Scattering Kernels in 3D

This is code to accompany the book:

A Hitchhiker's Guide to Multiple Scattering

www.eugenedeon.com/hitchhikers

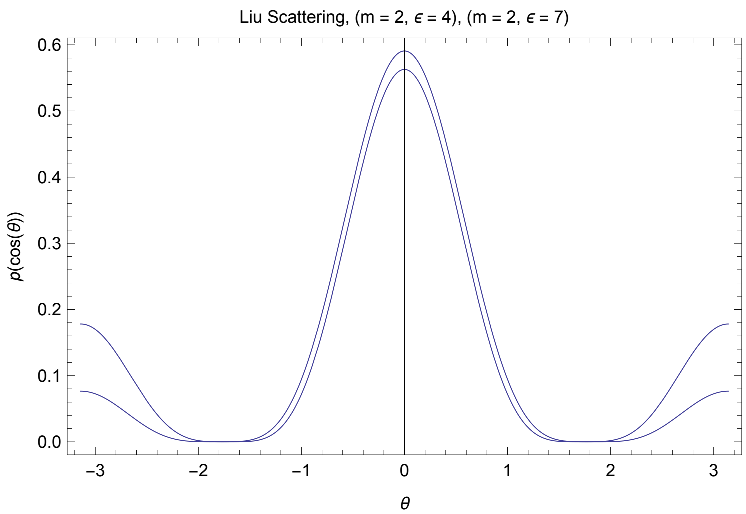
Liu Scattering

In[●]:=
```
pLiu[u_, e_, m_] := e (2 m + 1) (1 + e u)^(2 m) / (2 Pi ((1 + e)^(2 m+1) - (1 - e)^(2 m+1)))
```

$$pLiu[u\_, e\_, m\_] := \frac{e\,(2\,m+1)\,(1+e\,u)^{2\,m}}{2\,\mathrm{Pi}\,\left((1+e)^{2\,m+1}-(1-e)^{2\,m+1}\right)}$$

```
Clear[m]

pLiuplot = Show[
  Plot[pLiu[Cos[t], 4, 2], {t, -Pi, Pi}, PlotRange → All],
  Plot[pLiu[Cos[t], 7, 2], {t, -Pi, Pi}, PlotRange → All],
  Frame → True,
  ImageSize → 400,
  FrameLabel →
   {{p[Cos[θ]],}, {θ, "Liu Scattering, (m = 2, ϵ = 4), (m = 2, ϵ = 7)"}}]
```



Normalization condition

```
Integrate[2 Pi pLiu[u, e, m], {u, -1, 1}, Assumptions → e > 0 && m > 0 && m ∈ Integers]
1
```

Mean cosine (g)

```
Integrate[2 Pi u pLiu[u, e, m], {u, -1, 1},
 Assumptions → e > 0 && m > 0 && m ∈ Integers && e < 1]
```

$$\frac{(1+e)^{1+2m}\left(-1+e+2em\right)+(1-e)^{1+2m}\left(1+e+2em\right)}{2e\left(-(1-e)^{1+2m}+(1+e)^{1+2m}\right)(1+m)}$$

Legendre expansion coefficients

```
Integrate[2 Pi (2 k + 1) pLiu[u, e, m] LegendreP[k, u] /. k → 0, {u, -1, 1},
 Assumptions → m > 0 && m ∈ Integers && e ∈ Reals && e ≠ 0 && Abs[e] < 1]
```

1

```
Integrate[2 Pi (2 k + 1) pLiu[u, e, m] LegendreP[k, u] /. k → 2, {u, -1, 1},
 Assumptions → m > 0 && m ∈ Integers && e ∈ Reals && e ≠ 0 && Abs[e] < 1]
```

$$\left(5\left((1+e)^{1+2m}\left(3+e\left(-3+2m\left(-3+2e(1+m)\right)\right)\right)+(1-e)^{2m}(-1+e)\left(3+e\left(3+2m\left(3+2e(1+m)\right)\right)\right)\right)\right)\Big/\left(2e^2\left(-(1-e)^{1+2m}+(1+e)^{1+2m}\right)(1+m)(3+2m)\right)$$

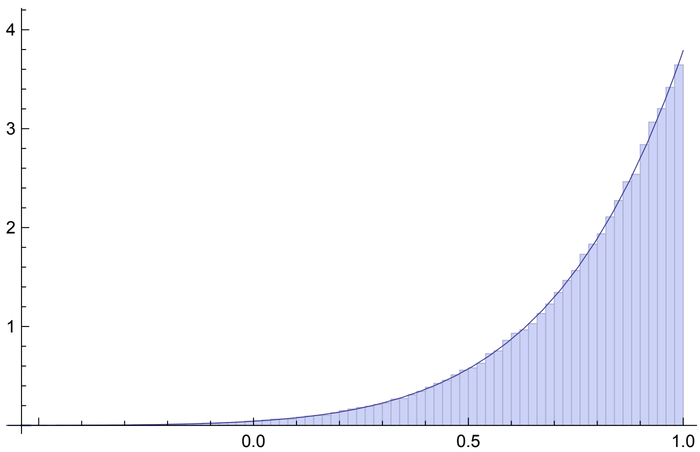
sampling

```
m = 3.5;
ϵ = 0.9;
```

$$\text{Show}\left[\text{Histogram}\left[\text{Map}\left[\frac{-1+\left((-1+\#)(1-\epsilon)^{2m}(-1+\epsilon)+\#(1+\epsilon)^{1+2m}\right)^{\frac{1}{1+2m}}}{\epsilon}\ \&,\right.\right.\right.$$

```
    Table[RandomReal[], {i, 1, 100 000}]], 50, "PDF"],
 Plot[2 Pi pLiu[u, ϵ, m], {u, -1, 1}, PlotRange → All]

]
Clear[m, ϵ];
```



When cosine u has been sampled with random variable ξ, what is the PDF at the sampled direction in terms of ξ?

In[•]:=
```
FullSimplify[pLiu[(-1 + ((-1 + #) (1 - ϵ)^(2 m) (-1 + ϵ) + # (1 + ϵ)^(1+2 m))^(1/(1+2 m)))/ϵ &[ξ], ϵ, m],
  Assumptions → ϵ > 0 && m > 0 && 0 < ξ < 1]
```

Out[•]=
$$\frac{(1+2\,m)\,\epsilon\,\left((1-\epsilon)^{2\,m}\,(-1+\epsilon)\,(-1+\xi)+(1+\epsilon)^{1+2\,m}\,\xi\right)^{\frac{2\,m}{1+2\,m}}}{2\,\pi\,\left(-(1-\epsilon)^{1+2\,m}+(1+\epsilon)^{1+2\,m}\right)}$$