Infinite 3D medium, Isotropic Point Source, Linearly-Anisotropic Scattering

Gamma-2 Random Flight

This is code to accompany the book:

A Hitchhiker's Guide to Multiple Scattering

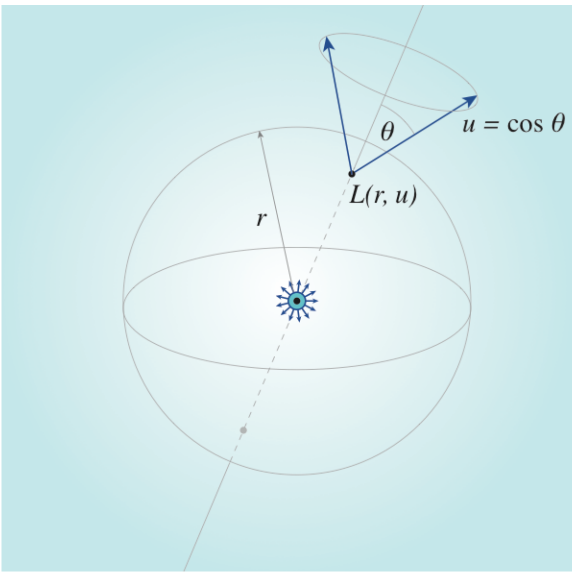
www.eugenedeon.com/hitchhikers

Path Setup

Put a file at ~/.hitchhikerpath with the path to your hitchhiker repo so that these worksheets can find the MC data from the C++ simulations for verification

In[911]:=
```
SetDirectory[Import["~/.hitchhikerpath"]]
```

Notation



c - single-scattering albedo
Σt - extinction coefficient

r - radial position coordinate in medium (distance from point source at origin)
u = cos θ - direction cosine
b - anisotropy parameter

Namespace

In[912]:=
```
Begin["inf3DisopointlinanisoscatterGamma2`"]
```

Out[912]=
```
inf3DisopointlinanisoscatterGamma2`
```

Analytical results

In[913]:=
```
LegPcoeff[k_, j_] :=
 DifferenceRoot[Function[{y, n}, {(-n + k) (1 + n + k) y[n] + (1 + n) (2 + n) y[2 + n] == 0,
     y[0] == LegendreP[k, 0], y[1] == -(1 + k) LegendreP[1 + k, 0]}]][j]
```

In[914]:=
```
Fpc[k_, m_, pc_] := Module[{integrand},
  integrand =
   I^m Sum[LegPcoeff[m, i] (1/I)^i D[SphericalBesselJ[k, x ], {x, i}], {i, 0, m}] /.
    x → u z;
  Integrate[pc[z] integrand, {z, 0, Infinity}, Assumptions → u > 0]
 ]
```

Collision rate density

collision rate density Cc[r] due to correlated emission:

derivation

In[1670]:=
```
Clear[A, b, c, r, h];
```

In[1671]:=
```
cpc[s_] := c Exp[-s] s
```

In[1672]:=
```
f00 = Fpc[0, 0, cpc];
f01 = Fpc[0, 1, cpc];
f11 = Fpc[1, 1, cpc];
o = 2;

A[0] := 1; A[1] := b;
hsystem =
  Table[h[k] == 2/Pi u F[k, 0] + Sum[A[m] × h[m] × F[k, m], {m, 0, o - 1}], {k, 0, o - 1}];
hsystemsolve = Simplify[Solve[hsystem, Table[h[i], {i, 0, o - 1}]] /. F[0, 0] → f00 /.
     F[0, 1] → f01 /. F[1, 1] → f11 /. F[1, 0] → -f01]
```

Out[1678]=
```
{{h[0] →
   (2 c u (b c u^2 + u^4 - b c (1 + u^2) ArcTan[u]^2)) / (π (u^4 (1 - c + u^2) + b c u^2 (2 - c + u^2) -
        2 b c u (1 + u^2) ArcTan[u] + b c^2 (1 + u^2) ArcTan[u]^2)),
  h[1] → (2 c u^3 (-u + (1 + u^2) ArcTan[u])) / (π (u^4 (1 - c + u^2) + b c u^2 (2 - c + u^2) -
        2 b c u (1 + u^2) ArcTan[u] + b c^2 (1 + u^2) ArcTan[u]^2))}}
```

In[∘]:=
```
Clear[r];
CF0 = (2 k + 1) 1/(4 Pi c) (h[k]) u SphericalBesselJ[k, r u] /. k → 0 /. hsystemsolve //
   FullSimplify // First
```

Out[∘]=
```
(u (b c u^2 + u^4 - b c (1 + u^2) ArcTan[u]^2) Sin[r u]) / (2 π^2 r
   (-b (-2 + c) c u^2 + (1 + (-1 + b) c) u^4 + u^6 + b c (1 + u^2) ArcTan[u] (-2 u + c ArcTan[u])))
```

In[∘]:=
```
CF0plus =
  (2 k + 1) 1/(4 Pi c) (h[k] - (2 u)/Pi f00) u SphericalBesselJ[k, r u] /. k → 0 /. hsystemsolve //
   FullSimplify // First
```

Out[∘]=

$$\frac{u^2\left(-\frac{1}{1+u^2}+\frac{b\,c\,u^2+u^4-b\,c\,(1+u^2)\,\mathrm{ArcTan}[u]^2}{-b\,(-2+c)\,c\,u^2+(1+(-1+b)\,c)\,u^4+u^6+b\,c\,(1+u^2)\,\mathrm{ArcTan}[u]\,(-2\,u+c\,\mathrm{ArcTan}[u])}\right)\mathrm{Sinc}[r\,u]}{2\,\pi^2}$$

result

In[1310]:=
```
Ccexact[r_, t_, c_, b_] :=
 NIntegrate[(u (b c u^2 + u^4 - b c (1 + u^2) ArcTan[u]^2) Sin[r u]) /
   (2 π^2 r (-b (-2 + c) c u^2 + (1 + (-1 + b) c) u^4 + u^6 + b c (1 + u^2) ArcTan[u]
        (-2 u + c ArcTan[u]))), {u, 0, Infinity}, Method → "LevinRule"]
```

In[∘]:=
```
Ccexact2[r_, t_, c_, b_] := Exp[-r] r/(4 Pi r^2) + NIntegrate[
```

$$\frac{u^2\left(-\frac{1}{1+u^2}+\frac{b\,c\,u^2+u^4-b\,c\,(1+u^2)\,\mathrm{ArcTan}[u]^2}{-b\,(-2+c)\,c\,u^2+(1+(-1+b)\,c)\,u^4+u^6+b\,c\,(1+u^2)\,\mathrm{ArcTan}[u]\,(-2\,u+c\,\mathrm{ArcTan}[u])}\right)\mathrm{Sinc}[r\,u]}{2\,\pi^2},$$

```
  {u, 0, Infinity}, Method → "LevinRule"]
```

uncorrelated emission

In[2203]:=
```
pu[s_] := c 1/2 Exp[-s] (1 + s)
```

In[2204]:=
```
f00u = Fpc[0, 0, pu];
f01u = Fpc[0, 1, pu];
f11u = Fpc[1, 1, pu];
```

In[•]:=
```
o = 2;
Clear[A, b, c, r, h];
A[0] := 1; A[1] := b;
hsystem =
  Table[h[k] == 2/Pi u Fu[k, 0] + Sum[A[m] × h[m] × F[k, m], {m, 0, o - 1}], {k, 0, o - 1}];
hsystemsolve = Simplify[
  Solve[hsystem, Table[h[i], {i, 0, o - 1}]] /. A[1] → b /. F[0, 0] → f00 /.
      F[0, 1] → f01 /. F[1, 1] → f11 /. F[1, 0] → -f01 /.
    Fu[1, 0] → -Fu[0, 1] /. Fu[0, 1] → f01u /. Fu[0, 0] → f00u]
```

Out[•]=
```
{{h[0] →
   (c u (2 b c u^2 + u^4 + (u^3 + u^5) ArcTan[u] - 2 b c (1 + u^2) ArcTan[u]^2)) / (π (u^4 (1 - c + u^2) +
         b c u^2 (2 - c + u^2) - 2 b c u (1 + u^2) ArcTan[u] + b c^2 (1 + u^2) ArcTan[u]^2)),
  h[1] → (c u^2 (-c u^2 + u^4 + c (1 + u^2) ArcTan[u]^2)) / (π (u^4 (1 - c + u^2) +
         b c u^2 (2 - c + u^2) - 2 b c u (1 + u^2) ArcTan[u] + b c^2 (1 + u^2) ArcTan[u]^2))}}
```

In[•]:=
```
Clear[r];
CF0 = (2 k + 1) 1/(4 Pi c) (h[k]) u SphericalBesselJ[k, r u] /. k → 0 /. hsystemsolve //
  FullSimplify
```

Out[•]=
```
{(u (2 b c u^2 + u^4 + (1 + u^2) ArcTan[u] (u^3 - 2 b c ArcTan[u])) Sin[r u]) /
  (4 π^2 r (-b (-2 + c) c u^2 + (1 + (-1 + b) c) u^4 +
       u^6 + b c (1 + u^2) ArcTan[u] (-2 u + c ArcTan[u])))}
```

In[•]:=
```
Cuexact[r_, t_, c_, b_] :=
 NIntegrate[(u (2 b c u^2 + u^4 + (1 + u^2) ArcTan[u] (u^3 - 2 b c ArcTan[u])) Sin[r u]) /
   (4 π^2 r (-b (-2 + c) c u^2 + (1 + (-1 + b) c) u^4 + u^6 + b c (1 + u^2) ArcTan[u]
        (-2 u + c ArcTan[u]))), {u, 0, Infinity}, Method → "LevinRule"]
```

In[•]:=
```
Clear[r];
CF0 = (2 k + 1) 1/(4 Pi c) (h[0] - 2 u/Pi f00u) u SphericalBesselJ[k, r u] /. k → 0 /.
   hsystemsolve // FullSimplify
```

Out[•]=
```
{(c (u^5 + b u^3 (c + u^2) - (1 + u^2) ArcTan[u]
         (-b c u^2 + (-1 + b) u^4 + b c ArcTan[u] (u + (1 + u^2) ArcTan[u]))) Sin[r u]) /
  (4 π^2 r (1 + u^2) (-b (-2 + c) c u^2 + (1 + (-1 + b) c) u^4 + u^6 +
       b c (1 + u^2) ArcTan[u] (-2 u + c ArcTan[u])))}
```

```
In[•]:= Cuexact2[r_, t_, c_, b_] :=
   (1/2 Exp[-r] (1 + r))/(4 Pi r^2) + NIntegrate[(c (u^5 + b u^3 (c + u^2) - (1 + u^2) ArcTan[u]
        (-b c u^2 + (-1 + b) u^4 + b c ArcTan[u] (u + (1 + u^2) ArcTan[u]))) Sin[r u]) /
     (4 π^2 r (1 + u^2) (-b (-2 + c) c u^2 + (1 + (-1 + b) c) u^4 + u^6 + b c (1 + u^2) ArcTan[u]
        (-2 u + c ArcTan[u]))), {u, 0, Infinity}, Method → "LevinRule"]
```

scalar flux / fluence

```
In[1679]:= cXc[s_] := c e^-s (1 + s)

In[1680]:= fX00 = Fpc[0, 0, cXc];
           fX01 = Fpc[0, 1, cXc];

In[1682]:= fluxh0 = Sum[h[m] × A[m] × FX[0, m], {m, 0, o - 1}] /. hsystemsolve /. FX[0, 0] → fX00 /.
              FX[0, 1] → fX01 // Simplify // First
Out[1682]= -((2 c^2 (-u^3 (u^2 + b (c + u^2)) - u^2 (1 + u^2) (u^2 + b (c - u^2)) ArcTan[u] +
              b c u (1 + u^2) ArcTan[u]^2 + b c (1 + u^2)^2 ArcTan[u]^3)) /
          (π (1 + u^2) (u^4 (1 - c + u^2) + b c u^2 (2 - c + u^2) - 2 b c u (1 + u^2) ArcTan[u] +
              b c^2 (1 + u^2) ArcTan[u]^2)))

In[1683]:= Clear[r];
           CF0 = (2 k + 1) 1/(4 Pi c) (fluxh0) u SphericalBesselJ[k, r u] /. k → 0
Out[1683]= -((c u (-u^3 (u^2 + b (c + u^2)) - u^2 (1 + u^2) (u^2 + b (c - u^2)) ArcTan[u] +
              b c u (1 + u^2) ArcTan[u]^2 + b c (1 + u^2)^2 ArcTan[u]^3) SphericalBesselJ[0, r u]) /
          (2 π^2 (1 + u^2) (u^4 (1 - c + u^2) + b c u^2 (2 - c + u^2) - 2 b c u (1 + u^2) ArcTan[u] +
              b c^2 (1 + u^2) ArcTan[u]^2)))

In[1714]:= ϕcexact[r_, t_, c_, b_] := (e^-r (1 + r))/(4 Pi r^2) +
     NIntegrate[-((c u (-u^3 (u^2 + b (c + u^2)) - u^2 (1 + u^2) (u^2 + b (c - u^2)) ArcTan[u] + b c u
              (1 + u^2) ArcTan[u]^2 + b c (1 + u^2)^2 ArcTan[u]^3) SphericalBesselJ[0, r u]) /
          (2 π^2 (1 + u^2) (u^4 (1 - c + u^2) + b c u^2 (2 - c + u^2) - 2 b c u (1 + u^2) ArcTan[u] +
              b c^2 (1 + u^2) ArcTan[u]^2))), {u, 0, Infinity}, Method → "LevinRule"]

In[1835]:= ϕcexactplus[r_, t_, c_, b_] :=
    NIntegrate[-((c u (-u^3 (u^2 + b (c + u^2)) - u^2 (1 + u^2) (u^2 + b (c - u^2)) ArcTan[u] + b c u
              (1 + u^2) ArcTan[u]^2 + b c (1 + u^2)^2 ArcTan[u]^3) SphericalBesselJ[0, r u]) /
          (2 π^2 (1 + u^2) (u^4 (1 - c + u^2) + b c u^2 (2 - c + u^2) - 2 b c u (1 + u^2) ArcTan[u] +
              b c^2 (1 + u^2) ArcTan[u]^2))), {u, 0, Infinity}, Method → "LevinRule"]
```

diffusion approximation

```
In[•]:= m3[v_, r_] := Exp[-r / v]/(4 Pi r v^2)
```

In[•]:= $\text{Limit}\left[\frac{\text{Pi}}{2\,c\,u}\,\text{fluxh0},\,u\to 0\right]$ // FullSimplify

Out[•]= $-\frac{2\,c}{-1+c}$

$\text{Limit}\left[\text{Simplify}\left[D\left[\frac{\text{Pi}}{2\,c\,u}\,\text{fluxh0},\,\{u,2\}\right]\right],\,u\to 0,\,\text{Direction}\to \text{"FromAbove"}\right]$

Out[•]= $\frac{4\,c\,\left(15-6\,c+b\,\left(3-4\,c+2\,c^{2}\right)\right)}{3\,(-1+c)^{2}\,(-3+b\,c)}$

In[•]:= $\text{Clear}[c,b];\ m0=-\frac{2\,c}{-1+c};\ m2=-3\left(\frac{4\,c\,\left(15-6\,c+b\,\left(3-4\,c+2\,c^{2}\right)\right)}{3\,(-1+c)^{2}\,(-3+b\,c)}\right)$

Out[•]= $-\frac{4\,c\,\left(15-6\,c+b\,\left(3-4\,c+2\,c^{2}\right)\right)}{(-1+c)^{2}\,(-3+b\,c)}$

In[•]:= $\text{m0 m3}\left[\sqrt{\frac{\text{m2}}{2\times 3\,\text{m0}}},\,r\right]$

Out[•]= $-\frac{3\,c\,(-3+b\,c)\,e^{-\frac{\sqrt{3}\,r}{\sqrt{\frac{15-6\,c+b\,\left(3-4\,c+2\,c^{2}\right)}{(-1+c)\,(-3+b\,c)}}}}}{2\,\left(15-6\,c+b\,\left(3-4\,c+2\,c^{2}\right)\right)\,\pi\,r}$

In[•]:= $\text{fluenceGrosjean}[r\_,\,c\_,\,b\_]:=\frac{e^{-r}\,(1+r)}{4\,\text{Pi}\,r^{2}}+-\frac{3\,c\,(-3+b\,c)\,e^{-\frac{\sqrt{3}\,r}{\sqrt{\frac{15-6\,c+b\,\left(3-4\,c+2\,c^{2}\right)}{(-1+c)\,(-3+b\,c)}}}}}{2\,\left(15-6\,c+b\,\left(3-4\,c+2\,c^{2}\right)\right)\,\pi\,r}$

nth collision rate density

```
Clear[h];
h[1, k_, u_] := 2 u c Fpc[k, 0, pc]
```

In[•]:= `h[n_, k_, u_] := Sum[A[m] × h[n - 1, m, u] c Fpc[k, m, pc], {m, 0, o - 1}]`

In[•]:= `h[1, 0, u] // Simplify`

Out[•]= $\frac{2\,u}{1+u^{2}}$

In[•]:= $\frac{1}{4\,\text{Pi}^{2}\,r^{2}}\,\text{Integrate}\left[r\left(\frac{2\,u}{1+u^{2}}\right)\text{Sin}[r\,u],\,\{u,0,\text{Infinity}\},\,\text{Assumptions}\to r>0\right]$

Out[•]= $\frac{e^{-r}}{4\,\pi\,r}$

$\text{CcSingle}[r\_,\,c\_,\,b\_]:=$
$\text{NIntegrate}\left[r\left(\frac{2\,u}{1+u^{2}}\right)\text{Sin}[r\,u],\,\{u,0,\text{Infinity}\},\,\text{Method}\to\text{"LevinRule"}\right]$

In[∘]:= `h[2, 0, u] // Simplify`

Out[∘]= $$-\frac{2\,c\,\left(u^2\,\left(b-u^2\right)-2\,b\,u\,\left(1+u^2\right)\,\text{ArcTan}[u]+b\,\left(1+u^2\right)^2\,\text{ArcTan}[u]^2\right)}{u^3\,\left(1+u^2\right)^2}$$

In[∘]:= $$\text{CcDouble}[r\_, c\_, b\_] := \frac{1}{4\,\text{Pi}^2\,r^2}\,\text{NIntegrate}\Big[r\left(-\frac{2\,c\,\left(u^2\,\left(b-u^2\right)-2\,b\,u\,\left(1+u^2\right)\,\text{ArcTan}[u]+b\,\left(1+u^2\right)^2\,\text{ArcTan}[u]^2\right)}{u^3\,\left(1+u^2\right)^2}\right)\,\text{Sin}[r\,u],\ \{u, 0, \text{Infinity}\},\ \text{Method}\to\text{"LevinRule"}\Big]$$

In[∘]:= `h[3, 0, u] // Simplify`

Out[∘]= $$\frac{1}{u^6\,\left(1+u^2\right)^3}\,2\,c^2\,\left(-2\,b\,u^5+u^7+b^2\,u^3\,\left(2+u^2\right)-2\,b\,u^2\,\left(1+u^2\right)\,\left(-2\,u^2+b\,\left(3+u^2\right)\right)\,\text{ArcTan}[u]+b\,u\,\left(1+u^2\right)^2\,\left(-2\,u^2+b\,\left(6+u^2\right)\right)\,\text{ArcTan}[u]^2-2\,b^2\,\left(1+u^2\right)^3\,\text{ArcTan}[u]^3\right)$$

In[∘]:= $$\text{CcTriple}[r\_, c\_, b\_] := \frac{1}{4\,\text{Pi}^2\,r^2}\,\text{NIntegrate}\Big[r\left(\frac{1}{u^6\,\left(1+u^2\right)^3}\,2\,c^2\,\left(-2\,b\,u^5+u^7+b^2\,u^3\,\left(2+u^2\right)-2\,b\,u^2\,\left(1+u^2\right)\,\left(-2\,u^2+b\,\left(3+u^2\right)\right)\,\text{ArcTan}[u]+b\,u\,\left(1+u^2\right)^2\,\left(-2\,u^2+b\,\left(6+u^2\right)\right)\,\text{ArcTan}[u]^2-2\,b^2\,\left(1+u^2\right)^3\,\text{ArcTan}[u]^3\right)\right)\,\text{Sin}[r\,u],\ \{u, 0, \text{Infinity}\},\ \text{Method}\to\text{"LevinRule"}\Big]$$

angular collision rate

In[1277]:= $$\text{Cl}[0, r\_, c\_, b\_] := \text{NIntegrate}\Big[-\Big(\Big(c\,u\,\left(-u^2\,\left(b\,(-1+c)+u^2\right)+b\,\left(1+u^2\right)\,\text{ArcTan}[u]\,\left(-2\,u+\left(1+c+u^2\right)\,\text{ArcTan}[u]\right)\right)\,\text{Sin}[r\,u]\Big)\Big/\Big(2\,\pi^2\,r\,\left(1+u^2\right)\,\left(-b\,\left(-2+c\right)\,c\,u^2+\left(1+\left(-1+b\right)\,c\right)\,u^4+u^6+b\,c\,\left(1+u^2\right)\,\text{ArcTan}[u]\,\left(-2\,u+c\,\text{ArcTan}[u]\right)\right)\Big)\Big),\ \{u, 0, \text{Infinity}\}\Big]$$

In[1280]:= $$\text{Cl}[1, r\_, c\_, b\_] := \text{NIntegrate}\Big[\Big(3\,c\,\left(-u+\left(1+u^2\right)\,\text{ArcTan}[u]\right)\,\left(-b\,\left(-2+c\right)\,u^2+\left(-1+b\right)\,u^4-b\,\left(1+u^2\right)\,\text{ArcTan}[u]\,\left(2\,u-c\,\text{ArcTan}[u]\right)\right)\,\left(r\,u\,\text{Cos}[r\,u]-\text{Sin}[r\,u]\right)\Big)\Big/\Big(2\,\pi^2\,r^2\,u^2\,\left(1+u^2\right)\,\left(-b\,\left(-2+c\right)\,c\,u^2+\left(1+\left(-1+b\right)\,c\right)\,u^4+u^6+b\,c\,\left(1+u^2\right)\,\text{ArcTan}[u]\,\left(-2\,u+c\,\text{ArcTan}[u]\right)\right)\Big),\ \{u, 0, \text{Infinity}\}\Big]$$

In[1289]:= $$\text{Cl}[2, r\_, c\_, b\_] := \text{NIntegrate}\Big[\frac{1}{4\,\pi^2\,r^3\,u^3}\,5\,\Big(-4-\frac{2}{1+u^2}+\frac{6\,\text{ArcTan}[u]}{u}+\left(6\,u^4+4\,u^6+b\,c\,u^2\,\left(3+4\,u^2\right)-\left(1+u^2\right)\,\text{ArcTan}[u]\,\left(6\,\left(b\,c\,u+u^3\right)+b\,c\,\left(-3+u^2\right)\,\text{ArcTan}[u]\right)\right)\Big/\left(-b\,\left(-2+c\right)\,c\,u^2+\left(1+\left(-1+b\right)\,c\right)\,u^4+u^6+b\,c\,\left(1+u^2\right)\,\text{ArcTan}[u]\,\left(-2\,u+c\,\text{ArcTan}[u]\right)\right)\Big)\,\left(-3\,r\,u\,\text{Cos}[r\,u]+\left(3-r^2\,u^2\right)\,\text{Sin}[r\,u]\right),\ \{u, 0, \text{Infinity}\}\Big]$$

```
In[1294]:= Cl[3, r_, c_, b_] :=
  NIntegrate[-((7 c (15 b (-4 + 3 c) u^3 + (45 + b (-58 + 39 c)) u^5 + (39 - 4 b) u^7 +
          u^2 (1 + u^2) (-9 b c (5 + u^2) - 9 u^2 (5 + u^2) + 4 b (30 + 19 u^2 + u^4)) ArcTan[u] -
          3 b (u + u^3) (5 (4 + 3 c) + 13 (2 + c) u^2 + 6 u^4) ArcTan[u]^2 + 9 b c (1 + u^2)^2
           (5 + u^2) ArcTan[u]^3) (r u (-15 + r^2 u^2) Cos[r u] + 3 (5 - 2 r^2 u^2) Sin[r u])) /
      (12 π^2 r^4 u^6 (1 + u^2) (-b (-2 + c) c u^2 + (1 + (-1 + b) c) u^4 + u^6 +
          b c (1 + u^2) ArcTan[u] (-2 u + c ArcTan[u])))), {u, 0, Infinity}]
```

angular flux / radiance

```
In[1688]:= fX11 = Fpc[1, 1, cXc];

In[1698]:= fX12 = Fpc[1, 2, cXc];

In[1699]:= fX02 = Fpc[0, 2, cXc];

In[1700]:= fX22 = Fpc[2, 2, cXc];

In[1930]:= fX03 = Fpc[0, 3, cXc];

In[1932]:= fX13 = Fpc[1, 3, cXc]
```

Out[1932]=

$$\frac{c\left(u\left(-13-\frac{2}{1+u^2}\right)+3\left(5+u^2\right)\mathrm{ArcTan}[u]\right)}{2\,u^5}$$

```
In[1689]:= fluxh1 = Sum[h[m] × A[m] × FX[1, m], {m, 0, o - 1}] /. hsystemsolve /. FX[0, 0] → fX00 /.
     FX[0, 1] → fX01 /. FX[1, 0] → -fX01 /. FX[1, 1] → fX11 // Simplify // First

Out[1689]= -((2 c^2 (-u^2 (b + b c u^2 + u^4) +
          2 b u (1 + u^2) ArcTan[u] + b (1 + u^2) (-1 + (-1 + c) u^2) ArcTan[u]^2)) /
     (π (1 + u^2) (u^4 (1 - c + u^2) + b c u^2 (2 - c + u^2) - 2 b c u (1 + u^2) ArcTan[u] +
          b c^2 (1 + u^2) ArcTan[u]^2)))

In[1702]:= Clear[r];
  CF0 = (2 k + 1) 1/(4 Pi c) (fluxh1) u SphericalBesselJ[k, r u] /. k → 1 //
     FunctionExpand // Simplify

Out[1702]= (3 c
     (-u^2 (b + b c u^2 + u^4) + 2 b u (1 + u^2) ArcTan[u] + b (1 + u^2) (-1 + (-1 + c) u^2) ArcTan[u]^2)
     (r u Cos[r u] - Sin[r u])) / (2 π^2 r^2 u (1 + u^2)
     (u^4 (1 - c + u^2) + b c u^2 (2 - c + u^2) - 2 b c u (1 + u^2) ArcTan[u] + b c^2 (1 + u^2) ArcTan[u]^2))

In[1703]:= ϕl[1, r_, c_, b_] :=
  NIntegrate[(3 c (-u^2 (b + b c u^2 + u^4) + 2 b u (1 + u^2) ArcTan[u] + b (1 + u^2)
         (-1 + (-1 + c) u^2) ArcTan[u]^2) (r u Cos[r u] - Sin[r u])) /
     (2 π^2 r^2 u (1 + u^2) (u^4 (1 - c + u^2) + b c u^2 (2 - c + u^2) - 2 b c u (1 + u^2) ArcTan[u] +
         b c^2 (1 + u^2) ArcTan[u]^2)), {u, 0, Infinity}]
```

```
In[1701]:= fluxh2 =
  Sum[h[m] × A[m] × FX[2, m], {m, 0, o - 1}] /. hsystemsolve /. FX[0, 0] → fX00 /. FX[
      0, 1] → fX01 /. FX[1, 0] → -fX01 /. FX[1, 1] → fX11 /.
    FX[2, 0] → fX02 /. FX[2, 1] → -fX12 // Simplify // First
```

$$\text{Out[1701]=}\ \Big(c^2 \Big(2 \left(u \left(3+u^2\right)+\left(-3-2 u^2+u^4\right) \text{ArcTan}[u]\right) \left(b c u^2+u^4-b c \left(1+u^2\right) \text{ArcTan}[u]^2\right)+ b u \left(-u+\left(1+u^2\right) \text{ArcTan}[u]\right) \Big(12 u+8 u^3+3 \left(1+u^2\right) \text{Arg}[-1-i u]+3 \left(1+u^2\right) \text{Arg}[1-i u]-3 \text{Arg}[-1+i u]- 3 u^2 \text{Arg}[-1+i u]-3 \text{Arg}[1+i u]-3 u^2 \text{Arg}[1+i u]+3 \text{Arg}\left[-\frac{i}{-i+u}\right]+ 3 u^2 \text{Arg}\left[-\frac{i}{-i+u}\right]-3 \text{Arg}\left[\frac{i}{-i+u}\right]-3 u^2 \text{Arg}\left[\frac{i}{-i+u}\right]+3 \text{Arg}\left[-\frac{i}{i+u}\right]+ 3 u^2 \text{Arg}\left[-\frac{i}{i+u}\right]-3 \text{Arg}\left[\frac{i}{i+u}\right]-3 u^2 \text{Arg}\left[\frac{i}{i+u}\right]+3 i \text{Log}[-1-i u]+ 3 i u^2 \text{Log}[-1-i u]-3 i \text{Log}[-1+i u]-3 i u^2 \text{Log}[-1+i u]-3 i \text{Log}\left[\frac{i}{-i+u}\right]- 3 i u^2 \text{Log}\left[\frac{i}{-i+u}\right]+3 i \text{Log}\left[-\frac{i}{i+u}\right]+3 i u^2 \text{Log}\left[-\frac{i}{i+u}\right]\Big)\Big)\Big)\Big/ \left(2 \pi u^2 \left(1+u^2\right) \left(u^4 \left(1-c+u^2\right)+b c u^2 \left(2-c+u^2\right)-2 b c u \left(1+u^2\right) \text{ArcTan}[u]+ b c^2 \left(1+u^2\right) \text{ArcTan}[u]^2\right)\right)$$

```
In[1934]:= Clear[r];
  CF0 = (2 k + 1) 1/(4 Pi c) (fluxh2) u SphericalBesselJ[k, r u] /. k → 2 //
    FunctionExpand // Simplify
```

$$\text{Out[1934]=}\ \Big(5 c \Big(2 \left(u \left(3+u^2\right)+\left(-3-2 u^2+u^4\right) \text{ArcTan}[u]\right) \left(b c u^2+u^4-b c \left(1+u^2\right) \text{ArcTan}[u]^2\right)+ b u \left(-u+\left(1+u^2\right) \text{ArcTan}[u]\right) \Big(12 u+8 u^3+3 \left(1+u^2\right) \text{Arg}[-1-i u]+3 \left(1+u^2\right) \text{Arg}[1-i u]-3 \text{Arg}[-1+i u]- 3 u^2 \text{Arg}[-1+i u]-3 \text{Arg}[1+i u]-3 u^2 \text{Arg}[1+i u]+3 \text{Arg}\left[-\frac{i}{-i+u}\right]+ 3 u^2 \text{Arg}\left[-\frac{i}{-i+u}\right]-3 \text{Arg}\left[\frac{i}{-i+u}\right]-3 u^2 \text{Arg}\left[\frac{i}{-i+u}\right]+3 \text{Arg}\left[-\frac{i}{i+u}\right]+ 3 u^2 \text{Arg}\left[-\frac{i}{i+u}\right]-3 \text{Arg}\left[\frac{i}{i+u}\right]-3 u^2 \text{Arg}\left[\frac{i}{i+u}\right]+3 i \text{Log}[-1-i u]+ 3 i u^2 \text{Log}[-1-i u]-3 i \text{Log}[-1+i u]-3 i u^2 \text{Log}[-1+i u]- 3 i \text{Log}\left[\frac{i}{-i+u}\right]-3 i u^2 \text{Log}\left[\frac{i}{-i+u}\right]+3 i \text{Log}\left[-\frac{i}{i+u}\right]+3 i u^2 \text{Log}\left[-\frac{i}{i+u}\right]\Big)\Big) \left(-3 r u \text{Cos}[r u]+\left(3-r^2 u^2\right) \text{Sin}[r u]\right)\Big)\Big/ \Big(8\ \pi^2\ r^3\ u^4\ \left(1+u^2\right) \left(u^4 \left(1-c+u^2\right)+b c u^2 \left(2-c+u^2\right)-2 b c u \left(1+u^2\right) \text{ArcTan}[u]+b c^2 \left(1+u^2\right) \text{ArcTan}[u]^2\right)\Big)$$

In[1935]:=
```
ϕl[2, r_, c_, b_] := NIntegrate[
  (5 c (2 (u (3 + u^2) + (-3 - 2 u^2 + u^4) ArcTan[u]) (b c u^2 + u^4 - b c (1 + u^2) ArcTan[u]^2) +
       b u (-u + (1 + u^2) ArcTan[u])
        (12 u + 8 u^3 + 3 (1 + u^2) Arg[-1 - ⅈ u] + 3 (1 + u^2) Arg[1 - ⅈ u] - 3 Arg[-1 + ⅈ u] -
          3 u^2 Arg[-1 + ⅈ u] - 3 Arg[1 + ⅈ u] - 3 u^2 Arg[1 + ⅈ u] + 3 Arg[-ⅈ/(-ⅈ + u)] +
          3 u^2 Arg[-ⅈ/(-ⅈ + u)] - 3 Arg[ⅈ/(-ⅈ + u)] - 3 u^2 Arg[ⅈ/(-ⅈ + u)] + 3 Arg[-ⅈ/(ⅈ + u)] +
          3 u^2 Arg[-ⅈ/(ⅈ + u)] - 3 Arg[ⅈ/(ⅈ + u)] - 3 u^2 Arg[ⅈ/(ⅈ + u)] + 3 ⅈ Log[-1 - ⅈ u] +
          3 ⅈ u^2 Log[-1 - ⅈ u] - 3 ⅈ Log[-1 + ⅈ u] - 3 ⅈ u^2 Log[-1 + ⅈ u] -
          3 ⅈ Log[ⅈ/(-ⅈ + u)] - 3 ⅈ u^2 Log[ⅈ/(-ⅈ + u)] + 3 ⅈ Log[-ⅈ/(ⅈ + u)] + 3 ⅈ u^2 Log[-ⅈ/(ⅈ + u)]))
     (-3 r u Cos[r u] + (3 - r^2 u^2) Sin[r u])) / (8 π^2 r^3 u^4 (1 + u^2)
     (u^4 (1 - c + u^2) + b c u^2 (2 - c + u^2) - 2 b c u (1 + u^2) ArcTan[u] +
       b c^2 (1 + u^2) ArcTan[u]^2)), {u, 0, Infinity}]
```

In[1982]:=
```
Clear[c, b];
fluxh3 = Sum[h[m] × A[m] × FX[3, m], {m, 0, o - 1}] /. hsystemsolve /. FX[0, 0] → fX00 /.
        FX[0, 1] → fX01 /. FX[1, 0] → -fX01 /. FX[1, 1] → fX11 /. FX[2, 0] → fX02 /.
     FX[2, 1] → -fX12 /. FX[3, 0] → -fX03 /. FX[3, 1] → fX13 // Simplify // First
```

Out[1982]=
```
(c^2 (6 b u (-u + (1 + u^2) ArcTan[u]) (u (-13 - 2/(1 + u^2)) + 3 (5 + u^2) ArcTan[u]) + 1/(1 + u^2)
      (b c u^2 + u^4 - b c (1 + u^2) ArcTan[u]^2) (-60 u - 40 u^3 + 8 u^5 - 15 (1 + u^2) Arg[-1 - ⅈ u] -
        15 (1 + u^2) Arg[1 - ⅈ u] + 15 Arg[-1 + ⅈ u] + 15 u^2 Arg[-1 + ⅈ u] +
        15 Arg[1 + ⅈ u] + 15 u^2 Arg[1 + ⅈ u] - 15 Arg[-ⅈ/(-ⅈ + u)] - 15 u^2 Arg[-ⅈ/(-ⅈ + u)] +
        15 Arg[ⅈ/(-ⅈ + u)] + 15 u^2 Arg[ⅈ/(-ⅈ + u)] - 15 Arg[-ⅈ/(ⅈ + u)] - 15 u^2 Arg[-ⅈ/(ⅈ + u)] +
        15 Arg[ⅈ/(ⅈ + u)] + 15 u^2 Arg[ⅈ/(ⅈ + u)] - 15 ⅈ Log[-1 - ⅈ u] - 15 ⅈ u^2 Log[-1 - ⅈ u] +
        15 ⅈ Log[-1 + ⅈ u] + 15 ⅈ u^2 Log[-1 + ⅈ u] + 15 ⅈ Log[ⅈ/(-ⅈ + u)] +
        15 ⅈ u^2 Log[ⅈ/(-ⅈ + u)] - 15 ⅈ Log[-ⅈ/(ⅈ + u)] - 15 ⅈ u^2 Log[-ⅈ/(ⅈ + u)]))) /
 (6 π u^3 (u^4 (1 - c + u^2) + b c u^2 (2 - c + u^2) - 2 b c u (1 + u^2) ArcTan[u] +
     b c^2 (1 + u^2) ArcTan[u]^2))
```

In[1983]:=
```
Clear[r];
CF0 = (2 k + 1) 1/(4 Pi c) (fluxh3) u SphericalBesselJ[k, r u] /. k → 3 //
    FunctionExpand // Simplify
```

Out[1983]=
```
(7 c (6 b u (-u + (1 + u^2) ArcTan[u]) (u (-13 - 2/(1 + u^2)) + 3 (5 + u^2) ArcTan[u]) +
     1/(1 + u^2) (b c u^2 + u^4 - b c (1 + u^2) ArcTan[u]^2)
      (-60 u - 40 u^3 + 8 u^5 - 15 (1 + u^2) Arg[-1 - ⅈ u] - 15 (1 + u^2) Arg[1 - ⅈ u] +
        15 Arg[-1 + ⅈ u] + 15 u^2 Arg[-1 + ⅈ u] + 15 Arg[1 + ⅈ u] + 15 u^2 Arg[1 + ⅈ u] -
        15 Arg[-ⅈ/(-ⅈ + u)] - 15 u^2 Arg[-ⅈ/(-ⅈ + u)] + 15 Arg[ⅈ/(-ⅈ + u)] + 15 u^2 Arg[ⅈ/(-ⅈ + u)] -
        15 Arg[-ⅈ/(ⅈ + u)] - 15 u^2 Arg[-ⅈ/(ⅈ + u)] + 15 Arg[ⅈ/(ⅈ + u)] + 15 u^2 Arg[ⅈ/(ⅈ + u)] - 15 ⅈ
         Log[-1 - ⅈ u] - 15 ⅈ u^2 Log[-1 - ⅈ u] + 15 ⅈ Log[-1 + ⅈ u] + 15 ⅈ u^2 Log[-1 + ⅈ u] +
        15 ⅈ Log[ⅈ/(-ⅈ + u)] + 15 ⅈ u^2 Log[ⅈ/(-ⅈ + u)] - 15 ⅈ Log[-ⅈ/(ⅈ + u)] - 15 ⅈ u^2 Log[-ⅈ/(ⅈ + u)]))
   (r u (-15 + r^2 u^2) Cos[r u] + 3 (5 - 2 r^2 u^2) Sin[r u])) / (24
   π^2
   r^4
   u^6
   (u^4 (1 - c + u^2) + b c u^2 (2 - c + u^2) -
      2 b c u (1 + u^2) ArcTan[u] + b c^2 (1 + u^2) ArcTan[u]^2))
```

In[1984]:=
```
ϕl[3, r_, c_, b_] := NIntegrate[
  (7 c (6 b u (-u + (1 + u^2) ArcTan[u]) (u (-13 - 2/(1 + u^2)) + 3 (5 + u^2) ArcTan[u]) +
       1/(1 + u^2) (b c u^2 + u^4 - b c (1 + u^2) ArcTan[u]^2) (-60 u - 40 u^3 + 8 u^5 -
          15 (1 + u^2) Arg[-1 - ⅈ u] - 15 (1 + u^2) Arg[1 - ⅈ u] + 15 Arg[-1 + ⅈ u] +
          15 u^2 Arg[-1 + ⅈ u] + 15 Arg[1 + ⅈ u] + 15 u^2 Arg[1 + ⅈ u] - 15 Arg[-ⅈ/(-ⅈ + u)] -
          15 u^2 Arg[-ⅈ/(-ⅈ + u)] + 15 Arg[ⅈ/(-ⅈ + u)] + 15 u^2 Arg[ⅈ/(-ⅈ + u)] - 15 Arg[-ⅈ/(ⅈ + u)] -
          15 u^2 Arg[-ⅈ/(ⅈ + u)] + 15 Arg[ⅈ/(ⅈ + u)] + 15 u^2 Arg[ⅈ/(ⅈ + u)] - 15 ⅈ Log[-1 - ⅈ u] -
          15 ⅈ u^2 Log[-1 - ⅈ u] + 15 ⅈ Log[-1 + ⅈ u] + 15 ⅈ u^2 Log[-1 + ⅈ u] + 15 ⅈ
           Log[ⅈ/(-ⅈ + u)] + 15 ⅈ u^2 Log[ⅈ/(-ⅈ + u)] - 15 ⅈ Log[-ⅈ/(ⅈ + u)] - 15 ⅈ u^2 Log[-ⅈ/(ⅈ + u)]))
     (r u (-15 + r^2 u^2) Cos[r u] + 3 (5 - 2 r^2 u^2) Sin[r u])) /
   (24 π^2 r^4 u^6 (u^4 (1 - c + u^2) + b c u^2 (2 - c + u^2) - 2 b c u (1 + u^2) ArcTan[u] +
       b c^2 (1 + u^2) ArcTan[u]^2)), {u, 0, Infinity}]
```

load MC data

In[915]:=
```
ppoints[xs_, dr_, maxx_] :=
 Table[{dr (i) - 0.5 dr, xs[[i]]}, {i, 1, Length[xs]}][[1 ;; -2]]
```

In[916]:=
```
ppointsu[xs_, du_, Σt_] :=
 Table[{-1.0 + du (i) - 0.5 du, xs[[i]] / (2 Σt)}, {i, 1, Length[xs]}][[1 ;; -1]]
```

In[917]:=
```
fs = FileNames["code/3D_medium/infinite3Dmedium/Isotropicpointsource/MCdata/
     inf3D_isotropicpoint_linanisoscatter_gamma2C*"];
```

In[918]:=
```
fsU = FileNames["code/3D_medium/infinite3Dmedium/Isotropicpointsource/MCdata/
    inf3D_isotropicpoint_linanisoscatter_gamma2U*"]
```

Out[918]=
```
{}
```

In[919]:=
```
index[x_] := Module[{data, c, mfp, b},
   data = Import[x, "Table"];
   mfp = data[[1, 13]];
   c = data[[2, 3]];
   b = data[[1, 16]];
   {c, mfp, b, data}];
simulations = index /@ fs;
simulationsU = index /@ fsU;
cs = Union[#[[1]] & /@ simulations]
```

Out[922]=
```
{0.01, 0.1, 0.3, 0.5, 0.7, 0.8, 0.9, 0.95, 0.99, 0.999}
```

In[923]:=
```
mfps = Union[#[[2]] & /@ simulations]
```

Out[923]=
```
{0.3, 1}
```

In[924]:=
```
bs = Union[#[[3]] & /@ simulations]
```

Out[924]=
```
{-0.9, 0.7}
```

In[925]:=
```
numcollorders = simulations[[1]][[-1]][[2, 13]];
```

Compare analytic and MC

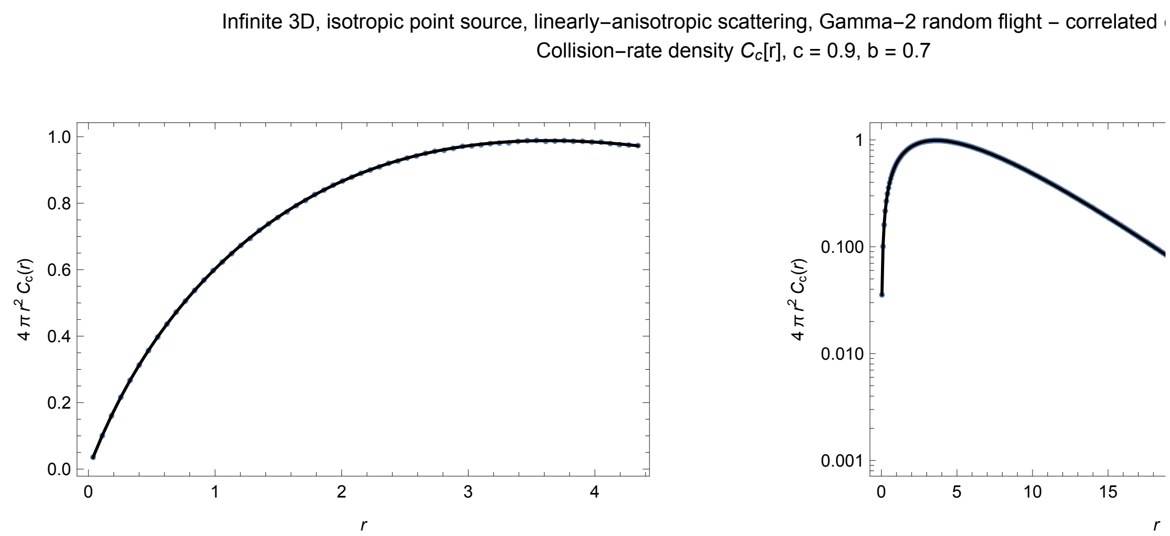
Collision-rate density - Correlated emission - Exact solution (1) comparison to MC

In[ ]:=
```
{{ActionMenu["Set c", "c = " <> ToString[#] :→ (c = #;) & /@ cs], Dynamic[c]},
 {ActionMenu["Set mfp", "mfp = " <> ToString[#] :→ (mfp = #;) & /@ mfps],
  Dynamic[mfp]},
 {ActionMenu["Set b", "b = " <> ToString[#] :→ (b = #;) & /@ bs], Dynamic[b]}}
```

Out[ ]=
```
{{Set c, 0.99}, {Set mfp, 1}, {Set b, 0.7}}
```

In[•]:=
```
data = SelectFirst[simulations, #[[1]] == c && #[[2]] == mfp && #[[3]] == b &][[4]];
maxr = data[[2, 5]];
dr = data[[2, 7]];
MCCollisionRate = ppoints[data[[4]], dr, maxr];
exact1CRShallow = Quiet[{#[[1]], 4 Pi #[[1]]^2 Ccexact[#[[1]], 1/mfp, c, b]}] & /@
   MCCollisionRate[[1 ;; 60]];
exact1CR = Quiet[{#[[1]], 4 Pi #[[1]]^2 Ccexact[#[[1]], 1/mfp, c, b]}] & /@
   MCCollisionRate[[61 ;; -1 ;; 10]];
plotϕshallow = Quiet[Show[
     ListPlot[MCCollisionRate[[1 ;; 60]],
      PlotRange → All, PlotStyle → PointSize[.01]],
     ListPlot[exact1CRShallow, PlotRange → All,
      Joined → True, PlotStyle → {Black}],
     Frame → True,
     FrameLabel -> {{4 π r^2 C"c"[r],}, {r,}}
   ]];
logplotϕ = Quiet[Show[
     ListLogPlot[MCCollisionRate, PlotRange → All, PlotStyle → PointSize[.01]],
     ListLogPlot[exact1CR, PlotRange → All, Joined → True, PlotStyle → {Black}],
     ListLogPlot[exact1CRShallow,
      PlotRange → All, Joined → True, PlotStyle → {Black}],
     Frame → True,
     FrameLabel -> {{4 π r^2 C"c"[r],}, {r,}}
   ]];
Show[GraphicsGrid[{{plotϕshallow, logplotϕ}}, ImageSize → 800], PlotLabel ->
  "Infinite 3D, isotropic point source, linearly-anisotropic scattering,
    Gamma-2 random flight - correlated emission\nCollision-rate
    density Cc[r], c = " <> ToString[c] <> ", b = " <> ToString[b]]
```

Out[•]=



Collision-rate density - Uncorrelated emission - Exact solution (2)

comparison to MC

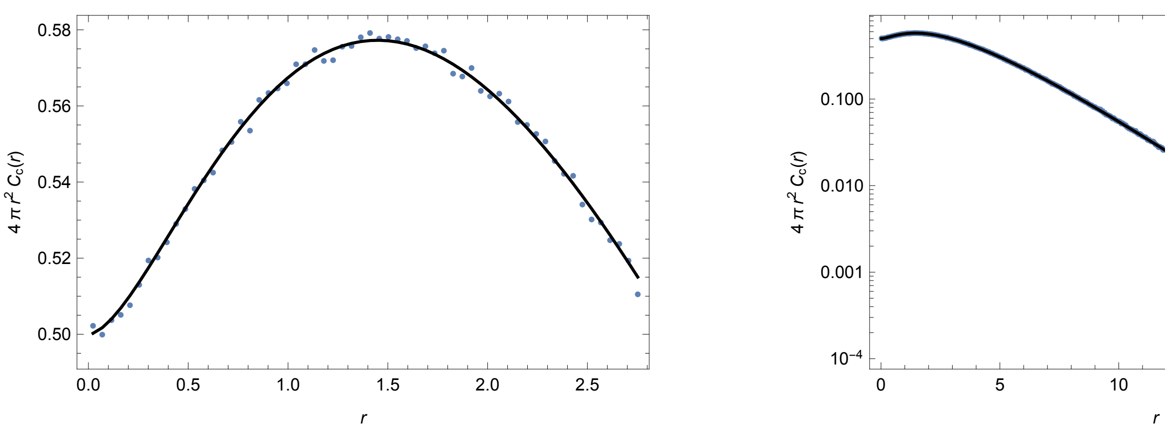
In[•]:=
```
{{ActionMenu["Set c", "c = " <> ToString[#] :> (c = #;) & /@ cs], Dynamic[c]},
 {ActionMenu["Set mfp", "mfp = " <> ToString[#] :> (mfp = #;) & /@ mfps],
  Dynamic[mfp]},
 {ActionMenu["Set b", "b = " <> ToString[#] :> (b = #;) & /@ bs], Dynamic[b]}}
```

Out[•]=
```
{{ Set c , 0.99}, { Set mfp , 1}, { Set b , 0.7}}
```

In[ ]:=
```
data = SelectFirst[simulationsU, #[[1]] == c && #[[2]] == mfp && #[[3]] == b &][[4]];
maxr = data[[2, 5]];
dr = data[[2, 7]];
MCCollisionRate = ppoints[data[[4]], dr, maxr];
exact1CRShallow =
  Quiet[{#[[1]], 4 Pi #[[1]]^2 Cuexact2[#[[1]], 1/mfp, c, b]}] & /@
   MCCollisionRate[[1 ;; 60]];
exact1CR = Quiet[{#[[1]], 4 Pi #[[1]]^2 Cuexact2[#[[1]], 1/mfp, c, b]}] & /@
   MCCollisionRate[[61 ;; -1 ;; 10]];
plotϕshallow = Quiet[Show[
     ListPlot[MCCollisionRate[[1 ;; 60]],
      PlotRange → All, PlotStyle → PointSize[.01]],
     ListPlot[exact1CRShallow, PlotRange → All,
      Joined → True, PlotStyle → {Black}],
     Frame → True,
     FrameLabel -> {{4 π r^2 C"c"[r],}, {r,}}
    ]];
logplotϕ = Quiet[Show[
     ListLogPlot[MCCollisionRate, PlotRange → All, PlotStyle → PointSize[.01]],
     ListLogPlot[exact1CR, PlotRange → All, Joined → True, PlotStyle → {Black}],
     ListLogPlot[exact1CRShallow,
      PlotRange → All, Joined → True, PlotStyle → {Black}],
     Frame → True,
     FrameLabel -> {{4 π r^2 C"c"[r],}, {r,}}
    ]];
Show[GraphicsGrid[{{plotϕshallow, logplotϕ}}, ImageSize → 800], PlotLabel ->
  "Infinite 3D, isotropic point source, linearly-anisotropic scattering,
    Gamma-2 random flight - uncorrelated emission\nCollision-rate
    density Cu[r], c = " <> ToString[c] <> ", b = " <> ToString[b]]
```

Out[ ]=

Infinite 3D, isotropic point source, linearly-anisotropic scattering, Gamma-2 random flight – uncorrelated
Collision-rate density C_u[r], c = 0.7, b = 0.7

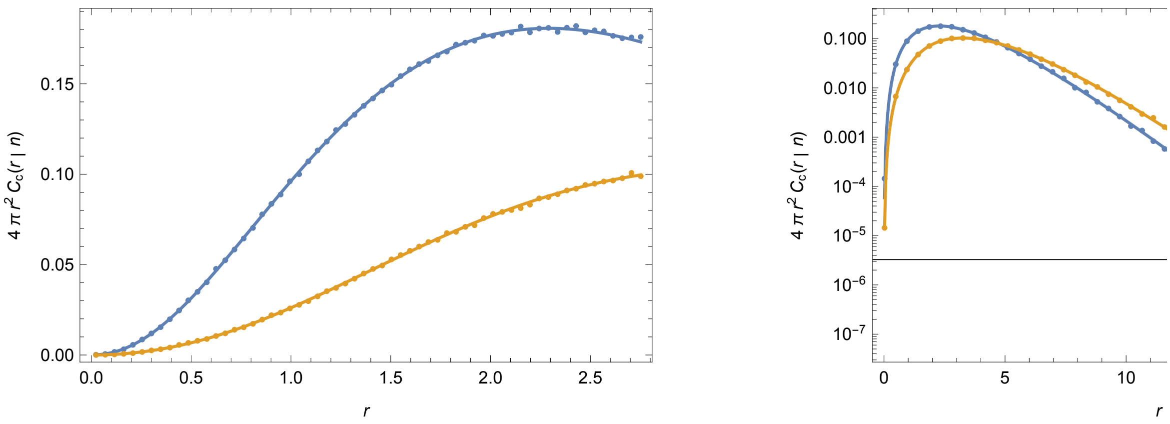


Collision-rate density - double and triple scattering

In[ ]:=
```
data = SelectFirst[simulations, #[[1]] == c && #[[2]] == mfp && #[[3]] == b &][[4]];
maxr = data[[2, 5]];
dr = data[[2, 7]];
MCCollisionRate2 = ppoints[data[[116]], dr, maxr];
MCCollisionRate3 = ppoints[data[[117]], dr, maxr];
exact1CR2Shallow = Quiet[{#[[1]], 4 Pi #[[1]]^2 CcDouble[#[[1]], c, b]} & /@
   MCCollisionRate2[[1 ;; 60]]];
exact1CR2 = Quiet[{#[[1]], 4 Pi #[[1]]^2 CcDouble[#[[1]], c, b]} & /@
   MCCollisionRate2[[61 ;; -1 ;; 10]]];
exact1CR3Shallow = Quiet[{#[[1]], 4 Pi #[[1]]^2 CcTriple[#[[1]], c, b]} & /@
   MCCollisionRate3[[1 ;; 60]]];
exact1CR3 = Quiet[{#[[1]], 4 Pi #[[1]]^2 CcTriple[#[[1]], c, b]} & /@
   MCCollisionRate3[[61 ;; -1 ;; 10]]];
plotϕshallow = Quiet[Show[
    ListPlot[{MCCollisionRate2[[1 ;; 60]], MCCollisionRate3[[1 ;; 60]]},
     PlotRange → All, PlotStyle → PointSize[.01]],
    ListPlot[{exact1CR2Shallow, exact1CR3Shallow},
     PlotRange → All, Joined → True],
    Frame → True,
    FrameLabel -> {{4 π r^2 C"c"[r | n],}, {r,}}
   ]];
logplotϕ = Quiet[Show[
    ListLogPlot[
     {MCCollisionRate2[[1 ;; -1 ;; 10]], MCCollisionRate3[[1 ;; -1 ;; 10]]},
     PlotRange → All, PlotStyle → PointSize[.01]],
    ListLogPlot[{exact1CR2, exact1CR3}, PlotRange → All, Joined → True],
    ListLogPlot[
     {exact1CR2Shallow, exact1CR3Shallow}, PlotRange → All, Joined → True],
    Frame → True,
    FrameLabel -> {{4 π r^2 C"c"[r | n],}, {r,}}, PlotRange → All
   ]];
Show[GraphicsGrid[{{plotϕshallow, logplotϕ}}, ImageSize → 800],
 PlotLabel -> "Exact solution (1)\nInfinite 3D, isotropic point source,
    linearly-anisotropic scattering, Gamma-2 random flight -
    correlated emission\n2nd and 3rd Collision-rate density Cc[r|2]
    and Cc[r|3], c = " <> ToString[c] <> ", b = " <> ToString[b]]
```

Exact solution (1)
Infinite 3D, isotropic point source, linearly−anisotropic scattering, Gamma−2 random flight − correlated
2nd and 3rd Collision−rate density C_c[r|2] and C_c[r|3], c = 0.7, b = 0.7

Out[ ]=



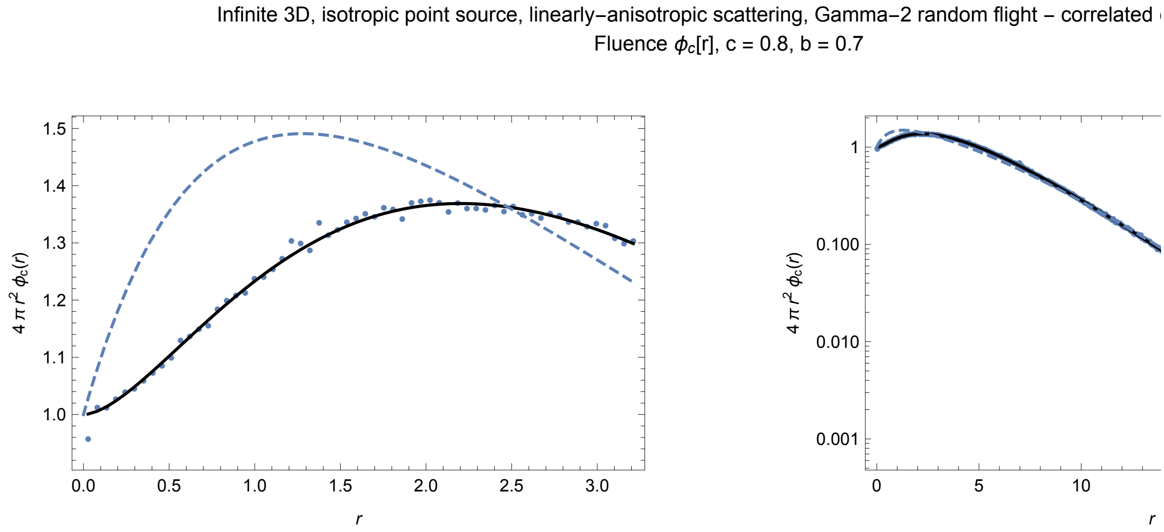
Fluence - Exact solution (1) comparison to MC

In[ ]:=
```
{{ActionMenu["Set c", "c = " <> ToString[#] :> (c = #;) & /@ cs], Dynamic[c]},
 {ActionMenu["Set mfp", "mfp = " <> ToString[#] :> (mfp = #;) & /@ mfps],
  Dynamic[mfp]},
 {ActionMenu["Set b", "b = " <> ToString[#] :> (b = #;) & /@ bs], Dynamic[b]}}
```

Out[ ]= {{ Set c , 0.99}, { Set mfp , 1}, { Set b , 0.7}}

```
In[•]:= data = SelectFirst[simulations, #[[1]] == c && #[[2]] == mfp && #[[3]] == b &][[4]];
maxr = data[[2, 5]];
dr = data[[2, 7]];
MCCollisionRate = ppoints[data[[6]], dr, maxr];
exact1CRShallow = Quiet[{#[[1]], 4 Pi #[[1]]^2 ϕcexact[#[[1]], 1/mfp, c, b]}] & /@
   MCCollisionRate[[1 ;; 60]];
exact1CR = Quiet[{#[[1]], 4 Pi #[[1]]^2 ϕcexact[#[[1]], 1/mfp, c, b]}] & /@
   MCCollisionRate[[61 ;; -1 ;; 10]];
plotϕshallow = Quiet[Show[
     ListPlot[MCCollisionRate[[1 ;; 60]],
      PlotRange → All, PlotStyle → PointSize[.01]],
     ListPlot[exact1CRShallow, PlotRange → All, Joined → True, PlotStyle → Black],
     Plot[4 Pi r^2 fluenceGrosjean[r, c, b],
      {r, 0, MCCollisionRate[[60, 1]]}, PlotStyle → Dashed],
     Frame → True, PlotRange → All,
     FrameLabel -> {{4 π r^2 ϕ"c"[r],}, {r,}}
    ]];
logplotϕ = Quiet[Show[
     ListLogPlot[MCCollisionRate, PlotRange → All, PlotStyle → PointSize[.01]],
     ListLogPlot[exact1CR, PlotRange → All, Joined → True, PlotStyle → Black],
     ListLogPlot[exact1CRShallow,
      PlotRange → All, Joined → True, PlotStyle → Black],
     LogPlot[4 Pi r^2 fluenceGrosjean[r, c, b], {r, 0, maxr},
      PlotStyle → Dashed, PlotRange → All],
     Frame → True,
     FrameLabel -> {{4 π r^2 ϕ"c"[r],}, {r,}}
    ]];
Show[GraphicsGrid[{{plotϕshallow, logplotϕ}}, ImageSize → 800], PlotLabel ->
  "Infinite 3D, isotropic point source, linearly-anisotropic scattering,
     Gamma-2 random flight - correlated emission\nFluence ϕc[r], c = " <>
   ToString[c] <> ", b = " <> ToString[b]]
```

Out[ ]=



In[ ]:= `fluenceGrosjean[.1, .8, 0.7]`

Out[ ]= 8.74066

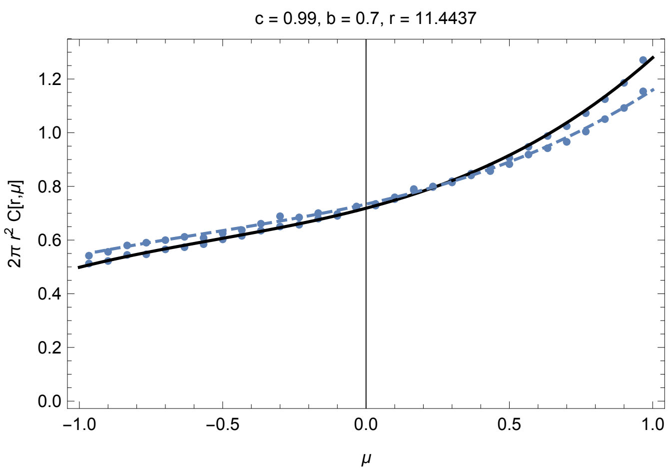
Angular distribution - collision rate density and Radiance compared

In[926]:=
```
{{ActionMenu["Set c", "c = " <> ToString[#] :> (c = #;) & /@ cs], Dynamic[c]},
 {ActionMenu["Set mfp", "mfp = " <> ToString[#] :> (mfp = #;) & /@ mfps],
  Dynamic[mfp]},
 {ActionMenu["Set b", "b = " <> ToString[#] :> (b = #;) & /@ bs], Dynamic[b]}}
```

Out[926]= {{ Set c , 0.99}, { Set mfp , 1}, { Set b , 0.7}}

In[2305]:=
```
depthi = 52;
Clear[u];
data = SelectFirst[simulations, #[[1]] == c && #[[2]] == mfp && #[[3]] == b &][[-1]];
du = data[[2, 9]];
maxr = data[[2, 5]];
dr = data[[2, 7]];
Ci = 15 + 4 numcollorders;
fluxi = 17 + 4 numcollorders + Floor[maxr / dr];
angularFlux = ppointsu[data[[fluxi + depthi]] / 2, du, 1];
angularC = ppointsu[data[[Ci + depthi]], du, 1];
r = dr * depthi - 0.5 dr;
af0 = Ccexact[r, 1, c, b];
af1 = Cl[1, r, c, b];
af2 = Cl[2, r, c, b];
af3 = Cl[3, r, c, b];
ff0 = ϕcexactplus[r, 1, c, b];
ff1 = ϕl[1, r, c, b];
ff2 = ϕl[2, r, c, b];
ff3 = ϕl[3, r, c, b];
Show[
 ListPlot[angularC, PlotRange → All,
  Frame → True,
  FrameLabel -> {{"2π r^2 C[r,μ]",}, {μ, "c = " <> ToString[c] <>
      ", b = " <> ToString[b] <> ", r = " <> ToString[r]}}],
 ListPlot[angularFlux, PlotRange → All],
 Plot[ Pi r^2 (af0 + LegendreP[1, u] af1 + LegendreP[2, u] af2 + LegendreP[3, u] af3),
  {u, -1, 1}, PlotRange → All, PlotStyle → Black],
 Plot[ 1/2 Pi r^2 (ff0 + LegendreP[1, u] ff1 +
      LegendreP[2, u] ff2 + LegendreP[3, u] ff3),
  {u, -1, 1}, PlotRange → All, PlotStyle → Dashed], PlotRange → All
]
```

Out[2324]=



Namespace

In[ ]:=
```
End[]
```

Out[ ]=
```
inf3DisopointlinanisoscatterGamma2`
```