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Magnification lateral/heigth/size $y = z$, longitudinal/distance $x \perp y$

$g \equiv$ distance from object, sender source (from Objectiv)

$b \equiv$ distance from observer, receiver detector (from Okular)

$l \equiv$ distance observer to next lense

$r \equiv$ radius of curvature (bended reflective surface, lense konkav vs. konvex)

$d \equiv$ max. thickness of curved lense enclosed between one konvex side from g and one konkav side from b with radi r_g and r_b

$s_0 \equiv$ sight/tubus length

$\varepsilon \equiv$ “visual angle” from observer, ε_0 smallest visual angle

$f \equiv$ focal length

$$g \parallel x \parallel b \parallel f \quad y_b \perp b \quad y_g \perp g \quad (1)$$

$$\tan(\varepsilon) = \frac{y_b}{b} \quad \tan(\varepsilon_0) = \frac{y_b}{g} \quad (2)$$

$$(\Delta\varphi)_{min} = \Delta\theta = 2\varepsilon_0 = 1.22 \frac{\lambda}{d_y}$$

“psychological visual angle”: $2\varepsilon_0^{min} = 1' = 60'' = \frac{1^\circ}{60} = \frac{2\pi}{360} \frac{1}{60} = \frac{\pi}{10800}$

konkav: $f > 0, r < 0$ konvex: $f < 0, r > 0$, virtual $b < 0$

$$\frac{\eta_g}{g} + \frac{\eta_b}{b} = \frac{\eta - \eta_g}{r_g} - \frac{\eta - \eta_b}{r_b} \quad (3)$$

$$\frac{1}{f_b} = \frac{1}{\eta_b} \left(\frac{\eta - \eta_g}{r_g} - \frac{\eta - \eta_b}{r_b} \right)$$

$$\frac{1}{f_g} = \frac{1}{\eta_g} \left(\frac{\eta - \eta_g}{r_g} - \frac{\eta - \eta_b}{r_b} \right)$$

thin lense: $f = f_g = f_b \iff d \rightarrow 0$

lense: $f = f_g = f_b \iff \eta_g = \eta_b$

spherical surface (panel), paraxial rays (linear approx., Gauss):

$$\frac{\eta_g}{g} + \frac{\eta_b}{b} = \frac{\eta_b - \eta_g}{r} \quad (4)$$

$$b = f_b = \frac{\eta_b}{\eta_b - \eta_g} \cdot r \quad g = f_g = \frac{\eta_g}{\eta_b - \eta_g} \cdot r \quad (5)$$

Focal Length:

$$f = \left(\frac{\Delta y}{\Delta \varepsilon} \right)_{min} = \frac{1}{r} \frac{\eta}{\Delta \eta} = \frac{r}{2} \frac{1}{\eta - 1} = -\frac{r}{2} = f_b = f_g \stackrel{conjungtion}{=} \frac{g \cdot b}{g + b} \quad (6)$$

where: $\hat{f}^2 = x_g x_b$ and $x_g = x_g(0) = g - f = -f \frac{y_g}{y_b} = +f \cdot \mathcal{G}_y$ and $x_b = x_b(0) = b - f = -f \frac{y_b}{y_g} = -f \cdot \mathcal{G}_y$

lense proportions:

$$x_g(r_b) = -f \cdot \frac{\eta - 1}{\eta} \cdot \frac{d}{r_b} \quad x_b(r_g) = -f \cdot \frac{\eta - 1}{\eta} \cdot \frac{d}{r_g} \quad (7)$$

$d = x_g(r_b) + x_b(r_g) + \Delta x$ where rays parallel within Δx inside lense

Dioptri (Refractive Power)¹:

$$D = \frac{\eta_g}{f_g} = \frac{\eta_b}{f_b} \stackrel{vacuum}{=} \frac{1}{f} = \frac{1}{g} + \frac{1}{b} \stackrel{paraxial}{=} \frac{\Delta \eta}{f} = \frac{2}{r} \stackrel{concave}{=} -\frac{2}{r} \stackrel{r_b = -r_g}{=} \frac{2}{r} (\eta - 1) = (\eta - 1) \left(\frac{1}{r_g} - \frac{1}{r_b} + \frac{(\eta - 1) \cdot d}{\eta \cdot r_g} \right) \quad (8)$$

where: $r < 0$

longitudinal magnification:

$$\mathcal{G}_x = \frac{g}{b}$$

lateral magnification:

$$\mathcal{G}_y \stackrel{konkav}{=} \frac{-y_b}{y_g} = -\frac{b}{g} \stackrel{konkvek}{=} \frac{y_b}{y_g} = -\frac{b - r}{g + r} = -\frac{f}{x_g} = -\frac{x_b}{f} = \frac{f}{f - g} \stackrel{aplanar}{=} \frac{\eta_g}{\eta_b} \cdot \frac{\sin(u_g)}{\sin(u_b)} \stackrel{kollinear}{=} \frac{\eta_g}{\eta_b} \cdot \frac{\tan(u_g)}{\tan(u_b)} \stackrel{paraxial}{=} \quad (9)$$

viewangle magnification:

$$\mathcal{G}_\varepsilon = \frac{\varepsilon}{\varepsilon_0} = \frac{\frac{y_b}{b+l}}{\frac{y_g}{s_0}} = \frac{s_0}{g} \frac{1}{1 + \frac{l}{b}} \cong \frac{\tan(\varepsilon)}{\tan(\varepsilon_0)} = \frac{y_b}{y_g} \cdot \frac{g}{b} = \mathcal{G}_y \cdot \mathcal{G}_x \stackrel{astronom.}{=} -\frac{f^{Obj}}{f^{Oku}} \stackrel{terrestr.}{=} +\frac{f^{Obj}}{f^{Oku}} \in \left[\frac{s_0}{f}, \frac{s_0}{f} + 1 \right] \quad (10)$$

¹Brechkraft

total magnification:

$$\mathcal{G} = \mathcal{G}_y^2 \cdot \mathcal{G}_x = \mathcal{G}_y \cdot \frac{\varepsilon}{\varepsilon_0} = \mathcal{G}_y \cdot \mathcal{G}_\varepsilon = -\frac{x_b^{Obj}}{f^{Obj}} \cdot \frac{s_0}{f^{Oku}} \quad (11)$$