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## Table of contents

Radiation (energy movement, charge acceleration, energytransport, poynting)

Hypothesis: Accelerated charge causes radiation (analogous to “force”)

I) Optical Radiation spectral range:  $\lambda = [100nm, 1mm]$ :

1. high frequency: UV spectral range:  $\lambda = [100nm, 200nm]$
2. visibility wavelength: light (Photon) range:  $\lambda = [380nm, 780nm]$
3. long wavelength: IR spectral range:  $\lambda = [5 \cdot 10^4nm, 10^6nm]$

II) Heat Radiation invisible (Black Body “B”),  $\mathcal{L}_{\lambda,B}(\lambda, T)$

Charge:

$$q = ne_0 \quad \sum q = Q = I \cdot t \quad n \in \mathbb{Z} \quad (1)$$

Radiation:

$$\mathcal{S} = \frac{d}{dt}p = \frac{1}{c} \frac{d}{dt}(E_{kin}) = q \cdot a \quad (2)$$

Optical Radiation:

Deviation angle of ray beam after refraction (transmission) and diffraction:

$$\varsigma = \alpha + \arcsin \left[ \sin(\varpi) \sqrt{\eta^2 - \sin^2(\alpha)} - \cos(\varpi) \sin(\alpha) \right] - \varpi = \alpha + \alpha_2 - \varpi \quad (3)$$

where  $\varpi = \beta + \beta_2 = const.$

Minimal aberration (minimal projection error):

$$\varsigma = \varsigma_{min} \iff$$

$$\begin{aligned} \varsigma_{min} &\iff \\ \frac{d}{d\alpha} \varsigma &= 0 \wedge \\ \varsigma_{min} &= 2\alpha - \varpi = 2(\alpha - \beta) \end{aligned} \quad (4)$$

Thermal Radiation:

Spectral Reflection:

$$\mathcal{R}(\lambda) = \frac{d^3\Phi_{\lambda,r}}{d^3\Phi_{\lambda,e}} \quad (5)$$

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Spectral Transmission:

$$\mathcal{T}(\lambda) = \frac{d^3\Phi_{\lambda,t}}{d^3\Phi_{\lambda,e}} \quad (6)$$

Spectral Absorbtion:

$$\mathcal{A}(\lambda) = \frac{d^3\Phi_{\lambda,a}}{d^3\Phi_{\lambda,e}} \quad (7)$$

Spectral Emission:

$$\mathcal{E}(\lambda) = \frac{\mathcal{L}(\lambda)}{\mathcal{L}(\lambda, B)} \quad (8)$$

$$\mathcal{R}(\lambda) + \mathcal{T}(\lambda) + \mathcal{A}(\lambda) = 1$$

Black Body “B”:

$$\mathcal{A}(\lambda, T) = \mathcal{E}(\lambda, T) = 1, \quad \mathcal{R}(\lambda, T) = 0, \quad \mathcal{T}(\lambda, T) = 0 \quad (9)$$

$\Rightarrow$  Heat Radiation (Black Body):

$$\mathcal{L}_B = \frac{c}{4\pi} \langle w_{EM} \rangle = \frac{c}{4\pi} w_B \quad (10)$$

$$\mathcal{L}_{\lambda,B}(\lambda, T) = \frac{\mathcal{L}_{\lambda}(\lambda, T)}{\mathcal{A}_{\lambda}(\lambda, T)} \stackrel{continuum}{=} \frac{2c}{\lambda^4} kT \stackrel{quantum}{=} \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} = const. \quad (11)$$

$$\mathcal{L}_{\nu,B}(\nu, T) \stackrel{quantum}{=} \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \quad (12)$$

$$\mathcal{L}_{\lambda,B}(\lambda_x, T_x) = \left( \frac{T_x}{T} \right)^5 \cdot \mathcal{L}_{\lambda,B}(\lambda, T) \quad (13)$$

$$T_x > T \Rightarrow \lambda_x < \lambda$$

$$\lambda_{max} \cdot T = \frac{h \cdot c}{5k} \frac{1}{1 - e^{-\lambda_{max} \cdot T}} = b = 2.897773 \cdot 10^{-3} m \cdot K \quad (14)$$

Heat Radiation Transfer (sent from source  $g$  to receiver  $b$  with areas  $\mathcal{A}$ ):

$$d^2\Phi \stackrel{vacuum}{=} \mathcal{L} \frac{dA_g \cos(\varepsilon_g)}{R} \frac{dA_b \cos(\varepsilon_b)}{R} = \frac{\mathcal{L}}{R^2} \cdot dA_g \cos(\varepsilon_g) \cdot dA_b \cos(\varepsilon_b) = \mathcal{L} \cos(\varepsilon_b) dA_b d\Omega_b \quad (15)$$

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Heat Radiation Return, Reflection Absorbtion (Feedback absorbed “a” at sender  $g$  received “e” at receiver  $b$ , with areas  $\mathcal{A}$  at wavelength  $\lambda$ ):

$$d^3\Phi_{\lambda,a,b} \stackrel{vacuum}{=} \mathcal{A}_b \cdot d^3\Phi_{\lambda,e,b} = \mathcal{A}_b \cdot \mathcal{L}_\lambda \cos(\varepsilon_b) d\mathcal{A}_b d\Omega_b d\lambda = \mathcal{L}_{\lambda,b} \cos(\varepsilon_g) d\mathcal{A}_g d\Omega_g d\lambda = d^3\Phi_{\lambda,b} \quad (16)$$

Energy Density  $w$  and Pressure  $P$ , Temperature  $T$ , Volume  $V$ , inner Energy  $U$ , Entropy density  $s$ , Radiance

$\mathcal{L}_B$ :

$$w_B = \frac{\mathcal{L}_B}{c} \int_0^{4\pi} d\Omega = \frac{4\pi}{c} \cdot \mathcal{L}_B = \frac{4}{c} \cdot \mathcal{M}_B = 2 \cdot 2\pi \cdot \frac{\mathcal{L}_B}{c} \stackrel{oriented}{=} P_{EM} \stackrel{unoriented}{=} 3w_{EM} = 3(\varepsilon_0 \varepsilon_r \mathcal{E}^2) = 3P_B = 3 \left( -\frac{dW}{dV} \right) \quad (17)$$

$$w_{\lambda,B} \stackrel{continuum}{=} -\frac{dn}{d\lambda} \overline{W} = \frac{8\pi}{\lambda^4} kT^{quantum} \frac{8\pi hc}{\lambda^5} \frac{1}{e^{\frac{hc}{kT}} - 1} = 8\pi h\nu \frac{1}{\lambda^4} \frac{1}{e^{\frac{hc}{kT}} - 1} \quad (18)$$

$$w_{\nu,B} \stackrel{continuum}{=} \frac{dn}{d\nu} \overline{W} = \frac{8\pi\nu^2}{c^3} kT^{quantum} \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{\frac{hc}{kT}} - 1} = 8\pi h \left( \frac{1}{\lambda} \right)^3 \frac{1}{e^{\frac{hc}{kT}} - 1} \quad (19)$$

where Radiant Exitance (Emittance)  $\mathcal{M}_B = \frac{d\Phi}{dA_g} = \sigma T^4$  and proportionality constants:

$$a = \frac{2^3}{3 \cdot 5} \frac{\pi^5 k^4}{h^3 c^3} = 7.5657 \cdot 10^{-16} Jm^{-3} K^{-4} \quad (a = \text{constant from integration})$$

$$\sigma = \frac{1}{4} \cdot a \cdot c = \frac{2}{3 \cdot 5} \frac{\pi^5 k^4}{h^3 c^3} = 5.6704 \cdot 10^{-8} Wm^{-2} K^{-4} \quad (c = \text{light speed})$$

$$\frac{d}{d\Omega} P_{EM} \stackrel{oriented}{=} \frac{d}{d\Omega} w_B = \frac{\mathcal{L}_B}{c} \quad (20)$$

Blackbody Entropy Density  $s_B = \frac{4}{3}aT^4$  Blackbody Entropy  $S_B = \frac{4}{3}aVT^3$  Blackbody Inner Energy  $U_B = aVT^4$  Blackbody Pressure  $P_B = \frac{1}{3}aT^4 \stackrel{unoriented}{=} \frac{1}{3}w_B$