Infinite 3D medium, Isotropic Point Source, Linearly-Anisotropic Scattering

Gamma-2 Random Flight

This is code to accompany the book:

A Hitchhiker’s Guide to Multiple Scattering

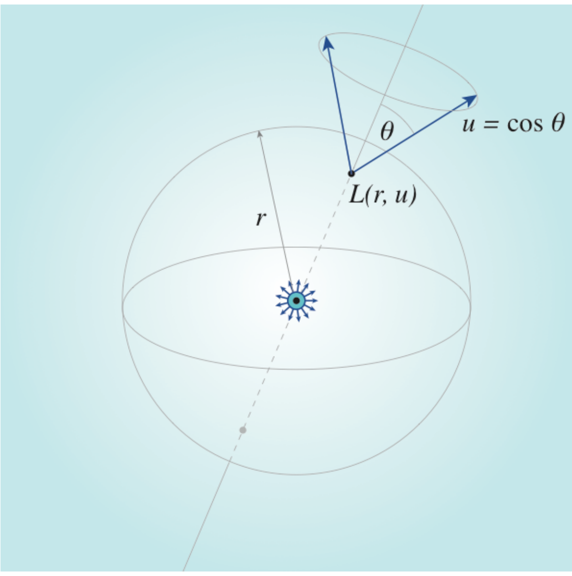
www.eugenedeon.com/hitchhikers

Path Setup

Put a file at ~/.hitchhikerpath with the path to your hitchhiker repo so that these worksheets can find the MC data from the C++ simulations for verification

```
SetDirectory[Import["~/.hitchhikerpath"]]
```

Notation



c - single-scattering albedo
Σt - extinction coefficient

r - radial position coordinate in medium (distance from point source at origin)
u = cos θ - direction cosine
b - anisotropy parameter

Namespace

In[9568]:=
```
Begin["inf3DisopointlinanisoscatterGamma4`"]
```
Out[9568]=
```
inf3DisopointlinanisoscatterGamma4`
```

Analytical results

Collision rate density

collision rate density Cc due to correlated emission:

derivation

In[9571]:=
```
f00 = Fpc[0, 0, 1/6 Exp[-#] #^3 &];
f01 = Fpc[0, 1, 1/6 Exp[-#] #^3 &];
f11 = Fpc[1, 1, 1/6 Exp[-#] #^3 &];
o = 2;
Clear[A, b, c, r, h];
A[0] := 1; A[1] := b;
hsystem = Table[
   h[k] == 2/Pi c u F[k, 0] + c Sum[A[m] × h[m] × F[k, m], {m, 0, o - 1}], {k, 0, o - 1}];
hsystemsolve = Simplify[Solve[hsystem, Table[h[i], {i, 0, o - 1}]] /. A[1] → b /.
     F[0, 0] → f00 /. F[0, 1] → f01 /. F[1, 1] → f11 /. F[1, 0] → -f01]
```
Out[9578]=
$$\left\{\left\{h[0] \to -\frac{2\,c\,u\,\left(-3+b\,c-2\,u^2+u^4\right)}{\pi\,\left(b\,c^2+3\,\left(1+u^2\right)^4+c\,\left(1+u^2\right)\,\left(-3+u^2+b\,\left(-1+3\,u^2\right)\right)\right)},\right.\right.$$
$$\left.\left.h[1] \to \frac{8\,c\,u^2\,\left(1+u^2\right)}{\pi\,\left(b\,c^2+3\,\left(1+u^2\right)^4+c\,\left(1+u^2\right)\,\left(-3+u^2+b\,\left(-1+3\,u^2\right)\right)\right)}\right\}\right\}$$

In[9579]:=
```
Clear[r];
(2 k + 1) 1/(4 Pi r c) (h[k]) j2[k, r u] /. k → 0 /. hsystemsolve // FullSimplify
```
Out[9579]=
$$\left\{-\frac{u\,\left(-3+b\,c-2\,u^2+u^4\right)\,\mathrm{Sin}[r\,u]}{2\,\pi^2\,r\,\left(b\,c^2+3\,\left(1+u^2\right)^4+c\,\left(1+u^2\right)\,\left(-3+u^2+b\,\left(-1+3\,u^2\right)\right)\right)}\right\}$$

result

In[9580]:=
```
Ccexact[r_, t_, c_, b_] :=
 NIntegrate[-(u (-3 + b c - 2 u^2 + u^4) Sin[r u])/(2 π^2 r (b c^2 + 3 (1 + u^2)^4 + c (1 + u^2) (-3 + u^2 + b (-1 + 3 u^2)))),
  {u, 0, Infinity}, Method → "LevinRule"]
```

In[*]:=
```
With[{c = 0.8, b = 0.5},
 Integrate[-(u (-3 + b c - 2 u^2 + u^4) Sin[r u])/(2 π^2 r (b c^2 + 3 (1 + u^2)^4 + c (1 + u^2) (-3 + u^2 + b (-1 + 3 u^2)))),
   {u, 0, Infinity}, Assumptions → r > 0] // Chop // FullSimplify
]
```

Out[*]=

$$\frac{1}{r}\left(\left(-0.011874-0.0251837\, i\right) e^{(-1.3078-0.448857\, i)\, r}-\left(0.011874-0.0251837\, i\right) e^{(-1.3078+0.448857\, i)\, r}+0.00229035\, e^{-0.965068\, r}+0.0214577\, e^{-0.225652\, r}\right)$$

In[9581]:=
```
TraditionalForm[HoldForm[Cc[r] = Integrate[
   -(u (-3 + b c - 2 u^2 + u^4) Sin[r u])/(2 π^2 r (b c^2 + 3 (1 + u^2)^4 + c (1 + u^2) (-3 + u^2 + b (-1 + 3 u^2)))), {u, 0, Infinity}]]]
```

Out[9581]//TraditionalForm=

$$C_c(r)=\int_0^{\infty}-\frac{u\left(-3+b\,c-2\,u^2+u^4\right)\sin(r\,u)}{2\,\pi^2\,r\left(b\,c^2+3\left(1+u^2\right)^4+c\left(1+u^2\right)\left(-3+u^2+b\left(-1+3\,u^2\right)\right)\right)}\,du$$

load MC data

In[9621]:=
```
ppoints[xs_, dr_, maxx_] :=
 Table[{dr (i) - 0.5 dr, xs[[i]]}, {i, 1, Length[xs]}][[1 ;; -2]]
```

In[9622]:=
```
ppointsu[xs_, du_, Σt_] :=
 Table[{-1.0 + du (i) - 0.5 du, xs[[i]] / (2 Σt)}, {i, 1, Length[xs]}][[1 ;; -1]]
```

In[9623]:=
```
fs = FileNames["code/3D_medium/infinite3Dmedium/Isotropicpointsource/MCdata/
    inf3D_isotropicpoint_linanisoscatter_gamma4C*"];
```

In[9624]:=
```
index[x_] := Module[{data, c, mfp, b},
   data = Import[x, "Table"];
   mfp = data[[1, 13]];
   c = data[[2, 3]];
   b = data[[1, 16]];
   {c, mfp, b, data}];
simulations = index /@ fs;
cs = Union[#[[1]] & /@ simulations]
```

Out[9626]=
```
{0.01, 0.1, 0.3, 0.5, 0.7, 0.8, 0.9, 0.95, 0.99, 0.999}
```

In[9627]:=
```
mfps = Union[#[[2]] & /@ simulations]
```
Out[9627]=
```
{0.3, 1}
```
In[9628]:=
```
bs = Union[#[[3]] & /@ simulations]
```
Out[9628]=
```
{-0.9, 0.7}
```
In[9629]:=
```
numcollorders = inf3Disopointlinanisoscatter`simulations[[1]][[-1]][[2, 13]];
```

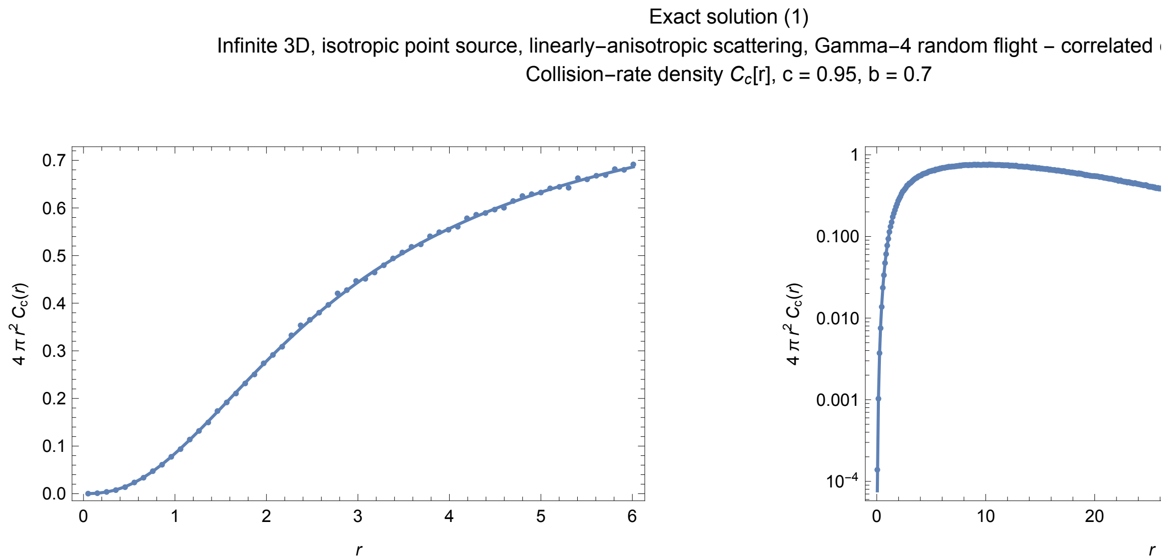
Compare analytic and MC

Collision-rate density - Exact solution (1) comparison to MC

In[9630]:=
```
{{ActionMenu["Set c", "c = " <> ToString[#] :→ (c = #;) & /@ cs], Dynamic[c]},
 {ActionMenu["Set mfp", "mfp = " <> ToString[#] :→ (mfp = #;) & /@ mfps],
  Dynamic[mfp]},
 {ActionMenu["Set b", "b = " <> ToString[#] :→ (b = #;) & /@ bs], Dynamic[b]}}
```
Out[9630]=
```
{{Set c, 0.95}, {Set mfp, 1}, {Set b, 0.7}}
```

In[9631]:=
```
data = SelectFirst[simulations, #[[1]] == c && #[[2]] == mfp && #[[3]] == b &][[4]];
maxr = data[[2, 5]];
dr = data[[2, 7]];
MCCollisionRate = ppoints[data[[4]], dr, maxr];
exact1CRShallow = Quiet[{#[[1]], 4 Pi #[[1]]^2 Ccexact[#[[1]], 1/mfp, c, b]}] & /@
   MCCollisionRate[[1 ;; 60]];
exact1CR = Quiet[{#[[1]], 4 Pi #[[1]]^2 Ccexact[#[[1]], 1/mfp, c, b]}] & /@
   MCCollisionRate[[61 ;; -1 ;; 10]];
plotϕshallow = Quiet[Show[
     ListPlot[MCCollisionRate[[1 ;; 60]],
      PlotRange → All, PlotStyle → PointSize[.01]],
     ListPlot[exact1CRShallow, PlotRange → All, Joined → True],
     Frame → True,
     FrameLabel -> {{4 π r^2 C"c"[r],}, {r,}}
    ]];
logplotϕ = Quiet[Show[
     ListLogPlot[MCCollisionRate, PlotRange → All, PlotStyle → PointSize[.01]],
     ListLogPlot[exact1CR, PlotRange → All, Joined → True],
     ListLogPlot[exact1CRShallow, PlotRange → All, Joined → True],
     Frame → True,
     FrameLabel -> {{4 π r^2 C"c"[r],}, {r,}}
    ]];
Show[GraphicsGrid[{{plotϕshallow, logplotϕ}}, ImageSize → 800],
 PlotLabel -> "Exact solution (1)\nInfinite 3D, isotropic point source,
     linearly-anisotropic scattering, Gamma-4 random flight -
     correlated emission\nCollision-rate density Cc[r], c = " <>
   ToString[c] <> ", b = " <> ToString[b]]
```

Out[9639]=



Collision-rate density - Exact solution (2) comparison to MC

In[•]:=
```
{{ActionMenu["Set c", "c = " <> ToString[#] :> (c = #;) & /@ cs], Dynamic[c]},
 {ActionMenu["Set mfp", "mfp = " <> ToString[#] :> (mfp = #;) & /@ mfps],
  Dynamic[mfp]},
 {ActionMenu["Set b", "b = " <> ToString[#] :> (b = #;) & /@ bs], Dynamic[b]}}
```

Out[•]=
```
{{ Set c , 0.95}, { Set mfp , 1}, { Set b , 0.7}}
```

In[9664]:=
```
data = SelectFirst[simulations, #[[1]] == c && #[[2]] == mfp && #[[3]] == b &][[4]];
maxr = data[[2, 5]];
dr = data[[2, 7]];
MCCollisionRate = ppoints[data[[4]], dr, maxr];
CCexactfr = FullSimplify[
```

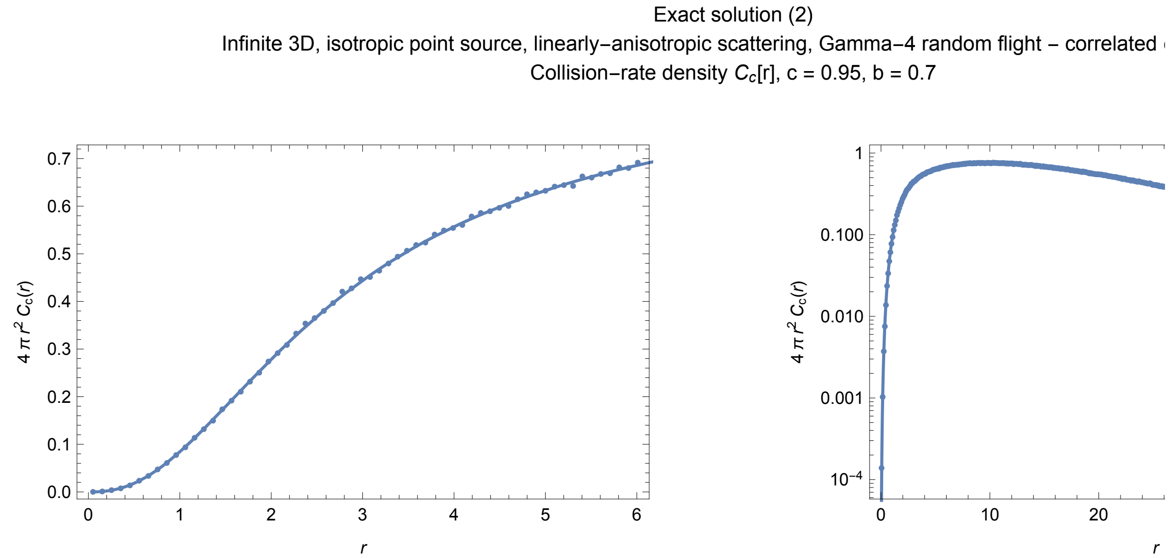
$$\text{Chop}\left[\text{Integrate}\left[-\frac{u\left(-3+b\,c-2\,u^2+u^4\right)\,\text{Sin}[r\,u]}{2\,\pi^2\,r\left(b\,c^2+3\left(1+u^2\right)^4+c\left(1+u^2\right)\left(-3+u^2+b\left(-1+3\,u^2\right)\right)\right)},\ \{u,0,\text{Infinity}\},\ \text{Assumptions}\to r>0\right]\right]\right]$$

```
plotϕshallow = Quiet[Show[
    ListPlot[MCCollisionRate[[1 ;; 60]],
     PlotRange → All, PlotStyle → PointSize[.01]],
    Plot[4 Pi r^2 CCexactfr, {r, 0, maxr}, PlotRange → All],
    Frame → True,
    FrameLabel -> {{4 π r^2 C"c"[r],}, {r,}}
   ]];
logplotϕ = Quiet[Show[
    ListLogPlot[MCCollisionRate, PlotRange → All, PlotStyle → PointSize[.01]],
    LogPlot[4 Pi r^2 CCexactfr, {r, 0, maxr}, PlotRange → All],
    Frame → True,
    FrameLabel -> {{4 π r^2 C"c"[r],}, {r,}}
   ]];
Show[GraphicsGrid[{{plotϕshallow, logplotϕ}}, ImageSize → 800],
 PlotLabel -> "Exact solution (2)\nInfinite 3D, isotropic point source,
   linearly-anisotropic scattering, Gamma-4 random flight -
   correlated emission\nCollision-rate density Cc[r], c = " <>
  ToString[c] <> ", b = " <> ToString[b]]
```

Out[9668]=

$$\frac{1}{r}\left(\left(-0.0101764-0.0204849\,i\right)e^{(-1.33922-0.497388\,i)\,r}-\left(0.0101764-0.0204849\,i\right)e^{(-1.33922+0.497388\,i)\,r}+0.00360865\,e^{-0.947286\,r}+0.0167441\,e^{-0.102038\,r}\right)$$

Out[9671]=



Namespace

In[9672]:= `End[]`

Out[9672]= `inf3DisopointlinanisoscatterGamma4``