Half Rod, Isotropic Point Source, Isotropic Scattering

Exponential Random Flight

This is code to accompany the book:

A Hitchhiker's Guide to Multiple Scattering

Path Setup

Put a file at ~/.hitchhikerpath with the path to your hitchhiker repo so that these worksheets can find the MC data from the C++ simulations for verification

```
In[673]:= SetDirectory[Import["~/.hitchhikerpath"]]
```

Notation

c - single-scattering albedo
x - position coordinate in rod (source at x = x_0)

Analytic solutions

Collision-Rate Density C[x]

```
In[683]:= halfrodPointSourceisoscatter`C[x_, c_, x0_] :=
```

$$\frac{1}{c}\left(\frac{e^{-\sqrt{1-c}\ (x+x0)}\left(-2+2\sqrt{1-c}+c+c\,e^{2\sqrt{1-c}\ \text{Min}[x0,x]}\right)}{2\sqrt{1-c}}\right)$$

```
In[684]:= halfrodPointSourceisoscatter`meanC[c_, x0_] :=
```
$$\frac{1}{1-c}-\frac{\left(1-\sqrt{1-c}\right)e^{-x0\sqrt{1-c}}}{c\,(1-c)}$$

```
In[685]:= halfrodPointSourceisoscatter`meanC[c, 0] // FullSimplify
```

$$Out[685]= \frac{1}{1+\sqrt{1-c}-c}$$

In[686]:= `c halfrodPointSourceisoscatter`meanC[c, 0] + 1 // FullSimplify`

Out[686]= $\dfrac{1}{\sqrt{1-c}}$

In[687]:= `halfrodPointSourceisoscatter`escapeprob[x0_, c_] :=` $\dfrac{\left(1-\sqrt{1-c}\right) e^{-\sqrt{1-c}\,x0}}{c}$

In[688]:= `FullSimplify[(halfrodPointSourceisoscatter`escapeprob[0, c])]`

Out[688]= $\dfrac{1}{1+\sqrt{1-c}}$

In[689]:= `Series[halfrodPointSourceisoscatter`escapeprob[x0, c], {c, 0, 4}]`

Out[689]= $\dfrac{e^{-x0}}{2} + \dfrac{1}{8} e^{-x0} \left(1+2\,x0\right) c + \dfrac{1}{16} e^{-x0} \left(1+x0\right)^2 c^2 + \dfrac{1}{384} e^{-x0} \left(15+30\,x0+18\,x0^2+4\,x0^3\right) c^3 +$
$\dfrac{1}{768} e^{-x0} \left(21+42\,x0+27\,x0^2+8\,x0^3+x0^4\right) c^4 + O[c]^5$

load MC data

In[674]:= `halfrodPointSourceisoscatter`ppoints[xs_, dx_, maxx_, Σt_] :=`
`    Table[{dx (i - 1) + 0.5 dx, (1 / Σt) xs[[i]]}, {i, 1, Length[xs]}][[1 ;; -2]]`

In[675]:= `halfrodPointSourceisoscatter`fs = FileNames[`
`        "code/rod/halfrod/pointSource/data/halfrod_pointsource_isotropicscatter_exp*`
`        "];`

In[676]:= `halfrodPointSourceisoscatter`index[x_] := Module[{data, c, x0},`
`        data = Import[x, "Table"];`
`        x0 = data[[1, 11]];`
`        c = data[[2, 3]];`
`        {c, x0, data}];`
`    halfrodPointSourceisoscatter`simulations =`
`      halfrodPointSourceisoscatter`index /@ halfrodPointSourceisoscatter`fs;`

In[678]:= `halfrodPointSourceisoscatter`cs =`
`    Union[#[[1]] & /@ halfrodPointSourceisoscatter`simulations]`

Out[678]= `{0.1, 0.3, 0.5, 0.7, 0.9, 0.95, 0.98, 0.99, 0.999}`

In[679]:= `halfrodPointSourceisoscatter`x0s =`
`    Union[#[[2]] & /@ halfrodPointSourceisoscatter`simulations]`

Out[679]= `{0.2, 1, 3}`

In[680]:= `halfrodPointSourceisoscatter`numcollorders =`
`    halfrodPointSourceisoscatter`simulations[[1]][[3]][[2, 11]]`

Out[680]= `20`

Compare Deterministic and MC

Collision-rate density

```
In[690]:= Clear[c, x0];
     Manipulate[
      If[Length[halfrodPointSourceisoscatter`simulations] > 0,
        Module[{data, maxx, dx, numcollorders, nummoments, pointsCL, plotpointsCL,
           pointsCR, plotpointsCR, plotpointsC, plotC, plotLL, plotLR, escapeMC},
         data = SelectFirst[halfrodPointSourceisoscatter`simulations,
             #[[1]] == c && #[[2]] == x0 &][[3]];
         maxx = data[[2, 5]];
         dx = data[[2, 7]];
         numcollorders = data[[2, 11]];
         nummoments = data[[2, 13]];

         escapeMC = data[[3, 3]];

         pointsCL = data[[7]];
         plotpointsCL = halfrodPointSourceisoscatter`ppoints[pointsCL, dx, maxx, 1];
         pointsCR = data[[9]];
         plotpointsCR = halfrodPointSourceisoscatter`ppoints[pointsCR, dx, maxx, 1];
         plotpointsC =
          halfrodPointSourceisoscatter`ppoints[pointsCL + pointsCR, dx, maxx, 1];

         plotC = Show[
           ListPlot[plotpointsC, PlotRange → All, PlotStyle → PointSize[.01]],
           Plot[
            halfrodPointSourceisoscatter`C[x, c, x0], {x, 0, maxx}, PlotRange → All]
           , Frame → True,
           FrameLabel ->
            {{C[x],}, {"x", "Semi-infinite rod, point source, isotropic scattering,
                collision density C[x], c = " <>
               ToString[c] <> ", x₀ = " <> ToString[x0] <> "\nEscape MC: " <>
               ToString[escapeMC] <> "\tEscape deterministic: " <>
               ToString[halfrodPointSourceisoscatter`escapeprob[x0, c]]}}
          ];
         Show[GraphicsGrid[{{plotC}}], ImageSize → 500]
        ]
        ,
        Text[
         "Uh oh!  Couldn't find MC data.  Try to evaluate this entire notebook and
           ensure the data path is setup correctly."]
       ]
      , {{c, 0.7}, halfrodPointSourceisoscatter`cs},
      {x0, halfrodPointSourceisoscatter`x0s}]
```

Out[691]=

Semi−infinite rod, point source, isotropic scattering, collision density C[x], c = 0.7, x_0 = 1
Escape MC: 0.373478   Escape deterministic: 0.373623