Table of contents

Constraints:

- initial state, starting conditions
- acting boundary conditions (to move within): e.g. demography, economical and political stability, labor force, education and know-how, health and technological level
- access to limited resources: e.g. chemical elements, minerals, fuels, commodities, trade and transportations, etc.
- contractual agreements and covenants, legal assurance
- juristic law framework

Modelling movement (change of state) under constraints:

1. Movement ist charecterized by change of energy, Lagrangian $\mathcal{L} \neq 0$:

$$\mathcal{L} = \mathcal{K} - \mathcal{U} \qquad \frac{\partial \mathcal{L}}{\partial r} = F \qquad \frac{\partial \mathcal{L}}{\partial v} = p \tag{1}$$

2. Movement happens within constraints of forces. Constrains are observable, whereas forces might be unknown. Therefore r can be determined from $\mathcal{L}$ (Energy and Work) of the system at rest $F = 0$, instead than from "searching" all acting forces F_i:

$$F_{total} = \hat{F}_i + \tilde{F}_i = \hat{F}_i + \sum_{k=1}^{s_d} \lambda_k a_{ki} = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial v_i}\right) - \frac{\partial \mathcal{L}}{\partial r_i} = \begin{cases} = 0 : \exists \text{ Equilibrium} \\ \neq 0 : \exists \text{ Movement} \end{cases} \tag{2}$$

and at same time solving for constraining conditions delimitating movement:

$$\sum_{l=1}^{f} a_{kl} v_l + b_k = 0 \tag{3}$$

3. Rest is characterized by unchanged energy, Hamiltonian $\mathcal{H} = E_{kin} + E_{pot} = const.$. There is movement from state of rest if $\mathcal{H} \neq 0$:

$$\mathcal{H} = \sum_i p_i v_i - \mathcal{L} = pv - \mathcal{L} = Fr - \mathcal{L} = W - \mathcal{L} = (\Delta E - \Delta H) - \mathcal{L} \tag{4}$$

4. Change of state/position is characterized by movement:

$$\dot{r}_i = v_i = \frac{\partial \mathcal{H}}{\partial p_i} \qquad \dot{p}_i = F_i = -\frac{\partial \mathcal{H}}{\partial r_i} \tag{5}$$

see description of "work" **?@eq-workgeom**

where:

$s_d =$ constraints $\quad i = 1, ..., f =$ dedrees of freedom $\quad k = 1, ..., s_d =$ constraints

$\hat{F}_i =$ known constraining forces $\quad \tilde{F}_i = \sum_{k=1}^{s_d} \lambda_k a_{ki} =$ unknown constraining forces

$\lambda_k(t) =$ Eigenvalues for transformation of reference system

$a_{ki} =$ Eigenvectors of transformation matrix

$\lambda_k \cdot \vec{r}_i = \lambda_k \cdot [a_{ki}]_{matrix}$