Scattering Kernels in 3D

This is code to accompany the book:

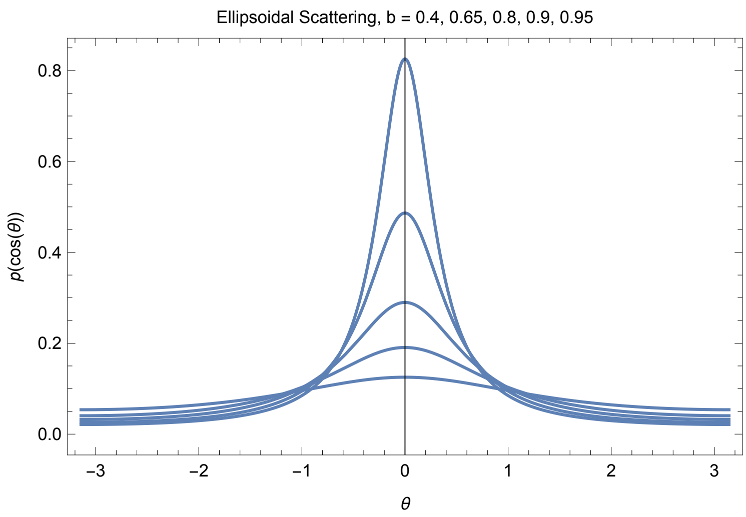
A Hitchhiker's Guide to Multiple Scattering

www.eugenedeon.com/hitchhikers

Kagiwada-Kalaba (Ellipsoidal) Scattering

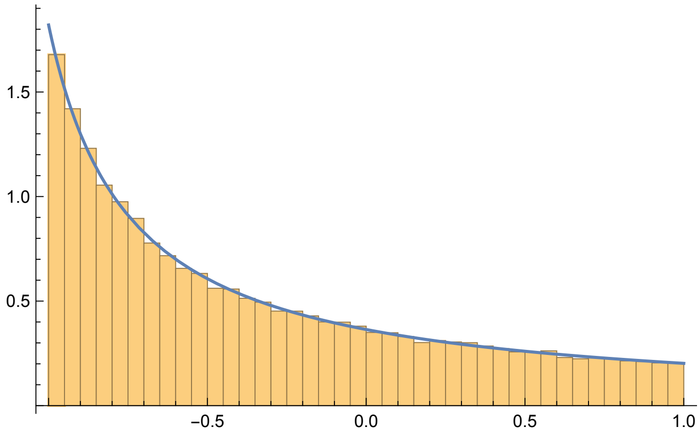
```
In[]:= pEllipsoidal[u_, b_] := b (2 Pi Log[(1 + b) / (1 - b)] (1 - b u))^-1

pEllplot = Show[
  Plot[pEllipsoidal[Cos[t], .9], {t, -Pi, Pi}, PlotRange → All],
  Plot[pEllipsoidal[Cos[t], .8], {t, -Pi, Pi}, PlotRange → All],
  Plot[pEllipsoidal[Cos[t], .65], {t, -Pi, Pi}, PlotRange → All],
  Plot[pEllipsoidal[Cos[t], .4], {t, -Pi, Pi}, PlotRange → All],
  Plot[pEllipsoidal[Cos[t], .95], {t, -Pi, Pi}, PlotRange → All],
  Frame → True,
  ImageSize → 400,
  FrameLabel → {{p[Cos[θ]],},
    {θ, "Ellipsoidal Scattering, b = 0.4, 0.65, 0.8, 0.9, 0.95"}}]
```



sampling

```
b = -0.8;
Show[Histogram[
  Map[(1 - (1 + b) ((1+b)/(1-b))^-#)/b &, Table[RandomReal[], {i, 1, 100 000}]], 50, "PDF"],
 Plot[2 Pi pEllipsoidal[u, b], {u, -1, 1}]
]
Clear[b];
```



When cosine u has been sampled with random variable ξ, what is the PDF at the sampled direction in terms of ξ?

In[∘]:=
```
FullSimplify[pEllipsoidal[(1 - (1 + b) ((1+b)/(1-b))^-#)/b &[ξ], b],
 Assumptions → 0 < b < 1 && 0 < ξ < 1]
```

Out[∘]= $$\frac{(1-b)^{-\xi}\, b\, (1+b)^{-1+\xi}}{2\pi \operatorname{Log}\left[\frac{1+b}{1-b}\right]}$$

Expansion coefficients

In[∘]:=
```
Integrate[2 Pi (2 k + 1) pEllipsoidal[u, b] LegendreP[k, u] /. k → 0,
  {u, -1, 1}, Assumptions → 0 < b < 1] /. Log[(1 + b)/(1 - b)] -> 2 ArcTanh[b]
```

Out[∘]= 1

In[∘]:=
```
Integrate[2 Pi (2 k + 1) pEllipsoidal[u, b] LegendreP[k, u] /. k → 1, {u, -1, 1},
   Assumptions → 0 < b < 1] /. Log[(1 + b)/(1 - b)] -> 2 ArcTanh[b] // FullSimplify
```

Out[∘]= $$\frac{3}{b} - \frac{3}{\operatorname{ArcTanh}[b]}$$

In[•]:=
```
Integrate[2 Pi (2 k + 1) pEllipsoidal[u, b] LegendreP[k, u] /. k → 2, {u, -1, 1},
    Assumptions → 0 < b < 1] /. Log[(1 + b)/(1 - b)] -> 2 ArcTanh[b] // FullSimplify
```

Out[•]=
$$\frac{5}{2}\left(-1+\frac{3}{b^2}-\frac{3}{b\,\text{ArcTanh}[b]}\right)$$

In[•]:=
```
Integrate[2 Pi (2 k + 1) pEllipsoidal[u, b] LegendreP[k, u] /. k → 3, {u, -1, 1},
    Assumptions → 0 < b < 1] /. Log[(1 + b)/(1 - b)] -> 2 ArcTanh[b] // FullSimplify
```

Out[•]=
$$\frac{7\left(b\left(-15+4\,b^2\right)+\left(15-9\,b^2\right)\text{ArcTanh}[b]\right)}{6\,b^3\,\text{ArcTanh}[b]}$$

In[•]:=
```
Integrate[2 Pi (2 k + 1) pEllipsoidal[u, b] LegendreP[k, u] /. k → 4, {u, -1, 1},
    Assumptions → 0 < b < 1] /. Log[(1 + b)/(1 - b)] -> 2 ArcTanh[b] // FullSimplify
```

Out[•]=
$$\frac{15\,b\left(-21+11\,b^2\right)+9\left(35-30\,b^2+3\,b^4\right)\text{ArcTanh}[b]}{8\,b^4\,\text{ArcTanh}[b]}$$