Infinite 3D medium, Isotropic Point Source, Henyey-Greenstein Scattering

Exponential Random Flight

This is code to accompany the book:

A Hitchhiker’s Guide to Multiple Scattering

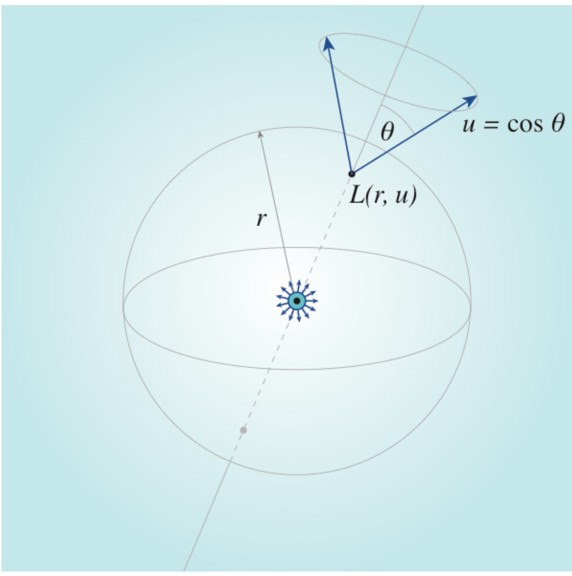
www.eugenedeon.com/hitchhikers

Path Setup

Put a file at ~/.hitchhikerpath with the path to your hitchhiker repo so that these worksheets can find the MC data from the C++ simulations for verification

```
SetDirectory[Import["~/.hitchhikerpath"]]
```

Notation



c - single-scattering albedo
Σt - extinction coefficient

r - radial position coordinate in medium (distance from point source at origin)
u = cos θ - direction cosine
b - anisotropy parameter

Namespace

In[5577]:=
```
Begin["inf3DisopointHGscatter`"]
```
Out[5577]=
```
inf3DisopointHGscatter`
```

Util

In[•]:=
$$\text{SA}[d\_, r\_] := d \frac{\text{Pi}^{d/2}}{\text{Gamma}\left[\frac{d}{2}+1\right]} r^{d-1}$$

Diffusion modes

In[•]:=
$$\text{diffusionMode}[v\_, d\_, r\_] := (2\pi)^{-d/2} r^{1-\frac{d}{2}} v^{-1-\frac{d}{2}} \text{BesselK}\left[\frac{1}{2}(-2+d), \frac{r}{v}\right]$$

Analytical solutions

Fluence: exact solution

[Grosjean 1963 - A New Approximate One-Velocity Theory for Treating both Isotropic and Anisotropic Multiple Scattering Problems, p. 37]

3-term expansion

In[5856]:=
$$\begin{aligned}&\phi\text{exactorder2}[r\_, \Sigma t\_, c\_, g\_] := \\ &\frac{\text{Exp}[-r\,\Sigma t]}{4\,\text{Pi}\,r^2} + \frac{c\,\Sigma t}{2\,\text{Pi}^2\,r}\,\text{NIntegrate}\Big[u\,\Big(\Big(-3\,g\,u^2\,\big(15\,g\,(-1+c\,g)+4\,u^2\big) + \\ &\quad 3\,g\,u\,\big(-15\,(-2+c)\,g\,(-1+c\,g)+\big(8-10\,g+c\,\big(-4+5\,g^2\big)\big)\,u^2\big)\,\text{ArcTan}[u] + \\ &\quad \big(12\,(-1+c)\,g\,u^2+4\,u^4+15\,(-1+c)\,c\,g^3\,\big(3+u^2\big)-5\,g^2\,\big(-3+3\,c-u^2\big)\,\big(3+u^2\big)\big) \\ &\quad \text{ArcTan}[u]^2\Big)\Big/ \\ &\quad \Big(u\,\Big(45\,(-1+c)\,c\,g^2\,(-1+c\,g)\,u+3\,c\,g\,(-4+4\,c+5\,g)\,u^3+4\,u^5-c\,\big(12\,(-1+c)\,g\,u^2+ \\ &\quad 4\,u^4+15\,(-1+c)\,c\,g^3\,\big(3+u^2\big)-5\,g^2\,\big(-3+3\,c-u^2\big)\,\big(3+u^2\big)\big)\,\text{ArcTan}[u]\Big)\Big)\Big) \\ &\quad \text{Sin}[r\,\Sigma t\,u], \{u, 0, \text{Infinity}\}, \text{Method} \to \text{"LevinRule"}\Big]\end{aligned}$$

4-term expansion

In[5877]:=
```
ϕexactorder3[r_, Σt_, c_, g_] := Exp[-r Σt]/(4 Pi r^2) + (c Σt)/(2 Pi^2 r)
  NIntegrate[u ((-g u^2 (-405 g u^2 + 108 u^4 - 105 c g^3 (15 + 4 u^2) - 105 c g^4 (15 + 4 u^2) +
          105 c^2 g^5 (15 + 4 u^2) + g^2 (1575 + 15 (56 + 27 c) u^2 + 112 u^4)) +
       g u (-108 (-2 + c) u^4 - 135 g u^2 (6 - 3 c + 2 u^2) + 35 c g^4
            (-90 + 45 c - 54 u^2 + 12 c u^2 - 4 u^4) + 105 c g^3 (-30 - 13 u^2 + c (15 + 4 u^2)) -
          105 c^2 g^5 (-30 - 13 u^2 + c (15 + 4 u^2)) + g^2 (-405 c^2 u^2 + 42
               (75 + 65 u^2 + 12 u^4) + c (-1575 - 30 u^2 + 23 u^4))) ArcTan[u] +
       3 (36 (-1 + c) g u^4 + 12 u^6 + 15 g^2 u^2 (3 + u^2) (3 - 3 c + u^2) - 105 (-1 + c) c
           g^4 (5 + 3 u^2) + 105 (-1 + c) c^2 g^6 (5 + 3 u^2) - 35 c g^5 (-3 + 3 c - u^2) (5 + 3 u^2) +
          g^3 (45 c^2 u^2 (3 + u^2) - 21 (5 + 3 u^2)^2 + c (525 + 320 u^2 + 39 u^4))) ArcTan[u]^2) /
     (u (u (36 u^6 + 105 c^4 g^6 (15 + 4 u^2) - 15 c^3 g^3 (-27 u^2 + 7 g (15 + 4 u^2) +
                7 g^2 (15 + 4 u^2) + 7 g^3 (15 + 4 u^2)) - 3 c g (36 u^4 - 45 g u^2 (3 + u^2) +
                7 g^2 (75 + 65 u^2 + 12 u^4)) + c^2 g (-405 g u^2 + 108 u^4 + 105 g^3 (15 + 4 u^2) +
                35 g^4 (45 + 27 u^2 + 4 u^4) + g^2 (1575 + 435 u^2 + 112 u^4))) -
          3 c (36 (-1 + c) g u^4 + 12 u^6 + 15 g^2 u^2 (3 + u^2) (3 - 3 c + u^2) -
             105 (-1 + c) c g^4 (5 + 3 u^2) + 105 (-1 + c) c^2 g^6 (5 + 3 u^2) -
             35 c g^5 (-3 + 3 c - u^2) (5 + 3 u^2) + g^3 (45 c^2 u^2 (3 + u^2) -
                21 (5 + 3 u^2)^2 + c (525 + 320 u^2 + 39 u^4))) ArcTan[u])))
    Sin[r Σt u], {u, 0, Infinity}, Method -> "LevinRule"]
```

5-term expansion

In[5879]:=
```
ϕexactorder4[r_, Σt_, c_, g_] :=
  Exp[-r Σt]/(4 Pi r^2) + (c Σt)/(2 Pi^2 r) NIntegrate[u ((-g u^2 (-6480 g u^4 + 1728 u^6 - 4725 c^2 g^7 (21 + 11 u^2) -
            4725 c^2 g^8 (21 + 11 u^2) + 4725 c^3 g^9 (21 + 11 u^2) - 315 c g^6 (-15 + 15 c - 4 u^2)
             (21 + 11 u^2) + 16 g^2 u^2 (1575 + 15 (56 + 27 c) u^2 + 112 u^4) +
            15 c g^4 (6615 + 3486 u^2 + 443 u^4) + 105 c g^5 (945 + 15 (33 + 16 c) u^2 + 64 c u^4) -
            15 g^3 (6615 + 210 (33 + 8 c) u^2 + (1815 + 448 c) u^4)) +
         g u (-1728 (-2 + c) u^6 - 2160 g u^4 (6 - 3 c + 2 u^2) + 315 c^2 g^8 (-630 + 315 c - 540
                 u^2 + 165 c u^2 - 82 u^4) + 945 c^2 g^7 (-210 - 145 u^2 - 9 u^4 + 5 c (21 + 11 u^2)) -
            945 c^3 g^9 (-210 - 145 u^2 - 9 u^4 + 5 c (21 + 11 u^2)) + 16 g^2 u^2
             (-405 c^2 u^2 + 42 (75 + 65 u^2 + 12 u^4) + c (-1575 - 30 u^2 + 23 u^4)) - 105 c g^5
             (16 c^2 u^2 (15 + 4 u^2) + c (945 + 15 u^2 - 208 u^4) - 6 (315 + 270 u^2 + 41 u^4)) +
            c g^4 (198450 + 137655 u^2 + 13500 u^4 - 53 u^6 - 15 c (6615 + 3486 u^2 + 443 u^4)) +
            63 c g^6 (75 c^2 (21 + 11 u^2) - 5 c (945 + 684 u^2 + 71 u^4) + 6
                 (525 + 590 u^2 + 165 u^4 + 6 u^6)) + 15 g^3 (112 c^2 u^2 (15 + 4 u^2) + c
                 (6615 + 3570 u^2 + 359 u^4) - 18 (735 + 1015 u^2 + 393 u^4 + 33 u^6)))
          ArcTan[u] + 3 (576 (-1 + c) g u^6 + 192 u^8 + 240 g^2 u^4 (3 + u^2) (3 - 3 c + u^2) -
            945 (-1 + c) c^2 g^8 (35 + 30 u^2 + 3 u^4) + 945 (-1 + c) c^3 g^10 (35 + 30 u^2 + 3 u^4) -
            315 c^2 g^9 (-3 + 3 c - u^2) (35 + 30 u^2 + 3 u^4) -
```

```
63 c g^7 (35 + 30 u^2 + 3 u^4) (15 + 15 c^2 + 9 u^2 - 2 c (15 + 2 u^2)) +
16 g^3 u^2 (45 c^2 u^2 (3 + u^2) - 21 (5 + 3 u^2)^2 + c (525 + 320 u^2 + 39 u^4)) + 105 c g^6
 (16 c^2 u^2 (5 + 3 u^2) + c (315 + 190 u^2 - 21 u^4) - 3 (105 + 125 u^2 + 39 u^4 + 3 u^6)) -
3 g^4 (560 c^2 u^2 (5 + 3 u^2) - 9 (35 + 30 u^2 + 3 u^4)^2 + 5 c
    (2205 + 2485 u^2 + 843 u^4 + 99 u^6)) + c g^5 (-33 075 - 28 455 u^2 - 2285
    u^4 + 951 u^6 + 3 c (11 025 + 9485 u^2 + 1695 u^4 + 243 u^6))) ArcTan[u]^2) /
(u (u (576 u^8 + 4725 c^5 g^10 (21 + 11 u^2) - 105 c^4 g^6 (-16 u^2 (15 + 4 u^2) +
    45 g (21 + 11 u^2) + 45 g^2 (21 + 11 u^2) + 45 g^3 (21 + 11 u^2) +
    45 g^4 (21 + 11 u^2)) + 15 c^3 g^3 (432 u^4 - 112 g u^2 (15 + 4 u^2) + 315 g^5
     (21 + 11 u^2) + 105 g^6 (63 + 54 u^2 + 11 u^4) + 42 g^4 (315 + 207 u^2 + 22 u^4) -
    7 g^3 (-945 - 255 u^2 + 64 u^4) + g^2 (6615 + 3486 u^2 + 443 u^4)) +
  3 c g (-576 u^6 + 720 g u^4 (3 + u^2) - 112 g^2 u^2 (75 + 65 u^2 + 12 u^4) +
    45 g^3 (735 + 1015 u^2 + 393 u^4 + 33 u^6)) - c^2 g (6480 g u^4 - 1728 u^6 +
    1575 g^5 (63 + 54 u^2 + 11 u^4) + 945 g^6 (105 + 118 u^2 + 33 u^4) -
    16 g^2 u^2 (1575 + 435 u^2 + 112 u^4) + 15 g^3 (6615 + 5250 u^2 + 1367 u^4) -
    5 g^4 (-19 845 - 10 458 u^2 + 351 u^4 + 448 u^6))) -
 3 c (576 (-1 + c) g u^6 + 192 u^8 + 240 g^2 u^4 (3 + u^2) (3 - 3 c + u^2) -
  945 (-1 + c) c^2 g^8 (35 + 30 u^2 + 3 u^4) + 945 (-1 + c) c^3 g^10 (35 + 30 u^2 + 3 u^4) -
  315 c^2 g^9 (-3 + 3 c - u^2) (35 + 30 u^2 + 3 u^4) -
  63 c g^7 (35 + 30 u^2 + 3 u^4) (15 + 15 c^2 + 9 u^2 - 2 c (15 + 2 u^2)) +
  16 g^3 u^2 (45 c^2 u^2 (3 + u^2) - 21 (5 + 3 u^2)^2 + c (525 + 320 u^2 + 39 u^4)) +
  105 c g^6 (16 c^2 u^2 (5 + 3 u^2) + c (315 + 190 u^2 - 21 u^4) - 3 (105 + 125 u^2 +
       39 u^4 + 3 u^6)) - 3 g^4 (560 c^2 u^2 (5 + 3 u^2) - 9 (35 + 30 u^2 + 3 u^4)^2 +
    5 c (2205 + 2485 u^2 + 843 u^4 + 99 u^6)) + c g^5 (-33 075 - 28 455 u^2 - 2285
     u^4 + 951 u^6 + 3 c (11 025 + 9485 u^2 + 1695 u^4 + 243 u^6))) ArcTan[u])))
Sin[r Σt u], {u, 0, Infinity}, Method → "LevinRule"]
```

load MC data

In[5594]:=
```
ppoints[xs_, dr_, maxx_] :=
  Table[{dr (i) - 0.5 dr, xs[[i]]}, {i, 1, Length[xs]}][[1 ;; -2]]
```

In[5595]:=
```
ppointsu[xs_, du_, Σt_] :=
  Table[{-1.0 + du (i) - 0.5 du, xs[[i]] / (2 Σt)}, {i, 1, Length[xs]}][[1 ;; -1]]
```

In[5596]:=
```
fs = FileNames["code/3D_medium/infinite3Dmedium/Isotropicpointsource/MCdata/
    inf3D_isotropicpoint_HG_*"];
```

In[5597]:=
```
index[x_] := Module[{data, c, mfp, g},
    data = Import[x, "Table"];
    mfp = data[[1, 13]];
    c = data[[2, 3]];
    g = data[[1, 16]];
    {c, mfp, g, data}];
simulations = index /@ fs;
cs = Union[#[[1]] & /@ simulations]
```

Out[5599]= {0.01, 0.1, 0.3, 0.5, 0.7, 0.8, 0.9, 0.95, 0.99, 0.999}

In[5600]:=
```
mfps = Union[#[[2]] & /@ simulations]
```

Out[5600]= {0.3, 1}

In[5601]:=
```
gs = Union[#[[3]] & /@ simulations]
```

Out[5601]= {-0.5, 0.3, 0.5, 0.7}

In[5602]:=
```
numcollorders = inf3Disopointlinanisoscatter`simulations[[1]][[-1]][[2, 13]];
```

Compare Deterministic and MC

In[5605]:=
```
Clear[g]
```

Mean Track Length

In[5614]:=
```
{{ActionMenu["Set c", "c = " <> ToString[#] :> (c = #;) & /@ cs], Dynamic[c]},
  {ActionMenu["Set mfp", "mfp = " <> ToString[#] :> (mfp = #;) & /@ mfps],
   Dynamic[mfp]},
  {ActionMenu["Set g", "g = " <> ToString[#] :> (g = #;) & /@ gs], Dynamic[g]}}
```

Out[5614]= {{ Set c , 0.95}, { Set mfp , 0.3}, { Set g , g}}

In[5615]:=
```
data = SelectFirst[simulations, #[[1]] == c && #[[2]] == mfp && #[[3]] == g &][[4]];
meanTL = data[[-1]]
mfp
───
1 - c
```

Out[5616]= {Mean, track, length:, 2.00001}

Out[5617]= 2.

Fluence - Exact solution (1) comparison to MC

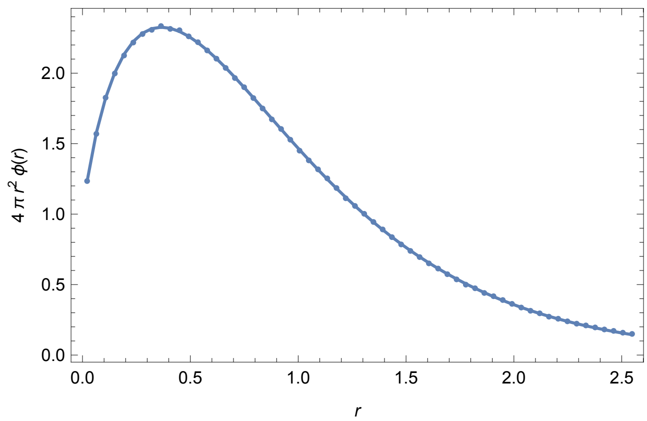
In[5618]:=
```
{{ActionMenu["Set c", "c = " <> ToString[#] :> (c = #;) & /@ cs], Dynamic[c]},
  {ActionMenu["Set mfp", "mfp = " <> ToString[#] :> (mfp = #;) & /@ mfps],
   Dynamic[mfp]},
  {ActionMenu["Set g", "g = " <> ToString[#] :> (g = #;) & /@ gs], Dynamic[g]}}
```

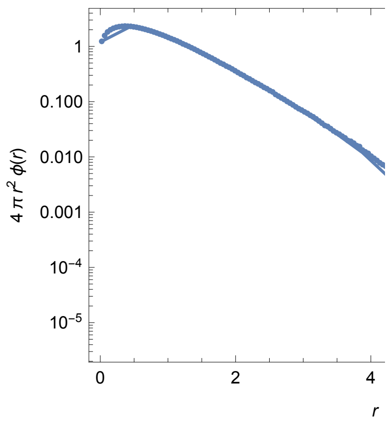
Out[5618]= {{ Set c , 0.95}, { Set mfp , 0.3}, { Set g , g}}

In[5907]:=
```
data = SelectFirst[simulations, #[[1]] == c && #[[2]] == mfp && #[[3]] == g &][[4]];
maxr = data[[2, 5]];
dr = data[[2, 7]];
pointsFluence = ppoints[data[[6]], dr, maxr];
exact1FluenceShallow =
  Quiet[{#[[1]], 4 Pi #[[1]]^2 ϕexactorder4[#[[1]], 1/mfp, c, g]}] & /@
   pointsFluence[[1 ;; 60]];
exact1Fluence = Quiet[{#[[1]], 4 Pi #[[1]]^2 ϕexactorder4[#[[1]], 1/mfp, c, g]}] & /@
   pointsFluence[[1 ;; -1 ;; 10]];
plotϕshallow = Quiet[Show[
    ListPlot[pointsFluence[[1 ;; 60]],
     PlotRange → All, PlotStyle → PointSize[.01]],
    ListPlot[exact1FluenceShallow, PlotRange → All, Joined → True],
    Frame → True,
    FrameLabel -> {{4 Pi r^2 ϕ[r],}, {r,}}
   ]];
logplotϕ = Quiet[Show[
    ListLogPlot[pointsFluence, PlotRange → All, PlotStyle → PointSize[.01]],
    ListLogPlot[exact1Fluence, PlotRange → All, Joined → True],
    Frame → True,
    FrameLabel -> {{4 Pi r^2 ϕ[r],}, {r,}}
   ]];
Show[GraphicsGrid[{{plotϕshallow, logplotϕ}}, ImageSize → 800],
 PlotLabel -> "Exact solution - expansion order 2\nInfinite 3D, isotropic point
    source, Henyey-Greenstein scattering, fluence ϕ[r], c = " <>
   ToString[c] <> ", Σt = " <> ToString[1/mfp] <> ", g = " <> ToString[g]]
```

Out[5915]=

Exact solution – expansion order 2
Infinite 3D, isotropic point source, Henyey–Greenstein scattering, fluence ϕ[r], c = 0.9, Σ_t = 3.33333, g





Close namespace

In[∘]:=
```
End[]
```