Statistical Inference Course Project - Part I

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Project directions

In this project you will investigate the exponential distribution in R and compare it with the Central Limit Theorem. The exponential distribution can be simulated in R with rexp(n, lambda) where lambda is the rate parameter. The mean of exponential distribution is 1/lambda and the standard deviation is also 1/lambda. Set lambda = 0.2 for all of the simulations. You will investigate the distribution of averages of 40 exponentials. Note that you will need to do a thousand simulations.

Overview:

The exponential distribution can be simulated in R with rexp(n, lambda) where lambda λ is the rate parameter. The mean of exponential distribution is $1/\lambda$ and the standard deviation is also $1/\lambda$. For this simulation, we set $\lambda = 0.2$. In this simulation, we investigate the distribution of averages of 40 numbers sampled from exponential distribution with $\lambda = 0.2$.

Simulations:

For this assignment we are performing 1,000 simulations for 40 exponetials. The code for that is as follows:

```
set.seed(3)
lambda <- 0.2
num_sim <- 1000
sample_size <- 40
sim <- matrix(rexp(num_sim*sample_size, rate=lambda), num_sim, sample_size)
row_means <- rowMeans(sim)
```

Sample Mean versus Theoretical Mean:

We can calculate and plot the distribution of the Sample Mean and compare it to the Theoretical Mean using the following code (see appendix for actual plot):

```
# plot the histogram of averages
hist(row_means, breaks=50, prob=TRUE,
     main="Distribution of averages of samples,
     drawn from exponential distribution with lambda=0.2",
     xlab="")
# density of the averages of samples
lines(density(row_means))
# theoretical center of distribution
abline(v=1/lambda, col="blue")
# theoretical density of the averages of samples
xfit <- seq(min(row_means), max(row_means), length=100)
yfit <- dnorm(xfit, mean=1/lambda, sd=(1/lambda/sqrt(sample_size)))
lines(xfit, yfit, pch=22, col="red", lty=2)
```

```r
# add legend
legend('topright', c("Sample Mean", "Theoretical Mean"), lty=c(1,2), col=c("black", "red"))
```

Sample Variance versus Theoretical Variance:

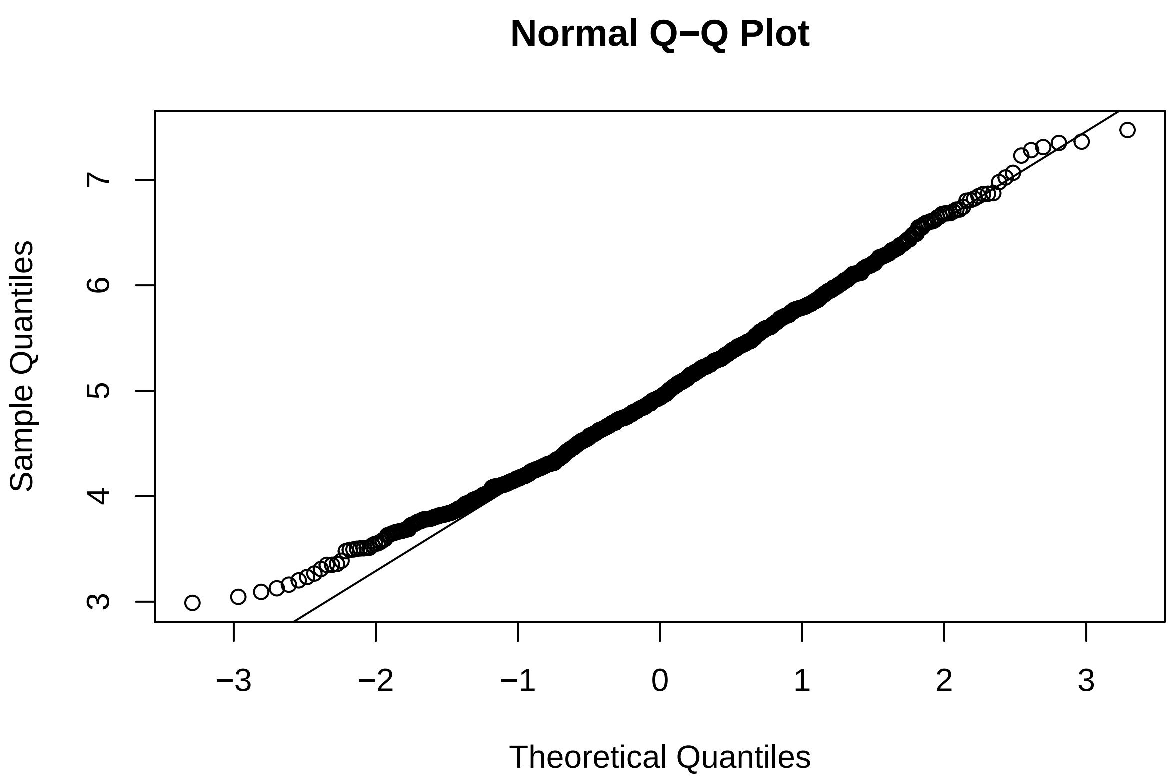
The distribution of sample means is centered at 4.9866 and the theoretical center of the distribution is $\lambda^{-1} = 5$. The variance of sample means is 0.6258 where the theoretical variance of the distribution is $\sigma^2/n = 1/(\lambda^2 n) = 1/(0.04 \times 40) = 0.625$.

Reference the figure above to viualize the variances.

Distribution:

Due to the central limit theorem, the averages of samples follow normal distribution. The figure above also shows the density computed using the histogram and the normal density plotted with theoretical mean and variance values. Also, the q-q plot below suggests the normality.

```r
qqnorm(row_means); qqline(row_means)
```



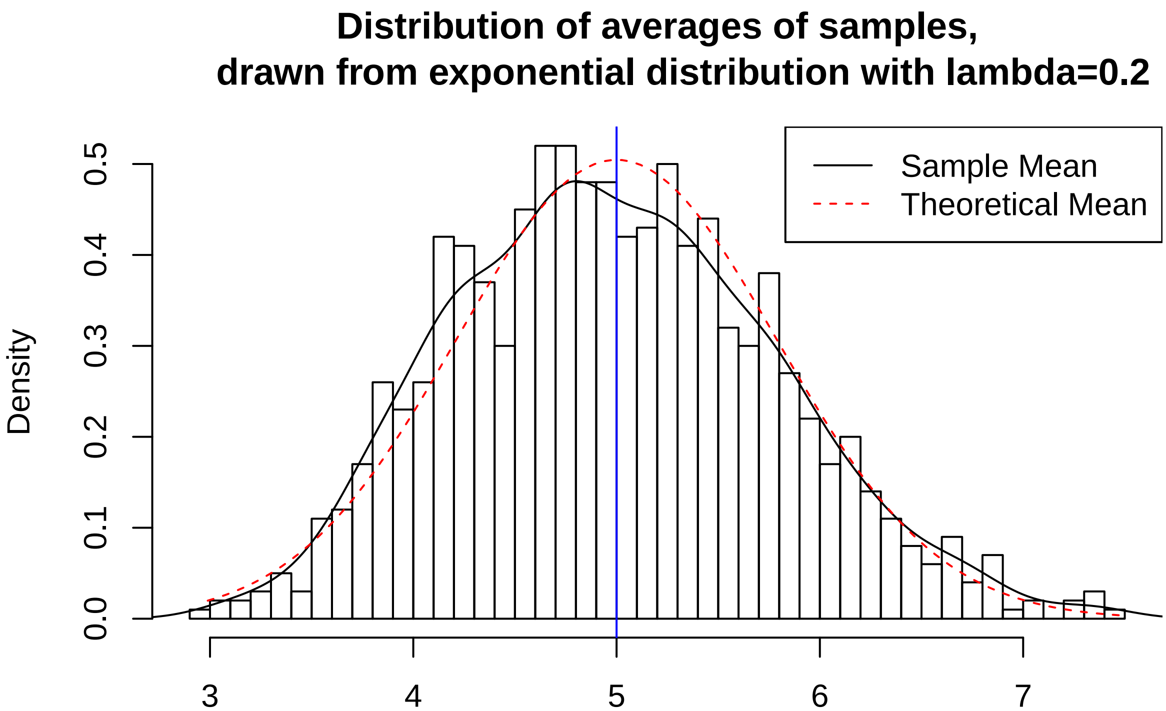
See appendix for plot showing the coverage of the confidence interval.

Conclusion

The 95% confidence intervals for the rate parameter (λ) to be estimated ($\hat{\lambda}$) are $\hat{\lambda}low = \hat{\lambda}(1 - \frac{1.96}{\sqrt{n}})$ agnd $\hat{\lambda}upp = \hat{\lambda}(1 + \frac{1.96}{\sqrt{n}})$. As can be seen from the plot above, for selection of $\hat{\lambda}$ around 5, the average of the sample mean falls within the confidence interval at least 95% of the time. Note that the true rate, λ is 5.

Appendix

Distribution of Sample Mean versus Theoretical Mean



Appendix

Confidence of Coverage Interval

The following plot displays the coverage of the confidence interval for $1/\lambda = \bar{X} \pm 1.96\frac{S}{\sqrt{n}}$