TECNICATURA UNIVERSITARIA EN PROGRAMACIÓN

TRABAJO PRÁCTICO N°2: ALGEBRA BOOLEANA (PARTE 1)

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EJERCICIO 1

1- Simplifique o demuestre (según corresponda) las siguientes expresiones booleanas, *indique en cada paso las propiedades que emplea.*

1-A

$(x + y + xy)\,(x + z)$ ABSORCIÓN

(X + Y). (X+Z) FACTOR COMÚN

X + (Y.Z)

1-B

$x[y + z(xy + xz)']$ DISTRIBUTIVA

X[Y+ Z. (X. (Y+Z))`] MORGAN

X.[Y+Z.(X` + (Y+Z)`)] DISTRIBUTIVA

X. [(Y+Z).X` + (Y+Z). (Y+Z)`] COMPLEMENTO

X. [(Y+Z) . X` + 0] IDENTIDAD

X. [(Y+Z) . X`] DISTRIBUTIVA

X. (Y+Z) . X .X` IDENTIDAD

X. (Y+Z) . 0

0

1-C

$(x + y)(x' + z) = xz + x'y + yz$ DISTRIBUTIVA

(X+Y).X` + (X+Y).Z = X.Z + X`Y + Y.Z DISTRIBUTIVA X 2

X.X` + Y.X` + X.Z + Y.Z = X.Z + X`Y + Y.Z COMPLEMENTO

0 + Y.X` + X.Z + Y.Z = X.Z + X`Y + Y.Z CONMUTATIVA CON MULTIPLICACION

X`.Y + X.Z + Y.Z = X.Z + X`Y + Y.Z CONMUTATIVA CON SUMA

X.Z + X`Y + Y.Z = X.Z + X`Y + Y.Z

1-D

$wx + \underline{\underline{x}z} + (y + \underline{z})^{`}$ MORGAN Y DOBLE NEGACIÓN

W.X + X + Z` + (Y+Z`) ABSORCIÓN

X + Z` + (Y+Z`) DISTRIBUTIVA

X + Z` + Y+Z` CONMUTATIVA

X + Z` + Z` + Y IDEMPOTENCIA

X + Z` + Y

1-E

AB+A(B+C)+B(B+C) Dibuje el circuito sin simplificar y simplificado

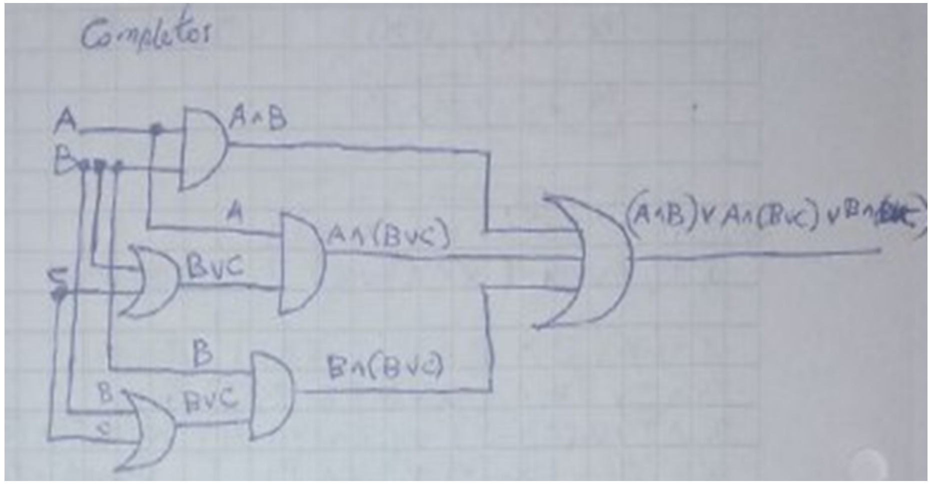
AB + A.(B+C) +B . (B+C) ABSORCION

A.B + A .(B+C) + B DISTRIBUTIVA

A.B + A.B + A.C + B IDEMPOTENCIA

A.B + A.C + B ABSORCIÓN

B+(A.C)



1-F

$(A+B)(A+C)=A+BC$ DISTRIBUTIVA

A + (B.C) = A + BC

EJERCICIO 2:

2- Aplicar los teoremas de Morgan a las siguientes expresiones

2-A

$\underline{(A + B + C)D}$ MORGAN

(A+B+C)` + D` MORGAN

A`.B`.C` + D`

2-B

$\underline{ABC + DEF}$ MORGAN

(A.B.C)` . (D.E.F)` MORGAN X 2

(A`+B`+C` ) . (D`+ E` + F`)

2-C

$\underline{A\underline{B} + \underline{C}D + EF}$ MORGAN

(A.B`)` . (C`.D)` . (E.F)` MORGAN Y DOBLE NEGACIÓN

(A` + B) . ( C + D`) . (E` + F`)

2-D

$\underline{\underline{(A + B)} + \underline{C}}$ MORGAN

((A+B)`)` . (C´)´ DOBLE NEGACIÓN

(A+B).C

2-E

$\underline{(\underline{A} + B) + CD}$ MORGAN

(A`+B)´ . (C.D)` MORGAN X 2

((A`)` . B`) . (C` + D`) DOBLE NEGACIÓN

(A . B`)  . (C` + D`)

2-F

$$\underline{(A+B)\underline{CD}+E+\underline{F}}$$

MORGAN

((A+B).(C.D)`)` . E` . (F`)`   DOBLE NEGACIÓN

((A+B).(C.D)`)` . E` . F   MORGAN

(A+B)` + ((C.D)`)` . E` .F   MORGAN Y DOBLE NEGACIÓN

A`.B` + C.D . E` .F

EJERCICIO 3:

3 - Encuentre las **formas normales disyuntivas (FND)** de las siguientes expresiones booleanas mediante el método algebraico y corrobore sus resultados a través de tabla de verdad. Una **FND** es una estandarización de una expresión lógica que es una disyunción de cláusulas conjuntivas, Por ejemplo: $(X_1 \vee Y_1) \wedge (X_2 \vee Y_2) \wedge \cdots \wedge (X_n \vee Y_n)$, donde X_1,Y_1, X_2 ,Y_2, $\cdots$, X_n,Y_n son proposiciones lógicas.

3-A

$$f_{(x,y,z)} = xy' + yz'$$

X.Y`.1 + 1.Y.Z`

X.Y`. (Z+Z`) + (X + X`) . Y. Z`

(X.Y`.Z) + (X.Y`.Z`) + ( X.Y.Z`) + (X`.Y.Z`)

	X	Y	Z	Y`	Z`	X.Y`	Y.Z`	X.Y`+Y.Z`
	0	0	0	1	1	0	0	0
	0	0	1	1	0	0	0	0
X`YZ`	0	1	0	0	1	0	1	1
	0	1	1	0	0	0	0	0
X.Y`.Z`	1	0	0	1	1	1	0	1
X.Y`.Z	1	0	1	1	0	1	0	1
X.Y.Z`	1	1	0	0	1	0	1	1
	1	1	1	0	0	0	0	0

3-B

$$f(x,y,z) = y' + [z' + x + (yz)'](z + x'y)$$

y`+(z`+x+y`+z`)(z+x`y) MORGAN

y`+(z(z`+x+y`+z`)) (x`y (z`+x+y`+z`)) DOBLE DISTRIBUTIVA

y`+z.z`+zx+zy`+zz`+x`yz`+x`yx+x`yy`+x`yz`

y`(0)+zx+zy`+(0)+x`yz`+y.0+x`.0+x`yz`

y`.1+zx.1+zy`.1+x`yz`

y`.(x+x`)+zx.(y+y`)+zy`(x+x`)+x`yz`

y`.x+y`x`+zxy +zxy`+zy`x+zy`x`+x`yz`

y`.x.1+y`x`1+zxy+zxy`+zy`x+zy`x`+x`yz`

y`x(z+z`)+y`x`(z+z`)+ zxy+zxy`+zy`x+zy`x`+x`yz`

(y`xz)+(y`xz`)+(y`x`z)+(y`x`z`)+ (zxy)+(zxy`)+(zy`x)+(zy`x`)+(x`yz`)

(xy`z)+(xy`z`)+(x`y`z)+(x`y`z`)+(xyz)+(xy`z) +(x`yz`)

(xy`z)+(xy`z`)+(x`y`z)+(x`y`z`)+(xyz) +(x`yz`)

	X	Y	Z	X`	Y`	Z`	X`Y	Z+X`Y	(Y.Z)`	X+(Y.Z)`	Z`+(X+(Y.Z)`)	[Z`+X+(Y.Z)`]. (Z+X`Y)	Y`+[Z`+X+(Y.Z)`]. ( Z+X`Y)
X`y`z`	0	0	0	1	1	1	0	0	1	1	1	0	1
X`y`z	0	0	1	1	1	0	0	1	1	1	1	1	1
X`YZ`	0	1	0	1	0	1	1	1	1	1	1	1	1
	0	1	1	1	0	0	1	1	0	0	0	0	0
XY`Z`	1	0	0	0	1	1	0	0	1	1	1	0	1
XY`Z	1	0	1	0	1	0	0	1	0	1	1	1	1
	1	1	0	0	0	1	0	0	1	1	1	0	0
XYZ	1	1	1	0	0	0	0	1	0	1	1	1	1

3-C

$$f_{(x,y,w,z)} = xy + yzw'$$

x.y.1+y.z.w’.1

$x.y(z+z')+y.z.w'(x.x')$

$x.y.z+x.y.z'+y.z.w'.x+y.z.w'.x'$

$x.y.z(w+w')+x.y.z'(w+w')+y.z.w'.x+y.z.w'.x'$

$x.y.z.w+x.y.z.w'+x.y.z'.w+x.y.z'.w'+ y.z.w'.x+y.z.w'.x'$

$(x.y.z.w)+(x.y.z.w')+(x.y.z'.w)+(x.y.z'.w') +(y.z.w'.x')$

	x	Y	z	w	w`	xy	Yzw`	Xy+yzw`
	0	0	0	0	1	0	0	0
	0	0	0	1	0	0	0	0
	0	0	1	0	1	0	0	0
	0	0	1	1	0	0	0	0
	0	1	0	0	1	0	0	0
	0	1	0	1	0	0	0	0
X`y z w`	0	1	1	0	1	0	1	1
	0	1	1	1	0	0	0	0
	1	0	0	0	1	0	0	0
	1	0	0	1	0	0	0	0
	1	0	1	0	1	0	0	0
	1	0	1	1	0	0	0	0
Xyz`w`	1	1	0	0	1	1	0	1
Xwz`w	1	1	0	1	0	1	0	1
Xyzw`	1	1	1	0	1	1	1	1
xyzw	1	1	1	1	0	1	0	1

3-D

$$f_{(x,y,z)} = xy' + z$$

= X.Y`.(1) + (1).Z

= X.Y`. (Z+Z`) + (Y+Y`).Z DISTRIBUTIVA

= (X.Y`.Z ) +  (X.Y`.Z`) +  (Y.Z.(1)  +  Y`.Z .(1)) DISTRIBUTIVA

= (X.Y`.Z ) +  (X.Y`.Z`) +  (Y.Z.(X+X`)) + (Y`.Z.(X+X`)) DISTRIBUTIVA Y CONMUTATIVA

= (X.Y`.Z ) +  (X.Y`.Z`) +  (X.Y.Z) + (X`.Y.Z) + (X.Y`.Z) + (X`.Y`.Z)

= (X.Y`.Z) + (X.Y`.Z`) +  (X.Y.Z) + (X`.Y.Z) + (X`.Y`.Z)

	X	Y	Z	Y`	X.Y`	X.Y`+Z
	0	0	0	1	0	0
X`.Y`.Z	0	0	1	1	0	1
	0	1	0	0	0	0
X`.Y.Z	0	1	1	0	0	1
X.Y`.Z`	1	0	0	1	1	1
X.Y`.Z	1	0	1	1	1	1
	1	1	0	0	0	0
X.Y.Z	1	1	1	0	0	1

3-E

$$f_{(x,y,w,z)} = w + x'y + y'z$$

(1).W + X’Y.(1) + (1).Y’Z
(X+X’).W + X’Y.(W+W’) + (X+X’).Y’Z

(XW)+(X’W).(1) + (X’YW)+(X’YW’).(1) + (XY’Z)+(X’Y’Z).(1)
(XW)+(X’W).(Y+Y’) + (X’YW)+(X’YW’).(Z+Z’) + (XY’Z)+(X’Y’Z).(W+W’)
(1).(XYW)+(X’YW)+(XY’W)+(X’Y’W) + (X’YWZ)+(X’YW’Z)+(X’YWZ’)+(X’YW’Z’)+(XY’WZ)+(X’Y’WZ)+(XY’W’Z)+(X’Y’W’Z)
(Z+Z’).(XYW)+(X’YW)+(XY’W)+(X’Y’W) +(X’YWZ)+(X’YW’Z)+(X’YWZ’)+(X’YW’Z’)+(XY’WZ)+(X’Y’WZ)+(XY’W’Z)+(X’Y’W’Z)
(XYWZ)+(X’YWZ)+(XY’WZ)+(X’Y’WZ)+(XYWZ’)+(X’YWZ’)+(XY’WZ)+(X’Y’WZ’)+(X’YWZ)+(X’YW’Z)+(X’YWZ’)+(X’YW’Z’)+(XY’WZ)+(X’Y’WZ)+(XY’W’Z)+(X’Y’W’Z)
(XYWZ)+(X’YWZ)+(XY’WZ)+(X’Y’WZ)+(XYWZ’)+(X’YWZ’)+(XY’WZ’)+(X’Y’WZ’)+(X’YW’Z)+(X’YW’Z’)+(XY’W’Z)+(X’Y’W’Z)

X	Y	W	Z	X’	Y’	X’Y	Y’Z	W+X’Y	W+X’Y+X’Z	
0	0	0	0	1	1	0	0	0	0	
0	0	0	1	1	1	0	1	0	1	(X’Y’W’Z)
0	0	1	0	1	1	0	0	1	1	(X’Y’WZ’)
0	0	1	1	1	1	0	1	1	1	(X’Y’WZ)
0	1	0	0	1	0	1	0	1	1	(X’YW’Z’)
0	1	0	1	1	0	1	0	1	1	(X’YW’Z)
0	1	1	0	1	0	1	0	1	1	(X’YWZ’)
0	1	1	1	1	0	1	0	1	1	(X’YWZ)
1	0	0	0	0	1	0	0	0	0	
1	0	0	1	0	1	0	1	0	1	(XY’W’Z)
1	0	1	0	0	1	0	0	1	1	(XY’WZ’)
1	0	1	1	0	1	0	1	1	1	(XY’WZ)
1	1	0	0	0	0	0	0	0	0	
1	1	0	1	0	0	0	0	0	0	

1	1	1	0	0	0	0	0	1	1	(XYWZ’)
1	1	1	1	0	0	0	0	1	1	(XYWZ)

EJERCICIO 4:

4-A:

$$f_{(x,y,z)} = (x+z)y$$

(X+Z+0) . (Y+O)

(X+Z+(Y.Y´)) . (Y+(X.X´))

(X+Z+Y).(X+Z+Y´) . (Y+X) . (Y+X´)

(X+Z+Y).(X+Z+Y´) . (Y+X+0) . (Y+X´+0)

(X+Z+Y).(X+Z+Y´) . (Y+X+(Z.Z´)) . (Y+X´+(Z.Z´))

(X+Z+Y).(X+Z+Y´) . (Y+X+Z).(Y+X+Z´).(Y+X´+Z).(Y+X´+Z´)

(X+Y+Z).(X+Y´+Z) . (X+Y+Z´).(X´+Y+Z).(X´+Y+Z´)

X	Y	Z	X´	Y´	Z´	(X+Z)	(X+Z)Y	RESULTADO
0	0	0	1	1	1	0	0	(X+Y+Z)
0	0	1	1	1	0	1	0	(X+Y+Z´)
0	1	0	1	0	1	0	0	(X+Y´+Z)
0	1	1	1	0	0	1	1	
1	0	0	0	1	1	1	0	(X´+Y+Z)
1	0	1	0	1	0	1	0	(X´+Y+Z´)
1	1	0	0	0	1	1	1	

1	1	1	0	0	0	1	1	

4-B

$$f(x,y,z) = x$$

= X + 0

= X + (Y+Y´) DISTRIBUTIVA

= (X+Y) . (X+Y´)

= (X+Y+0) . (X+Y´+0)

= (X+Y+(Z+Z´)) . (X+Y´+(Z+Z´))

= (X+Y+Z) . (X+Y+Z´) . (X+Y´+Z) . (X+Y´+Z´)

	X	Y	Z	X
(X+Y+Z)	0	0	0	0
(X+Y+Z´)	0	0	1	0
(X+Y´+Z)	0	1	0	0
(X+Y´+Z´)	0	1	1	0
	1	0	0	1
	1	0	1	1
	1	1	0	1
	1	1	1	1

4-C

$f_{(x,y,z)} = (yz + xz')(xy' + z)'$ DISTRIBUTIVA Y MORGAN

=((yz)+x) . (((yz)+z´) . (xy´)´ . z´ DISTRIBUTIVA X 2 Y MORGAN

=(x+z).(x+y) .(z´+z).(z´+y).(x´+y).(z´+(0)) COMPLEMENTO

=(x+z+0).(x+y+0).1. (z´+y+0).(x´+y+0).(z´+(x.x´))

=(x+z+(y.y´)) . (x+y+(z.z´)) .(z´+y+(x.x´)) .(x´+y+(z.z´)) . (z´+x+0).(z´+x´+0)

=(x+z+y).(x+z+y´).(x+y+z).(x+y+z´). (x+y+z´).(x´+y+z´).(x´+y+z).(x´+y+z´).(x+z´+(y-y´) .(x´+z´+(y.y´)

=(x+y+z).(x+y´+z).(x+y+z).(x+y+z´). (x+y+z´).(x´+y+z´).(x´+y+z).(x´+y+z´).(x+y+z´).(x+y´+z´).(x´+y+z´).(x´+y´+z´)

=(x+y+z).(x+y´+z).(x+y+z´).(x´+y+z´).(x´+y+z).(x+y´+z´).(x´+y´+z´)

	X	Y	Z	Y`	Z`	Y.Z	X.Z`	(Y.Z)+(X.Z`)	x.y`	(x.y`)+z	((x.y`)+z)`	((Y.Z)+(X.Z`)).((x.y`)+z)`
(x+y+z)	0	0	0	1	1	0	0	0	0	0	1	0
(x+y+z´)	0	0	1	1	0	0	0	0	0	1	0	0
(x+y´+z)	0	1	0	0	1	0	0	0	0	0	1	0
(x+y´+z´)	0	1	1	0	0	1	0	1	0	1	0	0
(x´+y+z)	1	0	0	1	1	0	1	1	1	1	0	0
(x´+y+z´)	1	0	1	1	0	0	0	0	1	1	0	0
	1	1	0	0	1	0	1	1	0	0	1	1
(x´+y´+z´)	1	1	1	0	0	1	0	1	0	1	0	0

4-D

$$f(x,y,z) = (x+y)(x'+z)(y+z')$$

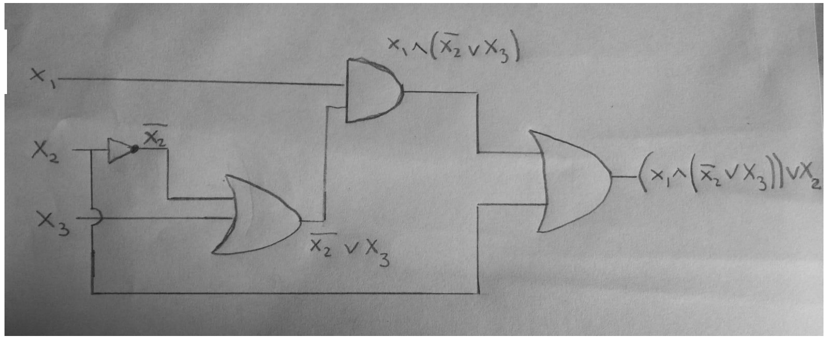
= (X+Y+0) . (X´+Z+0) . (Y+Z´+0)
= (X+Y+(Z.Z´)) . (X´+Z+(Y+Y´)) . (Y+Z´+(X+X´))
= (X+Y+Z).(X+Y+Z´).(X´+Z+Y).(X´+Z+Y´).(Y+Z´+X).(Y+Z´+X´)
= (X+Y+Z).(X+Y+Z´).(X´+Y+Z).(X´+Y´+Z).(X+Y+Z´).(X´+Y+Z´)
= (X+Y+Z) . (X+Y+Z´) . (X´+Y+Z) . (X´+Y´+Z) . (X´+Y+Z´)

	X	Y	Z	X´	Y´	Z´	(X+Y)	(X´+Z)	(Y+Z´)	(X´+Z).(Y+Z´)	(X+Y).(X´+Z).(Y++Z´)
(X+Y+Z)	0	0	0	1	1	1	0	1	1	1	0
(X+Y+Z´)	0	0	1	1	1	0	0	1	1	1	0
	0	1	0	1	0	1	1	1	1	1	1
	0	1	1	1	0	0	1	1	1	1	1
(X´+Y+Z)	1	0	0	0	1	1	1	0	1	0	0
(X´+Y+Z´)	1	0	1	0	1	0	1	1	0	0	0
(X´+Y´+Z)	1	1	0	0	0	1	1	0	1	0	0
	1	1	1	0	0	0	1	1	1	1	1

EJERCICIO 5:

A partir de la siguiente expresión booleana encuentre el circuito combinatorio

$$x_1 \wedge (\underline{x_2} \vee x_3) \vee x_2$$

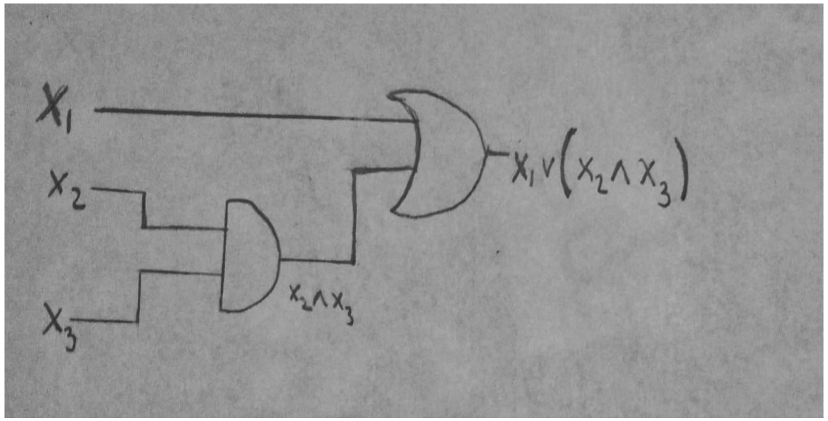


EJERCICIO 6:

Represente los siguientes circuitos

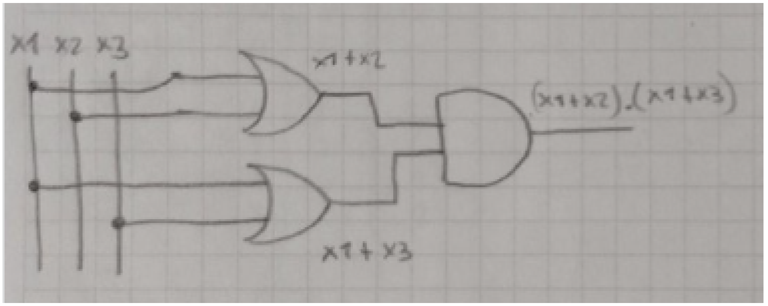
6-A

$$x_1 \vee (x_2 \ \wedge x_3)$$



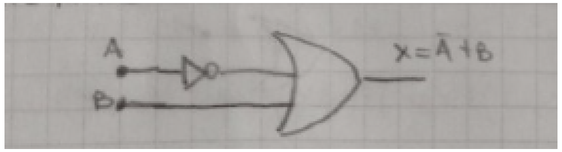
6-B

$$(x_1 \vee x_2) \wedge (x_1 \ \vee x_3)$$



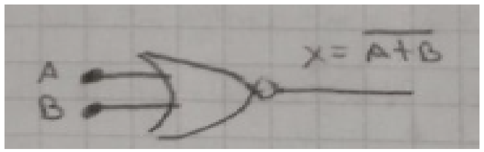
6-C

$$x = \underline{A} + B$$



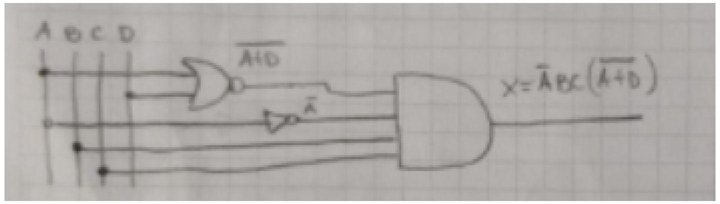
6-D

$$x = \underline{A + B}$$



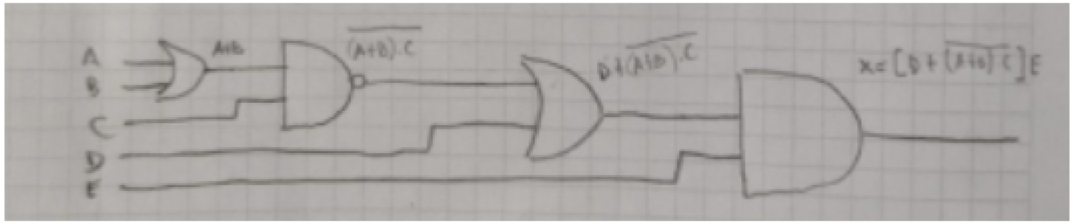
6-E

$x = \underline{A}BC(\underline{A + D})$



6-F

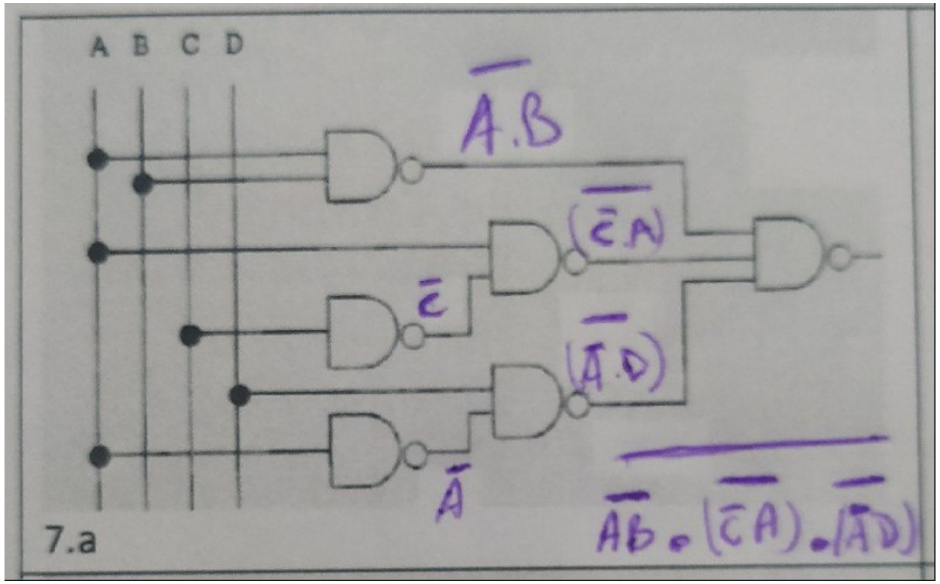
$$x = \left[D + \underline{(A + B)C}\right] E$$



EJERCICIO 7

Exprese las funciones booleanas de los siguientes circuitos

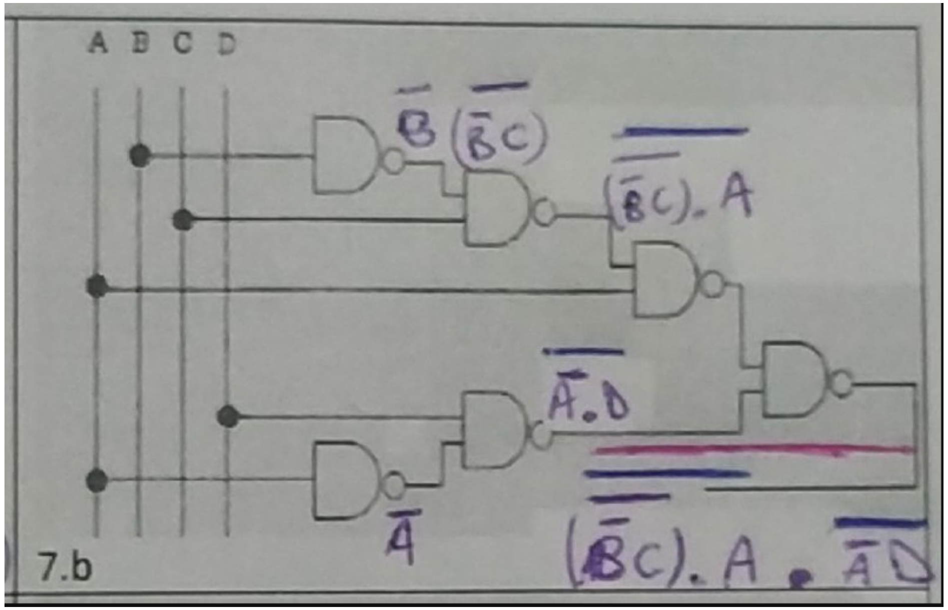
7-A



7.a

SIMPLIFICANDO DOBLE NEGACIÓN => AB .(C`.A).(A`.D)

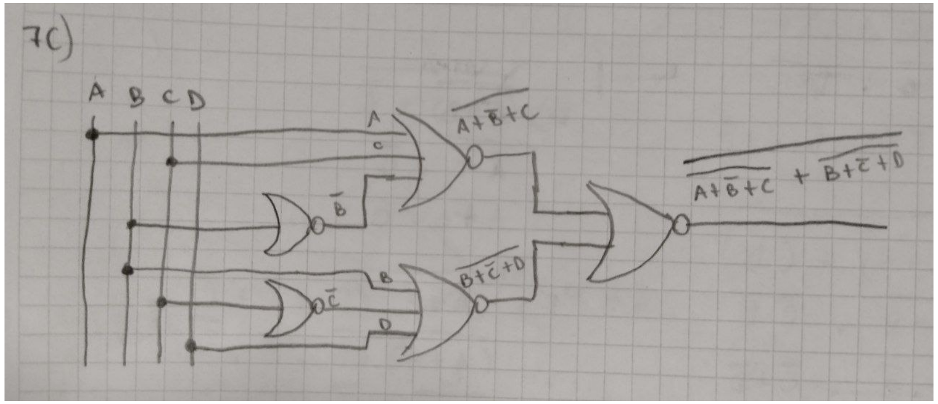
7-B



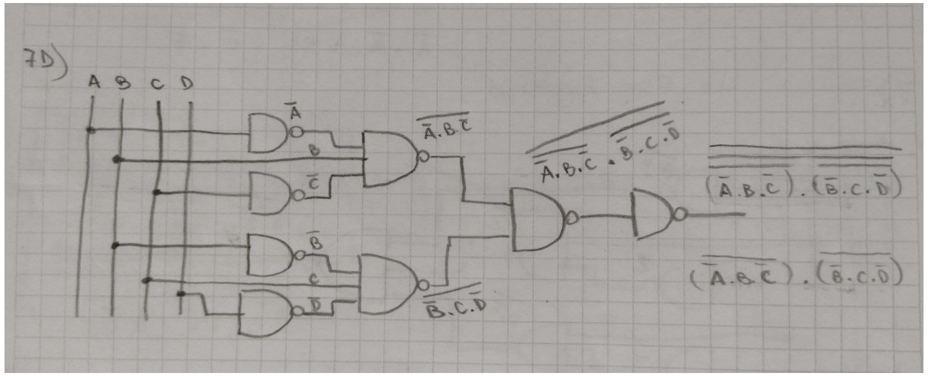
7.b

SIMPLIFICANDO DOBLE NEGACIÓN => (B`.C)`.A.(A`.D)

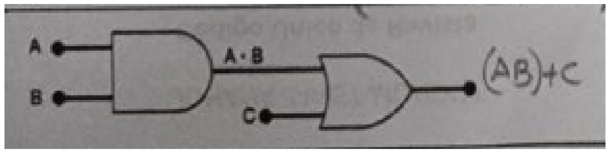
7-C



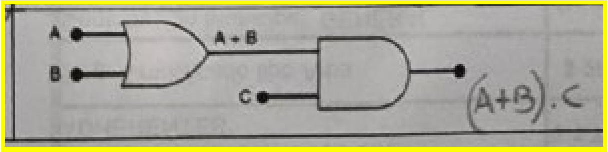
7-D



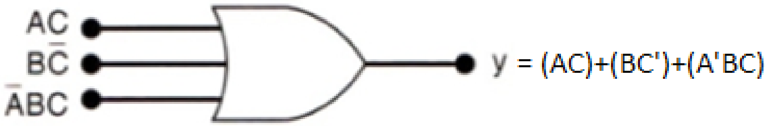
7-E



7-F



7-G



7-H