

A Dual-Beam 3D Searchlight BSSRDF

Eugene d'Eon

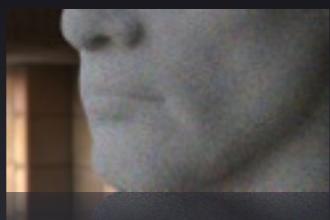


Thanks and welcome.

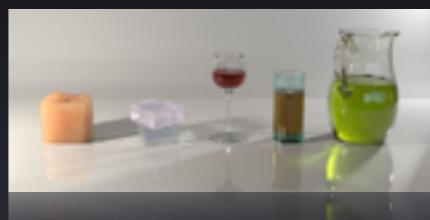
Monte Carlo Subsurface

- Easy exact solution
- Often impractical

Krivanek and d'Eon 2014



Krivanek et al. 2014



Zhao et al. 2014



Monte Carlo methods always, in theory, offer exact solutions to any subsurface scattering problem. In practice, however, the computation resources to achieve a low-noise solution are prohibitive. Thus, we continue to see new methods to reduce the variance of Monte Carlo approaches, like these three works presented at SIGGRAPH this week.

Deterministic Subsurface

Jensen et al. 2001



Borshukov and Lewis 2003



Donner and Jensen 2005



Deterministic approximations, such as these, remain highly relevant approaches to synthesising images of translucent media, such as human faces.

Today

- Question: why does the diffusion dipole work?
- New (old) ideas from neutron transport
- Propose:
 - Alternative form
 - Discard diffusion
 - 8D fully angular reciprocal BSSRDF
 - LOD chain



Today we describe a new approach to formulating an analytical approximate BSSRDF for rendering translucent materials that stems from asking the question: ‘why does the diffusion dipole work?’ We propose that the positive and negative source configuration works because it is an approximation application of an exact statement from old neutron transport literature. Taking this connection further, we propose an alternative form for analytic BSSRDFs that discards diffusion and produces a fully angular, reciprocal BSSRDF that places the BSSRDF carefully in the LOD chain between explicit structure and BRDF.

Recent Improved Methods

d'Eon and Irving 2011

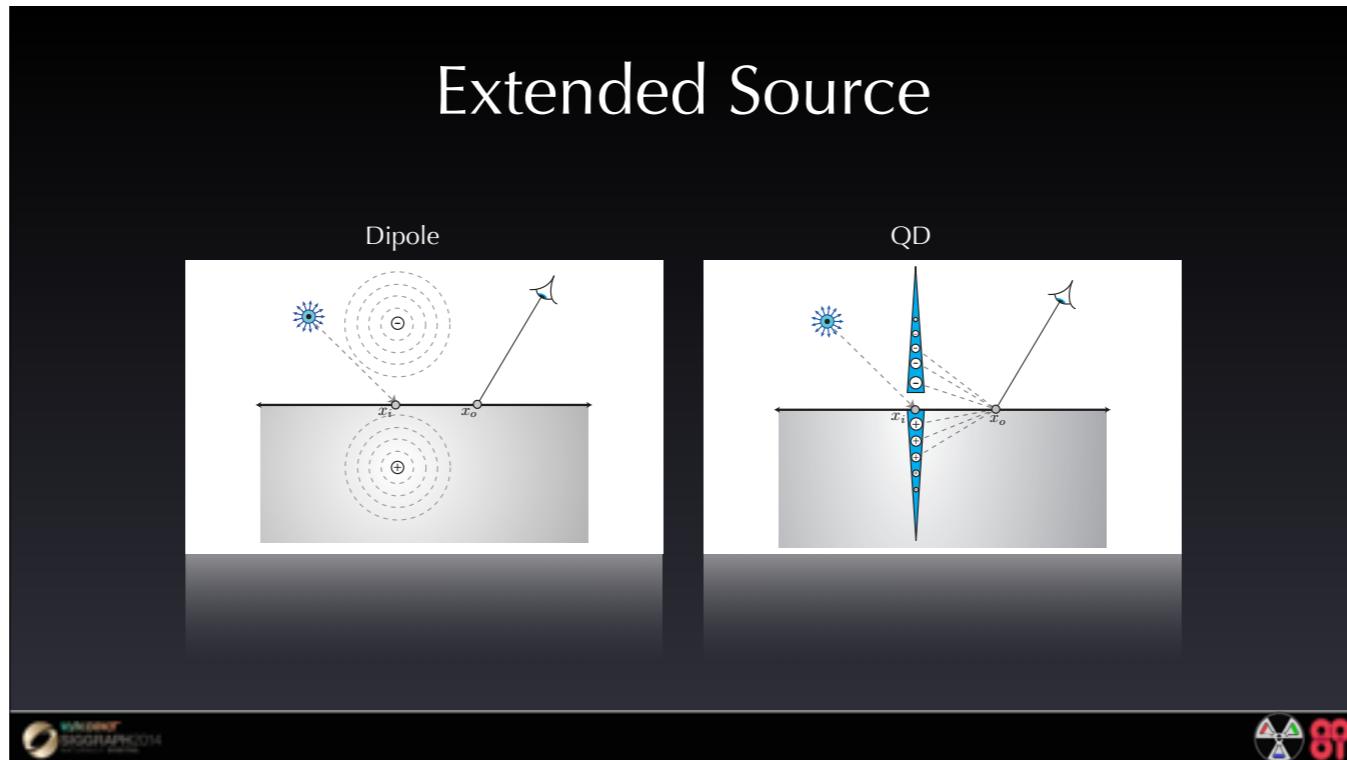


All frequency



We start by recalling a recent extension of the diffusion dipole, presented in 2011, called Quantized diffusion (QD).

Extended Source

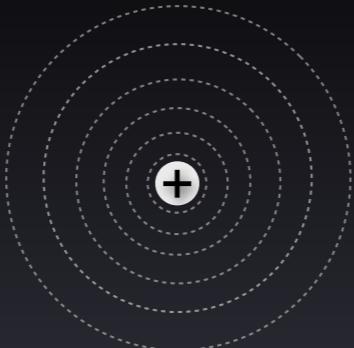


In QD, continuous distributions of positive and negative infinite-medium solutions combine to predict the existence flux at any surface location x_o where light is leaving the material due to illumination at x_i .

Point Source Green's Function

Grosjean 1954

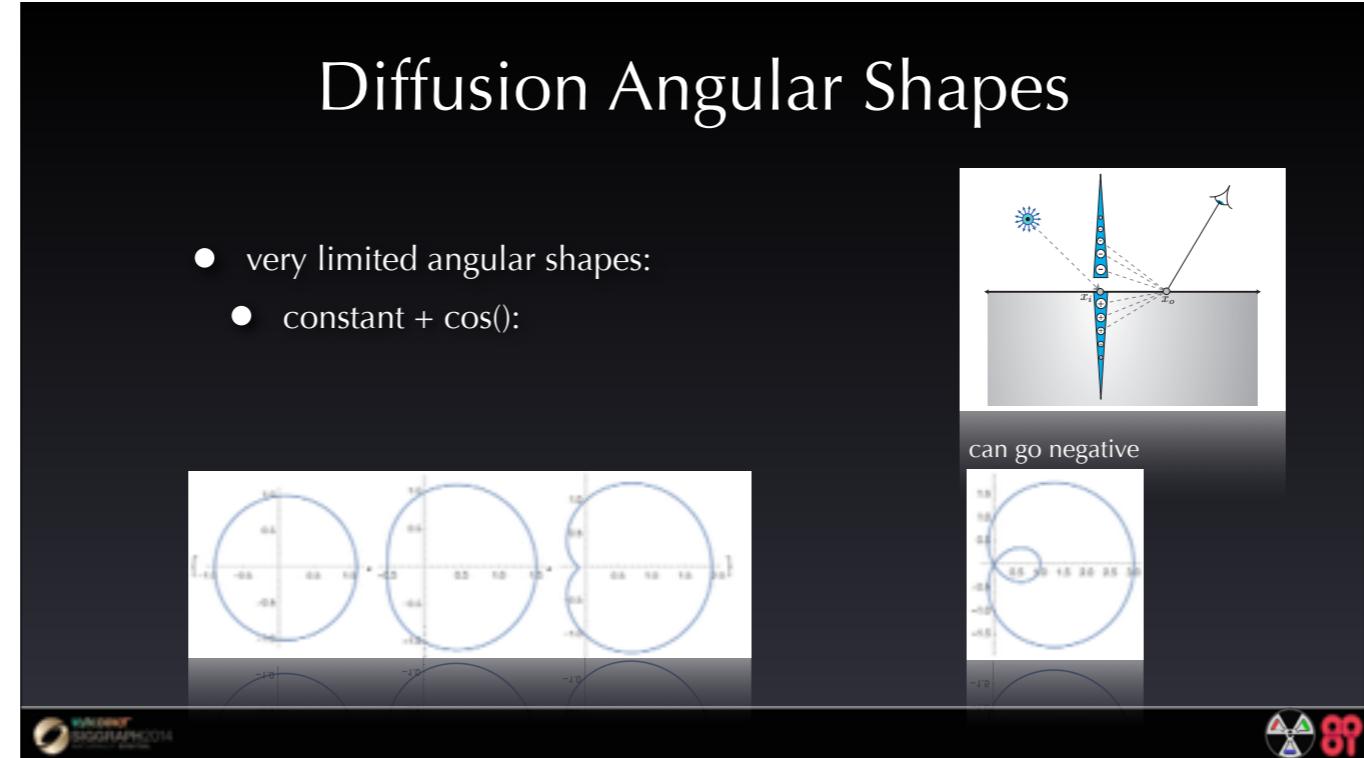
$$\phi(r) = \frac{e^{-\mu'_t r}}{4\pi r^2} + \frac{1}{4\pi} \frac{3\mu'_s \mu'_t}{2\mu_a + \mu'_s} \frac{e^{-r\sqrt{\frac{\mu_a}{D}}}}{r}$$



In the 2011 QD paper, Grosjean's 1954 analytic approximation to the fluence about a point source in infinite space was used as the foundation of the positive and negative source distributions defining the solutions for the semi-infinite medium.

Diffusion Angular Shapes

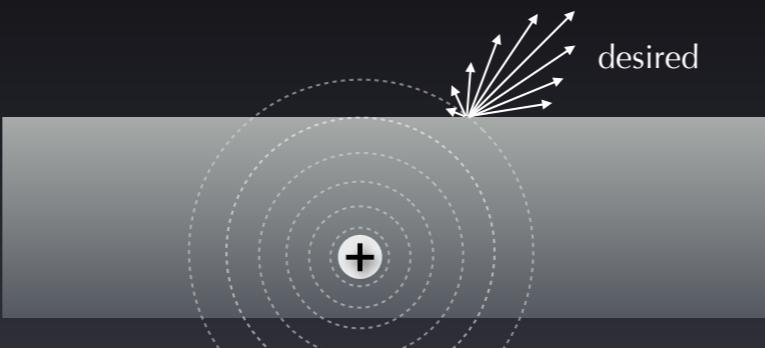
- very limited angular shapes:
 - constant + $\cos(\theta)$:



However, a particular issue with diffusion theory limits the angular accuracy of such approaches. This stems primarily from the limitation that diffusion solutions have an angular distribution everywhere that is restricted to a sum of a constant in angle and a cosine in angle. Worse, in some regions (such as very near a source) the radiance predicted by diffusion theory is actually negative, which is impossible physically.

Previous solution

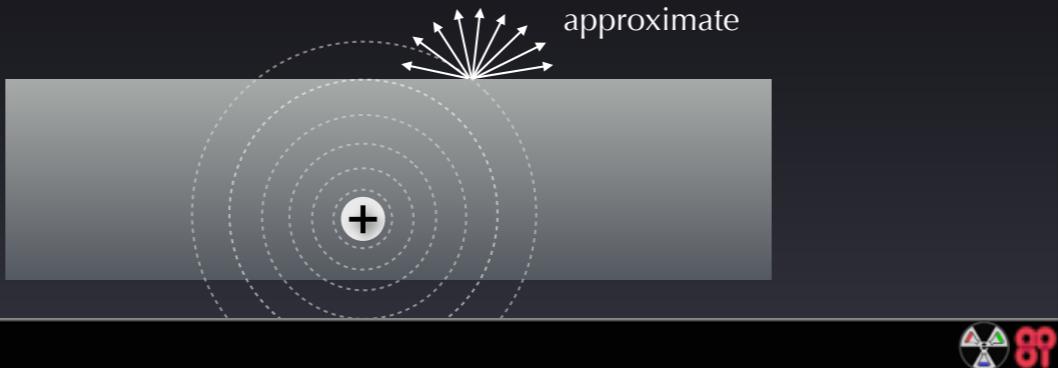
- Workaround: hemispherical integral
 - Put energy into Lambertian



To avoid this issue in previous applications of diffusion theory at a surface, the workaround was to integrate the total energy predicted to leave the surface at some location (which is always positive), and redistribute that outgoing energy into a Lambertian shape. So instead of getting an angular varying distribution like this...

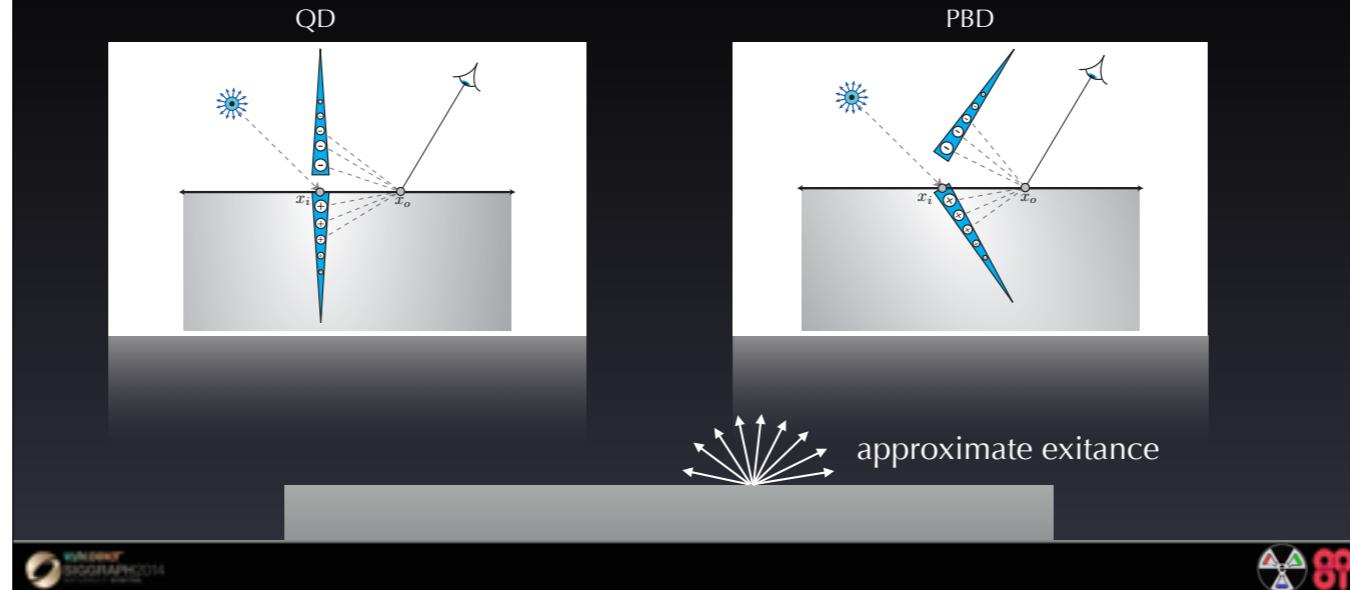
Previous solution

- Workaround: hemispherical integral
 - Put energy into Lambertian



...we instead suffer the following approximation to avoid negativity.

Photon Beam Diffusion



This limitation is inherent in the QD and Photon Beam Diffusion BSSRDFs. Both assume Lambertian existence from the surface to avoid the negativities predicted by the diffusion approximation.

Level of detail regimes



This is problematic when considering the BSSRDF and its neighbour, in the LOD chain, the BRDF, useful for rendering a surface where negligible lateral bleeding of light is visible (such as a very distant human face). We can imagine a continuous shot where a zoom from very close tissue (left) using explicit structure is blending into a BSSRDF model (middle) and finally to a BRDF model for efficiency in the distance (right). However, previous analytic BSSRDFs provide incredibly poor angular accuracy. The BRDF solution for multi-layered scattering layers is efficiently computable and highly accurate in angle (see, for example, this years paper by [Jakob et al. 2014]).

New Approach

1. Non diffusion exitance calculation
2. Generalize negative source placement
3. BRDF benchmark driven model



To produce a BSSRDF with improved angular accuracy, we propose to move away from a diffusion existence calculation. In addition, for better accuracy, we generalize previous placement of negative sources outside the medium. To ensure the most accurate match in the LOD chain to the BRDF, we drive step 2 by the known BRDF solution.

1. Non diffusion exitance

What's the exact point source solution?

Davison 1945 (MT-112)

$$\psi(r, \mu) = \frac{Q}{4\pi r^2} e^{-r} \delta(\mu - 1) + \\ + \frac{3Q}{8\pi} \sum_{n=0}^{\infty} P_n(\mu) \left(\frac{n!}{r^{n+1}} - \frac{2}{3} (2n+1) \frac{e^{-r}}{r^2} + \frac{2n+1}{3} \int_1^{\sqrt{\frac{2y}{\pi}}} \frac{\sqrt{2y} K_{n+1/2}(ry) \left[P_l\left(\frac{1}{y}\right) Q_s\left(\frac{1}{y}\right) - Q_l\left(\frac{1}{y}\right) P_s\left(\frac{1}{y}\right) \right]}{1 + \frac{1}{y} \log \frac{y-1}{y+1} + \frac{1}{4y^2} \left[\pi^2 + \log^2 \frac{y-1}{y+1} \right]} dy \right) \quad (3.16)$$

(3.16)

$$1 + \frac{\lambda}{10^6} + \frac{\lambda + 1}{10^6} + \frac{4\lambda^2}{10^6} \left[\frac{\lambda + 1}{10^6} \right]$$



Step 1: if we move away from diffusion angular limitation, what, then, is the exact solution, in infinite media, for the angular distribution of radiance anywhere relative to an isotropic point source (assuming isotropic scattering)? The solution was first written down by Davison in 1945. Yikes! It involves a delta function in angle (necessary: this is the uncollided energy attenuated as it moves away from the point source, and directed in a singular direction—directly away from the point source). The scattered solution is an infinite sum involving integrals of Bessel functions.

1. Non diffusion exitance

Or you could use the fluence (beam radiance estimate)

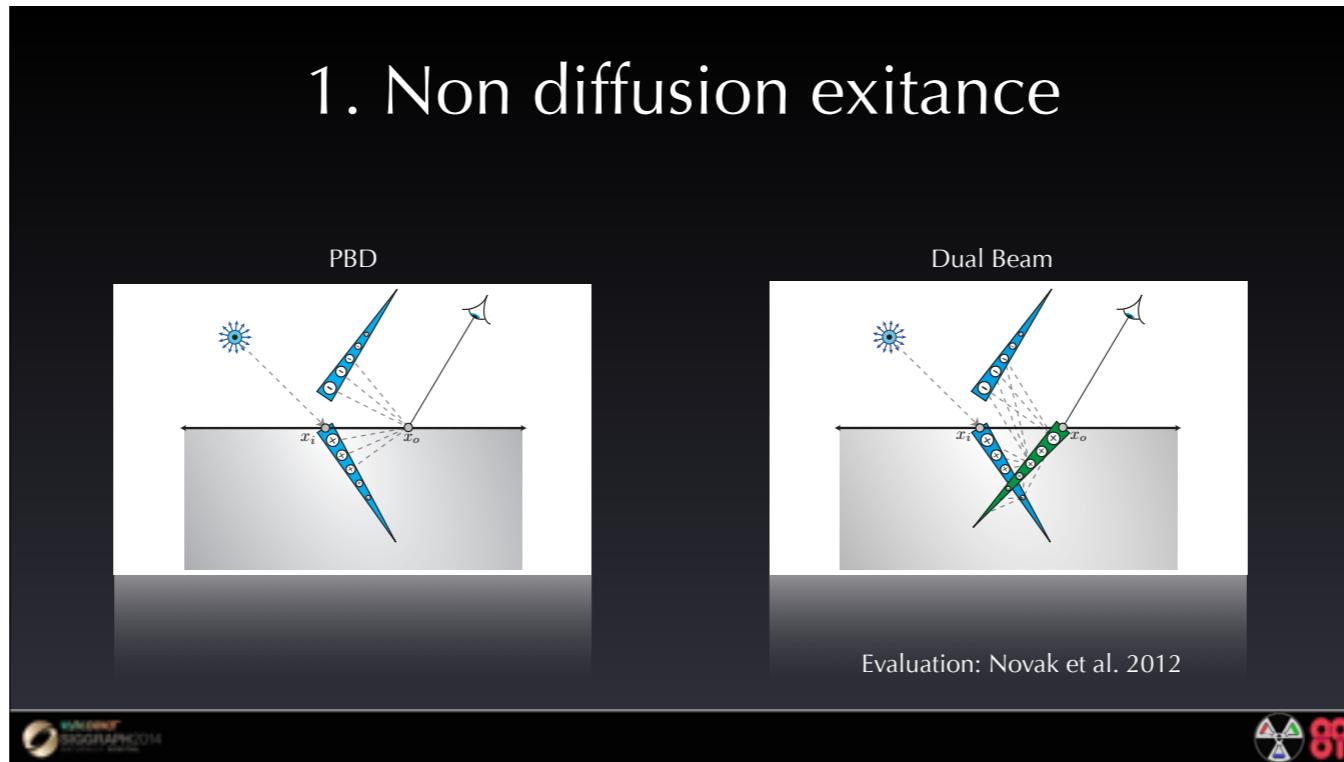
Davison 1945 (MT-112)

$$\psi(r, \mu) = \frac{Q}{4\pi r^2} e^{-r} \delta(\mu - 1) + \frac{1}{2} \int_0^\infty \psi_o(\sqrt{R^2 + r^2 - 2rR\mu}) e^{-R} dR \quad (3.17)$$



However, Davison noted in the same paper that the result is easily described (thanks to the assumption of isotropic scattering), as a 1D integral of the fluence (which we have a simple approximate form from Grosjean).

1. Non diffusion exitance



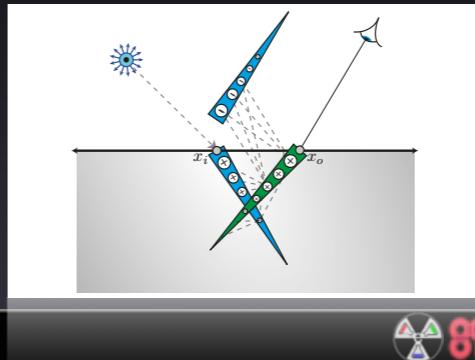
To apply this to the extended-source BSSRDF ideas of previous work, we replace the outgoing calculation of flux at x_o with Davison's line integral of the subsurface fluence. This is illustrated as the green continuum of positive 'detectors' inside the medium.

2. Generalize Negative Sources

Step 1: requires accurate fluence *everywhere inside* the medium

Previous methods: optimise for net flux at boundary (Milne problem)

Inspiration: Placzek's lemma



However, in contrast to previous work, we now require accurate fluence everywhere INSIDE the medium. This is where we leave diffusion theory behind, which imposed a boundary condition based on the Milne problem for solving for the negative source placement such that some property AT THE BOUNDARY was satisfied. To return somewhat to first principles, let's consider why the dipole idea works at all to begin with. Why is it that a negative copy of the internal source at just the right distance outside the medium does so well at predicting radial-exitant flux at the boundary (in particular, far from the incident location)? The first part of the answer is related to the linearity of the transport equation: if A and B are solutions to the equation, then so too is A+B. Thus, the sum of two infinite medium solutions, each satisfying the equation, are also solutions (except, of course, at boundaries, which must then reduce useful forms of their combination to only one or a few choices).

Placzek's Lemma

*"Introduction to the Theory of Neutron Diffusion",
Case, de Hoffman, Placzek 1953*

1. Given only infinite medium solutions
2. Construct solution for finite convex medium
 - A. Assume volume extends outside
 - B. Create a negative outgoing surface source



In 2011 we described the method of images (the placement of positive and negative infinite medium solutions to solve non-infinite problems) as an approximate application of Placeczek's Lemma. Placeczek's Lemma appears in a 1953 classic text on neutron transport (not easy to find) and indeed, sounds a lot like the method of images at first glance. However, it is an EXACT method for constructing solutions to finite problems (assuming a convex volume) using only infinite medium solutions.

Placzek's Lemma

17. Reduction of Finite Problems to an Infinite-Medium Problem

The main significance of the results derived in the preceding chapter lies in the fact that they may be used for the solution of a large class of problems involving finite media. Many, although not all, of these applications are based on a simple theorem. For the sake of clarity, we shall state it first in an unnecessarily restricted form, whereupon generalizations will become obvious.

THEOREM. *Given a uniform medium bounded by a surface S . Outside S , $c=\sigma=0$ (vacuum). Let the source distribution be prescribed both inside and outside S and let the angular density, defined by introduction of the solution of the integral equation (4) of Sec. 12 into (3) of Sec. 12, be $\phi(r,\Omega)$ and, in particular, at the surface $\phi(r_s,\Omega)$.*

The solution $\psi(r,\Omega)$ of this problem (problem I) inside S is given by the solution of the following infinite-medium problem (problem II): The uniform medium previously inside S now extends over all space, the sources outside S are removed, the sources inside S are retained, and a surface source at S of angular strength $[\Omega \cdot n(r_s)]\phi(r_s,\Omega)$ is added [$n(r_s)$ inside normal on S]. The asymptotic solution is chosen so that it vanishes outside S .

We also state

COROLLARY I. *The particular solution of problem II defined above vanishes over the entire region outside S .*

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SIGGRAPH
2014



The statement of it is not terribly complex...

Placzek's Lemma

following infinite-medium problem (problem II): The uniform medium previously inside S now extends over all space, the sources outside S are removed, the sources inside S are retained, and a surface source at S of angular strength $[\Omega \cdot \mathbf{n}(\mathbf{r}_s)]\psi(\mathbf{r}_s, \Omega)$ is added [$\mathbf{n}(\mathbf{r}_s)$ inside normal on S]. The asymptotic solution is chosen so that it vanishes outside S .

2]: The asymptotic solution is chosen so that it vanishes outside S .

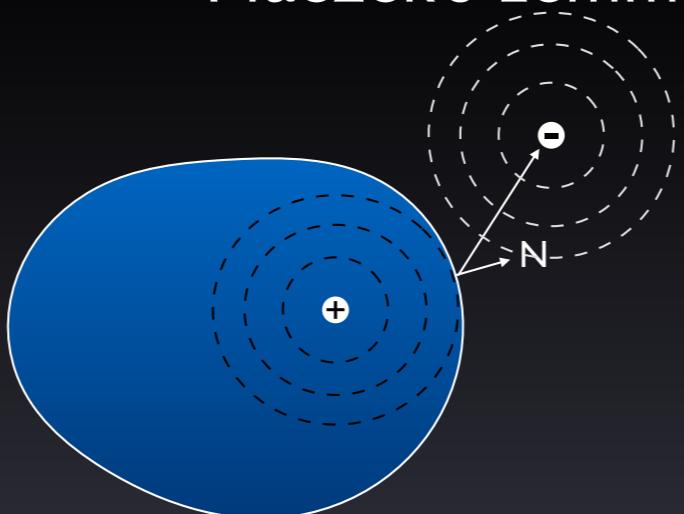
Negative surface source:

$$(\mathbf{N} \cdot \vec{\omega})L(\vec{x}, \vec{\omega})$$



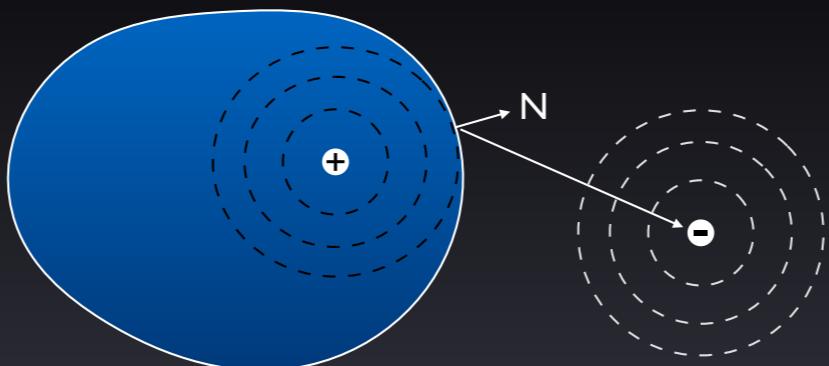
The construction is as follows: the scattering volume is now considered to extend everywhere in an infinite medium (i.e. outside the original volume). A new negative surface source is added to the system using this simple equation involving the normal dotted with the direction being considered. Now, the internal sources (the reduced-intensity beam in our case) and these new outward facing negative surface sources combine to produce the correct solution everywhere in the medium, using infinite medium green's functions. However, there is a chicken an egg problem: to know the magnitude of the negative surface source to produce this exact result requires knowing $L(x, \omega)$: the solution at the surface for the finite problem—the very solution we seek in the first place! However, it is still worthwhile to consider this Lemma.

Placzek's Lemma



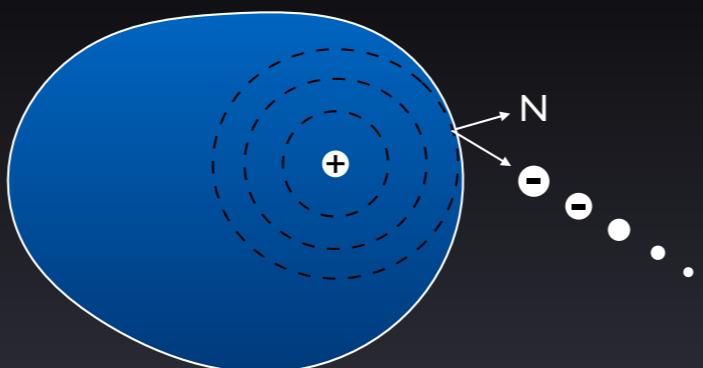
To see how this lemma works, consider a single point source inside the finite medium. (An extended reduced-intensity source is a superposition of these). For each surface location, and for each outgoing direction at that position, a negative source is added, which will first scatter at some point and place, effectively, a negative isotropic point source outside the volume.

Placzek's Lemma



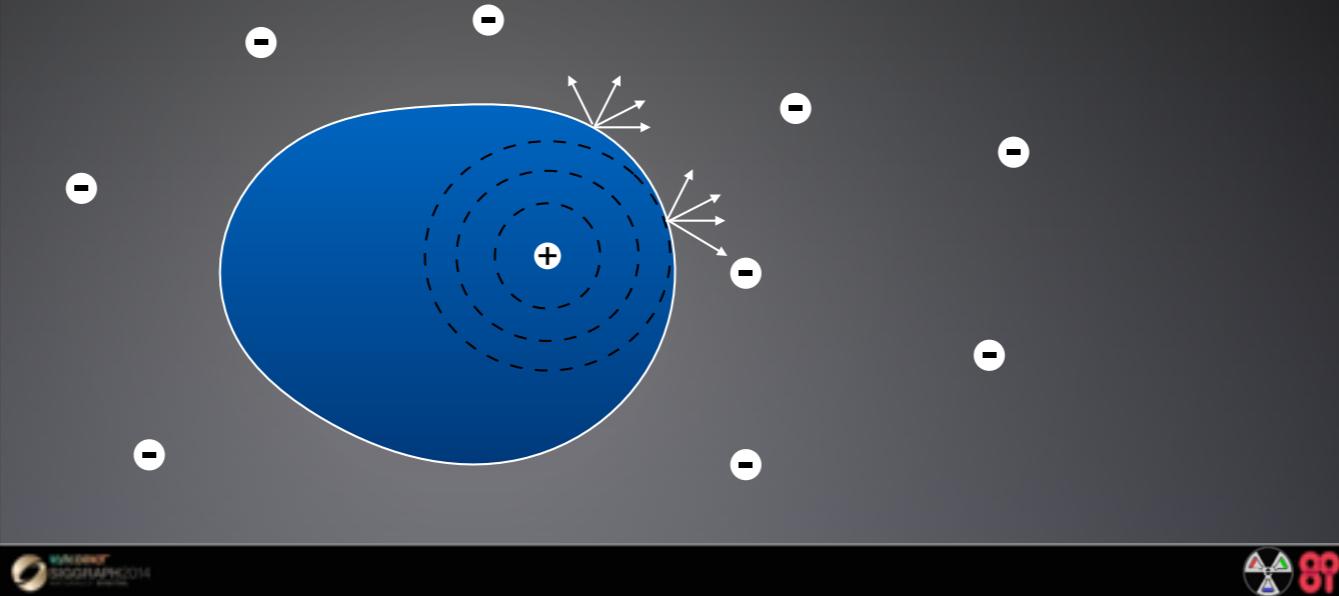
However, this happens for ALL outgoing directions, not just one, like in previous papers (like Jensen et al. 2011).

Placzek's Lemma



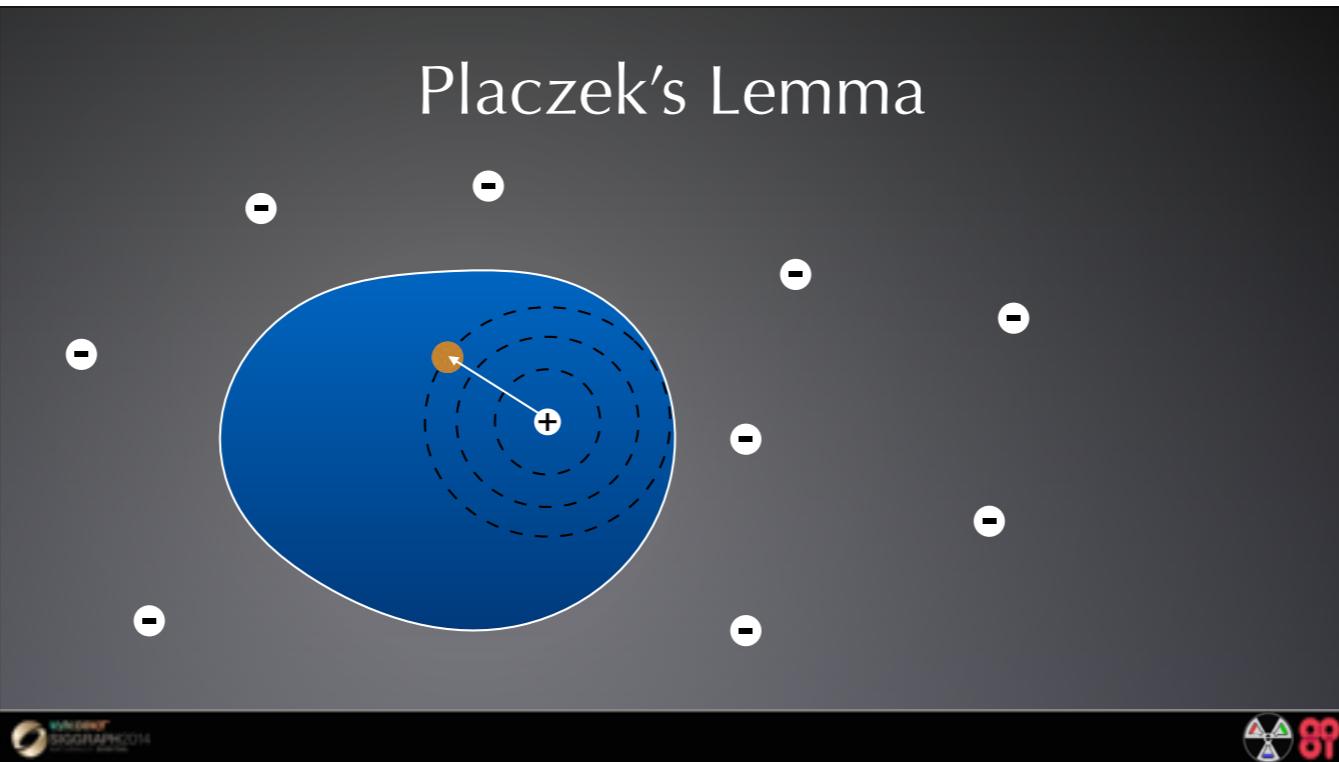
And since each of these surface sources begins at the surface, and the scattering medium extends outside the medium now, this effectively produces a continuous superposition of negative 'charges' outside the volume, due to the original point source inside.

Placzek's Lemma



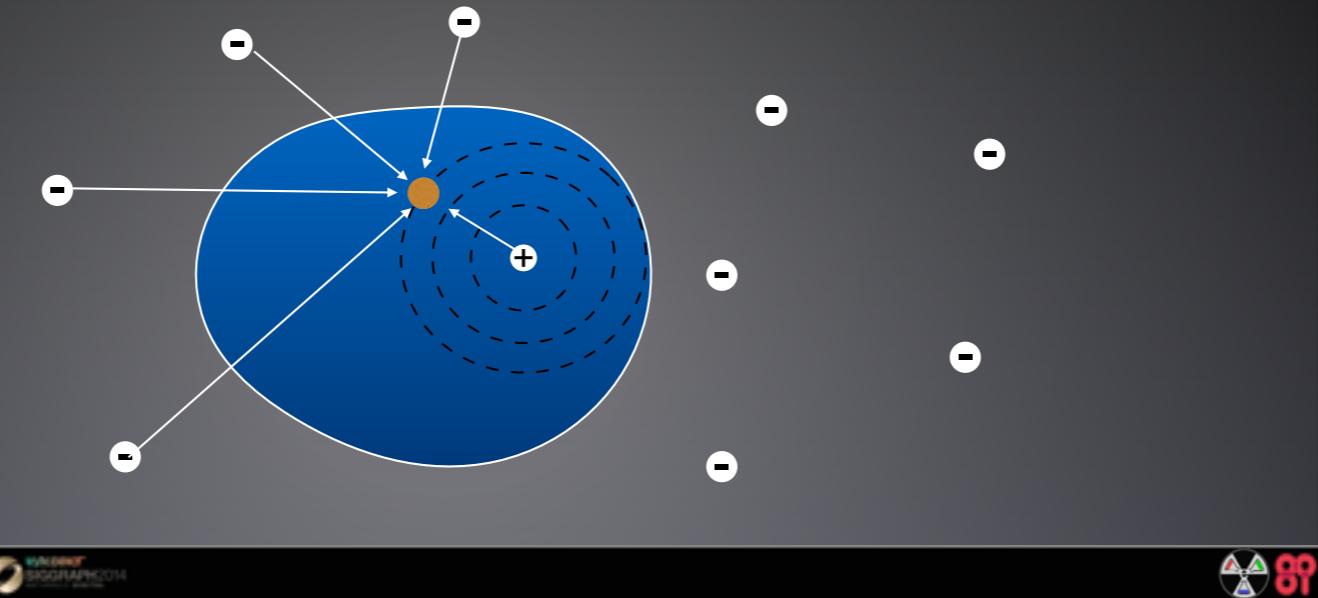
Considering all these extended beams, arising from all surface locations and angles, creates a continuous sea of negative energy outside the volume.

Placzek's Lemma



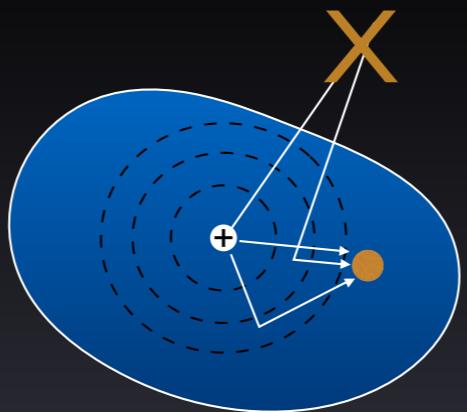
The exact solution anywhere in the medium is the sum of the original contribution from the interior positive source (computed with an infinite medium Green's function), and...

Placzek's Lemma



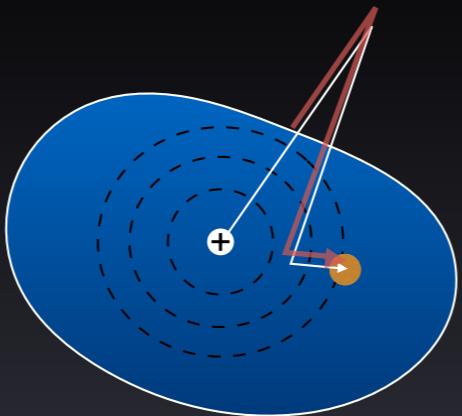
the integral of negative fluence from all of the exterior sources.

Proof Sketch



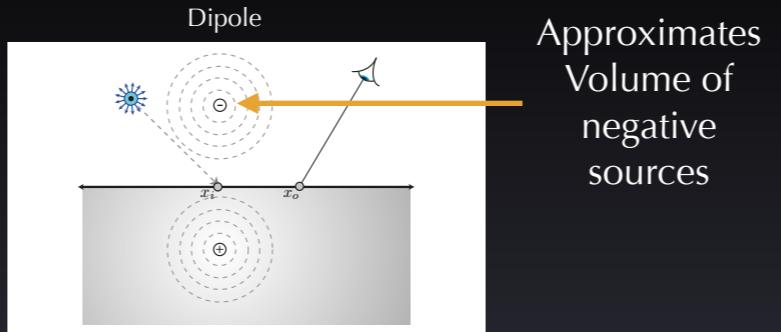
To convince yourself why something like this can work, consider that various paths described by the infinite solution used at the positive source will include some that exit and re-enter the original medium. As such, the infinite medium result will over predict the fluence at this orange location.

Proof Sketch



By adding a negative surface source just as this path leaves the volume the first time, this precisely cancels this extra energy, and the combination of positive source and negative external sources produces the finite medium desired answer.

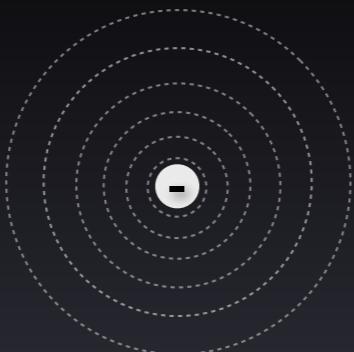
Look Familiar?



Previous work: same magnitude, no uncollided

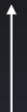
The original dipole is effectively an approximation of this idea in three ways: a) the continuous sea of negative energy is replaced by a single negative point source outside, b) the magnitude of that negative source is the same as the positive one, and b) the Green's function for the negative sources is missing the uncollided term.

Negative Uncollided Sources



Grosjean 1954

$$\phi(r) = \frac{e^{-\mu'_t r}}{4\pi r^2} + \frac{1}{4\pi} \frac{3\mu'_s \mu'_t}{2\mu_a + \mu'_s} \frac{e^{-r\sqrt{\frac{\mu_a}{D}}}}{r}$$



Uncollided (reduced-intensity) fluence



Placzek's lemma produces an exact solution by employing exact Green's functions. A highly accurate approximate Green's function contains an important uncollided term, which we can consider adding to the method of images for improved accuracy.

3. Discard Diffusion

How to determine negative source positions?

BRDF accuracy benchmark (lateral integration)

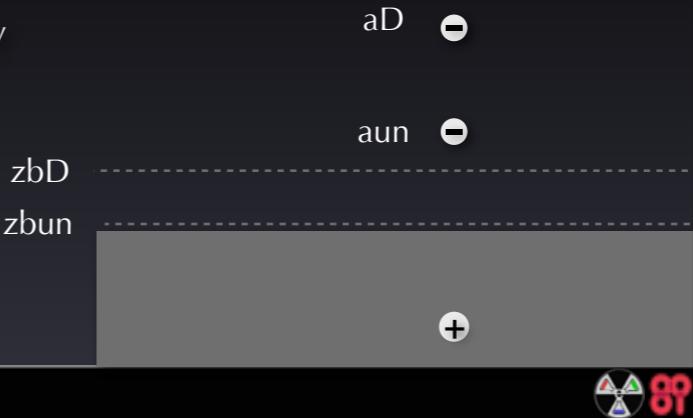
(Analytic / Jakob et al. 2014)



However, how do we maintain discrete negative approximate sources outside and place them in the most accurate way (without knowing the exact solution, as required by Placzek). To do this, we use the associated BRDF for the problem (computable using methods like [Jakob et al. 2014]).

Mirroring sources

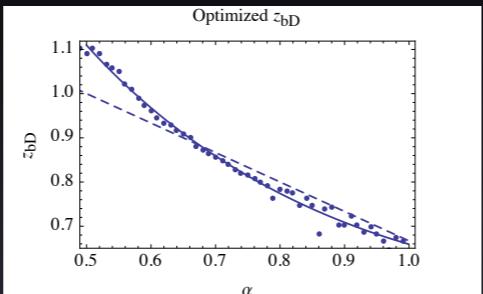
- Decouple negative ballistic and diffusive components
- Non linear optimisation for 4 values
 - diffuse reflection boundary
 - ballistic reflection boundary
 - general scale for each



For each positive source in a semi-infinite medium we propose placing a negative diffusive and negative uncollided source outside the medium, where the reflection boundary, and intensities of the negative sources are all determined by a non linear optimisation for 4 parameters such that the BRDF derived from this BSSRDF is as accurate as possible (more details in the supplemental material).

Findings (isotropic index-matched)

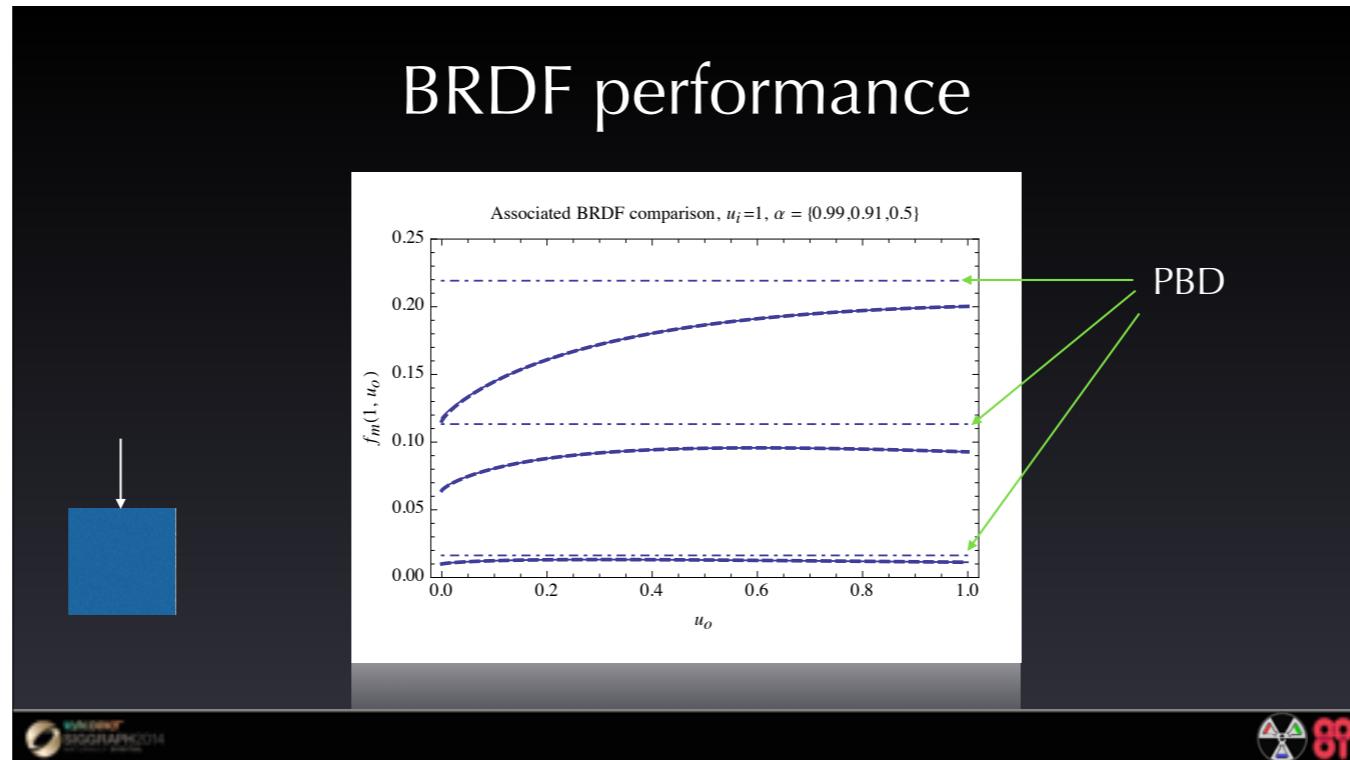
Previous work, $z_b = 2AD$ (dashed)



- High absorption:
 - Negative uncollided kicks in
 - Diffusive amplitude dives

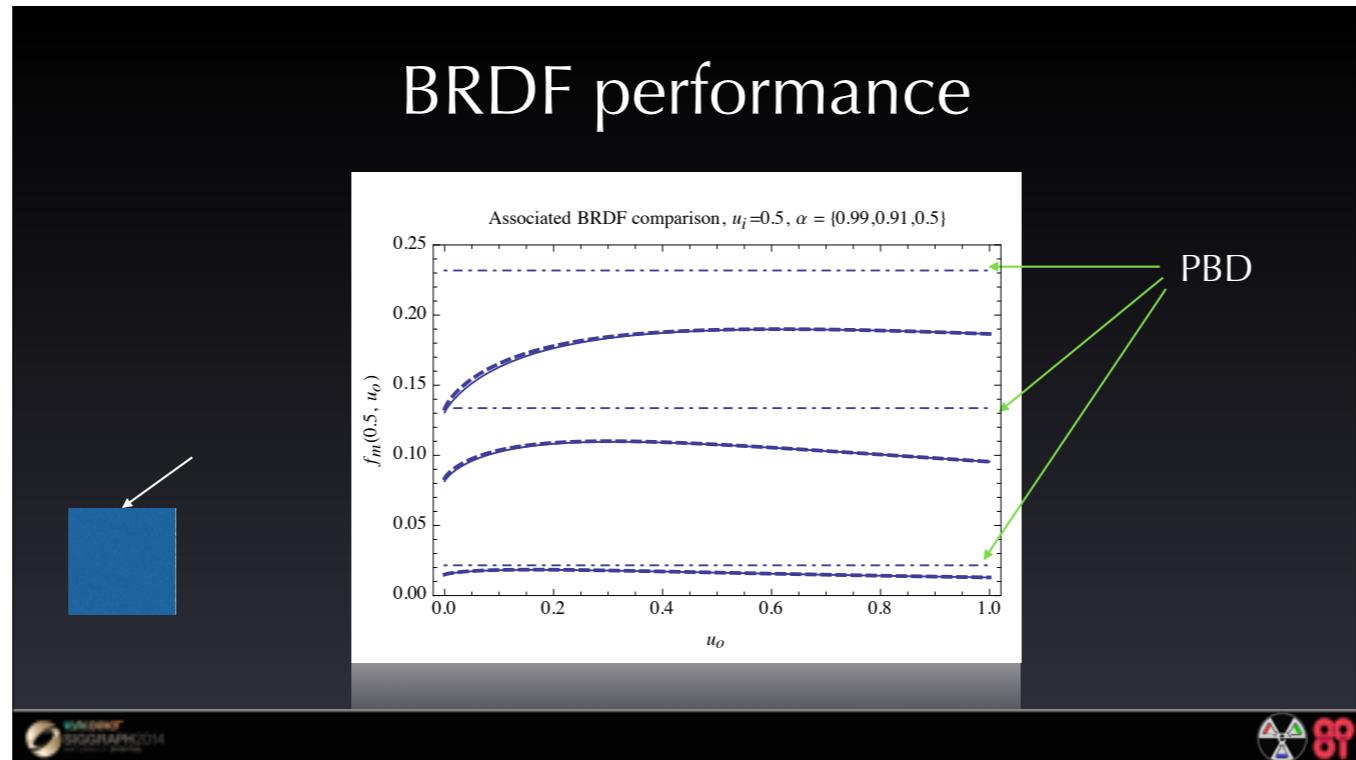
What we found when running the optimisation over a broad range of isotropic scattering media with single-scattering albedo alpha is: the optimal depth z_b about which to mirror the diffusive portion of the negative sources is not quite that predicted by diffusion theory. This and other findings in more depth in the supp. material.

BRDF performance



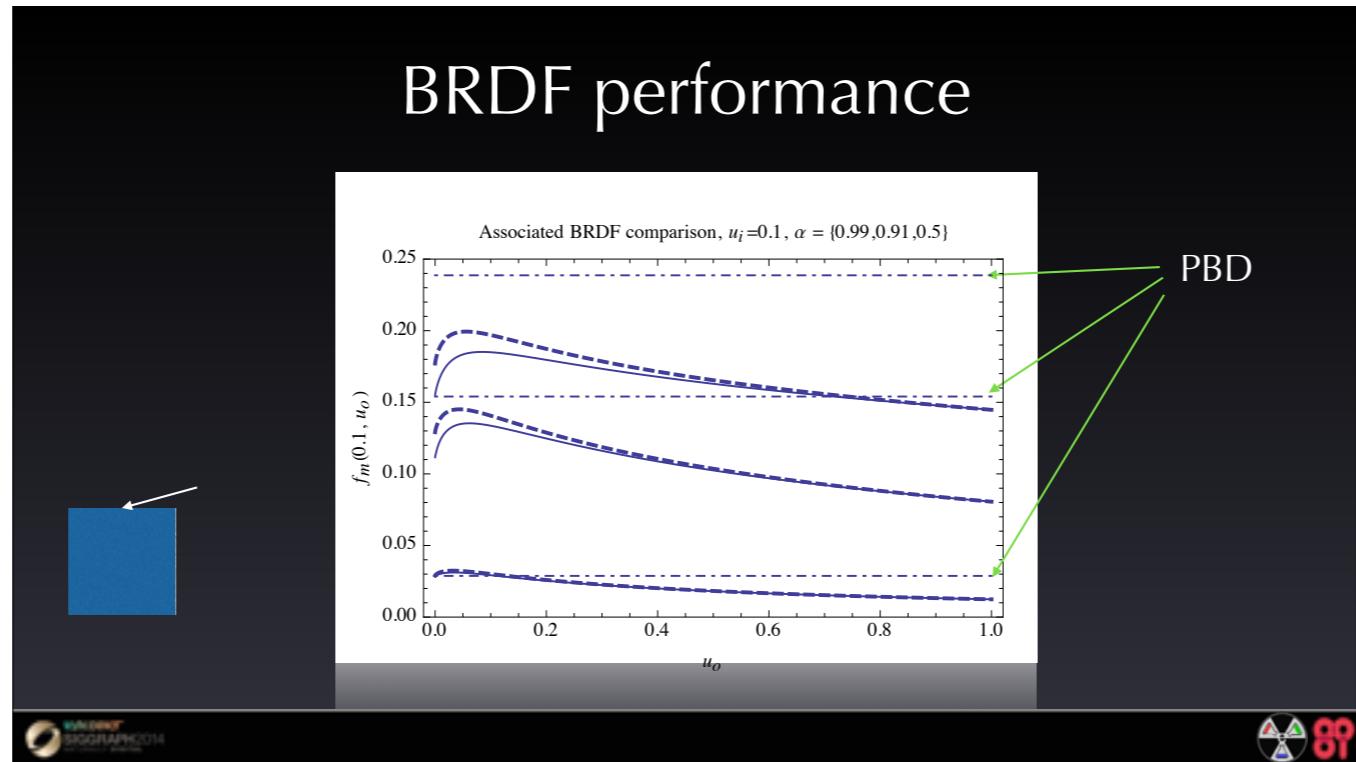
The BRDF of the new BSSRDF is much more accurate than previous Lambertian/diffusive approaches, like Photon Beam Diffusion.

BRDF performance

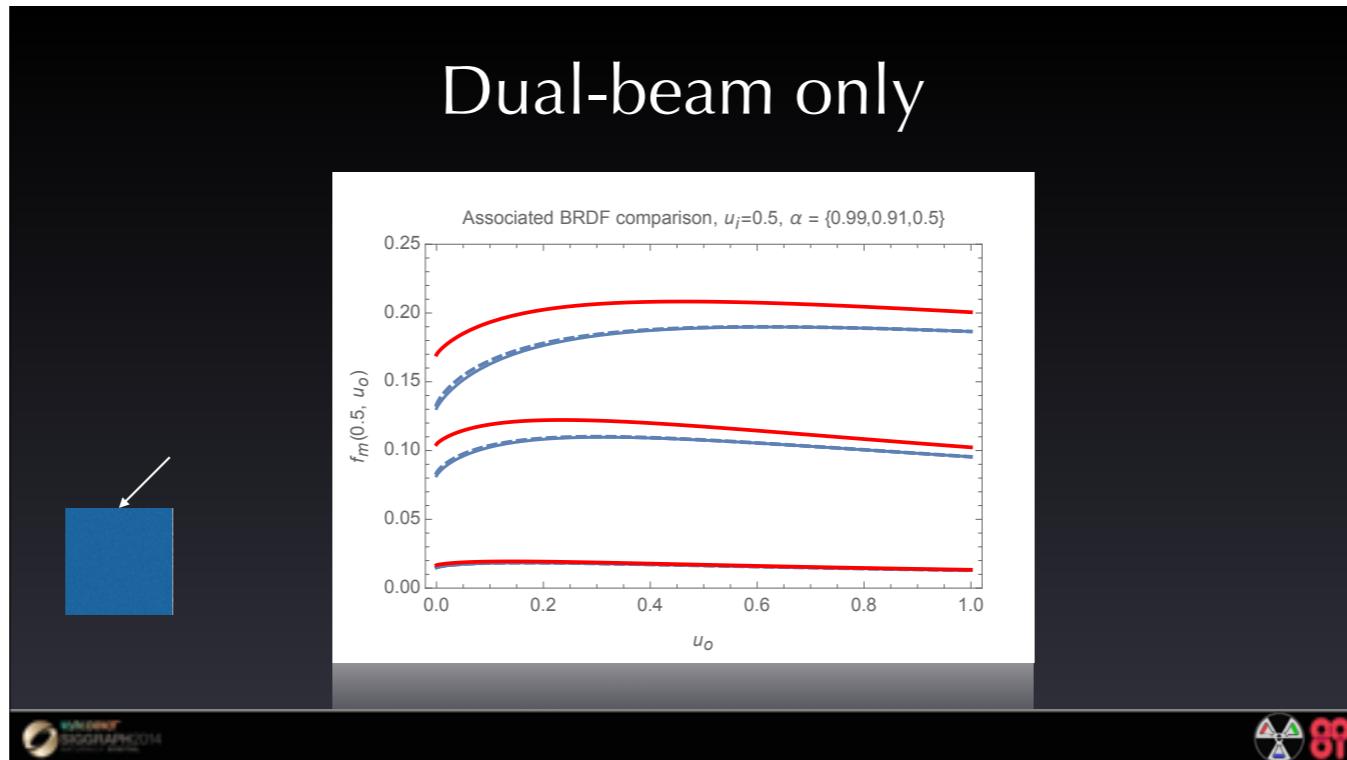


For a variety of incident angles...

BRDF performance

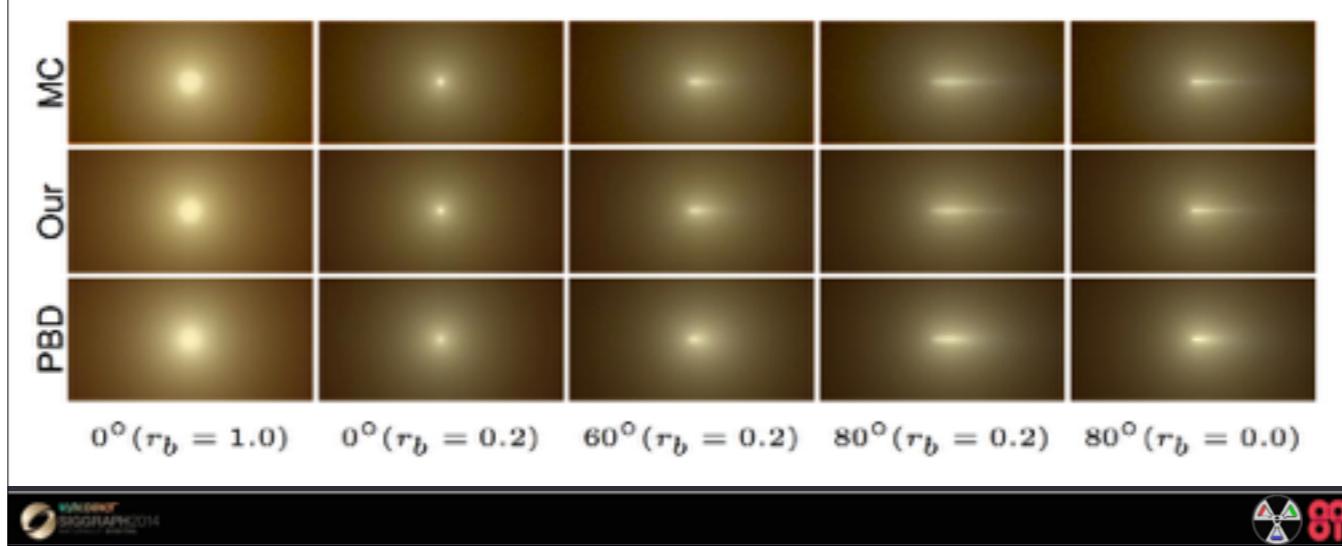


Dual-beam only



Here we show the improvement gained from only using the Davison-line-integral exitance calculation (but not the new source placements and uncollided term). We found both were required to get reasonable results over a variety of incident angles.

Results



Our new BSSRDF also produces more accurate BSSRDF behaviour than Photon Beam Diffusion.

Conclusion

- Dual-beam BSSRDF formulation
 - Reciprocal 8D
 - BRDF benchmark guided
 - Discarded diffusion



Future Work

- TODO
 - Moment driven?
 - Fresnel
 - Anisotropic scattering
 - Placzek inspired solutions for MC or curved geometry?
- Other related searchlight solutions
 - [Liemert and Kienle 2012,2013, Gardner 2013, Machida 2013]



It may be possible to derive the 4 BSSRDF parameters from moments of the BRDF solution, avoiding the non-linear optimization. We have yet to include Fresnel effects and it is unclear how to extend method-of-images solutions to anisotropic scattering. Further investigations along these lines should include performance and accuracy relationships to recent searchlight solutions such as those listed here.

THANK YOU

