Edge Colouring

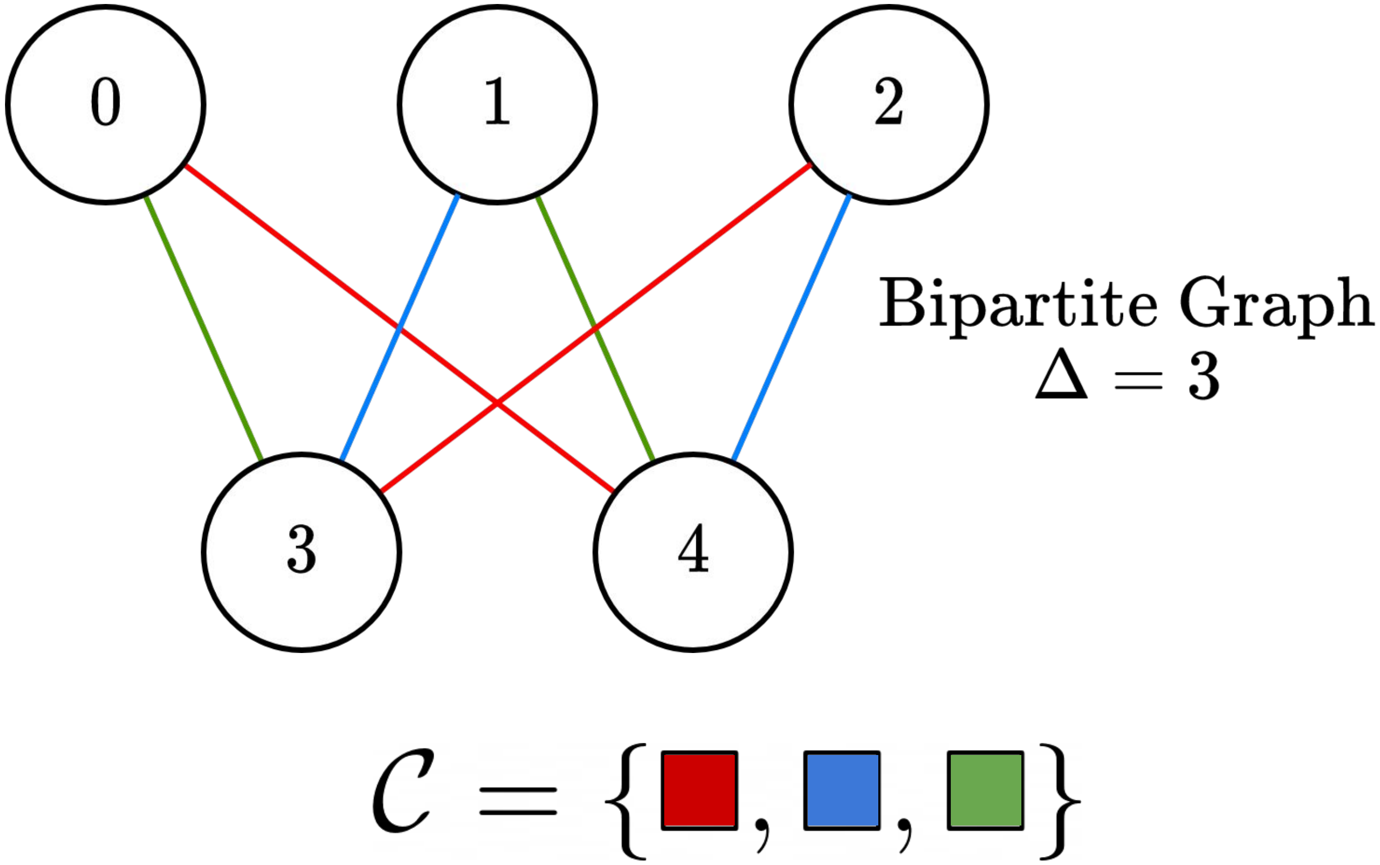
Eeo Jun

as small as possible

$$\varphi : E \mapsto \boxed{C}$$

$$\forall uv, ut \in E : \varphi(uv) \neq \varphi(ut)$$

(Adjacent edges should receive a different colour)


0
1
2
3
4
Bipartite Graph
$\Delta = 3$
$\mathcal{C} = \{\ ,\ ,\ \}$

Vizing's Theorem

$$\boxed{\Delta} \leq \underbrace{\chi'(G)}_{\text{Edge-Chromatic Index}} \leq \boxed{\Delta + 1}$$

Class 1

Class 2

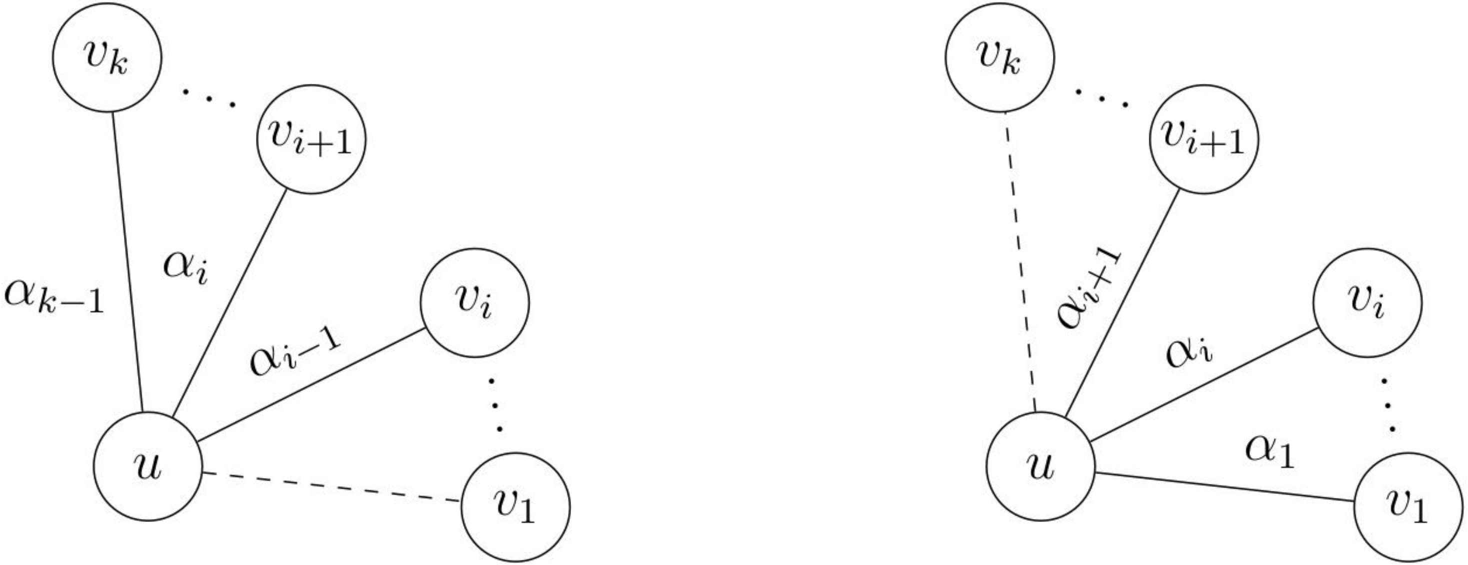
Deliverables

B.1 Implement MG Heuristic
B.2 Implement VH Heuristic
B.3 VH vs MG
B.4 Erdos & Wilson’s results

I.1 Investigate HZ Conjecture
I.2 Investigate Bad Cores Problem
I.3 Implement CB Heuristic

A.1 Use Hamilton
A.2 Derive Conjectures from **I.1**, **I.2**
A.3 Run on larger benchmark graphs

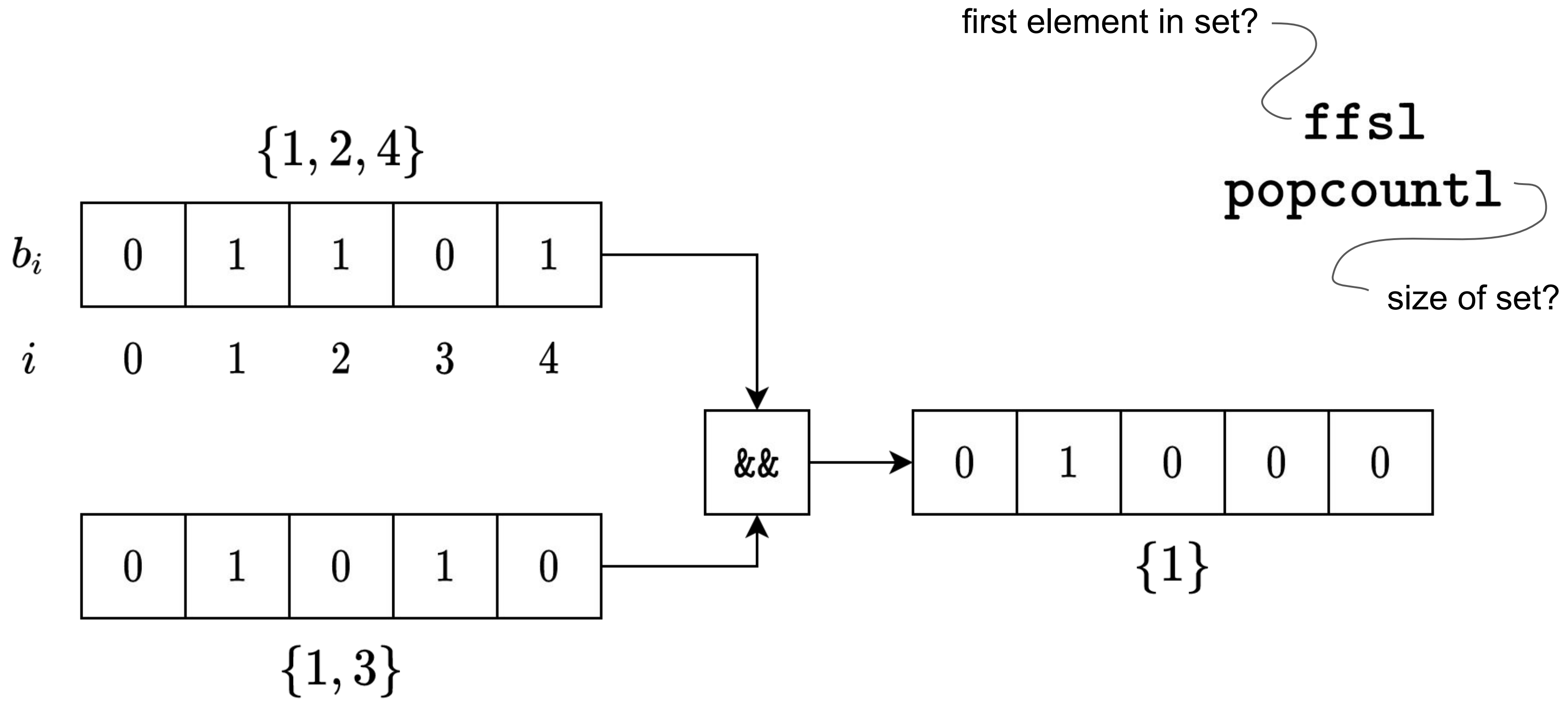
Misra & Gries Heuristic



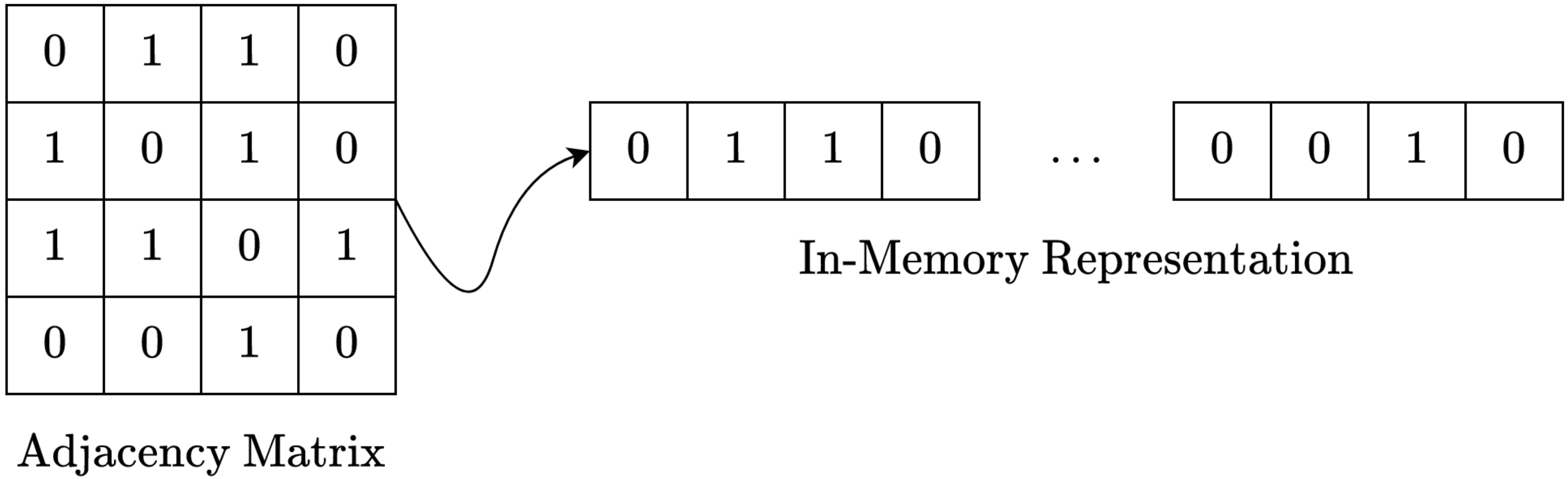
Vizing Heuristic

- Based on constructive proof in Gould 1998 (*Graph Theory*)
- **Idea:**
  - try to make an edge-colouring with only Δ colours
  - fail => use Δ+1

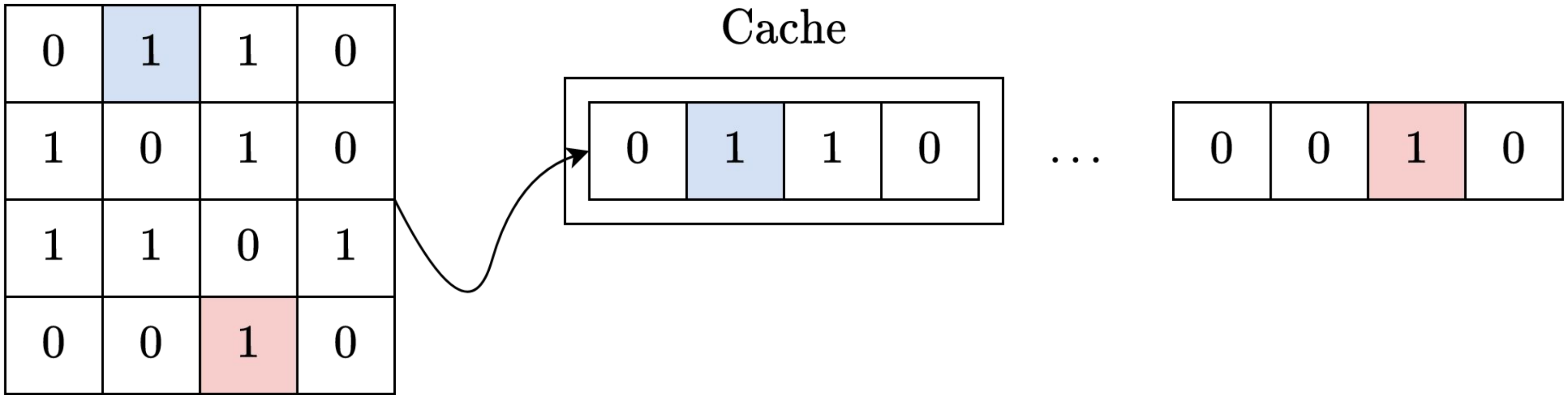
Bitsets


first element in set?
ffsl
popcountl
size of set?
{1, 2, 4}
b_i
0 1 1 0 1
i
0 1 2 3 4
&&
0 1 0 0 0
{1}
0 1 0 1 0
{1, 3}

Edge Sampling



Edge Sampling



VH vs MG

Benchmark	**Δ**	**ΔVh**	**MG**	**CB**	**Slowdown**
myciel3	5	5	**6**	5	1.56
myciel4	11	11	**12**	11	1.59
myciel5	23	23	23	23	7.28
myciel6	47	47	47	47	4.48
myciel7	95	95	95	95	6.51
le450_5a	42	42	42	42	4.16
le450_5b	42	42	43	42	3.94
le450_5c	66	66	**67**	66	6.43
le450_5d	68	68	**69**	68	5.97

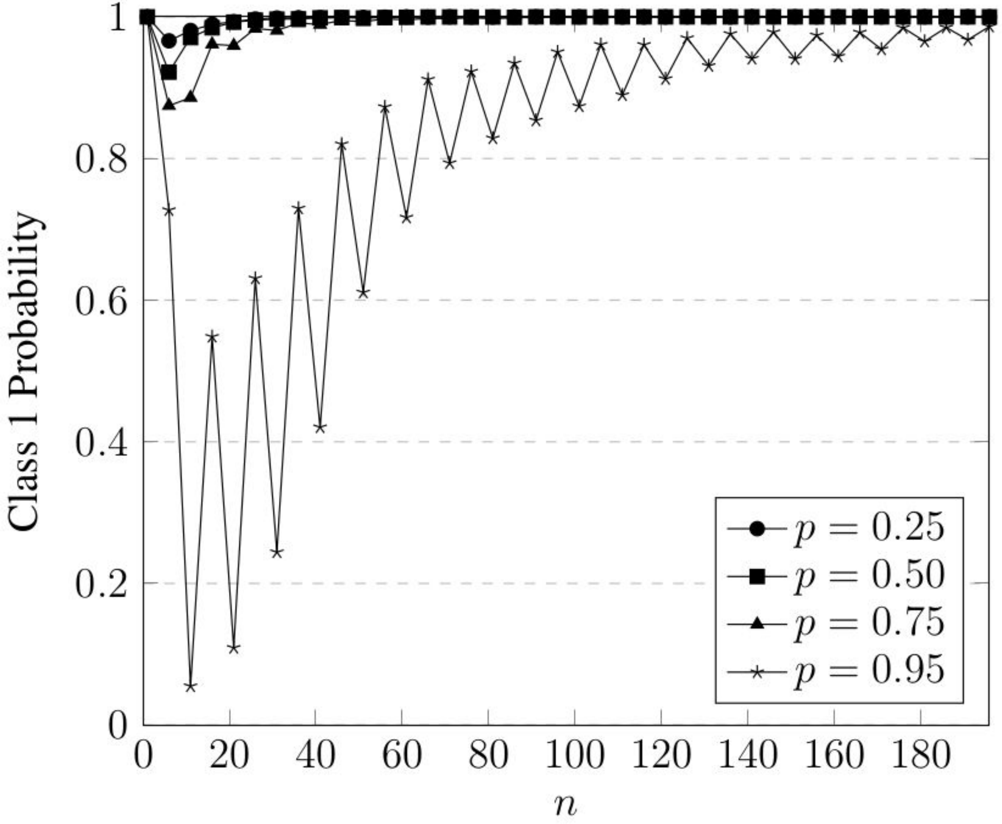
...

We got the same results!

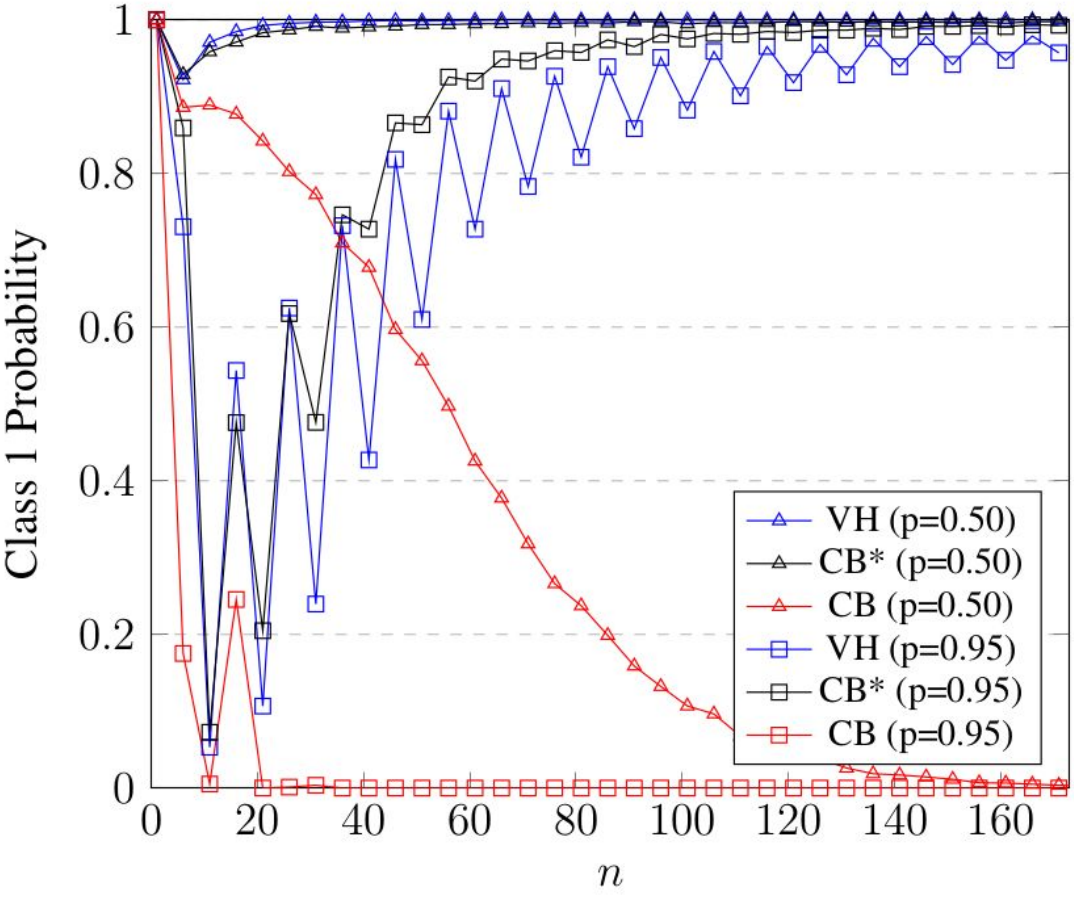
Erdos & Renyi Model

$$G \sim \mathcal{G}(n, p)$$

- Graph has **n** nodes
- P(*uv* ∈ E) = **p**

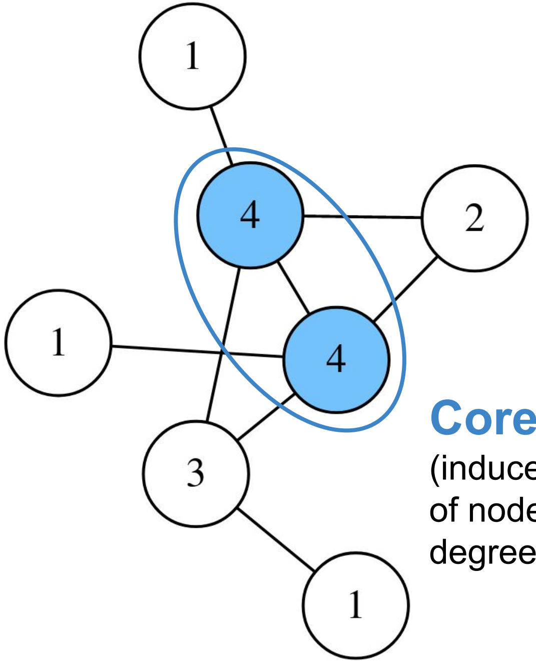


Implement Counting-Based Heuristic



Run on Large Benchmark Graphs

Hilton-Zhao Conjecture



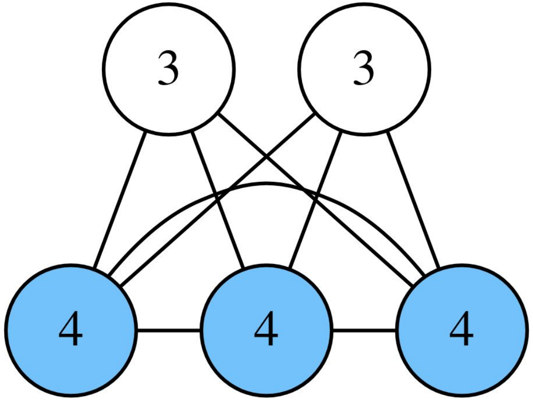
Core
(induced subgraph of nodes with max degree)

Any graph G with max degree Δ >= 4 and a core of degree 2 is Class 2 iff G satisfies:

$$|E| > \lfloor \frac{|V|}{2} \rfloor \Delta$$

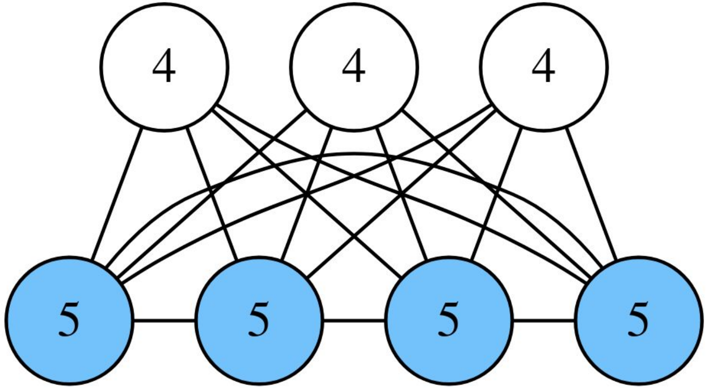
I.1 HZ conjecture
I.2 Bad cores
I.3 CB heuristic

Δ=4 case (Cranston & Rabern):



$C_3 \times 2P_1$

Δ=5, our experiments:



$C_4 \times 3P_1$

Δ			
6	$4P_1 \times C_5$	$C_4 \times C_3$	
7	$5P_1 \times C_6$ $5P_1 \times (C_3 + C_3)$	$C_5 \times C_4$	
8	$6P_1 \times C_7$ $6P_1 \times (C_3 + C_4)$	$C_6 \times C_5$ $(C_3 + C_3) \times C_5$	$C_3 \times H$

1 <= n <= 12: Brute force

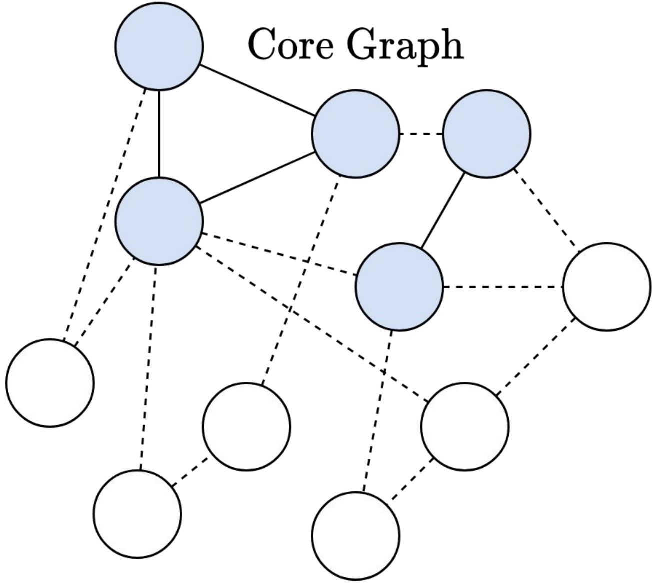
13 <= n <= 30:

Generate all core graphs + ExtendCore

Generate random degree sequences + graph realisation algorithm

n >= 30:

Generate random degree sequences + graph realisation algorithm

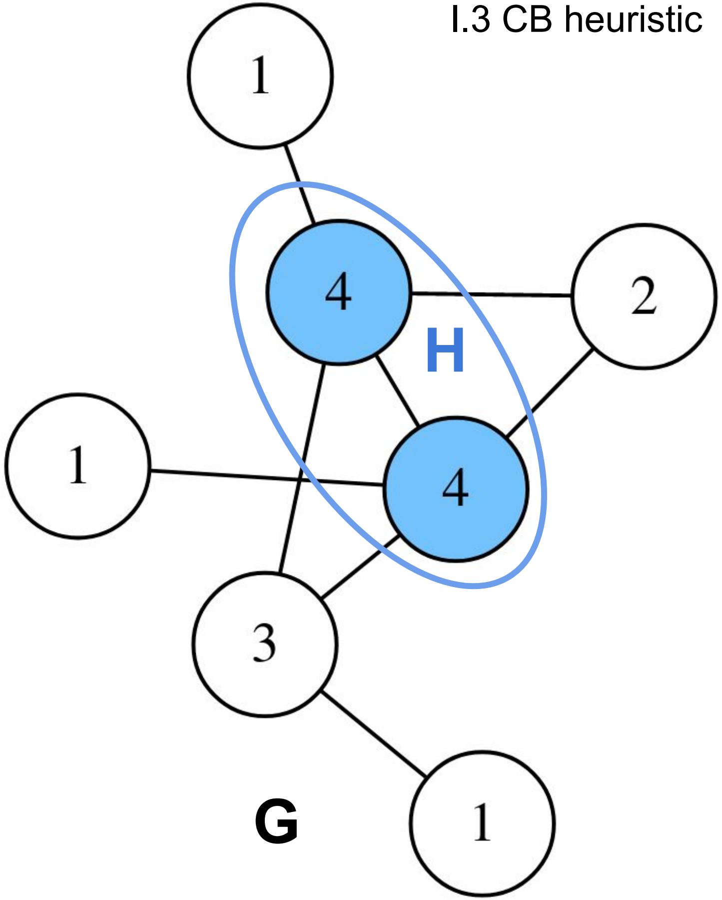


Bad Cores Problem

Graph **H** is a bad core if ∃ G s.t.:

1. G's core is **H**,
2. G is Class 2, and
3. G satisfies (*underfull*):

$$|E| \leq \lfloor \frac{|V|}{2} \rfloor \Delta$$



1 <= n <= 12: Brute force

13 <= n <= 30: Test random graphs from G(n,p) (p ∈ {0.25, 0.5, 0.75, 0.85})

For K_n (even n, n >= 4):

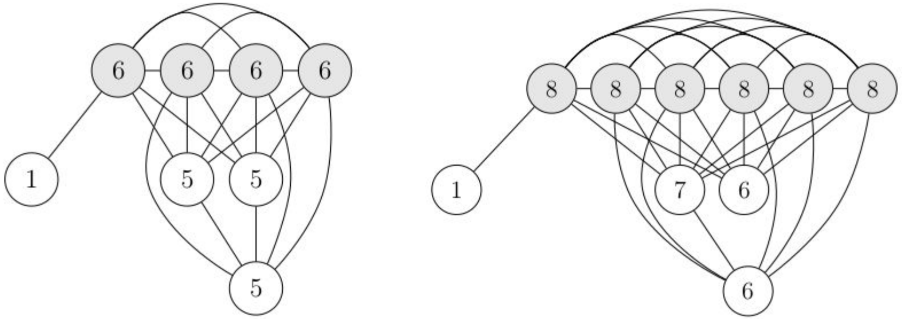


Figure 7: Extensions and cores (in grey) for K_4 and K_6, respectively.

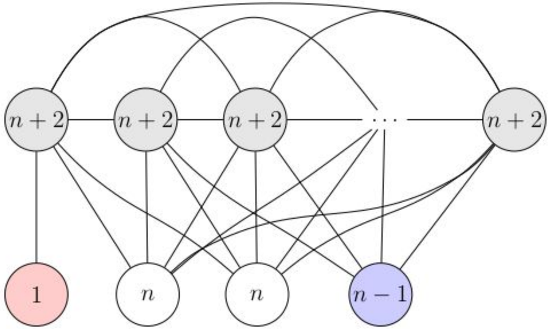


Figure 8: Extension and core for K_n (even $n \geq 8$)

For K_n (odd n, n >= 5)

$$K_5 : (1, 6^4, 7^5)$$

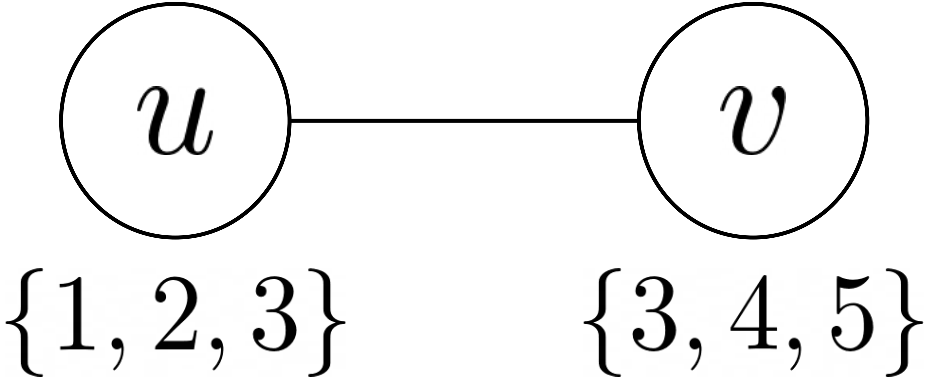
$$K_n : (1, n^2, (n+1)^2, (n+2)^n)$$

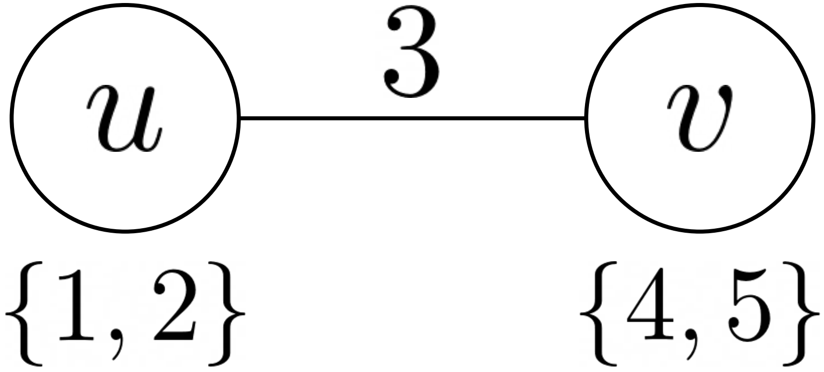
$$(n-1)P_1 \times C_n \quad (n \geq 3)$$

Use Hamilton

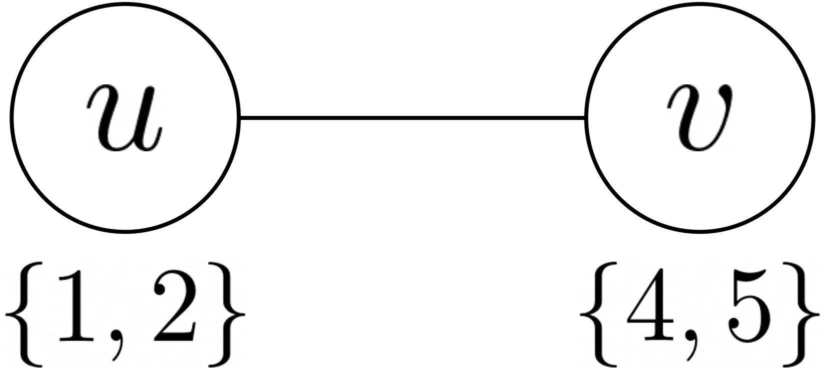
Extra: Heuristic Techniques

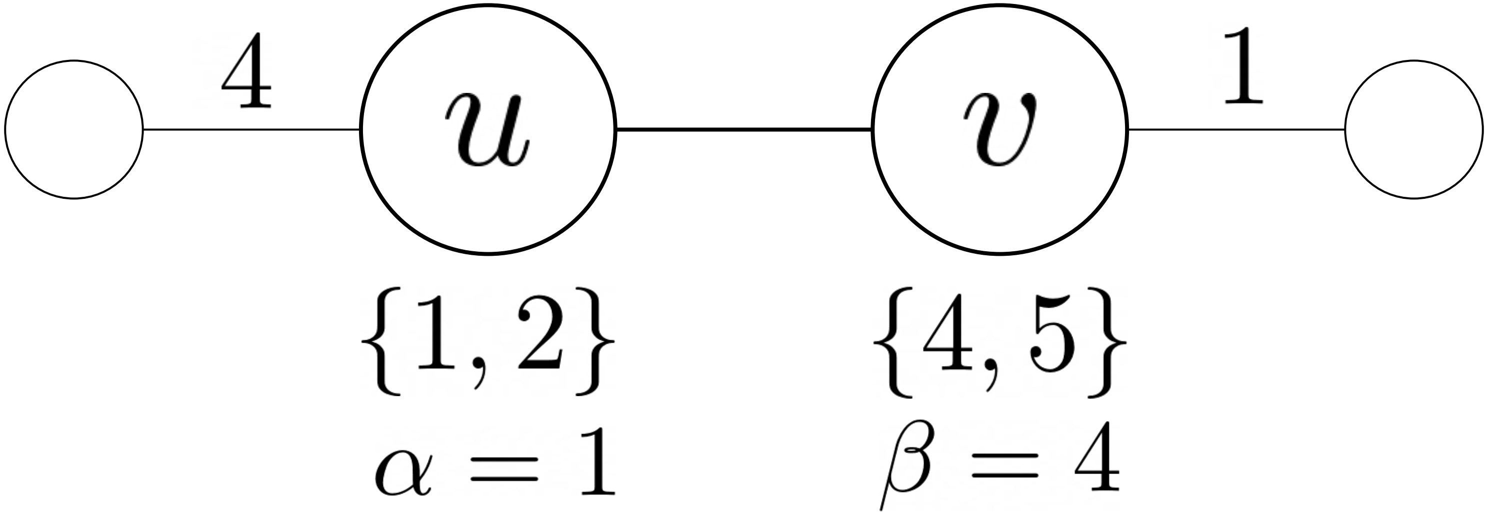
Simple Case

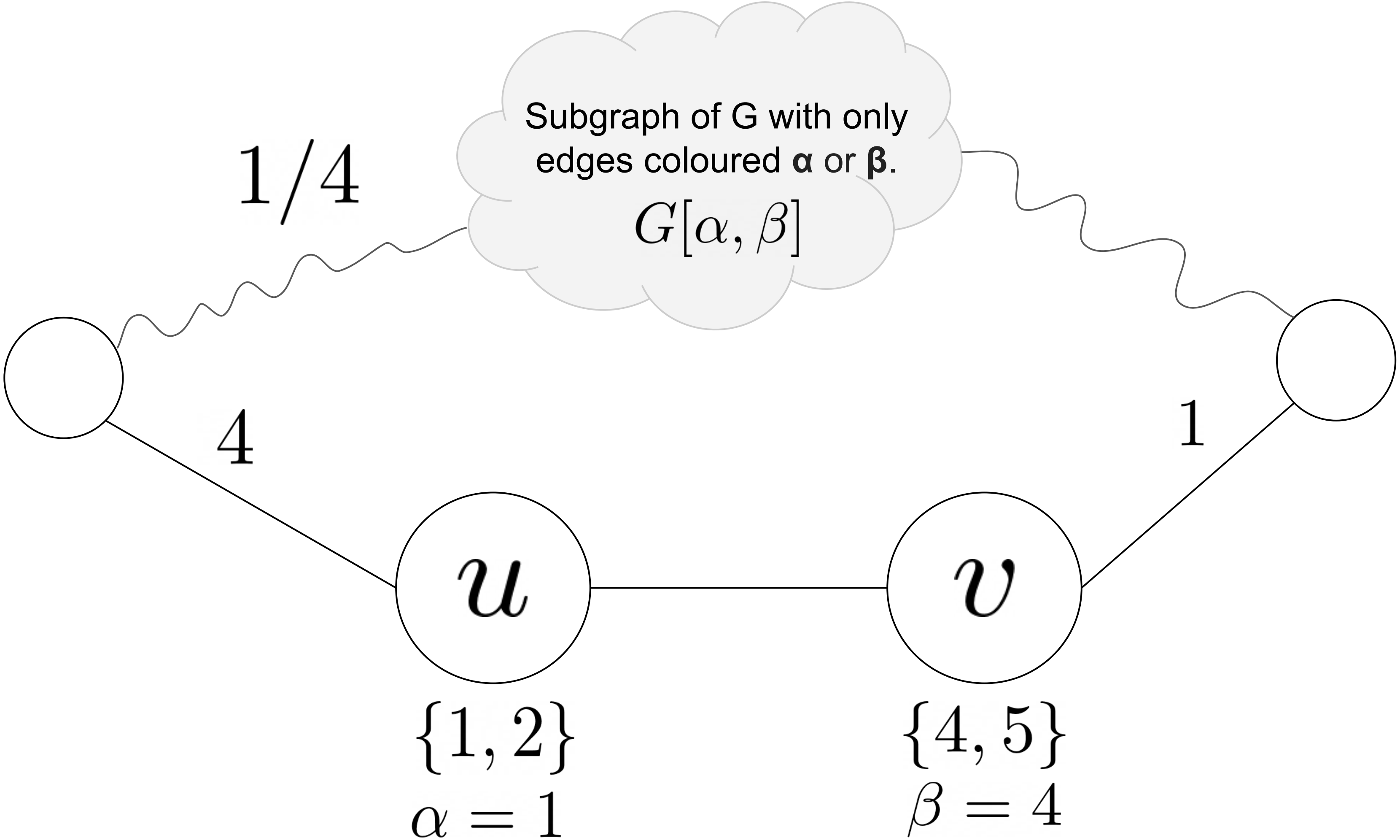


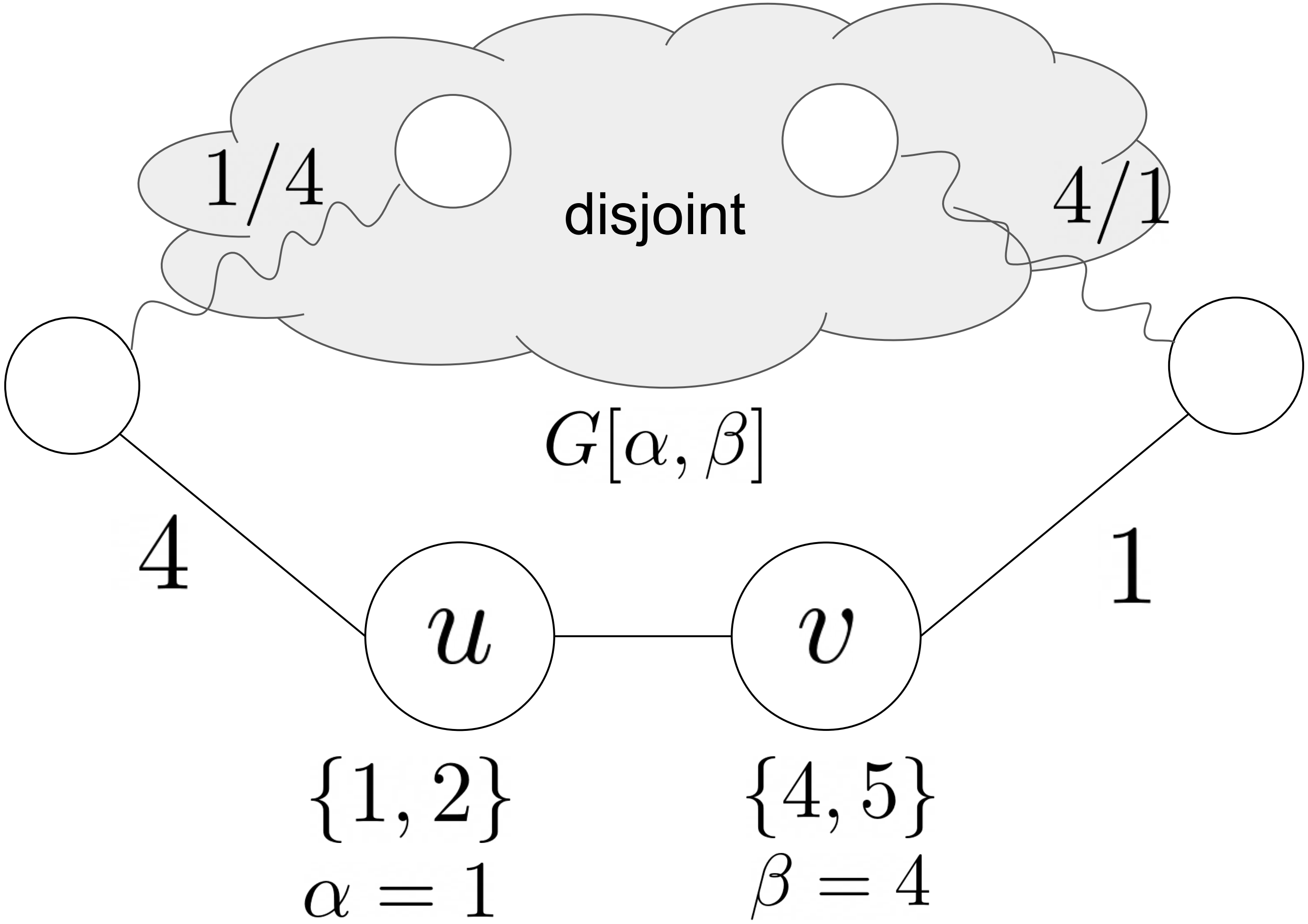

3
u
v
{1, 2}
{4, 5}

Harder Case

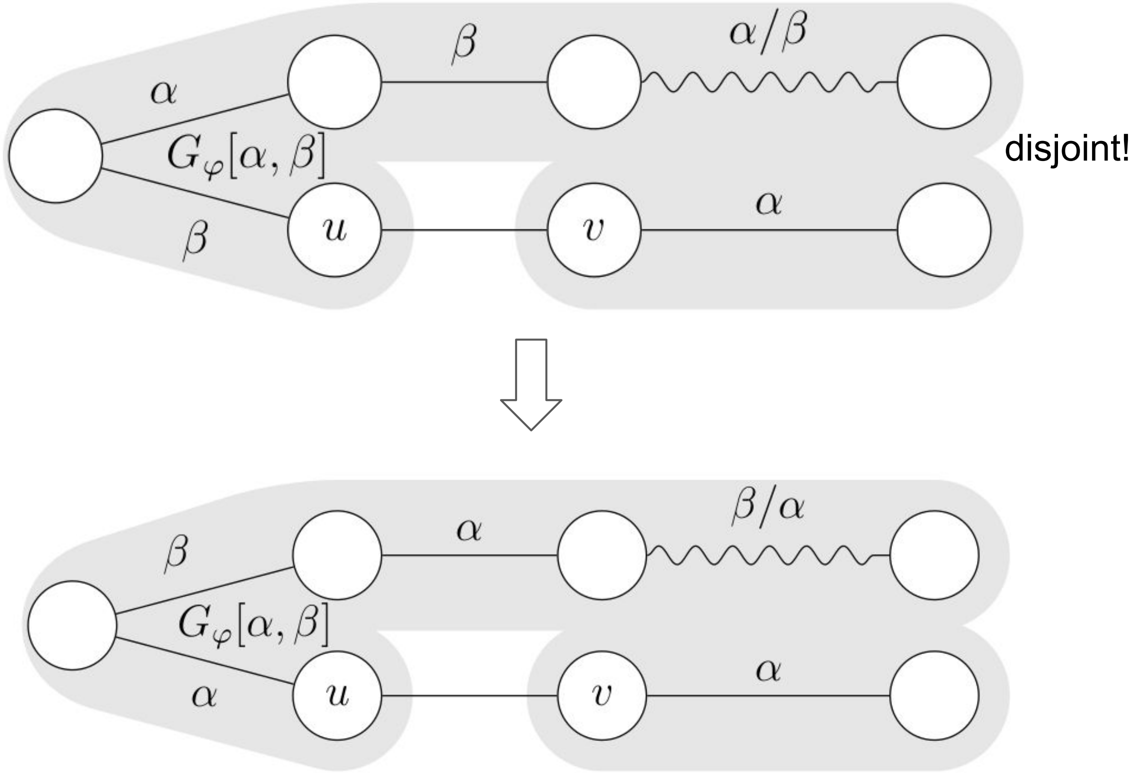


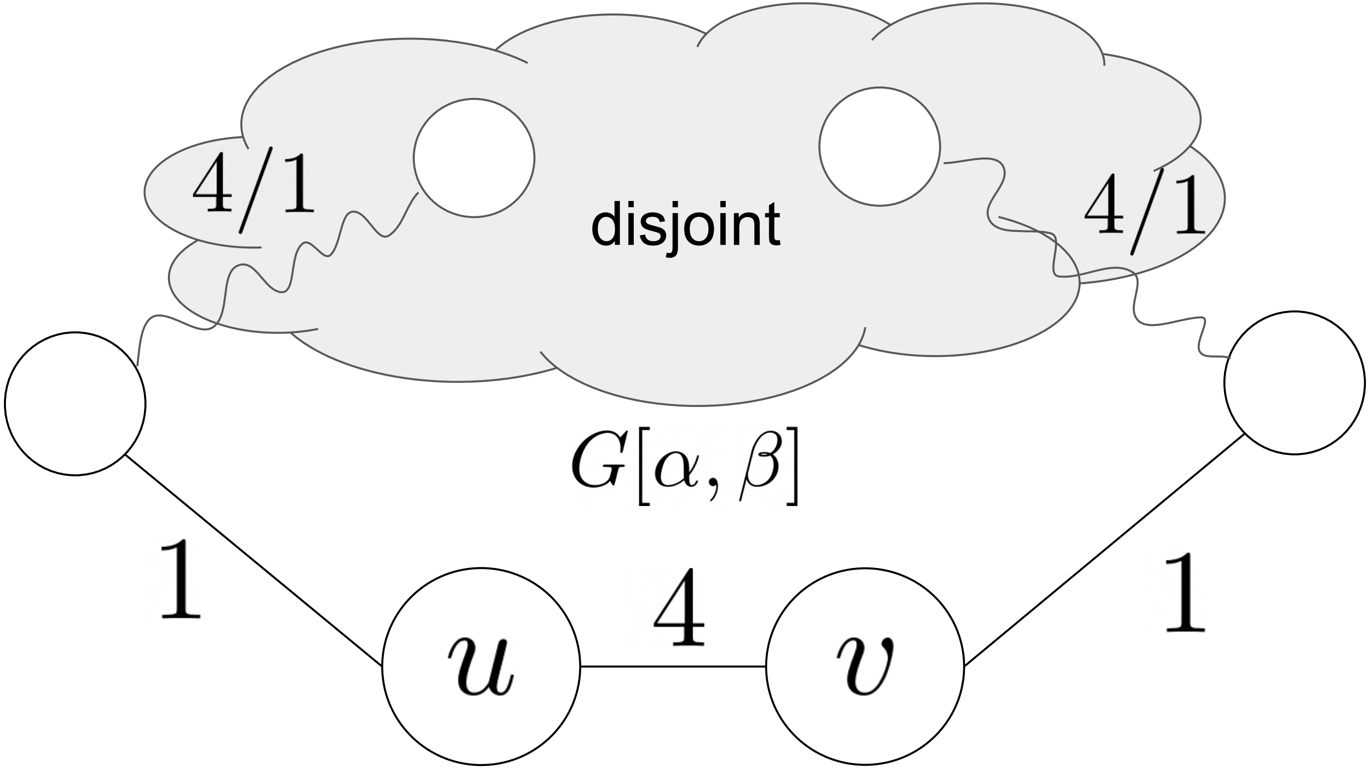

4
u
v
1
{1, 2}
α = 1
{4, 5}
β = 4

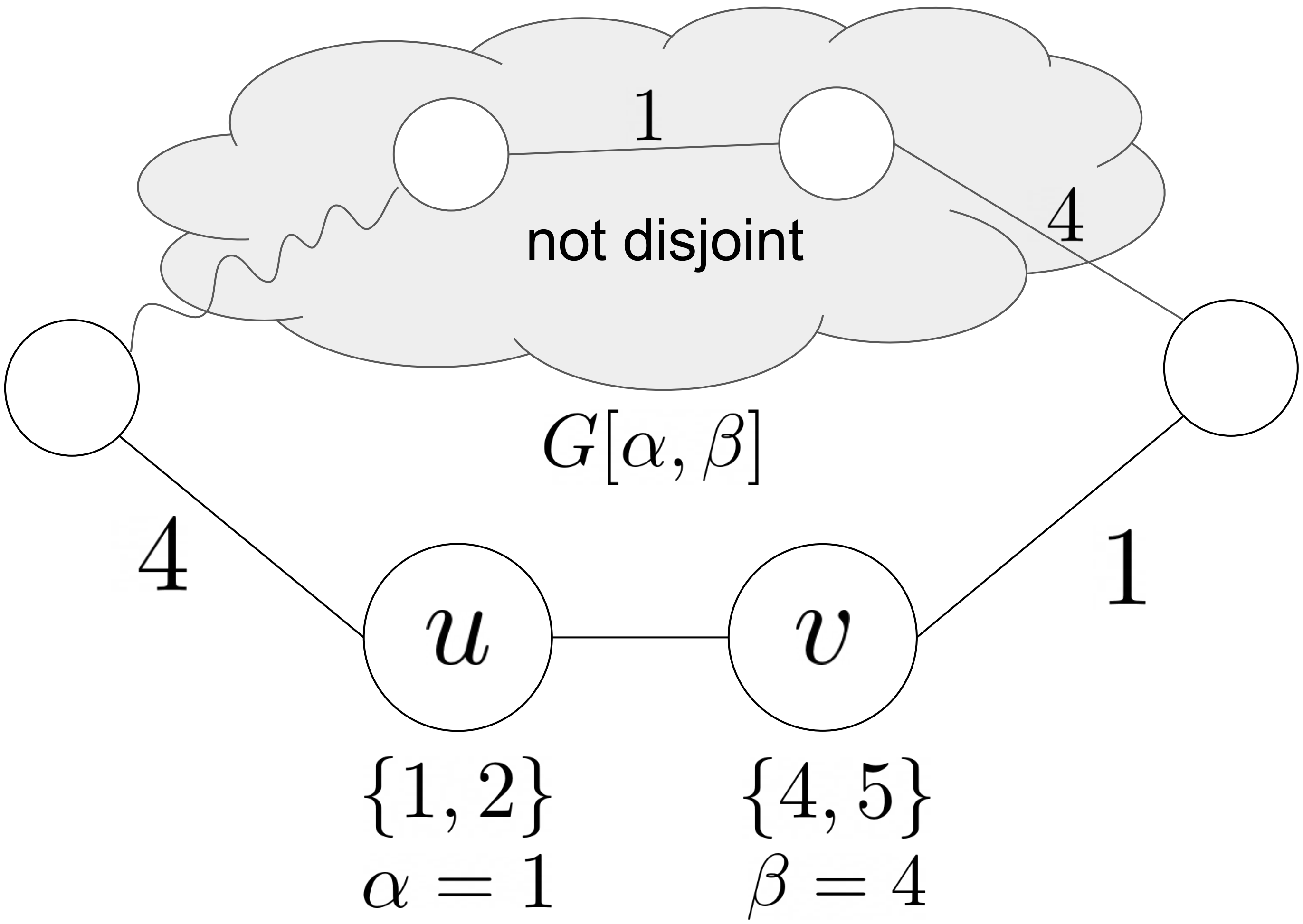

Subgraph of G with only edges coloured **α** or **β**.
$G[\alpha, \beta]$
$1/4$
4
1
u
v
$\{1, 2\}$
$\alpha = 1$
$\{4, 5\}$
$\beta = 4$

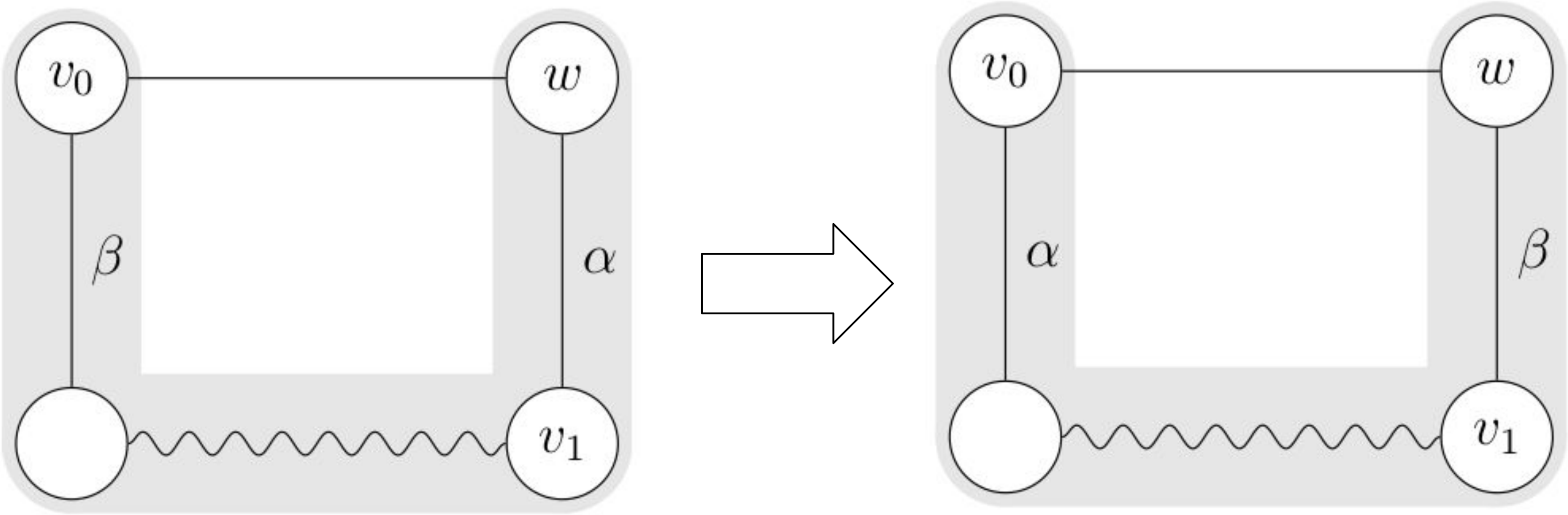

1/4
disjoint
4/1
$G[\alpha, \beta]$
4
1
u
v
$\{1, 2\}$
$\alpha = 1$
$\{4, 5\}$
$\beta = 4$

Kempe Exchange

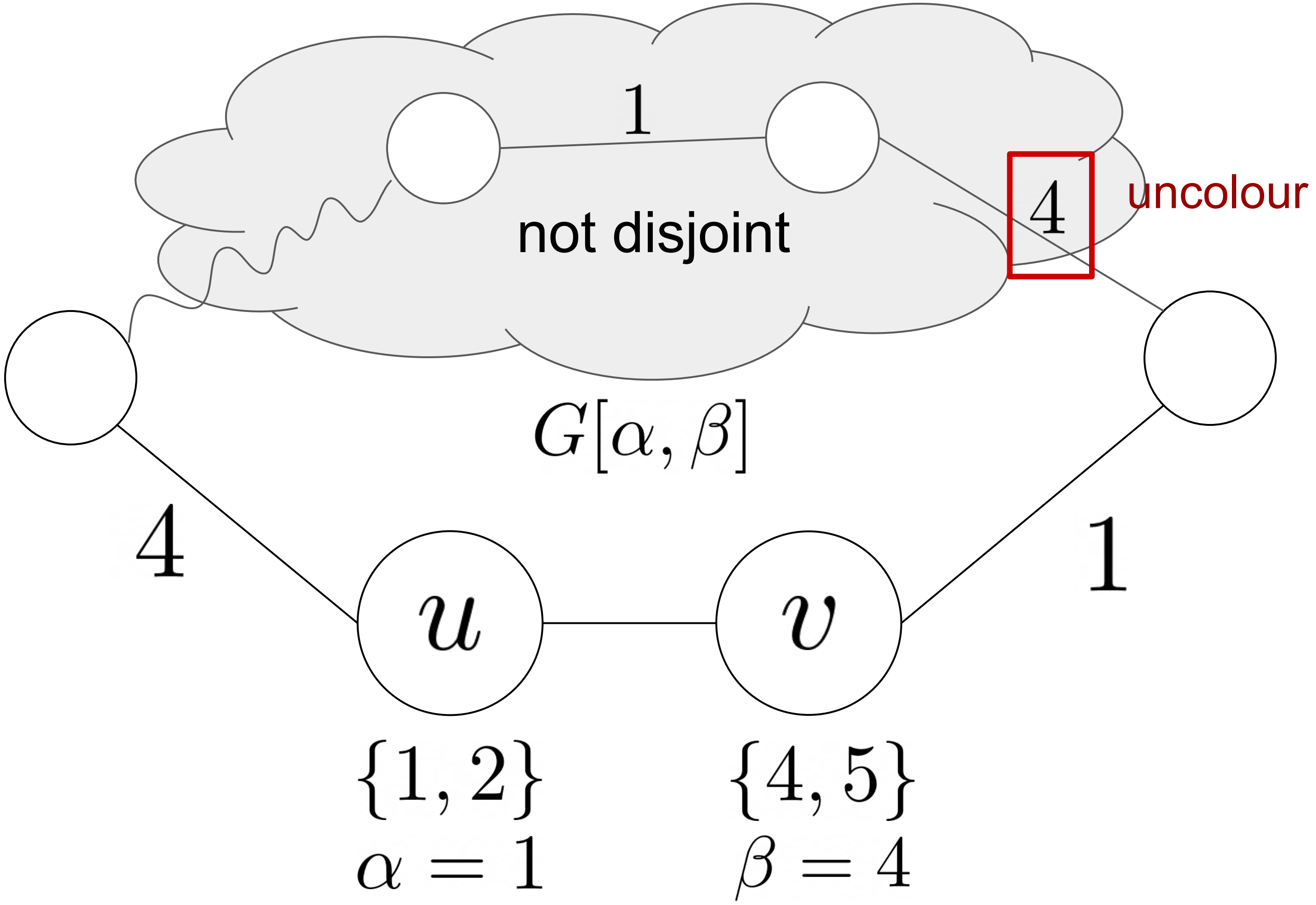



4/1
disjoint
4/1
$G[\alpha, \beta]$
1
u
4
v
1

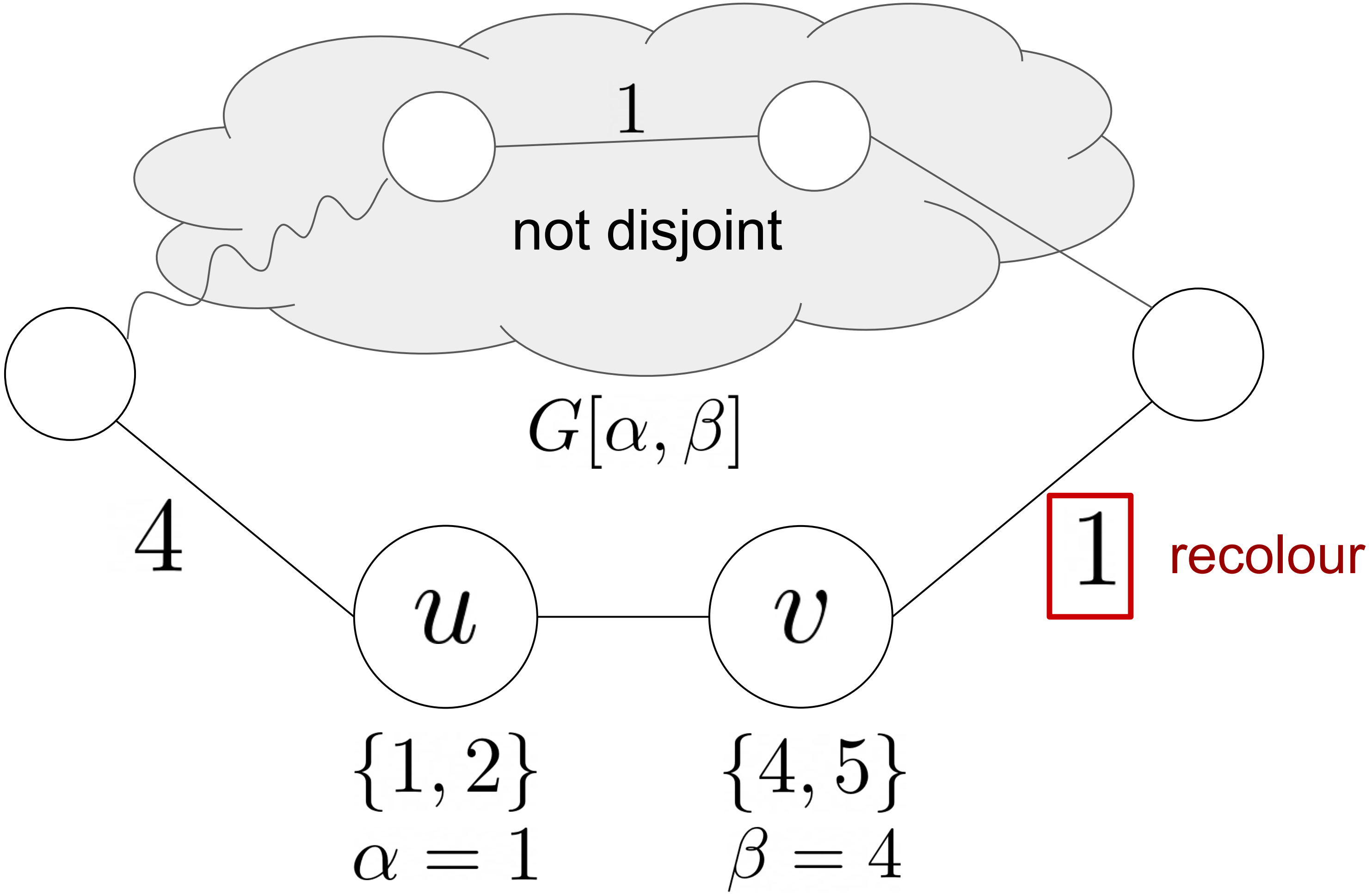

1
4
not disjoint
$G[\alpha, \beta]$
4
1
u
v
$\{1, 2\}$
$\alpha = 1$
$\{4, 5\}$
$\beta = 4$

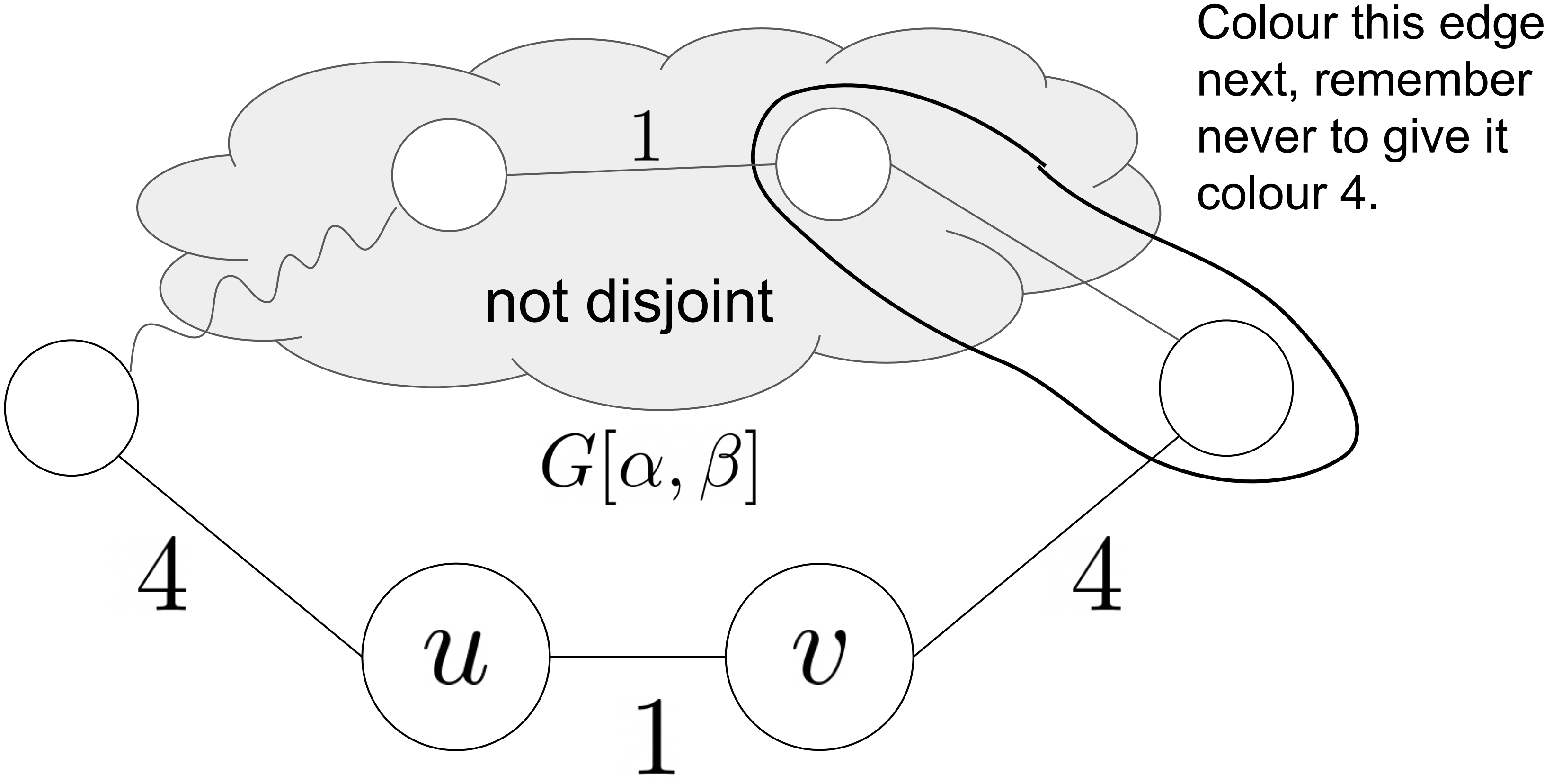


Not really helping...



1
4
uncolour
not disjoint
$G[\alpha, \beta]$
4
1
u
v
$\{1,2\}$
$\alpha = 1$
$\{4,5\}$
$\beta = 4$


1
not disjoint
$G[\alpha, \beta]$
4
u
v
1
recolour
$\{1, 2\}$
$\alpha = 1$
$\{4, 5\}$
$\beta = 4$



Colour this edge next, remember never to give it colour 4.
1
not disjoint
$G[\alpha, \beta]$
4
4
u
v
1