Scattering Kernels in 3D

This is code to accompany the book:

A Hitchhiker's Guide to Multiple Scattering

www.eugenedeon.com/hitchhikers

Haltrin

[Haltrin 1988] - a phase function such that the asymptotic mode in plane geometry has an exact solution:

Consequently, the phase function $p(\cos\chi) = p_H(\cos\chi)$, where

$$p_H(\cos\chi) \equiv 2g\,\delta(1-\cos\chi) + (1-g)\left[2(1-\cos\chi)\right]^{-1/2}, \quad (21)$$

In[ ]:=
```
pHaltrin[u_, g_] := 1/(4 Pi) (2 g DiracDelta[1 - u] + (1 - g)/Sqrt[2 (1 - u)])
```

Normalization condition

In[ ]:=
```
Integrate[2 Pi pHaltrin[u, g], {u, -1, 1}, Assumptions → g > -1 && g < 1] /.
  HeavisideTheta[0] → 1
```

Out[ ]=
```
1
```

Mean cosine (g)

In[ ]:=
```
Integrate[2 Pi pHaltrin[u, g] u, {u, -1, 1}] /. HeavisideTheta[0] → 1 // FullSimplify
```

Out[ ]=
```
1/3 (1 + 2 g)
```

Legendre expansion coefficients

In[ ]:=
```
Integrate[2 Pi (2 k + 1) pHaltrin[u, g] LegendreP[k, u] /. k → 0,
   {u, -1, 1}, Assumptions → g > -1 && g < 1] /. HeavisideTheta[0] → 1
```

Out[ ]=
```
1
```

In[ ]:=
```
Integrate[2 Pi (2 k + 1) pHaltrin[u, g] LegendreP[k, u] /. k → 1,
   {u, -1, 1}, Assumptions → g > -1 && g < 1] /. HeavisideTheta[0] → 1
```

Out[ ]=
```
1 + 2 g
```

In[ ]:=
```
Integrate[2 Pi (2 k + 1) pHaltrin[u, g] LegendreP[k, u] /. k → 2,
  {u, -1, 1}, Assumptions → g > -1 && g < 1] /. HeavisideTheta[0] → 1
```

Out[ ]= $1 + 4\,g$

In[ ]:=
```
Integrate[2 Pi (2 k + 1) pHaltrin[u, g] LegendreP[k, u] /. k → 3,
  {u, -1, 1}, Assumptions → g > -1 && g < 1] /. HeavisideTheta[0] → 1
```

Out[ ]= $1 + 6\,g$

In[ ]:=
```
Integrate[2 Pi (2 k + 1) pHaltrin[u, g] LegendreP[k, u] /. k → 4,
  {u, -1, 1}, Assumptions → g > -1 && g < 1] /. HeavisideTheta[0] → 1
```

Out[ ]= $1 + 8\,g$