Scattering Kernels in 3D

This is code to accompany the book:

A Hitchhiker's Guide to Multiple Scattering

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www.eugenedeon.com/hitchhikers

Rayleigh Scattering

General form:

$$\text{pRayleigh}[u\_, \gamma\_] := \frac{1}{4\,\text{Pi}}\,\frac{3}{4\,(1+2\,\gamma)}\,\left((1+3\,\gamma)+(1-\gamma)\,u^2\right)$$

Common special case ($\gamma = 0$):

In[971]:=
$$\text{pRayleigh}[u\_] := (1+u^2)\,\frac{3}{16\,\text{Pi}}$$

Normalization condition

```
Integrate[2 Pi pRayleigh[u], {u, -1, 1}]
```

```
1
```

```
Integrate[2 Pi pRayleigh[u, y], {u, -1, 1}, Assumptions → y > 0] // Simplify
```

```
1
```

Mean cosine (g)

```
Integrate[2 Pi pRayleigh[u] u, {u, -1, 1}]
```

```
0
```

```
Integrate[2 Pi pRayleigh[u, y] u, {u, -1, 1}, Assumptions → y > 0] // Simplify
```

```
0
```

Legendre expansion coefficients

```
Integrate[
 2 Pi (2 k + 1) pRayleigh[Cos[y]] LegendreP[k, Cos[y]] Sin[y] /. k → 0, {y, 0, Pi}]
```

```
1
```

```
Integrate[
 2 Pi (2 k + 1) pRayleigh[Cos[y]] LegendreP[k, Cos[y]] Sin[y] /. k → 1, {y, 0, Pi}]
```

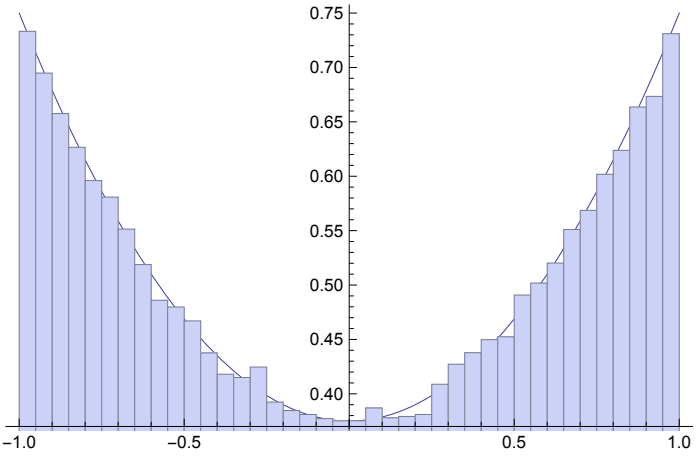
```
0
```

```
Integrate[
 2 Pi (2 k + 1) pRayleigh[Cos[y]] LegendreP[k, Cos[y]] Sin[y] /. k → 2, {y, 0, Pi}]
```

$$\frac{1}{2}$$

sampling

```
Show[
 Plot[2 Pi pRayleigh[u], {u, -1, 1}],
 Histogram[Map[(1 - (2 - 4 # + Sqrt[5 + 16 (-1 + #) #])^(2/3))/(2 - 4 # + Sqrt[5 + 16 (-1 + #) #])^(1/3) &,
   Table[RandomReal[], {i, 1, 100 000}]], 50, "PDF"]
]
Clear[b];
```



Adding/doubling integrals

In[972]:=
```
With[{l = 0},
 (2 - KroneckerDelta[l, 0])/Pi
  Integrate[pRayleigh[mui muo + Sqrt[(1 - mui^2) (1 - muo^2)] Cos[phi]] Cos[l phi],
   {phi, 0, Pi}, Assumptions → -1 < mui < 1 && -1 < muo < 1]
]
```

Out[972]=
$$\frac{3\left(3 - \text{muo}^2 + \text{mui}^2\left(-1 + 3\,\text{muo}^2\right)\right)}{32\,\pi}$$

In[973]:=
```
With[{l = 1},
 2 - KroneckerDelta[l, 0]
                Pi
  Integrate[pRayleigh[mui muo + Sqrt[(1 - mui^2) (1 - muo^2)] Cos[phi]] Cos[l phi],
   {phi, 0, Pi}, Assumptions → -1 < mui < 1 && -1 < muo < 1]

]
```

Out[973]=
$$\frac{3\,\text{mui}\,\text{muo}\,\sqrt{\left(-1+\text{mui}^2\right)\,\left(-1+\text{muo}^2\right)}}{8\,\pi}$$

In[974]:=
```
With[{l = 2},
 2 - KroneckerDelta[l, 0]
                Pi
  Integrate[pRayleigh[mui muo + Sqrt[(1 - mui^2) (1 - muo^2)] Cos[phi]] Cos[l phi],
   {phi, 0, Pi}, Assumptions → -1 < mui < 1 && -1 < muo < 1]

]
```

Out[974]=
$$\frac{3\,\left(-1+\text{mui}^2\right)\,\left(-1+\text{muo}^2\right)}{32\,\pi}$$

In[975]:=
```
With[{l = 3},
 2 - KroneckerDelta[l, 0]
                Pi
  Integrate[pRayleigh[mui muo + Sqrt[(1 - mui^2) (1 - muo^2)] Cos[phi]] Cos[l phi],
   {phi, 0, Pi}, Assumptions → -1 < mui < 1 && -1 < muo < 1]

]
```

Out[975]= 0