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Constraints given by:

- initial state
- boundary conditions
- access to limited resources
- contractual covenants
- juristic law

Modelling movement (change of state) under constraints:

1. Movement is characterized by change of energy, Lagrangian  $\mathcal{L} \neq 0$ :

$$\mathcal{L} = E_{kin} - E_{pot} \quad (1)$$

2. Movement happens within constraints of forces:

$$\hat{F}_i + \tilde{F}_i = \hat{F}_i + \sum_{k=1}^{s_d} \lambda_k a_{ki} = \frac{d}{dt} \left( \frac{\partial \mathcal{L}}{\partial v_i} \right) - \frac{\partial \mathcal{L}}{\partial r_i} \quad (2)$$

and at same time, constraining conditions delimitate movement:

$$\sum_{l=1}^f a_{kl} v_l + b_k = 0 \quad (3)$$

3. Rest is characterized by unchanged energy, Hamiltonian  $\mathcal{H} = E_{kin} + E_{pot} = const..$   
There is movement from state of rest if  $\mathcal{H} \neq 0$ :

$$\mathcal{H} = \sum_i p_i v_i - \mathcal{L} = p v - \mathcal{L} = F r - \mathcal{L} = W - \mathcal{L} = (\Delta E - \Delta H) - \mathcal{L} \quad (4)$$

4. Change of state/position is characterized by movement:

$$\dot{r}_i = v_i = \frac{\partial \mathcal{H}}{\partial p_i} \quad \dot{p}_i = F_i = -\frac{\partial \mathcal{H}}{\partial r_i} \quad (5)$$

see description of “work” ?@eq-workgeom

where:

$s_d$  = constraints  $i = 1, \dots, f$  = degrees of freedom  $k = 1, \dots, s_d$  = constraints

$\hat{F}_i$  = known constraining forces  $\tilde{F}_i = \sum_{k=1}^{s_d} \lambda_k a_{ki}$  = unknown constraining forces

$\lambda_k(t)$  = Eigenvalues for transformation of reference system

$a_{ki}$  = Eigenvectors of transformation matrix

$\lambda_k \cdot \vec{r}_i = \lambda_k \cdot [a_{ki}]_{matrix}$