Chapter 3

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Exercise 3.2

$$ds^2 = dr^2 + R^2 \sin^2(r/R) d\theta^2$$

as $dr = 0$ (assuming that "object" being at r from us implies that all its extremities are at r from us) we get

$$ds^2 = R^2 \sin^2(r/R) d\theta^2 \implies d\theta = \frac{ds}{R\sin(r/R)}$$

Exercise 3.3

Note that r is "spherical" distance. Let us put a radius from the center of the Earth to the center of the circle; the circle is visible from the center of Earth at angle α. Then the length of the arc L can be found as $L = \alpha R = 2r$. The length of the horde would be $2r' = 2R\sin(\alpha/2)$, where r' is the Euclidean radius of the circle.

The circumference of the circle equates to $2\pi r'$. Putting in our equations from above, we get: $2\pi r' = 2\pi R \sin(\alpha/2) = 2\pi R \sin(r/R)$.

Exercise 3.4

1. We can have an arbitrary large area on a plane;
2. $A = R^2(3\alpha - \pi)$ on a sphere, hence $2\pi R^2$ seems to be the maximum;[1]
3. $A = R^2(\pi - 3\alpha)$, hence the area is upper bounded by πR^2.

Exercise 3.5

$$\begin{aligned} x &= r\sin\theta\cos\phi, \\ y &= r\sin\theta\sin\phi, \\ z &= r\cos\theta, \end{aligned}$$

[1] Also see `https://blog.richmond.edu/physicsbunn/2014/02/01/spherical-triangles/`

hence

$$\begin{aligned}
dx &= dr \sin\theta \cos\phi + d\theta r \cos\phi \cos\theta - d\phi r \sin\phi \sin\theta \\
dy &= dr \sin\theta \sin\phi + d\theta r \cos\theta \sin\phi + d\phi r \sin\theta \cos\phi, \\
dz &= dr \cos\theta - d\theta r \sin\theta,
\end{aligned}$$

putting that into $ds^2 = dx^2 + dy^2 + dz^2$ and after a half-page of wresting trigonometry, we get the needed expression.