Scattering Kernels in 3D

This is code to accompany the book:

A Hitchhiker's Guide to Multiple Scattering

www.eugenedeon.com/hitchhikers

Rayleigh Scattering

General form:

$$\text{pRayleigh}[u\_, \gamma\_] := \frac{1}{4\,\text{Pi}}\,\frac{3}{4\,(1+2\,\gamma)}\,\left((1+3\,\gamma)+(1-\gamma)\,u^2\right)$$

Common special case ($\gamma = 0$):

$$\text{pRayleigh}[u\_] := \left(1+u^2\right)\,\frac{3}{16\,\text{Pi}}$$

Normalization condition

```
Integrate[2 Pi pRayleigh[u], {u, -1, 1}]
```

```
1
```

```
Integrate[2 Pi pRayleigh[u, y], {u, -1, 1}, Assumptions → y > 0] // Simplify
```

```
1
```

Mean cosine (g)

```
Integrate[2 Pi pRayleigh[u] u, {u, -1, 1}]
```

```
0
```

```
Integrate[2 Pi pRayleigh[u, y] u, {u, -1, 1}, Assumptions → y > 0] // Simplify
```

```
0
```

Legendre expansion coefficients

```
Integrate[
 2 Pi (2 k + 1) pRayleigh[Cos[y]] LegendreP[k, Cos[y]] Sin[y] /. k → 0, {y, 0, Pi}]
```

```
1
```

```
Integrate[
 2 Pi (2 k + 1) pRayleigh[Cos[y]] LegendreP[k, Cos[y]] Sin[y] /. k → 1, {y, 0, Pi}]
```

```
0
```

```
Integrate[
 2 Pi (2 k + 1) pRayleigh[Cos[y]] LegendreP[k, Cos[y]] Sin[y] /. k → 2, {y, 0, Pi}]
```

$$\frac{1}{2}$$

sampling

```
Show[
 Plot[2 Pi pRayleigh[u], {u, -1, 1}],
 Histogram[Map[(1 - (2 - 4 # + Sqrt[5 + 16 (-1 + #) #])^(2/3))/(2 - 4 # + Sqrt[5 + 16 (-1 + #) #])^(1/3) &,
   Table[RandomReal[], {i, 1, 100 000}]], 50, "PDF"]
]
Clear[b];
```