Dirac Delta NDF

This is code to accompany the book:

A Hitchhiker's Guide to Multiple Scattering

www.eugenedeon.com/hitchhikers

notation

$u = \mathbf{m} . \mathbf{n} = \mathrm{Cos}[\theta_m]$
α = roughness

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definitions and derivations

In[706]:= $\texttt{Dirac`D[u\_, ud\_] :=} \dfrac{\texttt{DiracDelta[u - ud]}}{\texttt{2 Pi ud}}$

In[713]:= $\texttt{Dirac`}\sigma\texttt{[u\_, ui\_] := Re}\left[2\left(\sqrt{1-u^2-ui^2} + u\,ui\,\mathrm{ArcCos}\left[-\dfrac{u\,ui}{\sqrt{1-u^2}\sqrt{1-ui^2}}\right]\right)\right]$

In[714]:= $\texttt{Dirac`}\Lambda\texttt{[u\_, ui\_] :=} \dfrac{\mathrm{Re}\left[2\left(\sqrt{1-u^2-ui^2} + u\,ui\,\mathrm{ArcCos}\left[-\frac{u\,ui}{\sqrt{1-u^2}\sqrt{1-ui^2}}\right]\right)\right]}{u} - 1$

In[716]:= $\left(1 + \texttt{Dirac`}\Lambda\texttt{[u, ud]}\right)\texttt{ u == Dirac`}\sigma\texttt{[u, ud]}$

Out[716]= True

In[722]:= $\texttt{FullSimplify}\left[\left(\texttt{Dirac`}\Lambda\texttt{[u, ud]}\right)\texttt{ u == Dirac`}\sigma\texttt{[-u, ud]},\right.$
$\left.\texttt{Assumptions} \to 0 < ud < 1\ \&\&\ -1 < u < 1\right]$

Out[722]= $2\,\pi\,u\,ud == u$

height field normalization

In[721]:= `Integrate[2 Pi u Dirac`D[u, ud], {u, 0, 1}, Assumptions → 0 < ud < 1]`

Out[721]= 1

distribution of slopes

In[735]:= $\texttt{Dirac`P22[p\_, q\_, ud\_] :=} \dfrac{\mathrm{DiracDelta}\left[\frac{1}{\sqrt{1+p^2+q^2}} - ud\right]}{2\,\pi\,\left(1+p^2+q^2\right)^2\,ud}$

In[736]:=
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Integrate[Dirac`P22[p, q, ud], {p, -Infinity, Infinity},
  {q, -Infinity, Infinity}, Assumptions → 0 < ud < 1]
```

Out[736]= 1

In[747]:=

$$\text{Dirac`P2[q\_, ud\_]} := \frac{\text{ud}}{2\,\pi\,\sqrt{\text{Abs}\left[-1+\left(1+\text{q}^2\right)\,\text{ud}^2\right]}}\ \text{HeavisideTheta}\left[\left(-1-\text{q}^2+\frac{1}{\text{ud}^2}\right)\right]$$

In[751]:=
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Plot[{Dirac`P2[q, .6]}, {q, -3, 3}]
```

Out[751]=