

# Characterizing subsurface gas through Bayesian inversion of a seismic attenuation model

Eugene Morgan

[eugene.morgan@psu.edu](mailto:eugene.morgan@psu.edu)

December 5, 2014

JOHN AND WILLIE LEONE FAMILY  
DEPARTMENT OF ENERGY AND MINERAL ENGINEERING

---

PETROLEUM AND NATURAL GAS ENGINEERING

---

COLLEGE OF EARTH AND MINERAL SCIENCES

# Table of Contents

## ① Introduction

Motivation

Attenuation

## ② Measuring Attenuation

Spectral Ratio Method

Finneid fjord

## ③ Hierarchical Model

Method

Finneid fjord

## ④ Bayesian Updating

CO<sub>2</sub> release experiment

Bayesian Updating

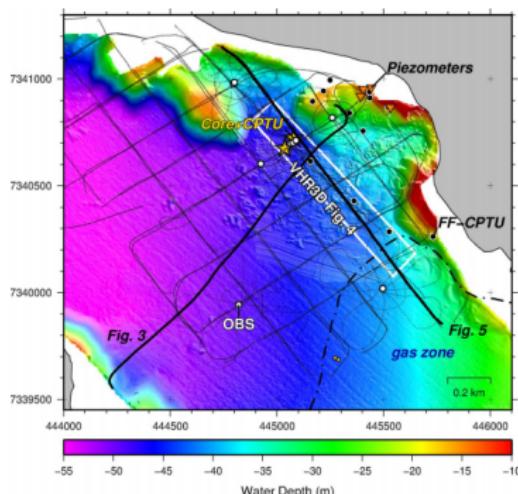
## ⑤ Genetic Algorithm

Method

Blake Ridge

Finneid fjord

## ⑥ Conclusions



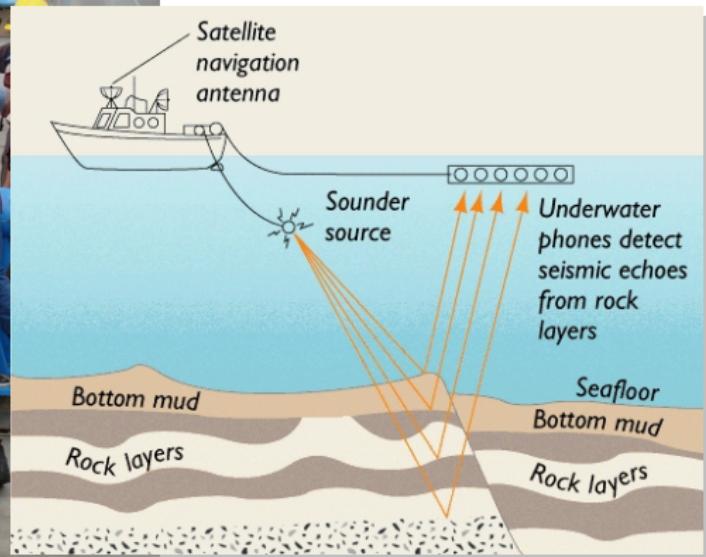
Maarten Vanneste, NGI

Mark Vardy, Southampton

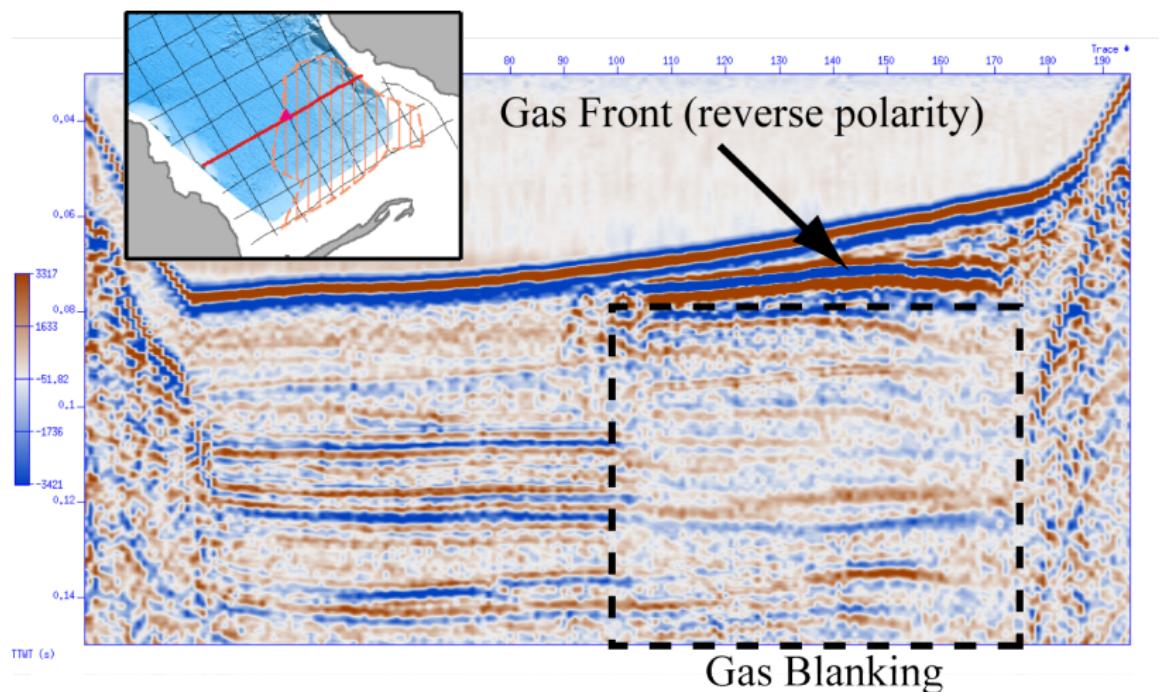
# Motivation



# Motivation



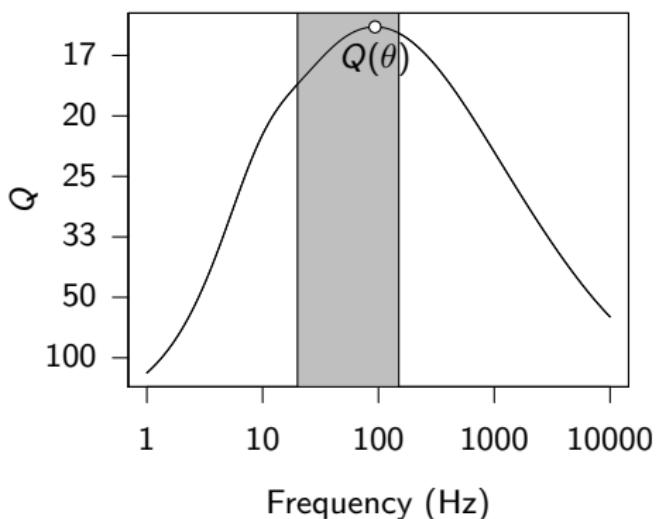
# Attenuation of seismic reflection signals



In: Morgan et al. (2010).

# The P-wave attenuation model of Carcione and Picotti (2006)

13 parameters in set  $\theta$  define the attenuation curve (left):

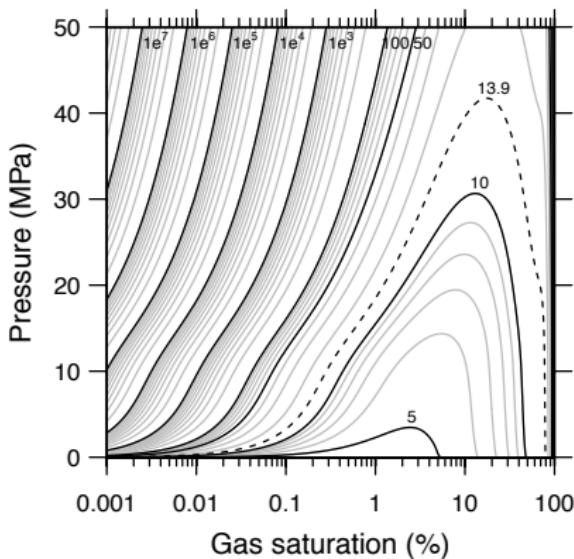
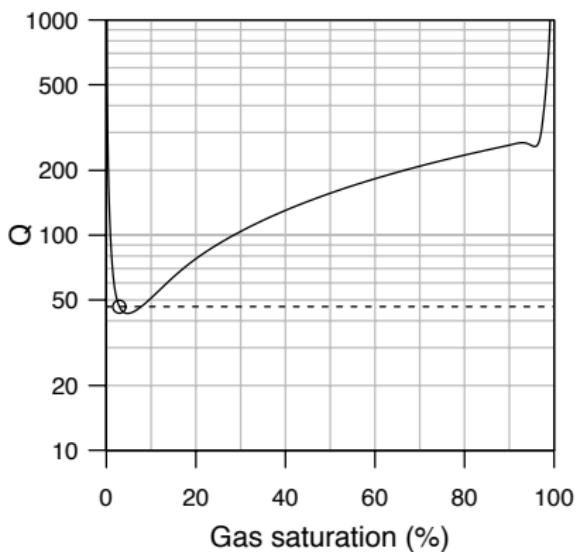


- 1  $\phi$ : soil porosity
- 2  $\kappa$ : soil permeability
- 3  $K_s$ : solid grain bulk moduli
- 4  $\mu_s$ : solid grain shear moduli
- 5  $\rho_s$ : solid grain density
- 6  $K_w$ : water bulk moduli
- 7  $\rho_w$ : water density
- 8  $\eta_w$ : viscosity of water
- 9  $d$ : total layer thickness
- 10  $S_g$ : gas saturation
- 11  $\eta_g$ : viscosity of gas
- 12  $P$ : total pressure
- 13  $T$ : temperature

I pick  $Q(\theta)$  as the minimum  $Q$  of the curve within the range of seismic frequencies.

# Model behavior

Attenuation model is nonlinear and multimodal



# Method overview

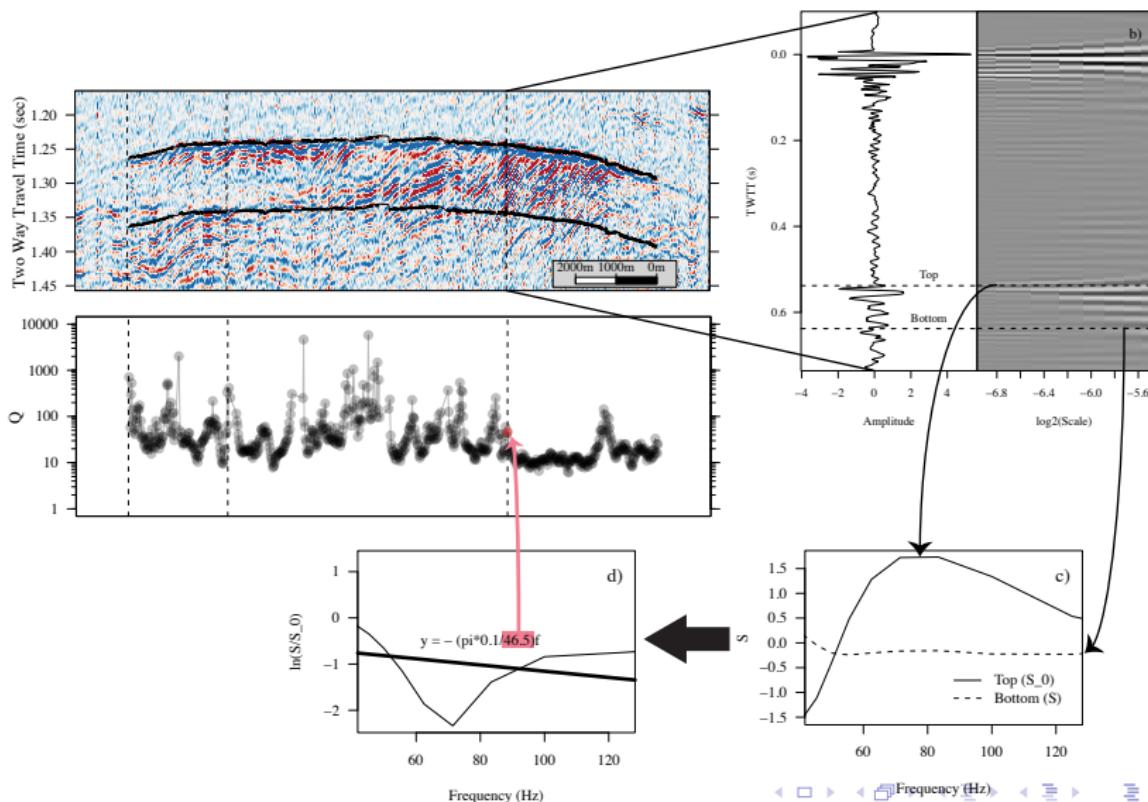
Our goal is to link the theoretical attenuation model to observations of attenuation from seismic data. This is a two step process:

- ① Measure quality factor ( $Q$ ) from seismic data
- ② Invert model: find set of parameters ( $\theta$ ) that match modeled  $Q(\theta)$  to measured  $\hat{Q}$ . Here, I explore two ways to do this:
  - ① Genetic algorithm (Morgan et al., 2012)
  - ② Bayesian hierarchical model (Morgan et al., 2014)

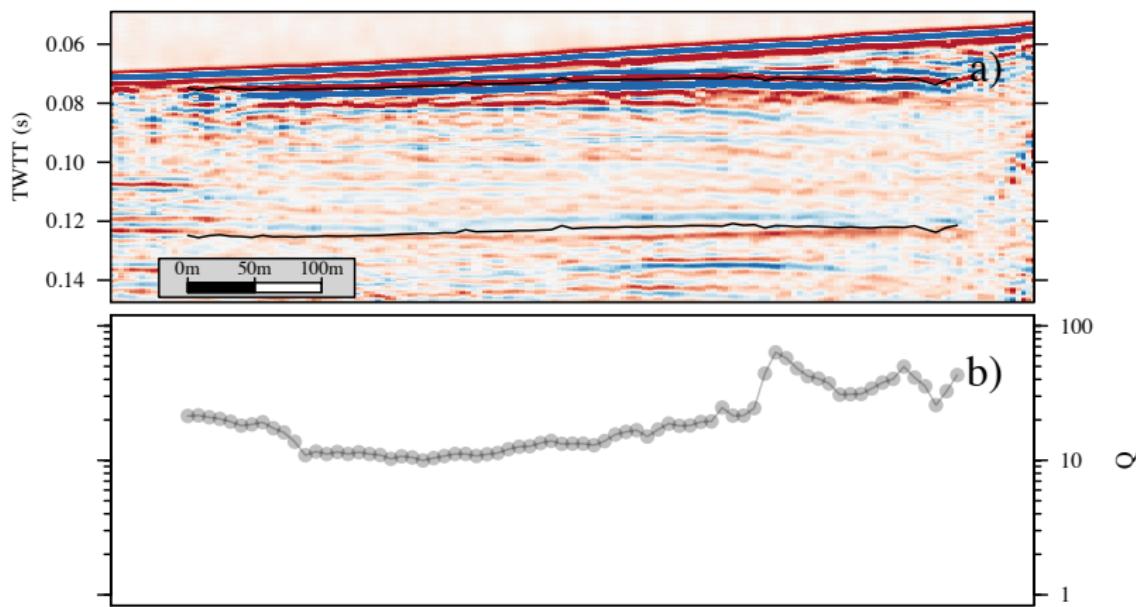
I validate the genetic algorithm method at the well-studied Blake Ridge (offshore the Carolinas) where free gas lies below gas hydrate. I test both inversion methods at Finneidfjord, Norway, where gas exists at a much shallower depth and in smaller concentration.

# Measuring attenuation

Measure quality factor  $Q$  from seismic data using spectral ratio method



# Finneidfjord



# Bayesian Hierarchical Model

With  $\hat{Q} = (\hat{Q}_1, \dots, \hat{Q}_m)^T$  measured at  $m$  locations (traces) with unknown parameter values  $\theta = (\theta_1, \dots, \theta_m)^T$ , where each  $\theta_i$  is a vector containing values for each of the 13 parameters,

$$\hat{Q} = Q(\theta) + e.$$

$Q(\theta)$  is the attenuation forward model ("simulator") and  $e$  contains both measurement error from  $\hat{Q}$  and model error from  $Q(\theta)$ . Then, our joint posterior is:

$$p(\theta|\hat{Q}) \propto L(\hat{Q}|Q(\theta)) \times \pi(\theta),$$

with likelihood coming from assuming  $e$  is Gaussian with zero mean:

$$L(\hat{Q}|Q(\theta)) \propto \exp\left(-\frac{1}{2\sigma^2}(\hat{Q} - Q(\theta))^T(\hat{Q} - Q(\theta))\right),$$

where  $\sigma$  is the standard deviation of  $\hat{Q}$ .

The prior contains the spatial smoothness constraint<sup>a</sup> that our random variables behave as a Markov random field over space:

$$\pi(\theta) \propto \exp \left( \beta \sum_{i \sim j} u(\theta_i - \theta_j) \right),$$

where the sum is over all sets of nearest neighbors ( $i \sim j$ ) and

$$u(d) = \begin{cases} \frac{1}{s}(1 - (d/s)^3)^3, & \text{if } -s < d < s \\ 0, & \text{if } |d| \geq s \end{cases}$$

is the tricube function. Pragmatically,  $\beta = 0.5$  and  $s = 0.3$  work well. The distance between vectors ( $d = \theta_i - \theta_j$ ) is found via Mahalanobis distance with the covariance matrix  $\Sigma_\theta$  set *a priori*.

---

<sup>a</sup>Higdon, D., Reese, C.S., Moulton, J.D., Vrugt, J.A., and Fox, C. 2008. Posterior exploration for computationally intensive forward models. In: Handbook of Markov Chain Monte Carlo. Boca Raton, Florida: CRC Press.

# MCMC: Single-site Metropolis Algorithm

Pseudo-code:

```
initialize  $\theta$ 
for  $k = 1 : niter$  do
    for  $i = 1 : m$  do
         $\theta'_i = \theta_i + z$ , where  $z \sim MVTN(0, \Sigma_\theta, a, b)$ 
        if  $u < \frac{p(\theta' | \hat{Q})}{p(\theta | \hat{Q})}$ , where  $u \sim U(0, 1)$  then
             $\theta_i \leftarrow \theta'_i$ 
        end if
    end for
end for
```

At Finneidfjord we have  $m = 83$  locations, set  $niter = 100,000$ , use burn-in of 10,000, and keep every 10<sup>th</sup>  $\theta_i$  after that. We get an acceptance rate of 49%.

# Sample Space

Initial guesses ( $\theta_1$ ) and bounds taken from literature or geometry of seismic data:

	Global (both sites)		
	Lower bound	Initial value	Upper bound
Gas saturation $S_g$ (%)	0	1	100
Porosity $\phi$	0.38 <sup>a</sup>	0.55	0.73 <sup>a</sup>
Permeability $\kappa$ (Darcy)	$10^{-8}$ <sup>a</sup>	$10^0$	$10^5$ <sup>a</sup>
Solid grain bulk modulus $K_s$ (GPa)	20 <sup>b</sup>	30	70 <sup>b</sup>
Solid grain shear modulus $\mu_s$ (GPa)	5 <sup>b</sup>	13	50 <sup>b</sup>
Solid grain density $\rho_s$ (g/cc)	2.55 <sup>b</sup>	2.65	2.71 <sup>b</sup>
Water bulk modulus $K_w$ (GPa)	2.00	2.25 <sup>c</sup>	2.50
Water density $\rho_w$ (g/cc)	1.000	1.025	1.030
Water viscosity $\eta_w$ (Pa·s)	0.001	0.003 <sup>c</sup>	0.005
Gas viscosity $\eta_g$ ( $10^{-4}$ Pa·s)	1.0	1.5 <sup>c</sup>	2.0
	Blake Ridge		
	Lower bound	Initial value	Upper bound
Total pressure $P$ (MPa)	~32.41	~32.93	~35.19
Temperature $T$ (°C)	11	12	16
Layer thickness $d$ (m)	70	75	80
	Finneidfjord		
	Lower bound	Initial value	Upper bound
Total pressure $P$ (MPa)	~0.55	~0.77	~2.93
Temperature $T$ (°C)	0	5	10
Layer thickness $d$ (m)	36.5	37.5	38.5

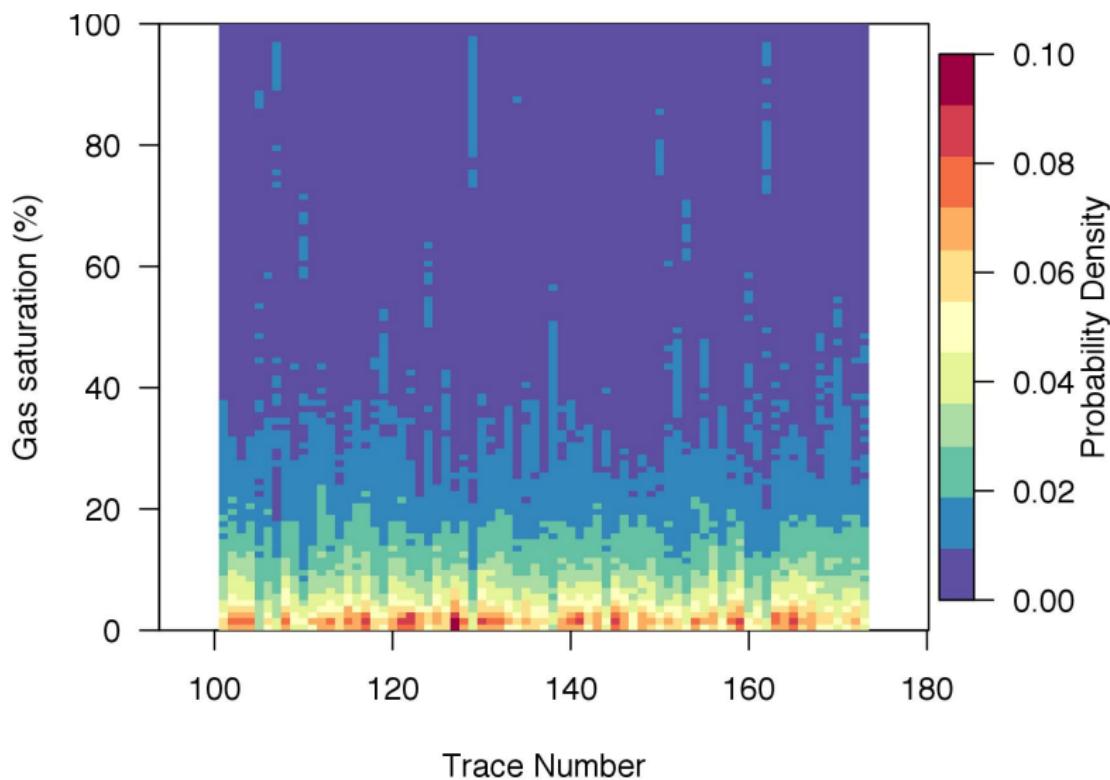
<sup>a</sup> Schön (1996)

<sup>b</sup> Mavko et al. (1998)

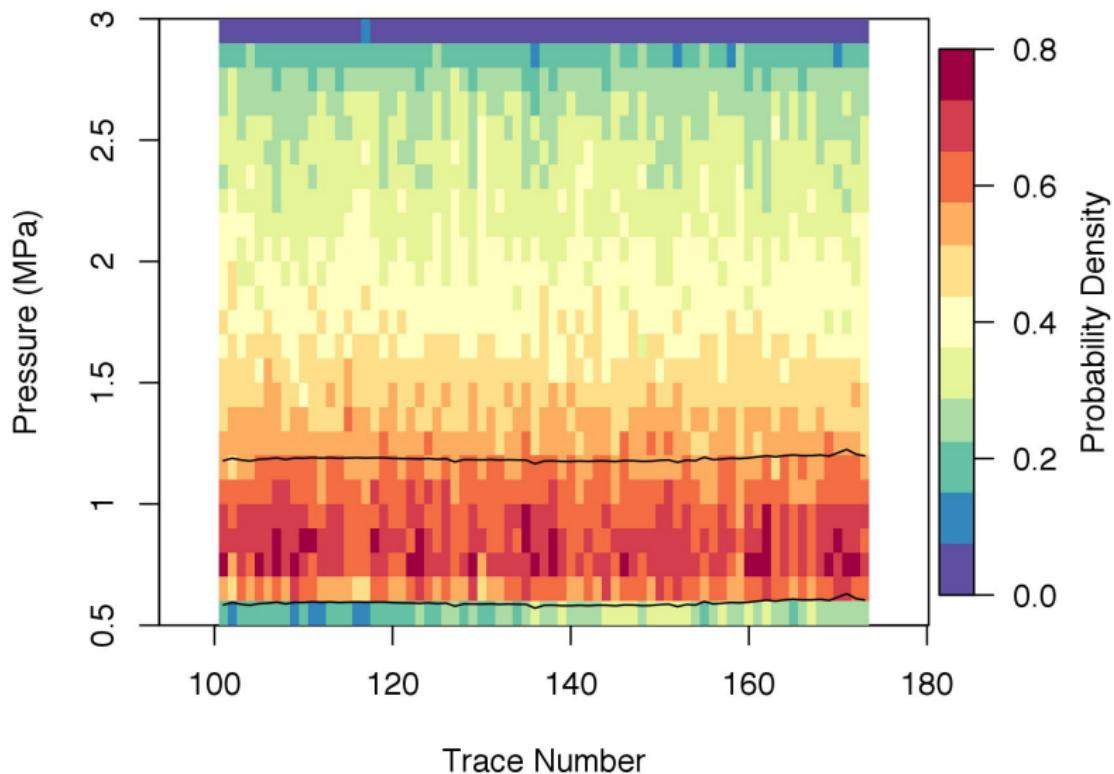
<sup>c</sup> Carcione and Picotti (2006)

Bounds represent likely or possible conditions, and make the inversion a constrained optimization problem.

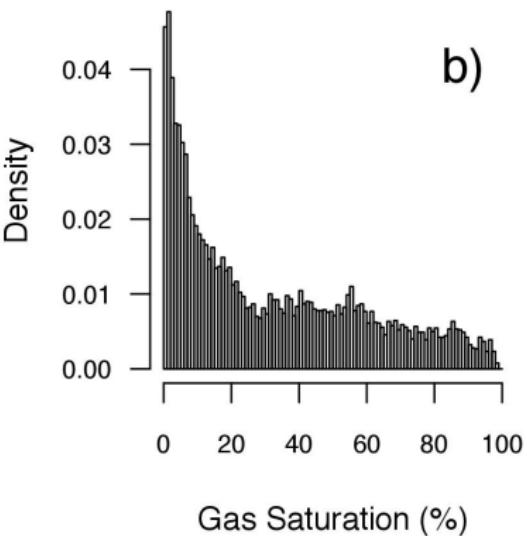
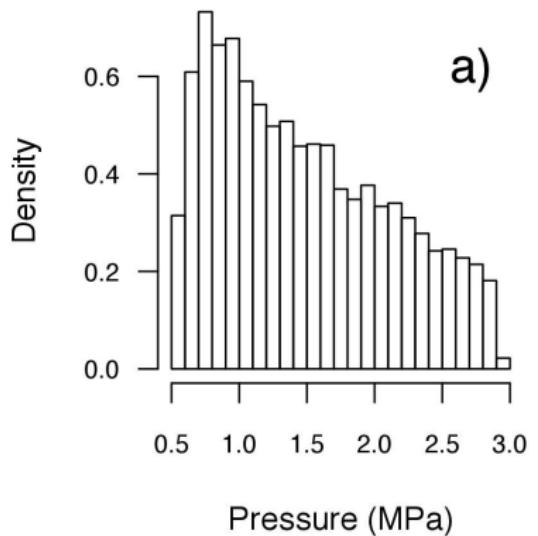
# Gas saturation posterior distributions



# Pressure posterior distributions



# Posterior distributions: Trace 160



# Ardmucknish Bay, Oban, Scotland: CO<sub>2</sub> release experiment

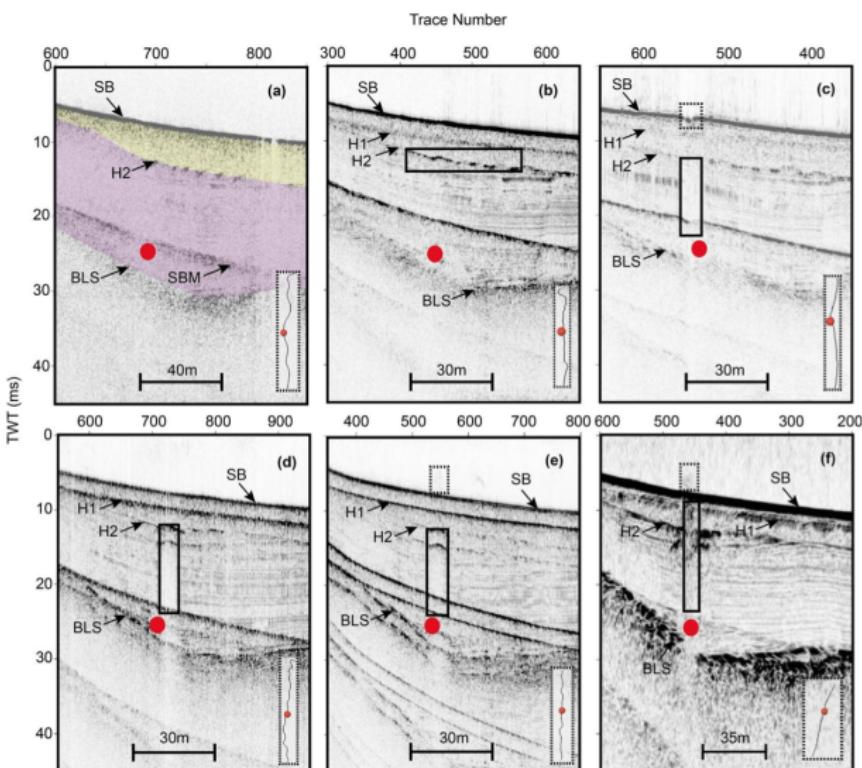
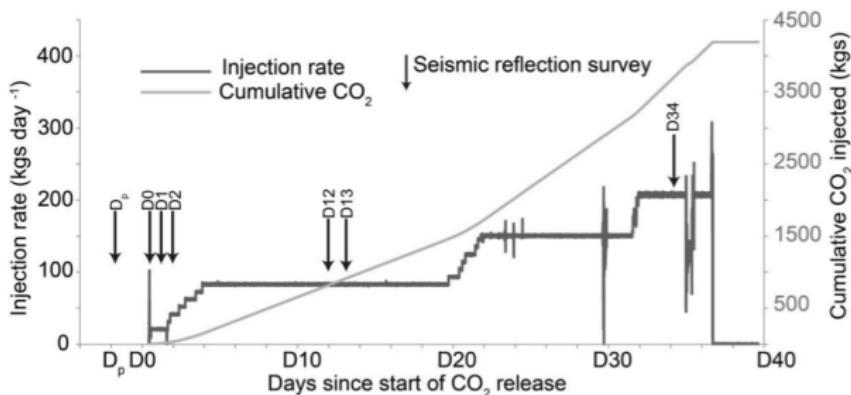


Figure from Cevatoglu et al. (*in review*)

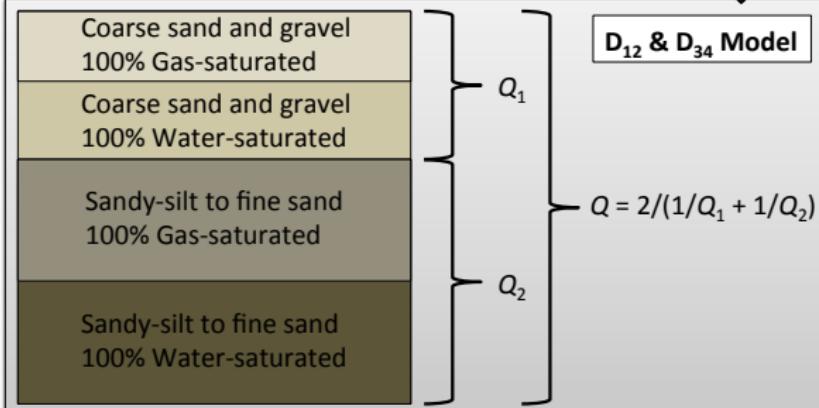
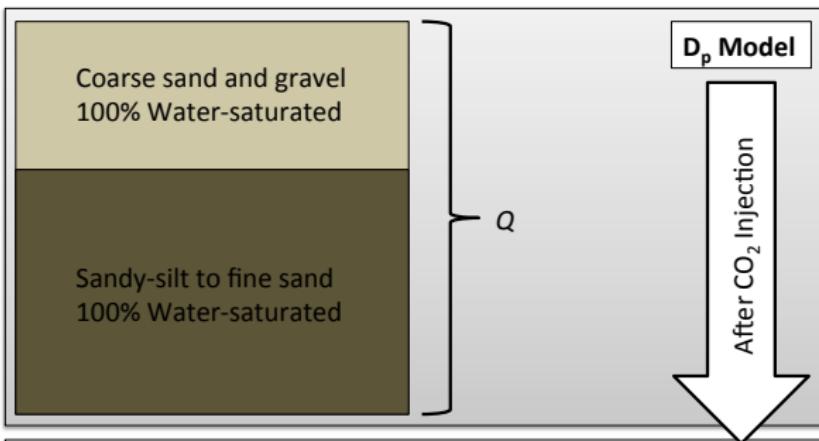
# Bayesian Updating for CO<sub>2</sub> Injection



Goal: Estimate gas storage over time.

- Use Bayesian Updating with seismic attenuation:
  - D<sub>p</sub>: feed in naive priors, get posteriors (without gas saturation)
  - D<sub>12</sub>: feed in posteriors from D<sub>p</sub> as priors, get new posteriors (with gas saturation)
  - Repeat over future surveys

# Stratigraphic Model



# Bayesian Updating scheme

Begin with prior based on geologic knowledge of system at D<sub>p</sub> with random variables  $\theta_p$ :

$$\pi(\theta_p) \sim MVTN(\mu_p, \Sigma_p, a_p, b_p) \quad (1)$$

Fit model to D<sub>p</sub> data:

$$p(\theta_p | \hat{Q}_p) \propto L(\hat{Q}_p | Q(\theta_p)) \times \pi(\theta_p) \quad (2)$$

where

$$L(\hat{Q} | Q(\theta)) \propto \exp\left(-\frac{1}{2\sigma^2}(\hat{Q} - Q(\theta))^T(\hat{Q} - Q(\theta))\right) \quad (3)$$

$$\sigma = 2 \quad (4)$$

# Bayesian Updating scheme (cont'd)

Update with D12 data, using both posterior of D<sub>p</sub> and expert knowledge of new variables ( $\theta_{new}$ ) to define prior:

$$p(\theta_{12} | \hat{Q}_{12}) \propto L(\hat{Q}_{12} | Q(\theta_{12})) \times \pi(\theta_{12}) \quad (5)$$

$$\pi(\theta_{12}) \sim MVTN(\mu_{12}, \Sigma_{12}, a_{12}, b_{12}) \quad (6)$$

$$\mu_{12} = [\overline{p(\theta_p)}, \mu_{new}] \quad (7)$$

$$\Sigma_{12} = \begin{bmatrix} \text{cov}(p(\theta_p)) & \Sigma_{p,new} \\ \Sigma_{new,p} & \Sigma_{new} \end{bmatrix} \quad (8)$$

$$a_{12} = [a_p, a_{new}] \quad (9)$$

$$b_{12} = [b_p, b_{new}] \quad (10)$$

# Bayesian Updating scheme (cont'd)

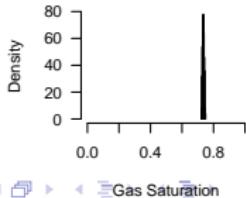
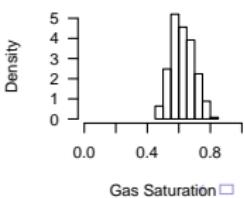
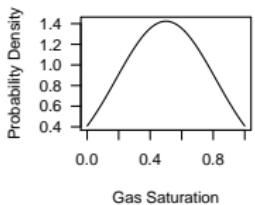
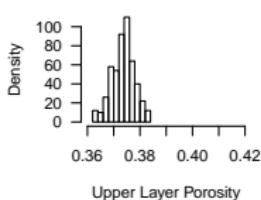
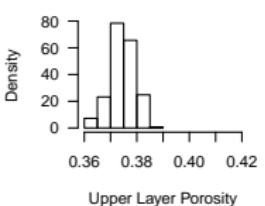
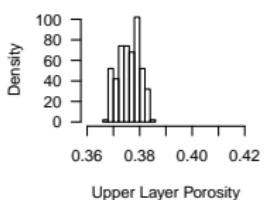
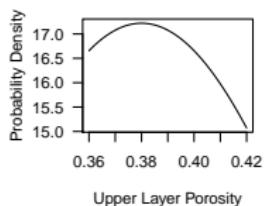
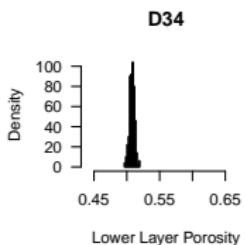
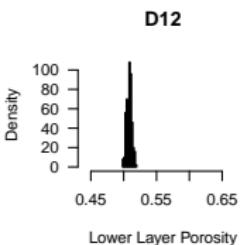
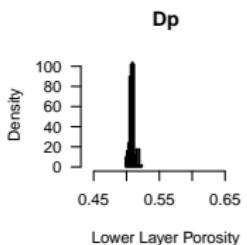
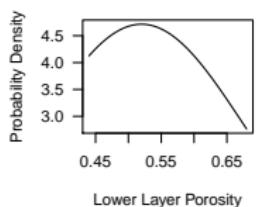
Repeat at next time step:

$$p(\theta_{34} | \hat{Q}_{34}) \propto L(\hat{Q}_{34} | Q(\theta_{34})) \times \pi(\theta_{34}) \quad (11)$$

$$\pi(\theta_{34}) \sim MVTN(\overline{p(\theta_{12})}, \text{cov}(p(\theta_{12})), a_{12}, b_{12}) \quad (12)$$

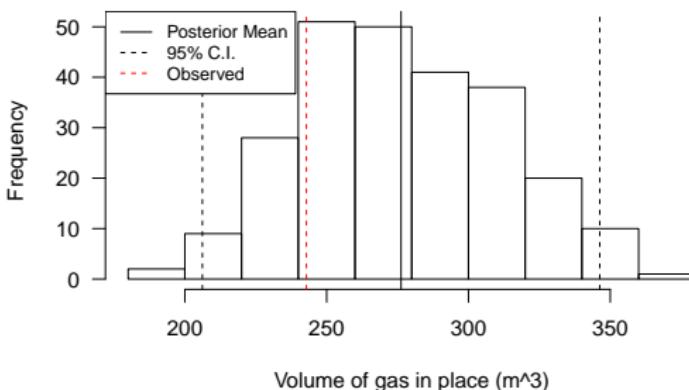
Continue with more data as it becomes available during gas injection. This process gains strength over time as priors become more informed, but is also flexible to adapt to changing conditions as they appear in the data ( $\hat{Q}$ ).

# Results: Prior/Posterior Comparison



# Results: Volume of Gas at D12

- **Observed:**  $(800 \text{ kg})(1 - 0.15 \text{ seepage rate}^{\text{a}}) / 2.8 \text{ kg/m}^3$  (for CO<sub>2</sub> at 0.15 MPa and 12 °C)
- Posterior:  $\sum_{i=\text{layers}} \phi_i \times S_{g,i} \times h_i \times (A = 10\text{m}^2)^{\text{b}}$

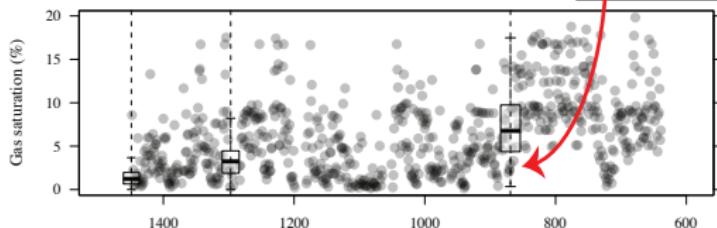
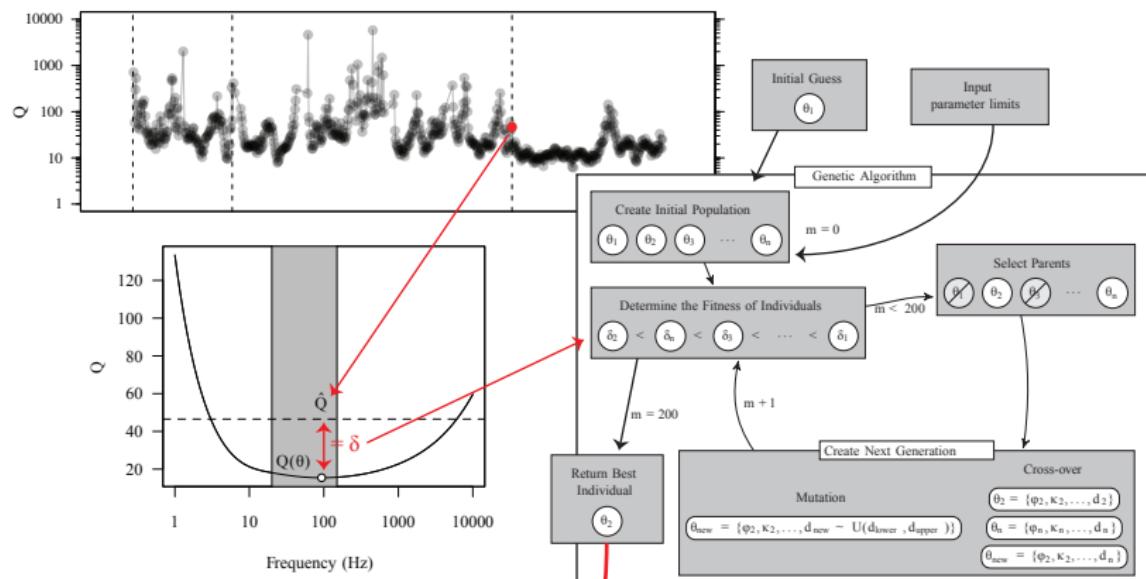


<sup>a</sup> Blackford et al. (2014). Detection and impacts of leakage from sub-seafloor deep geological carbon dioxide storage. *Nature Climate Change*. DOI: 10.1038/NCLIMATE2381.

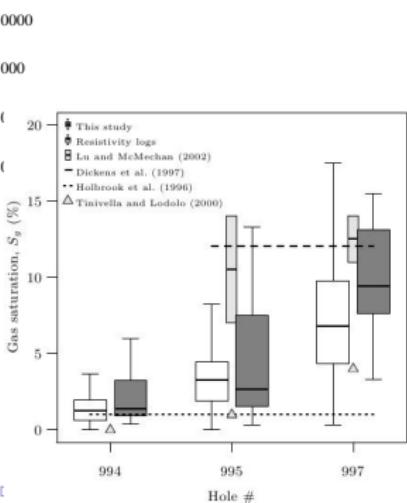
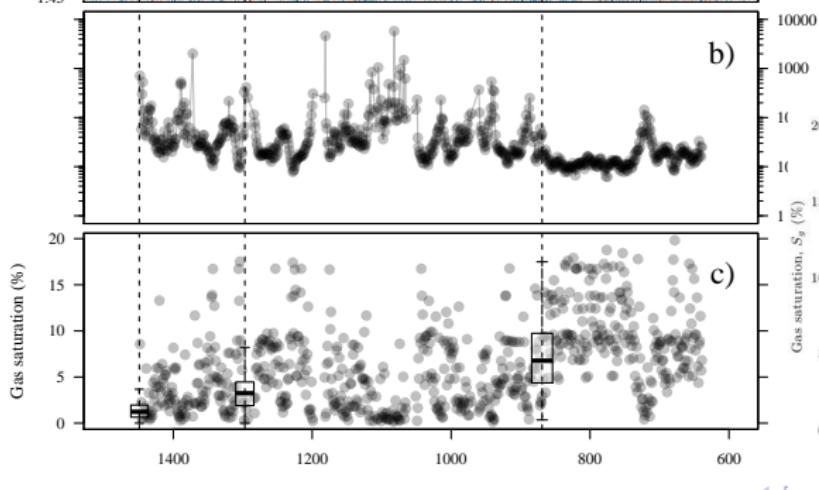
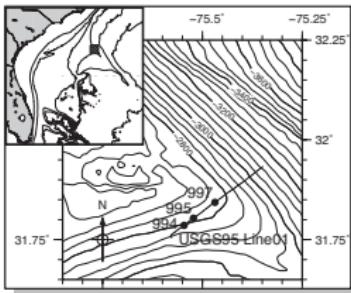
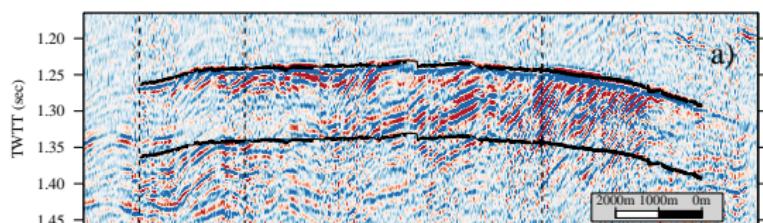
<sup>b</sup> Cevatoglu et al. (*in review*). Gas migration pathways, controlling mechanisms and changes in sediment physical properties observed in a controlled sub-seabed CO<sub>2</sub> release experiment

# Genetic Algorithm Inversion

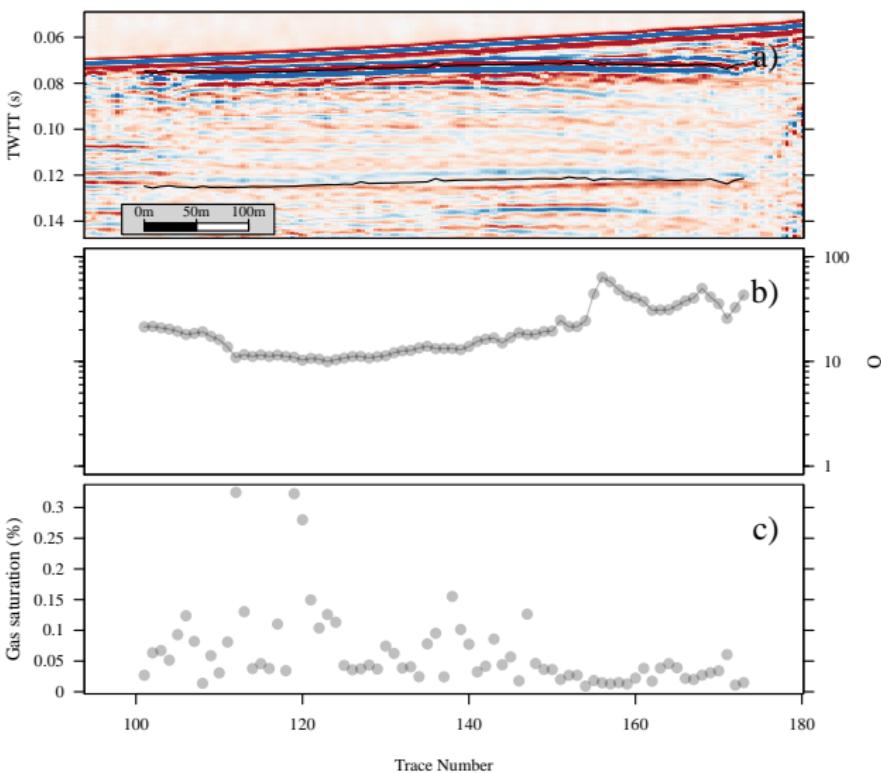
Invert attenuation model to get gas saturation estimates



# Blake Ridge



# Finneidfjord



# Conclusions

Seismic data can offer first-order estimates of gas properties over wide areas.

- Here, we estimate gas saturation and pressure by inverting attenuation model
  - No borehole data needed - no calibration required
  - Constrain the parameter space by realistic values from literature and seismic data
- Our estimates of  $S_g$  generally agree with borehole data at Blake Ridge, as well as between inversion methods
- Give reasonable gas volumes at Oban CO<sub>2</sub> injection site
- This methods work in shallower environments; are sensitive to small  $S_g$

# Conclusions

Compare how methods perform with attenuation model:

- Genetic Algorithm
  - Precise
  - Doesn't explore all modes well
  - Treats traces independently
  - Quick (can parallelize traces)
- Bayesian Hierarchical Model
  - Accounts for spatial correlation
  - Posteriors show modes
  - Slow (can't parallelize)

Bayesian Updating at single site gathers strength over time.

- More data is always better
- Priors refine over time
- Posterior uncertainty lessens over time

# References

- Carcione, J. M., Picotti, S., 2006. P-wave seismic attenuation by slow-wave diffusion: Effects of inhomogeneous rock properties. *Geophysics* 71 (3), no. 3, O1–O8.
- Mavko, G., Mukerji, T., Dvorkin, J., 1998. The rock physics handbook: tools for seismic analysis in porous media. Stanford-Cambridge program. Cambridge University Press.
- Morgan, E. C., Vanneste, M., Lecomte, I., Baise, L. G., Longva, O., McAdoo, B., 2012. Estimation of free gas saturation from seismic reflection surveys by the genetic algorithm inversion of a p-wave attenuation model. *Geophysics* 77 (4), R175–R187.  
URL <http://link.aip.org/link/?GPY/77/R175/1>
- Morgan, E. C., Vanneste, M., Longva, O., Lecomte, I., McAdoo, B., Baise, L., 2010. Evaluating gas-generated pore pressure with seismic reflection data in a landslide-prone area: An example from finneidfjord, norway. In: Mosher, D. C., Shipp, R. C., Moscardelli, L., Chaytor, J. D., Baxter, C. D. P., Lee, H. J., Urgeles, R. (Eds.), Submarine Mass Movements and Their Consequences. Vol. 28 of Advances in Natural and Technological Hazards Research. pp. 399–410, 4th International Symposium on Submarine Mass Movements and Their Consequences, Jackson Sch Geosci, Bur Econ Geol, Austin, TX, Nov 07-12, 2009.
- Morgan, E. C., Vanneste, M., Vardy, M., 2014. Characterization of the slope-destabilizing effects of gas-charged sediments via seismic surveys. In: OTC, 2014. p. 8, paper 25196.
- Schön, J., 1996. Physical properties of rocks: fundamentals and principles of petrophysics. Handbook of geophysical exploration: Seismic exploration. Pergamon.