Статистическая физика

Фомичев Сергей Венедиктович

Лекция №1

12.02.

N - число частиц ; $S = 3N$ - число степ. своб.

q_i, p_i - обобщ. коорд. $i \leq S$

$\{q_i\} = q \quad \{p_i\} = p$

$H(q, p)$ - гамильтониан

$$\dot{p}_i = -\frac{\partial H}{\partial q_i} \qquad \dot{q}_i = \frac{\partial H}{\partial p_i}$$

В системе многих частиц не имеет смысла ~~?~~ одну частицу.

Главнейший интерес: вер-ть нахожд. сист. в опр. сост.

$$dw = \rho(q, p, t)\, d\Gamma$$

$$d\Gamma = \frac{dq\, dp}{(2\pi\hbar)^s} \equiv \prod_{i=1}^{s} \frac{dq_i\, dp_i}{(2\pi\hbar)^s}$$

$$\int dw = \int \rho(q, p, t)\, d\Gamma = 1$$

$f(p, q)$

$$\langle f(t) \rangle = \int f(p, q)\, \rho(p, q, t)\, d\Gamma$$

$$\frac{\partial \rho}{\partial t} + \mathrm{div}_n \vec{j} = 0$$

$$\vec{j} = \rho \vec{v} = (\rho\{\dot{q}_i\}, \rho\{\dot{p}_i\})$$

$$\frac{\partial \rho}{\partial t} + \sum_{i=1}^{s} \left(\frac{\partial(\rho \dot{q}_i)}{\partial q_i} + \frac{\partial(\rho \dot{p}_i)}{\partial p_i} \right) = 0$$

$$\frac{\partial \rho}{\partial t} + \dot{q}_i \frac{\partial \rho}{\partial q_i} + \dot{p}_i \frac{\partial \rho}{\partial p_i} + \rho\left(\cancel{\frac{\partial^2 H}{\partial p_i \partial q_i} - \frac{\partial^2 H}{\partial q_i \partial p_i}}\right) = 0$$

$$\frac{\partial \rho}{\partial t} + \dot{q}_i \frac{\partial \rho}{\partial q_i} + \dot{p}_i \frac{\partial \rho}{\partial p_i} = 0 \Rightarrow \boxed{\frac{d\rho}{dt} = 0}$$

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \dot{q}_i \frac{\partial \rho}{\partial q_i} + \dot{p}_i \frac{\partial \rho}{\partial p_i}$$

$$\frac{\partial \rho}{\partial t} + \underbrace{\sum_{i=1}^{s}\left(\frac{\partial H}{\partial p_i}\cdot\frac{\partial \rho}{\partial q_i} - \frac{\partial H}{\partial q_i}\cdot\frac{\partial \rho}{\partial p_i}\right)}_{\{H,\rho\} \text{ – скобки Пуас.}} = 0$$

$$\boxed{\frac{d\rho}{dt} \equiv \frac{\partial \rho}{\partial t} + \{H, \rho\} = 0 \qquad i\hbar \frac{\partial \hat{\rho}(t)}{\partial t} = [\hat{H}, \hat{\rho}(t)]}$$ – класс. ур-ие Лиувилля для ф-ии распр.

Статфиз:

1) равновес. статфиз (2/3)

2) неравновес. (кинемат.) статфиз (1/3)

В равновес. сост. нет явной зав-ти от t.

$\rho(q,p,t) \equiv \rho(q,p) \Rightarrow \{H,\rho\} = 0$

$\rho(q,p) = F(H(q,p))$ – общее реш. ур-ия Лиувилля для равновесия.

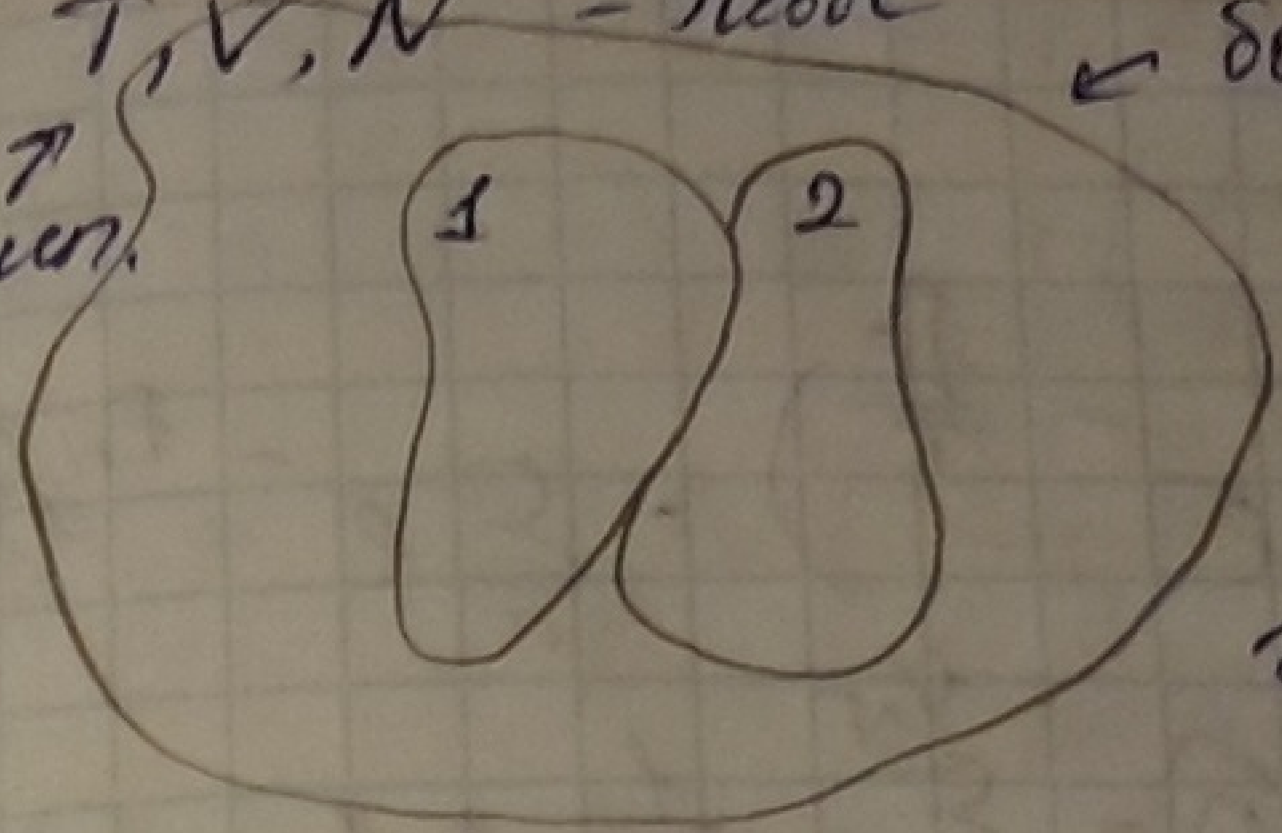
1) $\triangle$ замкнут. сист.

$E = const$, V, N – микроканонич. распр.

$$\boxed{\rho(q,p) \sim \delta(H(q,p) - E)} \qquad \rho_n = const = \frac{1}{\Delta\Gamma}$$

2) T, V, N – Гиббс

термост.



← большая замк. сист. в равнов.

1, 2 – квазинезавис. подсист. др. с др. и от сист.

соприкас-ся поверхностно

$$\mathcal{H}(q_1, p_1, q_2, p_2) = \mathcal{H}_1(q_1, p_1) + \mathcal{H}_2(q_2, p_2)$$

$$\rho(q_1, p_1, q_2, p_2) = \rho_1(q_1, p_1) * \rho_2(q_2, p_2)$$

$$\ln\rho = \ln\rho_1 + \ln\rho_2$$

выполнено, только если $\ln\rho_a(q,p) = \alpha_a + \beta\mathcal{H}_a(q,p)$

одинак. для каждой подсист.

$$\ln\rho = \alpha - \beta\mathcal{H}(q,p)$$

$$\beta = \frac{1}{T}$$

$$\boxed{\rho(q,p) = e^{\alpha - \frac{\mathcal{H}(q,p)}{T}} \equiv \frac{1}{Z} e^{-\frac{\mathcal{H}(q,p)}{T}}}$$

← каноническ. распр. Гиббса

$$\frac{1}{Z}\int e^{-\frac{\mathcal{H}(q,p)}{T}} d\Gamma = 1$$

$$\boxed{Z = \int e^{-\frac{\mathcal{H}(q,p)}{T}} d\Gamma}$$ – стат. интеграл (статсумма)

$$\mathcal{H}(q,p,\lambda) \to Z(T,\lambda) = \int e^{-\frac{\mathcal{H}(q,p,\lambda)}{T}} d\Gamma$$

$$\boxed{i\hbar \frac{\partial\hat{\rho}}{\partial t} = [\hat{\mathcal{H}}, \hat{\rho}]} \Rightarrow [\hat{\mathcal{H}}, \hat{\rho}] = 0$$ квант. ур. Лиувилля

$$Sp(\hat{\rho}) = 1 \qquad \langle f \rangle = \sum_{n,m} f_{nm}\rho_{mn} = Sp(\hat{f}\hat{\rho})$$

$$\hat{\rho} = F(\hat{\mathcal{H}})$$

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_1 + \hat{\mathcal{H}}_2 \qquad \underset{n\times n}{(\ )} \otimes \underset{m\times m}{(\ )} = \underset{nm\times nm}{(\ )}$$

$\rho = \rho_1 \otimes \rho_2$

$\hat{\rho} = \frac{1}{Z} e^{-\frac{\hat{H}(\lambda)}{T}}$

$Sp(\hat{\rho}) = 1 \Rightarrow Z = Sp\left(e^{-\frac{\hat{H}}{T}}\right) = \sum_n e^{-E_n/T}$

$\hat{\rho} = \frac{1}{\Delta\Gamma(E)} \Rightarrow Sp\,\hat{\rho} = \sum_{n=1}^{1/\Delta\Gamma} \frac{1}{\Delta\Gamma} = 1$

$F(T,\lambda)$

$Z = e^{-F/T}$

$\hat{\rho} = e^{\frac{F}{T} - \frac{\hat{H}}{T}} = e^{\frac{F-\hat{H}}{T}}$

$E = \sum_n \rho_n E_n$

Введём понятие энтропии

$$\boxed{S = -\int \rho(p,q) \ln\rho(p,q)\, d\Gamma \equiv -\langle \ln\rho \rangle}$$

$$\boxed{S = -\sum_n \rho_n \ln\rho_n \equiv -\langle \ln\hat{\rho} \rangle}$$

$\sum_n \rho_n = 1$

Ф-ие Лагранжа:

$L = S + \alpha_1 E + \alpha_2 = -\sum_n \rho_n \ln\rho_n + \alpha_1 \sum_n \rho_n E_n + \alpha_2 \sum \rho_n$

$\frac{\partial L}{\partial \rho_n} = -\ln\rho_n - 1 + \alpha_1 E_n + \alpha_2 = 0$

$\alpha_1 = -\frac{1}{T} \qquad \alpha_2 = \frac{E}{T} + 1$

тут $E \equiv Q$

$dL = dS + \alpha_1 dE = 0 \rightarrow \boxed{dE = TdS}$

$$F = -T\ln Z = -T\ln\sum_n e^{-E_n(\lambda)/T}$$

$$dF = \left(\frac{\partial F}{\partial T}\right)_\lambda dT + \left(\frac{\partial F}{\partial \lambda}\right)_T d\lambda$$

$$S = -\sum_n \rho_n \underbrace{\left(\frac{F-E_n}{T}\right)}_{\ln\rho_n} = \frac{E-F}{T} \Rightarrow \boxed{E = F + TS}$$

$$\left(\frac{\partial F}{\partial T}\right)_\lambda = -\ln Z - \frac{1}{TZ}\sum_n E_n e^{-E_n/T} = \frac{F}{T} - \frac{E}{T} = -S$$

$$\left(\frac{\partial F}{\partial \lambda}\right)_T = \frac{1}{Z}\sum_n \frac{\partial E_n}{\partial\lambda} e^{-E_n/T} = \sum_n \rho_n \frac{\partial E_n}{\partial \lambda} = \left\langle \frac{\partial E_n}{\partial\lambda}\right\rangle$$

$$dF = -SdT + \left(\frac{\partial F}{\partial\lambda}\right)_T d\lambda = -SdT + \left\langle\frac{\partial E_n}{\partial\lambda}\right\rangle d\lambda$$

$$dE = d(F+TS) = dF + TdS + SdT =$$

$$= -S\cancel{dT} + \left\langle\frac{\partial E_n}{d\lambda}\right\rangle d\lambda + TdS + \cancel{SdT} =$$

$$= TdS + \left\langle\frac{\partial E}{\partial\lambda}\right\rangle_S d\lambda$$

$$dE = \sum_n E_n d\rho_n + \sum_n \rho_n \frac{\partial E_n}{\partial\lambda} d\lambda \equiv$$

$$\equiv (dQ = TdS) \equiv \underbrace{\sum_n E_n d\rho_n}_{TdS} + \left\langle\frac{\partial E_n}{\partial\lambda}\right\rangle d\lambda$$

Получили первое начало термодинамики.

$$\left(\frac{\partial E}{\partial\lambda}\right)_S = \left\langle\frac{\partial E_n}{\partial\lambda}\right\rangle = \left(\frac{\partial F}{\partial\lambda}\right)_T$$

$$\lambda \equiv V \Rightarrow \left(\frac{\partial E}{\partial V}\right)_S = \left(\frac{\partial F}{\partial V}\right)_T = -P$$

$$dE = TdS - PdV + \mu dN$$

$dF = -SdT - PdV + \mu dN$

μ - хим. потенциал сист.

$E(S,V,N)$ $F(T,V,N)$

$\mu = \left(\frac{\partial E}{\partial N}\right)_{S,V} = \left(\frac{\partial F}{\partial N}\right)_{T,V}$

1) $Z(T,V) = \sum_n \exp\left(-\frac{E_n(V)}{T}\right)$

$F(T,V) - T \ln Z$

$S = -\left(\frac{\partial F}{\partial T}\right)_V$ $P = -\left(\frac{\partial F}{\partial V}\right)_T$

2) $\Delta\Gamma(E,V) \Rightarrow S(E,V)$

$dS = \frac{dE}{T} + \frac{P}{T}dV \Rightarrow \frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_V$, $\frac{P}{T} = \left(\frac{\partial S}{\partial V}\right)_E$

$\rho_n = \frac{1}{\Delta\Gamma}$

$S = \sum_n \rho_n \ln \rho_n = \sum_{n=1}^{\Delta\Gamma} \frac{1}{\Delta\Gamma} \ln \frac{1}{\Delta\Gamma} = \ln \Delta\Gamma$

$\boxed{S = \ln \Delta\Gamma}$

$F = E - TS$

энтальпия

$W = E + PV$ $W(S,P,N)$

$dW = TdS + VdP + \mu dN$

пот. Гиббса

$\Phi = F + PV$

$d\Phi = -SdT + VdP + \mu dN$

$\Phi(T,P,N)$

$E,\ F,\ W,\ \Phi$

$T,\ S,\ P,\ V$

$P = P(T, V, N)$

$C_V(T, V)$

$dF = -S\,dT - P\,dV$

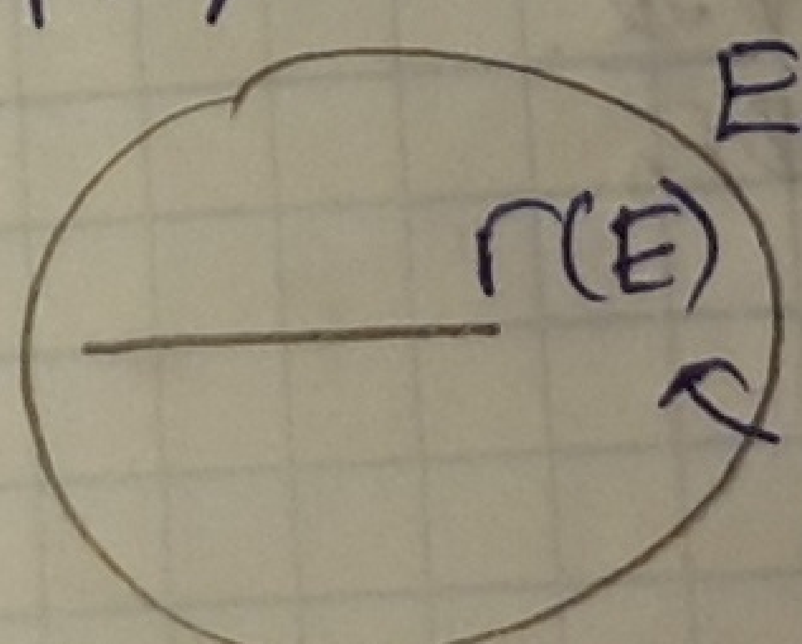
$$\left(\frac{\partial S}{\partial T}\right)_V ;\ \left(\frac{\partial S}{\partial V}\right)_T = -\frac{\partial^2 F}{\partial T \partial V} = \left(\frac{\partial P}{\partial T}\right)_V ;\ \left(\frac{\partial P}{\partial V}\right)_T$$

$$\left(\frac{\partial S}{\partial T}\right)_V \equiv \frac{C_V}{T} \qquad C_V = T\left(\frac{\partial S}{\partial T}\right)_V$$

$$\frac{\partial C_V}{\partial V} = T\frac{\partial^2 S}{\partial T \partial V} = T\frac{\partial^2 P}{\partial T^2}$$

Семинар №1. 12.02.16г.

$\rho(p, q, t)$ — плотность распр. Все реализации равновер.



$\omega \sim \frac{1}{\Delta\Gamma(E)}$ — микроканон. распр.

$$\frac{dp\,dq}{(2\pi\hbar)^{3N}} \qquad \rho(p, q, t) \to \delta(E(p, q) - E_0)$$

Т-ма Лиувилля:

Для замкн-х систем $\boxed{\frac{d\rho}{dt} = 0}$

$$\rho(\underset{E_1+E_2}{E}, \underset{\vec{p}_1+\vec{p}_2}{\vec{P}}, \vec{M}) \quad \rho \to \rho_1 * \rho_2$$

$\ln \rho_I \rho_{II} = \ln \rho_I + \ln \rho_{II} =$

Лекция №2 19.02.16

$$H(q,p)=\sum_{i=1}^{N}\frac{p_i^2}{2m}=E$$

$$\Gamma(E,V,N)=\frac{1}{N!}\int\limits_{H(q,p)\le E\ (1)}\frac{d^{3N}p\,d^{3N}q}{(2\pi\hbar)^{3N}} \;=$$

Интегрируем по (1), т.к. иначе $\int=0$
основ вклад дает $H\sim E$, $H<E$ — малый вклад

$$= \frac{V^N}{(2\pi\hbar)^{3N}N!}\int\limits_{\sum_{i=1}^{3N}p_i^2\le 2mE=R^2} d^{3N}p = \frac{V^N C_{3N}(2mE)^{\frac{3N}{2}}}{(2\pi\hbar)^{3N}N!}$$

$$R=\sqrt{2mE}$$

$$\Omega_{3N}(R)=C_{3N}R^{3N}$$

$$\int\limits_{-\infty}^{+\infty}\frac{d\Omega_{3N}(R)}{dp_1\dots dp_{3N}}e^{-\overbrace{(p_1^2+\dots+p_{3N}^2)}^{R^2}} = \quad R^2=t$$

$$=\left(\int\limits_{-\infty}^{+\infty}e^{-p^2}dp\right)^{3N}=\pi^{\frac{3N}{2}} \;=$$

$$R^{3N-2}$$

$$=\int\limits_0^\infty e^{-R^2}\cdot\frac{1}{2R}\underbrace{C_{3N}\cdot 3NR\,dR}_{d\Omega_{3N}(R)}\cdot\overbrace{2R}^{dR^2}=\frac{3N}{2}C_{3N}\int\limits_0^\infty dt\cdot t^{\frac{3N}{2}-1}e^{-t}=$$

$$=\frac{3N}{2}C_{3N}\Gamma\left(\frac{3N}{2}\right)$$

$$C_{3N}=\frac{\pi^{3N/2}}{\frac{3N}{2}\Gamma\left(\frac{3N}{2}\right)}=\frac{\pi^{3N/2}}{\Gamma\left(\frac{3N}{2}+1\right)}$$

$$x\Gamma(x)=\Gamma(x+1) \qquad \Gamma(x)=\int\limits_0^\infty dt\,e^{-t}t^{x-1}$$

$$\boxed{\Gamma(E,V,N)=\frac{V^N\pi^{\frac{3N}{2}}(2mE)^{3N/2}}{(2\pi\hbar)^{3N}N!\,\Gamma\left(\frac{3N}{2}+1\right)}}\sim V^N E^{3N/2}$$

число сост. с энергией $<E$

$N >> 1 \quad N! \simeq \sqrt{2\pi N}\left(\frac{N}{e}\right)^N$

энтропия $S(E,V,N) = \ln \Gamma(E,V,N) \simeq$

$$\simeq \ln\left[\frac{V^N \pi^{3N/2} (2mE)^{3N/2}}{(2\pi\hbar)^{3N} \cdot \left(\frac{N}{e}\right)^N \left(\frac{3N}{2e}\right)^{3N/2}}\right] \simeq$$

Для больших макросист. $\sqrt{2\pi N}$ не нужен (для $N!$)

$$\simeq \boxed{\frac{3N}{2} + N\ln\left[\frac{eV}{N}\left(\frac{E}{3N/2}\cdot\left(\frac{m}{2\pi\hbar^2}\right)\right)^{3/2}\right]}$$

$$\boxed{S = N\ln V + \frac{3}{2}N\ln E}$$

$dE = TdS - PdV \Rightarrow dS = \frac{dE}{T} + \frac{P}{T}dV$

$$\left(\frac{\partial S}{\partial E}\right)_V = \frac{1}{T} \Rightarrow \frac{3N}{2E} = \frac{1}{T} \Rightarrow E = \frac{3N}{2}T$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \frac{P}{T} = \frac{N}{V} \Rightarrow PV = NT$$

$C_V = \frac{3}{2}N$

$$\boxed{E(S,V,N) = \frac{3N}{2e}\cdot\frac{(2\pi\hbar)^2}{m}\left(\frac{N}{eV}\right)^{2/3} e^{\frac{2S}{3N}}}$$

$$F = E - TS = \cancel{\frac{3N}{2}T} - \cancel{\frac{3N}{2}T} - NT\ln\left(\frac{eV}{N}\left(\frac{mT}{2\pi\hbar^2}\right)^{3/2}\right) =$$

$$= F(T,V,N)$$

$\mathcal{H}_{1+2} = \mathcal{H}_1 + \mathcal{H}_2$

$E_n = E_{1n_1} + E_{2n_2}$

$$Z = \sum_n e^{-\frac{E_n}{T}} = \sum_n e^{-\frac{E_{1n_1}}{T}} e^{-\frac{E_{2n_2}}{T}} = \sum_{n_1} e^{-\frac{E_{1n_1}}{T}} \sum_{n_2} e^{-\frac{E_{2n_2}}{T}} =$$

$= Z_1 \cdot Z_2$

$$F = -T\ln Z = -T\ln Z_1 Z_2 = -T\ln Z_1 - T\ln Z_2 = F_1 + F_2$$

$C = C_1 + C_2 \qquad S = S_1 + S_2$

Аддитивность для незав. подсист.

Для идеального газа кажд. атом можно как независ. подсист.

$$\mathcal{E} = \mathcal{E}_{\text{поступ.}} + \mathcal{E}_{\text{внутр.}}$$

$$Z = \frac{1}{N!}\Bigg(\underbrace{\int \frac{dp\,dq}{(2\pi\hbar)^3} e^{-\frac{p^2}{2mT}}}_{Z_{\text{пост}}} \underbrace{\sum_n \mathcal{E}_{n\,\text{вн}}}_{Z_{\text{вн}}}\Bigg)^N$$

$$Z = \frac{1}{N!}\left(Z_{\text{пост}} \cdot Z_{\text{вн}}\right)^N = \left(\frac{e}{N} \cdot Z_{\text{пост}} Z_{\text{вн}}\right)^N$$

$$N! \sim \left(\frac{N}{e}\right)^N$$

$$Z_{\text{пост}} = \frac{V}{(2\pi\hbar)^{3/2}}\left(\int_{-\infty}^{+\infty} e^{-\frac{p_x^2}{2mT}} dp_x\right)^3 = \frac{V(2\pi mT)^{3/2}}{(2\pi\hbar)^3} =$$

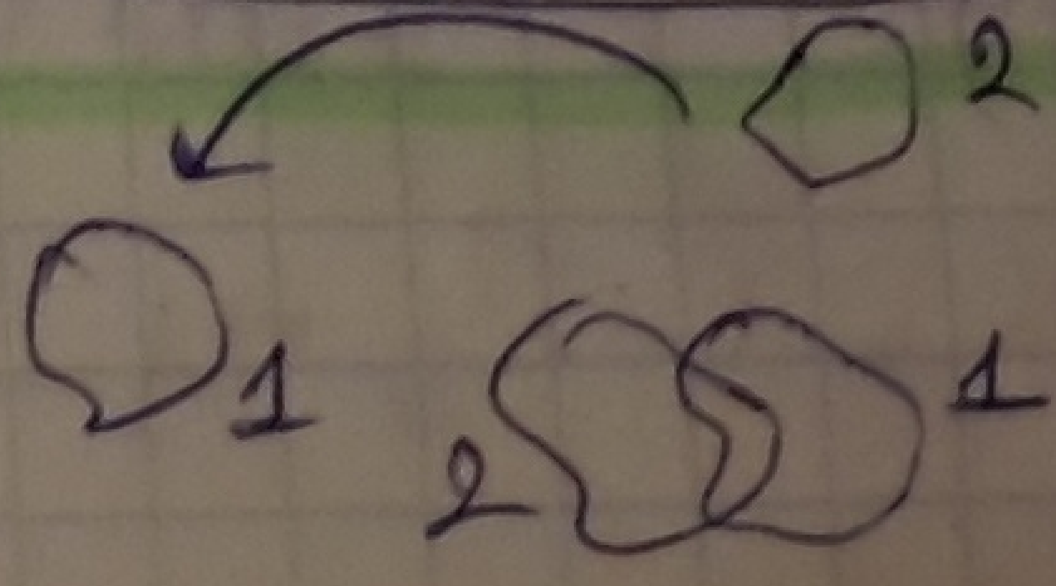
$$= V\left(\frac{mT}{2\pi\hbar^2}\right)^{3/2}$$

$$Z = \left(\frac{eV}{N}\left(\frac{mT}{2\pi\hbar^2}\right)^{3/2} Z_{\text{вн}}\right)^N$$

$$F = -T\ln Z = -TN\ln\left(\frac{eV}{N} Z_{\text{вн}} \left(\frac{mT}{2\pi\hbar^2}\right)^{3/2}\right) =$$

$$= \boxed{\underbrace{-TN\ln\left(\frac{eV}{N}\cdot\left(\frac{mT}{2\pi\hbar^2}\right)^{3/2}\right)}_{F_{\text{пост}}}} - TN\ln Z_{\text{вн}}$$

$$\frac{dp\,dq}{(2\pi\hbar)^3} \sim 1$$



$$\frac{p^2}{2m} \qquad p \sim \sqrt{mT}$$

$$\lambda_T \sim \frac{\hbar}{p} \sim \frac{\hbar}{\sqrt{mT}}$$

$F = -NT \ln \frac{e Z_1}{N}$ свобод. энергия

$Z_1 = \frac{V(2\pi m T)^{3/2}}{(2\pi\hbar)^3}$ $\qquad \sum e^{-\varepsilon_k/T}$

$\frac{F}{\varepsilon}$ для одной молек.

$$\lambda_T \sim \frac{\hbar}{\sqrt{mT}} << \left(\frac{V}{N}\right)^3$$

$$\boxed{T >> \frac{\hbar^2}{m}\left(\frac{N}{V}\right)^{2/3}} \sim T_{вырожд}$$

в противном случае надо учитывать квантовые эффекты и просто делить на $N!$ уже не верно

$$\boxed{\mu_{пост} = -T\ln\left(\frac{V}{N}\left(\frac{mT}{2\pi\hbar^2}\right)^{3/2}\right)}$$

$$\boxed{\mu_{пост} \underset{T>>T_{вырожд}}{\simeq} -T\ln\left(\frac{T}{T_{вырожд}}\right)^{3/2}}$$ химический потенциал

$\mu_{пост} < 0 \qquad |\mu_{пост}| >> T$

$$Z_{вн} = g e^{-\varepsilon_0/T} \qquad // g = 2S+1 //$$

$$F = -TN\ln\left(\frac{eV}{N} g e^{-\frac{\varepsilon_0}{T}}\left(\frac{mT}{2\pi\hbar}\right)^{3/2}\right) =$$

$$= -TN\ln\frac{eV}{N} - \underbrace{\frac{3}{2}}_{C_v} NT\ln T + N\varepsilon_0 - TN S_{пост}$$

$$\varepsilon = \varepsilon_0 + \frac{p^2}{2m} + \varepsilon_{кол} + \varepsilon_{вр.}$$

$$Z = g e^{-\frac{\varepsilon_0}{T}} \cdot Z_{пост} \cdot Z_{вр} \cdot Z_{кол}$$

$$F = N\varepsilon_0 + F_{пост} + F_{вр} + F_{кол}$$

$$\varepsilon_{кол} = \hbar\omega\left(n_v + \frac{1}{2}\right)$$

$$\varepsilon_{вр} = \frac{\hbar^2}{2I}K(K+1)$$

$$\bar{Z}_{кол} = \sum_{n_v=0}^{\infty} \exp\left(-\frac{\hbar\omega(n_v + 1/2)}{T}\right) =$$

$$= \frac{e^{-\hbar\omega/2T}}{1 - e^{-\hbar\omega/T}}$$

Распр. Максв-Больц.

$$n_{\vec{p}} = \exp\left(\frac{\mu - \varepsilon_{\vec{p}}}{T}\right)$$

$$\varepsilon_{\vec{p}} = \frac{p^2}{2m}$$

$$n_{\vec{p}} \ll 1$$

$$F_{кол} = -NT \ln \bar{Z}_{кол} = -NT \ln \frac{e^{-\frac{\hbar\omega}{2T}}}{\left(1 - e^{-\hbar\omega/T}\right)}$$

$$F_{кол} = N\frac{\hbar\omega}{2} + NT\ln\left(1 - e^{-\hbar\omega/T}\right)$$

$$S_{кол} = -\left(\frac{\partial F_{кол}}{\partial T}\right)_T = -N\ln\left(1 - e^{-\frac{\hbar\omega}{T}}\right) +$$

$$+ \frac{N\hbar\omega}{T} \cdot \frac{e^{-\frac{\hbar\omega}{T}}}{1 - e^{-\hbar\omega/T}} = \frac{N\hbar\omega}{T} \cdot \frac{1}{e^{\frac{\hbar\omega}{T}} - 1}$$

$$E_{кол} = F_{кол} + TS_{кол} = N\frac{\hbar\omega}{2} + N\frac{\hbar\omega}{e^{\frac{\hbar\omega}{T}} - 1}$$

$$\bar{\varepsilon} = \hbar\omega\left(\bar{n}_v + \frac{1}{2}\right) \qquad \boxed{\bar{n}_v = \frac{1}{e^{\frac{\hbar\omega}{T}} - 1}}$$

$$C_V = \left(\frac{\partial E}{\partial T}\right)_V = N\left(\frac{\hbar\omega}{T}\right)^2 \cdot \frac{e^{\frac{\hbar\omega}{T}}}{\left(e^{\frac{\hbar\omega}{T}} - 1\right)^2}$$

$$T \to 0 \quad C_V \sim N\left(\frac{\hbar\omega}{T}\right)^2 e^{-\frac{\hbar\omega}{T}} \to 0$$

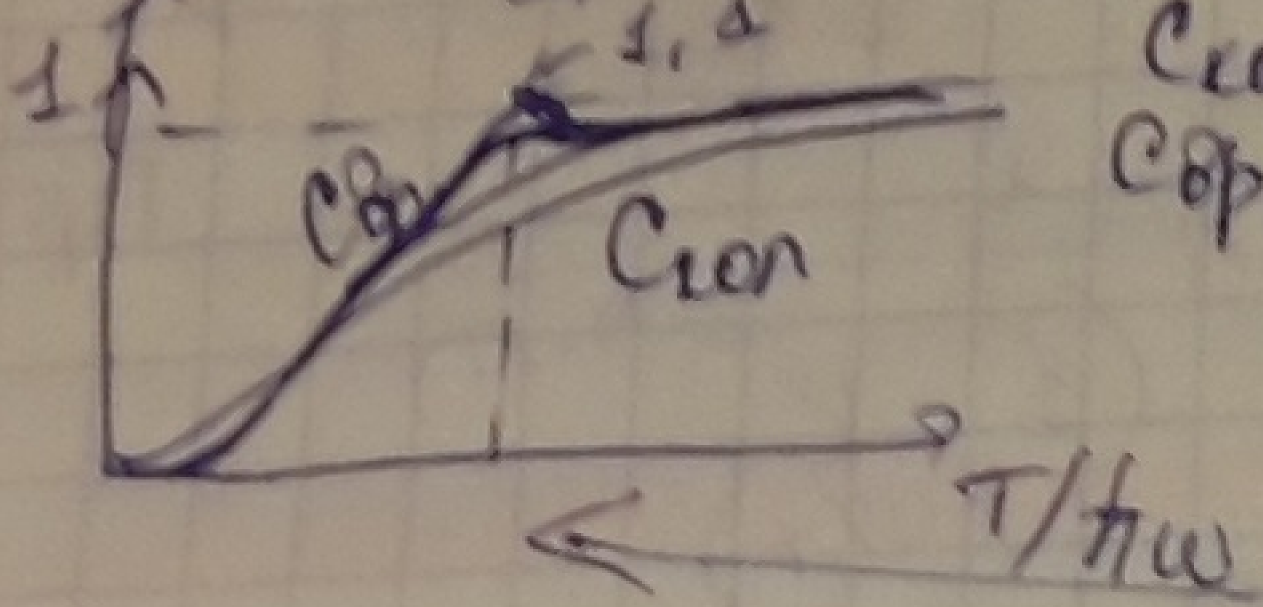
$$T \to \infty \quad C_V \sim N\left(\frac{\hbar\omega}{T}\right)^2 \cdot \frac{1}{\left(1 + \frac{\hbar\omega}{T} - 1\right)^2} \Rightarrow \frac{C_V}{N} \to 1$$

$$F_{кол} = N\frac{\hbar\omega}{2} + NT\ln\left(1 - e^{-\frac{\hbar\omega}{T}}\right)$$

$$F_{кол} \sim \frac{N\hbar\omega}{2} - NTe^{-\frac{\hbar\omega}{T}} \quad T \to 0$$

$$F_{кол} \simeq \frac{N\hbar\omega}{2} + NT\ln\left(1 - 1 + \frac{\hbar\omega}{T} - \frac{1}{2}\left(\frac{\hbar\omega}{T}\right)^2\right) =$$

$$= \frac{N\hbar\omega}{2} + NT\ln\frac{\hbar\omega}{T}\left(1 - \frac{\hbar\omega}{2T}\right) \simeq \underline{NT\ln\frac{\hbar\omega}{T}}$$



$C_{кол} \to 1$ снизу

$C_{вр} \to 1$ сверху

$S_{кол} = -\ln\hbar\omega$

$$\mathcal{E}_{вр} = \frac{\hbar^2}{2I}K(K+1)$$

$$Z_{вр} = \sum_{K=0}^{\infty}(2K+1)e^{-\frac{\hbar^2}{2IT}K(K+1)}$$

↑ точно не рассчит-ся. В крайние случ.

1) $\frac{\hbar^2}{IT} \gg 1$ (низк. темп.) $K=1$

$$Z_{вр} = 1 + 3e^{-\frac{\hbar^2}{IT}}$$

$$F_{вр} = -NT\ln\left(1 + 3e^{-\frac{\hbar^2}{IT}}\right) \simeq -3NTe^{-\frac{\hbar^2}{IT}}$$

$$S_{вр} = -\frac{\partial F}{\partial T} = 3Ne^{-\frac{\hbar^2}{IT}} + 3N\frac{\hbar^2}{IT}e^{-\frac{\hbar^2}{IT}}$$

$$E_{вр} = F_{вр} + TS_{вр} = 3N\frac{\hbar^2}{I}e^{-\frac{\hbar^2}{IT}}$$

$$C_{вр} = \frac{\partial E_{вр}}{\partial T} = 3N\left(\frac{\hbar^2}{IT}\right)e^{-\frac{\hbar^2}{IT}} \to 0$$

2) $\frac{\hbar^2}{IT} \ll 1$ (выс. темп.)

$$Z_{вр} = \int_0^\infty \frac{dk\,(2k+1)}{d(k+k^2)}\, e^{-\frac{\hbar^2}{2IT}k(k+1)} =$$

$$= \int_0^\infty dx \cdot e^{-\frac{\hbar^2}{2IT}x} = \frac{2IT}{\hbar^2}$$

Аналогично считаются $S_{вр}$, $C_{вр}$.

$$F_{вр} = -NT\ln\frac{2IT}{\hbar^2}.$$

$$\frac{C_V}{N} = 1$$

$$\frac{2IT}{\hbar^2} \sim 0{,}8$$

4. Многоатомные молекулы:

$C_{пост} = 3/2$ (на одну молекулу)

~~При вр. темп.~~

$$C = 3n - 5 + \frac{5}{2} = 3n - \frac{5}{2}$$

1) линейные: $n_{вр} = 2$ $\quad C_{вр} = 1$ $\quad$ всего: $3n-5$

2) нелинейные $n_{вр} = 3$ $\quad C_{вр} = 3/2$ $\quad$ всего: $3n-6$

$$C = 3n - 3$$

$$\mathcal{H} = \sum_{i=1}^{3N} \frac{1}{2m}\left(P_i - \frac{e}{c}A_i\right)^2 + U(r_1, r_2, \dots r_N)$$

$$\vec{A} = \frac{1}{2}[\vec{H} \times \vec{r}]$$

$$Z = \frac{1}{N!}\int_{p\in(-\infty;+\infty)} \frac{d^{3N}p\, d^{3N}\vec{r}}{(2\pi\hbar)^{3N}}\, e^{-\frac{\mathcal{H}}{T}(P_i - \frac{e}{c}A_i)} =$$

$$= \frac{1}{N!}\iint \frac{d^{3N}p\, d^{3N}\vec{r}}{(2\pi\hbar)^{3N}}\, e^{-\frac{\mathcal{H}(p,\vec{r})}{T}}$$

В маг. поле $dF = -SdT - PdV - MdH$

$\frac{\partial F}{\partial H} = M = 0$ $\chi = \frac{\partial M}{\partial H} = 0$

Отсутств. маг. в сист. с класс. мех и класс. статист.

$$\begin{cases} F = F_{поступ} + F_{маг} \\ S = -\left(\frac{\partial F}{\partial T}\right) = const \\ F_H(H) = -\frac{1}{2}\chi(T) H^2 \end{cases}$$

$$S = S_{пост}(T_0) + S_H(T_0) = S_{пост}(T_1)$$

02.16

Лекция №3.

N_k - число частиц в N квант. сост.

$n_k = \overline{N_k}$ $\quad \boxed{n_k << 1}$ - квант. эфф. не существенны

получаем ф.р.

$\Rightarrow$ для классич. газа

$n_k = a \cdot e^{-\varepsilon_k/T}$ $\leftarrow$ квазикл. сист.

$\varepsilon_k = \frac{p^2}{2m} + U(\vec{r}) \Rightarrow n_k \sim \exp\left(-\frac{\vec{p}^2}{2mT} - \frac{U(\vec{r})}{T}\right)$

распр. Максв.-Больцмана

$n \sim e^{-U/T}$

$U: n_{\vec{p}} = e^{-\frac{p^2}{2mT}} \sim e^{-\frac{mv^2}{2T}}$ - распр. Максвелла

$n_p = a e^{-\frac{\vec{p}^2}{2mT}}$

$\int n_p \, d\Gamma = N = a \int e^{-\frac{\vec{p}^2}{2mT}} \frac{g \, d^3\vec{p} \, d^3\vec{r}}{(2\pi\hbar)^3} =$

!!! $\boxed{g = 2s+1}$ $= \frac{g a V}{(2\pi\hbar)^3} \left\{ \int_{-\infty}^{+\infty} \exp\left(-\frac{p_x^2}{2mT}\right) dp_x \right\}^3 =$

$= a \frac{gV}{(2\pi\hbar)^3} \left(\sqrt{2\pi m T}\right)^3 \Rightarrow$

$\Rightarrow \boxed{a = \frac{(2\pi\hbar)^3}{(2\pi m T)^{3/2}} \cdot \frac{N}{gV}}$ $\leftarrow$ нормир. коэф.

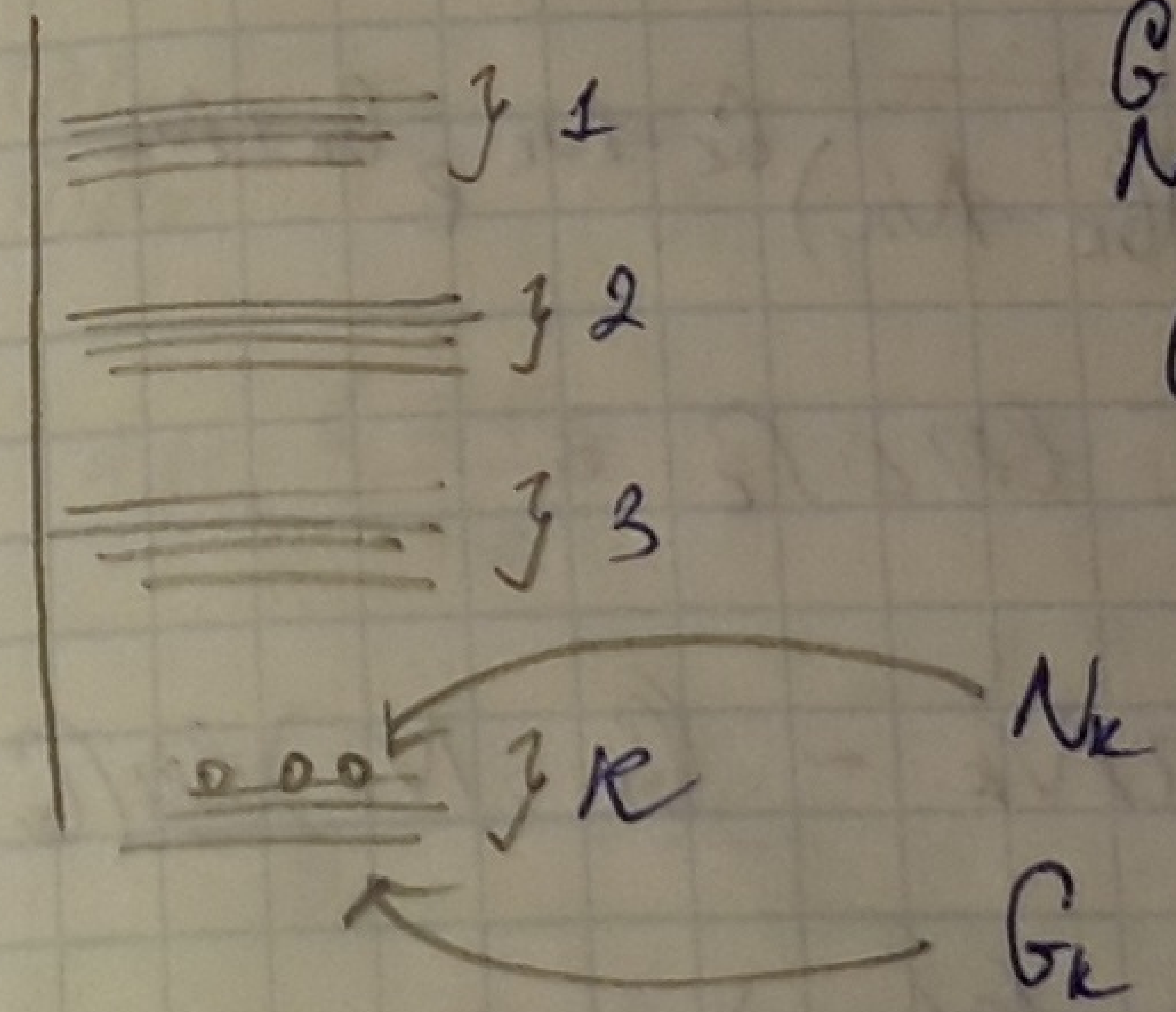
$$\ln a = -\ln\left[\frac{gV}{N}\left(\frac{mT}{2\pi\hbar^2}\right)^{3/2}\right] = \frac{\mu}{T}$$

$$\mu = -T\ln\left(\frac{gV}{N}\cdot\left(\frac{mT}{2\pi\hbar^2}\right)^{3/2}\right)$$

$$a = e^{\mu/T} \to n_k = a e^{-\varepsilon_k/T} = e^{\frac{\mu-\varepsilon_k}{T}}$$ Максв.-Больц.

$$n_k \ll 1 \Rightarrow e^{\mu/T} \ll 1 \Rightarrow |\mu| \gg T \quad \mu < 0$$

∃ в ид. газе мн-ся сост. Разобьем их на группы



G_k – число ур. на k гр.
N_k – число част. на k ур.

$G_k \gg 1 \quad N_k \gg 1$

0 0/0 0 0 0/0 : 0 0

$$\Delta\Gamma = \prod_{k=1}\Delta\Gamma_k$$

$$S = \ln\Delta\Gamma = \ln\prod_{k=1}\Delta\Gamma_k = \sum_k \ln\Delta\Gamma_k \;(=)$$

$$\Delta\Gamma_k = C_{N_k+G_k-1}^{G_k-1} = \frac{(N_k+G_k-1)!}{(G_k-1)!\,N_k!}$$ для бозонов

$$N! \sim \left(\frac{N}{e}\right)^N \quad N \gg 1$$

$$\ln\Delta\Gamma_k = \ln\frac{(N_k+G_k)^{N_k+G_k}\cdot\cancel{e^{-N_k-G_k}}}{G_k^{G_k}\cdot\cancel{e^{-G_k}}\,N_k^{N_k}\,\cancel{e^{-N_k}}} \Rightarrow$$

$$(=)\sum_k\{(N_k+G_k)\ln(N_k+G_k) - N_k\ln N_k - G_k\ln G_k\} =$$

$$\left|n_k = \frac{N_k}{G_k}\right| = \sum_k G_k\{(1+n_k)\ln G_k(1+n_k) -$$

$- n_k \ln(g_k n_k) - \ln g_k \} =$

$$\boxed{S = \sum_k g_k \{(1+n_k)\ln(1+n_k) - n_k \ln n_k\}}$$

↑ для бозонов

У фермионов на одном ур. о-ии 1 частица!

$$\Delta\Gamma_k = C_{g_k}^{N_k} = \frac{g_k!}{N_k!(g_k-N_k)!}$$

$$\ln \Delta\Gamma_k = \ln \frac{g_k^{g_k} e^{-g_k}}{N_k^{N_k} e^{-N_k} (g_k-N_k)^{g_k-N_k} e^{-g_k+N_k}}$$

$$S = \ln\Delta\Gamma = \ln \prod_k \Delta\Gamma_k = \sum_k \ln \Delta\Gamma_k =$$

$$= \sum_k \{ g_k \ln g_k - N_k \ln N_k - (g_k - N_k)\ln(g_k - N_k)\} =$$

$$= \sum_k g_k \{ 1\cdot \ln \cancel{g_k} - n_k \ln(\cancel{g_k} n_k) - (1-n_k) \cdot$$

$$\cdot \ln(\cancel{g_k}(1-n_k))\} \Rightarrow$$

$$\boxed{S = \sum_k g_k \{-(1-n_k)\ln(1-n_k) - n_k \ln n_k\}}$$

↑ для фермионов

Итак,

$$\boxed{S = \sum_k g_k \{\mp(1\mp n_k)\ln(1\mp n_k) - n_k \ln n_k\}}$$

"+" - бозон

"-" - фермион

$n_k = \frac{N_k}{G_k}$ $\quad N = \sum_k N_k \equiv \sum n_k G_k$

$E = \sum_k \varepsilon_k N_k = \sum \varepsilon_k n_k G_k$

$L = S + \alpha_1 E + \alpha_2 N =$ – ищем экстр. энтр. для при $E = const$ и $N = const$

$= \sum_k G_k \{ \mp (1 \mp n_k) \ln (1 \mp n_k) - n_k \ln n_k + \alpha_1 \varepsilon_k n_k + \alpha_2 n_k \}$

$\frac{\partial L}{\partial n_k} = 0 = \cancel{1} + \ln(1 \mp n_k) - \cancel{1} - \ln n_k + \alpha_1 \varepsilon_k + \alpha_2$

$\ln \frac{1 \mp n_k}{n_k} = -\alpha_1 \varepsilon_k - \alpha_2$

$\frac{1 \mp n_k}{n_k} = e^{-\alpha_1 \varepsilon_k - \alpha_2}$ ф-р Бозе-Дир/ Ферми-Эйнш.

$$\boxed{n_k = \frac{1}{e^{-\alpha_1 \varepsilon_k - \alpha_2} \pm 1} = \frac{1}{e^{(\varepsilon_k - \mu)/T} \pm 1}}$$ ферми, бозе

$dL = dS + \alpha_1 dE + \alpha_2 dN = 0$

$dE = -\frac{dS}{\alpha_1} - \frac{\alpha_2}{\alpha_1} dN \equiv T dS + \mu dN$

$\alpha_1 = -\frac{1}{T}$ $\quad \alpha_2 = \frac{\mu}{T}$

$e^{\frac{\varepsilon_k - \mu}{T}} \gg 1 \to n_k \approx e^{\frac{\mu - \varepsilon_k}{T}}$

При $\varepsilon_0 = 0$ $\quad e^{-\mu/T} \gg 1 \to \boxed{e^{\mu/T} \ll 1} \Leftrightarrow T \gg T_{\text{вырожд.}}$

Знаем, что $E \equiv E(S, V, N) = N f_\varepsilon\left(\frac{S}{N}, \frac{V}{N}\right)$

S и V – аддитивные величины

$$F(T,V,N) = N f_F\left(T, \frac{V}{N}\right)$$

$$W(S,P,N) = N f_W\left(\frac{S}{N}, P\right)$$

$$\Phi(T,P,N) = N f_\Phi(T,P)$$

$$\mu = \left(\frac{\partial E}{\partial N}\right)_{S,V} = \dots = \left(\frac{\partial \Phi}{\partial N}\right)_{T,P}$$

$$\mu = \left(\frac{\partial \Phi}{\partial N}\right)_{T,P} = f_\Phi(T,P) = \frac{\Phi}{N}$$

$$\Phi = \mu N$$

$$d\Phi = -SdT + VdP + \cancel{\mu dN} = d(\mu N) =$$

$$= \cancel{\mu dN} + N d\mu$$

$$d\mu = -\left(\frac{S}{N}\right)dT + \left(\frac{V}{N}\right)dP$$

$$\boxed{\mu(T,P)}$$

$$dE = TdS - PdV = d(TS) - SdT - PdV$$

$$dF = d(E - TS) = -SdT - PdV$$

$$dF = -SdT - PdV + \underbrace{\mu dN + N d\mu}_{d(N\cdot\mu)} - N d\mu$$

$$d(F - \mu N) = -SdT - PdV - N d\mu$$

$$\Omega(T,V,\mu)$$

$$d\Omega = -SdT - PdV - N d\mu$$

$$\Omega = F - \mu N = F - \Phi = F - (F + PV) = -PV$$

$$\Omega = -PV$$

$$\frac{\partial \Omega}{\partial V} = f_V = -P$$

$$\Omega(T, V, \mu) = V f_\Omega(T, \mu) = -PV$$

$$\hat{\rho} = \frac{1}{Q} \exp\left(\frac{\mu \hat{N} - \hat{H}}{T}\right) = \exp\left(\frac{\Omega + \mu \hat{N} - \hat{H}}{T}\right)$$

$$\rho_{n,N}(T, \lambda) = \exp\left(\frac{F(T, N, \lambda) - E_{nN}(\lambda)}{T}\right)$$

$$F = \Omega + \mu N$$

$$\rho_{nN}(T, \lambda) = \exp\left(\frac{\Omega(T, \mu, N) + \mu N - E_{nN}(\lambda)}{T}\right) = \frac{1}{Q} e^{\frac{\mu N - E_{nN}}{T}}$$

Будем ~~из~~ систему с переменным числом частиц

$$\sum_{N=0}^{\infty} \sum_n \rho_{nN} = \sum_{N=0}^{\infty} \sum_n e^{\frac{\Omega + \mu N - E_{nN}}{T}} = 1$$

$$e^{\Omega/T} \sum_{N=0}^{\infty} \sum_n e^{(\mu N - E_{nN})/T} = 1$$

$$\boxed{\Omega = -T \ln Q} = -T \ln \sum_{N=0}^{\infty} \sum_n e^{\frac{\mu N - E_{nN}}{T}}$$

большая статсумма

$$\Omega = -T \ln \sum_{N=0}^{\infty} \sum_n e^{\frac{\mu N - E_{nN}}{T}} = -T \ln \sum_{N_k} e^{\frac{\mu \sum_k N_k - \sum_k \varepsilon_k N_k}{T}} =$$

$$= -T \ln \sum_{N_k} e^{\sum_k \frac{N_k (\mu - \varepsilon_k)}{T}} = -T \ln \sum_{N_k} \prod_k e^{\frac{N_k(\mu - \varepsilon_k)}{T}} =$$

$$= -T \ln \prod_k \sum_{N_k} e^{\frac{N_k(\mu - \varepsilon_k)}{T}} = -T \sum_k \ln \sum_{N_k} e^{\frac{N_k(\mu - \varepsilon_k)}{T}}$$

для фермионов:

$N_k = 0, 1$

$$\Omega = -T\sum_k \ln\left(1 + e^{\frac{\mu - \varepsilon_k}{T}}\right)$$

для бозонов:

$$\Omega = -T\sum_k \ln \sum_{N_k=0}^{\infty} e^{\frac{N_k(\mu-\varepsilon_k)}{T}} =$$

$$= T\sum_k \ln\left(1 - e^{\frac{\mu-\varepsilon_k}{T}}\right)^{-1} \qquad e^{\frac{\mu}{T}} < 1$$

$\mu < \varepsilon_0$

$\mu < 0$

$$\boxed{\Omega = \mp T\sum_k \ln\left(1 \pm e^{\frac{\mu-\varepsilon_k}{T}}\right)}$$

ферми, бозе

$$\bar{N} \equiv N = -\frac{\partial\Omega}{\partial\mu} = \sum_k \frac{e^{\frac{\mu-\varepsilon_k}{T}}}{1 \pm e^{\frac{\mu-\varepsilon_k}{T}}} = \sum_k \frac{1}{e^{\frac{\varepsilon_k-\mu}{T}} \pm 1} =$$

$$= \overline{\sum_k N_k} = \sum_k \bar{N}_k \equiv \sum_k n_k$$

$dE \leq TdS - PdV$

$E = const \quad V = const \quad TdS \geq 0$

$T = const \quad V = const$

$d(E - TS) \equiv dF \leq 0$

$T = const \quad P = const$

$d(E - TS + PV) \leq 0$

$F + PV = \Phi \qquad d\Phi \leq 0$

$$A \to A^+ + e$$

$$d\Phi = -SdT + VdP + \sum_i \mu_i dN_i$$

$T = const \quad P = const$

$$\boxed{d\Phi = \sum_i \mu_i dN_i = 0}$$

$dN_i \sim \nu_i$

$$\sum \nu_i A_i = 0$$

$$A - A^+ - e^- = 0$$

$1 \quad -1 \quad -1$

$$\sum \nu_i \mu_i = 0$$

$$\mu = -T \ln \left\{ \frac{V \cancel{g}}{N} \left(\frac{m_i T}{2\pi\hbar^2} \right)^{3/2} g_i e^{-\frac{\varepsilon_{0i}}{T}} \right\}$$

$$\frac{V}{N_i} = \frac{V}{N} \cdot \frac{N}{N_i} = \frac{T}{P x_i} \qquad x_i = \frac{N_i}{N}$$

$$\mu_i = -T \ln \left\{ \frac{T}{P x_i} \left(\frac{m_i T}{2\pi\hbar^2} \right)^{3/2} g_i e^{-\varepsilon_{0i}/T} \right\} =$$

$$= T \ln (P x_i) - \frac{5}{2} T \ln T + \varepsilon_{0i} - T \ln g_i \left(\frac{m_i}{2\pi\hbar^2} \right)^{3/2} =$$

$$= T \ln P_i + \chi_i(T)$$

$$T \ln (P x_0) + \chi_0(T) - T \ln (P x_1) - \chi_1(T) -$$

$$- T \ln (P x_e) + \chi_e(T) = 0$$

$$\ln \frac{x_0}{p x_1 x_e} = \frac{\chi_e \cancel{+} \chi_0 + \chi_1}{T}$$

$$\frac{x_0}{x_1 x_e} = p \exp \left(\frac{\chi_e + \chi_1 - \chi_0}{T} \right)$$

$$\chi_e + \chi_1 - \chi_0 = -\frac{5}{2} T \ln T - \frac{\overbrace{\varepsilon_{01} - \varepsilon_{00}}}{I} -$$

потенц. ионизации

$$- T \ln \frac{g_0}{g_e g_1} \left(\frac{m_e}{2\pi \hbar^2} \right)^{3/2}$$

Формула Саха:

$$\boxed{\frac{x_0}{x_1 x_e} = \frac{P}{T} \cdot \frac{g_0}{2 g_1} \left(\frac{2\pi \hbar^2}{m_e T} \right)^{3/2} e^{I/T}}$$

$g_e = 2$

$x_1 = x_e$

$x_0 + x_1 + x_e = 1$

4.03.16

Лекция № 4.

$$C_\lambda = \left(\frac{\partial E}{\partial T}\right)_\lambda = \frac{\partial}{\partial T}\left(\frac{\sum_n E_n e^{-E_n/T}}{\sum_n e^{-E_n/T}}\right) =$$

$$= \frac{1}{T^2}\,\frac{\sum_n E_n^2 e^{-E_n/T}}{\sum_n e^{-E_n/T}} - \frac{1}{T^2}\,\frac{\left(\sum_n E_n e^{-E_n/T}\right)^2}{\left(\sum_n e^{-E_n/T}\right)^2} =$$

$$= \frac{1}{T^2}\left(\langle E_n^2\rangle - \langle E_n\rangle^2\right) = \frac{1}{T^2}\left(\langle (E_n - \langle E_n\rangle)^2\rangle\right)$$

$$\langle E_n^2\rangle - 2\langle E_n\rangle^2 + \langle E_n\rangle^2 = \langle E_n^2\rangle - \langle E_n\rangle^2$$

$$\boxed{\langle \Delta E^2\rangle = T^2 C_\lambda > 0}$$ флукт. эн-ии в канон. ансамбле

$C_\lambda > 0$

$\Delta E \to E_n - \langle E_n\rangle$

$\lambda = V$ $\quad C_V > 0$ Первое термодин. нерав-во

$N \to \infty$

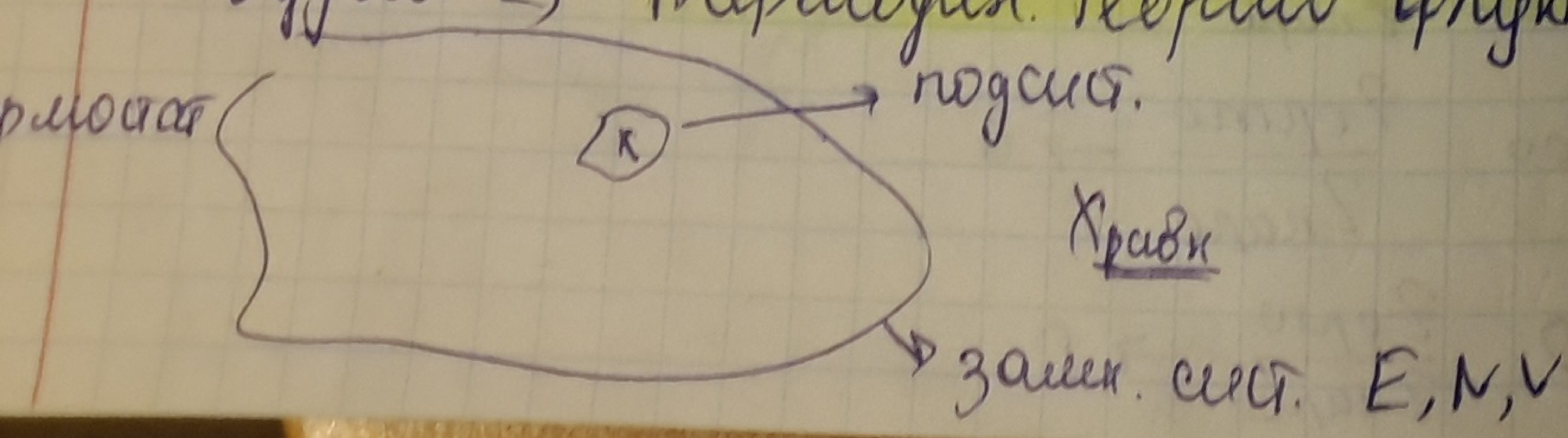
$$\langle \Delta E^2\rangle \simeq T^2 C \sim T^2 N$$

$$\langle E\rangle \sim TN$$

$$\frac{\sqrt{\langle \Delta E^2\rangle}}{\langle E\rangle} \sim \frac{T\sqrt{N}}{TN} \sim \frac{1}{\sqrt{N}}$$

$$\boxed{N \to \infty \quad V \to \infty \quad \frac{N}{V} = \text{const}}$$ термодин. предел

Будем Δя термодин. теорию флук-ий



$w(x) \sim \Delta\Gamma(x) \sim e^{S(x)} \sim e^{S(x) - S(x_{равн})} =$

$= e^{\Delta S_{полн}(x)} = e^{\Delta S_{терм} + \Delta S(x)}$ ⊜

$\Delta S_{полн} = S_{полн}(x_{равн}) - \frac{\beta}{2}(x - x_{равн})^2$

$w(x) \sim \exp\left(-\frac{\beta}{2}(x - x_{равн})^2\right)$

$\Delta E = T\Delta S - P\Delta V + \mu\Delta N$

$\Delta S_{терм} = \frac{\Delta E_{терм}}{T} + \frac{P}{T}\Delta V_{терм} - \frac{\mu}{T}\Delta N_{терм}.$

т. к. сист. замкн. $\to \Delta E_{терм} + \Delta E = 0$

$\Delta V_{терм} + \Delta V = 0$ $\quad$ $\Delta N_{терм} + \Delta N = 0$ $\quad$ отн-ся к подсист.

⊜ $\exp\left(\Delta S - \frac{\Delta E}{T} - \frac{P}{T}\Delta V + \frac{\mu}{T}\Delta N\right)$

Вероятность флуктуации

$$\boxed{w \sim \exp\left\{\frac{T\Delta S - \Delta E - P\Delta V + \mu\Delta N}{T}\right\}}$$

⊜

$E = E(S, V, N)$

$\Delta E = \underbrace{\left(\frac{\partial E}{\partial S}\right)_{V,N}}_{=T}\Delta S + \underbrace{\left(\frac{\partial E}{\partial V}\right)_{S,N}}_{=-P}\Delta V + \underbrace{\left(\frac{\partial E}{\partial N}\right)_{S,V}}_{=\mu}\Delta N \oplus$ // иначе = 0

$\oplus \left\{2\frac{\partial^2 E}{\partial S\partial V}\Delta S\Delta V + 2\frac{\partial^2 E}{\partial S\partial N}\Delta S\Delta N + 2\frac{\partial^2 E}{\partial V\partial N}\Delta V\Delta N + \right.$

$\left. + \frac{\partial^2 E}{\partial S^2}(\Delta S)^2 + \frac{\partial^2 E}{\partial V^2}(\Delta V)^2 + \frac{\partial^2 E}{\partial N^2}(\Delta N)^2\right\}\cdot\frac{1}{2}$

$w \sim \exp\left\{-\frac{\text{кв. форма } 2\text{-го пор.}}{2T}\right\}$

$T = \left(\frac{\partial E}{\partial S}\right)_{V,N}$ $\qquad$ $\Delta T = \frac{\partial^2 E}{\partial S^2}\Delta S + \frac{\partial^2 E}{\partial S\partial V}\Delta S\Delta V + \frac{\partial^2 E}{\partial S\partial N}\Delta N$

$P = -\left(\frac{\partial E}{\partial V}\right)_{S,N}$ $\Delta P = -\frac{\partial^2 E}{\partial V^2}\Delta V - \frac{\partial^2 E}{\partial V \partial S}\Delta S - \frac{\partial^2 E}{\partial V \partial N}\Delta N$

$\mu = \left(\frac{\partial E}{\partial N}\right)_{S,V}$ $\Delta \mu = \frac{\partial^2 E}{\partial N^2}\Delta N + \frac{\partial^2 E}{\partial N \partial S}\Delta S + \frac{\partial^2 E}{\partial N \partial V}\Delta V$

$\Delta E = T\Delta S - P\Delta V + \mu \Delta N$

$$\Delta T \cdot \Delta S - \Delta P \Delta V + \Delta \mu \Delta N = \frac{\partial^2 E}{\partial S^2}\Delta S^2 + 2\frac{\partial^2 E}{\partial S \partial V}\Delta V \Delta S + 2\frac{\partial^2 E}{\partial S \partial N}\Delta N \Delta S + \frac{\partial^2 E}{\partial V^2}(\Delta V)^2 + 2\frac{\partial^2 E}{\partial V \partial N}\Delta V \Delta N + \frac{\partial^2 E}{\partial N^2}(\Delta N)^2$$

$$\boxed{w \sim \exp\left\{\frac{\Delta P \Delta V - \Delta T \Delta S - \Delta \mu \Delta N}{2T}\right\}}$$ ф-ла Эйнштейна

$n = \frac{N}{V}$ 1) $\Delta N = 0$
2) $\Delta V = 0$

1) $\Delta N = 0$

$$w \sim \exp\left(\frac{\Delta P \Delta V - \Delta T \Delta S}{2T}\right) \; ⊜$$

T, V - независ. переменные
P и S выразим через них

$$\Delta P = \left(\frac{\partial P}{\partial T}\right)_V \Delta T + \left(\frac{\partial P}{\partial V}\right)_T \Delta V$$

$$\Delta S = \left(\frac{\partial S}{\partial T}\right)_V \Delta T + \left(\frac{\partial S}{\partial V}\right)_T \Delta V$$

$$⊜ \exp\left\{\frac{1}{2T}\left(\left(\frac{\partial P}{\partial V}\right)_T (\Delta V)^2 + \left(\frac{\partial P}{\partial T}\right)_V \Delta T \Delta V - \left(\frac{\partial S}{\partial T}\right)_V (\Delta T)^2 - \left(\frac{\partial S}{\partial V}\right)_T \Delta V \Delta T\right)\right\} = \Big/ \left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T = -\frac{\partial^2 F}{\partial T \partial V} \Big/ =$$

$$= \exp\left\{ \frac{1}{2T}\left(\left(\frac{\partial P}{\partial V}\right)_T (\Delta V)^2 - \left(\frac{\partial S}{\partial T}\right)_V (\Delta T)^2 \right) \right\} \sim$$

$$\sim \exp \frac{1}{2T}\left(\frac{\partial P}{\partial V}\right)_T (\Delta V)^2 \cdot \exp \frac{1}{2T}\left(-\left(\frac{\partial S}{\partial T}\right)_V\right)(\Delta T)^2$$

|| Флукт. объема и темп. статист. независ.

$w_{гаусс} \sim e^{-x^2/2D}$ $\quad D = \langle x^2 \rangle$

$$\langle \Delta T^2 \rangle = \frac{T}{\left(\frac{\partial S}{\partial T}\right)_V} = \frac{T^2}{\underbrace{T\left(\frac{\partial S}{\partial T}\right)_V}_{C_V}} = \frac{T^2}{C_V} > 0 \to C_V > 0$$

второе термодин. нерав-во

$$\langle \Delta V^2 \rangle = -\frac{T}{\left(\frac{\partial P}{\partial V}\right)_T} = -T\cdot\left(\frac{\partial V}{\partial P}\right)_T > 0 \to \left(\frac{\partial V}{\partial P}\right)_T < 0$$

$\exists$ другая пара пер-х: P, S.

Получим: $\langle \Delta P \Delta S \rangle = 0$

2) $\Delta V = 0$

$$w \sim \exp\left\{ \frac{-\Delta T \Delta S - \Delta\mu \Delta N}{2T} \right\}$$

$E(T, V) \qquad \Delta E = \left(\frac{\partial E}{\partial T}\right)\Delta T + \left(\frac{\partial E}{\partial V}\right)\Delta V$

$$\langle \Delta E^2 \rangle = \underbrace{\left(\frac{\partial E}{\partial T}\right)_V^2 \langle \Delta T^2 \rangle}_{C_V^2 \frac{T^2}{C_V} = C_V T^2} + \left(\frac{\partial E}{\partial V}\right)_T^2 \langle \Delta V^2 \rangle + \cancel{2\frac{\partial E}{\partial T}\cdot\frac{\partial E}{\partial V}\langle \Delta T \Delta V \rangle}^{=0}$$

$$\Delta S = \left(\frac{\partial S}{\partial T}\right)_N \Delta T + \left(\frac{\partial S}{\partial N}\right)_T \Delta N$$

$$\Delta \mu = \left(\frac{\partial \mu}{\partial T}\right)_N \Delta T + \left(\frac{\partial \mu}{\partial N}\right)_T \Delta N$$

$$w \sim \exp\left\{\frac{1}{2T}\left(\left(\frac{\partial S}{\partial T}\right)_N (\Delta T)^2 - \cancel{\left(\frac{\partial S}{\partial N}\right)_T \Delta N \Delta T} - \right.\right.$$

$$\left.\left. - \cancel{\left(\frac{\partial \mu}{\partial T}\right)_N \Delta T \Delta N} - \left(\frac{\partial \mu}{\partial N}\right)_T (\Delta N)^2\right)\right\} =$$

$$= \Big/ \left(\frac{\partial S}{\partial N}\right)_T = -\left(\frac{\partial \mu}{\partial T}\right)_N = -\frac{\partial^2 F}{\partial T \partial N} \Big/ =$$

$$dF = -S dT + \mu dN$$

$$= \exp\left\{\frac{1}{2T}\left(\left(\frac{\partial S}{\partial T}\right)_N (\Delta T)^2 - \left(\frac{\partial \mu}{\partial N}\right)_T (\Delta N)^2\right)\right\}$$

$$\langle \Delta T \Delta N \rangle = 0$$

$$\langle \Delta T^2 \rangle = \frac{T}{\left(\frac{\partial S}{\partial T}\right)_{N,V}} \equiv \frac{T^2}{C_V}$$

$$\langle \delta N^2 \rangle = \frac{T}{\left(\frac{\partial \mu}{\partial N}\right)_T} = T\left(\frac{\partial N}{\partial \mu}\right)_T$$

$$\overline{N} = \frac{\sum_N \sum_n N \exp\left(\frac{\mu N - E_{nN}}{T}\right)}{\sum_N \sum_n \exp\left(\frac{\mu N - E_{nN}}{T}\right)}$$

$$\frac{\partial \overline{N}}{\partial \mu} = \frac{1}{T}\left(\frac{\sum_{n,N} N^2 \exp\left(\frac{\mu N - E_{nN}}{T}\right)}{\sum_{n,N} \exp\left(\frac{\mu N - E_{nN}}{T}\right)} - \frac{\left(\sum_{n,N} N e^{\frac{\mu N - E_{nN}}{T}}\right)^2}{\left(\sum_{n,N} \exp\left(\frac{\mu N - E_{nN}}{T}\right)\right)^2}\right)$$

$$\Rightarrow \langle N^2 \rangle - \langle N \rangle^2 = \langle \Delta N^2 \rangle$$

Квантовые идеальные газы.

$$N = \sum_k n_k = \sum_k \frac{1}{e^{\frac{\varepsilon-\mu}{T}} \pm 1}$$ ферми / бозе

$$E = \sum_k \varepsilon_k n_k = \sum_k \frac{\varepsilon_k}{\exp\left(\frac{\varepsilon-\mu}{T}\right) \pm 1}$$

$$\Omega = \mp T \sum_k \ln\left(1 \pm e^{\frac{\mu-\varepsilon_k}{T}}\right)$$

$g = 2s+1 \quad \varepsilon = \frac{p^2}{2m} \quad \underline{D = 3} \quad p = \sqrt{2m\varepsilon}$

$$\sum_k \to g\int \frac{d^3r\, d^3\vec{p}}{(2\pi\hbar)^3} = \frac{gV}{(2\pi\hbar)^3}\int_0^\infty 4\pi p^2 dp =$$

$$= \frac{gV 2\pi (2m)^{3/2}}{(2\pi\hbar)^3}\int_0^\infty \sqrt{\varepsilon}\, d\varepsilon$$

$$N = \frac{gVm^{3/2}}{\sqrt{2}\pi^2\hbar^3}\int_0^\infty \frac{\varepsilon^{1/2} d\varepsilon}{\exp\left(\frac{\varepsilon-\mu}{T}\right) \pm 1}$$

$$E = \frac{gVm^{3/2}}{\sqrt{2}\pi^2\hbar^3}\int_0^\infty \frac{\varepsilon^{3/2} d\varepsilon}{\exp\left(\frac{\varepsilon-\mu}{T}\right) \pm 1}$$

$$\Omega = \mp \frac{gVm^{3/2}}{\sqrt{2}\pi^2\hbar^3} T\int_0^\infty \ln\left(1 \pm e^{\frac{\mu-\varepsilon}{T}}\right) \underbrace{\sqrt{\varepsilon}\, d\varepsilon}_{\frac{2}{3}d(\varepsilon^{3/2})} =$$

$$= -\frac{2}{3}\cdot\frac{gVm^{3/2}}{\sqrt{2}\pi^2\hbar^3}\int_0^\infty \frac{\varepsilon^{3/2} e^{\frac{\mu-\varepsilon}{T}}}{1 \pm e^{\frac{\mu-\varepsilon}{T}}}\, d\varepsilon =$$

$$= -\frac{2}{3}\frac{gVm^{3/2}}{\sqrt{2}\pi^2\hbar^3}\int_0^\infty \frac{\varepsilon^{3/2} d\varepsilon}{e^{\frac{\varepsilon-\mu}{T}} \pm 1} = -PV = -\frac{2}{3}E$$

$$P = \frac{2}{3}\cdot\frac{gm^{3/2}}{\sqrt{2}\pi^2\hbar^3}\int_0^\infty \frac{\varepsilon^{3/2} d\varepsilon}{e^{\frac{\varepsilon-\mu}{T}} \pm 1}$$

$$E = \frac{3}{2} PV \qquad P = \frac{2}{3}\frac{E}{V}$$

$$\frac{N}{V} = n(T,\mu) \Rightarrow \mu = \mu(T,n) \rightarrow$$

$$\rightarrow P(T,\mu) \rightarrow P(T,n)$$

$$e^{-\frac{\mu}{T}} \gg 1 \qquad e^{\frac{\mu}{T}} \ll 1$$

$$n = \frac{g m^{3/2} T^{3/2}}{\sqrt{2}\,\pi^2 \hbar^3} \int_0^\infty \frac{z^{1/2}\,dz}{e^{z-\frac{\mu}{T}} \pm 1} = \left| z = \frac{\varepsilon}{T} \right| =$$

$$= \frac{g m^{3/2} T^{3/2}}{\sqrt{2}\,\pi^2 \hbar^3} \int_0^\infty \frac{z^{1/2} e^{\frac{\mu}{T}-z}\,dz}{1 \pm \exp\left(\frac{\mu}{T}-z\right)}$$

$$n = \frac{g m^{3/2} T^{3/2}}{\sqrt{2}\,\pi^2 \hbar^3} \int_0^\infty z^{1/2} e^{\frac{\mu}{T}-z}\,dz \left(1 \mp e^{\frac{\mu}{T}-z} \ldots\right)$$

$$n = \frac{g m^{3/2} T^{3/2}}{\sqrt{2}\,\pi^2 \hbar^3} e^{\frac{\mu}{T}} \underbrace{\int_0^\infty z^{1/2} e^{-z}\,dz}_{\Gamma\left(\frac{3}{2}\right) = \sqrt{\pi}/2}$$

$$P = \frac{2}{3} \cdot \frac{g m^{3/2} T^{3/2}}{\sqrt{2}\,\pi^2 \hbar^3} e^{\frac{\mu}{T}} \underbrace{\int_0^\infty z^{3/2} e^{-z}\,dz}_{\Gamma\left(\frac{5}{2}\right) = \frac{3}{2}\Gamma\left(\frac{3}{2}\right)}$$

$$n = \boxed{\frac{g (mT)^{3/2}}{\sqrt{2}\,\pi^2 \hbar^3} \cdot \frac{\sqrt{\pi}}{2}} e^{\mu/T} = A e^{\mu/T}$$

$$P = \frac{g (mT)^{3/2} T}{\sqrt{2}\,\pi^2 \hbar^3} \cdot \frac{\sqrt{\pi}}{2} e^{\mu/T} = A T e^{\mu/T}$$

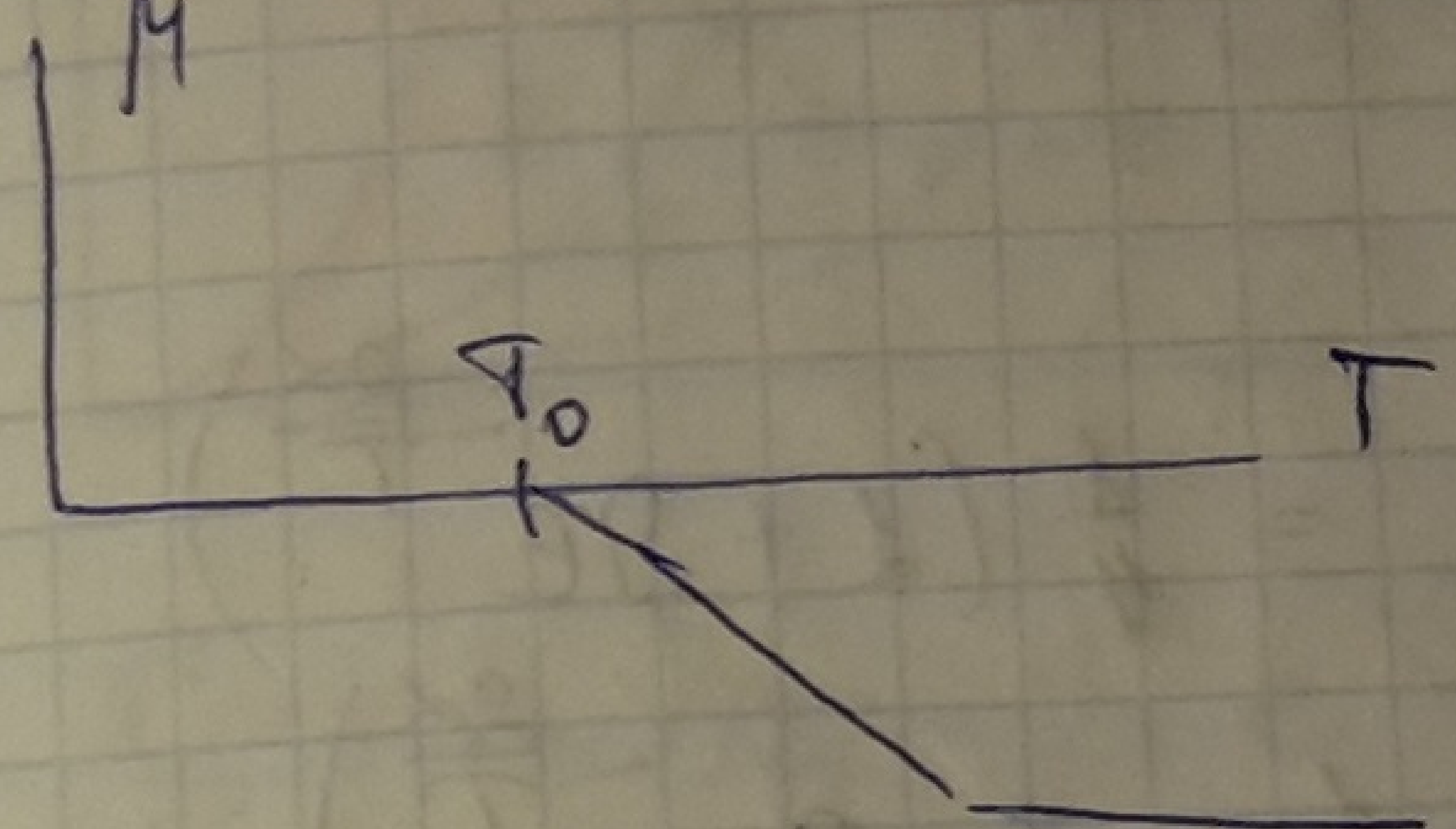
$$e^{\frac{\mu}{T}} = \frac{n}{A} \qquad \mu = T \ln \frac{n}{A}$$

$$A = \frac{g(mT)^{3/2}}{\sqrt{2}\,\pi^2 \hbar^3}$$

$$P = ATe^{\mu/T} = A \cdot T \cdot \frac{n}{A} = nT = \frac{N}{V}T$$

$$\boxed{PV = NT}$$

$$\mu \simeq -T\ln\left(\frac{T}{T_{вр}}\right)^{3/2} \simeq -\frac{3}{2}T\ln T$$



Будем ~~до~~ бозе-газ!

$$n = \frac{gm^{3/2}}{\sqrt{2}\,\pi^2\hbar^3}\int_0^\infty \frac{\varepsilon^{1/2}\,d\varepsilon}{e^{\frac{\varepsilon-\mu}{T}}-1}$$

$\mu = 0$ T_0

$$n = \frac{gm^{3/2}}{\sqrt{2}\,\pi^2\hbar^3}\int_0^\infty \frac{\varepsilon^{1/2}\,d\varepsilon}{e^{\varepsilon/T_0}-1}$$

$$n = \frac{gm^{3/2}T_0^{3/2}}{\sqrt{2}\,\pi^2\hbar^3}\int_0^\infty \frac{z^{1/2}\,dz}{e^z-1} =$$ /"В Ландау посмотрите, там в принципе не так сложно..."/ =

$$= \Gamma\left(\frac{3}{2}\right)\zeta\left(\frac{3}{2}\right)$$

$$\int_0^\infty \frac{z^{x-1}\,dz}{e^z-1} = \Gamma(x)\zeta(x) \qquad \zeta(x) = \sum_{n=1}^\infty \frac{1}{n^x}$$

$$T_0 = \left[\frac{\sqrt{2}\,\pi^2\hbar^3 n}{gm^{3/2}\Gamma\left(\frac{3}{2}\right)\zeta\left(\frac{3}{2}\right)}\right]^{2/3} \sim 3{,}31\,\frac{\hbar^2 n^{2/3}}{g^{2/3}m}$$

Что будет при дальнейшем понижении T?

11.03.16

$$N = \sum_k n_k = \sum_k \frac{1}{e^{\frac{\varepsilon_k-\mu}{T}}-1}$$

$$D=3 \qquad \varepsilon = \frac{p^2}{2m}$$

$$n = \frac{N}{V} = \frac{g m^{3/2}}{\sqrt{2}\pi^2\hbar^3}\int_0^\infty \frac{\varepsilon^{1/2}\,d\varepsilon}{e^{\frac{\varepsilon-\mu}{T}}-1}$$

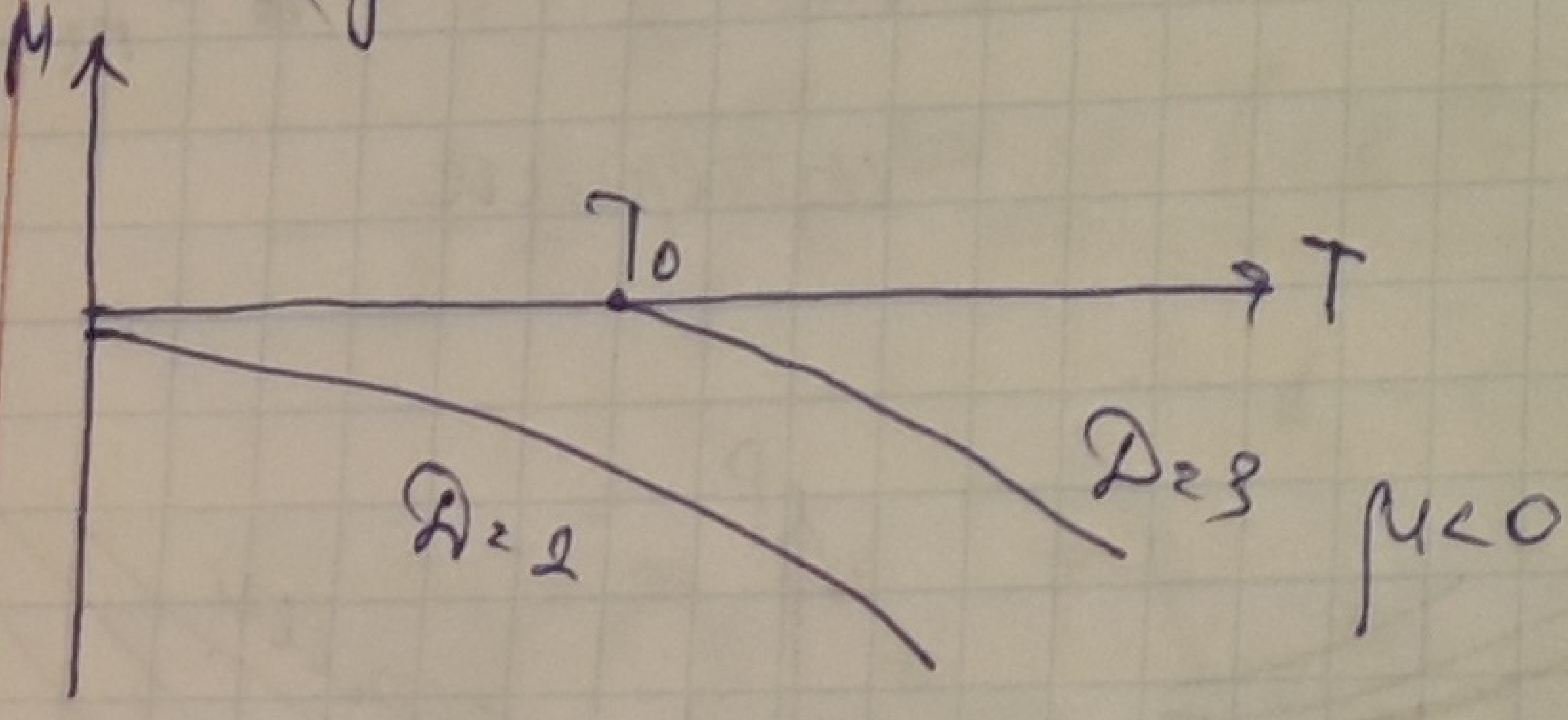
$$P = \frac{2}{3}\frac{g m^{3/2}}{\sqrt{2}\pi^2\hbar^3}\int_0^\infty \frac{\varepsilon^{1/2}\,d\varepsilon}{e^{\frac{\varepsilon-\mu}{T}}-1} \cdot \frac{2}{3}\cdot\frac{E}{V}$$

$\mu=0$
Г-кция

$$n = \frac{N}{V} = \frac{g m^{3/2}}{\sqrt{2}\pi^2\hbar^3}\int_0^\infty \frac{\varepsilon^{1/2}\,d\varepsilon}{e^{\varepsilon/T}-1} = \frac{g m^{3/2}T_0}{\sqrt{2}\pi^2\hbar^3}\underbrace{\int_0^\infty \frac{z^{1/2}\,dz}{e^{z}-1}}_{\Gamma(3/2)}$$

$$\Gamma\left(\frac{3}{2}\right) = \frac{\sqrt{\pi}}{2} \qquad \zeta\left(\frac{3}{2}\right) = \sum_n^\infty \frac{1}{n^x}$$ — дзета-функ. Римана

$$T_0 = \left(\frac{\sqrt{2}\pi^2\hbar^3 n}{g m^{3/2}\,\Gamma\left(\frac{3}{2}\right)\zeta\left(\frac{3}{2}\right)}\right)^{2/3} \simeq \frac{3{,}31}{g^{2/3}}\cdot\frac{\hbar^2}{m}n^{2/3}$$



$T<T_0(n) \qquad \mu>0$

$$N_{\varepsilon>0} = \frac{g m^{3/2}}{\sqrt{2}\pi^2\hbar^3}\int_0^\infty \frac{\varepsilon^{1/2}\,d\varepsilon}{e^{\varepsilon/T}-1} = \left|z=\frac{\varepsilon}{T}\right| =$$

$$= \frac{g m^{3/2}}{\sqrt{2}\pi^2\hbar^3}\int_0^\infty \frac{z^{1/2}\,dz}{e^{z}-1} - N\left(\frac{T}{T_0}\right)^{3/2}$$

$$n_{\varepsilon>0} = n\left(\frac{T}{T_0}\right)^{3/2}$$

$$N_{\varepsilon=0} = N - N_{\varepsilon>0} = N\left(1-\left(\frac{T}{T_0}\right)^{3/2}\right)$$

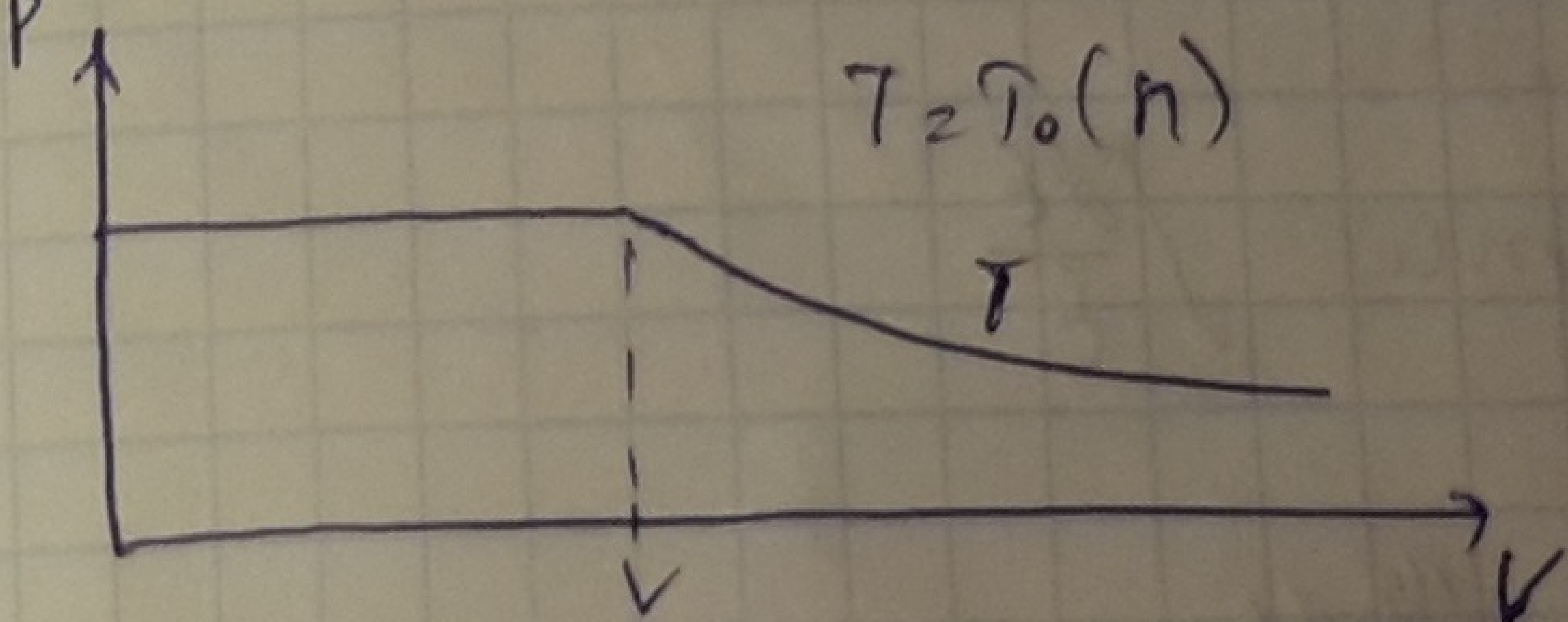
$$N = \frac{1}{e^{-\frac{\mu-\varepsilon_0}{T}}-1} + \underbrace{\sum_{k=1}^{\infty}\frac{1}{e^{\frac{\varepsilon_k-\mu}{T}}-1}}_{\varepsilon_k>0}$$

$$N_{\varepsilon=0} = \frac{1}{e^{-\frac{\mu}{T}}-1} \simeq \frac{T}{|\mu|} \to$$

$$\Rightarrow |\mu| = \frac{T}{N_{\varepsilon=0}} = \frac{T}{N\left(1-\left(\frac{T}{T_0}\right)^{3/2}\right)}$$

$$P = \frac{2}{3}\frac{g m^{3/2}}{\sqrt{2}\pi^2\hbar^3}\int_0^\infty \frac{\varepsilon^{3/2}\,d\varepsilon}{e^{\varepsilon/T}-1} = \frac{2}{3}\cdot\frac{g m^{3/2}T^{5/2}}{\sqrt{2}\pi^2\hbar^3}\int_0^\infty \frac{z^{3/2}\,dz}{e^{z}-1}$$

Давление не зависит от объёма



$$PV = NT \qquad P = \frac{NT}{V}$$

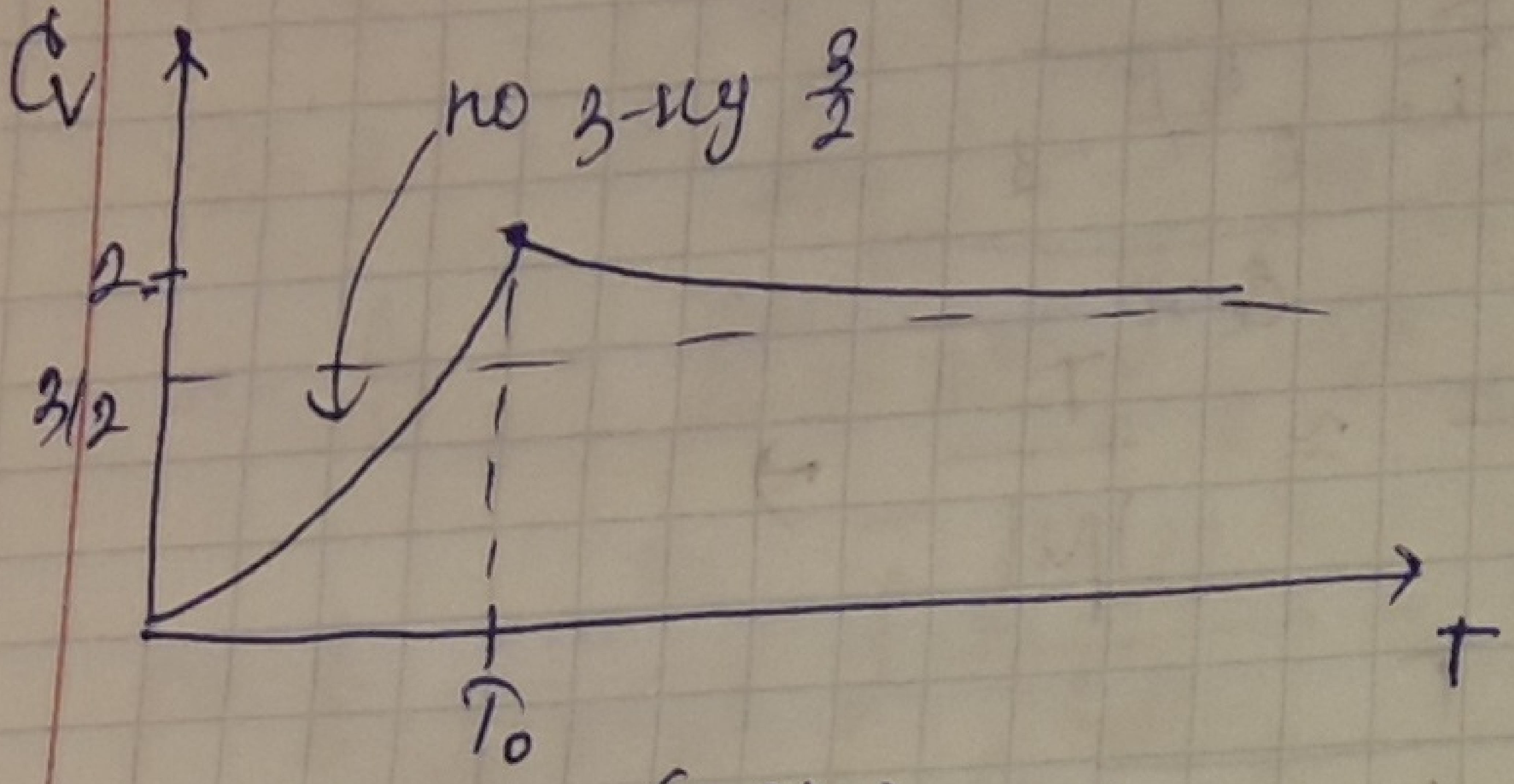
$$E = \frac{3}{2}PV = \frac{g m^{3/2}T^{5/2}V}{\sqrt{2}\pi^2\hbar^3}\Gamma\left(\frac{5}{2}\right)\zeta\left(\frac{5}{2}\right)$$

$$C = \left(\frac{\partial E}{\partial T}\right)_V = \frac{5}{2}\cdot\frac{g m^{3/2}T^{3/2}V}{\sqrt{2}\pi^2\hbar^3}\Gamma\left(\frac{5}{2}\right)\zeta\left(\frac{5}{2}\right) =$$

$$= N\left(\frac{T}{T_0}\right)^{3/2}\cdot\frac{5}{2}\cdot\frac{\Gamma(5/2)\,\zeta(5/2)}{\Gamma(3/2)\,\zeta(3/2)}$$

$$C_V = \frac{3N}{2}\left(\frac{T}{T_0}\right)^{3/2}\left[\frac{5}{2}\cdot\frac{\zeta(5/2)}{\zeta(3/2)}\right]$$

$2{,}5\cdot 0{,}5 \to \boxed{1{,}25}$



$$C_p - C_V = \frac{T\left(\frac{\partial P}{\partial T}\right)_V^2}{\left(\frac{\partial P}{\partial V}\right)_T}$$

$$\sum_k \sim \int d^2 s\, d^2 p \sim \int\int d\varepsilon \to$$

$$\frac{N}{S} \sim \int_0^\infty \frac{d\varepsilon}{e^{\frac{\varepsilon-\mu}{T}}-1}$$

в $D=2$ $\to$ расходится $\int \frac{d\varepsilon}{\varepsilon}$

Ферми-газы:

$$\frac{N}{V} = n = \frac{g m^{3/2}}{\sqrt{2}\,\pi^2\hbar^3}\int_0^\infty \frac{\varepsilon^{1/2}\, d\varepsilon}{e^{\frac{\varepsilon-\mu}{T}}+1}$$

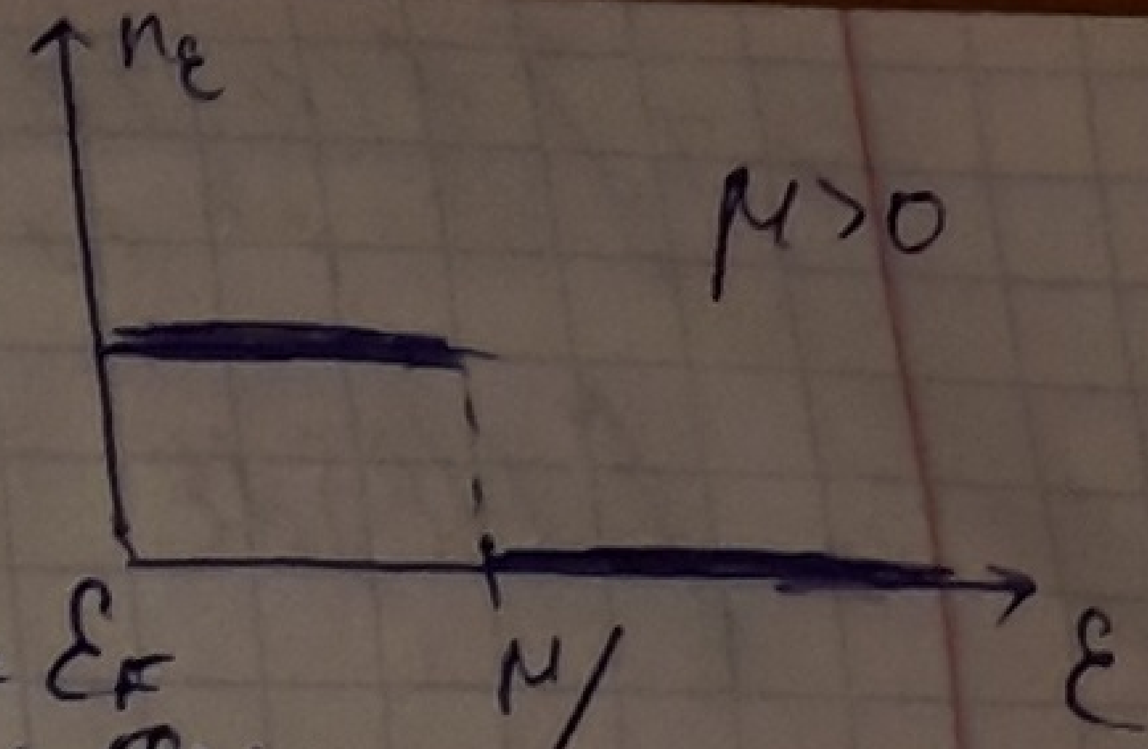
$$P = \frac{2}{3}\cdot\frac{g m^{3/2}}{\sqrt{2}\,\pi^2\hbar^3}\int_0^\infty \frac{\varepsilon^{3/2}\, d\varepsilon}{e^{\frac{\varepsilon-\mu}{T}}+1} = \frac{2}{3}\frac{E}{V}$$

$$\Rightarrow \int n_\varepsilon\, \varepsilon^{1/2}\, d\varepsilon$$

$$\boxed{n_\varepsilon = \frac{1}{e^{\frac{\varepsilon-\mu}{T}}+1}}$$

$T \to +0 \ (0)$

$$n_\varepsilon(T\to 0) = \begin{cases} \varepsilon>\mu & n_\varepsilon=0 \\ \varepsilon<\mu & n_\varepsilon = 1\end{cases}$$



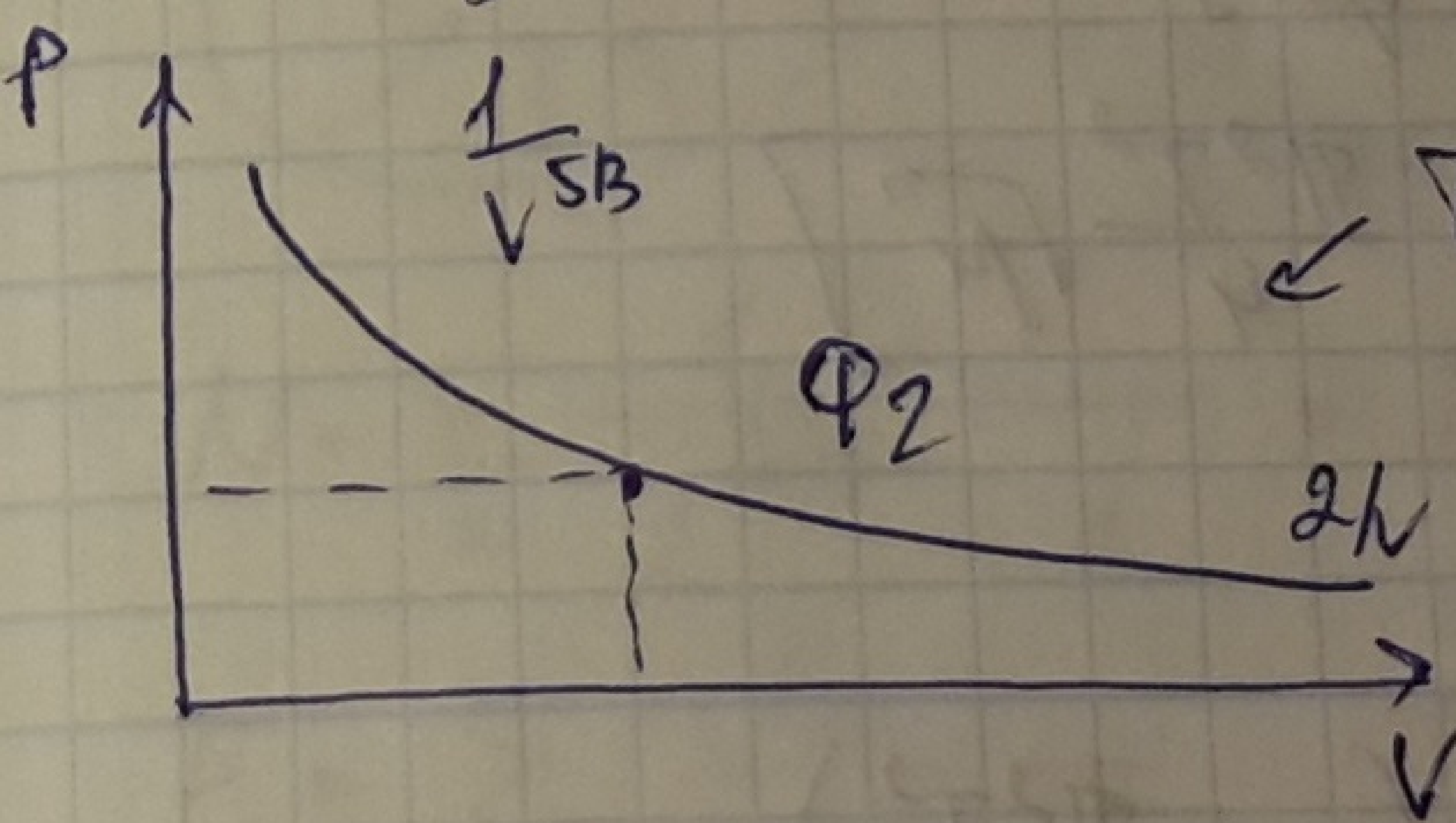
$$n = \frac{gm^{3/2}}{\sqrt{2}\pi^2\hbar^3}\int_0^{\mu(T=0)} \varepsilon^{1/2}\, d\varepsilon = \Big/ \mu(T=0) = \varepsilon_F \text{ энергия Ферми} \Big/$$

$$= \frac{gm^{3/2}}{\sqrt{2}\pi^2\hbar^3}\cdot\frac{2}{3}\varepsilon_F^{3/2}$$

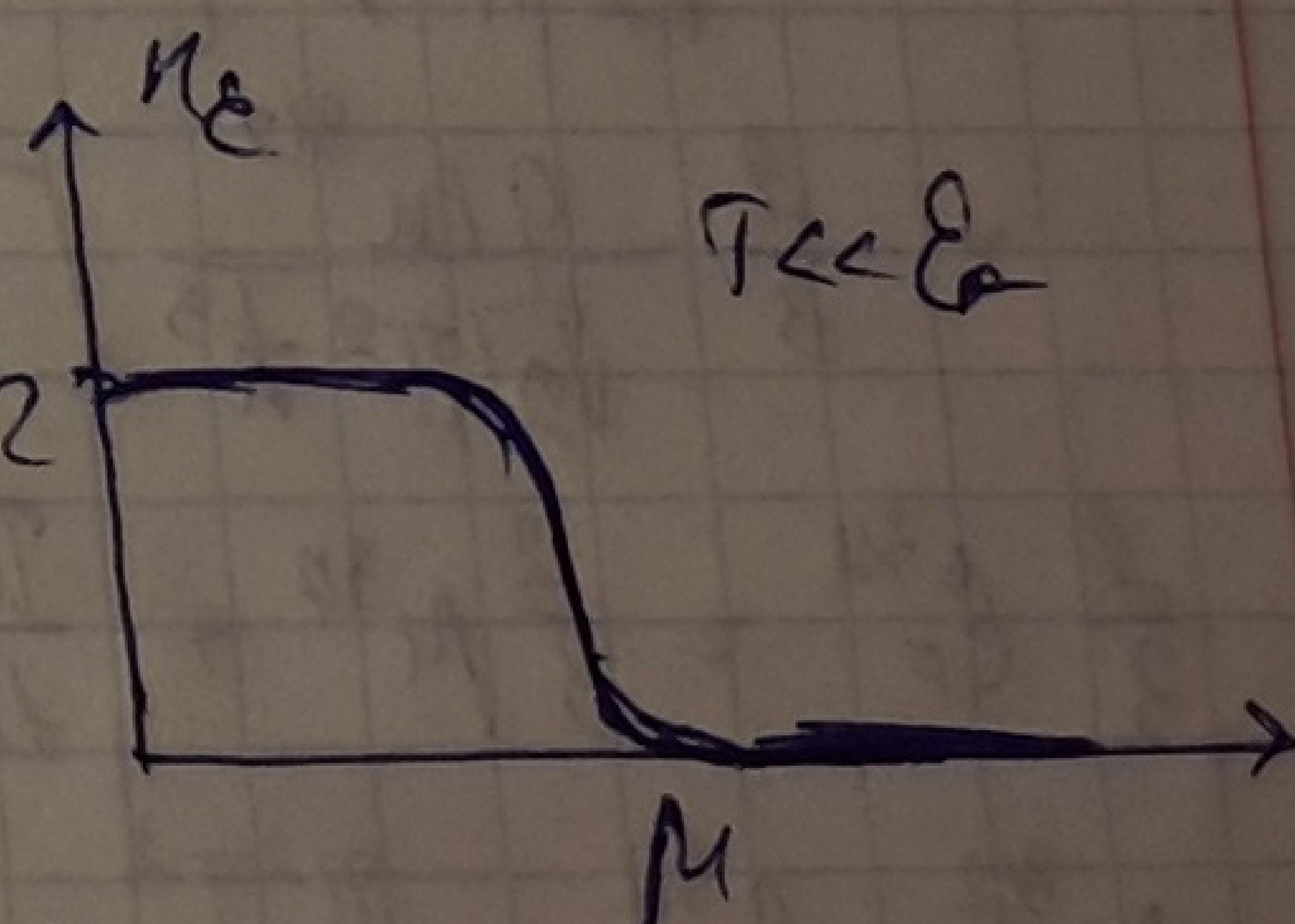
$$\varepsilon_F = \left(\frac{3\pi^2\hbar^3 n}{\sqrt{2}\, g m^{3/2}}\right)^{2/3} = \left(\frac{6\pi^2}{g}\right)^{2/3}\cdot\frac{\hbar^2}{2m}\cdot n^{2/3}$$

$$P = \frac{2}{3}\cdot\frac{gm^{3/2}}{\sqrt{2}\pi^2\hbar^3}\int_0^{\varepsilon_F}\varepsilon^{3/2}\, d\varepsilon = \frac{2}{3}\cdot\frac{gm^{3/2}}{\sqrt{2}\pi^2\hbar^3}\cdot\frac{2}{5}\varepsilon_F^{5/2} =$$

$$= \frac{2}{5} n\varepsilon_F \sim n^{5/3}$$



$T \ll \varepsilon_F$



$$\int_0^\infty \frac{f(\varepsilon)\,d\varepsilon}{e^{\frac{\varepsilon-\mu}{T}}+1} = \int_0^\mu f(\varepsilon)\,d\varepsilon + \int_0^\mu f(\varepsilon)\,d\varepsilon\left[\frac{1}{e^{\frac{\varepsilon-\mu}{T}}+1}-1\right] +$$

$$+ \int_\mu^\infty \frac{f(\varepsilon)\,d\varepsilon}{e^{\frac{\varepsilon-\mu}{T}}+1}$$

$$\int_0^\infty \frac{f(\varepsilon)\,d\varepsilon}{e^{\frac{\varepsilon-\mu}{T}}+1} = \int_0^\mu f(\varepsilon)\,d\varepsilon + T\int_{\mu/T\to-\infty}^{0} \frac{f(\mu-Tz)\,dz}{e^{z}+1} +$$

$$+T\int_0^\infty \frac{f(\mu+Tz)\,dz}{e^z+1} =$$

$$= \int_0^\mu f(\varepsilon)d\varepsilon + T\int_0^\infty dz\,\frac{f(\mu+Tz)-f(\mu-Tz)}{e^z+1} =$$

$f(\mu)-f(z)$

$$= \int_0^\mu f(\varepsilon)d\varepsilon + 2T^2 f'(\mu)\int_0^\infty \frac{z\,dz}{e^z}$$

$$= \int_0^\mu f(\varepsilon)d\varepsilon + \frac{\pi^2T^2}{6}f'(\mu)$$

$$\frac{N}{V} = n = \frac{gm^{3/2}}{\sqrt{2}\pi^2\hbar^3}\int_0^\infty \frac{\varepsilon^{1/2}d\varepsilon}{e^{\frac{\varepsilon-\mu}{T}}+1} =$$

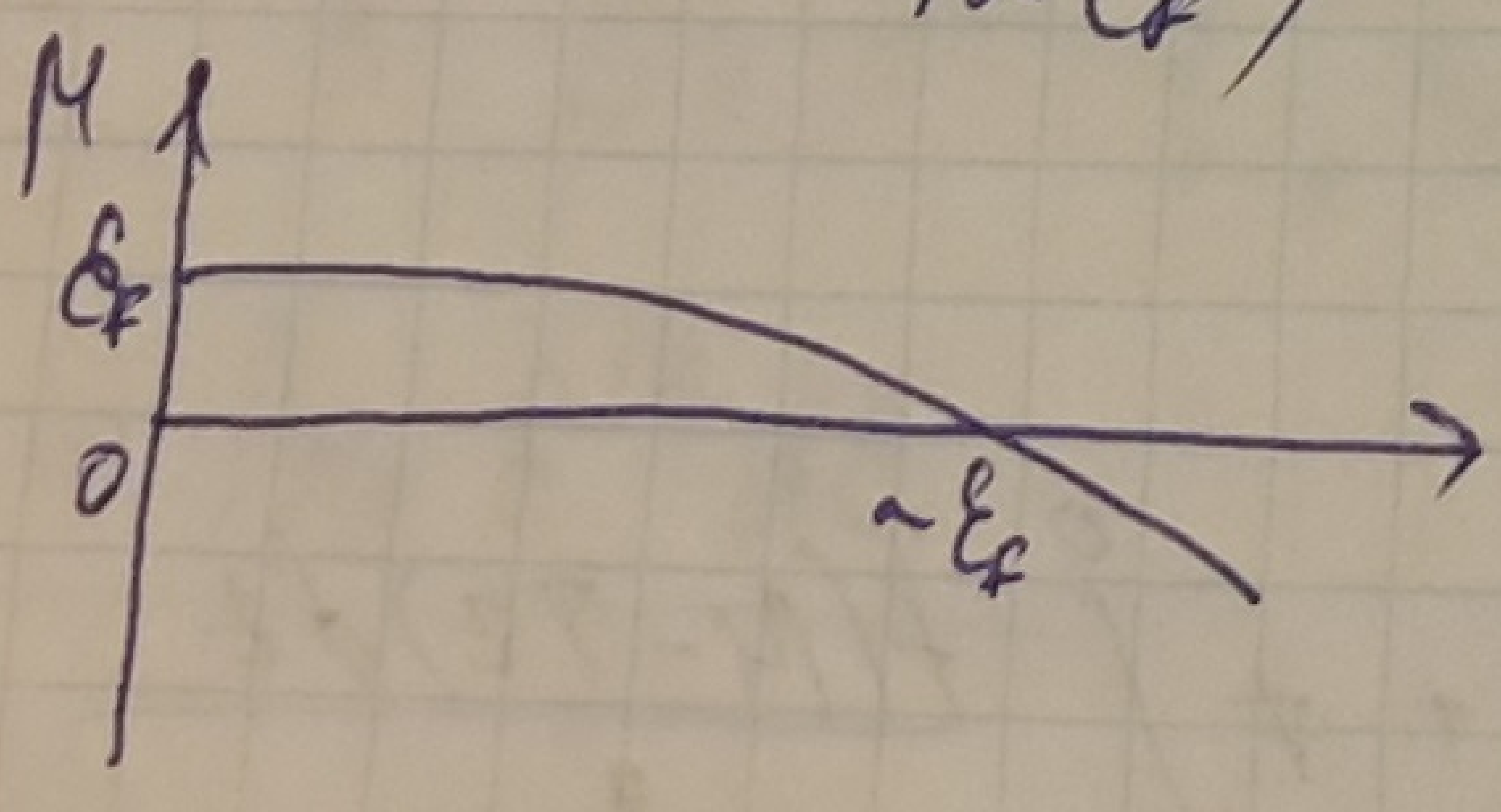
$$= \frac{gm^{3/2}}{\sqrt{2}\pi^2\hbar^3}\left[\frac{2}{3}\mu^{3/2} + \frac{\pi^2T^2}{12\sqrt{\mu}}\right] =$$

$$= \frac{2}{3}\frac{gm^{3/2}}{\sqrt{2}\pi^2\hbar^3}\left[\frac{2}{5}\mu^{5/2} + \frac{\pi^2T^2}{4}\sqrt{\mu}\right]$$

$$\frac{2}{3}\varepsilon_F^{3/2} = \frac{2}{3}\mu^{3/2} + \frac{\pi^2T^2}{12\sqrt{\mu}}$$

$$\mu^{3/2} = \varepsilon^{3/2} - \frac{\pi^2T^2}{8\sqrt{\varepsilon_F}} = \varepsilon_F^{3/2}\left(1 - \frac{\pi^2T^2}{8\varepsilon_F^2}\right)$$

$$\mu \simeq \varepsilon_F\left(1 - \frac{\pi^2T^2}{12\varepsilon_F^2}\right)$$



$$P = \frac{2}{3}\cdot\frac{g m^{3/2}}{\sqrt{2}\,\pi^2\hbar^3}\left[\frac{2}{5}\varepsilon_F^{5/2}\left(1-\frac{5\pi^2T^2}{24\varepsilon_F^2}\right)+\right.$$

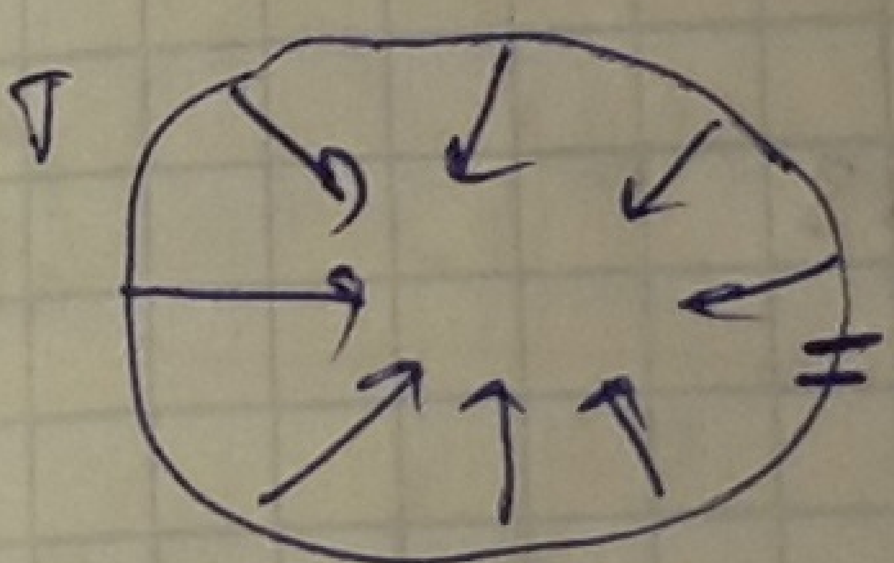
$$\left.+\frac{\pi^2T^2}{4}\sqrt{\varepsilon_F}\right] = \frac{2}{3}\frac{g m^{3/2}}{\sqrt{2}\,\pi^2\hbar^3}\cdot\frac{2}{5}\varepsilon_F^{5/2}\left(1+\right.$$

$$\left.+\frac{5\pi^2}{12}\cdot\frac{T^2}{\varepsilon_F^2}\right)$$

$$E = \frac{3}{2}PV = \frac{3}{2}N\varepsilon_F\left(1+\frac{5}{12}\pi^2\cdot\frac{T^2}{\varepsilon_F^2}\right)$$

$$C_V = \left(\frac{\partial E}{\partial T}\right)_V = \frac{2}{5}N\varepsilon_F\cdot\frac{5}{12}\pi^2\cdot\frac{2T}{\varepsilon_F^2} = N\frac{\pi^2}{2}\cdot\frac{T}{\varepsilon_F}$$

$T \ll \varepsilon_F$



$$\left(\frac{\partial F}{\partial N}\right)_{T,V} = \mu$$

$$\frac{\partial F}{\partial N} = 0 \Rightarrow \mu = 0$$

$$N = \sum_k \frac{1}{e^{\varepsilon_k/T}-1} = \sum_k \frac{1}{e^{\frac{\hbar\omega_k}{T}}-1}$$

$$E = \sum_k \frac{\varepsilon_k}{e^{\varepsilon_k/T}-1}$$

$$\Omega = T\sum_k \ln\left(1-e^{-\varepsilon_k/T}\right)$$

$\omega = ck$ $\quad\varepsilon = \hbar\omega$ $\quad\boxed{g=2}$

$\varepsilon = cp$ $\quad p = \hbar k$

$$\sum_k \to \int \frac{g\,d^3p\,d^3q}{(2\pi\hbar)^3} = \frac{gV}{(2\pi\hbar)^3}\int_0^\infty 4\pi p^2\,dp = \frac{gV\cdot 4\pi}{(2\pi\hbar c)^3}\int_0^\infty \varepsilon^2\,d\varepsilon$$

$$N = \frac{4\pi g V}{(2\pi\hbar c)^3}\int_0^\infty \frac{\varepsilon^2 d\varepsilon}{e^{\varepsilon/T}-1}$$

$$E = \frac{4\pi g V}{(2\pi\hbar c)^3}\int_0^\infty \frac{\varepsilon^3 d\varepsilon}{e^{\varepsilon/T}-1}$$

$$\Omega = T\sum_k \ln\left(1-e^{-\varepsilon_k/T}\right) =$$

$$= \frac{4\pi g V T}{(2\pi\hbar c)^3}\int_0^\infty \ln\left(1-e^{-\frac{\varepsilon}{T}}\right)\varepsilon^2 d\varepsilon =$$

$$= -\frac{1}{3}\cdot\frac{4\pi g V}{(2\pi\hbar c)^3}\int_0^\infty \frac{\varepsilon^3 d\varepsilon}{e^{\varepsilon/T}-1} \Rightarrow -\frac{E}{3} = -PV$$

$$\boxed{P = \frac{E}{3V}}$$

$$C_V = \left(\frac{dE}{dT}\right)_V \sim T^3$$

$$C_p = \infty$$

~~18~~ 18.03.16 Лекция №6

Фононы:

а) 3N-6 степеней свободы у N атомов

$N>>1 \sim 3N$

$$E = \sum_{k=1}^{3N} \hbar\omega_k \left(N_k + \frac{1}{2}\right)$$

$$\langle N_k \rangle = n_k = \frac{\sum\limits_{N_k=0}^{\infty} N_k e^{-\frac{\hbar\omega_k}{T}\left(N_k+\frac{1}{2}\right)}}{\sum\limits_{N_k=0}^{\infty} e^{-\frac{\hbar\omega_k}{T}\left(N_k+\frac{1}{2}\right)}} =$$

$$= -\frac{\partial}{\partial\alpha} \ln \sum_{N_k=0}^{\infty} e^{-\alpha N_k} = \frac{\partial}{\partial\alpha} \ln\left(1-e^{-\alpha}\right) =$$

$$= \frac{1}{e^{\alpha}-1} \qquad // \; \alpha = \frac{\hbar\omega_k}{T}$$

$$\langle N_k \rangle = n_k = \frac{1}{e^{\frac{\hbar\omega_k}{T}}-1}$$

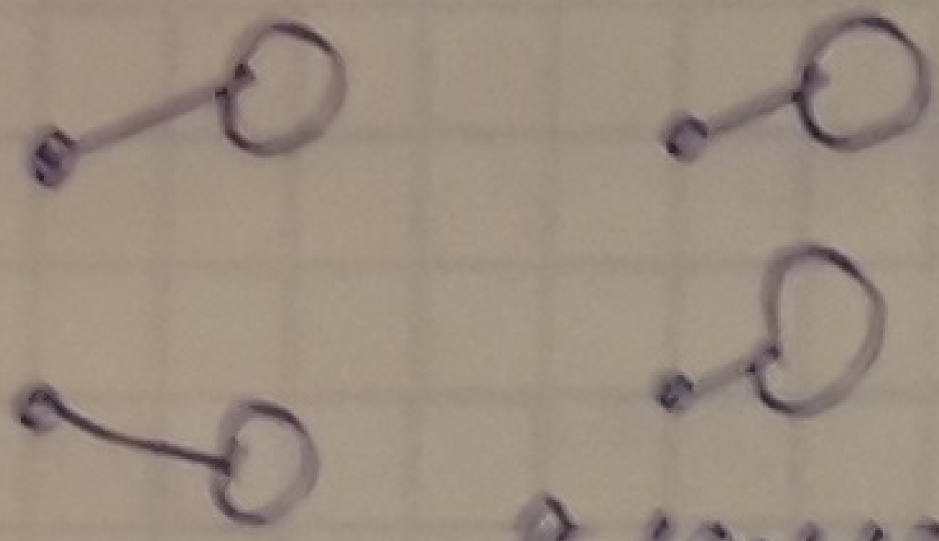
$$\langle E \rangle = \sum_{k=1}^{3N} \frac{\hbar\omega_k}{2} + \sum_{k=1}^{3N} \hbar\omega_k = E_0 + \sum_{k=1}^{3N} \frac{\hbar\omega_k}{e^{\hbar\omega_k/T}-1}$$

Бозе: $\varepsilon_k = \hbar\omega_k \qquad \mu = 0$

$\frac{pa}{\hbar} \ll 1$ $\quad \varepsilon(p) = \varepsilon\hbar\sqrt{\frac{\alpha}{M}}\,\frac{pa}{2\hbar} = cp$

$c = \sqrt{\frac{\alpha}{m}}\,a$

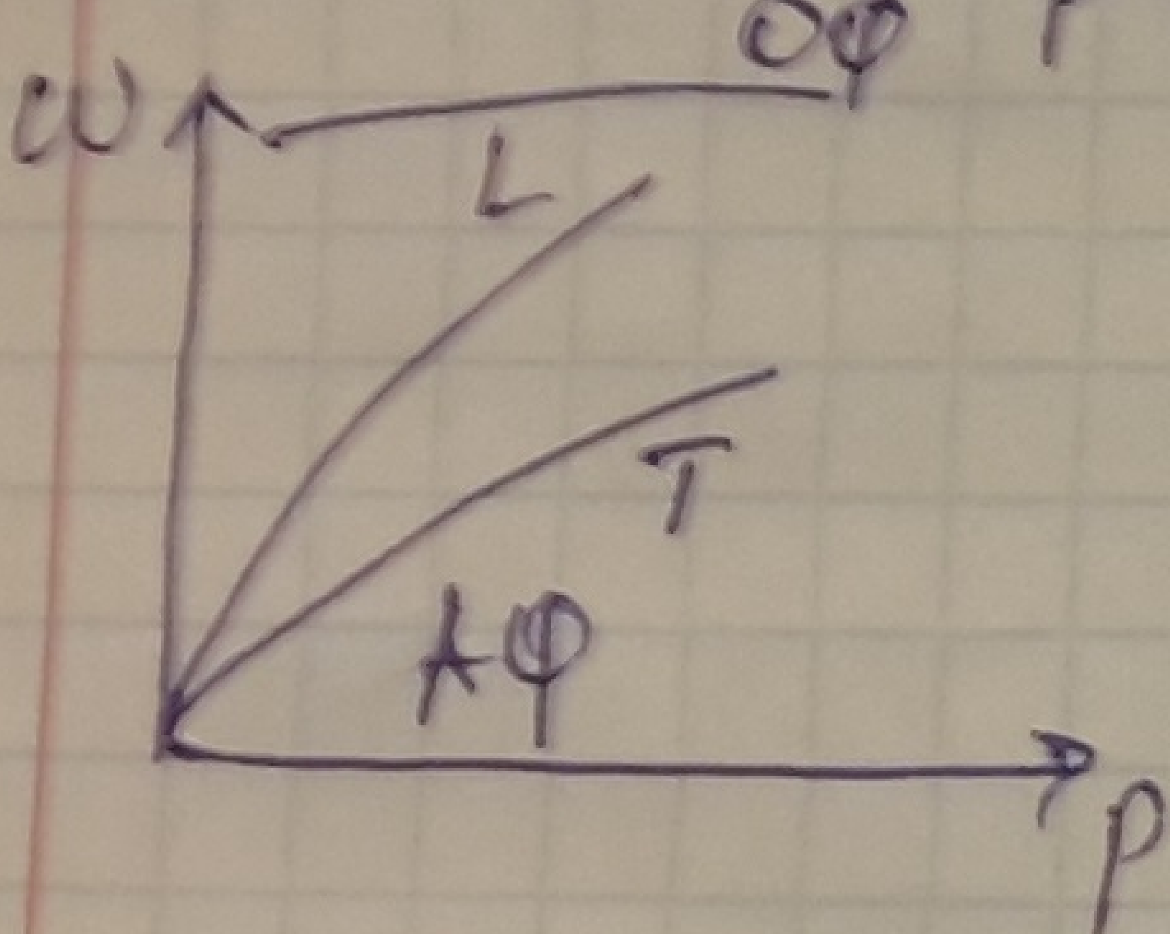
b)



элемент. ячейке $\quad \varepsilon(p) \sim cp \quad 3N\nu \quad$ АФ $\to 3$

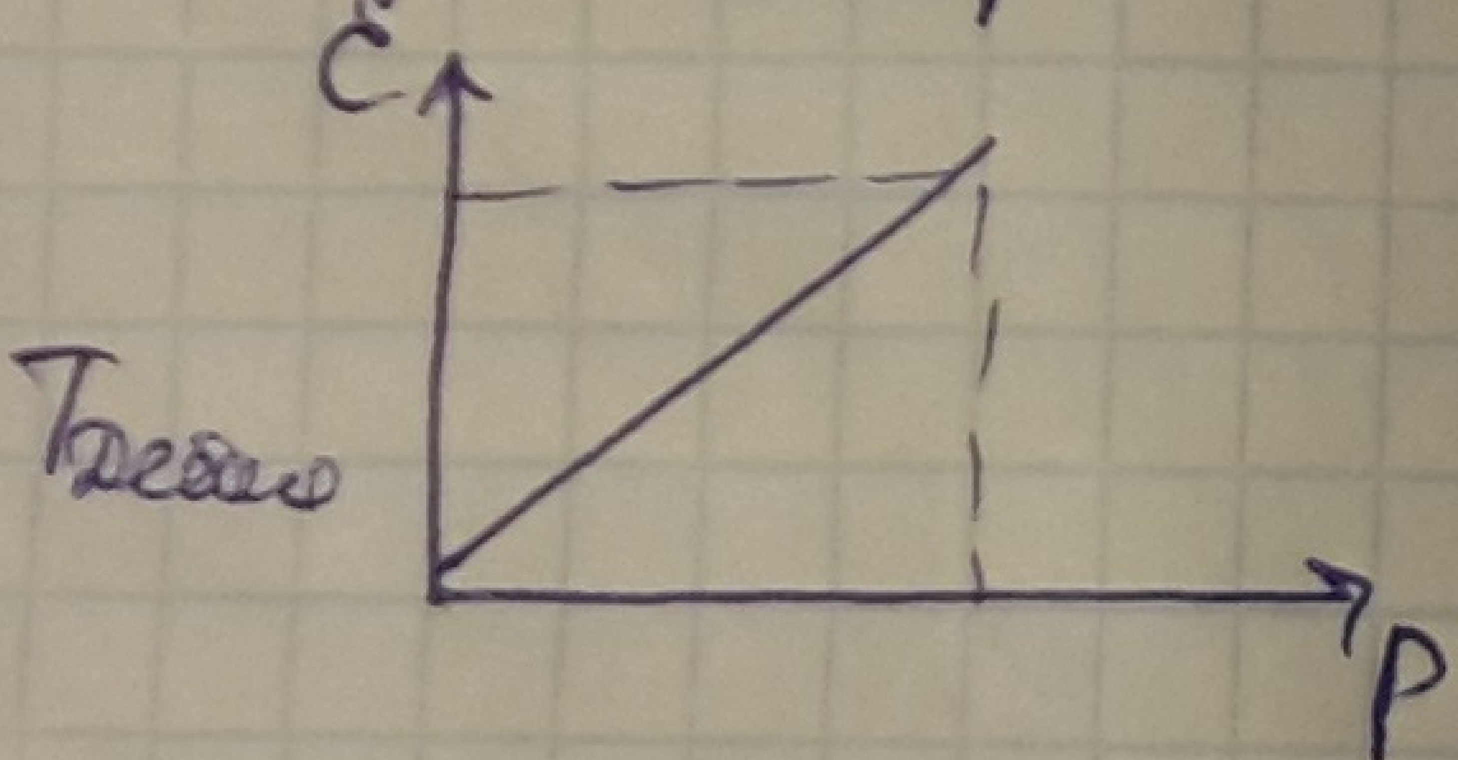
C_T, C_L — разные

оптич. фотоны ОФ $\quad \varepsilon(p) \sim const$, $p \to 0$ $\quad$ Всего 3ν мод



Модель Дебая

$\varepsilon = cp$



Модель Дебая:

2) $3N = \sum_{k=1}^{3N} 1 \Rightarrow (1+2)\int \frac{d^3p\, d^3V}{(2\pi\hbar)^3} = \frac{V(1+2)}{(2\pi\hbar)^3}\int_0^{\infty} 4\pi p^2 dp =$

$= \|\varepsilon = cp\| \underbrace{\left(\frac{1}{C_L^3} + \frac{2}{C_T^3}\right)}_{3/c^3} \cdot \frac{V}{(2\pi\hbar)^3}\int_0^{T_D} 4\pi\varepsilon^2 d\varepsilon$

$$3N = \frac{3}{c^3}\cdot\frac{V\cdot 4\pi}{(2\pi\hbar)^3}\cdot\frac{T_D^3}{3} \Rightarrow T_D = \hbar c\left(6\pi^2\cdot\frac{N}{V}\right)^{1/3}$$

Тогда

$$E = E_0 + \int \frac{\varepsilon}{e^{\varepsilon/T}-1}\cdot\frac{3}{c^3}\cdot\frac{V\cdot 4\pi\varepsilon^2 d\varepsilon}{(2\pi\hbar)^3} =$$

$$= E_0 + \frac{3}{c^3}\cdot\frac{V\cdot 4\pi}{(2\pi\hbar)^3}\int_0^{T_D}\frac{\varepsilon^3 d\varepsilon}{e^{\varepsilon/T}-1} = E_0 + \frac{9V\cdot T^4}{8\pi^2(\hbar c)^3}\int_0^{T_D/T}\frac{z^3 dz}{e^z-1}$$

$$E = E_0 + 3TN\mathcal{D}\left(\frac{T_D}{T}\right)$$, где

$$\mathcal{D}(x) = \frac{3}{x^3}\int_0^x \frac{z^3 dz}{e^z-1}$$ — ф-ция Дебая

$$\approx \begin{cases} x \gg 1 & \frac{3}{x^3}\int_0^\infty \frac{z^3 dz}{e^z-1} = \frac{\pi^4}{5x^3} \\ x \ll 1 & \frac{3}{x^3}\int_0^x \frac{z^3 dz}{z+\frac{z^2}{2}+\frac{z^3}{6}} \end{cases}$$

$T \ll T_D \quad x \gg 1$

$$E = E_0 + 3TN\cdot\frac{\pi^4}{5}\cdot\left(\frac{T}{T_D}\right)^3$$

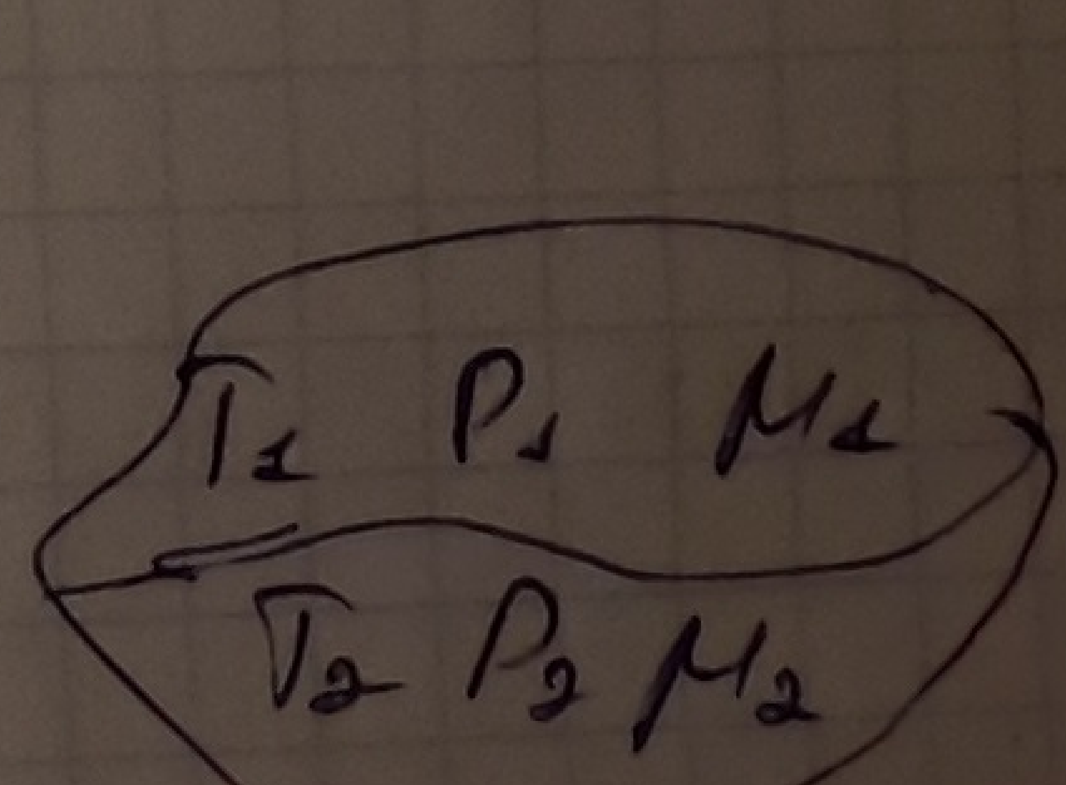
$$C_V = \frac{12\pi^4}{5}N\left(\frac{T}{T_D}\right)^3$$

$T \gg T_D \quad x \ll 1$

$$E = E_0 + 3TN\left(1 - 3\frac{T_D}{T} + \frac{1}{20}\left(\frac{T_D}{T}\right)^2\right)$$

$$C_V = 3N\left(1 - \frac{1}{20}\left(\frac{T_D}{T}\right)^2\right)$$

д) Фазовые переходы



$$dE = TdS - PdV + \mu dN$$

$$dS = \frac{dE}{T} + \frac{P}{T}dV - \frac{\mu}{T}dN$$

$$dS = dS_1 + dS_2 = \ldots$$

$$dE_1 + dE_2 = 0$$

$dV = 0 \quad dN = 0 \to \quad dS = \ldots = 0 \Leftarrow$ в равновесии

$$\Downarrow$$

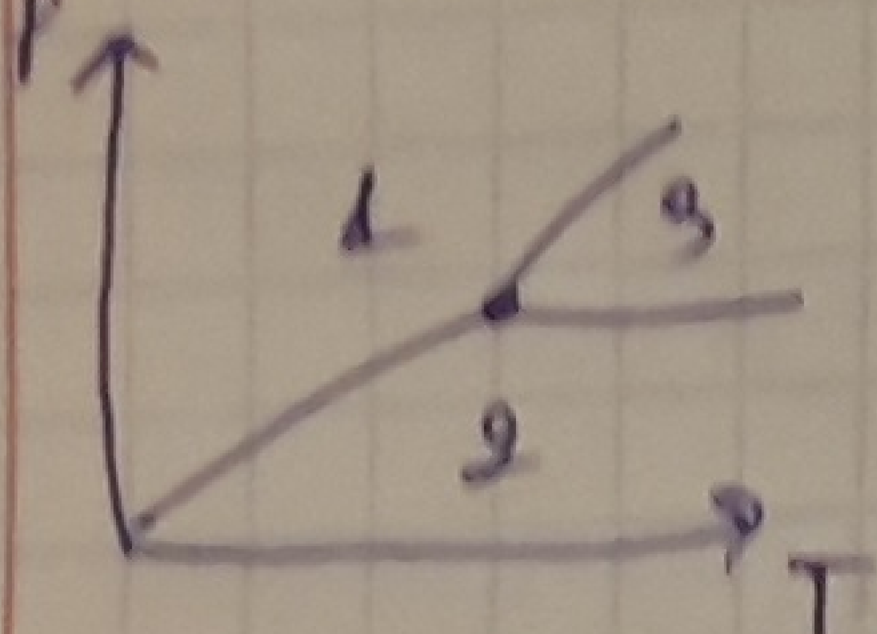
$T_1 = T_2$

$P_1 = P_2$

$\mu_1 = \mu_2$

имеем

$\mu_1(P_1, T_1) = \mu_2(T_2, P_2) \Leftarrow$



Это люда!

$$\frac{\partial \mu_1}{\partial T} \neq \frac{\partial \mu_2}{\partial T} \qquad \frac{\partial \mu_1}{\partial P} \neq \frac{\partial \mu_2}{\partial P}$$

$$\frac{\partial \mu_1}{\partial T} + \frac{\partial \mu_1}{\partial P} \cdot \frac{dP}{dT} = \frac{\partial \mu_2}{\partial T} + \frac{\partial \mu_2}{\partial P} \cdot \frac{dP}{dT}$$

$$-s_1 + v_1 \frac{dP}{dT} = -s_2 + v_2 \frac{dP}{dT}$$

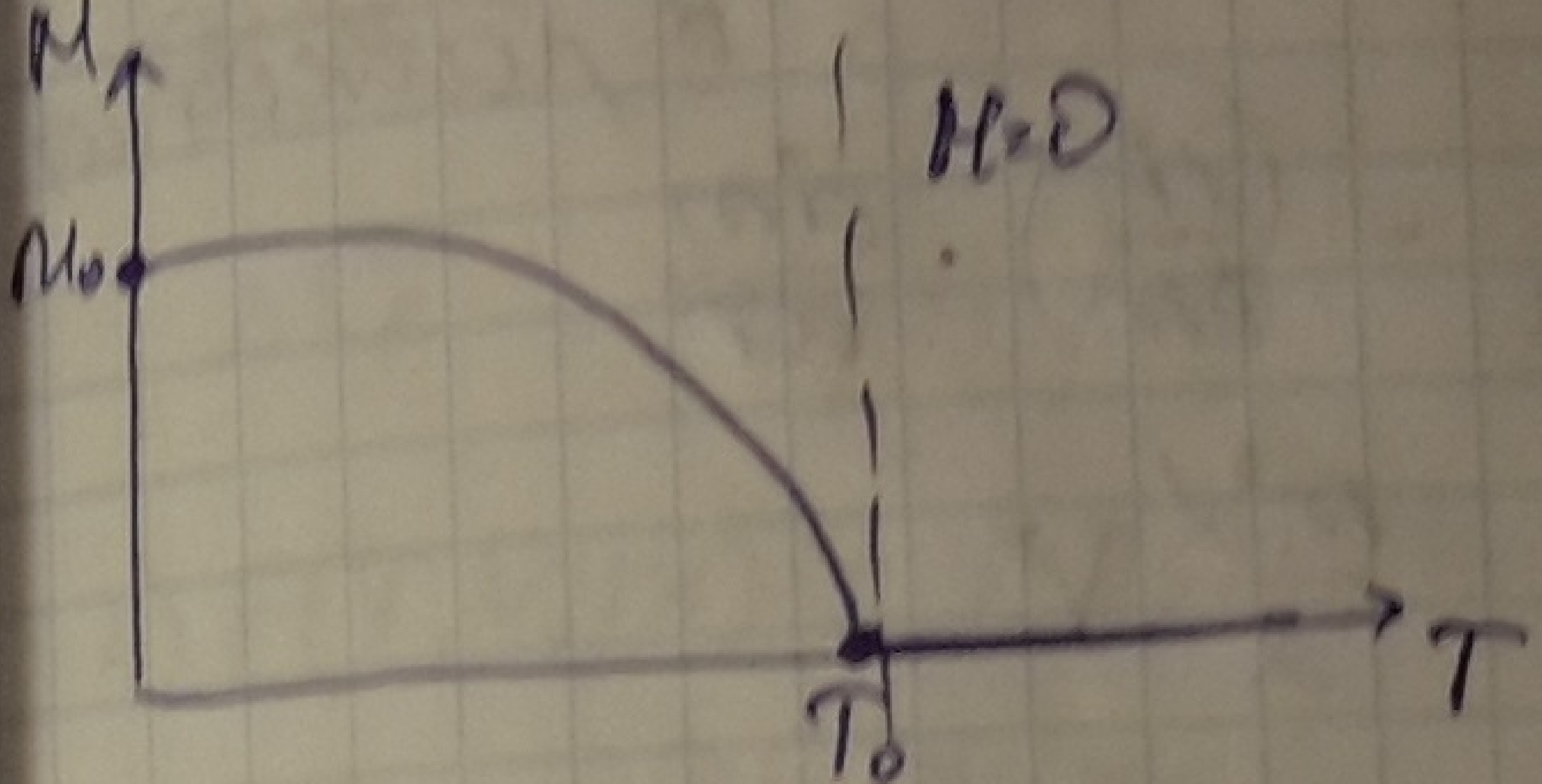
$$d\mu = -\frac{S}{N} dT + \frac{V}{N} dP \nearrow$$

$$\frac{dP}{dT} = \frac{q}{T(v_2 - v_1)} \qquad \frac{dP}{dT} = \frac{s_2 - s_1}{v_2 - v_1}$$

$$P = P_0 e^{-q/T} \Leftarrow \quad | v_2 \gg v_1 \quad \frac{T}{P} = \frac{V}{N}$$

Лекция №7 01.04.2016

Фазовые переходы 2го рода

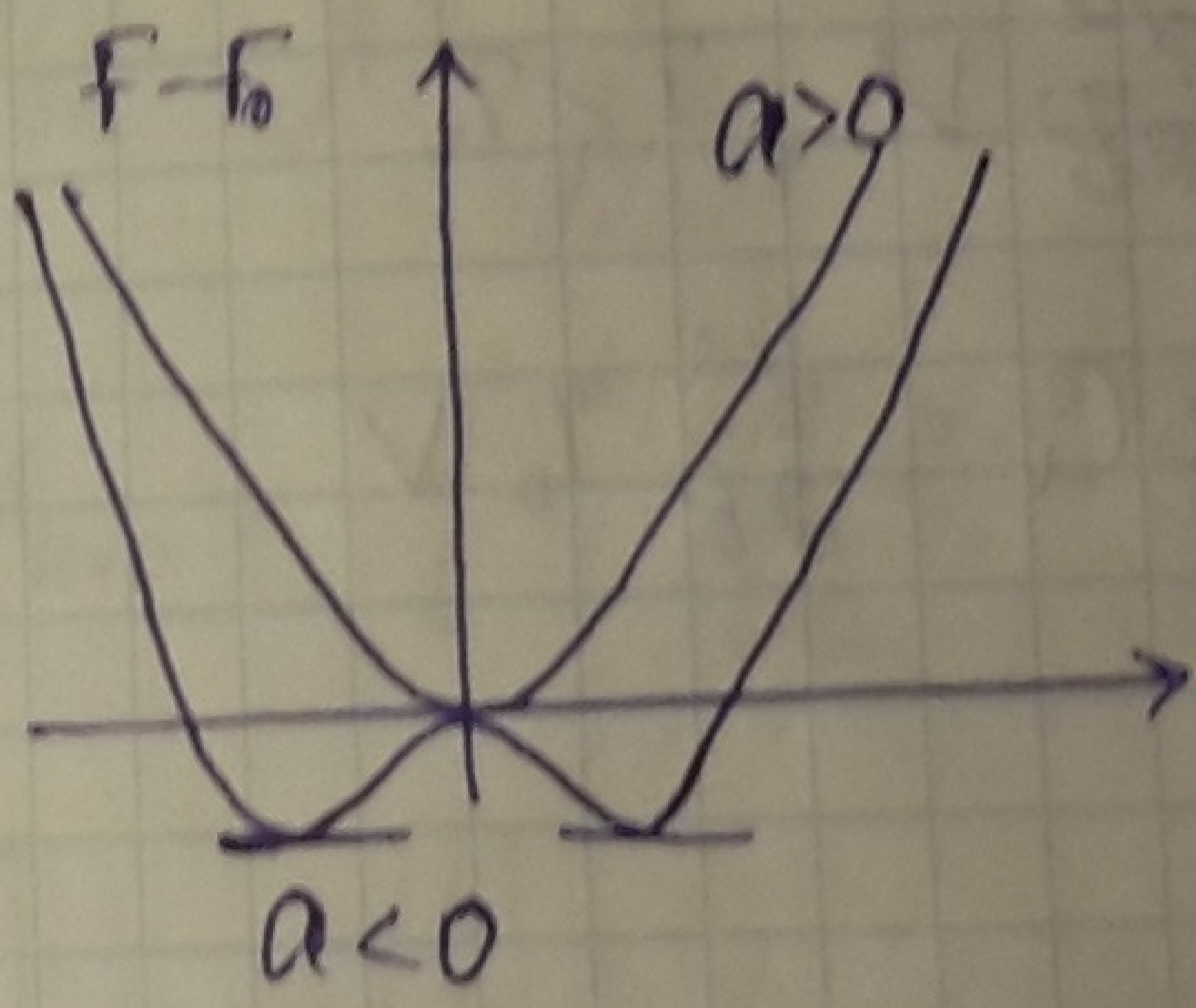


$m = \frac{M}{V}$

$$F = F(T,V,m) = F_0(T,V) + \left(\frac{a}{2}m^2 + \frac{b}{4}m^4 - Hm\right)V$$

$$a = a(T),\ b = b(T)$$

В равновесии $\frac{\partial F}{\partial m} = 0$



$$b(T) \approx b(T_0) \equiv b > 0$$

$$a(T) = \alpha(T - T_0)$$

$$\left(\frac{\partial F}{\partial m}\right)_{T,V} = 0 \Rightarrow am + bm^3 = H$$

а) $H = 0 \Rightarrow (a + bm^2)m = 0$ → $m = 0$; → $m^2 = -\frac{a}{b^2}$, $a < 0$

Устойчивость:

$$\frac{1}{V} \cdot \frac{\partial^2 F}{\partial m^2} = a + 3bm^2 > 0$$

1) $m = 0$: $\frac{1}{V} \cdot \frac{\partial^2 F}{\partial m^2} = a > 0$ при $a > 0$

2) $m^2 = -\frac{a}{b}$ ⇒ $a + 3b\left(-\frac{a}{b^2}\right) = -2a > 0$

при $a < 0$

$m = \sqrt{\frac{\alpha(T_0 - T)}{b^2}}$

в равновесии

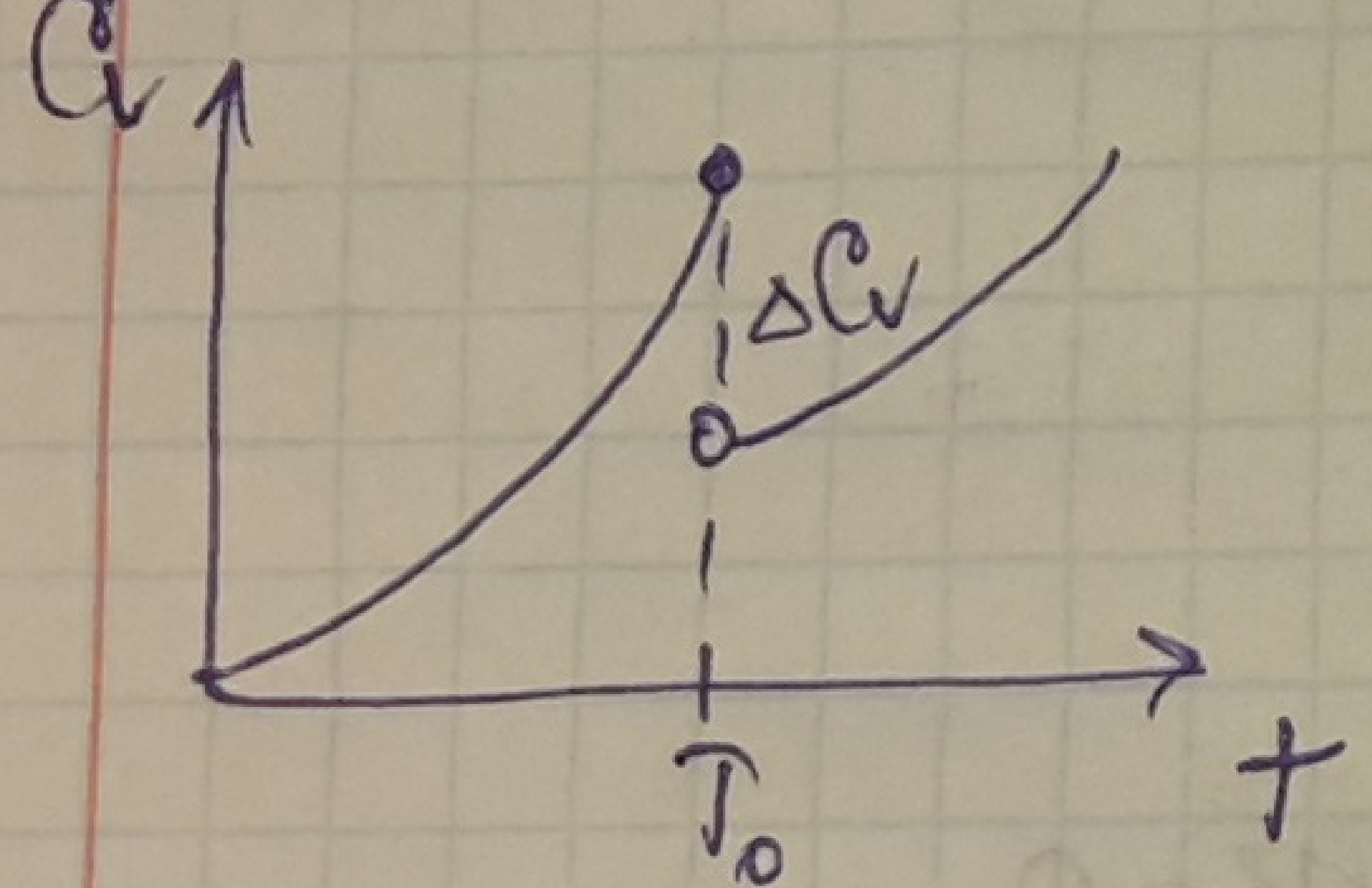
$$S = -\left(\frac{\partial F}{\partial T}\right)_{T,V} = -\left(\frac{\partial F}{\partial T}\right)_m - \overbrace{\left(\frac{\partial F}{\partial m}\right)}^{=0} \cdot \frac{dm}{dT}$$

$$S_{равн} = S = -\left(\frac{\partial F}{\partial T}\right)_m = S_0 - \frac{m^2\alpha}{2} V$$

$$S = S_0 + \begin{cases} 0, & T > T_0 \\ -\frac{\alpha^2(T_0 - T)}{2B} V, & T < T_0 \end{cases}$$

$$C_V = T\frac{\partial S}{\partial T} = C_0(T) + \begin{cases} 0, & T > T_0 \\ \frac{\alpha^2}{2B} V, & T < T_0 \end{cases}$$

$$C_V(T_0 - 0) - C_V(T_0 + 0) = \Delta C_V = \frac{\alpha^2}{2B} T_0 V$$



3) $\Delta S = 0$ $\quad \Delta P = 0$ при фазовом переходе

$$\Delta\left(\frac{\partial S}{\partial T}\right)_V + \Delta\left(\frac{\partial S}{\partial V}\right)_T \cdot \frac{dV}{dT} = 0$$

$$\Delta\left(\frac{\partial P}{\partial T}\right)_V + \Delta\left(\frac{\partial P}{\partial V}\right)_T \cdot \frac{dV}{dT} = 0$$

$$C_V = T\left(\frac{\partial S}{\partial T}\right)_V \Rightarrow \Delta\frac{C_V}{T} - \Delta\left(\frac{\partial P}{\partial V}\right)_T \cdot \left(\frac{dV}{dT}\right)^2 = 0$$

$$\Delta C_V = -T\cdot\left(\frac{dV}{dT}\right)^2 \Delta\left(-\frac{\partial P}{\partial V}\right)_T$$

$$\Delta C_P = T\left(\frac{dP}{dT}\right)^2 \Delta\left(-\frac{\partial V}{\partial P}\right)_T$$

соотношения Эренфеста

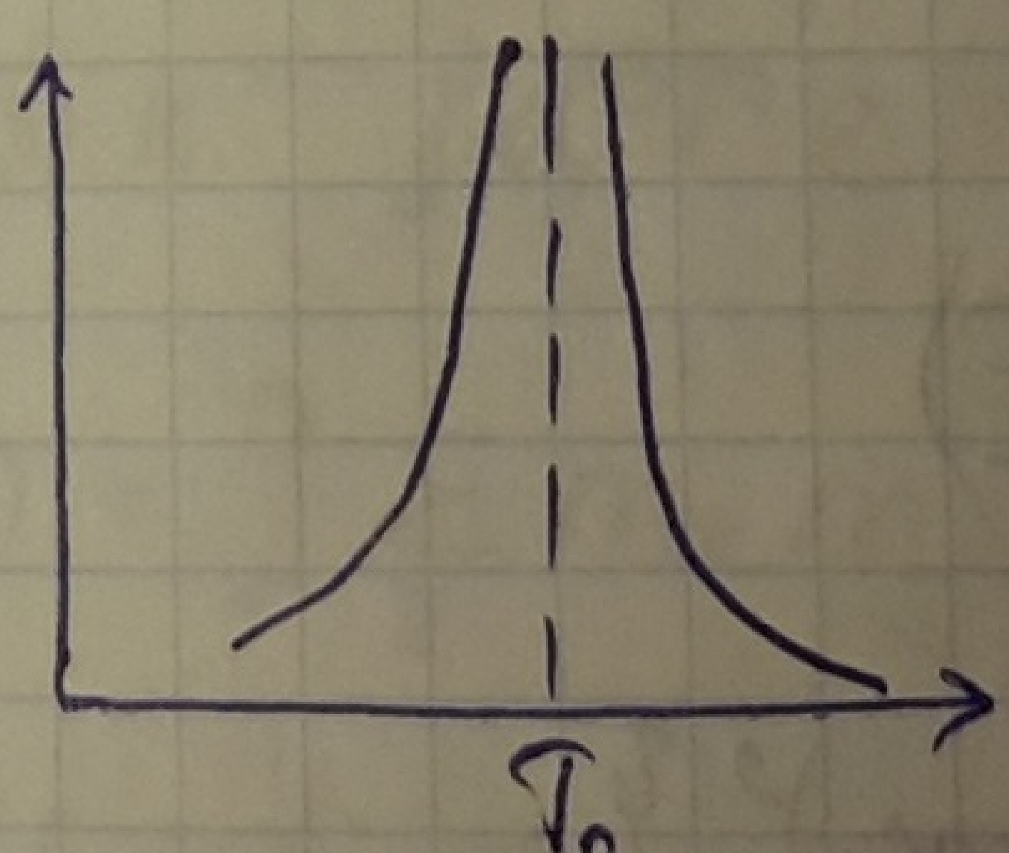
4) $F(T,V,m) = F(T,V,H=0) - HmV$

$$\frac{\partial F}{\partial m} = 0 \Rightarrow \frac{\partial F(m, H=0)}{\partial m} - HV = 0$$

$$\frac{\partial^2 F}{\partial m^2} = V\frac{\partial H}{\partial m} = \frac{V}{\chi}$$

$$\frac{\partial m}{\partial H} = \chi = \frac{V}{\frac{\partial^2 F}{\partial m^2}} = \frac{1}{a + 3bm^2}$$

$$\chi_{H=0} = \begin{cases} 1/a, & T > T_0 \\ -1/2a, & T < T_0 \end{cases}$$

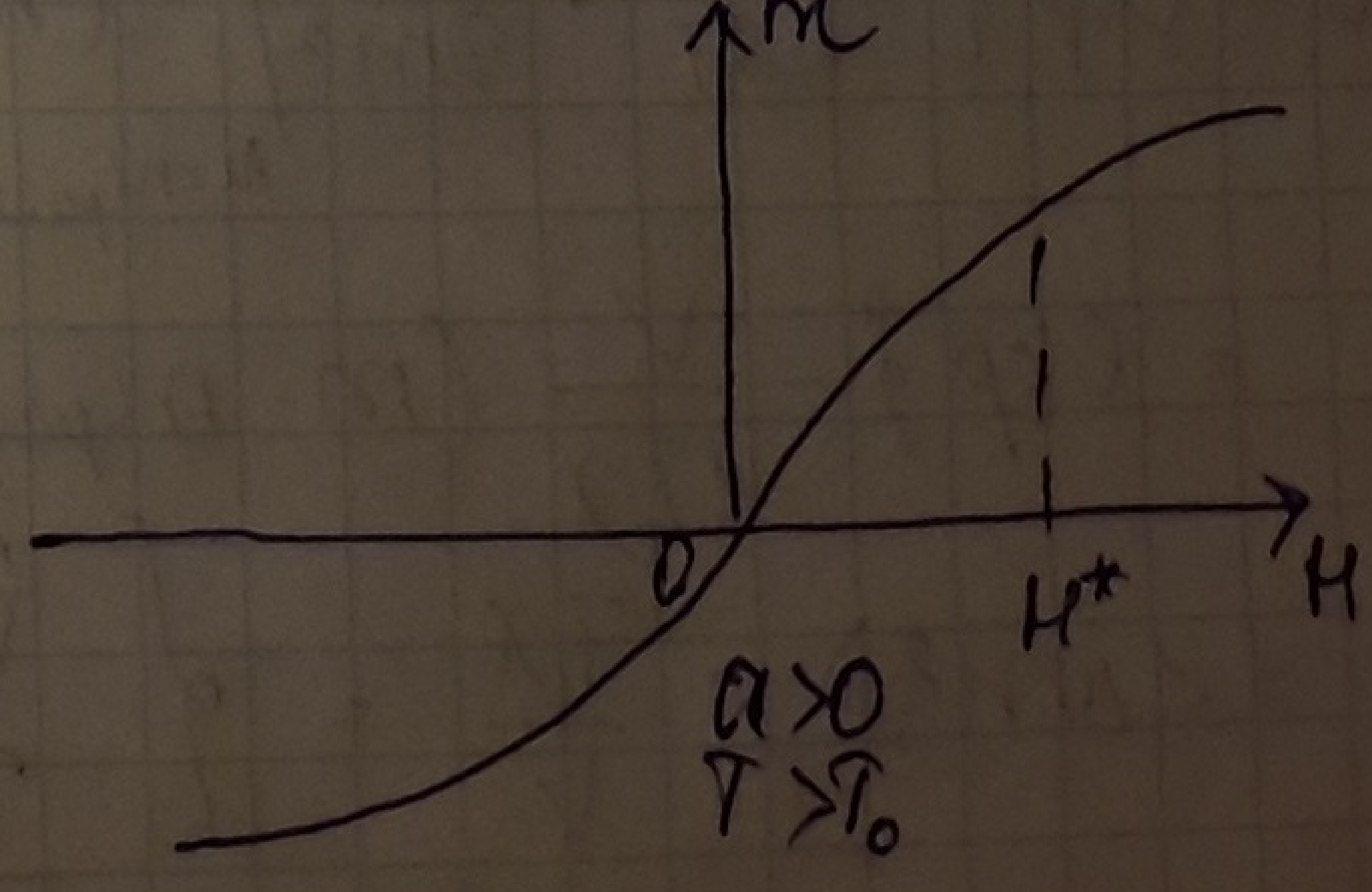


$$\chi(T)_{H=0} = \begin{cases} \frac{1}{\alpha(T-T_0)}, & T > T_0 \\ \frac{1}{2\alpha(T_0 - T)}, & T < T_0 \end{cases}$$

5) $am + bm^3 = H$

$a + 3bm^2 > 0$

$m(H)$



$H \ll H^*$

$am = H$

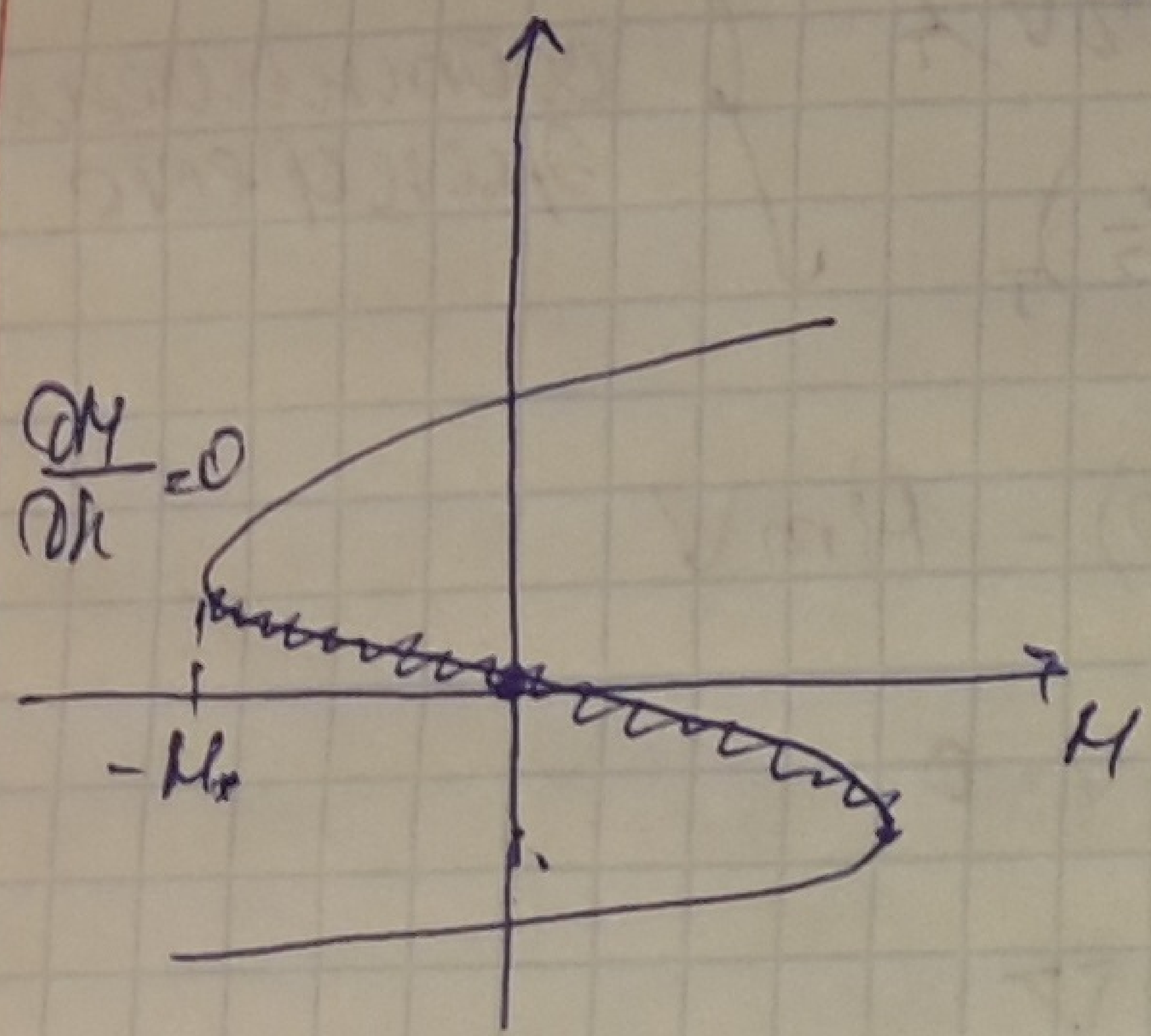
$m = \frac{H}{a}$

$H \gg H^*$

$m = \sqrt[3]{\frac{H}{b}}$

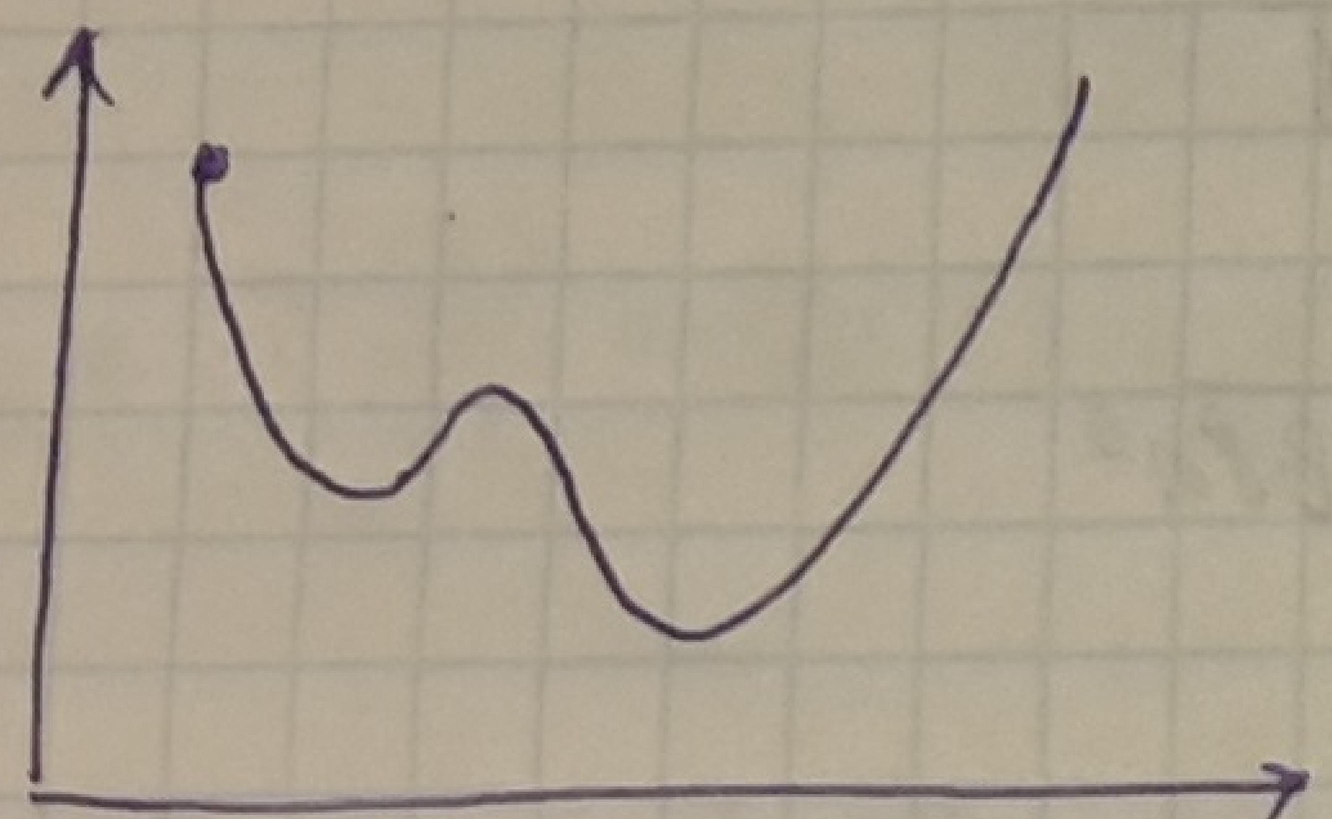
$$\frac{M^*}{a} = \left(\frac{M^*}{B}\right)^{1/3} \Rightarrow M^* = \frac{a^{3/2}}{B^{1/2}} = \frac{\alpha^{3/2}}{B^{1/2}}(T-T_0)^{3/2}$$

при $T = T_0 \Rightarrow a = 0$



$$\chi = \frac{V}{\frac{\partial^2 F}{\partial m^2}}$$

$$M_* = \left(\frac{2a}{3}\right)\left(-\frac{a}{3B}\right)^{1/2}$$



$$w \sim \exp\left\{\frac{T\Delta S - P\Delta V + \mu\Delta N - \Delta E}{T}\right\}$$

$T, V, N = const$

$$w \sim \exp\left(\frac{\Delta(TS - E)}{T}\right) = \exp\left(\frac{\Delta F}{T}\right)$$

$$w \sim \exp\left\{-\frac{1}{2T} \cdot \frac{\partial^2 E}{\partial m^2}\Big|_{m = m_{равн}} (m - m_{равн})^2\right\} =$$

$$= \exp\left\{-\frac{V}{2T\chi}(m - m_{равн})^2\right\}$$

$$D = \langle \Delta m^2 \rangle = \frac{T\chi}{V} = \begin{cases} \frac{T_0}{Va} = \frac{T_0}{2V(T-T_0)}, & T > T_0 \\ -\frac{T_0}{V \cdot 2a} = \frac{T_0}{2\alpha V(T_0 - T)}, & T < T_0 \end{cases}$$

в) Условие применимости

$$F(T,V,m(\vec{r})) = F_0(T,V) + \int d^3\vec{r}\left(\frac{a}{2}m^2 + \frac{b}{4}m^4 + \right.$$
$$\left. + \frac{c}{2}(\nabla m)^2 - Hm\right)$$

1) $H=0,\ T>T_0$

$$F = F_0(T,V) + \int d^3\vec{r}\left(\frac{a}{2}m^2 + \frac{c}{2}(\nabla m)^2\right)$$

$$\langle m(\vec{r})m(\vec{r}\,')\rangle = G(\vec{r}-\vec{r}\,')$$

$$m(\vec{r}) = \frac{1}{\sqrt{V}}\sum_k m_{\vec{k}}\, e^{i\vec{k}\cdot\vec{r}} = \left(\frac{1}{\sqrt{V}}\sum_k m_{\vec{k}}\cdot e^{-i\vec{k}\cdot\vec{r}}\right)^* =$$
$$= \frac{1}{\sqrt{V}}\sum_k m_{\vec{k}}\cdot e^{-i\vec{k}\cdot\vec{r}} = \frac{1}{\sqrt{V}}\sum_k m_{-\vec{k}}\, e^{-i\vec{k}\cdot\vec{r}} \Rightarrow$$

$$\Rightarrow m_{\vec{k}}^* = m_{-\vec{k}}$$

Тогда $F - F_0 = \int d^3\vec{r}\left(\frac{a}{2}\cdot\frac{1}{V}\sum_{k,k'} m_{\vec{k}}\, m_{\vec{k}'}\,* \right.$

$$\left. * e^{i(\vec{k}+\vec{k}')\vec{r}} + \frac{c}{2V}\sum_{k,k'}(i\vec{k})(i\vec{k}')\, m_{\vec{k}}\, m_{\vec{k}'}\, e^{i(\vec{k}+\vec{k}')\vec{r}}\right) (*)$$

т.к. $\int d^3\vec{r}\cdot e^{i(\vec{k}+\vec{k}')\vec{r}} = V\delta_{\vec{k}\,-\vec{k}'}$

$$(*) = \frac{a}{2}\sum_k m_{\vec{k}}\,\underbrace{m_{-\vec{k}}}_{m_{\vec{k}}^*} + \frac{c}{2}\sum_k k^2 m_{\vec{k}}\, m_{-\vec{k}}$$

Но $F - F_0 = \Delta F = \sum_k\left(\frac{a}{2} + \frac{c}{2}k^2\right)|m_{\vec{k}}|^2 =$

$$= \sum_{k>0}(a + ck^2)|m_{\vec{k}}|^2$$

$$w \sim e^{-\frac{\Delta F}{T}} = \exp\left(-\sum_{\vec{k}>0}\frac{a+ck^2}{T}\underbrace{|m_{\vec{k}}|^2}_{m_k'^2 + m_k''^2}\right) \Rightarrow$$

Re Im

$$\Rightarrow \langle m_k'^2\rangle = \langle m_k''^2\rangle = \frac{T}{2(a+ck^2)}$$

$$\langle |m_k|^2\rangle = \langle m_k'^2\rangle + \langle m_k''^2\rangle = \frac{T}{a+ck^2}$$

$$\langle m(\vec{r})m(\vec{r}')\rangle = \frac{1}{V}\sum_{\vec{k},\vec{k}'} \overbrace{\langle m_k m_{k'}\rangle}^{k'=-k} e^{i\vec{k}\cdot\vec{r} + i\vec{k}'\cdot\vec{r}'}$$

$$= \frac{1}{V}\sum_k \langle |m_k|^2\rangle e^{i\vec{k}\cdot(\vec{r}-\vec{r}')}$$

$$G(\vec{r}-\vec{r}') = \frac{1}{\cancel{V}}\int \frac{\cancel{V}\, d^3\vec{k}}{(2\pi)^3}\cdot \frac{T_0^{\sim T}}{a+ck^2} e^{i\vec{k}(\vec{r}-\vec{r}')} =$$

$$= \frac{T_0}{c}\int \frac{d^3\vec{k}}{(2\pi)^3}\cdot \frac{e^{i\vec{k}\cdot(\vec{r}-\vec{r}')}}{k^2+a/c} = \frac{T_0}{4\pi c^4} e^{-|\vec{r}-\vec{r}'|\sqrt{\frac{a}{c}}}$$

$$= \frac{T_0 e^{-|\vec{r}-\vec{r}'|/r_c}}{4\pi c r}$$

где r_c - кор. радиус

$$r_c = \sqrt{\frac{c}{a}} = \sqrt{\frac{c}{\alpha(T-T_0)}} \xrightarrow[T\to T_0]{} \infty$$

Применимо при $\langle m(\vec{r})m(\vec{r}')\rangle \ll \frac{a}{b}$

$$\langle m(\vec{r})m(\vec{r}')\rangle = \frac{1}{c r_c} \qquad |\vec{r}-\vec{r}'| \leq r_c$$

для оценки при $T > T_0$

$$m^2 = -\frac{a}{b^2}$$

$$\frac{1}{c r_c} \ll \frac{a}{b}$$

$$\Downarrow$$

$$\frac{T_0\sqrt{a}}{c^{3/2}} \ll \frac{a}{b} \qquad \frac{T_0}{c^{3/2}} \ll \frac{\sqrt{a}}{b} \Rightarrow \sqrt{a} \gg \frac{b T_0}{c^{3/2}}$$

$$a = \alpha(T-T_0) \gg \frac{b^2 T_0^2}{c^3} \qquad \frac{T-T_0}{T_0} = \tau \gg \frac{b^2 T_0}{\alpha c^3} = Gi$$

число Гинзбурга

$\tau \ll 1$, т.е. $\frac{b^2 T_0}{\alpha c^3} \ll 1$

Лекция №8.

<u>08.04.16</u>

1) $C \sim |\tau|^{-\alpha}$ $\quad G(r) \sim \dfrac{1}{r^{d-2+\varsigma}}$

$\chi \sim |\tau|^{-\gamma}$ $\quad d = 3$ в $\mathbb{R}^3$

$m \sim (-\tau)^{\beta}$ $\quad r_c \sim |\tau|^{\nu}$ в критич. области

в теории Ландау $\quad \alpha = 0 \quad \nu = 1/2$

$\beta = 1/2 \quad \varsigma = 0$

$\gamma = 1$

2) Тепловая энергия: $C\tau^2 \sim mHV = \dfrac{m^2}{m/H}V \sim \dfrac{m^2}{\chi}V$

$\tau^{2-\alpha} \sim \tau^{2\beta+\gamma}$

$\Downarrow$

$2 - \alpha = 2\beta + \gamma \Leftrightarrow \alpha + \gamma + 2\beta = 2$

$$\langle m(\bar r) m(\bar r') \rangle \Big|_{|\bar r - \bar r'| \le r_c} \sim \frac{1}{r_c^{d-2+\varsigma}} \sim \frac{T\chi}{V_c} \sim$$

$$\frac{T|\tau|^{-\gamma}}{r_c^{d}} \quad (*) \Rightarrow \nu(d-2+\varsigma) = \nu d - \gamma$$

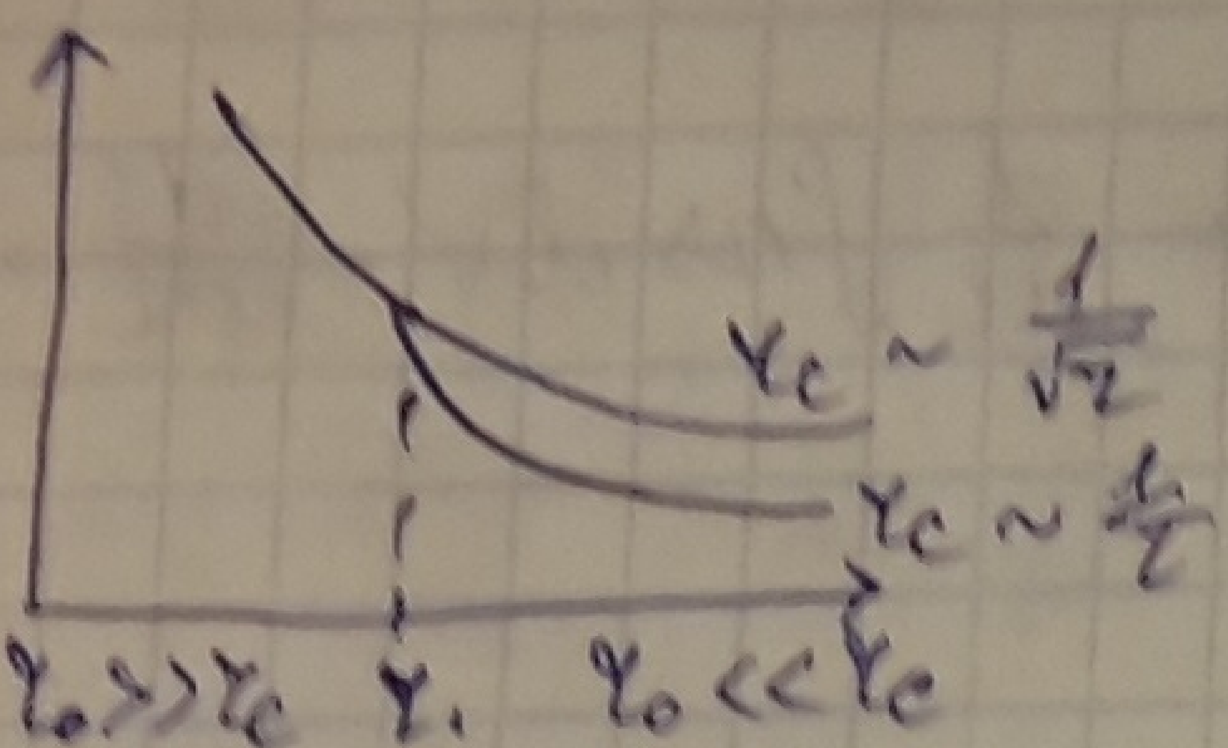
$\gamma = \nu(2-\varsigma)$

Предположим: $(*) \sim (-\tau)^{2\beta}$ $\quad \nu d - \gamma = 2\beta$

не вып. в теории Ландау $\quad \nu d = 2\beta + \gamma = 2 - \alpha$

$\rightarrow \boxed{\nu d = 2 - \alpha}$

Т.к. $m^2 = \frac{a}{b} = \frac{2tT_0}{\tau} \sim \frac{T}{c r_0}$



3) экспер.: $\alpha \sim 0{,}1$, $\xi \sim 0{,}03$

$d = 3$, $\nu = \frac{2}{3}$, $\gamma = 4/3$; $\beta = 1/3$

Вторичное квантование:

4) $|\ldots N_{k-1}, N_k \ldots\rangle$

Бозоны: $\hat{a}_k^+ |\ldots N_k \ldots\rangle = \sqrt{N_k+1}\, |\ldots N_k+1 \ldots\rangle$

$\hat{a}_k |\ldots N_k \ldots\rangle = \sqrt{N_k}\, |\ldots N_k-1 \ldots\rangle$

$\hat{a}_k^+ \hat{a}_k |\ldots N_k \ldots\rangle = N_k |\ldots N_k \ldots\rangle$

бозе/ферми: $\hat{a}_k \hat{a}_{k'}^+ \mp \hat{a}_{k'}^+ \hat{a}_k = \delta_{kk'}$ $\quad \hat{a}_k \hat{a}_{k'} \mp \hat{a}_{k'} \hat{a}_k = 0$

$\hat{a}_k^+ \hat{a}_{k'}^+ \mp \hat{a}_{k'}^+ \hat{a}_k^+ = 0$

Ферми: $\hat{a}_k^+ |\ldots N_k \ldots\rangle = (-1)^{\nu_k} (1-N_k) |\ldots 1-N_k \ldots\rangle$

$\hat{a}_k |\ldots N_k \ldots\rangle = (-1)^{\nu_k} N_k |\ldots 1-N_k \ldots\rangle$

$\hat{a}_k^+ \hat{a}_k |\ldots N_k \ldots\rangle = N_k |\ldots N_k \ldots\rangle$

$\nu_k = \sum_{s=1}^{k-1} N_s$

$\hat{\psi}(\vec{r}) = \sum_k \hat{a}_k \varphi_k(\vec{r})$ $\quad \hat{\psi}^+(\vec{r}) = \sum_k \hat{a}_k^+ \varphi_k^*(\vec{r})$

5) $\hat{H} = \sum_{i=1}^{N} \frac{\hat{p}_i^2}{2m} + \frac{1}{2} \sum_{i \neq j} \hat{U}(\vec{r}_i - \vec{r}_j)$ – слабо-неид бозе-газ

$\boxed{na^3 << 1}$

$$\hat{H} = \int_V d^3r\, \hat{\psi}^+(\vec{r}) \cdot \frac{\hat{p}^2}{2m} \hat{\psi}(\vec{r}) +$$

$$+ \frac{1}{2} \iint_V d^3\vec{r}_1 d^3\vec{r}_2\, \hat{\psi}^+(\vec{r}_1) \hat{\psi}^+(\vec{r}_2) U(\vec{r}_1 - \vec{r}_2) \hat{\psi}(\vec{r}_2) \hat{\psi}(\vec{r}_1)$$

$\varphi_k(\vec{r}) = \varphi_p(\vec{r}) = \frac{1}{\sqrt{V}} e^{\frac{i(\vec{p}, \vec{r})}{\hbar}}$ подстановке $\psi \to \hat{H}$ даст:

$$\hat{H} = \sum_p \frac{p^2}{2m} \hat{a}_p^+ \hat{a}_p + \frac{1}{2} \sum_{\vec{p}_1, \vec{p}_2, \vec{p}_1', \vec{p}_2'} \langle \vec{p}_1' \vec{p}_2' | U | \vec{p}_2 \vec{p}_1 \rangle \hat{a}_{p_1'}^+ \hat{a}_{p_2'}^+ \hat{a}_{p_2} \hat{a}_{p_1}$$

Замечание:

$\exists$ $\left.\begin{aligned} \vec{r} &= \vec{r}_1 - \vec{r}_2 \\ \vec{R} &= \frac{\vec{r}_1 + \vec{r}_2}{2} \end{aligned}\right\}$ $\begin{aligned} \vec{r}_1 &= \vec{R} + \frac{\vec{r}}{2} \\ \vec{r}_2 &= \vec{R} - \frac{\vec{r}}{2} \end{aligned}$

$$U(\vec{r}_1 - \vec{r}_2) = \frac{1}{V^2} \int d^3R\, d^3r\, U(\vec{r}) e^{\frac{i}{\hbar} \vec{R} (\vec{p}_1 + \vec{p}_2 - \vec{p}_1' - \vec{p}_2')} \cdot$$

$$\cdot e^{\frac{i}{\hbar} \cdot \frac{\vec{r}}{2} (\vec{p}_1 - \vec{p}_1' - \vec{p}_2 + \vec{p}_2')} = \frac{1}{V} \delta_{\vec{p}_1 + \vec{p}_2, \vec{p}_1' + \vec{p}_2'} \int d^3\vec{r} \cdot$$

$$\cdot U(r) e^{-\frac{i\vec{q} \cdot r}{\hbar}}, \text{ где } \vec{q} = \vec{p}_1' - \vec{p}_1$$

Считаем $\frac{a p_0}{\hbar} \ll 1 \Rightarrow U(\vec{q}) \approx U(\vec{q} = 0) = U_0 = \int d^3r\, U(r)$

В бозе-газе: $\hat{a}_0^+ \hat{a}_0 = N_0 \approx N$

$a_0 a_0^+ - a_0^+ a_0 = \sim \quad \hat{a}_0 \approx \hat{a}_0^+ \approx \sqrt{N}$

Выделим бозе-конденсат:

$$\hat{H} = \sum_p \frac{p^2}{2m} \hat{a}_p^+ \hat{a}_p + \frac{U_0}{2V} \begin{cases} \hat{a}_0^+ \hat{a}_0^+ \hat{a}_0 \hat{a}_0 + \hat{a}_0 \hat{a}_0 \sum_{p \neq 0} \hat{a}_p^+ \hat{a}_{-p}^+ + \\ + \hat{a}_0^+ \hat{a}_0^+ \sum_{p \neq 0} \hat{a}_p \hat{a}_{-p} + 4 \hat{a}_0^+ \hat{a}_0 \end{cases}$$

Знаем $\hat{a}_0^+\hat{a}_0 = N - \sum\limits_{p\neq 0} \hat{a}_p^+\hat{a}_p \sum \hat{a}_p^+\hat{a}_p$

т.к.

$\sum a_p^+ a_p = N$

$\hat{a}_0^+\hat{a}_0\hat{a}_0^+\hat{a}_0 = N^2 - 2N\sum\limits_{p\neq 0}\hat{a}_p^+\hat{a}_p = \hat{a}_0^+\hat{a}_0^+\hat{a}_0\hat{a}_0 + \frac{+\hat{a}_0^+\hat{a}_0}{N}$

$$\hat{H} = \sum_p \frac{p^2}{2m}\hat{a}_p^+\hat{a}_p + \frac{U_0}{2V}\Big\{N^2 - N + N\Big(\sum_{p\neq 0}\hat{a}_p^+\hat{a}_{-p}^+ + \sum_{p\neq 0}\hat{a}_p\hat{a}_{-p}\Big) + 2N\sum_{p\neq 0}\hat{a}_p^+\hat{a}_p\Big\}$$

$$\hat{H} = E_0 + \sum_{p\neq 0}\hat{a}_p^+\hat{a}_p\Big(\frac{p^2}{2m} + U_0\frac{N}{V}\Big) + \frac{U_0}{2}\cdot\frac{V}{N} * \sum_{p\neq 0}\Big(\hat{a}_p^+\hat{a}_{-p}^+ + \hat{a}_p\hat{a}_{-p}\Big), \text{ где } E_0 \simeq \frac{U_0 N^2}{2V}$$

Идеальный газ $E = \sum\limits_k \varepsilon_k N_k \Rightarrow \hat{H} = \sum\limits_k \varepsilon_k \hat{a}_k^+\hat{a}_k$

Приведем к этому виду:

$$\left.\begin{aligned}\hat{a}_p &= u_p\hat{b}_p + v_p\hat{b}_{-p}^+\\ \hat{a}_p^+ &= u_p\hat{b}_p^+ + v_p\hat{b}_{-p}\end{aligned}\right\} \text{ преобр. Боголюбова}$$

$$\hat{a}_p\hat{a}_p^+ - \hat{a}_p^+\hat{a}_p = 1 = u_p^2 - v_p^2$$

Знаем

$$\left.\begin{aligned}\hat{b}_p &= u_p\hat{a}_p - v_p\hat{a}_{-p}^+\\ \hat{b}_p^+ &= u_p\hat{a}_p^+ - v_p\hat{a}_{-p}\end{aligned}\right\} \qquad u_p = \frac{1}{\sqrt{1-L_p^2}} \qquad v_p = \frac{L_p}{\sqrt{1-L_p^2}}$$

$$\hat{H} = E_0 + \sum_{p\neq 0}\varepsilon_p\big(u_p\hat{a}_p^+ - v_p\hat{a}_{-p}\big)\big(u_p\hat{a}_p - v_p\hat{a}_{-p}^+\big) =$$

$$= E_0 + \sum_{p\neq 0} \varepsilon_p \{ u_p^2 \hat{a}_p^+ \hat{a}_p - v_p u_p \hat{a}_p \hat{a}_{-p} - v_p u_p \hat{a}_p^+ \hat{a}_{-p}^+ + v_p^2 \hat{a}_{-p} \hat{a}_{-p}^+ \}$$

с учетом ком. соотнош. и замен $\vec{p} \to -\vec{p}$

$$E_0' + \sum_p \varepsilon_p v_p^2 = E_0$$

$$\varepsilon_p (u_p^2 + v_p^2) = \frac{p^2}{2m} + \underbrace{U_0 \frac{N}{V}}_{mu^2} \qquad mu^2 \equiv U_0 n$$

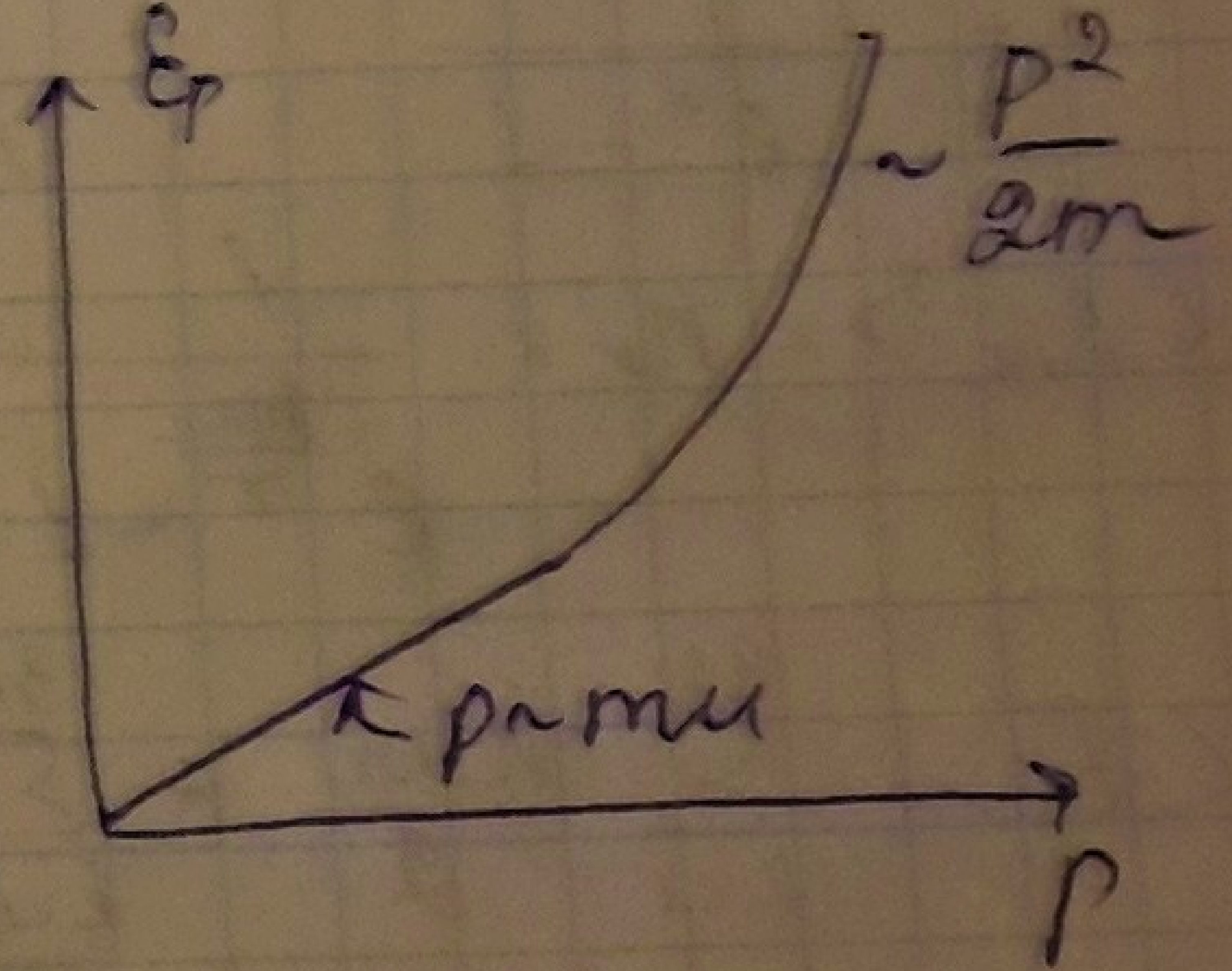
Г две члена

mu^2 кв. зв. скор

$$\sum_p u_p v_p = -\frac{U_0 n}{2} \equiv -\frac{mu^2}{2}$$

$$\begin{cases} \varepsilon_p \dfrac{1+L_p^2}{1-L_p^2} = \dfrac{p^2}{2m} + mu^2 \\ \varepsilon_p \dfrac{L_p}{1-L_p^2} = -mu^2 \end{cases} \Rightarrow \varepsilon_p^2 = \frac{p^2}{2m}\left(\frac{p^2}{2m} + 2mu^2\right)$$

$$\varepsilon_p = \sqrt{\left(\frac{p^2}{2m}\right)^2 + (pu)^2}$$



$p \ll mu$ – линейно

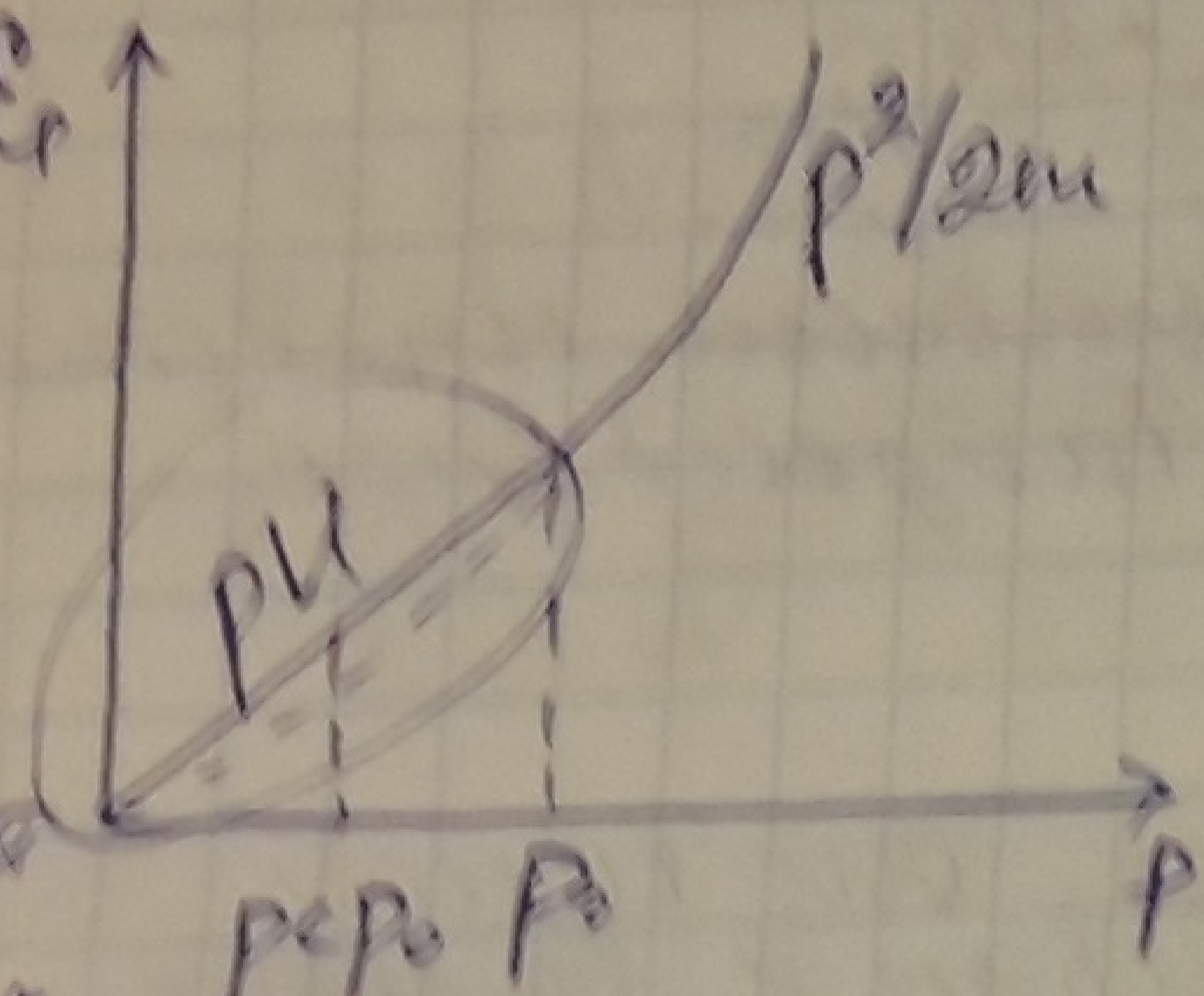
$p \gg mu$ квадрат.

15.04.16

Лекция № 9

$$\varepsilon_p = \sqrt{\left(\frac{p^2}{2m}\right)^2 + (pu)^2}$$

$mu^2 = U_0 n$
$\uparrow$
фаз. ск. элементарн. возбужд.



приводит к сверхтекучести бозе-газа

$p_0 \sim mu$

$$p_0 \sim m\sqrt{\frac{U_0 n}{m}} \sim \sqrt{U_0 n m}$$

$$\frac{p_0 a}{\hbar} \ll 1 \qquad U_0 = \frac{4\pi\hbar^2 a}{m} \qquad \frac{p_0 a}{\hbar} \ll 1$$

$$p_0 \sim \sqrt{an}\,\hbar \qquad \sqrt{a^3 n} \ll 1$$

$na^3 \ll 1$

$$n = \frac{N}{V} \qquad na^3 \qquad \left(a\Big/\left(\frac{V}{N}\right)^{1/3}\right)^3$$

газ. пар-р

$$u_T^2 = \frac{\partial P}{\partial \rho} \qquad E_0 = \frac{NU_0 n}{2} = \frac{N^2 U_0}{2V}$$

гидродин.

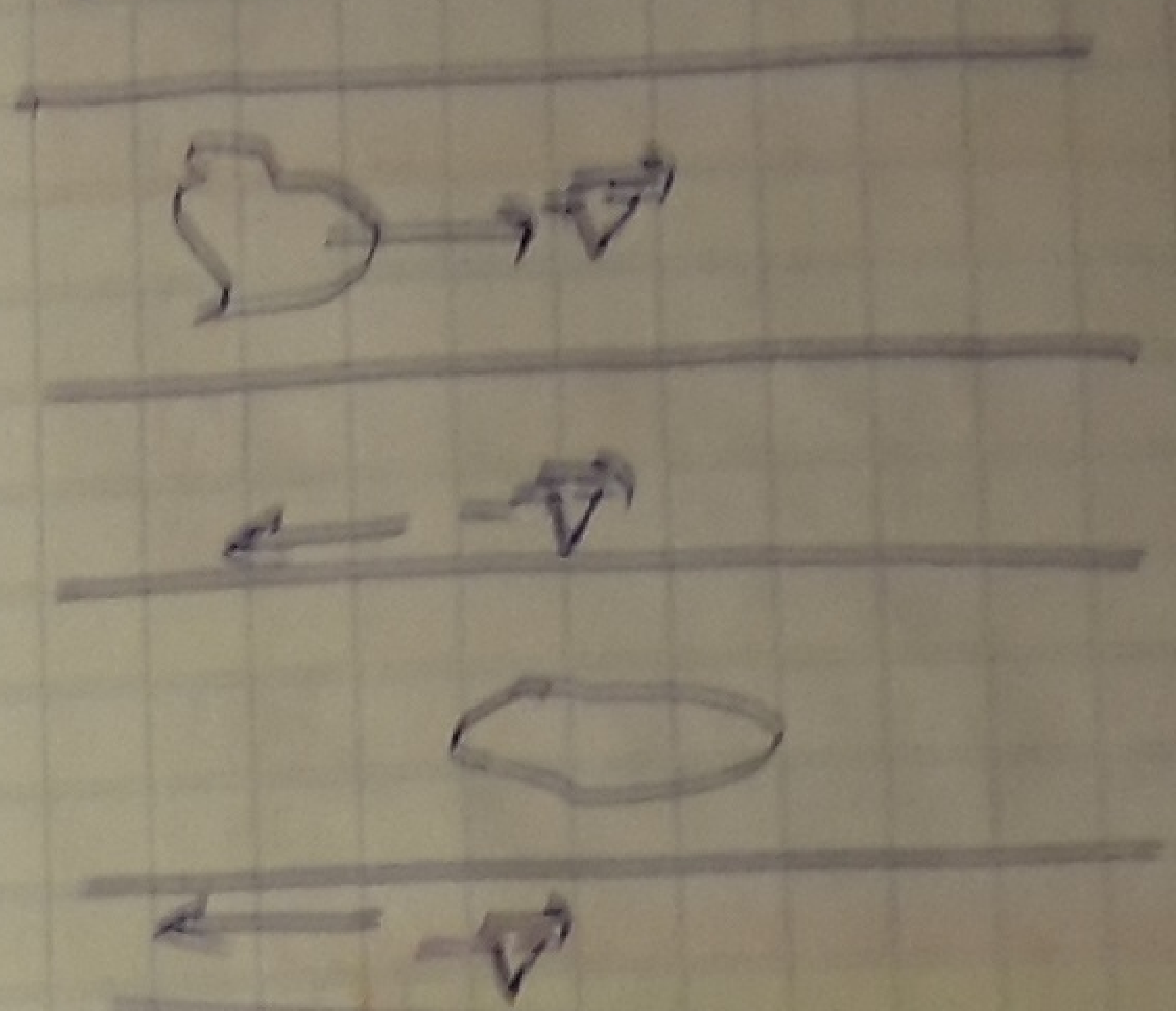
$$P = -\frac{\partial E_0}{\partial V} = \frac{N^2 U_0}{2V^2} = \frac{n^2 U_0}{2}$$

$$P = \frac{\rho^2 U_0}{2m^2}$$

$$u_T^2 = \frac{\partial P}{\partial \rho} = \frac{U_0 \rho}{m^2} = \frac{U_0 n}{m} = u^2$$ ск. звука в газе

$T = 0$

В ходе теч. трения со стенками



K $\quad \varepsilon' + Vp' < 0$ [illegible]

$\varepsilon' < Vp'$

$\downarrow \qquad V > \dfrac{\varepsilon'}{p'}$

K' $\quad \varepsilon = \varepsilon' + \mathbf{v} \cdot \mathbf{p}'$

$$\boxed{\mathbf{v} = \mathbf{V} + \mathbf{v}'}$$

$$E = \sum \frac{m}{2}(\mathbf{V} + \mathbf{v}')^2 + U(r) =$$

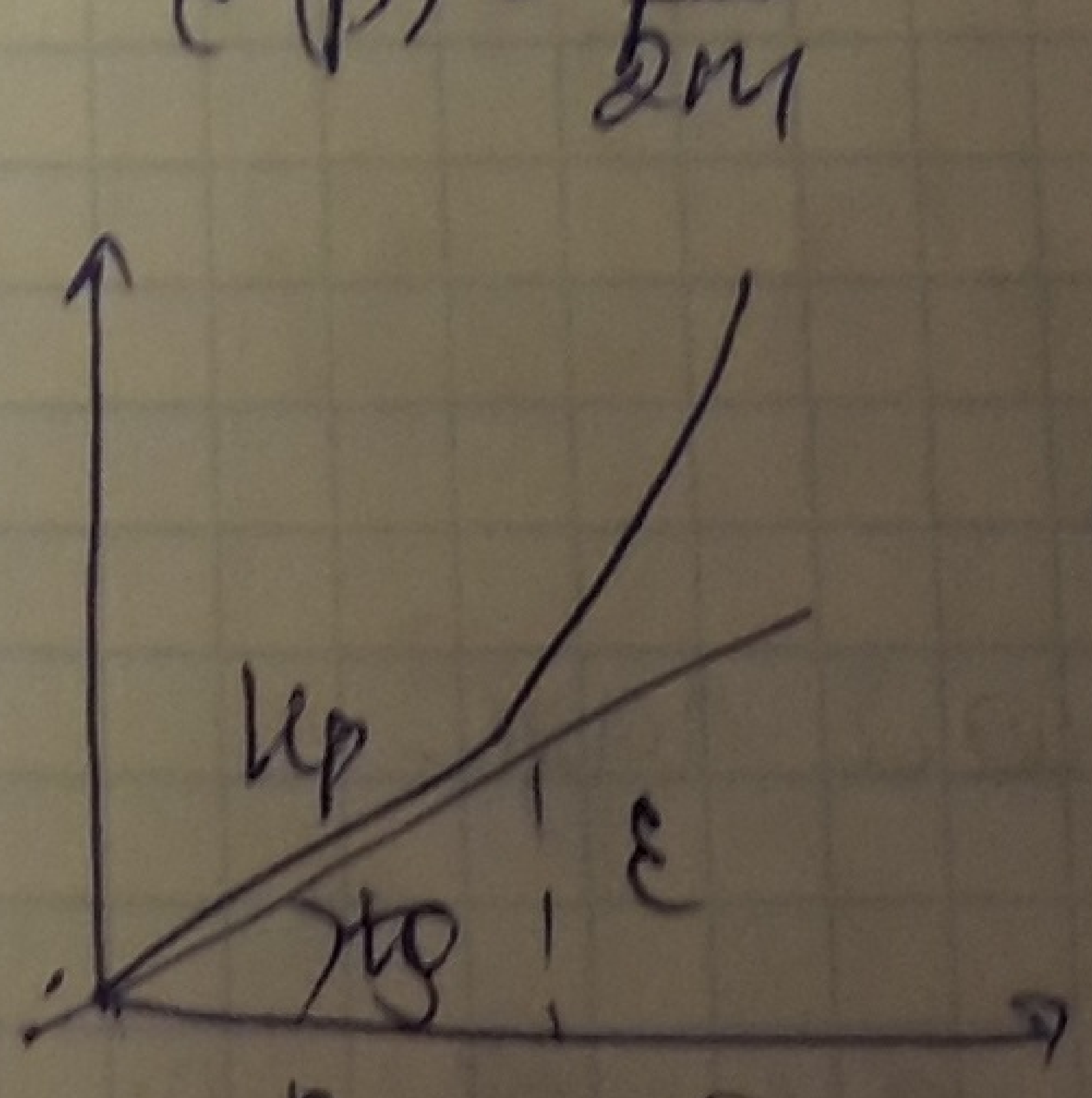
$$= \sum \left(\frac{m V^2}{2} + m \mathbf{V} \cdot \mathbf{v}' + \underbrace{\frac{m}{2} v'^2 \right) + U} =$$

$$= \frac{M V^2}{2} + \underbrace{\mathbf{V} \cdot \mathbf{p}' + \varepsilon'}_{\varepsilon}$$

$V > \min \dfrac{\varepsilon(p)}{p}$ – условие того, что жидк. будет торм-ся

$$\boxed{V < V_{кр} = \min \frac{\varepsilon(p)}{p}}$$ – критерий сверхтекуч. Ландау

$\varepsilon(p) = \dfrac{p^2}{2m}$ $\qquad \min \dfrac{\varepsilon(p)}{p} = 0$

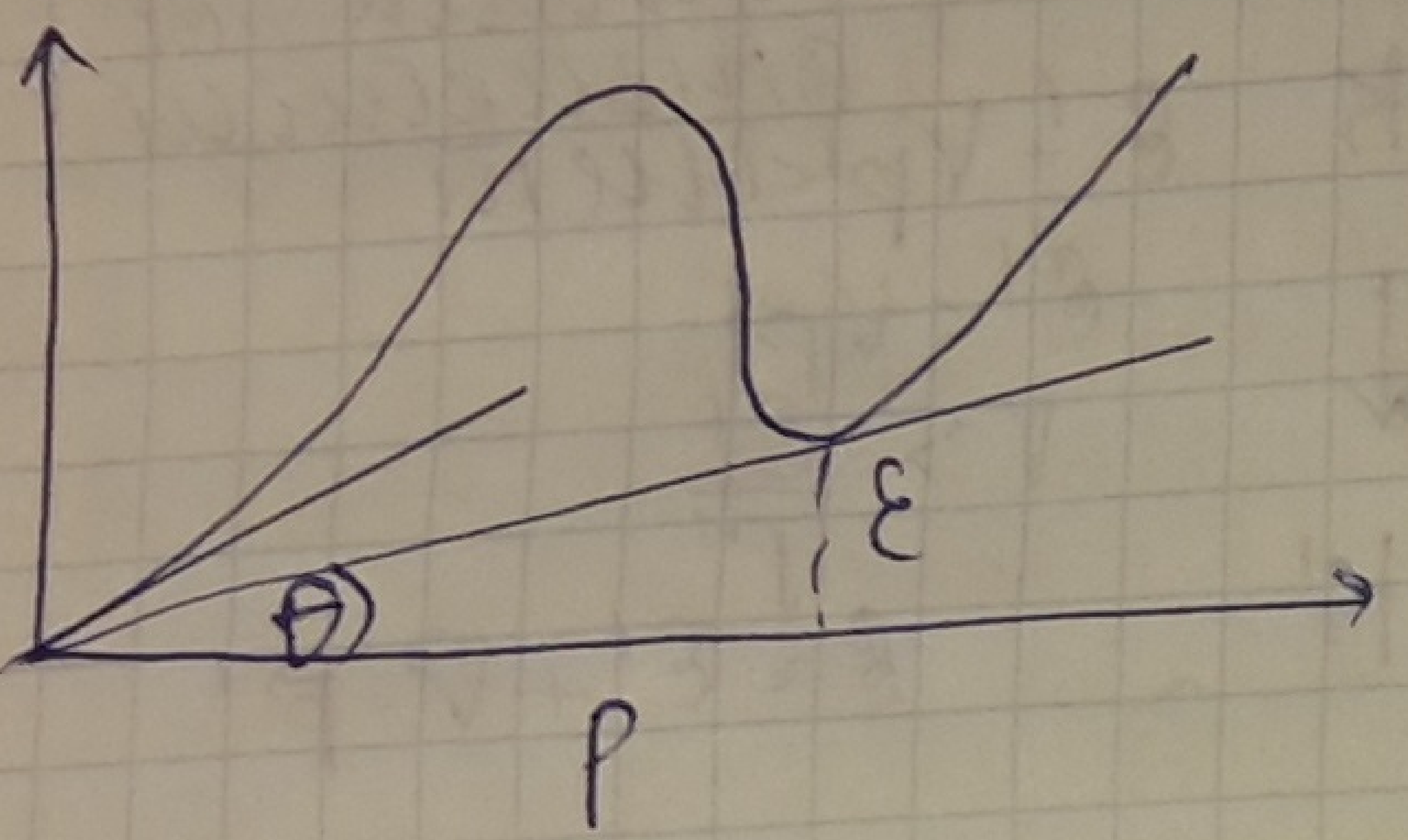


$\min \dfrac{\varepsilon(p)}{p} = u$

p слабо жид. бозе-газ

$\boxed{V < u}$ – сверхтек.

В реальной жидкости



Неравновесная статфизика

$na^3 \ll 1$

△ на примере больц. слабо неид. газа

Полная ф.р. $f(t, \vec{r}_1, \vec{p}_1, \vec{r}_2, \vec{p}_2, \ldots \vec{r}_N, \vec{p}_N)$

При $N \gg 1$ с такой ф.р. работать сложно, но можно △ част. как квазинезав. Поэтому △ $f(t, \vec{r}, \vec{p})$

$$N_a = \int \frac{f_a(t, \vec{r}_a, \vec{p}_a)\, d^3\vec{r}_a d^3\vec{p}_a}{\cancel{(2\pi\hbar)^3}}$$ — число част. в точке $\vec{r}$ с имп. $\vec{p}$

не будем писать

$$n_a(\vec{r}_a, t) = \int f_a(t, \vec{r}_a, \vec{p}_a)\, d^3\vec{p}_a$$

Для начала △ ид. случай газа без взаимодейств.

И для этой одночастич. ф.р. верно:

$$\boxed{\frac{df_a}{dt} = \frac{\partial f_a}{\partial t} + \underbrace{\vec{V}_a}_{\frac{d\vec r_a}{dt}} \frac{\partial f}{\partial \vec r_a} + \dot{\vec p}_a \frac{\partial f}{\partial \vec p_a} = 0}$$ ур. Лиувилля

$$\frac{\partial f_a}{\partial t} + \frac{\partial (V_{a\alpha} f)}{\partial r_{a\alpha}} + \frac{\partial (f \dot p_{a\alpha})}{\partial p_{a\alpha}} = 0$$

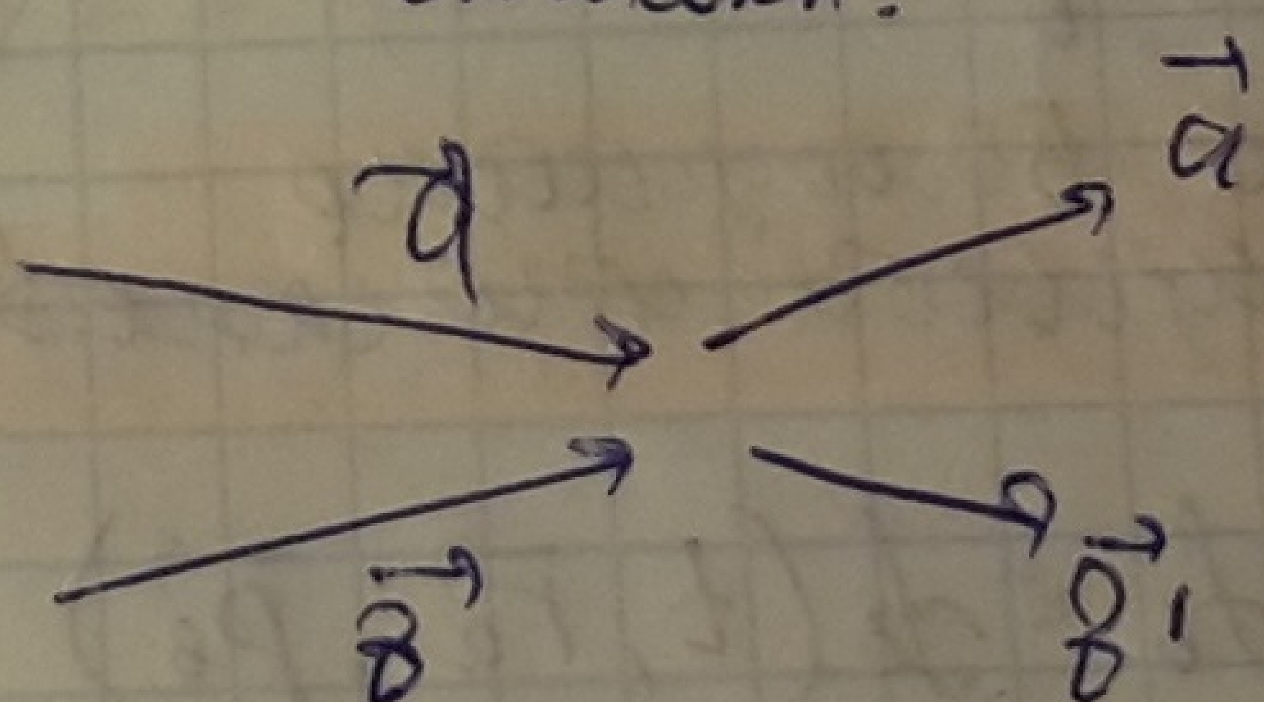
$$\frac{\partial f_a}{\partial t} + \operatorname{div} \vec V_a f + \operatorname{div} \dot{\vec p}_a f = 0$$

$N_{вход} = N_{выход}$

$$\frac{df_a}{dt} = \frac{\partial f_a}{\partial t} + \vec V_a \frac{\partial f}{\partial \vec r_a} + \dot{\vec p}_a \frac{\partial f}{\partial \vec p_a} = \left[\frac{\partial f_a}{\partial t}\right]_{ст} = I_{ст}(f_a) \; (=)$$

т.к. взаимод. сил., учитываем парн. взаимодейств.

$$(=) \; \underbrace{\sum_b}_{\text{по сортам частиц}} \underbrace{I_{ст}}_{\text{интегр. столкн.}} [f_a, f_b]$$

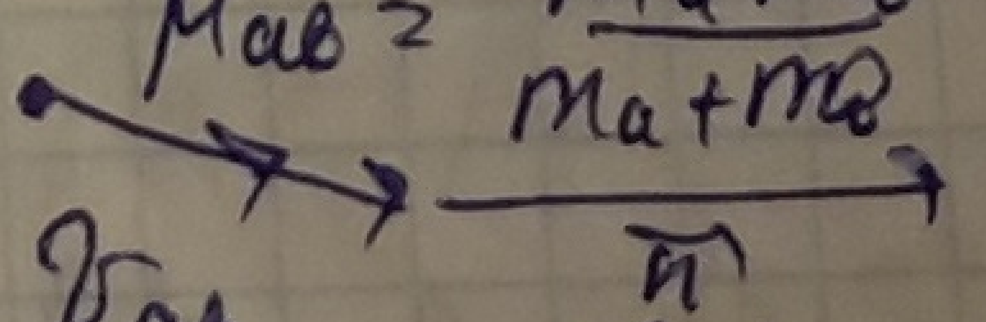


$$\vec p_a + \vec p_b = \vec p_a{}' + \vec p_b{}'$$

$$\frac{p_a^2}{2m_a} + \frac{p_b^2}{2m_b} = \frac{p_a'^2}{2m_a} + \frac{p_b'^2}{2m_b}$$

$$\vec V_{цм} = \frac{\vec p_a + \vec p_b}{m_a + m_b} = \frac{\vec p_a{}' + \vec p_b{}'}{m_a + m_b} = \vec V_{цм}{}'$$

$$\mu_{ab} = \frac{m_a m_b}{m_a + m_b}$$



$$\vec v_{ab}{}' = n \vec v_{ab}$$

$$\frac{\mu_{ab} v_{ab}^2}{2} = \frac{\mu_{ab} v_{ab}'^2}{2}$$

$$\Downarrow$$

$$v_{ab} = v_{ab}'$$

$$|\vec{v}_a - \vec{v}_b| = v_{ab} = \left|\frac{\vec{p}_a}{m_a} + \frac{\vec{p}_b}{m_b}\right| = \left|\frac{\vec{p}_a{}'}{m_a} - \frac{\vec{p}_b{}'}{m_b}\right| = v_{ab}'$$

$d\sigma_{ab}(v_{ab}, \theta, \varphi)$

$$\left[\frac{\partial f_a}{\partial t}\right]_{cm} d^3\vec{p} = \underbrace{d\sigma_{ab} v_{ab}}_{\frac{\text{см}^3}{\text{с}}} \underbrace{f_a d^3\vec{p}_a}_{\frac{1}{\text{см}^3}} \underbrace{f_b d^3\vec{p}_b}_{\frac{1}{\text{см}^3}} =$$

$$= d\sigma_{ab} v_{ab} f_a' d^3\vec{p}_a{}' f_b' d^3\vec{p}_b{}' =$$

$$= d^3\vec{p}_a{}' d^3\vec{p}_b{}' f_a' f_b' v_{ab} d\sigma_{ab} - d^3\vec{p}_a d^3\vec{p}_b f_a f_b v_{ab} d\sigma_{ab}$$

$$\boxed{d^3\vec{p}_a{}' d^3\vec{p}_b{}' = d^3\vec{p}_a d^3\vec{p}_b}$$

$$d^3\vec{p}_a d^3\vec{p}_b = m_a^3 m_b^3 d^3\underset{v_{ab}}{\vec{v}_a} d^3\vec{v}_b = m_a^3 m_b^3 d^3\underset{v_{ab}'}{v_{ab}} d^3 v_{ц.м.} =$$

$$= m_a^3 m_b^3 d^3\vec{v}_{ab}{}' d^3\vec{v}_{ц.м}{}'$$

$$I_{cm}[f_a, f_b] = \int d^3\vec{p}_b v_{ab} d\sigma_{ab} (f_a' f_b' - f_a f_b)$$

$\dot{\vec{p}}_a = \vec{F}_a$

$$\boxed{\frac{\partial f_a}{\partial t} + \vec{v}_a \cdot \frac{\partial f_a}{\partial \vec{r}_a} + \vec{F}_a \cdot \frac{\partial f_a}{\partial \vec{p}_a} = \sum_b \int d^3\vec{p}_b \vec{v}_{ab} d\sigma_{ab} (f_a' f_b' - f_a f_b)}$$

ур-ие Больцмана для парных столкнов.

$f_a' = f_a'(t, \vec{r}_a, \vec{p}_a{}')$ $\qquad f_b = f_b(t, \vec{r}_a, \vec{p}_b)$

$f_a = f_a(t, \vec{r}_a, \vec{p}_a)$ $\qquad f_b' = f_b'(t, \vec{r}_a, \vec{p}_b{}')$

Следствие. Без внешнего поля. $\vec{F}_a = 0$

$f_a^{равн} \equiv f_a^{равн}(\vec{p}_a)$

$$л.ч. = 0 = \sum_b \int d^3\vec{p}_b v_{ab} d\sigma_{ab} (f_a'^{равн} f_b'^{равн} - f_a^{равн} f_b^{равн})$$

$$f_a^{\prime\,равн} f_b^{\prime\,равн} = f_a^{равн} f_b^{равн}$$

$$\ln f_a^{\prime\,равн} + \ln f_b^{\prime\,равн} = \ln f_a^{равн} + \ln f_b^{равн}$$

$$\ln f_a^{равн}(p_a) = C_1 \frac{p_a^2}{2m_a} + \vec{C_2}\vec{p_a} + C_3(a)$$

$$f_a^{равн}(\vec{p_a}) = \frac{n_a}{(2\pi m_a T)^{3/2}} e^{-\frac{(\vec{p_a} - m_a \vec{u})^2}{2Tm_a}}$$

$$(C_1, C_2, C_3) \to (T, \underset{T}{\vec{u}}, n_a)$$ концентр. частиц

сред. ск. теч. газа

$$\int f^{равн}(\vec{p_a})\, d^3\vec{p_a} = n_a$$

$$\int f^{равн}(\vec{p_a}) \cdot \vec{p_a}\, d^3\vec{p_a} = m_a \vec{u}\, n_a$$

$$S = -\sum_a \int d^3\vec{p_a}\, d^3\vec{r_a}\, f_a(t, \vec{r_a}, \vec{p_a}) \ln f_a(t, \vec{r_a}, \vec{p_a})$$

$$\vec{F_a} = 0 \qquad f_a(t, \vec{p_a})$$

$$S = -V \sum_a \int d^3\vec{p_a}\, f_a(t, \vec{p_a}) \ln f_a(t, \vec{p_a})$$

$$\frac{d(S/V)}{dt} = -\sum_a d^3\vec{p_a}\,(1 + \ln f_a)\frac{\partial f_a}{\partial t} =$$

$$a \leftrightarrow b$$

$$= -\sum_{a,b} \int d^3\vec{p_a}\, d^3\vec{p_b}\, v_{ab}\, d\sigma_{ab}\, (f_a' f_b' - f_a f_b)(1 + \ln f_a) =$$

$$= -\frac{1}{2}\sum_{a,b} \int d^3\vec{p_a}\, d^3\vec{p_b}\, v_{ab}\, d\sigma_{ab}\, (f_a' f_b' - f_a f_b)(2 + \ln f_a + \ln f_b)$$

$$p_a \leftrightarrow p_a' \qquad p_b \leftrightarrow p_b'$$

$$d(S/V) = -\frac{1}{2}\sum_{a,b} \int d\vec{p_a'}\, d^3\vec{p_b'}\, v_{ab}\, d\sigma_{ab}\, (f_a f_b - f_a' f_b') \cdot$$

$$\cdot (2 + \ln f_a' + \ln f_b')$$

$$\frac{d(S/V)}{dt} = -\frac{1}{4}\sum_{a,b}\int d^3\vec{p}_a\, d^3\vec{p}_b\, v_{ab}\, d\sigma_{ab}\, (f_a' f_b' - f_a f_b)(2 + \ln(f_a f_b) - 2\ln f_a' f_b') =$$

$$= -\frac{1}{4}\sum_{a,b}\int d^3\vec{p}_a\, d^3\vec{p}_b\, v_{ab}\, d\sigma_{ab}\, \underbrace{(f_a' f_b' - f_a f_b)\ln\frac{f_a f_b}{f_a' f_b'}}_{y - x\ln\frac{x}{y} \leq 0}$$

$$\frac{d(S/V)}{dt} \geq 0$$

$$f_a f_b = f_a' f_b' \Leftarrow \frac{d(S/V)}{dt} = 0$$

ур. равн.

Положение о регулярном хаосе.

Однокомп. газ: с $\vec{F} = 0$

$$\varphi(\vec{p})\; \frac{\partial f}{\partial t} + \vec{v}\cdot\frac{\partial f}{\partial \vec{r}} = \int d^3\vec{p}_1\, v_{отн}\, d\sigma\, (f' f_1' - f f_1)$$

$f(t, \vec{r}, \vec{p})$ $\quad f_1(t, \vec{r}, \vec{p}_1)$

$f'(t, \vec{r}, \vec{p}')$ $\quad f_1'(t, \vec{r}, \vec{p}_1')$

$$\vec{p} + \vec{p}_1 = \vec{p}' + \vec{p}_1'$$

$$d\sigma(v_{отн}, \theta, \varphi)$$

$$\frac{\partial}{\partial t}\int d^3\vec{p}\, f\varphi(\vec{p}) + \frac{\partial}{\partial \vec{r}}\int d^3\vec{p}\cdot\vec{v}\cdot f\varphi(\vec{r}) =$$

$$= \int d^3\vec{p}\, d^3\vec{p}_1\, v_{отн}\, d\sigma\, (f' f_1' - f f_1)\,\varphi(\vec{p}) = \ldots$$

... Аналогичными рассуждениями ... =

$$= \frac{1}{4}\int d^3\vec{p}\, d^3\vec{p}_1\, v_{отн}\, d\sigma\, (f' f_1' - f f_1)(\varphi(\vec{p}) + \varphi(\vec{p}_1) - \varphi(\vec{p}') - \varphi(\vec{p}_1'))$$

$$\begin{cases} \varphi(\vec{p}) = const\,(1) \\ \varphi(\vec{p}) = p_\alpha \\ \varphi(\vec{p}) = \dfrac{\vec{p}^{\,2}}{2m} \end{cases} \Rightarrow \text{в прав. ч.} = 0$$

$\varphi(p) = 1$

$\Downarrow$

$$\frac{\partial}{\partial t}\underbrace{\int d^3\vec{p}\, f}_{n} + \frac{\partial}{\partial \vec{r}}\underbrace{\int d^3\vec{p}\, \vec{v}\cdot f}_{= n\vec{u}} = 0$$

$$\langle \vec{v} \rangle = \vec{u} = \frac{\int d^3 p \cdot \vec{v} \cdot f}{\int d^3 p \cdot f}$$

$$\boxed{\frac{\partial n}{\partial t} + div(n \cdot \vec{u}) = 0}$$

Лекция № 10.

22.04.16

$$\vec{p} + \vec{p}_1 = \vec{p}\,' + \vec{p}_1\,'$$

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{r}} = I_{ст}(f) =$$

$$= \int d^3\vec{p}_1\, v_{отн}\, d\sigma_{\vec{p}\vec{p}_1} (f' f_1' - f f_1)$$

$$\frac{\partial}{\partial t} \int d^3\vec{p} f \varphi(\vec{p}) + \frac{\partial}{\partial \vec{r}} \int d^3\vec{p} f \vec{v} \varphi(\vec{p}) = \Big| \begin{matrix} d^3\vec{p}_1 \\ \varphi(\vec{p}) \end{matrix}$$

$$= \frac{1}{4} \int d^3\vec{p}\, d^3\vec{p}_1\, \vec{v}_{отн}\, d\sigma_{\vec{p}\vec{p}_1} (f' f_1' - f f_1) [\varphi(\vec{p})$$

$$+ \varphi(\vec{p}_1) - \varphi(\vec{p}_1\,') - \varphi(\vec{p}\,')]$$

ЗСЭ и ЗСИ

$$\varphi(\vec{p}) = 1 \Rightarrow \boxed{\frac{\partial n}{\partial t} + \mathrm{div}(n\vec{u}) = 0}, \text{ где}$$

ЗС числа частиц

$$n(t, \vec{r}) = \int d^3\vec{p} \cdot f(t, \vec{r}, \vec{p})$$

$$u(t, \vec{r}) = \frac{1}{n} \int d^3\vec{p} \cdot \vec{v} \cdot f(t, \vec{r}, \vec{p})$$

$$\varphi(p) = m v_\alpha = p_\alpha$$

$$\frac{\partial}{\partial t}\underbrace{\int d^3\vec{p}\cdot f\cdot v_\alpha m}_{mnu_\alpha} + \frac{\partial}{\partial x_\beta}\underbrace{\int d^3\vec{p}\, f\, v_\beta m v_\alpha}_{\Pi_{\alpha\beta} = nm\langle v_\alpha v_\beta\rangle}$$

$$\langle A\rangle = \frac{\int d^3\vec{p}\, f A}{\int d^3\vec{p}\, f}$$

$$\frac{\partial(mnu_\alpha)}{\partial t} + \frac{\partial \Pi_{\alpha\beta}}{\partial x_\beta} = 0$$

тензор плотности потока импульса

$$\Pi_{\alpha\beta} = nm\langle v_\alpha v_\beta\rangle = mnu_\alpha u_\beta + mn\langle(v_\alpha - u_\alpha)\cdot(v_\beta - u_\beta)\rangle = mnu_\alpha u_\beta + \underbrace{mn\langle(v_\alpha - u_\alpha)(v_\beta - u_\beta) - \frac{\delta_{\alpha\beta}}{3}(\vec{v}-\vec{u})^2\rangle}_{-\sigma'_{\alpha\beta}} + \underbrace{\frac{mn}{3}\delta_{\alpha\beta}\langle(\vec{v}-\vec{u})^2\rangle}_{p\delta_{\alpha\beta}}$$

тензор напряж. вязких — давление

$$\sigma'_{\alpha\alpha} = 0$$

$$\frac{\partial(mnu_\alpha)}{\partial t} + \frac{\partial p}{\partial x_\alpha} + \frac{\partial(mnu_\alpha u_\beta)}{\partial x_\beta} = \frac{\partial\sigma'_{\alpha\beta}}{\partial x_\beta}$$

$$mn\frac{\partial u_\alpha}{\partial t} + mnu_\beta\frac{\partial u_\alpha}{\partial x_\beta} + \frac{\partial p}{\partial x_\alpha} + \underbrace{mu_\alpha\frac{\partial n}{\partial t} + mu_\alpha\frac{\partial(nu_\beta)}{\partial x_\beta}}_{} = \frac{\partial\sigma'_{\alpha\beta}}{\partial x_\beta}$$

$$\frac{\partial n}{\partial t} + \operatorname{div}(\vec{n}\cdot\vec{u}) = 0$$

$$\boxed{mn\left(\frac{\partial u_\alpha}{\partial t} + u_\beta\frac{\partial u_\alpha}{\partial x_\beta}\right) + \frac{\partial p}{\partial x_\alpha} = \frac{\partial\sigma'_{\alpha\beta}}{\partial x_\beta}}$$ ЗСИ

Ур-ие Ньютона! Т $mn = \rho$

$$\boxed{\rho \frac{du_\alpha}{dt} + \frac{\partial P}{\partial x_\alpha} = \frac{\partial \sigma'_{\alpha\beta}}{\partial x_\beta}}$$, где $\frac{d}{dt} = \frac{\partial}{\partial t} + u_\beta \frac{\partial}{\partial x_\beta} \equiv$

$\boxed{\text{ЗСИ}}$

$\equiv \frac{\partial}{\partial t} + \vec{u} \cdot \nabla$ субстанц. произв.

← первый интеграл

$$\varphi(\vec{p}) = \frac{\vec{p}^{\,2}}{2m} \Rightarrow \frac{\partial \varepsilon}{\partial t} + \operatorname{div} \vec{q} = 0$$

$$\varepsilon = n \left\langle \frac{mv^2}{2} \right\rangle \qquad \vec{q} = n \left\langle \vec{v} \cdot \frac{mv^2}{2} \right\rangle$$

$$\varepsilon = \frac{mn}{2} \langle (\vec{v} - \vec{u})^2 + \vec{u}^{\,2} \rangle = \frac{3P}{2} + \frac{mnu^2}{2}$$

$$\vec{q} = \frac{mn}{2} \langle (\vec{v} - \vec{u} + \vec{u})((\vec{v} - \vec{u})^2 + 2\vec{v}\cdot\vec{u} - \vec{u}^{\,2}) \rangle \Rightarrow$$

$$\rightarrow q_\alpha = \frac{mn}{2} \Big(\langle (\vec{v} - \vec{u})_\alpha (\vec{v} - \vec{u})^2 \rangle + u_\alpha \langle (\vec{v} - \vec{u})^2 \rangle +$$

$$+ 2 (\vec{v} - \vec{u})_\alpha u_\beta (\vec{v} - \vec{u})_\beta + u_\alpha u^2$$

$$q_\alpha = \frac{mn}{2} \langle (\vec{v} - \vec{u})_\alpha (\vec{v} - \vec{u})^2 \rangle + \frac{mn}{2} u_\alpha u^2 + nmu_\beta \cdot$$

$$\cdot \langle (v-u)_\alpha (v-u)_\beta - \frac{\delta_{\alpha\beta}}{3} (\vec{v} - \vec{u})^2 \rangle + \frac{5}{3 \cdot 2} nmu_\alpha \langle (\vec{v} - \vec{u})^2 \rangle$$

$$\boxed{q'_\alpha = \frac{mn}{2} \langle (\vec{v} - \vec{u})_\alpha (\vec{v} - \vec{u})^2 \rangle}$$ диссипативный поток тепла

$$\boxed{q_\alpha = q'_\alpha - u_\beta \sigma'_{\alpha\beta} + u_\alpha \left(\frac{mnu^2}{2} + \frac{5P}{2} \right)}$$

Тогда:

$$\frac{\partial}{\partial t} \left(\frac{3P}{2} + \frac{nmu^2}{2} \right) + \operatorname{div} \left\{ \vec{u} \cdot \left(\frac{5P}{2} + \frac{nmu^2}{2} \right) \right\} =$$

$$= \frac{\partial (u_\beta \sigma'_{\alpha\beta})}{\partial x_\beta} - \operatorname{div} \vec{q}\,'$$

$$\frac{3}{2}\cdot\frac{\partial P}{\partial t}+\frac{u^2}{2}\cdot\frac{\partial(mn)}{\partial t}+nmu_\alpha\frac{\partial u_\alpha}{\partial t}+\frac{5P}{2}\operatorname{div}\vec{u}+$$

$$+\frac{5u_\alpha}{2}\cdot\frac{\partial P}{\partial x_\alpha}+\cancel{\frac{mnu^2}{2}\operatorname{div}\vec{u}}+u_\alpha\left(\frac{u^2}{2}\cancel{\frac{\partial(mn)}{\partial x_\alpha}}+\right.$$

$$\left.+nmu_\beta\frac{\partial u_\alpha}{\partial x_\beta}\right)=u_\beta\frac{\partial\sigma'_{\alpha\beta}}{\partial x_\alpha}+\sigma'_{\alpha\beta}\frac{\partial u_\beta}{\partial x_\alpha}-\operatorname{div}q'$$

Итог:
$$\boxed{\frac{3}{2}\cdot\frac{\partial P}{\partial t}+\frac{5P}{2}\operatorname{div}\vec{u}+\frac{3u_\alpha}{2}\cdot\frac{\partial P}{\partial x_\alpha}=\sigma'_{\alpha\beta}\cdot\frac{\partial u_\beta}{\partial x_\alpha}-\operatorname{div}q'}$$
ЗСЭ

Знаем, что $P=\frac{nm}{3}\langle(\vec{v}-\vec{u})^2\rangle$

Введём $\frac{3T}{2}=\frac{m\langle(\vec{v}-\vec{u})^2\rangle}{2}$.

$$P(t,\vec{r})=n(t,\vec{r})\cdot T(t,\vec{r})$$

Упрощаем:

$$\frac{\partial n}{\partial t}+n\operatorname{div}\vec{u}=\frac{\partial n}{\partial t}+\operatorname{div}(\vec{n}\cdot\vec{u})=0$$
$$\downarrow$$
$$\frac{3}{2}n\frac{\partial T}{\partial t}+\underbrace{\frac{3}{2}T\frac{\partial n}{\partial t}+\frac{3}{2}nT\operatorname{div}u}_{0}+nT\operatorname{div}\vec{u}=$$

$$=\sigma'_{\alpha\beta}\frac{\partial u_\beta}{\partial x_\alpha}-\operatorname{div}q'$$

$$\frac{3}{2}n\frac{dT}{dt}+nT\operatorname{div}\vec{u}=\frac{3}{2}n\left(\underbrace{\frac{\partial T}{\partial t}+(\vec{u}\cdot\vec{\nabla})T}_{\frac{dT}{dt}}+\frac{2}{3}T\operatorname{div}\vec{u}\right)=$$

$$=\sigma'_{\alpha\beta}\frac{\partial u_\beta}{\partial x_\alpha}-\operatorname{div}\vec{q}'$$

б) $\frac{\partial f}{\partial t} + \vec{v}\cdot\frac{\partial f}{\partial \vec{r}} = \frac{1}{\varepsilon} I_{ст}(f) = \frac{1}{\varepsilon}\int d^3\vec{p}_1 \vec{v}_{отн}\cdot$

$\cdot d\sigma_{\vec{p}\vec{p}_1}(f' f_1' - f f_1)$

Разложение $f = f^{(0)} + \varepsilon\delta f^{(1)} + \varepsilon^2 \delta f^{(2)}$

$\varepsilon >> 1$ то $= 0$

$\Downarrow$

$F = f(\vec{r} - \vec{v}t, \vec{v})$

① $\int d^3\vec{p}_1 \vec{v}_{отн} d\sigma_{\vec{p}\vec{p}_1}(f^{(0)\prime} f_1^{(0)\prime} - f^{(0)} f_1^{(0)}) = 0$

② $\frac{\partial f^{(0)}}{\partial t} + \vec{v}\cdot\frac{\partial f^{(0)}}{\partial \vec{r}} = \int d^3\vec{p}_1 \cdot v_{отн} d\sigma_{\vec{p}\vec{p}_1}(f^{(0)\prime} f_1^{(1)\prime} +$

$+ \delta f^{(1)\prime} f_1^{(0)\prime} - f^{(0)}\delta f_1^{(1)} - \delta f^{(1)} f_1^{(0)})$

① $f^{(0)}(\vec{r}, t) = \frac{1}{(2\pi m T)^{3/2}} e^{-\frac{(\vec{p} - m\vec{u})^2}{2mT}}$

Будем считать:

$\int f d^3\vec{p} \equiv n \equiv \int f^{(0)} d^3\vec{p} \Rightarrow \int \delta f d^3\vec{p} = 0$

$\vec{u} = \frac{1}{n}\int f\vec{v} d^3\vec{p} = \ldots f^{(0)} \Rightarrow \int \delta f \vec{v} d^3\vec{p} = 0$

аналогично $T\cdot\frac{3}{2} = \frac{1}{n}\int f \frac{m(\vec{v} - \vec{u})^2}{2} d^3\vec{p} \Rightarrow$

$\Rightarrow \int \delta f \ldots = 0$

$\delta f^{(1)} << f^{(0)}$ при $\frac{\delta f^{(1)}}{f^{(0)}} \sim \frac{\tau}{t_{макро}} \simeq \frac{1}{n v \sigma t_{макро}} \sim$

$\sim \frac{l}{L_{макро}}$

$\frac{L}{l} >> 1 \Rightarrow \varepsilon >> 1$

В приближ. $\sigma'_{\alpha\beta} = 0$. Первая поправка (по $f^{(0)}$ сфер. сим.) $q'_\alpha = 0$

$$\text{л.ч.} = \frac{\partial f^{(0)}}{\partial t} + \vec{v}\cdot\frac{\partial f^{(0)}}{\partial \vec{r}} = \frac{f^{(0)}}{n}\left(\frac{\partial n}{\partial t} + \vec{u}\frac{\partial n}{\partial \vec{r}}\right) -$$

$$- \frac{3f^{(0)}}{2T}\left(\frac{\partial T}{\partial t} + \vec{v}\cdot\frac{\partial T}{\partial \vec{r}}\right) + \frac{m(\vec{v}-\vec{u})^2}{2T}\cdot\frac{f^{(0)}}{T}\left(\frac{\partial T}{\partial t} + \right.$$

$$\left. + \vec{v}\cdot\frac{\partial T}{\partial \vec{r}}\right) + f^{(0)}\frac{m(\vec{v}-\vec{u})}{T}\left(\frac{\partial \vec{u}}{\partial t} + (\vec{v}\cdot\nabla)\vec{u}\right)$$

$$\frac{\partial n}{\partial t} + \operatorname{div}(n\vec{u}) = 0 \qquad \rho\frac{d\vec{u}}{dt} + \nabla P = 0$$

$$\frac{dT}{dt} + \frac{2}{3}T\operatorname{div}\vec{u} = 0$$

$$\text{л.ч.} = \frac{f^{(0)}}{n}\left(-\operatorname{div}(n\vec{u}) + \vec{v}\frac{\partial n}{\partial \vec{r}}\right) + \left(-\frac{3f^{(0)}}{2T} + \right.$$

$$\left. + \frac{m(\vec{v}-\vec{u})^2}{2T}\cdot\frac{f^{(0)}}{T}\right)\left(-\frac{2}{3}T\operatorname{div}\vec{u} - \vec{u}\frac{\partial T}{\partial \vec{r}} + \right.$$

$$\left. + \vec{v}\frac{\partial T}{\partial \vec{r}}\right) + f^{(0)}\frac{m(\vec{v}-\vec{u})}{T}\left(-(\vec{u}\nabla)\vec{v} - \frac{\nabla(nT)}{mn} + \right.$$

$$\left. + (\vec{v}\cdot\nabla)\vec{u}\right)$$

$$\text{л.ч.} = \left((\vec{v}-\vec{u})\nabla T\right)\left(-\frac{5f^{(0)}}{2T} + \frac{m(\vec{v}-\vec{u})^2}{2T}\cdot\frac{f^{(0)}}{T}\right) -$$

$$- \frac{m(\vec{v}-\vec{u})^2}{3}\cdot\frac{f^{(0)}}{T}\operatorname{div}\vec{u} + f^{(0)}\cdot\frac{m(\vec{v}-\vec{u})}{T} *$$

$$* \left(\left((\vec{v}-\vec{u})\nabla\right)\vec{u}\right) =$$

$$= \frac{f^{(0)}}{T}\left((\vec{v}-\vec{u})\nabla T\right)\left(\frac{m(\vec{v}-\vec{u})^2}{2T} - \frac{5}{2}\right) +$$

$$+ m(\vec{v}-\vec{u})_\alpha(\vec{v}-\vec{u})_\beta\left[\frac{1}{2}\left(\frac{\partial u_\alpha}{\partial x_\beta} + \frac{\partial u_\beta}{\partial x_\alpha}\right) - \right.$$

$- \frac{\delta_{\alpha\beta}}{3} \operatorname{div} \vec{u} \,]$

$] \, u_{\alpha\beta} = \frac{\partial u_\alpha}{\partial x_\beta} + \frac{\partial u_\beta}{\partial x_\alpha} - \frac{2}{3} \delta_{\alpha\beta} \operatorname{div} \vec{u}$

$\vec{v}' = v - \vec{u}$

$\text{п.ч.} = \frac{f^{(0)}}{T} \left\{ (\vec{v}' \nabla T) \left(\frac{mv'^2}{2T} - \frac{5}{2} \right) + \frac{m}{2} v'_\alpha v'_\beta u_{\alpha\beta} \right\}$

Семинар № 11. 29.04.16

$\frac{3}{2} \cdot \frac{dT}{dt} + T \operatorname{div} \vec{u} = 0 \; (*) \Leftrightarrow \frac{ds}{dt} = 0$

$\left.\begin{aligned} \vec{q}' &= 0 \\ \sigma'_{\alpha\beta} &= 0 \end{aligned}\right\} f_{л.р.}$

$s = \frac{S}{V}$!

$\frac{dE}{V} = \frac{T dS}{V} + \frac{\mu dN}{V} \qquad (dV = 0)$

$\frac{d\mathcal{E}}{dt} = T \frac{ds}{dt} + \mu \frac{dn}{dt}$

$\frac{\partial n}{\partial t} + n \operatorname{div} \vec{u} = 0$

$\frac{d\mathcal{E}}{dt} + \frac{5}{2} nT \operatorname{div} \vec{u} = 0$

$\frac{ds}{dt} = \frac{\partial s}{\partial t} + \vec{u} \cdot \vec{\nabla} s$

$T \left(\frac{\partial s}{\partial t} + \vec{u} \nabla s \right) = -\frac{5}{2} nT \operatorname{div} \vec{u} + \mu n \operatorname{div} \vec{u} \; \Leftrightarrow$

29.04.16

~~Лекция № 12~~. Лекция № 12.

а) Было:

$$f = f^{(0)} + \delta f^{(1)}, \text{ где } \frac{\partial f^{(0)}}{\partial t} + \vec{v}\cdot\frac{\partial f^{(0)}}{\partial \vec{r}} = (*)$$

$$(*) = \int d^3\vec{p}_1 \cdot v_{отн}\, d\sigma_{\vec{p}\vec{p}_1} \cdot \left(f^{(0)\prime}\delta f_1^{(1)\prime} + f_1^{(0)\prime}\delta f^{(1)\prime} - f^{(0)}\delta f_1^{(1)} - f_1^{(0)}\delta f^{(1)}\right)$$

$$\boxed{f_a^{(0)} = \frac{n}{(2\pi m T)^{3/2}} e^{-\frac{m(\vec{v}-\vec{u})^2}{2T}}}$$

Вычислим:

$$!!\ л.ч. = \frac{f^{(0)}}{T}\left\{(\vec{v}'\cdot\nabla T)\left(\frac{m\vec{v}'^2}{2T} - \frac{5}{2}\right) + \frac{m}{2} v_\alpha' v_\beta' u_{\alpha\beta}\right\}$$

где $u_{\alpha\beta} = \frac{\partial u_\alpha}{\partial x_\beta} + \frac{\partial u_\beta}{\partial x_\alpha} - \frac{2}{3}\delta_{\alpha\beta}\,\mathrm{div}\,\vec{u}$

Ищем в виде:

$$\delta f^{(1)} = \frac{f^{(0)}}{T}\chi^{(1)}$$

$$п.ч. = \frac{1}{T}\int d^3\vec{p}_1 v_{отн}\, d\sigma_{\vec{p}\vec{p}_1}\left(f^{(0)\prime} f_1^{(0)\prime}\chi^{(1)} + f_1^{(0)\prime} f^{(0)\prime}\chi_1^{(1)} - f^{(0)} f_1^{(0)}\chi_1^{(1)} - f^{(0)} f_1^{(0)}\chi^{(1)}\right)$$

Из равновес. $\boxed{f^{(0)\prime} f_1^{(0)\prime} = f^{(0)} f_1^{(0)}}$

$$\frac{\partial f^{(0)}}{\partial t} + \vec{v}\frac{\partial f^{(0)}}{\partial \vec{r}} = \frac{f}{T}\int d^3\vec{p}_1 v_{отн}\, d\sigma_{\vec{p}\vec{p}_1} f_1^{(0)}\left(\chi^{(1)\prime} + \chi_1^{(1)\prime} - \chi_1^{(1)} - \chi^{(1)}\right)$$

б) Пренебрежение

п.ч.: $-\frac{f^{(0)}}{T}\cdot\frac{\chi^{(1)}}{\tau}$, где $\tau(v') = t(|\vec{v}-\vec{u}|)$, $\tau \sim \frac{1}{n v_T \sigma}$

$$\frac{\delta f^{(1)}}{f^{(0)}} \ll 1 \qquad \frac{\tau v_T}{L} \sim \frac{l}{L} \ll 1$$

$\frac{l}{L} \sim (Kn)$ – п-р Кнудсена

Тогда

$$\chi^{(1)} = -t\,\{(\vec{v}'\cdot\nabla T)\left(\frac{mv'^2}{2T}-\frac{5}{2}\right)+\frac{m}{2}v_\alpha' v_\beta' u_{\alpha\beta}$$

Вычислим диссипативные потоки:

$$\vec{q}\,' = n\left\langle \vec{v}\,'\frac{mv'^2}{2}\right\rangle \qquad \vec{v}\,' = \vec{v}-\vec{u} \qquad f = f^{(0)}+\delta f^{(1)}$$

поток тепла $$q'_\alpha = \int d^3\vec{p}\,\delta f^{(1)} v_\alpha' \frac{mv'^2}{2} =$$

$$= -\int d^3\vec{p}\,\underbrace{\frac{f^{(0)}}{T}\tau}_{\delta f^{(0)}}\,\underbrace{v_\beta' v_\alpha'}_{\frac{1}{3}\delta_{\alpha\beta}v'^2}\,\nabla_\beta T\left(\frac{mv'^2}{2T}-\frac{5}{2}\right)\frac{mv'^2}{2}$$

$$q'_\alpha = -\nabla_\alpha T\,\overbrace{\int d^3\vec{p}\,\frac{f^{(0)}t_m}{6T}\left(\frac{mv'^6}{2T}-\frac{5}{2}v'^4\right)}^{\varkappa} =$$

$$= -\varkappa\nabla_\alpha T \Rightarrow \vec{q}\,' = -\varkappa\nabla T$$

$$\varkappa = n\left\langle \frac{\tau m}{6T}\left(\frac{mv'^6}{2T}-\frac{5}{2}v'^4\right)\right\rangle_{(0)}$$

$$\langle v'^{2n}\rangle_{(0)} = \frac{(T/m)^n}{(2\pi)^{3/2}}\int \underbrace{d^3x}_{4\pi x^2dx}\cdot x^{2n}e^{-x^2/2} = \frac{(T/m)^n}{(2\pi)^{3/2}}\cdot 2\pi\int_0^\infty dx^2\cdot x^{2n+1}e^{-x^2/2} =$$

$$= (2n+1)!!\left(\frac{T}{m}\right)^n$$

Если $\tau = \mathrm{const}$, то $$\varkappa = \frac{n\tau m}{6T}\left(\frac{m}{2T}\left(\frac{T}{m}\right)^3\cdot 7\cdot 15-\frac{5}{2}\left(\frac{T}{m}\right)^2\cdot 15\right) =$$

$$= \frac{5}{2}\cdot\frac{n\tau T}{m} \sim \frac{nmv_T^2}{mnv_T\sigma} \sim \frac{v_T}{\sigma}$$

$$\sigma'_{\alpha\beta} = -nm\left\langle v_\alpha' v_\beta' - \frac{1}{3}\delta_{\alpha\beta}v'^2\right\rangle$$

$$= -m\int d^3\vec{p}\left(-\frac{f^{(0)}\chi^{(1)}}{T}\right)\left(v_\alpha' v_\beta' - \frac{1}{3}\delta_{\alpha\beta}v'^2\right) =$$

$$= m\int d^3\vec{p}\,\frac{f^{(0)}}{T}\tau\cdot\frac{m}{2}v_\mu' v_\nu' u_{\mu\nu}\left(v_\alpha' v_\beta' - \frac{1}{3}\delta_{\alpha\beta}v'^2\right) =$$

$$= \frac{m^2}{2T} u_{\mu\nu} \int d^3p' f^{(0)} \tau v_\mu' v_\nu' \left(v_\alpha' v_\beta' - \frac{1}{3}\delta_{\alpha\beta} v'^2 \right) =$$

$$= \frac{nm^2}{2T} u_{\mu\nu} \left\langle \tau v_\mu' v_\nu' \left(v_\alpha' v_\beta' - \overset{0}{\frac{\delta_{\alpha\beta}}{3}} v'^2 \right) \right\rangle_{(0)} = (**)$$

где $\langle v_\alpha' v_\beta' \rangle_{(0)} = \frac{1}{3}\delta_{\alpha\beta} \langle v'^2 \rangle_{(0)}$

$$\langle v_\alpha' v_\beta' v_\mu' v_\nu' \rangle = \frac{1}{15} (\delta_{\alpha\beta}\delta_{\mu\nu} + \delta_{\alpha\mu}\delta_{\beta\nu} + \delta_{\alpha\nu}\delta_{\beta\mu}) \langle v'^4 \rangle_{(0)}$$

$u_{\alpha\alpha} = 0$

$$(**) = \frac{m^2 n}{2T} u_{\mu\nu} \langle \tau v'^4 \rangle_{(0)} \frac{1}{15} \ldots =$$

$$= \frac{nm^2}{2T} \cdot 2 \cdot \frac{\langle \tau v^4 \rangle_{(0)}}{15} u_{\alpha\beta} = \Big/ u_{\alpha\beta} \text{ при } \tau = \text{const}$$

$$= \frac{nm^2}{15T} \cdot 15 \left(\frac{T}{m}\right)^2 = n\tau T \sim \frac{v_T m}{\sigma}$$

$$\sigma_{\alpha\beta}' = \eta \left(\frac{\partial u_\alpha}{\partial x_\beta} + \frac{\partial u_\beta}{\partial x_\alpha} - \frac{2}{3}\delta_{\alpha\beta} \operatorname{div}\vec{u} \right) + \zeta \delta_{\alpha\beta} \operatorname{div}\vec{u}$$

у нас $\eta = n\tau T$, $\zeta = 0$ (где $\zeta \ll \eta$)

в) Ур-ние с потоками:

$$\frac{\partial n}{\partial t} + \operatorname{div} n\vec{u} = 0$$

$$\rho \frac{du_\alpha}{dt} = \rho \left(\frac{\partial u_\alpha}{\partial t} + u_\beta \frac{\partial u_\alpha}{\partial x_\beta} \right) = -\frac{\partial p}{\partial x_\alpha} + \frac{\partial \sigma_{\alpha\beta}'}{\partial x_\beta}$$

т.к. $\frac{\partial^2 u_\alpha}{\partial x_\alpha \partial x_\beta} \equiv \Delta u_\alpha$ $\quad \frac{\partial \sigma_{\alpha\beta}}{\partial x_\beta} = \eta \Big(\Delta u_\alpha + \nabla_\alpha \operatorname{div}\vec{u} -$

$$- \frac{2}{3} \nabla_\alpha \operatorname{div}\vec{u} \Big) + \zeta \nabla_\alpha \operatorname{div}\vec{u} \}$$

$$\rho\frac{d\vec{u}}{dt} = -\nabla P + \eta\Delta\vec{u} + \left(\frac{\eta}{3}+\zeta\right)\operatorname{grad}\operatorname{div}\vec{u}$$ Ур-ие Навье-Стокса

3) $$\frac{3n}{2}\cdot\frac{dT}{dt} + nT\operatorname{div}\vec{u} = \sigma'_{\alpha\beta}\frac{\partial u_\beta}{\partial x_\alpha} - \operatorname{div}q'$$ Ур-ие переноса энергии

$$\frac{3n}{2}\cdot\frac{dT}{dt} + nT\operatorname{div}\vec{u} = \operatorname{div}(\varkappa\nabla T) + \frac{\eta}{2}\left(\frac{\partial u_\alpha}{\partial x_\beta} + \frac{\partial u_\beta}{\partial x_\alpha} - \frac{2}{3}\delta_{\alpha\beta}\operatorname{div}\vec{u}\right)^2 + \zeta(\operatorname{div}\vec{u})^2$$

$$\sigma_{\alpha\beta}\left(-\frac{1}{2}\left(\frac{\partial u_\beta}{\partial x_\alpha} + \frac{\partial u_\alpha}{\partial x_\beta}\right) - \frac{2}{3}\delta_{\alpha\beta}\operatorname{div}\vec{u}\right)$$

Если $\vec{u}=0$, $C_V\cdot\frac{\partial T}{\partial t} = \operatorname{div}(\varkappa\nabla T)$

$\frac{3n}{2}$

2) $$= s' = \frac{S}{V} = -\int d^3\vec{p}\, f(t,\vec{r},\vec{p})\ln f(t,\vec{r},\vec{p})$$

$$s(\vec{r},t) = \frac{s'}{n}$$

$$ns = -\int d^3\vec{p}\,(f_0+\delta f^{(1)})\ln(f_0+\delta f^{(1)}) =$$

$$= -\int d^3\vec{p}\,(f_0+\delta f^{(1)})\left(\ln f_0 + \frac{\delta f^{(1)}}{f_0}\right) =$$

$$= -\int d^3\vec{p}\,(f_0\ln f_0 + \delta f^{(1)}\cancel{\ln f_0} + 1)$$

Но $\ln f_0 = C_1 + \vec{C_2}\cdot\vec{p} + C_3\frac{p^2}{2m}$ $\exists$ по усл ортогон.

$$\int ..\, \delta f^{(1)}(\ln f^{(0)} + 1) = 0$$

$$s = -\frac{1}{n}\int d^3\vec{p}\cdot f_0\ln f_0 = -\langle \ln f_0\rangle_{(0)}$$ верно с первой поправкой

$$f^{(0)} = \frac{n}{(2\pi m T)^{3/2}} e^{-\frac{m(\vec{v}-\vec{u})^2}{2T}}$$

$$s = \frac{3}{2} - \ln\frac{n}{(2\pi m T)^{3/2}}$$

$$nT\frac{ds}{dt} = \left(-\frac{1}{n}\frac{dn}{dt} + \frac{3}{2}\cdot\frac{1}{T}\cdot\frac{dT}{dt}\right)nT = -T\frac{dn}{dt} +$$

$$+ \frac{3n}{2}\cdot\frac{dT}{dt} = \text{// пункт 3 //} = -T\frac{dn}{dt} - nT\,\mathrm{div}\,\vec{u} +$$

$$+ \sigma'_{\alpha\beta}\frac{\partial u_\beta}{\partial x_\alpha} - \mathrm{div}\,\vec{q}\,' =$$

$$= -T\underbrace{\left(\frac{\partial n}{\partial t} + (\vec{u}\cdot\nabla)n + n\,\mathrm{div}\,\vec{u}\right)}_{\frac{\partial n}{\partial t} + \mathrm{div}\,n\vec{u} = 0} + \sigma'_{\alpha\beta}\frac{\partial u_\beta}{\partial x_\alpha} - \mathrm{div}\,\vec{q}\,'$$

$$\boxed{nT\frac{ds}{dt} \equiv nT\left(\frac{\partial s}{\partial t} + (\vec{u}\cdot\nabla)s\right) = \sigma'_{\alpha\beta}\frac{\partial u_\beta}{\partial x_\alpha} - \mathrm{div}\,\vec{q}\,'}$$

ур-ие переноса одночастичной энергии

т.е. $\left(\frac{ds}{dt}\right)_{(0)} \equiv 0$

$$\frac{\partial(ns)}{\partial t} + \mathrm{div}(ns\vec{u}) = s\left(\frac{\partial n}{\partial t} + \mathrm{div}\,n\vec{u}\right) + n\left(\frac{\partial s}{\partial t} + (\vec{u}\cdot\nabla)s\right) =$$

$$= \frac{1}{T}\sigma'_{\alpha\beta}\frac{\partial u_\beta}{\partial x_\alpha} - \frac{1}{T}\mathrm{div}\,\vec{q}\,'$$

$$\frac{\partial(ns)}{\partial t} + \mathrm{div}(ns\vec{u}) = \frac{1}{T}\sigma'_{\alpha\beta}\frac{\partial u_\beta}{\partial x_\alpha} - \mathrm{div}\left(\frac{\vec{q}\,'}{T}\right) + \vec{q}\,'\cdot\nabla\frac{1}{T} \Rightarrow$$

$$\Rightarrow (A) \equiv \frac{\partial(ns)}{\partial t} + \mathrm{div}(ns\vec{u} + \vec{q}\,'_s) = \frac{1}{T}\sigma'_{\alpha\beta}\frac{\partial u_\beta}{\partial x_\alpha} - \frac{\nabla T\cdot\vec{q}\,'}{T^2},$$

где $\vec{q}\,'_s = \frac{q'}{T} = -\frac{\varkappa\nabla T}{T}$

Учитывая потоки:

$(A) = Q_s$, где $Q_s = \frac{\varkappa(\nabla T)^2}{T^2} + \frac{\eta}{2T}\left(\frac{\partial u_\alpha}{\partial x_\beta} + \frac{\partial u_\beta}{\partial x_\alpha} - \right.$

$\left. - \frac{2}{3}\delta_{\alpha\beta}\,\mathrm{div}\,\vec{u}\right)^2 + \frac{\zeta}{T}(\mathrm{div}\,\vec{u})^2 > 0$

$$\int\left(\frac{\partial(ns)}{\partial t} + \underbrace{\mathrm{div}(ns\vec{u} + q'_s)}_{0}\right)dV = \frac{d}{dt}\int ns\,dV = \frac{dS}{dt} =$$

$$= \int Q_s\,dV$$