Infinite 3D medium, Isotropic Point Source, Isotropic Scattering

Exponential Random Flight

This is code to accompany the book:

A Hitchhiker's Guide to Multiple Scattering

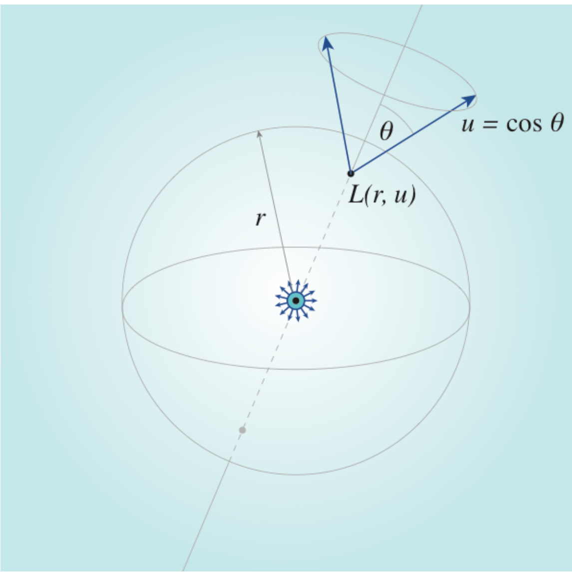
www.eugenedeon.com/hitchhikers

Path Setup

Put a file at ~/.hitchhikerpath with the path to your hitchhiker repo so that these worksheets can find the MC data from the C++ simulations for verification

```
In[725]:= SetDirectory[Import["~/.hitchhikerpath"]]
```

Notation



c - single-scattering albedo

Σt - extinction coefficient

r - radial position coordinate in medium (distance from point source at origin)

u = cos θ - direction cosine

Namespace

In[6706]:=
```
Begin["inf3Disopointisoscatter`"]
```
Out[6706]=
```
inf3Disopointisoscatter`
```

Util

In[6770]:= $\texttt{SA[d\_, r\_] := d}\ \frac{\texttt{Pi}^{d/2}}{\texttt{Gamma}\left[\frac{d}{2}+1\right]}\ r^{d-1}$

Diffusion modes

In[6771]:= $\texttt{diffusionMode[v\_, d\_, r\_] :=}\ (2\pi)^{-d/2}\ r^{1-\frac{d}{2}}\ v^{-1-\frac{d}{2}}\ \texttt{BesselK}\left[\frac{1}{2}(-2+d), \frac{r}{v}\right]$

Analytic solutions

Caseology quantities

In[6739]:= $\texttt{CaseN0[c\_, v0\_] :=}\ \frac{1}{2}\ c\ v0^3 \left(\frac{c}{v0^2-1} - \frac{1}{v0^2}\right)$

In[6740]:= $\texttt{Casev0[c\_?NumericQ] :=}$
$\texttt{FindRoot}\left[c\ v\ \texttt{ArcTanh}\left[\frac{1}{v}\right] - 1 == 0, \{v, 1.00000000001, 10^{10}\}, \texttt{Method} \to \texttt{"Brent"}\right][[1]][[2]]$

In[6741]:= $\texttt{Casev0approx[c\_] :=}\ 1\Big/\sqrt{1 - c^{2.4429445001914587` + \frac{0.5786368322364553`}{c} - 0.021581332427913873`\ c}}$

In[6742]:= $\texttt{CaseN[c\_, v\_] :=}\ v\left(\texttt{Case}\lambda[v, c]^2 + \left(\frac{\pi\ c\ v}{2}\right)^2\right)$

In[6743]:= $\texttt{Case}\lambda\texttt{[v\_, c\_] :=}\ 1 - c\ v\ \texttt{ArcTanh}[v]$

Fluence: exact solution (1)

[Bothe 1942]

In[6744]:= $\phi\texttt{exact1a[r\_, }\Sigma\texttt{t\_, c\_] :=}\ \frac{1}{2\ \texttt{Pi}^2\ r}\ \texttt{NIntegrate}\left[\frac{z\ \texttt{ArcTan}[z/\Sigma t]}{z - c\ \Sigma t\ \texttt{ArcTan}[z/\Sigma t]}\ \texttt{Sin}[r\ z], \{z, 0, \texttt{Infinity}\}, \texttt{Method} \to \texttt{"ExtrapolatingOscillatory"}\right]$

[Case et al. 1953]

In[6745]:= $\phi\texttt{exact1b[r\_, }\Sigma\texttt{t\_, c\_] :=}\ \frac{\texttt{Exp}[-\Sigma t\ r]}{4\ \texttt{Pi}\ r^2} + c\ \frac{\Sigma t}{2\ \texttt{Pi}^2\ r}\ \texttt{NIntegrate}\left[\frac{\texttt{ArcTan}[z]^2}{z - c\ \texttt{ArcTan}[z]}\ \texttt{Sin}[r\ \Sigma t\ z], \{z, 0, \texttt{Infinity}\}, \texttt{Method} \to \texttt{"LevinRule"}\right]$

Rigorous diffusion approximation

In[6746]:= $$\phi\text{rigourousDiffusion}[r\_,\ \Sigma t\_,\ c\_] := \frac{\Sigma t}{4\,\text{Pi}\,r}\ \frac{E^{-r\,\Sigma t/\#}}{\#\ \text{CaseN0}[c,\ \#]}\ \&[\text{Casev0}[c]]$$

In[6747]:= $$\phi\text{transient}[r\_,\ \Sigma t\_,\ c\_] := \frac{\Sigma t}{4\,\text{Pi}\,r}\ \text{NIntegrate}\left[\frac{e^{-\Sigma t\,r\,/v}}{v\ \text{CaseN}[c,\ v]},\ \{v,\ 0,\ 1\}\right]$$

Expansion of transient term [Case et al. 1953]

In[6748]:= $$\phi\text{transient2}[r\_,\ \Sigma t\_,\ c\_,\ M\_] := \frac{\text{Exp}[-r\,\Sigma t]}{4\,\text{Pi}\,r^2} + \frac{1}{4\,\text{Pi}\,r}\ \text{Sum}\big[\text{ExpIntegralE}[2\,n,\ r\,\Sigma t]\ \text{SeriesCoefficient}\big[v\,/\,\text{CaseN}[c,\ v],\ \{v,\ 0,\ 2\,n\}\big],\ \{n,\ 1,\ M\}\big]$$

Fluence: exact solution (2)

[Davison 1947]

In[6749]:= $$\phi\text{exact2a}[r\_,\ \Sigma t\_,\ c\_] := \phi\text{rigourousDiffusion}[r,\ \Sigma t,\ c] + \frac{\Sigma t}{4\,\text{Pi}\,r}\ \text{NIntegrate}\left[\frac{e^{-\Sigma t\,r\,y}}{\frac{c^2\,\pi^2}{4\,y^2} + \left(1 - \frac{c}{2\,y}\ \text{Log}\left[\frac{y+1}{y-1}\right]\right)^2},\ \{y,\ 1,\ \text{Infinity}\}\right]$$

[Case and Zwiefel 1967]

In[6750]:= $$\phi\text{exact2b}[r\_,\ \Sigma t\_,\ c\_] := \phi\text{rigourousDiffusion}[r,\ \Sigma t,\ c] + \frac{\Sigma t}{4\,\text{Pi}\,r}\ \text{NIntegrate}\left[\frac{e^{-\Sigma t\,r\,/v}}{v\ \text{CaseN}[c,\ v]},\ \{v,\ 0,\ 1\}\right]$$

n-th scattered fluence

In[6751]:= $$\phi\text{exact1}[r\_,\ \Sigma t\_,\ c\_,\ n\_] := \frac{(c\,\Sigma t)^n}{2\,\pi^2\,r}\ \text{NIntegrate}\left[\frac{\text{ArcTan}\left[\frac{z}{\Sigma t}\right]^{1+n}\ \text{Sin}[r\,z]}{z^n},\ \{z,\ 0,\ \text{Infinity}\},\ \text{Method} \to \text{"ExtrapolatingOscillatory"}\right]$$

In[6752]:= $$\phi\text{exact2}[r\_,\ \Sigma t\_,\ c\_,\ n\_] := \frac{c^n\,\Sigma t}{2^{n+3}\,\text{Pi}^2\,\text{I}\,r}\ \text{Chop}\left[\text{NIntegrate}\left[\frac{\text{Exp}[-r\,z\,\Sigma t]}{z^n}\left(\left(\text{Log}\left[\frac{z+1}{z-1}\right] + \text{I}\,\text{Pi}\right)^{n+1} - \left(\text{Log}\left[\frac{z+1}{z-1}\right] - \text{I}\,\text{Pi}\right)^{n+1}\right),\ \{z,\ 1,\ \text{Infinity}\}\right]\right]$$

In[6753]:= $$\phi\text{Gaussian}[r\_,\ \Sigma t\_,\ c\_,\ n\_] := \frac{3\,\sqrt{3}\ e^{-\frac{3\,r^2\,\Sigma t^2}{4\,(1+n)}}\ c^n\,\Sigma t^2}{8\,\sqrt{(1+n)^3}\ \pi^{3/2}}$$

Moments

In[6754]:= $$\phi\text{m}[c\_,\ \Sigma t\_,\ m\_?\text{IntegerQ},\ n\_] := \text{Limit}\left[\text{Simplify}\left[(-1)^{m/2}\left(\frac{2\ \text{Gamma}\left[\frac{3+m}{2}\right]}{\text{Gamma}\left[\frac{1+m}{2}\right]}\ \text{D}\left[\frac{\left(\frac{c\,\Sigma t\ \text{ArcTan}\left[\frac{z}{\Sigma t}\right]}{z}\right)^{1+n}}{c\,\Sigma t},\ \{z,\ m\}\right]\right)\right],\ z \to 0\right]$$

In[6755]:= TableForm[Table[ϕm[c, Σt, m, n], {m, 0, 6, 2}]]

Out[6755]//TableForm=

$$\frac{c^n}{\Sigma t}$$
$$\frac{2\, c^n\, (1+n)}{\Sigma t^3}$$
$$\frac{4\, c^n\, (1+n)\, (18+5\, n)}{3\, \Sigma t^5}$$
$$\frac{8\, c^n\, (1+n)\, \left(810+343\, n+35\, n^2\right)}{9\, \Sigma t^7}$$

In[6756]:=
$$\phi m[c\_, \Sigma t\_, m\_?IntegerQ] := \mathrm{Limit}\left[\mathrm{Simplify}\left[(-1)^{m/2}\left(\frac{2\,\mathrm{Gamma}\left[\frac{3+m}{2}\right]}{\mathrm{Gamma}\left[\frac{1+m}{2}\right]}\, \mathrm{D}\left[\frac{\mathrm{ArcTan}\left[\frac{z}{\Sigma t}\right]}{z - c\, \Sigma t\, \mathrm{ArcTan}\left[\frac{z}{\Sigma t}\right]}, \{z, m\}\right]\right)\right], z \to 0\right]$$

In[6757]:= TableForm[Table[ϕm[c, Σt, m], {m, 0, 6, 2}]]

Out[6757]//TableForm=

$$\frac{1}{\Sigma t - c\, \Sigma t}$$
$$\frac{2}{(-1+c)^2\, \Sigma t^3}$$
$$\frac{8\, (-9+4\, c)}{3\, (-1+c)^3\, \Sigma t^5}$$
$$\frac{16\, \left(135-144\, c+44\, c^2\right)}{3\, (-1+c)^4\, \Sigma t^7}$$

Recurrence derivation [Case et al. 1953]

In[6758]:=
$$\mathrm{CaseB}[0, c\_] := \frac{1}{1-c};$$
$$\mathrm{CaseB}[m\_, c\_] := \frac{1}{(1-c)^2}\, \mathrm{Sum}\left[\mathrm{Caseb}[m, s]\left(\frac{c}{1-c}\right)^{s-1}, \{s, 1, m\}\right];$$
$$\mathrm{Caseb}[m\_, 1] := \frac{1}{2\, m+1};$$
$$\mathrm{Caseb}[m\_, s\_] := \mathrm{Sum}\left[\frac{\mathrm{Caseb}[n, s-1]}{1+2\, (m-n)}, \{n, s-1, m-1\}\right]$$

In[6772]:=
$$\phi mCase[c\_, \Sigma t\_, m\_?IntegerQ] := \frac{1}{\Sigma t^{m+1}}\, \mathrm{CaseB}\left[m/2, c\right]\, \mathrm{Factorial}[m+1]$$

In[6773]:= TableForm[Table[FullSimplify[ϕmCase[c, Σt, m]], {m, 0, 6, 2}]]

Out[6773]//TableForm=

$$\frac{1}{\Sigma t - c\, \Sigma t}$$
$$\frac{2}{(-1+c)^2\, \Sigma t^3}$$
$$\frac{8\, (-9+4\, c)}{3\, (-1+c)^3\, \Sigma t^5}$$
$$\frac{16\, (135+4\, c\, (-36+11\, c))}{3\, (-1+c)^4\, \Sigma t^7}$$

Classical diffusion approximation

In[6774]:=
$$\phi Diffusion[r\_, \Sigma t\_, c\_] := \frac{1}{\Sigma t\, (1-c)}\, \mathrm{diffusionMode}\left[\frac{1}{\sqrt{3\, (1-c)}\, \Sigma t}, 3, r\right]$$

In[6775]:= FullSimplify[ϕDiffusion[r, Σt, c], Assumptions → 0 < c < 1 && Σt > 0]

Out[6775]=
$$\frac{3\, e^{-\sqrt{3-3\, c}\; r\, \Sigma t}\, \Sigma t}{4\, \pi\, r}$$

Grosjean-style diffusion approximation

In[6776]:=
$$\phi\text{Grosjean}[r\_, \Sigma t\_, c\_] := \frac{\text{Exp}[-r\,\Sigma t]}{4\,\text{Pi}\,r^2} + \frac{c}{\Sigma t\,(1-c)}\,\text{diffusionMode}\left[\frac{\sqrt{2-c}}{\sqrt{3\,(1-c)}\;\Sigma t}, 3, r\right]$$

In[6777]:=
```
FullSimplify[ϕGrosjean[r, Σt, c], Assumptions → 0 < c < 1 && Σt > 0]
```

Out[6777]=
$$\frac{e^{-r\,\Sigma t} - \dfrac{3\,c\,e^{-\sqrt{3+\frac{3}{-2+c}}\;r\,\Sigma t}\,r\,\Sigma t}{-2+c}}{4\,\pi\,r^2}$$

Angular ϕ Integral

Note: this form leaves out the singular term $\frac{e^{-r\Sigma_t}}{4\pi r^2}\delta(u-1)$, because it doesn't plot:

In[6778]:=
$$\text{Lintegral}[r\_, u\_, \Sigma t\_, c\_, \phi\_] := \frac{c\,\Sigma t}{4\,\text{Pi}}\,\text{NIntegrate}\left[\phi\left[\sqrt{r^2+t^2-2\,r\,t\,u}, \Sigma t, c\right]\text{Exp}[-\Sigma t\,t], \{t, 0, \text{Infinity}\}\right]$$

Angular Classical diffusion approximation

In[6779]:=
$$\text{Ldiffusion}[r\_, u\_, \Sigma t\_, c\_] := \frac{1}{4\,\text{Pi}}\,\phi\text{Diffusion}[r, \Sigma t, c] + \frac{1}{4\,\text{Pi}}\,u\,\frac{3\,e^{-r\sqrt{3-3c}\;\Sigma t}\left(1+r\sqrt{3-3\,c}\;\Sigma t\right)}{4\,\pi\,r^2}$$

---

load MC data

In[6780]:=
```
ppoints[xs_, dr_, maxx_] :=
  Table[{dr (i) - 0.5 dr, xs[[i]]}, {i, 1, Length[xs]}][[1 ;; -2]]
```

In[6781]:=
```
ppointsu[xs_, du_, Σt_] :=
  Table[{-1.0 + du (i) - 0.5 du, xs[[i]] / (2 Σt)}, {i, 1, Length[xs]}][[1 ;; -1]]
```

In[6782]:=
```
fs = FileNames["code/3D_medium/infinite3Dmedium/Isotropicpointsource/MCdata/
      inf3D_isotropicpoint_isotropicscatter*"];
```

In[6783]:=
```
index[x_] := Module[{data, α, Σt},
    data = Import[x, "Table"];
    Σt = data[[1, 13]];
    α = data[[2, 3]];
    {α, Σt, data}];
simulations = index /@ fs;
cs = Union[#[[1]] & /@ simulations]
```

Out[6785]=
```
{0.01, 0.1, 0.3, 0.5, 0.7, 0.8, 0.9, 0.95, 0.99, 0.999}
```

In[6786]:=
```
mfps = Union[#[[2]] & /@ simulations]
```

Out[6786]=
```
{0.3, 1}
```

In[6787]:=
```
numcollorders = simulations[[1]][[3]][[2, 13]];
maxr = simulations[[1]][[3]][[2, 5]];
dr = simulations[[1]][[3]][[2, 7]];
numr = Floor[maxr / dr];
```

Compare Deterministic and MC

Mean Track Length

In[6872]:=
```
{{ActionMenu["Set c", "c = " <> ToString[#] :> (c = #;) & /@ cs], Dynamic[c]},
  {ActionMenu["Set mfp", "mfp = " <> ToString[#] :> (mfp = #;) & /@ mfps],
   Dynamic[mfp]}} // TableForm
```

Out[6872]//TableForm=

Set c	0.8
Set mfp	0.3

In[6791]:=
```
data = SelectFirst[simulations, #[[1]] == c && #[[2]] == mfp &][[3]];
meanTL = data[[-1]]
```

$$\frac{\text{mfp}}{1 - c}$$

Out[6792]= {Mean, track, length:, 1.00046}

Out[6793]= 1.

Fluence - Exact solution (1a) comparison to MC

In[6873]:=
```
{{ActionMenu["Set c", "c = " <> ToString[#] :> (c = #;) & /@ cs], Dynamic[c]},
  {ActionMenu["Set mfp", "mfp = " <> ToString[#] :> (mfp = #;) & /@ mfps],
   Dynamic[mfp]}} // TableForm
```

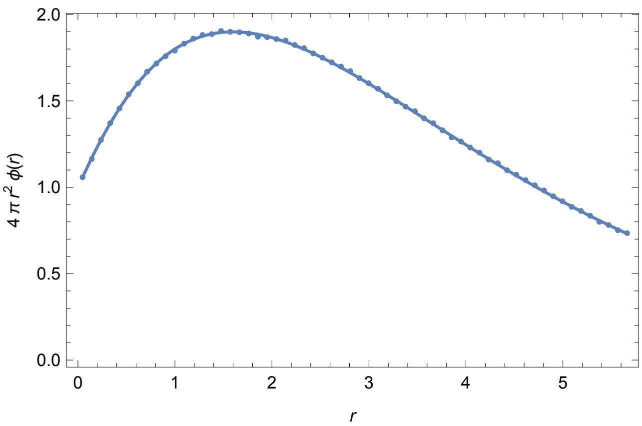
Out[6873]//TableForm=

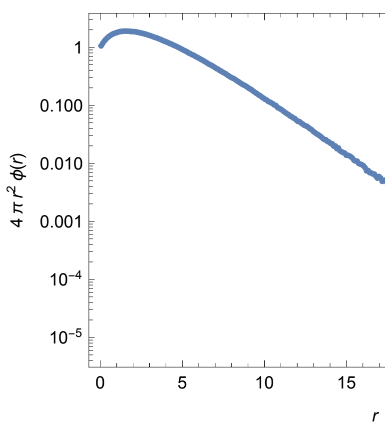
Set c	0.8
Set mfp	0.3

In[6812]:=
```
data = SelectFirst[simulations, #[[1]] == c && #[[2]] == mfp &][[3]];
maxr = data[[2, 5]];
dr = data[[2, 7]];
MCFluence = ppoints[data[[6]], dr, maxr];
exact1FluenceShallow =
  Quiet[{#[[1]], 4 Pi #[[1]]^2 ϕexact1a[#[[1]], 1/mfp, c]}] & /@
   MCFluence[[1 ;; 60]];
exact1Fluence = Quiet[{#[[1]], 4 Pi #[[1]]^2 ϕexact1a[#[[1]], 1/mfp, c]}] & /@
   MCFluence[[60 ;; -1 ;; 10]];
plotϕshallow = Quiet[Show[
    ListPlot[MCFluence[[1 ;; 60]], PlotRange → All, PlotStyle → PointSize[.01]],
    ListPlot[exact1FluenceShallow, PlotRange → All, Joined → True],
    Frame → True,
    FrameLabel -> {{4 Pi r^2 ϕ[r],}, {r,}}
   ]];
logplotϕ = Quiet[Show[
    ListLogPlot[MCFluence, PlotRange → All, PlotStyle → PointSize[.01]],
    ListLogPlot[exact1Fluence, PlotRange → All, Joined → True],
    ListLogPlot[exact1FluenceShallow, PlotRange → All, Joined → True],
    Frame → True,
    FrameLabel -> {{4 Pi r^2 ϕ[r],}, {r,}}
   ]];
Show[GraphicsGrid[{{plotϕshallow, logplotϕ}}, ImageSize → 800],
 PlotLabel -> "Exact solution (1a)\nInfinite 3D, isotropic
    point source, isotropic scattering, fluence ϕ[r], c = " <>
   ToString[c] <> ", Σt = " <> ToString[1/mfp]]
```

Out[6820]=

Exact solution (1a)
Infinite 3D, isotropic point source, isotropic scattering, fluence ϕ[r], c = 0.9, Σ_t = 1





Fluence - Exact solution (1b) comparison to MC

In[6874]:=
```
{{ActionMenu["Set c", "c = " <> ToString[#] :→ (c = #;) & /@ cs], Dynamic[c]},
  {ActionMenu["Set mfp", "mfp = " <> ToString[#] :→ (mfp = #;) & /@ mfps],
   Dynamic[mfp]}} // TableForm
```

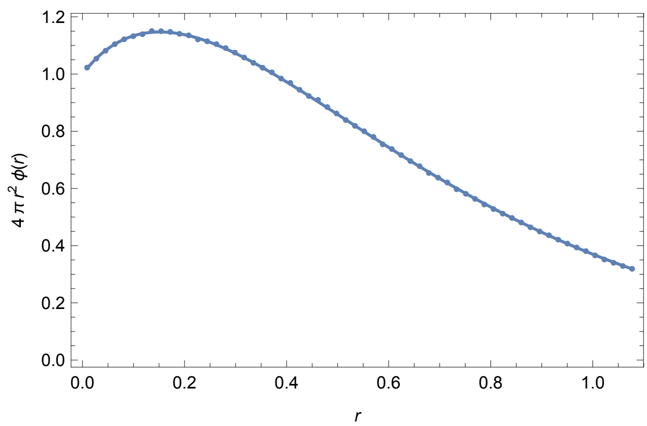
Out[6874]//TableForm=

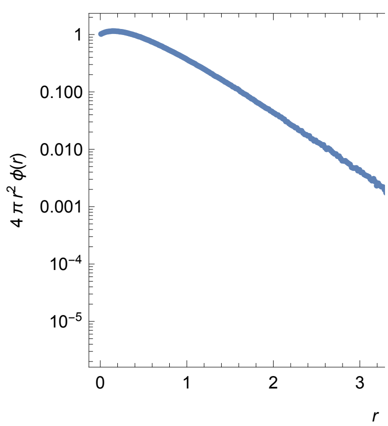
Set c	0.8
Set mfp	0.3

In[6830]:=
```
data = SelectFirst[simulations, #[[1]] == c && #[[2]] == mfp &][[3]];
maxr = data[[2, 5]];
dr = data[[2, 7]];
MCFluence = ppoints[data[[6]], dr, maxr];
exact1FluenceShallow =
  Quiet[{#[[1]], 4 Pi #[[1]]^2 ϕexact1b[#[[1]], 1/mfp, c]}] & /@
   MCFluence[[1 ;; 60]];
exact1Fluence = Quiet[{#[[1]], 4 Pi #[[1]]^2 ϕexact1b[#[[1]], 1/mfp, c]}] & /@
   MCFluence[[60 ;; -1 ;; 10]];
plotϕshallow = Quiet[Show[
    ListPlot[MCFluence[[1 ;; 60]], PlotRange → All, PlotStyle → PointSize[.01]],
    ListPlot[exact1FluenceShallow, PlotRange → All, Joined → True],
    Frame → True,
    FrameLabel -> {{4 Pi r^2 ϕ[r],}, {r,}}
   ]];
logplotϕ = Quiet[Show[
    ListLogPlot[MCFluence, PlotRange → All, PlotStyle → PointSize[.01]],
    ListLogPlot[exact1Fluence, PlotRange → All, Joined → True],
    ListLogPlot[exact1FluenceShallow, PlotRange → All, Joined → True],
    Frame → True,
    FrameLabel -> {{4 Pi r^2 ϕ[r],}, {r,}}
   ]];
Show[GraphicsGrid[{{plotϕshallow, logplotϕ}}, ImageSize → 800],
 PlotLabel -> "Exact solution (1b)\nInfinite 3D, isotropic
    point source, isotropic scattering, fluence ϕ[r], c = " <>
   ToString[c] <> ", Σt = " <> ToString[1/mfp]]
```

Out[6838]=

Exact solution (1b)
Infinite 3D, isotropic point source, isotropic scattering, fluence ϕ[r], c = 0.7, Σ_t = 3.33333





Fluence - Exact solution (2a) comparison to MC

In[6875]:=
```
{{ActionMenu["Set c", "c = " <> ToString[#] :→ (c = #;) & /@ cs], Dynamic[c]},
  {ActionMenu["Set mfp", "mfp = " <> ToString[#] :→ (mfp = #;) & /@ mfps],
   Dynamic[mfp]}} // TableForm
```

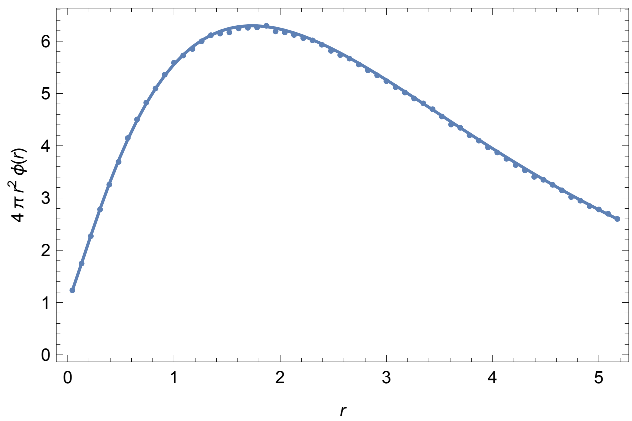
Out[6875]//TableForm=

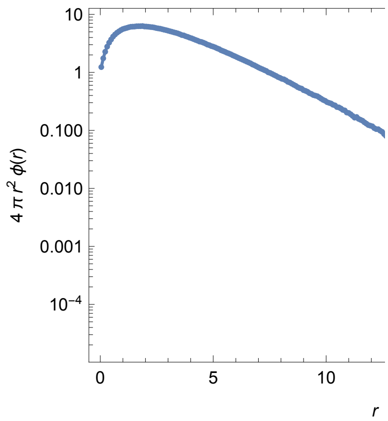
Set c	0.8
Set mfp	0.3

In[6839]:=
```
data = SelectFirst[simulations, #[[1]] == c && #[[2]] == mfp &][[3]];
maxr = data[[2, 5]];
dr = data[[2, 7]];
MCFluence = ppoints[data[[6]], dr, maxr];
exact1FluenceShallow =
  Quiet[{#[[1]], 4 Pi #[[1]]^2 ϕexact2a[#[[1]], 1/mfp, c]}] & /@
   MCFluence[[1 ;; 60]];
exact1Fluence = Quiet[{#[[1]], 4 Pi #[[1]]^2 ϕexact2a[#[[1]], 1/mfp, c]}] & /@
   MCFluence[[60 ;; -1 ;; 10]];
plotϕshallow = Quiet[Show[
    ListPlot[MCFluence[[1 ;; 60]], PlotRange → All, PlotStyle → PointSize[.01]],
    ListPlot[exact1FluenceShallow, PlotRange → All, Joined → True],
    Frame → True,
    FrameLabel -> {{4 Pi r^2 ϕ[r],}, {r,}}
   ]];
logplotϕ = Quiet[Show[
    ListLogPlot[MCFluence, PlotRange → All, PlotStyle → PointSize[.01]],
    ListLogPlot[exact1Fluence, PlotRange → All, Joined → True],
    ListLogPlot[exact1FluenceShallow, PlotRange → All, Joined → True],
    Frame → True,
    FrameLabel -> {{4 Pi r^2 ϕ[r],}, {r,}}
   ]];
Show[GraphicsGrid[{{plotϕshallow, logplotϕ}}, ImageSize → 800],
 PlotLabel -> "Exact solution (2a)\nInfinite 3D, isotropic
    point source, isotropic scattering, fluence ϕ[r], c = " <>
   ToString[c] <> ", Σt = " <> ToString[1/mfp]]
```

Out[6847]=

Exact solution (2a)
Infinite 3D, isotropic point source, isotropic scattering, fluence ϕ[r], c = 0.99, Σ_t = 3.33333





Fluence - Exact solution (2b) comparison to MC

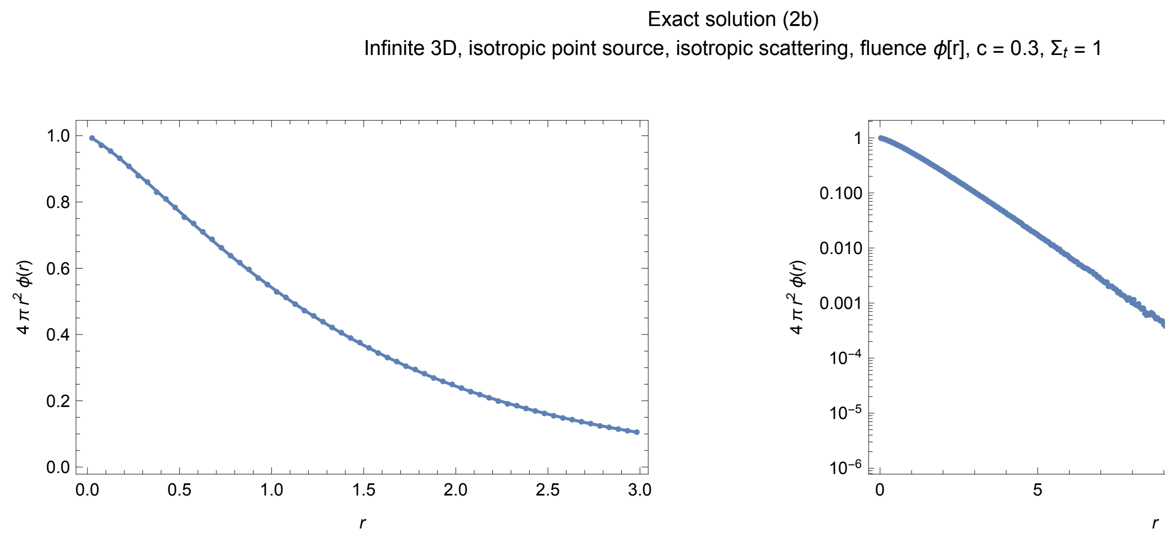
In[6876]:=
```
{{ActionMenu["Set c", "c = " <> ToString[#] :→ (c = #;) & /@ cs], Dynamic[c]},
  {ActionMenu["Set mfp", "mfp = " <> ToString[#] :→ (mfp = #;) & /@ mfps],
   Dynamic[mfp]}} // TableForm
```

Out[6876]//TableForm=

Set c	0.8
Set mfp	0.3

In[6848]:=
```
data = SelectFirst[simulations, #[[1]] == c && #[[2]] == mfp &][[3]];
maxr = data[[2, 5]];
dr = data[[2, 7]];
MCFluence = ppoints[data[[6]], dr, maxr];
exact1FluenceShallow =
  Quiet[{#[[1]], 4 Pi #[[1]]^2 ϕexact2b[#[[1]], 1/mfp, c]}] & /@
   MCFluence[[1 ;; 60]];
exact1Fluence = Quiet[{#[[1]], 4 Pi #[[1]]^2 ϕexact2b[#[[1]], 1/mfp, c]}] & /@
   MCFluence[[60 ;; -1 ;; 10]];
plotϕshallow = Quiet[Show[
    ListPlot[MCFluence[[1 ;; 60]], PlotRange → All, PlotStyle → PointSize[.01]],
    ListPlot[exact1FluenceShallow, PlotRange → All, Joined → True],
    Frame → True,
    FrameLabel -> {{4 Pi r^2 ϕ[r],}, {r,}}
   ]];
logplotϕ = Quiet[Show[
    ListLogPlot[MCFluence, PlotRange → All, PlotStyle → PointSize[.01]],
    ListLogPlot[exact1Fluence, PlotRange → All, Joined → True],
    ListLogPlot[exact1FluenceShallow, PlotRange → All, Joined → True],
    Frame → True,
    FrameLabel -> {{4 Pi r^2 ϕ[r],}, {r,}}
   ]];
Show[GraphicsGrid[{{plotϕshallow, logplotϕ}}, ImageSize → 800],
 PlotLabel -> "Exact solution (2b)\nInfinite 3D, isotropic
    point source, isotropic scattering, fluence ϕ[r], c = " <>
   ToString[c] <> ", Σt = " <> ToString[1/mfp]]
```

Out[6856]=



Fluence - Diffusion approximations (Classical and Grosjean) comparison to MC

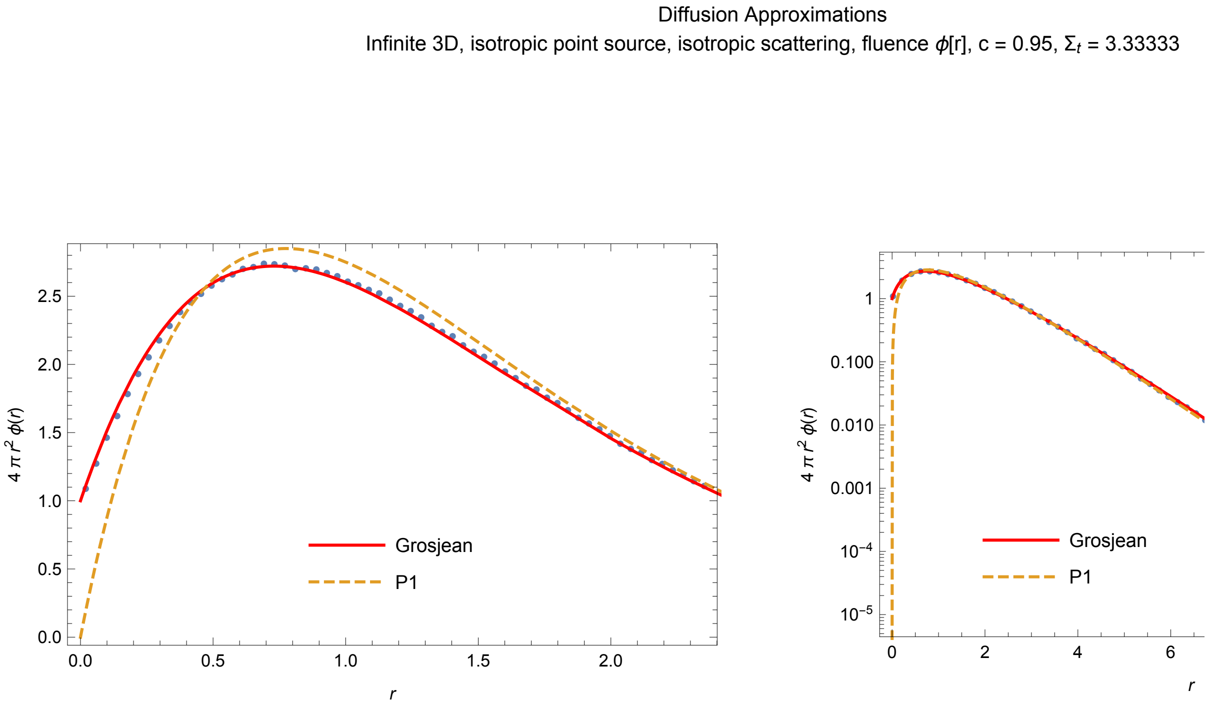
In[6877]:=
```
{{ActionMenu["Set c", "c = " <> ToString[#] :> (c = #;) & /@ cs], Dynamic[c]},
  {ActionMenu["Set mfp", "mfp = " <> ToString[#] :> (mfp = #;) & /@ mfps],
   Dynamic[mfp]}} // TableForm
```

Out[6877]//TableForm=

Set c	0.8
Set mfp	0.3

In[6857]:=
```
data = SelectFirst[simulations, #[[1]] == c && #[[2]] == mfp &][[3]];
maxr = data[[2, 5]];
dr = data[[2, 7]];
MCFluence = ppoints[data[[6]], dr, maxr];
plotϕshallow = Quiet[Show[
    ListPlot[MCFluence[[1 ;; 60]], PlotRange → All, PlotStyle → PointSize[.01]],
    Plot[{
      4 Pi r^2 ϕGrosjean[r, 1 / mfp, c],
      4 Pi r^2 ϕDiffusion[r, 1 / mfp, c]
     }
     , {r, 0, maxr}, PlotRange → All, PlotStyle → {Red, Dashed},
     PlotLegends → Placed[{"Grosjean", "P1"}, {0.5, .2}]],
    Frame → True,
    FrameLabel -> {{4 Pi r^2 ϕ[r],}, {r,}}
   ]];
logplotϕ = Quiet[Show[
    ListLogPlot[MCFluence[[1 ;; -1 ;; 5]],
     PlotRange → All, PlotStyle → PointSize[.01]],
    LogPlot[{
      4 Pi r^2 ϕGrosjean[r, 1 / mfp, c],
      4 Pi r^2 ϕDiffusion[r, 1 / mfp, c]
     }
     , {r, 0, maxr}, PlotRange → All, PlotStyle → {Red, Dashed},
     PlotLegends → Placed[{"Grosjean", "P1"}, {0.3, .2}]],
    Frame → True,
    FrameLabel -> {{4 Pi r^2 ϕ[r],}, {r,}}
   ]];
Show[GraphicsGrid[{{plotϕshallow, logplotϕ}}, ImageSize → 800],
 PlotLabel -> "Diffusion Approximations\nInfinite 3D, isotropic
     point source, isotropic scattering, fluence ϕ[r], c = " <>
   ToString[c] <> ", Σt = " <> ToString[1 / mfp]]
```

Out[6863]=



Fluence - Diffusion approximation (Rigorous) comparison to MC

In[6878]:=

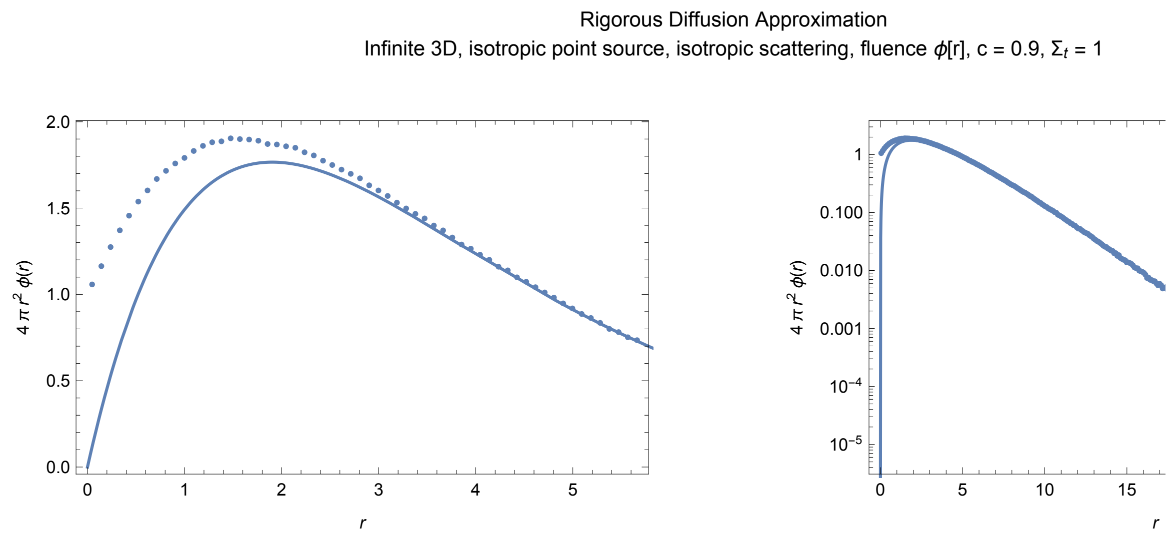
```
{{ActionMenu["Set c", "c = " <> ToString[#] :> (c = #;) & /@ cs], Dynamic[c]},
  {ActionMenu["Set mfp", "mfp = " <> ToString[#] :> (mfp = #;) & /@ mfps],
   Dynamic[mfp]}} // TableForm
```

Out[6878]//TableForm=

Set c	0.8
Set mfp	0.3

In[6864]:=
```
data = SelectFirst[simulations, #[[1]] == c && #[[2]] == mfp &][[3]];
maxr = data[[2, 5]];
dr = data[[2, 7]];
MCFluence = ppoints[data[[6]], dr, maxr];
plotϕshallow = Quiet[Show[
    ListPlot[MCFluence[[1 ;; 60]], PlotRange → All, PlotStyle → PointSize[.01]],
    Plot[4 Pi r^2 ϕrigourousDiffusion[r, 1 / mfp, c], {r, 0, maxr}, PlotRange → All],
    Frame → True,
    FrameLabel -> {{4 Pi r^2 ϕ[r],}, {r,}}
   ]];
logplotϕ = Quiet[Show[
    ListLogPlot[MCFluence, PlotRange → All, PlotStyle → PointSize[.01]],
    LogPlot[
     4 Pi r^2 ϕrigourousDiffusion[r, 1 / mfp, c], {r, 0, maxr}, PlotRange → All],
    Frame → True,
    FrameLabel -> {{4 Pi r^2 ϕ[r],}, {r,}}
   ]];
Show[GraphicsGrid[{{plotϕshallow, logplotϕ}}, ImageSize → 800],
 PlotLabel -> "Rigorous Diffusion Approximation\nInfinite 3D, isotropic
    point source, isotropic scattering, fluence ϕ[r], c = " <>
   ToString[c] <> ", Σt = " <> ToString[1 / mfp]]
```

Out[6870]=



N-th order fluence / scalar flux

N-th collided Fluence - Exact solution (1) comparison to MC

In[6871]:=
```
{{ActionMenu["Set c", "c = " <> ToString[#] :→ (c = #;) & /@ cs], Dynamic[c]},
  {ActionMenu["Set mfp", "mfp = " <> ToString[#] :→ (mfp = #;) & /@ mfps],
   Dynamic[mfp]},
  {ActionMenu["Set collision order",
    "collisionOrder = " <> ToString[#] :→ (collisionOrder = #;) & /@
     Range[0, numcollorders - 1]], Dynamic[collisionOrder]}} // TableForm
```

Out[6871]//TableForm=

Set c	0.8
Set mfp	0.3
Set collision order	2

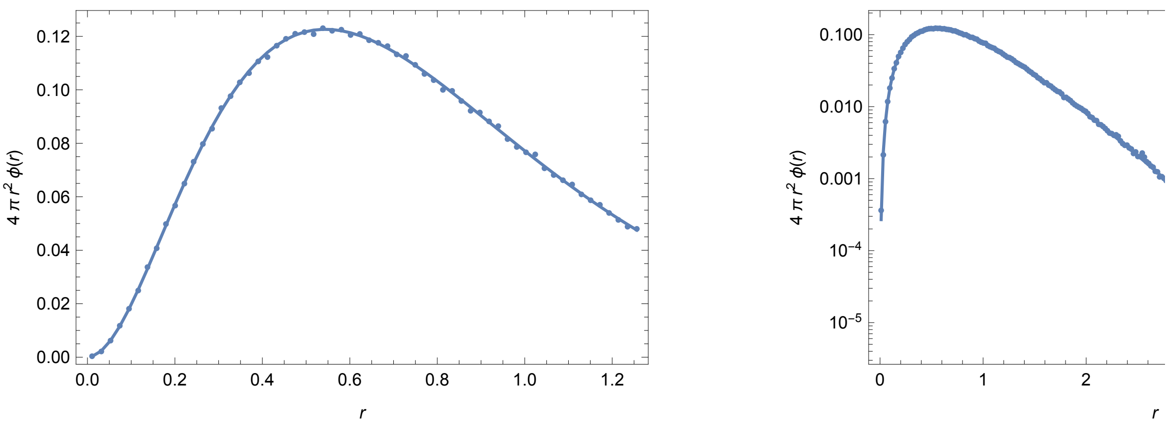
In[6879]:=
```
data = SelectFirst[simulations, #[[1]] == c && #[[2]] == mfp &][[3]];
maxr = data[[2, 5]];
dr = data[[2, 7]];
fluencei = 3 numcollorders + 15 + collisionOrder;

MCFluence = ppoints[data[[fluencei]], dr, maxr];
exact1FluenceShallow =
  Quiet[{#[[1]], 4 Pi #[[1]]^2 ϕexact1[#[[1]], 1/mfp, c, collisionOrder]}] & /@
   MCFluence[[1 ;; 60]];
exact1Fluence = Quiet[{#[[1]], 4 Pi #[[1]]^2 ϕexact1[#[[1]], 1/mfp,
        c, collisionOrder]}] & /@ MCFluence[[61 ;; -1 ;; 10]];
plotϕshallow = Quiet[Show[
    ListPlot[MCFluence[[1 ;; 60]], PlotRange → All, PlotStyle → PointSize[.01]],
    ListPlot[exact1FluenceShallow, PlotRange → All, Joined → True],
    Frame → True,
    FrameLabel -> {{4 Pi r^2 ϕ[r],}, {r,}}
   ]];
logplotϕ = Quiet[Show[
    ListLogPlot[MCFluence, PlotRange → All, PlotStyle → PointSize[.01]],
    ListLogPlot[exact1FluenceShallow, PlotRange → All, Joined → True],
    ListLogPlot[exact1Fluence, PlotRange → All, Joined → True],
    Frame → True,
    FrameLabel -> {{4 Pi r^2 ϕ[r],}, {r,}}
   ]];
Show[GraphicsGrid[{{plotϕshallow, logplotϕ}}, ImageSize → 800],
 PlotLabel -> "Exact solution (1)\nInfinite 3D medium, isotropic point source,
     isotropic scattering, n-th scattered fluence ϕ[r|" <>
   ToString[collisionOrder] <> "], c =" <> ToString[c] <>
   ", Σt = " <> ToString[1/mfp]]
```

Exact solution (1)

Infinite 3D medium, isotropic point source, isotropic scattering, n–th scattered fluence ϕ[r|4], c =0.8, Σ_t =

Out[6888]=



N-th collided Fluence - Exact solution (2) comparison to MC

In[6889]:=
```
{{ActionMenu["Set c", "c = " <> ToString[#] :> (c = #;) & /@ cs], Dynamic[c]},
  {ActionMenu["Set mfp", "mfp = " <> ToString[#] :> (mfp = #;) & /@ mfps],
   Dynamic[mfp]},
  {ActionMenu["Set collision order",
    "collisionOrder = " <> ToString[#] :> (collisionOrder = #;) & /@
     Range[0, numcollorders - 1]], Dynamic[collisionOrder]}} // TableForm
```

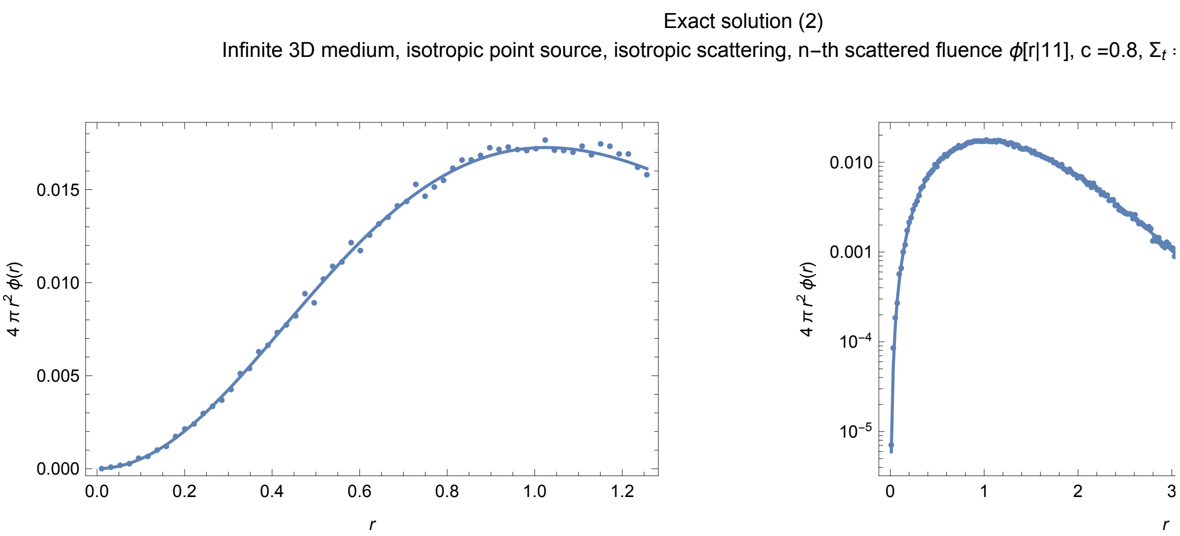
Out[6889]//TableForm=

Set c	0.8
Set mfp	0.3
Set collision order	2

In[6890]:=
```
data = SelectFirst[simulations, #[[1]] == c && #[[2]] == mfp &][[3]];
maxr = data[[2, 5]];
dr = data[[2, 7]];
fluencei = 3 numcollorders + 15 + collisionOrder;

MCFluence = ppoints[data[[fluencei]], dr, maxr];
exact1FluenceShallow =
  Quiet[{#[[1]], 4 Pi #[[1]]^2 ϕexact2[#[[1]], 1/mfp, c, collisionOrder]}] & /@
   MCFluence[[1 ;; 60]];
exact1Fluence = Quiet[{#[[1]], 4 Pi #[[1]]^2 ϕexact2[#[[1]], 1/mfp,
        c, collisionOrder]}] & /@ MCFluence[[61 ;; -1 ;; 10]];
plotϕshallow = Quiet[Show[
    ListPlot[MCFluence[[1 ;; 60]], PlotRange → All, PlotStyle → PointSize[.01]],
    ListPlot[exact1FluenceShallow, PlotRange → All, Joined → True],
    Frame → True,
    FrameLabel -> {{4 Pi r^2 ϕ[r],}, {r,}}
   ]];
logplotϕ = Quiet[Show[
    ListLogPlot[MCFluence, PlotRange → All, PlotStyle → PointSize[.01]],
    ListLogPlot[exact1FluenceShallow, PlotRange → All, Joined → True],
    ListLogPlot[exact1Fluence, PlotRange → All, Joined → True],
    Frame → True,
    FrameLabel -> {{4 Pi r^2 ϕ[r],}, {r,}}
   ]];
Show[GraphicsGrid[{{plotϕshallow, logplotϕ}}, ImageSize → 800],
 PlotLabel -> "Exact solution (2)\nInfinite 3D medium, isotropic point source,
     isotropic scattering, n-th scattered fluence ϕ[r|" <>
   ToString[collisionOrder] <> "], c =" <> ToString[c] <>
   ", Σt = " <> ToString[1/mfp]]
```

Out[6899]=



N-th collided Fluence - Approximations

In[6900]:=
```
{{ActionMenu["Set c", "c = " <> ToString[#] :> (c = #;) & /@ cs], Dynamic[c]},
  {ActionMenu["Set mfp", "mfp = " <> ToString[#] :> (mfp = #;) & /@ mfps],
   Dynamic[mfp]},
  {ActionMenu["Set collision order",
    "collisionOrder = " <> ToString[#] :> (collisionOrder = #;) & /@
     Range[0, numcollorders - 1]], Dynamic[collisionOrder]}} // TableForm
```

Out[6900]//TableForm=

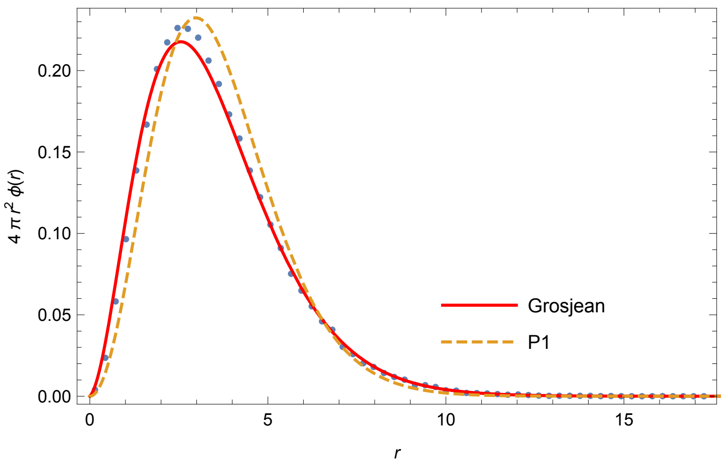
Set c	0.8
Set mfp	0.3
Set collision order	2

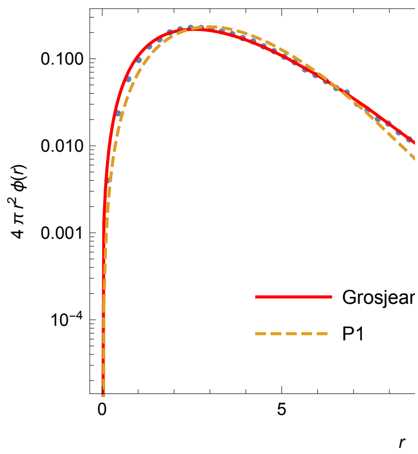
In[6931]:=
```
data = SelectFirst[simulations, #[[1]] == c && #[[2]] == mfp &][[3]];
maxr = data[[2, 5]];
dr = data[[2, 7]];
fluencei = 3 numcollorders + 15 + collisionOrder;

MCFluence = ppoints[data[[fluencei]], dr, maxr];
seriesclassical = c^collisionOrder
  SeriesCoefficient[ϕDiffusion[r, 1/mfp, C], {C, 0, collisionOrder}];
seriesG = c^collisionOrder SeriesCoefficient[
   ϕGrosjean[r, 1/mfp, C], {C, 0, collisionOrder}];
plotϕshallow = Quiet[Show[
    ListPlot[MCFluence[[1 ;; 60]], PlotRange → All, PlotStyle → PointSize[.01]],
    Plot[{4 Pi r^2 seriesG, 4 Pi r^2 seriesclassical},
     {r, 0, maxr}, PlotRange → All, PlotStyle → {Red, Dashed},
     PlotLegends → Placed[{"Grosjean", "P1"}, {0.7, .2}]],
    Frame → True,
    FrameLabel -> {{4 Pi r^2 ϕ[r],}, {r,}}
   ]];
logplotϕ = Quiet[Show[
    ListLogPlot[MCFluence, PlotRange → All, PlotStyle → PointSize[.01]],
    LogPlot[{4 Pi r^2 seriesG, 4 Pi r^2 seriesclassical},
     {r, 0, maxr}, PlotRange → All, PlotStyle → {Red, Dashed},
     PlotLegends → Placed[{"Grosjean", "P1"}, {0.4, .2}]],
    Frame → True,
    FrameLabel -> {{4 Pi r^2 ϕ[r],}, {r,}}
   ]];
Show[GraphicsGrid[{{plotϕshallow, logplotϕ}}, ImageSize → 800], PlotLabel ->
  "Diffusion Approximations\nInfinite 3D medium, isotropic point source,
     isotropic scattering, n-th scattered fluence ϕ[r|" <>
   ToString[collisionOrder] <> "], c =" <> ToString[c] <>
   ", Σt = " <> ToString[1/mfp]]
```

Diffusion Approximations
Infinite 3D medium, isotropic point source, isotropic scattering, n–th scattered fluence ϕ[r|7], c =0.99,

Out[6940]=





Compare moments of ϕ

In[6941]:=
```
{{ActionMenu["Set c", "c = " <> ToString[#] :> (c = #;) & /@ cs], Dynamic[c]},
  {ActionMenu["Set mfp", "mfp = " <> ToString[#] :> (mfp = #;) & /@ mfps],
   Dynamic[mfp]}} // TableForm
```

Out[6941]//TableForm=

Set c	0.8
Set mfp	0.3

In[6950]:=
```
data = SelectFirst[simulations, #[[1]] == c && #[[2]] == mfp &][[3]];
nummoments = data[[2, 15]];
ϕmoments = {data[[10]]};
ks = Table[k, {k, 0, nummoments - 1}];
analytic = Table[ϕm[c, 1 / mfp, k], {k, ks}];
j = Join[{ks}, {analytic}, ϕmoments];
TableForm[
 Join[{{"n", "analytic", "MC"}}, Transpose[j]]
]
```

Out[6956]//TableForm=

n	analytic	MC
0	2.	2.00096
1	0. + 0. i	2.98567
2	8.	8.01171
3	0. + 0. i	30.4943
4	149.333	149.478
5	0. + 0. i	894.305
6	6314.67	6305.37
7	0. + 0. i	51 051.9
8	472 269.	464 772
9	Indeterminate	4.67159×10^{6}

In[6957]:=
```
{{ActionMenu["Set mfp", "mfp = " <> ToString[#] :→ (mfp = #;) & /@ mfps],
   Dynamic[mfp]}} // TableForm
```

Out[6957]//TableForm=

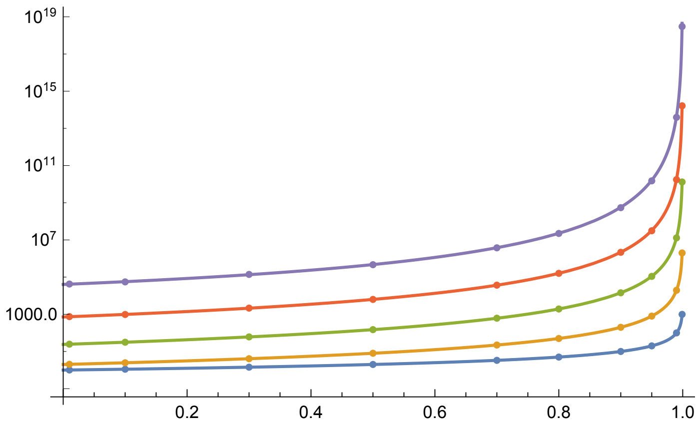
Set mfp   0.3

In[6995]:=
```
sims1 = Select[simulations, #[[2]] == mfp &];
Show[
 ListLogPlot[{
   {#[[-1, 2, 3]], #[[-1, 10, 1]]} & /@ sims1,
   {#[[-1, 2, 3]], #[[-1, 10, 3]]} & /@ sims1,
   {#[[-1, 2, 3]], #[[-1, 10, 5]]} & /@ sims1,
   {#[[-1, 2, 3]], #[[-1, 10, 7]]} & /@ sims1,
   {#[[-1, 2, 3]], #[[-1, 10, 9]]} & /@ sims1
  }],
```

$$\texttt{LogPlot}\Big[\Big\{-\frac{\texttt{mfp}}{-1+\texttt{c}},\ \frac{2\,\texttt{mfp}^3}{(-1+\texttt{c})^2},\ \frac{8\,(-9+4\,\texttt{c})\,\texttt{mfp}^5}{3\,(-1+\texttt{c})^3},\ \frac{16\,(135-144\,\texttt{c}+44\,\texttt{c}^2)\,\texttt{mfp}^7}{3\,(-1+\texttt{c})^4},\ \frac{128\,(-1575+2808\,\texttt{c}-1836\,\texttt{c}^2+428\,\texttt{c}^3)\,\texttt{mfp}^9}{5\,(-1+\texttt{c})^5}\Big\},\ \{\texttt{c},\,0,\,.999\},\ \texttt{PlotRange}\to\texttt{All}\Big]$$

```
]
```

Out[6996]=



n-th collided moments of ϕ

In[6942]:=
```
{{ActionMenu["Set c", "c = " <> ToString[#] :> (c = #;) & /@ cs], Dynamic[c]},
  {ActionMenu["Set mfp", "mfp = " <> ToString[#] :> (mfp = #;) & /@ mfps],
   Dynamic[mfp]},
  {ActionMenu["Set collision order",
    "collisionOrder = " <> ToString[#] :> (collisionOrder = #;) & /@
     Range[0, numcollorders - 1]], Dynamic[collisionOrder]}} // TableForm
```

Out[6942]//TableForm=

Set c	0.8
Set mfp	0.3
Set collision order	2

In[7000]:=
```
data = SelectFirst[simulations, #[[1]] == c && #[[2]] == mfp &][[3]];
nummoments = data[[2, 15]];
ϕmoments = N[{data[[numcollorders + 13 + collisionOrder]]}];
ks = Table[k, {k, 0, nummoments - 1}];
analytic = Table[ϕm[c, 1 / mfp, k, collisionOrder], {k, ks}];
j = Join[{ks}, {analytic}, ϕmoments];
TableForm[
 Join[{{"n", "analytic", "MC"}}, Transpose[j]]
]
```

Out[7006]//TableForm=

n	analytic	MC
0	0.192	0.192008
1	0. + 0. i	0.11745
2	0.10368	0.103751
3	0.	0.120756
4	0.174182	0.175155
5	0.	0.305055
6	0.610634	0.621431
7	0.	1.45293
8	3.68312	3.84418
9	0.	11.3783

Angular Distributions

In[7007]:=
```
{{ActionMenu["Set c", "c = " <> ToString[#] :> (c = #;) & /@ cs], Dynamic[c]},
  {ActionMenu["Set mfp", "mfp = " <> ToString[#] :> (mfp = #;) & /@ mfps],
   Dynamic[mfp]}} // TableForm
```

Out[7007]//TableForm=

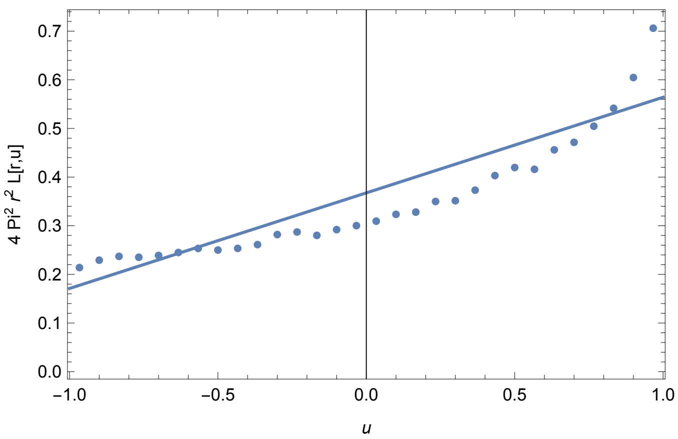
Set c	0.8
Set mfp	0.3

In[2437]:=
```
depthi = 52
```

Out[2437]= 52

In[2438]:=
```
Clear[u];
data = SelectFirst[simulations, #[[1]] == c && #[[2]] == mfp &][[3]];
du = data[[2, 9]];
maxr = data[[2, 5]];
dr = data[[2, 7]];
fluxi = 17 + 4 numcollorders + Floor[maxr / dr];
angularFlux = ppointsu[data[[fluxi + depthi]], du, 1];
r = dr * depthi - 0.5 dr;
Show[
 ListPlot[angularFlux, PlotRange → All,
  Frame → True,
  FrameLabel -> {{"4 Pi² r² L[r,u]",}, {u,}}],
 Plot[4 Pi r² Pi Ldiffusion[r, u, 1 / mfp, c], {u, -1, 1}, PlotRange → All]
]
```

Out[2446]=



Angular Distribution: Integral of Grosjean’s Diffusion Approximation

In[7008]:=
```
{{ActionMenu["Set c", "c = " <> ToString[#] :> (c = #;) & /@ cs], Dynamic[c]},
  {ActionMenu["Set mfp", "mfp = " <> ToString[#] :> (mfp = #;) & /@ mfps],
   Dynamic[mfp]}} // TableForm
```

Out[7008]//TableForm=

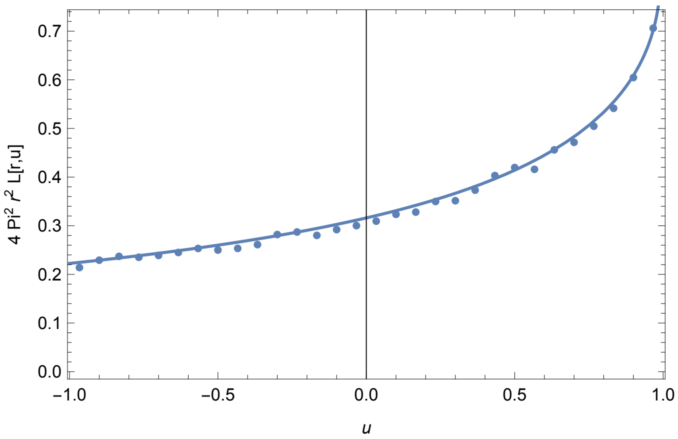
Set c	0.8
Set mfp	0.3

In[∘]:=
```
depthi = 52
```

Out[∘]= 52

In[2447]:=
```
Clear[u];
data = SelectFirst[simulations, #[[1]] == c && #[[2]] == mfp &][[3]];
du = data[[2, 9]];
maxr = data[[2, 5]];
dr = data[[2, 7]];
fluxi = 17 + 4 numcollorders + Floor[maxr / dr];
angularFlux = ppointsu[data[[fluxi + depthi]], du, 1];
r = dr * depthi - 0.5 dr;
Show[
 ListPlot[angularFlux, PlotRange → All,
  Frame → True,
  FrameLabel -> {{"4 Pi² r² L[r,u]",}, {u,}}],
 Plot[4 Pi r² Pi Lintegral[r, u, 1 / mfp, c, ϕGrosjean], {u, -1, 1}, PlotRange → All]
]
```

Out[2455]=



End context

In[7009]:=
```
End[]
```

Out[7009]=
```
inf3Disopointisoscatter`
```