COMP6247 Lab 2: Kalman Filter

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1 Introduction

This report presents the findings and results for Lab 2 of COMP6247 of University of Southampton [1]. The code implementation is stored in a Github repository [2].

First, a random excitation signal is generated using Numpy sampled from a normal distribution, $X \sim \mathcal{N}(0, 1)$. The excitation signal is then transformed into a second order autoregressive (AR) time series.

Two AR series are generated. One with constant parameters, $\mathbf{a} = (1.2, -0.4)^T$. Another is with time-varying parameters defined in Equation (1). The signal plots will not be shown as it is identical to the ones in [1].

$$a_0(t) = \frac{1}{10}\cos(\frac{2\pi t}{400}) + 1.2 \tag{1a}$$

$$a_1(t) = \frac{1}{10}\sin(\frac{\pi t}{400}) - 0.4 \tag{1b}$$

A Kalman Filter is used for state estimation of the parameters of the both the stationary and time-varying AR process. The effects of initial conditions, process noise covariance and measurement noise variance on the convergence on the state estimation are explored.

2 Effects of Initial Conditions

In this section, the effects of initial conditions $\boldsymbol{\theta}(0|0)$ and $\boldsymbol{P}(0|0)$ on the convergence of the parameter estimation is explored. Throughout the experiment in this section, the process noise covariance $\boldsymbol{Q}$ and measurement noise variance R are set to $0.01I$ and the variance of the AR process, respectively. Both stationary and non-stationary plots have similar effects when initial conditions are changed, hence only the stationary plots will be shown in this section.

2.1 Initial State

Five different initial values of initial state $\boldsymbol{\theta}(0|0)$ will be explored. The initial value of the posteriori estimate covariance $\boldsymbol{P}(0|0)$ is set to $0.001I$. Figure 1 shows the convergence of five different initial state values, namely:

1. Random values generated from Numpy, $\boldsymbol{\theta}(0|0) \sim \mathcal{N}(0, 1)$;

2. $(0,0)^T$;
3. $(100,100)^T$;
4. $(1.2,-0.4)^T$;
5. $(-100,-100)^T$;

It is shown that all five different initial values of $\boldsymbol{\theta}(0|0)$ eventually converges to the same values after certain number of iterations. However, initialising the values closer to the true values allow for a faster convergence. Hence, it can be concluded that $\boldsymbol{\theta}(0|0)$ does not affect the convergence of the parameter estimation, but tuning it will increase the convergence speed.

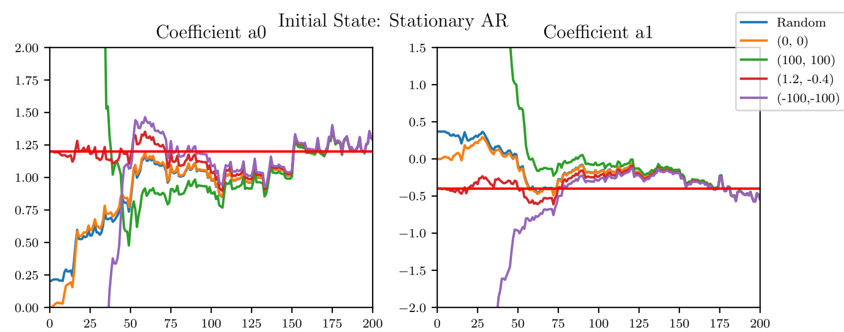


Figure 1: Stationary AR parameter estimation with different initial states.

2.2 Initial Parameter Covariance

Five different initial values of initial state covariance $\boldsymbol{P}(0|0)$ will be explored. The initial value of the posteriori state estimate $\boldsymbol{\theta}(0|0)$ is set to $\boldsymbol{\theta}(0|0) \sim \mathcal{N}(0,1)$. Figure 2 shows the convergence of five different initial covariance values, namely:

1. $0.001I$;
2. $Var(ex)I$, the variance of the excitation signal;
3. $1000I$;
4. $0I$;
5. I;

Similar to Section 2.1, all five different initial values of $\boldsymbol{P}(0|0)$ eventually converges to the same values after certain number of iterations. $\boldsymbol{P}(0|0)$ signifies the estimation confidence of the initial parameter estimate $\boldsymbol{\theta}(0|0)$. Setting $\boldsymbol{P}(0|0)$ to a large value allows more fluctuations in the initial iterations but faster convergence, as shown in the green line in Figure 2.

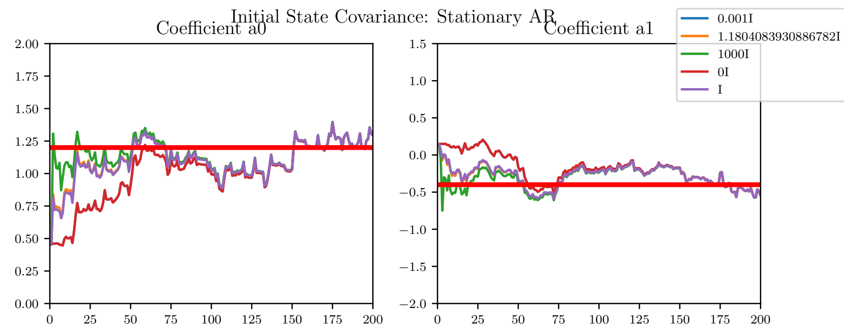


Figure 2: Stationary AR parameter estimation with different initial state covariances.

3 Effects of Process Noise

Figure 3 shows the convergence of parameters with different values of process noise covariance Q. For this section's experiment, the following initial values and parameters are set:

- $\boldsymbol{\theta}(0|0) \sim \mathcal{N}(0, 1)$;
- $\boldsymbol{P}(0|0) = 100I$;
- $R = Var(Observation)$;

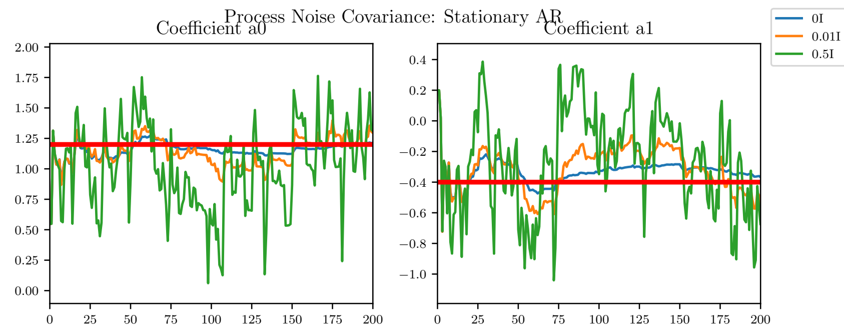
For the stationary AR process, three values of Q are chosen, namely: $0I$, $0.01I$, $0.5I$. For the non-stationary AR process, only two values of Q are chosen, namely: $0I$ and $0.01I$. From Figure 3, we can see that for both stationary and non-stationary AR processes, increasing the value of Q does not contribute to the convergence. Rather, it introduces more variance and fluctuations of the convergence. Setting Q to $0I$ provides the most stable and better convergence. This is reasonable as no noise was introduced when the AR time series is generated from the excitation signal.

For non-stationary AR process, with $Q = 0I$, Figure 3b show that the estimation is able to capture the time-varying parameter. However, there is a small lag with the parameter estimation, thus being insensitive to time-varying parameters.

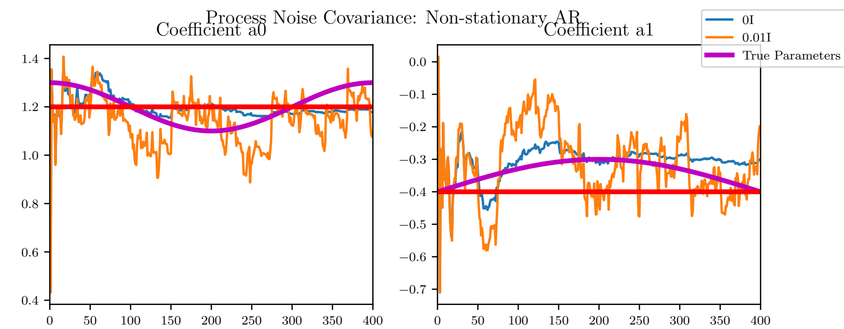
4 Effects of Observation Noise

Figure 4 shows the convergence of parameters with different values of observation noise variance R. For this section's experiment, the following initial values and parameters are set:

- $\boldsymbol{\theta}(0|0) \sim \mathcal{N}(0, 1)$;



(a) Stationary AR



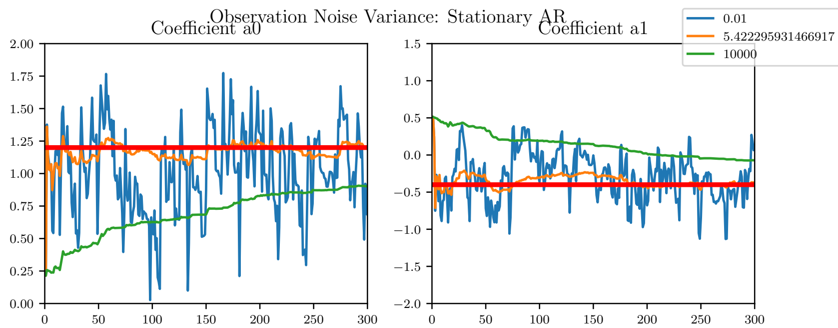
(b) Non-statinoary AR

Figure 3: AR parameter estimation with different process noise covariance Q.

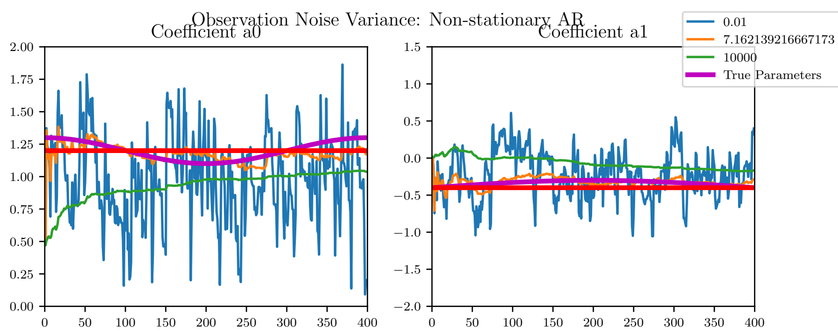
- $\boldsymbol{P}(0|0) = 100I$;
- $\boldsymbol{Q} = 0.001I$

For both stationary and non-stationary AR process, three parameters are set, namely: 0.01, $Var(Observation)$, 10000, where $Observation$ is the AR process. From Figure 4, it is shown that smaller values enables faster convergence. However, for values that are too small, in this case $R < 0.1$, more fluctuation and variance of the parameter estimation is introduced. For larger values, there are less fluctuations and more stable convergence at the expense of slower convergence.

In many applications, R is set the the variance of the observation if it is known [3, p. 265]. Figure 4 shows that by using the variance of the observation, it is able to provide reasonable convergence speed with less noise. However, R is a parameter to be further tuned from prior knowledge of the problem.



(a) Stationary AR



(b) Non-stationary AR

Figure 4: AR parameter estimation with different observation noise variance R.

References

[1] Mahesan Niranjan. *COMP6247(2020/21): Reinforcement and Online Learning, Kalman Filter*. School of Electronics and Computer Science, University of Southampton, 16 February 2021.

[2] Eugene Teoh. *COMP6247 Labs*. 4 March 2021. URL: `https://github.com/eugeneteoh/COMP6247-Labs`.

[3] Roger Labbe. *Kalman and Bayesian Filters in Python*. `https://github.com/rlabbe/Kalman-and-Bayesian-Filters-in-Python`. 2020.