

# Infinite 3D medium, Isotropic Point Source, Rayleigh Scattering

BesselK0 Random Flight

This is code to accompany the book:

A Hitchhiker's Guide to Multiple Scattering

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[www.eugenedeon.com/hitchhikers](http://www.eugenedeon.com/hitchhikers)

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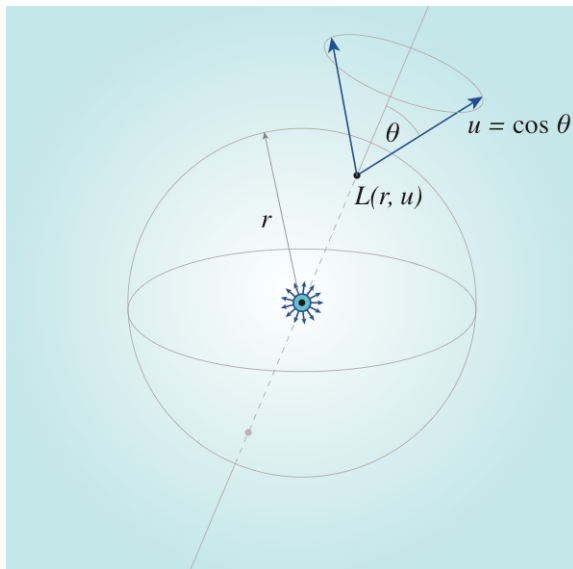
## Path Setup

Put a file at `~/hitchhikerpath` with the path to your hitchhiker repo so that these worksheets can find the MC data from the C++ simulations for verification

```
In[ ]:= SetDirectory[Import["~/hitchhikerpath"]]
```

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## Notation



$c$  - single-scattering albedo

$r$  - radial position coordinate in medium (distance from point source at origin)

$u = \cos \theta$  - direction cosine

## Namespace

```
In[ ]:= Begin["inf3DisopointRayleighscatterK0`"]
```

```
Out[ ]:= inf3DisopointRayleighscatterK0`
```

## Analytical results

### Collision rate density

collision rate density  $C_c$  due to correlated emission:

#### derivation

#### result

This is a highly oscillatory integrand and this does *not* run fast...

```
In[ ]:= Ccexact[r_, c_] := NIntegrate[
$$\left( u \left( 18 c u^2 (1 + u^2) - \text{ArcSinh}[u] \right. \right. \\ \left. \left. - 64 u^5 + 9 c u \sqrt{1 + u^2} + 18 c u^3 \sqrt{1 + u^2} + 9 c \text{ArcSinh}[u] \right) \right) \text{Sin}[r u] \Bigg/ \\ \left( 2 \pi^2 r \left( u^2 \left( 64 u^4 - 18 c^2 (1 + u^2) + 3 c \sqrt{1 + u^2} (9 + 2 u^2) \right) + 3 c \text{ArcSinh}[u] \right. \right. \\ \left. \left. \left( u \left( -9 - 24 u^4 + 3 c \sqrt{1 + u^2} + u^2 (-8 + 6 c \sqrt{1 + u^2}) \right) + 3 c \text{ArcSinh}[u] \right) \right) \right), \\ \{u, 0, \text{Infinity}\}, \text{Method} \rightarrow \text{"ExtrapolatingOscillatory"}]$$

```

## load MC data

```
In[ ]:= ppoints[xs_, dr_, maxx_] :=
  Table[{dr (i) - 0.5 dr, xs[[i]]}, {i, 1, Length[xs]}][[1 ;; -2]]

In[ ]:= ppointsu[xs_, du_, st_] :=
  Table[{-1.0 + du (i) - 0.5 du, xs[[i]] / (2 st)}, {i, 1, Length[xs]}][[1 ;; -1]]

In[ ]:= fs = FileNames["code/3D_medium/infinite3Dmedium/Isotropicpointsource/MCdata/
  inf3D_isotropicpoint_rayleighscatter_bessel*"];

In[ ]:= index[x_] := Module[{data, c},
  data = Import[x, "Table"];
  c = data[[2, 3]];
  {c, data}];
simulations = index /@ fs;
cs = Union[#[[1]] & /@ simulations]

Out[ ]:= {0.01, 0.1, 0.3, 0.5, 0.7, 0.8, 0.9, 0.95, 0.99, 0.999}

In[ ]:= numcollorders = simulations[[1]][[-1]][[2, 13]];
```

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# Compare analytic and MC

## Collision-rate density - Exact solution - comparison to MC

```
In[ ]:= {ActionMenu["Set c", "c = " <> ToString[#] => (c = #;) & /@ cs], Dynamic[c]}
```

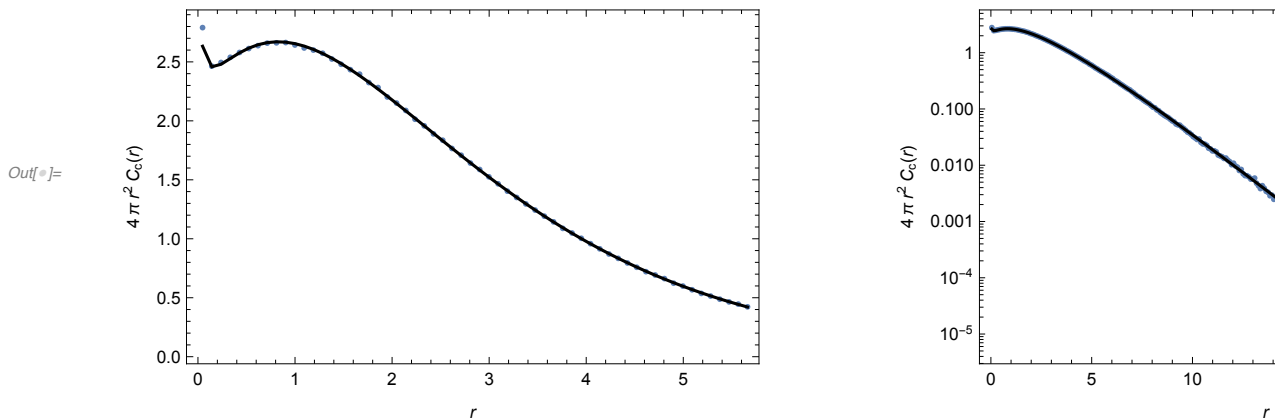
```
Out[ ]:= { Set c, 0.9 }
```

```

In[ ]:= data = SelectFirst[simulations, #[[1]] == c &][[2]];
maxr = data[[2, 5]];
dr = data[[2, 7]];
MCCollisionRate = ppoints[data[[4]], dr, maxr];
exact1CRShallow =
  Quiet[{#[[1]], 4 Pi #[[1]]^2 Ccexact#[[1]], c}] & /@ MCCollisionRate[[1 ;; 60]];
exact1CR = Quiet[{#[[1]], 4 Pi #[[1]]^2 Ccexact#[[1]], c}] & /@
  MCCollisionRate[[61 ;; -1 ;; 10]];
plotϕshallow = Quiet[Show[
  ListPlot[MCCollisionRate[[1 ;; 60]],
    PlotRange → All, PlotStyle → PointSize[.01]],
  ListPlot[exact1CRShallow, PlotRange → All, Joined → True, PlotStyle → Black],
  Frame → True,
  FrameLabel -> {{4 π r2 Cc"[r],}, {r,}}
]];
logplotϕ = Quiet[Show[
  ListLogPlot[MCCollisionRate, PlotRange → All, PlotStyle → PointSize[.01]],
  ListLogPlot[exact1CR, PlotRange → All, Joined → True, PlotStyle → Black],
  ListLogPlot[exact1CRShallow,
    PlotRange → All, Joined → True, PlotStyle → Black],
  Frame → True,
  FrameLabel -> {{4 π r2 Cc"[r],}, {r,}}
]];
Show[GraphicsGrid[{{plotϕshallow, logplotϕ}}, ImageSize → 800],
  PlotLabel -> "Infinite 3D, isotropic point source,
  Rayleigh scattering, BesselK0 random flight - correlated
  emission\nCollision-rate density Cc[r], c = "<>ToString[c]]

```

Infinite 3D, isotropic point source, Rayleigh scattering, BesselK<sub>0</sub> random flight – correlated emission  
Collision-rate density C<sub>c</sub>[r], c = 0.9



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## Namespace

In[ ]:= **End[]**

Out[ ]:= inf3DisopointRayleighscatterK0`