Physics Concepts as Indicative Framework to Model Systems of Economies Ex-ante

Coincise Notes and Conjectures Explorations of Natural Laws to Preliminarly Structure Economies

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Abstract

An exploratory attempt …

Interconnection network between movement, force, momentum, energy, and work. Describing systems with Lagrangian and Hamltonian when constraints and starting conditions exist. Modeling systems and optimisation with constraints and conditions. Interwoven system of macro- and microeconomie under constraints, limitations, and conditions.

Introduction

Nature strives through Physics to behave economical by using its resources in an austere way: Principle of Least Action.

Economy - not pre-given by natural laws - is constructed as social set of co-living rules to allocate, distribute and share (natural and non-natural) resources. Ex-ante modelling economic systems by rules (contractual, legislative, judicative, executive).

The state (situation, position) of the Economy can be described by e.g. Assets, Liabilities, Expenses, Incomes, Profitability, Productivity, etc.. The Economy changes through time from a state r_0 = "start" to a state r_1 = "end". For the state to change, work is applied by processing limited resources (e.g. access to raw materials and availability of intermediate goods), constrained capital (e.g. financial funds, interest rates, and inflation), as well as labor capacity (e.g machines and human force).

How can a trajectory $r = r_1 - r_0$ between two states be modelled by work W, which is applied for the change of the state to occure: searching the condition by defing target state through "path of chage" $= r$?

$$r = f(x_i) = \int A = \int \int W = \int \int (\mathcal{L} + \mathcal{H}) = \int \int (\Delta E - \Delta H) \tag{1}$$

Motion and State Change

- motion of matter in space and time
- change of state
- displacement
- diffusion
- dissipation ad refraction
- distance, velocity, momentum, acceleration (curvature), force, energy, work, and action
- isotropy and homomenity

Motion:
$$r = r(t) = r_x(t_x) - r_0(t_0) = \Delta r(\Delta t)$$
$$\overset{Path}{=} State(arrival) - State(start) = Trajectory(Change) \tag{2}$$

$$v = \dot{r} = \frac{d}{dt}r \qquad p = \frac{1}{v}\sqrt{E^2 - (mv^2)^2} \qquad F = \dot{p} = \frac{d}{dt}p \tag{3}$$

Energy total:
$$E = Motion_{Energy} + Rest_{Energy} = E_{kin} + E_{pot} \tag{4}$$

Work and Energy

$\exists$ Work done or received $W \neq 0 \Rightarrow \exists$ Change of State ("Displacement", "Deformation") $r \neq 0$:

$$E = \mathcal{K} + \mathcal{U} \qquad E_{kin} := \mathcal{K} = \frac{1}{2}pv \qquad E_{pot} := \mathcal{U} = \mathcal{V} + \mathcal{Z} \tag{5}$$

Energy is conserved in konservative systems; There is no work in closed path between two points (loop) of any shape under path variation, hence the "Principle of Action" applies (extremities of bumps and saddles exist): $\delta A = 0$

Potentials:

$$\mathcal{U} = \mathcal{V} + \mathcal{Z}$$

$$\mathcal{V}\begin{cases} = \frac{1}{2}kr^2 : \text{isotrop harmonical oscilator central field Potential,} \\ \quad \text{closed bounded path, "spring constant" } k = m\omega^2 = 2\pi \cdot \nu \\ = -\frac{1}{r}\alpha : \text{gravitational central field Potential,} \\ \quad \text{closed bounded path, } \alpha = \left(\frac{2\pi}{\Delta\varphi}\right)^2 = const. \\ \sim \frac{1}{r}e^{\mu r} : \text{Yukawa Potential, "Photon mass" } m_\gamma = \mu\frac{\hbar}{c} \\ ... \end{cases} \tag{6}$$

Centrifugal Potential:

$$\mathcal{Z} = \frac{1}{2}\frac{1}{m}\left(\frac{L_0}{r}\right)^2 , \text{ angular momentum} = L_0 \tag{7}$$

Work:

$$W = \partial_t A = \partial_t(E \cdot t) = \partial_t(p \cdot r) = F_1 \cdot r_1 + p_2 \cdot v_2 = \begin{cases} F \cdot r \\ p \cdot v \end{cases} \tag{8}$$

Geometrical:

$$W = F \cdot r = mr \cdot a = I \cdot \frac{v}{t} = \frac{p}{t} \cdot tv = Ft \cdot v = pv = mv^2 \tag{9}$$

Analytical:

$$\mathcal{L} = \mathcal{L}(r, \dot{r}, t) = E_{kin} - E_{pot} \tag{10}$$

$$\mathcal{H} = \mathcal{H}(r, p, t) = \sum_i^f \dot{r}_i p_i - \mathcal{L} \stackrel{konserv.}{=} E_{kin} + E_{pot} = const. \tag{11}$$

$$A = E \cdot t = p \cdot r \stackrel{\mathcal{H}=0}{=} \int_{t_0}^{t_x} \mathcal{L} dt \tag{12}$$

$$W = p \cdot v = \mathcal{L} + \mathcal{H} = (\mathcal{K} - \mathcal{U}) + \mathcal{H}$$
$$\stackrel{konserv.}{=} (\mathcal{K} - \mathcal{U}) + (\mathcal{K} + \mathcal{U}) = (\mathcal{K} - \mathcal{U}) + E \tag{13}$$

Electro Magnetic:

$$W = \mathcal{L} + \mathcal{H} = \left(\frac{1}{2}mv^2 - q(\Phi - \vec{\mathcal{A}} \cdot \vec{v})\right) + \left(\frac{1}{2}\frac{1}{m}(\vec{p} - q\vec{\mathcal{A}})^2 + q\Phi\right) \tag{14}$$

Relativistic:

$$W = p \cdot v = \frac{v}{c}\sqrt{E^2 - (mc^2)^2} \tag{15}$$

Thermal:

$$W = Fr = PV^\iota = T \cdot V^{\frac{f+2}{f}} = T \cdot V^{(\frac{c_P}{c_V}-1)} = TV^{\kappa-1}$$
$$= T \cdot l \cdot g_0 = T \cdot n \cdot b_0 = T \cdot m \cdot c_0 \tag{16}$$

Thermo Statistical:

$$W = \mathcal{L} + \mathcal{H} = \Delta U - \Delta H = \Delta(T \cdot \Delta S - P \cdot \Delta V) - \Delta(T \cdot S) \tag{17}$$

Quantistic:

$$W = p \cdot v = k \cdot \omega = h\nu k = Ek \tag{18}$$

$$\Delta A = \Delta p \cdot \Delta r = \Delta E \cdot \Delta t \geq \frac{1}{2}\hbar \tag{19}$$

see description of "movement" Equation

acronyms see Section

Constrains and Conditions

Constraints:

- initial state, starting conditions
- acting boundary conditions (to move within): e.g. demography, economical and political stability, labor force, education and know-how, health and technological level
- access to limited resources: e.g. chemical elements, minerals, fuels, commodities, trade and transportations, etc.
- contractual agreements and covenants, legal assurance
- juristic law framework

Modelling movement (change of state) under constraints:

1. Movement ist charecterized by change of energy, Lagrangian $\mathcal{L} \neq 0$:

$$\mathcal{L} = \mathcal{K} - \mathcal{U} \qquad \frac{\partial \mathcal{L}}{\partial r} = F \qquad \frac{\partial \mathcal{L}}{\partial v} = p \tag{20}$$

2. Movement happens within constraints of forces. Constrains are observable, whereas forces might be unknown. Therefore, r can be determined from $\mathcal{L}$ (Energy and Work) of the **system at rest** $F = 0$, instead than from "searching" all acting forces F_i. In general, unknow forces at motion are defined by solving the Eigenvalueproblem. Solving for $\mathcal{L}$ at equilibirium:

$$F_{total} = \hat{F}_i + \tilde{F}_i = \hat{F}_i + \sum_{k=1}^{s_d} \lambda_k a_{ki} = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial v_i}\right) - \frac{\partial \mathcal{L}}{\partial r_i} = \begin{cases} = 0 : & \text{Equilibrium} \\ \neq 0 : & \text{Movement} \end{cases} \tag{21}$$

and at same time solving for constraining conditions that hold the system at rest (equilibrium):

$$\sum_{l=1}^{f} a_{kl} v_l + b_k = 0 \tag{22}$$

3. Rest is characterized by unchanged energy, Hamiltonian $\mathcal{H} = E_{kin} + E_{pot} = const.$. There is movement from state of rest if the new $\mathcal{H} \neq 0$ differs from the initial one. Using the found solution for the Lagrangian $\mathcal{L}$ containing the constraints, the Hamiltonian for the energy is deduced:

$$\mathcal{H} = \sum_i p_i v_i - \mathcal{L} = pv - \mathcal{L} = Fr - \mathcal{L} = W - \mathcal{L} = (\Delta E - \Delta H) - \mathcal{L} \tag{23}$$

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4. Change of state/position is characterized by movement. Eventually, by solving the new system of differential equations, from the found solution of the energy $\mathcal{H}$, the set r, v, p which describe the movement (and thus the change of state), is deduced:

$$\dot{r}_i = v_i = \frac{\partial \mathcal{H}}{\partial p_i} \qquad \dot{p}_i = F_i = -\frac{\partial \mathcal{H}}{\partial r_i} \tag{24}$$

see description of "work" Equation 9

where:

$s_d = $ constraints $\quad i = 1, ..., f = $ dedrees of freedom $\quad k = 1, ..., s_d = $ constraints

$\hat{F}_i = $ known constraining forces $\quad \tilde{F}_i = \sum_{k=1}^{s_d} \lambda_k a_{ki} = $ unknown constraining forces

$\lambda_k(t) = $ Eigenvalues for transformation of reference system

$a_{ki} = $ Eigenvectors of transformation matrix

$\lambda_k \cdot \vec{r}_i = \lambda_k \cdot [a_{ki}]_{matrix}$

Determinism and Uncertainity

- Determinism through contratcting & jurisdiction
- Uncertainity minimizsations
- Statistical approximation
- Prognosis matching Determinism

see Equation 19

Ex-ante Framing and Post-ante Optimization

Framing contracts and jurisdiction ahead to match $p = $ "Economy Funds" and $v = $ "Economy Velocity" that can be modelled with typical approaches for fuction $z = $ "arrival state" depending on $x = $ "starting state", so that "work" $W = F \cdot r = F \cdot (z - x)$:

Table 1: Modelling Functions

Type	Model	Linearisation
Linear	$z = a + bx$	$z = a + bx$
Gemetric	$z = a \cdot x^b$	$\log z = \log a + b \cdot (\log x)$
Exponential deca.	$z = a \cdot b^x$	$\log z = \log a + (\log b) \cdot x$
Exponential nat.	$z = a \cdot e^{bx}$	$\ln z = \ln a + bx$
Periodical		
Polynomial		
Taylor		

Linear optimization to achieve target states "z, h":

1. Oneness situaiton modelling:

   minimize efforts $= z$ or maximizie return $= z$

$$
\begin{aligned}
&\text{target:} && z = c^T \cdot x \\
&\text{min. constraints:} && [a_{ki}] \cdot x \geq b \\
&\text{max. constraints:} && [a_{ki}] \cdot x \leq b \\
&\text{condition:} && x \geq 0
\end{aligned}
\tag{25}
$$

2. Duality situation modelling:

   minimize efforts $= z$ and maximize return $= h$

$$
\begin{aligned}
&\text{target min.:} && z = c^T \cdot x \\
&\text{min. constraints:} && [a_{ki}] \cdot x \geq b \\
&\text{condition:} && x \geq 0 \\
&\ \ and \\
&\text{target max.:} && h = b^T \cdot y \\
&\text{max. constraints:} && [a_{ki}]^T \cdot y \leq c \\
&\text{condition:} && y \geq 0
\end{aligned}
\tag{26}
$$

3. Normalization:

$$
\begin{aligned}
\text{target:} && z &= c^T \cdot x \\
\text{min. constraints:} && \left([a_{ki}] - x_{n+k}\right) \cdot x &= b \\
\text{max. constraints:} && \left([a_{ki}] + x_{n+k}\right) \cdot x &= b \\
\text{condition:} && x &\geq 0
\end{aligned}
\tag{27}
$$

4. Solution with Simplex algorithm

multilinear and exponential models ...

Productiviy and Value

Society's living requires:

- "Resources" are processed to create goods to be consumed
- "Labor" is used to process goods and services
- "Capital" intermediates value between goods and services

Society organizes in groups of households:

- natural households
- juristic corporations
- governmental institutions

Households cooperate though exchange (trade) of goods and services between "Demand" and "Supply" using monetary valuation:

- Goods Market Production, Inventory, Trade → "Price" for value of production
- Labor Market Employment, Demographic Structure, Work Capabilities → "Wage" for value of labor
- Capital Market Savings, Investments, Exchange → "Interest" for value of capital

acronyms see Section

Supply is provided by the income Y:

$$Y = L + Y_H + Y_A + T_A + Y_G - Z_G \tag{28}$$

$$Y = C + S + T + M - X \tag{29}$$

Demand is satisfied by the production V_I:

$$V_I = C + I + G + X - M \tag{30}$$

Equilibrium:

$$
\begin{aligned}
Y &= V_I \\
L - C &= S = I \\
C + S &= L \\
C + I &= R
\end{aligned}
\tag{31}
$$

Domestic Production & International Trade:

$$V_S = Y + T - G_A + D_A \tag{32}$$

$$V_N = V_S - D_A \tag{33}$$

$$V_I = V_S - R_M + R_X \tag{34}$$

Monetary Value of Production:

$$P \cdot V_I = Q \cdot N \tag{35}$$

Macro Productivity:

$$Productivity = \frac{Output}{Input} = \frac{V_I}{Y} \tag{36}$$

We track production at microeconomic level through (acronyms see Section):

A = Assets represent "Property Capital" which consist of "Current Capital" and "Structural Capital". Those investments are required for the production processes.

B = Liabilities represent "Financial Capital" which consist of "Exogen Funds" and "Endogen Funds". Later includes "Equity Capital" and "Retained Gains". Those funds do finance Assets.

W = Expenses represent "Work Efforts" to "Supply Production" with use of Assets.

Y = Income represents "Received Earnings" achieved with production to "Satisfy Demand". Parts of the gains will be retained to finance Assets.

R = Gain, Profit or Loss from executed work (resultant value from input efforts) is represented by $R = Y - W$

The "transfer of energy" between all subsystems is mapped in balancing out theirs capital (usage of capital for production though possessed assets): $A = B + R$

Micro Productivity:

$$Productivity = \frac{Output}{Input} = \frac{Y}{W} \tag{37}$$

Hypothesis and Conjecture

Conjecture of Equation 35 and Equation 16

How consistently map economic measures to the quantities of nature to model the economical system?

Input:
$$Input = f_0(Resouces, Labor, Capital) = \text{Work} = \Delta\text{Energy} = p \cdot v \tag{38}$$

Output:
$$Output = \text{Goods} + \text{Services} = \text{Production} = \Delta\text{State} = V_I \tag{39}$$

Productivity:

$$Productivity = \frac{Output}{Input} = \frac{\Delta State}{\Delta Energy} = \frac{Production}{Work} = \frac{V_I}{pv} \tag{40}$$

see description of "movement" Equation 24

Table 2: Core Measurements

Economy	$\rightarrow$			$\leftarrow$	Physics
Resources	Assets			Force	Matter
Capital	Liabilities			Momentum	Space
Labor	Expenses			Work	
	Income	Production		Energy	Time

As a first approach, considering the geometrical perspective of the concept of "work" (Equation 9), applied to economical measurements, so to satisfy the following system:

$f_A(Assets) = f_B(Liabilities + Gain) =$ "momentum" p from initial state r_0 to new state r_1, where the change of state is $r = r_1 - r_0$

Matter substance, object movement/change:

$$A = f_A = F \cdot t = p \tag{41}$$

Space of movement, maneuverability:

$$B + R = f_B = m \cdot v = p \tag{42}$$

Work input (efforts, costs) through labor force, machines and infrastructure applied, and materials consumed for production:

$$W = f_0 = F \cdot r = m \cdot \frac{v}{t} \cdot r = p \cdot v \tag{43}$$

Energy transfer, energy change, profit or loss:

$$Y = \Delta E = \frac{1}{2}pv + \mathcal{U} \tag{44}$$

Production value inflation adjusted:

$$P \cdot V_I = \sum Y_{adjust} = \sum (W + R)_{adjust} = \Delta U - \Delta H = Q \cdot N \tag{45}$$

see Equation 17 and Equation 9

Thus, to be determined are v (economy velocity) and p (financial structure). Eventually, determine $r =$ "state change through trajectory" (substance change to the new assets structure)

by given $v = $ "velocity of movement" (Business speed = monetary value, price with inflation), or determine v by given r. The individual positions $\{i, j, l, m\}$ of Liabilities $\{B_i + R_j\}_k$, Assets $\{A_m\}_k$, and Expenses $\{W_l\}_k$ shall be grouped in sets $\{k\}$, so to satisfy $f_k(i,j) = f_k(m) = f_k(l) = \{pv\}_k$.

The constraints shall be modelled with the Lagrangian. Hence, "Work" shall be derived from the Hamiltonian which leads to the identification of the "movement" (= change, transformation) of the trajectory (= path to new state) with $\dot{r} = v$ and $\dot{p} = F$.

Experiment design: p **and** v **are modelled ex-ante with means of CONSTRAINTS from contractual agreement and from given juristic legislation, in such a way that physics laws are satisfied. Decisions making occure within this ex-ante frame.**

Acronyms

Acronyms of Physics: Chapter

$A = $ Action	$E = E_{kin} + E_{pot} = $ total energy
$t = $ time	$r = $ dpace (distance between two points, one-dimensional length)
$v = $ velocity	$c = $ light speed
$p = $ momentum	$F = $ force
$I = $ inertia	$\mathcal{L} = $ Lagrangian
$\mathcal{H} = $ Hamiltonian	$\mathcal{K} = $ kinetic Energy
$\mathcal{U} = $ effective potential Energy $(\in E_{pot})$	$\mathcal{V} = $ potential Energy $(\in E_{pot})$
$\mathcal{Z} = \frac{1}{2}\frac{L_0^2}{mr^2} = $ Centrifugal Potential	$V = r_1 \cdot r_2 \cdot r_3 = $ Volume
$k = $ Wave Vector ("curvature")	$W = $ Work (done vs. received)
$P = $ Pressure	$L_0 = $ angular momentum
$T = $ endogen Temperature	$H = $ exogen Heat
$U = $ endogen Energy $(E_{kin} + E_{pot})$	$\Phi = \frac{\mathcal{V}}{q} = $ Electric Potential
$\mathcal{A} = $ Magnetic Potential	$b_0 = $ Boltzmann constant
$g_0 = $ Gas constant	$m = $ mass
$q = $ charge	$\$n = $ amount of objectes (particles density $n = \frac{N}{V}$), $n = 2$ and $f = 1$
$\epsilon_0 = $ electric constant	$f = 3n \pm z = $ degrees of freedom, $f = 1$ and $n = 2$
$\mu_0 = $ magnetic constant	$\mathcal{B} = $ Magnetic Field
$\mathcal{E} = $ Electric Field	$\text{H}\mathcal{C} = m \cdot c_0 = \frac{\Delta H}{\Delta T} = $ heat capacity
$c_0 = \frac{1}{m}\frac{\Delta H}{\Delta T} = $ specific heat	$S = b_0 \cdot \ln(\Omega) = $ Entropy (macro state)

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l = Moll quantity	Ω = micro states
z = amount of constraints (boundry conditions)	b_0 = Boltzmann constant
$\kappa = \frac{c_P}{c_V} = \frac{f+2}{f}$ = adiabaty	$\iota = \kappa$ adiabat
$\iota = 0$ isobar	$\iota = 1$ isotherm
$\iota = \infty$ isochor	...
...	...

Acronyms of Economy: Chapter

T = Taxes	M = Import of Goods and Services from foreing symstes
G = Goverent Expenses, incl. Social Insurances	X = Export of Goods and Services to foreign system
Y = Income of Economy from Turnover	G_A = Governental Subsidies
D_A = Depreciations (Reinvestments) on Assets	V_N = Net Naötional Production, Society NNP
N = Monetary Quantity	Q = Monetary Turnover Velocity
V_I = Gross Domestic Product GDP = $\frac{Output}{Input}$, Tradevolume	P = Price niveau (Inflation adjusted Value)
L = Wages from Labor Work (Salaries, ...)	R = Returns, Earnings, Gains
Y_A = Income of priv. Business Households (Companies, Services, Real Estate Rentals, Retained Profits)	Y_H = Income from priv. Capital Households (Interests, Coupons, Dividends, ... of priv. Assets, Investmens, Credits, Debits, Bonds, Equity)
T_A = Tax on Capital of Corprate Compaies (Business Assets)	Y_G = Governmental Income from Assets, Services, Social Institutions/Insurances
Z_G = Interests on Governmental Debt	V_S = Gross National Produkt, Society GNP
I = Investments on Assets, incl. Storage Change	R_M = Capital Earnings and Wages from Abroad (from Foreign System)
R_X Capital Earnings and Wages to Abroad (to Foreign System)	W = Expensens, costs from human and machinary work efforts

see also [1]

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