{cases} \overbrace{\boxed{n-1}}^{\text{Class 1}} & \text{if } n \text{ is even} \\ \underbrace{\boxed{n}}_{\text{Class 2}} & \text{otherwise} \end{cases}$$

Complete graph on n vertices
(All nodes linked to each other)

B.1 MG Heuristic
B.2 VH Heuristic
B.3 VH vs MG
B.4 Random graphs



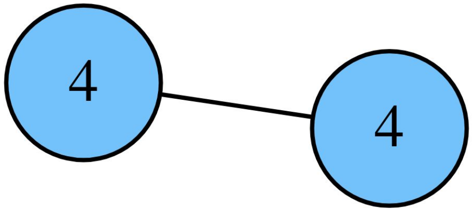
Deliverables

Intermediate

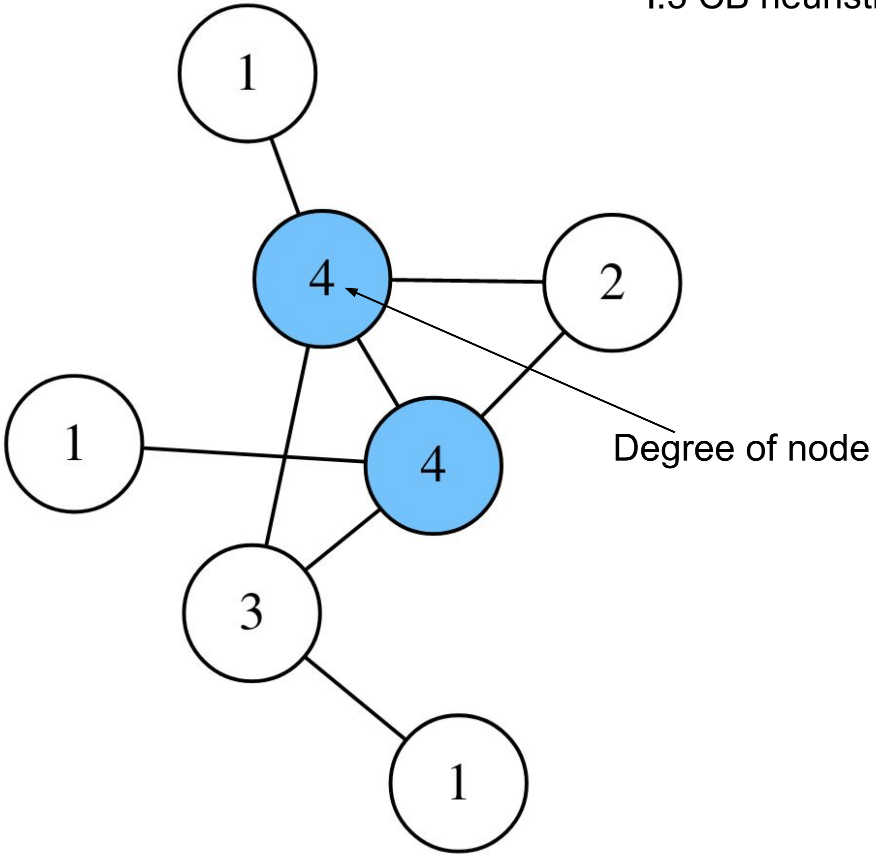
- **I.1:** Investigate case $\Delta=5$ of Hilton Zhao Conjecture. (✓)
- **I.2:** Find bad cores. (✓)
- **I.3:** Implement counting-based heuristic. (✓)

Hilton Zhao Conjecture

Core of a graph = induced subgraph of nodes with degree Δ (blue nodes)



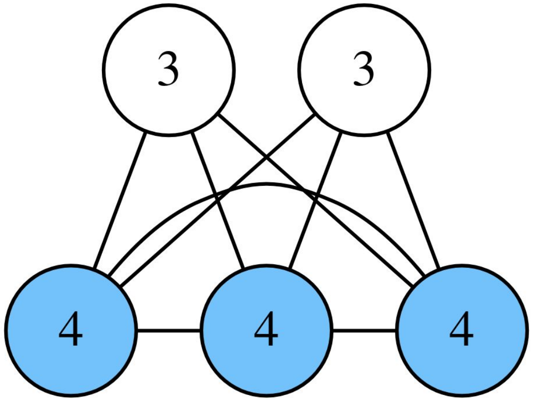
Core



Hilton Zhao Conjecture

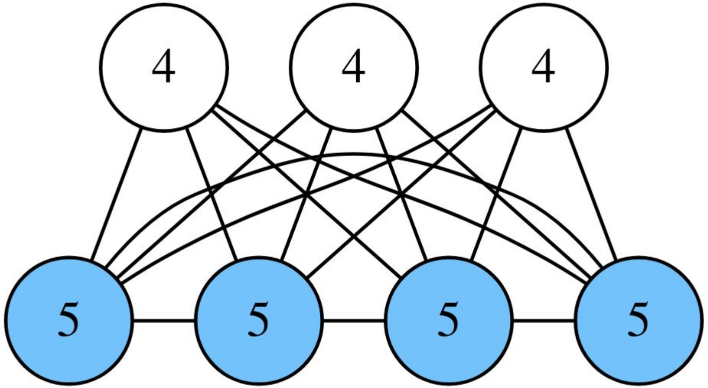
Graphs with Δ(core) = 2 and $\Delta \geq 4$ are **Class 2** iff $|E| > \Delta\lfloor |V|/2 \rfloor$ (*overfull*).

Δ=4 is proven by Cranston & Rabern.



$C_3 \times 2P_1$

Our experiments (Δ=5):

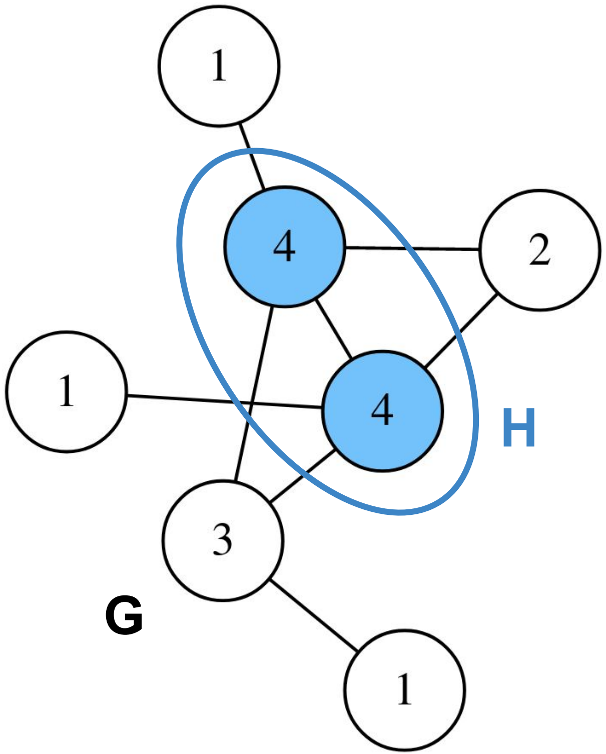


$C_4 \times 3P_1$

Bad Cores

Graph **H** is a **bad core** if ∃ G s.t.:

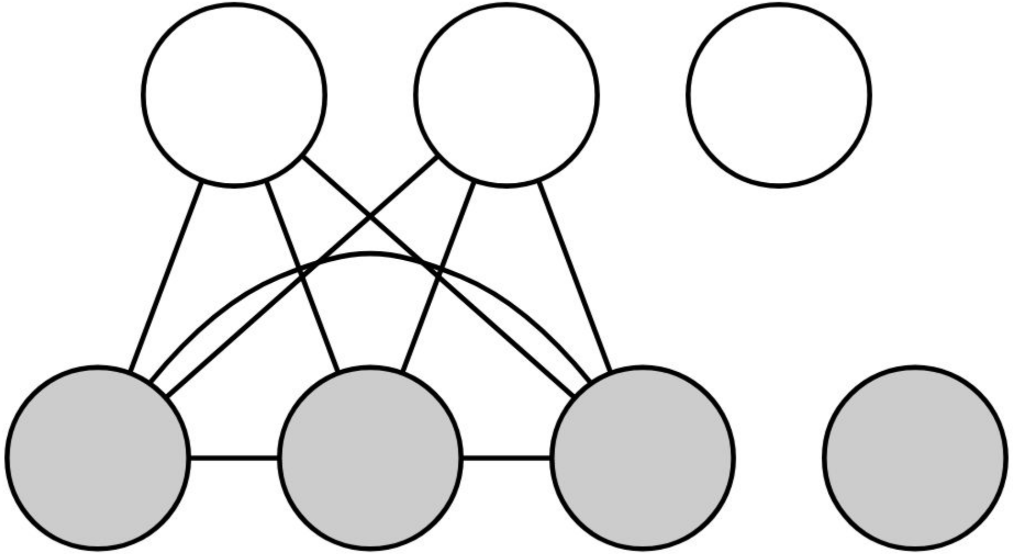
1. G's core is **H**,
2. $|E(G)| \leq \Delta \lfloor |V(G)|/2 \rfloor$ (*underfull*),
3. G is **Class 2**.



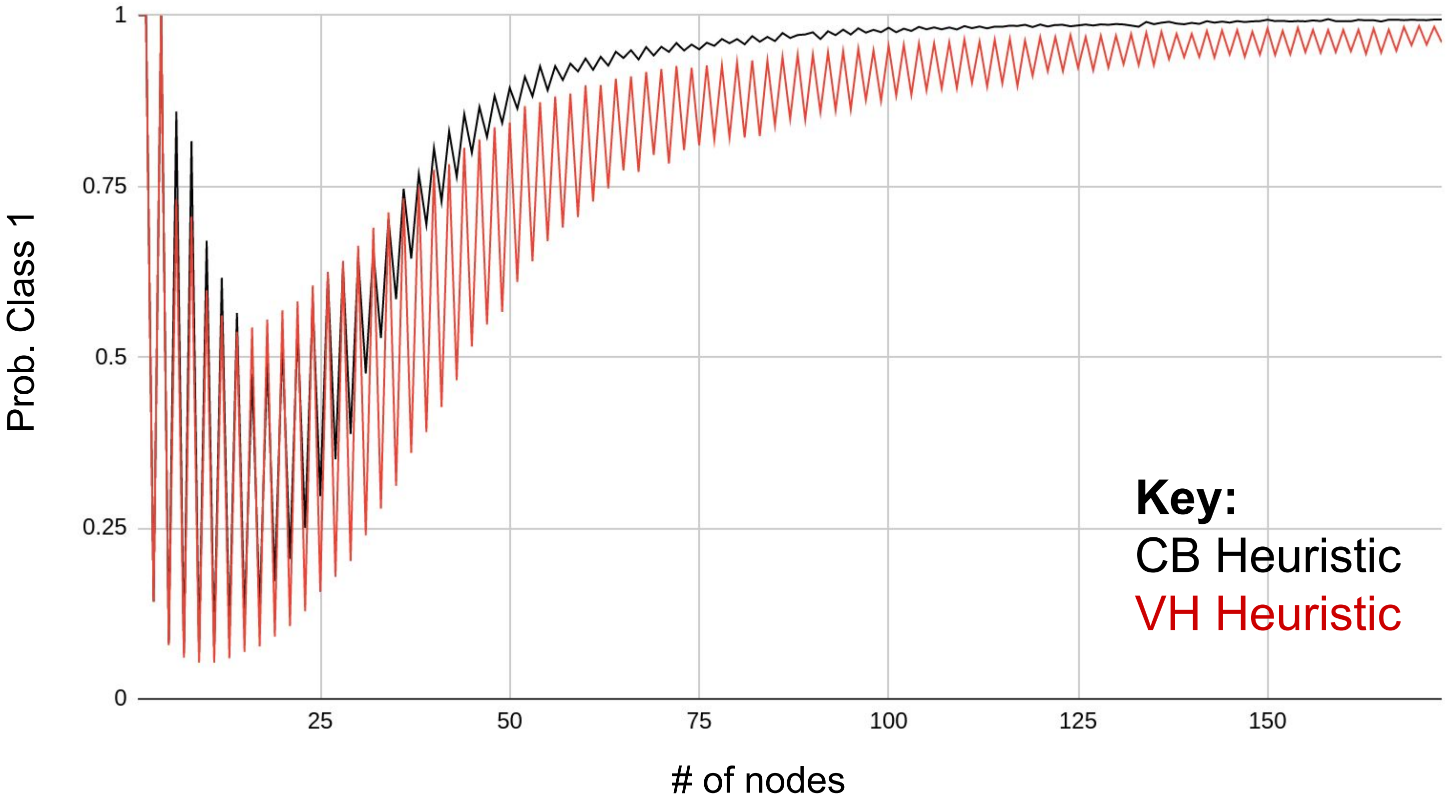
Bad Cores

One family:

- n=5, $C_3 \times 2P_1$
- n=7, $C_4 \times 3P_1$
- n=9, $C_5 \times 4P_1$
- …(?)



Counting Based Heuristic (Ehrenfeucht et al.)



Deliverables

Advanced

For Hilton Zhao conjecture and Bad Cores:

- **A.1:** Use Hamilton supercomputer to get more results. (~WIP)
- **A.2:** Deduce theoretical conjectures, and try to prove them. (~WIP)

Thanks for listening!