Scattering Kernels in 3D

This is code to accompany the book:

A Hitchhiker's Guide to Multiple Scattering

www.eugenedeon.com/hitchhikers

Cornette-Shanks

[Cornette and Shanks 1992] - *Physically reasonable analytic expression for the single-scattering phase function.*

Independently proposed [Liu and Weng 2006]

```
In[•]:= pCornetteShanks[u_, g_] := 3/(8 Pi) ((1 - g^2) (1 + u^2))/((2 + g^2) (1 + g^2 - 2 g u)^(3/2))
```

Normalization condition

```
In[•]:= Integrate[2 Pi pCornetteShanks[u, g], {u, -1, 1}, Assumptions → -1 < g < 1]
Out[•]= 1
```

Mean-cosine

```
In[•]:= Integrate[2 Pi pCornetteShanks[u, g] u, {u, -1, 1}, Assumptions → -1 < g < 1]
```

Out[•]= $\frac{3\,g\,(4+g^2)}{5\,(2+g^2)}$

Legendre expansion coefficients

```
In[•]:= Integrate[
          2 Pi (2 k + 1) pCornetteShanks[Cos[y], g] LegendreP[k, Cos[y]] Sin[y] /. k → 0,
          {y, 0, Pi}, Assumptions → -1 < g < 1]
Out[•]= 1
```

```
In[•]:= Integrate[
          2 Pi (2 k + 1) pCornetteShanks[Cos[y], g] LegendreP[k, Cos[y]] Sin[y] /. k → 1,
          {y, 0, Pi}, Assumptions → -1 < g < 1]
```

Out[•]= $\frac{9\,g\,(4+g^2)}{5\,(2+g^2)}$

In[•]:=
```
Integrate[
  2 Pi (2 k + 1) pCornetteShanks[Cos[y], g] LegendreP[k, Cos[y]] Sin[y] /. k → 2,
  {y, 0, Pi}, Assumptions → -1 < g < 1]
```

Out[•]=
$$\frac{7 + 80\, g^2 + 18\, g^4}{14 + 7\, g^2}$$

In[•]:=
```
Integrate[
  2 Pi (2 k + 1) pCornetteShanks[Cos[y], g] LegendreP[k, Cos[y]] Sin[y] /. k → 3,
  {y, 0, Pi}, Assumptions → -1 < g < 1]
```

Out[•]=
$$\frac{g \left(27 + 238\, g^2 + 50\, g^4\right)}{15 \left(2 + g^2\right)}$$

sampling

In[•]:=
```
cdf = Integrate[2 Pi pCornetteShanks[u, g],
   {u, -1, x}, Assumptions → -1 < g < 1 && 0 < x < 1]
```

Out[•]=
$$\frac{1}{4\, g^3 \left(2 + g^2\right) \sqrt{1 + g^2 - 2\, g\, x}} \left(2 - 2\, g^6 - 2\, g\, x - 2\, \sqrt{1 + g^2 - 2\, g\, x} + 4\, g^3\, \sqrt{1 + g^2 - 2\, g\, x} + g^4 \left(-5 + x^2\right) + 2\, g^5 \left(x + \sqrt{1 + g^2 - 2\, g\, x}\right) - g^2 \left(-5 + x^2 + 4\, \sqrt{1 + g^2 - 2\, g\, x}\right)\right)$$