Infinite 3D medium, Isotropic Point Source, Lambert Sphere Scattering

Exponential Random Flight

This is code to accompany the book:

A Hitchhiker's Guide to Multiple Scattering

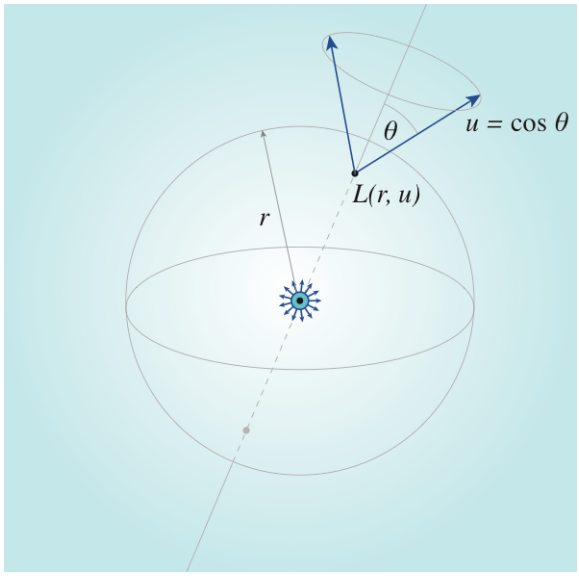
www.eugenedeon.com/hitchhikers

Path Setup

Put a file at ~/.hitchhikerpath with the path to your hitchhiker repo so that these worksheets can find the MC data from the C++ simulations for verification

In[•]:=
```
SetDirectory[Import["~/.hitchhikerpath"]]
```

Notation



c - single-scattering albedo

Σt - extinction coefficient

r - radial position coordinate in medium (distance from point source at origin)

u = cos θ - direction cosine

Namespace

In[4627]:=
```
Begin["inf3DisopointLambertSpherescatter`"]
```
Out[4627]=
```
inf3DisopointLambertSpherescatter`
```

Util

In[4628]:=
```
SA[d_, r_] := d Pi^(d/2) / Gamma[d/2 + 1] r^(d-1)
```

Diffusion modes

In[4629]:=
```
diffusionMode[v_, d_, r_] := (2 π)^(-d/2) r^(1-d/2) v^(-1-d/2) BesselK[1/2 (-2 + d), r/v]
```

Analytical solutions

Fluence: exact solution

[Grosjean 1963 - A New Approximate One-Velocity Theory for Treating both Isotropic and Anisotropic Multiple Scattering Problems, p. 37]

In[4649]:=
```
ϕexactTruncatedFourierOrder7[r_, Σt_, c_] :=
 Exp[-r Σt]/(4 Pi r^2) + c Σt/(2 Pi^2 r) NIntegrate[u ((1.3333333333333333` u^2 -
      1.3999999999999997` u ArcTan[u] - 2.337311630789803`*^-16 u^3 ArcTan[u] +
      0.06666666666666664` ArcTan[u]^2 + 0.9999999999999998` u^2 ArcTan[u]^2 -
      0.00030924479166666665` u^2 Hypergeometric2F1[7/2, 4, 15/2, -u^2] -
      0.0004216974431818181` u^4 Hypergeometric2F1[7/2, 4, 15/2, -u^2] -
      0.0001405658143939394` u^6 Hypergeometric2F1[7/2, 4, 15/2, -u^2] -
      6.693610209235209`*^-6 u^8 Hypergeometric2F1[7/2, 4, 15/2, -u^2] +
      0.00032470703125` u ArcTan[u] Hypergeometric2F1[7/2, 4, 15/2, -u^2] +
      0.000442782315340909` u^3 ArcTan[u] Hypergeometric2F1[7/2, 4, 15/2, -u^2] +
      0.0001475941051136363 8` u^5 ArcTan[u] Hypergeometric2F1[7/2, 4, 15/2, -u^2] +
      7.028290719696983`*^-6 u^7 ArcTan[u] Hypergeometric2F1[7/2, 4, 15/2, -u^2] +
      8.916136286887371`*^-22 u^9 ArcTan[u] Hypergeometric2F1[7/2, 4, 15/2, -u^2] -
      0.00001546223958333328` ArcTan[u]^2 Hypergeometric2F1[7/2, 4, 15/2, -u^2] -
```

```
0.00025301846590909086` u^2 ArcTan[u]^2 Hypergeometric2F1[7/2, 4, 15/2, -u^2] -
0.00032330137310606055` u^4 ArcTan[u]^2 Hypergeometric2F1[7/2, 4, 15/2, -u^2] -
0.0001057590413059163` u^6 ArcTan[u]^2 Hypergeometric2F1[7/2, 4, 15/2, -u^2] -
5.020207656926406`*^-6 u^8 ArcTan[u]^2 Hypergeometric2F1[7/2, 4, 15/2, -u^2] -
3.242461239176438`*^-8 u^6 Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 -
5.558504981445322`*^-8 u^8 Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 -
1.0561159464746111`*^-7 u^10 Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 -
5.5585049814453225`*^-8 u^12 Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 -
1.228318152177984`*^-7 u^14 Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 +
9.00974926948953`*^-8 u^16 Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 +
3.40458430113526`*^-8 u^5 ArcTan[u] Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 +
5.836430230517588`*^-8 u^7 ArcTan[u] Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 +
1.1089217437983416`*^-7 u^9 ArcTan[u] Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 +
5.8364302305175895`*^-8 u^11 ArcTan[u]
 Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 +
1.484991523182093`*^-8 u^13 ArcTan[u] Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 +
8.559261806015054`*^-8 u^15 ArcTan[u] Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 -
1.6212306195882188`*^-9 u^4 ArcTan[u]^2
 Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 - 2.7097711784545943`*^-8
 u^6 ArcTan[u]^2 Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 -
4.696936709321298`*^-8 u^8 ArcTan[u]^2 Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 -
8.19879484763185`*^-8 u^10 ArcTan[u]^2 Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 -
4.212420358438646`*^-8 u^12 ArcTan[u]^2 Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 -
6.531243353198253`*^-9 u^14 ArcTan[u]^2 Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 +
u^4 Hypergeometric2F1[5/2, 3, 11/2, -u^2]^2
 (-8.527021919879064`*^-6 u^2 - 5.684681279919375`*^-6 u^4 -
    0.000013713046947173927` u^6 + 0.000010078105316200553` u^8 +
    8.953373015873015`*^-6 u ArcTan[u] + 5.968915343915344`*^-6
     u^3 ArcTan[u] + 1.633099227345259`*^-6 u^5 ArcTan[u] +
```

$$9.574200050390523`*^-6\,u^7\,\text{ArcTan}[u] - 4.2635109599395304`*^-7\,\text{ArcTan}[u]^2 - 6.679500503905266`*^-6\,u^2\,\text{ArcTan}[u]^2 - 4.310883303938859`*^-6\,u^4\,\text{ArcTan}[u]^2 - 7.105851599899218`*^-7\,u^6\,\text{ArcTan}[u]^2 + \Big(u^2\,\big(1.9777028378625758`*^-9 + 4.015336064751289`*^-9\,u^2 + 5.877383765896755`*^-9\,u^4 + 2.6417277537085793`*^-9\,u^6 - 1.7132059435166385`*^-9\,u^8 - 9.93635467705722`*^-10\,u^{10} - 5.059418147570075`*^-11\,u^{12}\big) + \big(-2.0765879797557045`*^-9\,u - 4.216102867988854`*^-9\,u^3 - 3.210481454233586`*^-9\,u^5 - 3.4113008279878395`*^-9\,u^7 - 3.2301953979921833`*^-9\,u^9 - 1.017552417685869`*^-9\,u^{11} - 4.806447240191571`*^-11\,u^{13}\big)\,\text{ArcTan}[u] + \big(9.888514189312876`*^-11 + 1.6840439316344963`*^-9\,u^2 + 3.1573326618604042`*^-9\,u^4 + 2.234547362260312`*^-9\,u^6 + 7.127435552037204`*^-10\,u^8 + 9.655451565322337`*^-11\,u^{10} + 3.5672850610796814`*^-12\,u^{12}\big)\,\text{ArcTan}[u]^2\Big)\,\text{Hypergeometric2F1}\Big[\frac{7}{2}, 4, \frac{15}{2}, -u^2\Big] + 2.3600816241260076`*^-44\,u^{13}\,\text{ArcTan}[u]\,\text{Hypergeometric2F1}\Big[\frac{7}{2}, 4, \frac{15}{2}, -u^2\Big]^2\Big) + \text{Hypergeometric2F1}\Big[\frac{5}{2}, 3, \frac{11}{2}, -u^2\Big]\,\Big(-0.002199074074074074`\,u^2 - 0.0018849206349206352`\,u^4 - 0.00018849206349206342`\,u^6 + 0.002309027777777778`\,u\,\text{ArcTan}[u] + 0.001979166666666667`\,u^3\,\text{ArcTan}[u] + 0.0001979166666666655`\,u^5\,\text{ArcTan}[u] - 0.00010995370370370366`\,\text{ArcTan}[u]^2 - 0.0017435515873015868`\,u^2\,\text{ArcTan}[u]^2 - 0.0014231150793650794`\,u^4\,\text{ArcTan}[u]^2 - 0.00014136904761904762`\,u^6\,\text{ArcTan}[u]^2 + \big(u^2\,\big(5.100391529224537`*^-7 + 1.1326843525940206`*^-6\,u^2 + 8.717032795401936`*^-7\,u^4 + 2.693713275174864`*^-7\,u^6 + 2.933434831279418`*^-8\,u^8 + 9.462693004127154`*^-10\,u^{10}\big) + \big(-5.355411105685764`*^-7\,u - 1.1893185702237215`*^-6\,u^3 - 9.152884435172032`*^-7\,u^5 - 2.8283989389336067`*^-7\,u^7 - 3.080106572843393`*^-8\,u^9 - 9.935827654333515`*^-10\,u^{11} - 1.088395542832931`*^-25\,u^{13}\big)\,\text{ArcTan}[u] + \big(2.550195764612268`*^-8 + 4.3916358232154127`*^-7\,u^2 + 8.930984284225248`*^-7\,u^4 + 6.672460260310193`*^-7\,u^6 + 2.0349521305375447`*^-7\,u^8 + 2.2048074699616273`*^-8\,u^{10} + 7.097019753095366`*^-10\,u^{12}\big)\,\text{ArcTan}[u]^2\Big)\,\text{Hypergeometric2F1}\Big[\frac{7}{2}, 4, \frac{15}{2}, -u^2\Big] + u^8\,\big(u^2\,\big(-1.2572808886602517`*^-10 - 1.9158565922441929`*^-10\,u^2 - 2.8481561050051276`*^-10\,u^4 - 3.1895562657583395`*^-11\,u^6 + 1.1003902778341604`*^-10\,u^8 + 1.2736996735141446`*^-11\,u^{10}\big) + \big(1.3201449330932643`*^-10\,u + 2.0116494218564024`*^-10\,u^3 + 1.10831883717337`*^-10\,u^5 + 1.6935163929863798`*^-10\,u^7 + 1.230654263323963`*^-10\,u^9 + 1.2100146898384375`*^-11\,u^{11}\big)\,\text{ArcTan}[u] + \big(-6.286404443301257`*^-12 - 1.0387534961073984`*^-10\,u^2 - 1.4851879957793006`*^-10\,u^4 - 7.340125569035563`*^-11\,u^6 - 1.4428794960339077`*^-11\,u^8 - 8.980577776144653`*^-13\,u^{10}\big)$$

```
        ArcTan[u]^2) Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2) +
u^5 Hypergeometric2F1[3/2, 2, 7/2, -u^2]^2 (-0.007037037037037035` u +
   0.005555555555555555` u^3 + 0.0003518518518518523` ArcTan[u] +
   0.005277777777777776` u^2 ArcTan[u] +
   (1.6321252893518515`*^-6 u + 9.371054292929284`*^-7
       u^3 - 1.015197548400673`*^-6 u^5 - 5.503635060926727`*^-7
       u^7 - 2.7890042538480034`*^-8 u^9 + (-8.160626446759271`*^-8 -
          1.3353752367424243`*^-6 u^2 - 1.7063128025042084`*^-6 u^4 -
          5.58172718003447`*^-7 u^6 - 2.6495540411556034`*^-8 u^8)
       ArcTan[u]) Hypergeometric2F1[7/2, 4, 15/2, -u^2] +
   u^4 (1.7112989873431197`*^-10 u + 1.582629890550404`*^-10
       u^3 - 7.653973129212405`*^-11 u^5 - 9.72738371752931`*^-11
       u^7 - 1.859452261650161`*^-11 u^9 - 9.92590175258093`*^-13
       u^11 + (-8.556494936715614`*^-12 - 1.430157010851036`*^-10 u^2 -
          2.2777738766228217`*^-10 u^4 - 1.175557630897335`*^-10 u^6 -
          1.892207737433678`*^-11 u^8 - 9.429606664951885`*^-13 u^10)
       ArcTan[u]) Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 +
   Hypergeometric2F1[5/2, 3, 11/2, -u^2] (0.000011606224279835386`
       u + 7.853835978835977`*^-7 u^3 - 6.859016754850088`*^-6 u^5 -
      7.853835978835977`*^-7 u^7 - 5.803112139917703`*^-7 ArcTan[u] -
      9.202077821869489`*^-6 u^2 ArcTan[u] - 7.5108851410934715`*^-6
       u^4 ArcTan[u] - 7.461144179894178`*^-7 u^6 ArcTan[u] +
      (-2.6918733070907274`*^-9 u - 3.8528931681804386`*^-9 u^3 +
          1.1886193823517555`*^-10 u^5 + 2.2104149917418506`*^-9 u^7 +
          9.675603596720017`*^-10 u^9 + 1.1723225221779753`*^-10 u^11 +
          3.942788751719648`*^-12 u^13 + (1.345936653545366`*^-10 +
             2.3178077955859128`*^-9 u^2 + 4.713575038896659`*^-9 u^4 +
             3.521576248497046`*^-9 u^6 + 1.0740025133392595`*^-9 u^8 +
             1.1636483869241919`*^-10 u^10 + 3.745649314133665`*^-12 u^12)
           ArcTan[u]) Hypergeometric2F1[7/2, 4, 15/2, -u^2])) +
Hypergeometric2F1[3/2, 2, 7/2, -u^2] (-0.07916666666666665` u^2 -
   0.026388888888888882` u^4 + 0.08312499999999999` u ArcTan[u] +
   0.027708333333333324` u^3 ArcTan[u] + 3.652049423109067`*^-18 u^5
    ArcTan[u] - 0.003958333333333333` ArcTan[u]^2 - 0.06069444444444443`
    u^2 ArcTan[u]^2 - 0.019791666666666662` u^4 ArcTan[u]^2 +
   0.00001836140950520833` u^2 Hypergeometric2F1[7/2, 4, 15/2, -u^2] +
   0.00003115875552398989` u^4 Hypergeometric2F1[7/2, 4, 15/2, -u^2] +
   0.000016692190459280 3` u^6 Hypergeometric2F1[7/2, 4, 15/2, -u^2] +
```

$$3.179464849386724`*^-6\,u^8\,\text{Hypergeometric2F1}\left[\frac{7}{2},4,\frac{15}{2},-u^2\right]+$$
$$1.3247770205778017`*^-7\,u^{10}\,\text{Hypergeometric2F1}\left[\frac{7}{2},4,\frac{15}{2},-u^2\right]-$$
$$0.00001927947998046875`\,u\,\text{ArcTan}[u]\,\text{Hypergeometric2F1}\left[\frac{7}{2},4,\frac{15}{2},-u^2\right]-0.00003271669330018938`\,u^3\,\text{ArcTan}[u]\,\text{Hypergeometric2F1}\left[\frac{7}{2},4,\frac{15}{2},-u^2\right]-0.00017526799982244312`\,u^5\,\text{ArcTan}[u]\,\text{Hypergeometric2F1}\left[\frac{7}{2},4,\frac{15}{2},-u^2\right]-$$
$$3.33843809185606`*^-6\,u^7\,\text{ArcTan}[u]\,\text{Hypergeometric2F1}\left[\frac{7}{2},4,\frac{15}{2},-u^2\right]-$$
$$1.391015871606694`*^-7\,u^9\,\text{ArcTan}[u]\,\text{Hypergeometric2F1}\left[\frac{7}{2},4,\frac{15}{2},-u^2\right]+$$
$$9.180704752604166`*^-7\,\text{ArcTan}[u]^2\,\text{Hypergeometric2F1}\left[\frac{7}{2},4,\frac{15}{2},-u^2\right]+$$
$$0.00001532894905105745`\,u^2\,\text{ArcTan}[u]^2\,\text{Hypergeometric2F1}\left[\frac{7}{2},4,\frac{15}{2},-u^2\right]+0.00002420367616595643 6`\,u^4\,\text{ArcTan}[u]^2\,\text{Hypergeometric2F1}\left[\frac{7}{2},4,\frac{15}{2},-u^2\right]+$$
$$0.00012678116086929564`\,u^6\,\text{ArcTan}[u]^2\,\text{Hypergeometric2F1}\left[\frac{7}{2},4,\frac{15}{2},-u^2\right]+2.3912225221429324`*^-6\,u^8\,\text{ArcTan}[u]^2\,\text{Hypergeometric2F1}\left[\frac{7}{2},4,\frac{15}{2},-u^2\right]+9.935827654333512`*^-8\,u^{10}\,\text{ArcTan}[u]^2\,\text{Hypergeometric2F1}\left[\frac{7}{2},4,\frac{15}{2},-u^2\right]+$$
$$1.92521136076101`*^-9\,u^6\,\text{Hypergeometric2F1}\left[\frac{7}{2},4,\frac{15}{2},-u^2\right]^2+$$
$$3.94209945298683`*^-9\,u^8\,\text{Hypergeometric2F1}\left[\frac{7}{2},4,\frac{15}{2},-u^2\right]^2+$$
$$2.844598010593818`*^-9\,u^{10}\,\text{Hypergeometric2F1}\left[\frac{7}{2},4,\frac{15}{2},-u^2\right]^2+$$
$$8.643806109539228`*^-10\,u^{12}\,\text{Hypergeometric2F1}\left[\frac{7}{2},4,\frac{15}{2},-u^2\right]^2-$$
$$6.667641786448301`*^-9\,u^{14}\,\text{Hypergeometric2F1}\left[\frac{7}{2},4,\frac{15}{2},-u^2\right]^2+$$
$$3.095559344393466`*^-9\,u^{16}\,\text{Hypergeometric2F1}\left[\frac{7}{2},4,\frac{15}{2},-u^2\right]^2+$$
$$1.7831795429198027`*^-9\,u^{18}\,\text{Hypergeometric2F1}\left[\frac{7}{2},4,\frac{15}{2},-u^2\right]^2-$$
$$2.0214719287990605`*^-9\,u^5\,\text{ArcTan}[u]\,\text{Hypergeometric2F1}\left[\frac{7}{2},4,\frac{15}{2},-u^2\right]^2-4.1392044256361715`*^-9\,u^7\,\text{ArcTan}[u]\,\text{Hypergeometric2F1}\left[\frac{7}{2},4,\frac{15}{2},-u^2\right]^2-$$
$$2.9868279111235095`*^-9\,u^9\,\text{ArcTan}[u]\,\text{Hypergeometric2F1}\left[\frac{7}{2},4,\frac{15}{2},-u^2\right]^2-9.075996415016188`*^-10$$

```
 u^11 ArcTan[u] Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 +
2.2494161267546796`*^-10 u^13 ArcTan[u]
 Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 + 5.190045858207257`*^-9
 u^15 ArcTan[u] Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 +
1.6940205657738122`*^-9 u^17 ArcTan[u]
 Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 + 9.626056803805048`*^-11
 u^4 ArcTan[u]^2 Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 +
1.6410134932200989`*^-9 u^6 ArcTan[u]^2
 Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 + 3.0988044902698135`*^-9
 u^8 ArcTan[u]^2 Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 +
2.1766675384930604`*^-9 u^10 ArcTan[u]^2
 Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 + 6.537074820477894`*^-10
 u^12 ArcTan[u]^2 Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 +
8.156609765183379`*^-11 u^14 ArcTan[u]^2
 Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 + 3.536102499356957`*^-12
 u^16 ArcTan[u]^2 Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 +
u^9 Hypergeometric2F1[5/2, 3, 11/2, -u^2]^2 (-7.5795750398925`*^-7
    u + 3.45735001819658`*^-7 u^3 + 1.994625010498026`*^-7
    u^5 + 3.789787519946254`*^-8 ArcTan[u] + 5.81100753058425`*^-7
    u^2 ArcTan[u] + 1.8948937599731244`*^-7 u^4 ArcTan[u] +
   (1.7579580781000673`*^-10 u + 1.5953399464735848`*^-10 u^3 -
      7.570154403301727`*^-11 u^5 - 9.572840753381547`*^-11 u^7 -
      2.2763868179240637`*^-11 u^9 - 1.0013431750399107`*^-12 u^11 +
      (-8.789790390500344`*^-12 - 1.4676286379289958`*^-10 u^2 -
         2.3173083756773613`*^-10 u^4 - 1.2138281967833794`*^-10 u^6 -
         2.289404279199582`*^-11 u^8 - 9.51276016287915`*^-13 u^10)
       ArcTan[u]) Hypergeometric2F1[7/2, 4, 15/2, -u^2]) +
Hypergeometric2F1[5/2, 3, 11/2, -u^2] (0.00013057002314814813`
    u^2 + 0.00015544050374779537` u^4 + 0.00004849743716931215` u^6 +
   3.730572089947088`*^-6 u^8 - 0.00013709852430555554` u ArcTan[u] -
   0.00016321252893518516` u^3 ArcTan[u] - 0.00005092230902777777`
    u^5 ArcTan[u] - 3.9171006944444275`*^-6 u^7 ArcTan[u] -
   4.458068143443686`*^-22 u^9 ArcTan[u] + 6.528501157407406`*^-6
    ArcTan[u]^2 + 0.00010569954254850087` u^2 ArcTan[u]^2 +
   0.00011900524966931214` u^4 ArcTan[u]^2 + 0.00003659606481481464`
    u^6 ArcTan[u]^2 + 2.7979290674603167`*^-6 u^8 ArcTan[u]^2 +
```

$$\Big(u^2 \Big(-3.028357470477069`*^-8 - 7.734765833686019`*^-8\, u^2 - 7.417509336778895`*^-8\, u^4 - 3.324638331225041`*^-8\, u^6 - 7.073034454855736`*^-9\, u^8 - 6.36760383402723`*^-10\, u^{10} - 1.8728246570668323`*^-11\, u^{12}\Big) + \Big(3.1797753440009226`*^-8\, u + 8.121504125370321`*^-8\, u^3 + 7.788384803617842`*^-8\, u^5 + 3.490870247786293`*^-8\, u^7 + 7.426686177598523`*^-9\, u^9 + 6.685984025728589`*^-10\, u^{11} + 1.966465889920171`*^-11\, u^{13} + 1.700618035676455`*^-27\, u^{15}\Big)\, \text{ArcTan}[u] + \Big(-1.5141787352385342`*^-9 - 2.658006394542103`*^-8\, u^2 - 6.171949842103459`*^-8\, u^4 - 5.7293639191454246`*^-8\, u^6 - 2.5288439206930598`*^-8\, u^8 - 5.33661386031194`*^-9\, u^{10} - 4.785066998805757`*^-10\, u^{12} - 1.4046184928001245`*^-11\, u^{14}\Big)\, \text{ArcTan}[u]^2\Big)\, \text{Hypergeometric2F1}\Big[\frac{7}{2}, 4, \frac{15}{2}, -u^2\Big] + u^{13} \Big(-1.1175830121424458`*^-11\, u - 4.481535888290506`*^-12\, u^3 + 6.352577121651799`*^-12\, u^5 + 2.957813686271736`*^-12\, u^7 + 2.520863937163412`*^-13\, u^9 + \Big(5.587915060712234`*^-13 + 9.047100574486466`*^-12\, u^2 + 1.0185970882098291`*^-11\, u^4 + 3.129232433998848`*^-12\, u^6 + 2.3948207403052407`*^-13\, u^8\Big)\, \text{ArcTan}[u]\Big)\, \text{Hypergeometric2F1}\Big[\frac{7}{2}, 4, \frac{15}{2}, -u^2\Big]^2\Big)\Big)\Big)\Big/$$

$$\Big(u \Big(0.06333333333333331`\, u + 1.`\, u^3 - 0.06333333333333331`\, \text{ArcTan}[u] - 0.9499999999999998`\, u^2\, \text{ArcTan}[u] - 0.00014689127604166663`\, u\, \text{Hypergeometric2F1}\Big[\frac{7}{2}, 4, \frac{15}{2}, -u^2\Big] - 0.0002519642223011364`\, u^3\, \text{Hypergeometric2F1}\Big[\frac{7}{2}, 4, \frac{15}{2}, -u^2\Big] - 0.0003229499585700757`\, u^5\, \text{Hypergeometric2F1}\Big[\frac{7}{2}, 4, \frac{15}{2}, -u^2\Big] - 0.0001057423072803932`\, u^7\, \text{Hypergeometric2F1}\Big[\frac{7}{2}, 4, \frac{15}{2}, -u^2\Big] - 5.020207656926407`*^-6\, u^9\, \text{Hypergeometric2F1}\Big[\frac{7}{2}, 4, \frac{15}{2}, -u^2\Big] + 0.00014689127604166663`\, \text{ArcTan}[u]\, \text{Hypergeometric2F1}\Big[\frac{7}{2}, 4, \frac{15}{2}, -u^2\Big] + 0.0002403675426136364`\, u^2\, \text{ArcTan}[u]\, \text{Hypergeometric2F1}\Big[\frac{7}{2}, 4, \frac{15}{2}, -u^2\Big] + 0.0003071363044507575`\, u^4\, \text{ArcTan}[u]\, \text{Hypergeometric2F1}\Big[\frac{7}{2}, 4, \frac{15}{2}, -u^2\Big] + 0.00010047108924062049`\, u^6\, \text{ArcTan}[u]\, \text{Hypergeometric2F1}\Big[\frac{7}{2}, 4, \frac{15}{2}, -u^2\Big] + 4.769197274080086`*^-6\, u^8\, \text{ArcTan}[u]\, \text{Hypergeometric2F1}\Big[\frac{7}{2}, 4, \frac{15}{2}, -u^2\Big] - 1.5401690886088079`*^-9\, u^5\, \text{Hypergeometric2F1}\Big[\frac{7}{2}, 4, \frac{15}{2}, -u^2\Big]^2 - 2.695874916000981`*^-8\, u^7\, \text{Hypergeometric2F1}\Big[\frac{7}{2}, 4, \frac{15}{2}, -u^2\Big]^2 -$$

```
4.670533810659433`*^-8 u^9 Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 -
8.184898585178239`*^-8 u^11 Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 -
4.7523298583685345`*^-8 u^13 Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 -
8.784423051034127`*^-8 u^15 Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 +
1.5401690886088079`*^-9 u^4 ArcTan[u]
 Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 + 2.5742826195318646`*^-8
 u^6 ArcTan[u] Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 +
4.462089873855233`*^-8 u^8 ArcTan[u]
 Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 + 7.788855105250257`*^-8
 u^10 ArcTan[u] Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 +
4.001799340516714`*^-8 u^12 ArcTan[u]
 Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 + 6.204681185538341`*^-9
 u^14 ArcTan[u] Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 +
u^4 Hypergeometric2F1[5/2, 3, 11/2, -u^2]^2 (-4.050335411942554`*^-7 u -
   6.665288800705468`*^-6 u^3 - 4.914880689930294`*^-6 u^5 -
   9.806075207860919`*^-6 u^7 + 4.050335411942554`*^-7 ArcTan[u] +
   6.3455254787100024`*^-6 u^2 ArcTan[u] + 4.095339138741916`*^-6
    u^4 ArcTan[u] + 6.750559019904257`*^-7 u^6 ArcTan[u] +
   (9.394088479847233`*^-11 u + 1.6740055914726182`*^-9 u^3 +
      3.290677777443564`*^-9 u^5 + 4.533519892723724`*^-9 u^7 +
      3.6530069210974262`*^-9 u^9 + 1.0584729323733594`*^-9 u^11 +
      4.9228533842899604`*^-11 u^13 + (-9.394088479847233`*^-11 -
         1.5998417350527715`*^-9 u^2 - 2.9994660287673843`*^-9 u^4 -
         2.1228199941472967`*^-9 u^6 - 6.771063774435344`*^-10 u^8 -
         9.172678987056221`*^-11 u^10 - 3.3889208080256974`*^-12 u^12)
       ArcTan[u]) Hypergeometric2F1[7/2, 4, 15/2, -u^2] -
   2.2420775429197073`*^-44 u^13 Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2) +
Hypergeometric2F1[5/2, 3, 11/2, -u^2] (-0.00010445601851851848` u -
   0.0017388392857142856` u^3 - 0.001422643849206349` u^5 -
   0.0001413690476190476` u^7 + 0.00010445601851851848` ArcTan[u] +
   0.0016563740079365075` u^2 ArcTan[u] + 0.0013519593253968254`
    u^4 ArcTan[u] + 0.00013430059523809523` u^6 ArcTan[u] +
   (2.4226859763816547`*^-8 u + 4.3633187144005633`*^-7 u^3 +
      8.909191702236746`*^-7 u^5 + 6.665725977122258`*^-7 u^7 +
      2.0342187718297252`*^-7 u^9 + 2.204570902636524`*^-8 u^11 +
      7.097019753095367`*^-10 u^13 + (-2.4226859763816547`*^-8 -
```

```
      4.172054032054642`*^-7 u^2 - 8.484435070013986`*^-7 u^4 -
      6.338837247294684`*^-7 u^6 - 1.9332045240106675`*^-7 u^8 -
      2.094567096463546`*^-8 u^10 - 6.742168765440598`*^-10 u^12)
    ArcTan[u]) Hypergeometric2F1[7/2, 4, 15/2, -u^2] +
 u^8 (-5.972084221136194`*^-12 u - 1.0339638546267878`*^-10 u^3 -
    1.572179859170888`*^-10 u^5 - 2.151267471016198`*^-10 u^7 -
    1.3014353960596295`*^-10 u^9 - 1.2393197331079623`*^-11 u^11 +
    (5.972084221136194`*^-12 + 9.868158213020285`*^-11 u^2 +
      1.4109285959903355`*^-10 u^4 + 6.973119290583785`*^-11 u^6 +
      1.3707355212322124`*^-11 u^8 + 8.531548887337421`*^-13 u^10)
    ArcTan[u]) Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2) +
u^5 Hypergeometric2F1[3/2, 2, 7/2, -u^2]^2 (-0.00033425925925925973` -
    0.005013888888888888` u^2 + (7.752595124421308`*^-8 +
      1.268606474905303`*^-6 u^2 + 1.620997162378998`*^-6 u^4 +
      5.302640821032747`*^-7 u^6 + 2.5170763390978234`*^-8 u^8)
    Hypergeometric2F1[7/2, 4, 15/2, -u^2] + u^4 (8.128670189879833`*^-12 +
      1.3586491603084844`*^-10 u^2 + 2.1638851827916808`*^-10 u^4 +
      1.1167797493524682`*^-10 u^6 + 1.7975973505619944`*^-11 u^8 +
      8.95812633170429`*^-13 u^10) Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 +
    Hypergeometric2F1[5/2, 3, 11/2, -u^2] (5.512956532921818`*^-7 +
      8.741973930776015`*^-6 u^2 + 7.135340884038798`*^-6 u^4 +
      7.08808697089947`*^-7 u^6 + (-1.2786398208680978`*^-10 -
        2.2019174058066174`*^-9 u^2 - 4.477896286951826`*^-9 u^4 -
        3.345497436072194`*^-9 u^6 - 1.0203023876722966`*^-9 u^8 -
        1.1054659675779824`*^-10 u^10 - 3.558366848426982`*^-12 u^12)
      Hypergeometric2F1[7/2, 4, 15/2, -u^2])) +
Hypergeometric2F1[3/2, 2, 7/2, -u^2] (-0.0037604166666666662` u -
    0.06062847222222222` u^3 - 0.01979166666666667` u^5 +
    0.0037604166666666662` ArcTan[u] + 0.057659722222222216`
     u^2 ArcTan[u] + 0.01880208333333333` u^4 ArcTan[u] +
    (8.721669514973958`*^-7 u + 0.000015251098016295774` u^3 +
      0.00002416194568980823 6` u^5 + 0.000012670167424806096` u^7 +
      2.3908913278877878`*^-6 u^9 + 9.935827654333514`*^-8 u^11 +
      (-8.721669514973958`*^-7 - 0.000014562545159850 46` u^2 -
        0.00002299349235765861 6` u^4 - 0.000012044210282583086` u^6 -
        2.271661396035786`*^-6 u^8 - 9.439036271616837`*^-8 u^10)
      ArcTan[u]) Hypergeometric2F1[7/2, 4, 15/2, -u^2] +
    u^4 (9.144753963614797`*^-11 u + 1.6311582445876322`*^-9 u^3 +
      3.091692995243329`*^-9 u^5 + 2.1745065869656758`*^-9 u^7 +
```

```
3.315724733591484`*^-10 u^9 - 4.853692270977536`*^-9 u^11 -
 1.6057834349857647`*^-9 u^13 + (-9.144753963614797`*^-11 -
    1.558962818559094`*^-9 u^2 - 2.943864265756323`*^-9 u^4 -
    2.0678341615684074`*^-9 u^6 - 6.210221079454`*^-10 u^8 -
    7.74877927692421`*^-11 u^10 - 3.359297374389109`*^-12 u^12)
  ArcTan[u]) Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2 +
u^9 Hypergeometric2F1[5/2, 3, 11/2, -u^2]^2 (-3.600298143948941`*^-8 -
    5.520457154055038`*^-7 u^2 - 1.8001490719744684`*^-7 u^4 +
    (8.350300870975327`*^-12 + 1.394247206032546`*^-10 u^2 +
       2.2014429568934935`*^-10 u^4 + 1.1531367869442106`*^-10 u^6 +
       2.174934065239603`*^-11 u^8 + 9.037122154735193`*^-13 u^10)
     Hypergeometric2F1[7/2, 4, 15/2, -u^2]) + Hypergeometric2F1[5/2, 3,
   11/2, -u^2] (6.202076099537036`*^-6 u + 0.0001053109412891314` u^3 +
    0.00011888400607638889` u^5 + 0.00003655028005125606` u^7 +
    2.797929067460317`*^-6 u^9 - 6.202076099537036`*^-6 ArcTan[u] -
    0.00010041456542107584` u^2 ArcTan[u] -
    0.00011305498718584654` u^4 ArcTan[u] - 0.00003473162615740739` 
     u^6 ArcTan[u] - 2.658032614087301`*^-6 u^8 ArcTan[u] +
    (-1.4384697984766076`*^-9 u - 2.638669479957888`*^-8 u^3 -
       6.153406068761513`*^-8 u^5 - 5.721052323317363`*^-8 u^7 -
       2.5270756620793458`*^-8 u^9 - 5.335021959353433`*^-9 u^11 -
       4.784598792641491`*^-10 u^13 - 1.4046184928001247`*^-11 u^15 +
       (1.4384697984766076`*^-9 + 2.525106074814998`*^-8 u^2 +
          5.863352349998286`*^-8 u^4 + 5.4428957231881536`*^-8 u^6 +
          2.402401724658407`*^-8 u^8 + 5.069783167296343`*^-9 u^10 +
          4.54581364886547`*^-10 u^12 + 1.3343875681601184`*^-11
           u^14) ArcTan[u]) Hypergeometric2F1[7/2, 4, 15/2, -u^2] +
    u^13 (-5.308519307676622`*^-13 - 8.594745545762143`*^-12
        u^2 - 9.676672337993377`*^-12 u^4 -
       2.9727708122989056`*^-12 u^6 - 2.2750797032899787`*^-13 u^8)
     Hypergeometric2F1[7/2, 4, 15/2, -u^2]^2)))))
Sin[r Σt u], {u, 0, Infinity}, Method → "LevinRule"]
```

load MC data

In[4650]:=
```
ppoints[xs_, dr_, maxx_] :=
 Table[{dr (i) - 0.5 dr, xs[[i]]}, {i, 1, Length[xs]}][[1 ;; -2]]
```

In[4651]:=
```
ppointsu[xs_, du_, Σt_] :=
 Table[{-1.0 + du (i) - 0.5 du, xs[[i]] / (2 Σt)}, {i, 1, Length[xs]}][[1 ;; -1]]
```

In[4652]:=
```
fs = FileNames["code/3D_medium/infinite3Dmedium/Isotropicpointsource/MCdata/
      inf3D_isotropicpoint_LSscatter*"];
```

In[4653]:=
```
index[x_] := Module[{data, α, Σt},
   data = Import[x, "Table"];
   Σt = data[[1, 13]];
   α = data[[2, 3]];
   {α, Σt, data}];
simulations = index /@ fs;
cs = Union[#[[1]] & /@ simulations]
```

Out[4655]= {0.01, 0.1, 0.3, 0.5, 0.7, 0.8, 0.9, 0.95, 0.99, 0.999}

In[4656]:=
```
mfps = Union[#[[2]] & /@ simulations]
```

Out[4656]= {0.3, 1}

In[4657]:=
```
numcollorders = simulations[[1]][[3]][[2, 13]];
maxr = simulations[[1]][[3]][[2, 5]];
dr = simulations[[1]][[3]][[2, 7]];
numr = Floor[maxr / dr];
```

Compare MC and deterministic

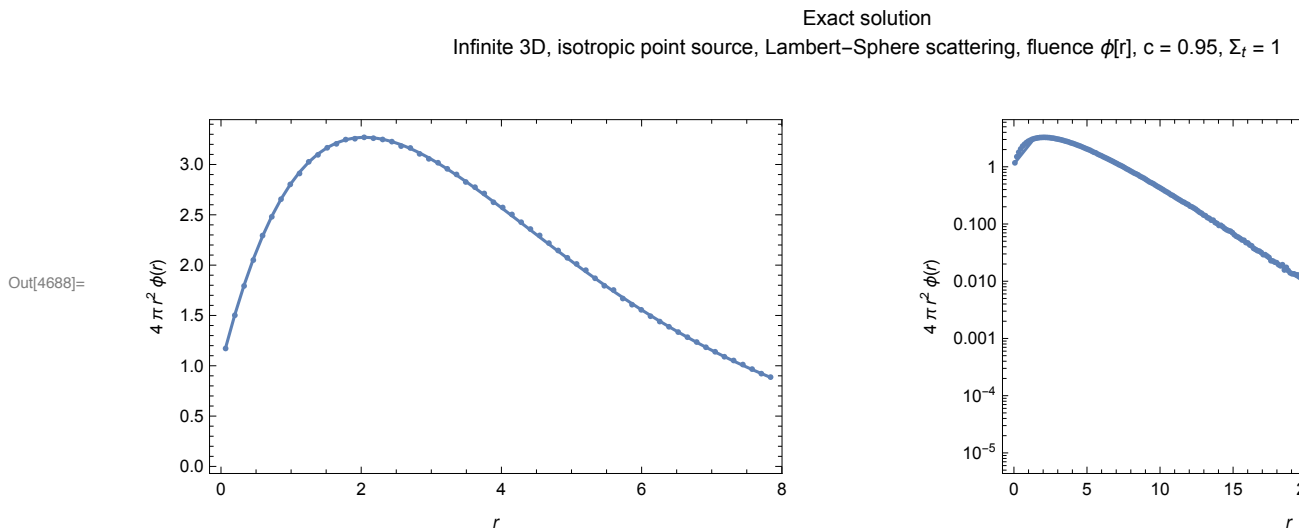
Fluence - Exact solution comparison to MC

In[4670]:=
```
{{ActionMenu["Set c", "c = " <> ToString[#] :> (c = #;) & /@ cs], Dynamic[c]},
 {ActionMenu["Set mfp", "mfp = " <> ToString[#] :> (mfp = #;) & /@ mfps],
  Dynamic[mfp]}}
```

Out[4670]= {{Set c, 0.95}, {Set mfp, 1}}

In[4680]:=
```
data = SelectFirst[simulations, #[[1]] == c && #[[2]] == mfp &][[-1]];
maxr = data[[2, 5]];
dr = data[[2, 7]];
pointsFluence = ppoints[data[[6]], dr, maxr];
exact1FluenceShallow =
  Quiet[{#[[1]], 4 Pi #[[1]]^2 ϕexactTruncatedFourierOrder7[#[[1]],
        1 / mfp, c]}] & /@ pointsFluence[[1 ;; 60]];
exact1Fluence = Quiet[{#[[1]], 4 Pi #[[1]]^2 ϕexactTruncatedFourierOrder7[
        #[[1]], 1 / mfp, c]}] & /@ pointsFluence[[1 ;; -1 ;; 10]];
plotϕshallow = Quiet[Show[
    ListPlot[pointsFluence[[1 ;; 60]],
     PlotRange → All, PlotStyle → PointSize[.01]],
    ListPlot[exact1FluenceShallow, PlotRange → All, Joined → True],
    Frame → True,
    FrameLabel -> {{4 Pi r^2 ϕ[r],}, {r,}}
   ]];
logplotϕ = Quiet[Show[
    ListLogPlot[pointsFluence, PlotRange → All, PlotStyle → PointSize[.01]],
    ListLogPlot[exact1Fluence, PlotRange → All, Joined → True],
    Frame → True,
    FrameLabel -> {{4 Pi r^2 ϕ[r],}, {r,}}
   ]];
Show[GraphicsGrid[{{plotϕshallow, logplotϕ}}, ImageSize → 800],
 PlotLabel -> "Exact solution\nInfinite 3D, isotropic point
     source, Lambert-Sphere scattering, fluence ϕ[r], c = " <>
   ToString[c] <> ", Σt = " <> ToString[1 / mfp]]
```

Out[4688]=



Compare moments of ϕ

In[4231]:=
```
{{ActionMenu["Set c", "c = " <> ToString[#] :> (c = #;) & /@ cs], Dynamic[c]},
 {ActionMenu["Set mfp", "mfp = " <> ToString[#] :> (mfp = #;) & /@ mfps],
  Dynamic[mfp]}}
```

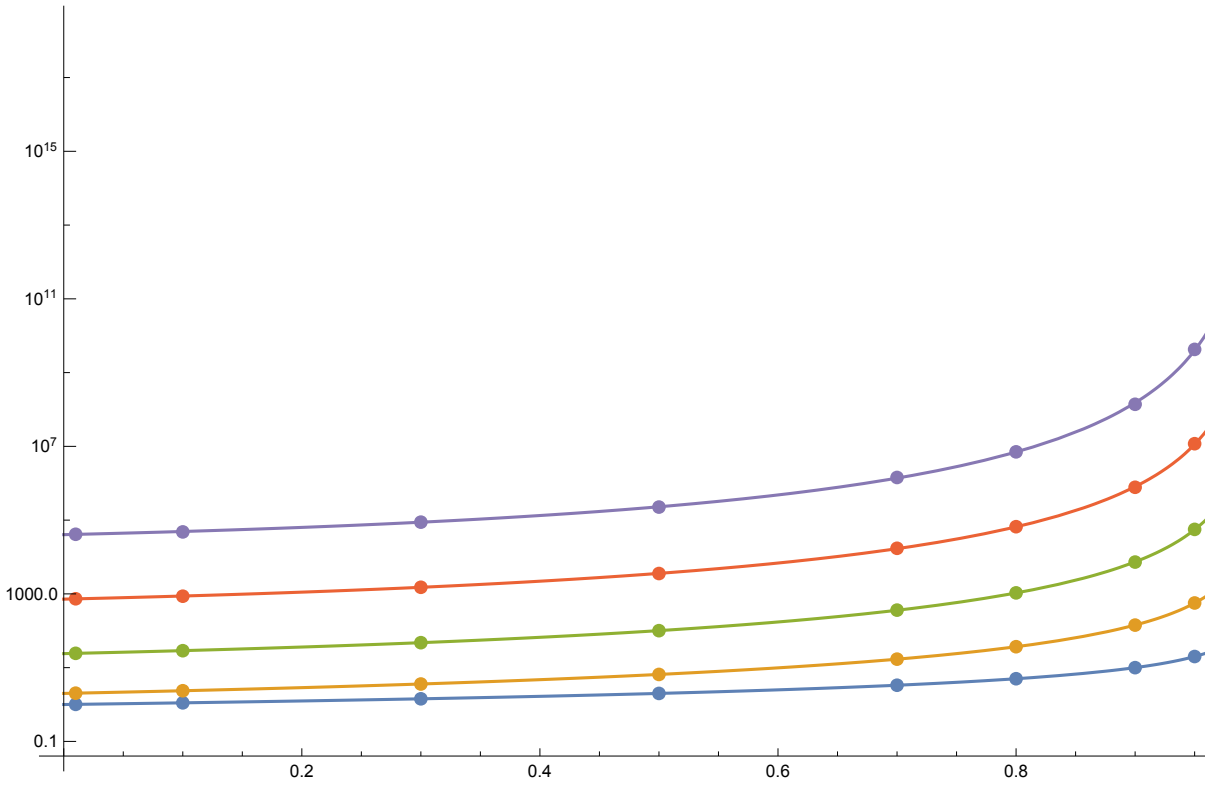
Out[4231]= {{Set c, 0.95}, {Set mfp, 1}}

mfp 1

In[4245]:=
```
mfp = 1;
sims1 = Select[simulations, #[[2]] == mfp &];
```

In[4254]:=
```
Show[
 ListLogPlot[{
   {#[[-1, 2, 3]], #[[-1, 10, 1]]} & /@ sims1,
   {#[[-1, 2, 3]], #[[-1, 10, 3]]} & /@ sims1,
   {#[[-1, 2, 3]], #[[-1, 10, 5]]} & /@ sims1,
   {#[[-1, 2, 3]], #[[-1, 10, 7]]} & /@ sims1,
   {#[[-1, 2, 3]], #[[-1, 10, 9]]} & /@ sims1
  }],
 LogPlot[{mfp/(1 - c), -3! mfp mfp^2/((-3 - 4 c/3) (-1 + c)^2),
   5! mfp (9 - 69 c/16) mfp^4/((-3 - 4 c/3)^2 (-5 + 5 c/16) (-1 + c)^3), mfp 7! (-675 + 5691 c/8 - 36399 c^2/256 - 48 c^3) mfp^6/(7 (-3 - 4 c/3)^3 (-5 + 5 c/16)^2 (-1 + c)^4),
   mfp 9! (((-496125 + 52729425 c/64 - 367391171 c^2/1024 -
         1099685881 c^3/16384 + 11363604819 c^4/262144 + 312487 c^5/32 - 329 c^6) mfp^8)/
     (49 (-3 - 4 c/3)^4 (-9 + c/64) (-5 + 5 c/16)^3 (-1 + c)^5)}, {c, 0, .999}, PlotRange -> All]
]
```

Out[4254]=



mfp 0.3

In[4255]:=
```
mfp = 0.3;
sims1 = Select[simulations, #[[2]] == mfp &];
```

In[4257]:=
```
Show[
 ListLogPlot[{
   {#[[-1, 2, 3]], #[[-1, 10, 1]]} & /@ sims1,
   {#[[-1, 2, 3]], #[[-1, 10, 3]]} & /@ sims1,
   {#[[-1, 2, 3]], #[[-1, 10, 5]]} & /@ sims1,
   {#[[-1, 2, 3]], #[[-1, 10, 7]]} & /@ sims1,
   {#[[-1, 2, 3]], #[[-1, 10, 9]]} & /@ sims1
  }],
 LogPlot[{mfp/(1 - c), -3! mfp mfp^2/((-3 - 4 c/3) (-1 + c)^2),
   5! mfp (9 - 69 c/16) mfp^4/((-3 - 4 c/3)^2 (-5 + 5 c/16) (-1 + c)^3), mfp 7! (-675 + 5691 c/8 - 36 399 c^2/256 - 48 c^3) mfp^6/(7 (-3 - 4 c/3)^3 (-5 + 5 c/16)^2 (-1 + c)^4),
   mfp 9! (((-496 125 + 52 729 425 c/64 - 367 391 171 c^2/1024 -
         1 099 685 881 c^3/16 384 + 11 363 604 819 c^4/262 144 + 312 487 c^5/32 - 329 c^6) mfp^8)/
     (49 (-3 - 4 c/3)^4 (-9 + c/64) (-5 + 5 c/16)^3 (-1 + c)^5)}, {c, 0, .999}, PlotRange → All]
]
```

Out[4257]=