Generalized Exponential NDF

This is code to accompany the book:

A Hitchhiker’s Guide to Multiple Scattering

www.eugenedeon.com/hitchhikers

In[2747]:=
$$\texttt{GenExp`D[u\_, }\alpha\texttt{\_, }\gamma\texttt{\_] :=} \frac{e^{-(1-u^2)^{\gamma/2}\,(u\,\alpha)^{-\gamma}}}{\pi\, u^4\, \alpha^2\, \texttt{Gamma}\left[\frac{2+\gamma}{\gamma}\right]}\ \texttt{HeavisideTheta[u]}$$

In[2347]:=
```
GenExp`D[u, α, 2] == Beckmann`D[u, α] // FullSimplify
```

Out[2347]=
```
True
```

In[2420]:=
```
GenExp`D[u, α, 1] == Exponential`D[u, 2 α] // FullSimplify
```

Out[2420]=
```
True
```

distribution of slopes

In[2352]:=
$$\texttt{FullSimplify}\left[\texttt{GenExp`D}\left[\frac{1}{\sqrt{p^2+q^2+1}}, \alpha, \gamma\right] \left(\frac{1}{\sqrt{p^2+q^2+1}}\right)^4,\ \texttt{Assumptions} \to 0 < \alpha < 1\ \&\&\ \gamma > 0\ \&\&\ q > 0\right]$$

Out[2352]=
$$\frac{e^{-\left(1-\frac{1}{1+p^2+q^2}\right)^{\gamma/2} \left(\frac{\alpha}{\sqrt{1+p^2+q^2}}\right)^{-\gamma}}}{\pi\, \alpha^2\, \texttt{Gamma}\left[\frac{2+\gamma}{\gamma}\right]}$$

In[2353]:=
$$\texttt{GenExp`P22[p\_, q\_, }\alpha\texttt{\_, }\gamma\texttt{\_] :=} \frac{e^{-\left(1-\frac{1}{1+p^2+q^2}\right)^{\gamma/2} \left(\frac{\alpha}{\sqrt{1+p^2+q^2}}\right)^{-\gamma}}}{\pi\, \alpha^2\, \texttt{Gamma}\left[\frac{2+\gamma}{\gamma}\right]}$$

In[2354]:=
```
Integrate[GenExp`P22[p, q, α, γ], {p, -Infinity, Infinity},
  {q, -Infinity, Infinity}, Assumptions → 0 < α < 1 && γ > 0]
```

Out[2354]=
```
1
```

In[2368]:=
```
Integrate[GenExp`P22[p, q, α, 2],
  {q, -Infinity, Infinity}, Assumptions → α > 0 && p > 0 && γ > 0]
```

Out[2368]=
$$\frac{e^{-\frac{p^2}{\alpha^2}}}{\sqrt{\pi}\ \alpha}$$

In[2360]:=
```
Integrate[GenExp`P22[p, q, α, 4],
 {q, -Infinity, Infinity}, Assumptions → α > 0 && p > 0 && γ > 0]
```

Out[2360]= $\frac{\sqrt{2}\, e^{-\frac{p^4}{2\alpha^4}}\, p\, \mathrm{BesselK}\left[\frac{1}{4}, \frac{p^4}{2\alpha^4}\right]}{\pi^{3/2}\alpha^2}$

derivation

In[•]:= $f[x\_] := \mathrm{Exp}[-x^p]$

In[•]:= $\mathrm{FullSimplify}\left[\frac{f\left[\frac{\sqrt{1-u^2}}{\alpha u}\right]}{\alpha^2 u^4}\left(\frac{1}{\sqrt{p^2+q^2+1}}\right)^4 \text{ /. } u \text{ -> } \frac{1}{\sqrt{p^2+q^2+1}} \text{ /. } p^2+q^2 \to r^2,\right.$
 $\left.\mathrm{Assumptions} \to r > 0 \,\&\&\, p > 0 \,\&\&\, \alpha > 0\right]$

Out[•]= $\frac{e^{-\left(\frac{r}{\alpha}\right)^p}}{\alpha^2}$

In[•]:= $\mathrm{Integrate}\left[2\,\mathrm{Pi}\, r\,\frac{e^{-\left(\frac{r}{\alpha}\right)^p}}{\alpha^2}, \{r, 0, \mathrm{Infinity}\}\right]$

Out[•]= $\mathrm{ConditionalExpression}\left[\frac{2\pi\left(\left(\frac{1}{\alpha}\right)^p\right)^{-2/p}\mathrm{Gamma}\left[\frac{2}{p}\right]}{p\,\alpha^2}, \mathrm{Re}[p] > 0 \,\&\&\, \mathrm{Re}\left[\left(\frac{1}{\alpha}\right)^p\right] > 0\right]$

In[2699]:= $f[x\_, \alpha\_, p\_] := \frac{e^{-x^p}}{\pi\,\mathrm{Gamma}\left[\frac{2+p}{p}\right]}$

In[2700]:= $\mathrm{FullSimplify}\left[\frac{f\left[\frac{\sqrt{1-u^2}}{\alpha u}, \alpha, \gamma\right]}{\alpha^2 u^4}, \mathrm{Assumptions} \to 0 < u < 1 \,\&\&\, \gamma > 0 \,\&\&\, \alpha > 0\right]$

Out[2700]= $\frac{e^{-\left(1-u^2\right)^{\gamma/2}(u\,\alpha)^{-\gamma}}}{\pi\, u^4\,\alpha^2\,\mathrm{Gamma}\left[\frac{2+\gamma}{\gamma}\right]}$