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Constraints:

- initial state
- boundry conditions
- access to limited resources
- contractual covenants
- juristic law

Modelling movement (change of state) under constraints:

1. Movement ist charecterized by change of energy, Lagrangian $\mathcal{L} \neq 0$:

$$\mathcal{L} = E_{kin} - E_{pot} \tag{1}$$

2. Movement happens within constraints of forces:

$$\hat{F}_i + \tilde{F}_i = \hat{F}_i + \sum_{k=1}^{s_d} \lambda_k a_{ki} = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial v_i}\right) - \frac{\partial \mathcal{L}}{\partial r_i} \tag{2}$$

   and at same time, constraining conditions delimitate movement:

$$\sum_{l=1}^{f} a_{kl} v_l + b_k = 0 \tag{3}$$

3. Rest is characterized by unchanged energy, Hamiltonian $\mathcal{H} = E_{kin} + E_{pot} = const.$. There is movement from state of rest if $\mathcal{H} \neq 0$:

$$\mathcal{H} = \sum_i p_i v_i - \mathcal{L} = pv - \mathcal{L} = Fr - \mathcal{L} = W - \mathcal{L} = (\Delta E - \Delta H) - \mathcal{L} \tag{4}$$

4. Change of state/position is characterized by movement:

$$\dot{r}_i = v_i = \frac{\partial \mathcal{H}}{\partial p_i} \qquad \dot{p}_i = F_i = -\frac{\partial \mathcal{H}}{\partial r_i} \tag{5}$$

see description of “work” **?@eq-workgeom**

where:

s_d = constraints $\quad i = 1, ..., f$ = dedrees of freedom $\quad k = 1, ..., s_d$ = constraints

$\hat{F}_i$ = known constraining forces $\quad \tilde{F}_i = \sum_{k=1}^{s_d} \lambda_k a_{ki}$ = unknown constraining forces

$\lambda_k(t)$ = Eigenvalues for transformation of reference system

a_{ki} = Eigenvectors of transformation matrix

$\lambda_k \cdot \vec{r}_i = \lambda_k \cdot [a_{ki}]_{matrix}$