Chapter 2

Eugene Kharitonov

May 16, 2021

Exercise 2.1

Consider an arrow trajectory line. If a there is a tree center less than R to the left of it, then the arrow would hit it. Similarly, if there is a tree center within R to the right, it will be hit. Hence, the (one-dimensional) area of the cross-section would be $2 \cdot R$. When the volume swept by the cross-section becomes of the order sufficient to fit one tree, a hit becomes likely to happen.

$$\Sigma \cdot (2R) \cdot l \approx 1 \Longrightarrow l = \frac{1}{2 \cdot \Sigma \cdot R} \tag{1.1}$$

Exercise 2.2

As above, but cross-section becomes πR^2.

$$S \cdot l \cdot n_* \approx 1 \Longrightarrow l = \frac{1}{n_* S} = \frac{1}{n_* \pi R^2} \tag{2.1}$$

Exercise 2.3

(it is a bit shady, but the previous exercises hint that this is the expected way of reasoning)

Assuming photon density of n, in time t a spherical person of radius R would "sweep" $N = n \cdot \pi R^2 \cdot ct$ photons, each carrying energy E. As a result

$$w = \frac{d}{dt} E = \frac{d}{dt}\left(\pi c R^2 n E t\right) = \pi R^2 c n E \tag{3.1}$$

Exercise 2.5

We are given that

$$\frac{dE}{dr} = -KE \Longrightarrow E = E_0 e^{-Kr} \Longrightarrow \frac{E}{E_0} = e^{Kr} \tag{4.1}$$

At the same time, we can express redshift z via the emitted f_e and observed f_o frequencies:

$$z = \frac{f_e}{f_o} - 1 = \frac{hf_e}{hf_o} - 1 = \frac{E}{E_o} - 1, \tag{4.2}$$

where h is Plank constant. Putting (4.1)

$$z = \frac{E}{E_o} - 1 = e^{Kr} - 1 \Longrightarrow Kr = \ln(z+1) \approx z \quad \blacksquare \tag{4.3}$$

From the Hubble's law[1] we know that $z \approx rH_0/c$.. Comparing with (4.3) we obtain:

$$Kr \approx \frac{rH_0}{c} \Longrightarrow K = H_0 c^{-1} \tag{4.4}$$

[1]While it is "known", we can derive it starting from $v = H_0 r$: $\frac{f_0}{f} - 1 = \frac{1}{1-\Delta v/c} - 1 \approx 1 + \Delta v/c - 1$