

# Computer Vision

CS-E4850, 5 study credits

Lecturer: Juho Kannala

# Lecture 7:

## Optical flow and keypoint tracking

- Given two subsequent frames of a video, the optical flow field indicates the apparent motion of each pixel
- If we have more than two frames, we can track features from one frame to the next by following the optical flow

**Acknowledgement:** many slides from Svetlana Lazebnik, Derek Hoiem, Steve Seitz, Rick Szeliski, M. Pollefeys, and others (detailed credits on individual slides)

# Reading & software

- Szeliski's book, Chapter 8
- Baker & Matthews: Lucas-Kanade 20 years on, a unifying framework, 2004
  - <https://www.ri.cmu.edu/publications/lucas-kanade-20-years-on-a-unifying-framework/>
- Shi & Tomasi: Good features to track, 1994
  - <http://www.ai.mit.edu/courses/6.891/handouts/shi94good.pdf>
- OpenCV software:
  - [http://docs.opencv.org/3.1.0/d7/d8b/tutorial\\_py\\_lucas\\_kanade.html](http://docs.opencv.org/3.1.0/d7/d8b/tutorial_py_lucas_kanade.html)

Motivation: glimpse to the state of the art

# FlowNet 2.0: Evolution of Optical Flow Estimation with Deep Networks

Eddy Ilg, Nikolaus Mayer, Tonmoy Saikia, Margret Keuper, Alexey Dosovitskiy, Thomas Brox

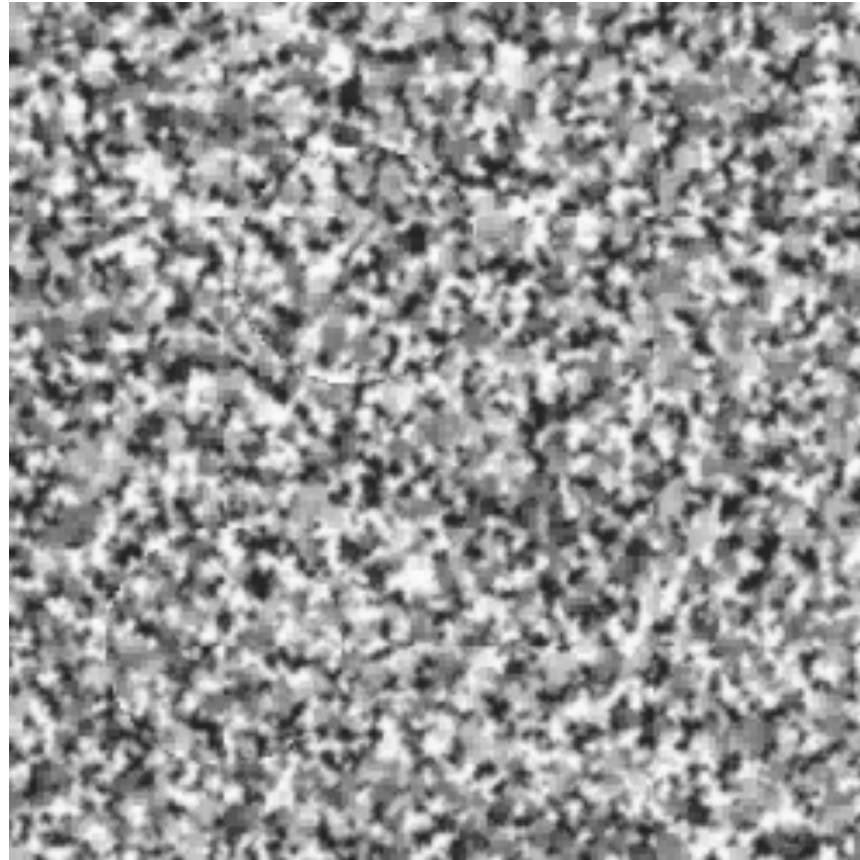
University of Freiburg, Germany

—— Supplementary Material ——

# Motion is a powerful perceptual cue

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- Sometimes, it is the only cue



# Motion is a powerful perceptual cue

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- Even “impoverished” motion data can evoke a strong percept



G. Johansson, “Visual Perception of Biological Motion and a Model For Its Analysis”,  
*Perception and Psychophysics* 14, 201-211, 1973.

# Motion is a powerful perceptual cue

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# Uses of motion in computer vision

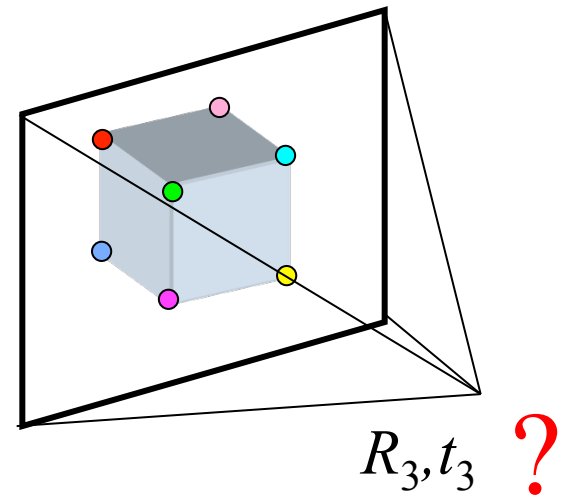
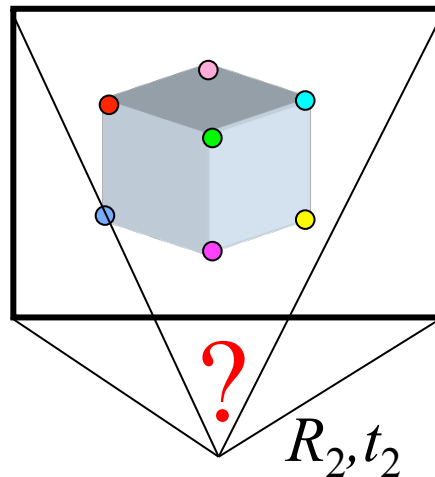
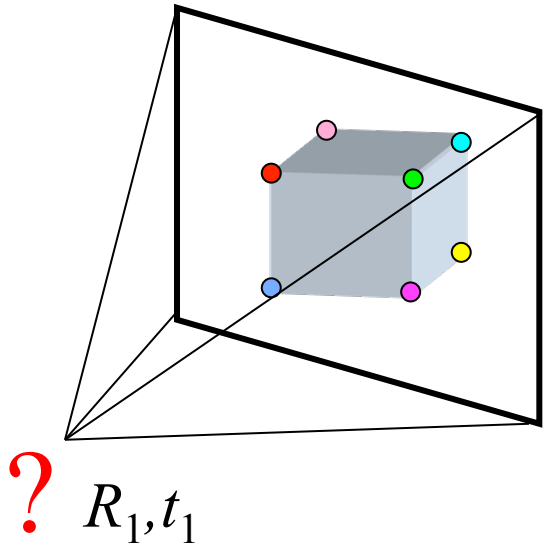
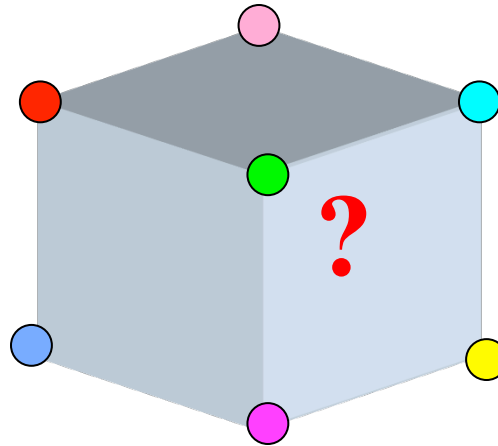
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- 3D shape reconstruction
- Object segmentation
- Learning and tracking of dynamical models
- Event and activity recognition



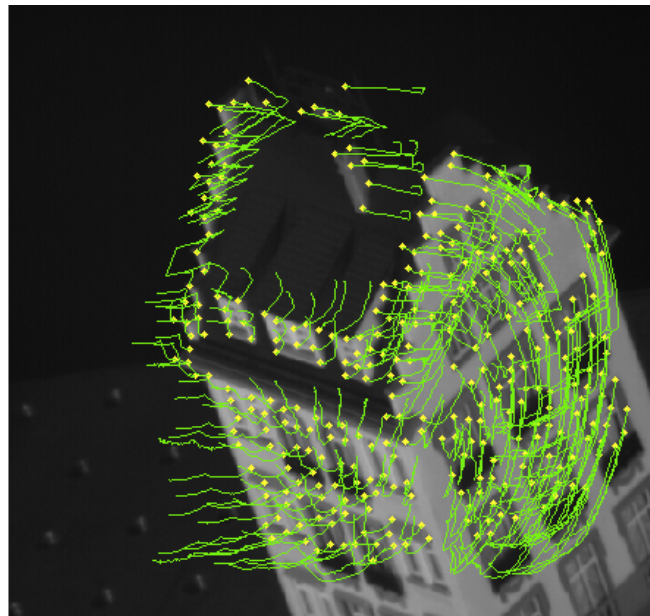
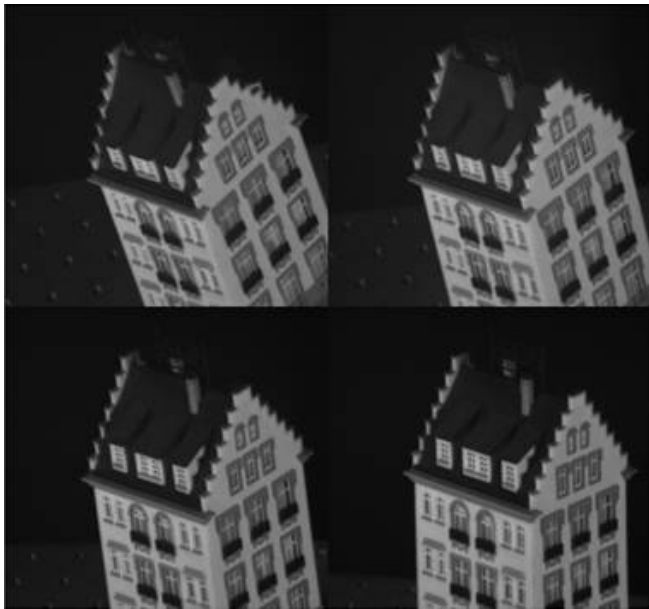
# Preview: Structure from motion

- Given a set of corresponding points in two or more images, compute the camera parameters and the 3D point coordinates



# Keypoint tracking

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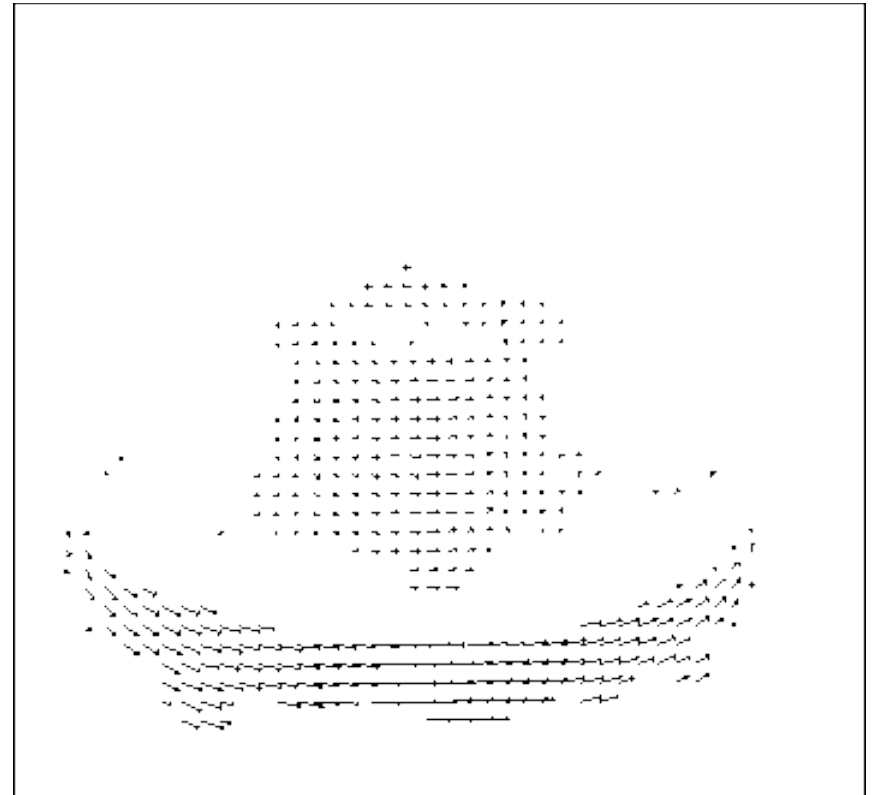
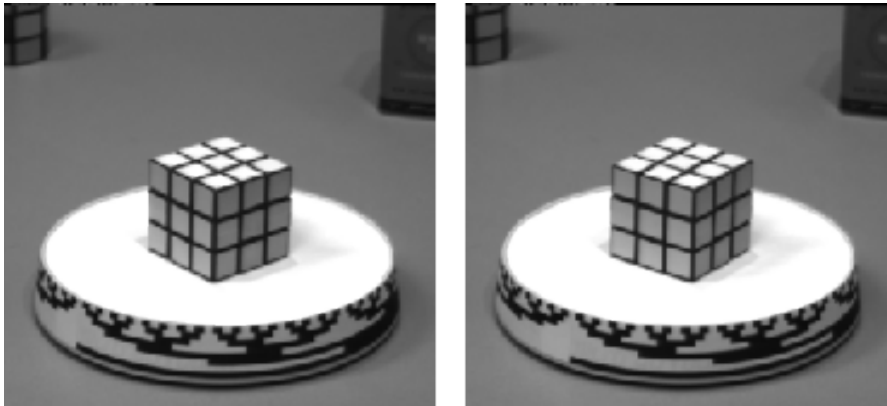


C. Tomasi and T. Kanade. [Shape and motion from image streams under orthography: A factorization method](#). *IJCV*, 9(2):137-154, November 1992.

# Motion field

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- The motion field is the projection of the 3D scene motion into the image



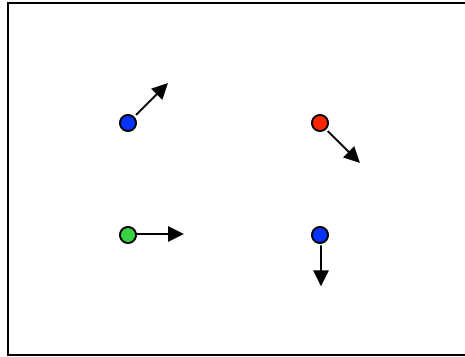
# Optical flow

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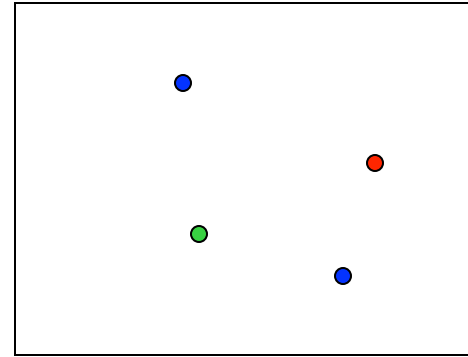
- Definition: optical flow is the *apparent* motion of brightness patterns in the image
- Ideally, optical flow would be the same as the motion field
- Have to be careful: apparent motion can be caused by lighting changes without any actual motion
  - Think of a uniform rotating sphere under fixed lighting vs. a stationary sphere under moving illumination

# Estimating optical flow

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$I(x,y,t-1)$

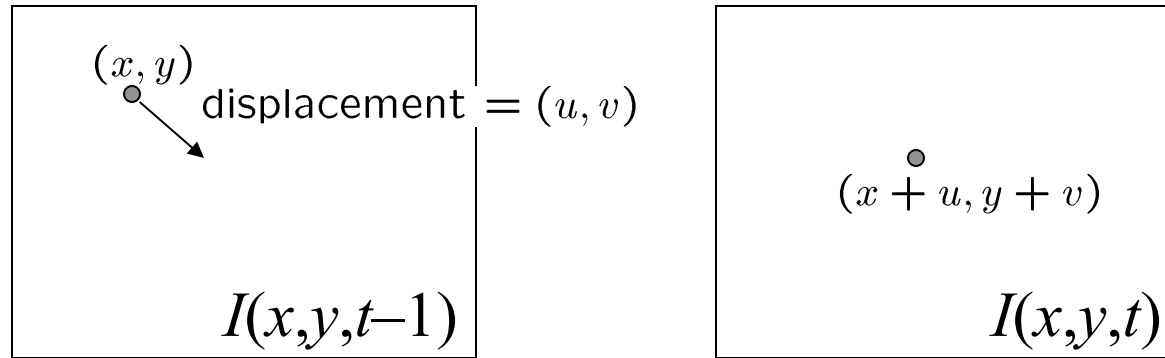


$I(x,y,t)$

- Given two subsequent frames, estimate the apparent motion field  $u(x,y)$  and  $v(x,y)$  between them
- Key assumptions
  - **Brightness constancy:** projection of the same point looks the same in every frame
  - **Small motion:** points do not move very far
  - **Spatial coherence:** points move like their neighbors

# The brightness constancy constraint

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Brightness Constancy Equation:

$$I(x, y, t - 1) = I(x + u(x, y), y + v(x, y), t)$$

Linearizing the right side using Taylor expansion:

$$I(x, y, t - 1) \approx I(x, y, t) + I_x u(x, y) + I_y v(x, y)$$

Hence, 
$$I_x u + I_y v + I_t \approx 0$$

# The brightness constancy constraint

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$$I_x u + I_y v + I_t = 0$$

- How many equations and unknowns per pixel?
  - One equation, two unknowns

- What does this constraint mean?

$$\nabla I \cdot (u, v) + I_t = 0$$

- The component of the flow perpendicular to the gradient (i.e., parallel to the edge) is unknown

# The brightness constancy constraint

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$$I_x u + I_y v + I_t = 0$$

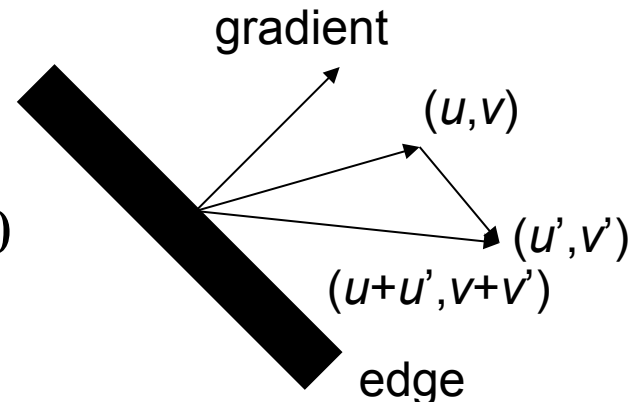
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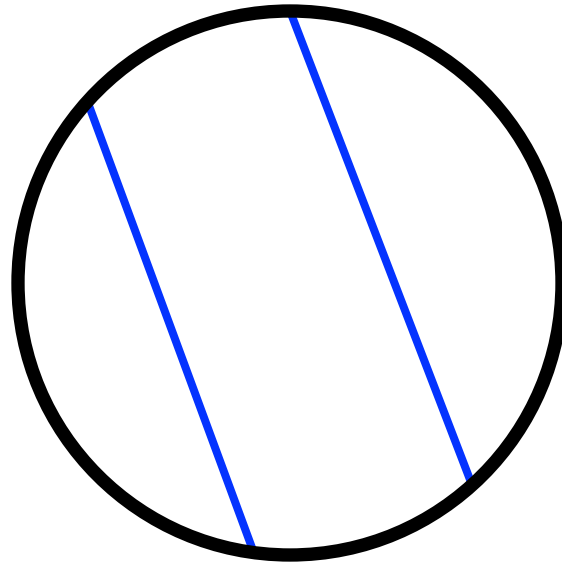
If  $(u, v)$  satisfies the equation,  
so does  $(u+u', v+v')$  if  $\nabla I \cdot (u', v') = 0$





# The aperture problem

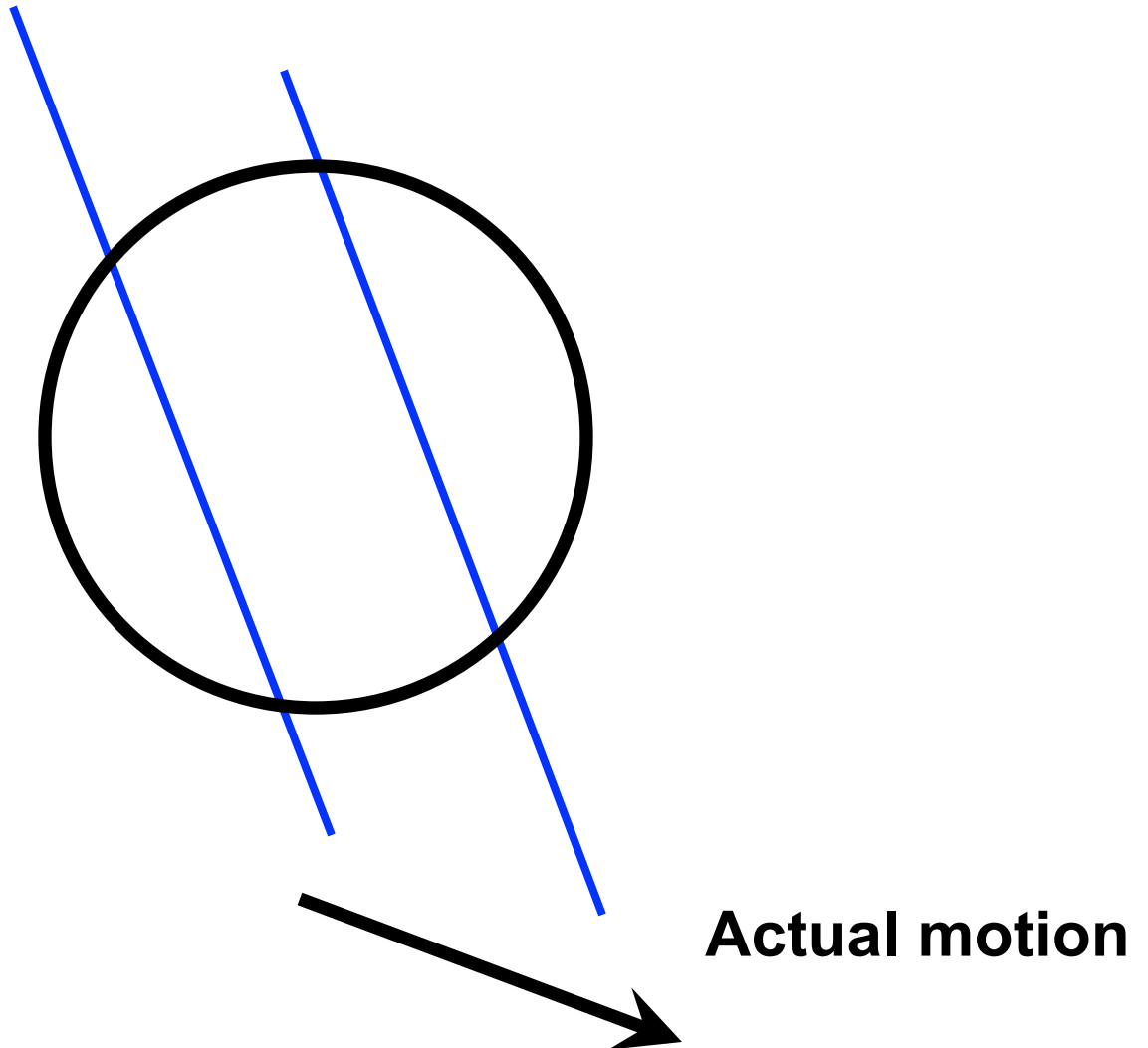
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**Perceived motion**

# The aperture problem

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# The barber pole illusion

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[http://en.wikipedia.org/wiki/Barberpole\\_illusion](http://en.wikipedia.org/wiki/Barberpole_illusion)

# The barber pole illusion

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[http://en.wikipedia.org/wiki/Barberpole\\_illusion](http://en.wikipedia.org/wiki/Barberpole_illusion)

# Solving the aperture problem

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- How to get more equations for a pixel?
- **Spatial coherence constraint:** pretend the pixel's neighbors have the same  $(u,v)$ 
  - E.g., if we use a 5x5 window, that gives us 25 equations per pixel

$$\nabla I(\mathbf{x}_i) \cdot [u, v] + I_t(\mathbf{x}_i) = 0$$

$$\begin{bmatrix} I_x(\mathbf{x}_1) & I_y(\mathbf{x}_1) \\ I_x(\mathbf{x}_2) & I_y(\mathbf{x}_2) \\ \vdots & \vdots \\ I_x(\mathbf{x}_n) & I_y(\mathbf{x}_n) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathbf{x}_1) \\ I_t(\mathbf{x}_2) \\ \vdots \\ I_t(\mathbf{x}_n) \end{bmatrix}$$

B. Lucas and T. Kanade. [An iterative image registration technique with an application to stereo vision](#). In *Proceedings of the International Joint Conference on Artificial Intelligence*, pp. 674–679, 1981.

# Solving the aperture problem

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- Least squares problem:

$$\begin{bmatrix} I_x(\mathbf{x}_1) & I_y(\mathbf{x}_1) \\ I_x(\mathbf{x}_2) & I_y(\mathbf{x}_2) \\ \vdots & \vdots \\ I_x(\mathbf{x}_n) & I_y(\mathbf{x}_n) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathbf{x}_1) \\ I_t(\mathbf{x}_2) \\ \vdots \\ I_t(\mathbf{x}_n) \end{bmatrix}$$

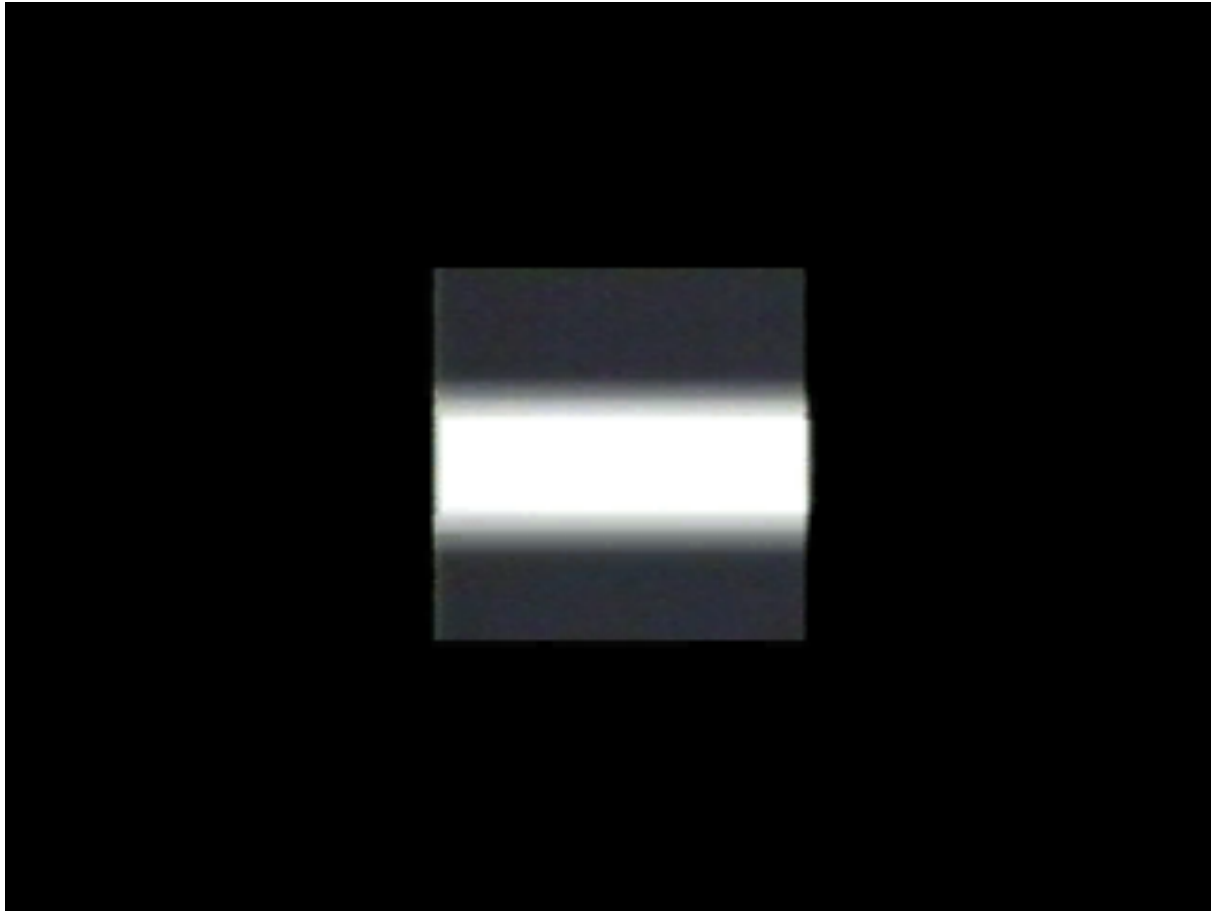
- When is this system solvable?
  - What if the window contains just a single straight edge?

B. Lucas and T. Kanade. [An iterative image registration technique with an application to stereo vision](#). In *Proceedings of the International Joint Conference on Artificial Intelligence*, pp. 674–679, 1981.

# Conditions for solvability

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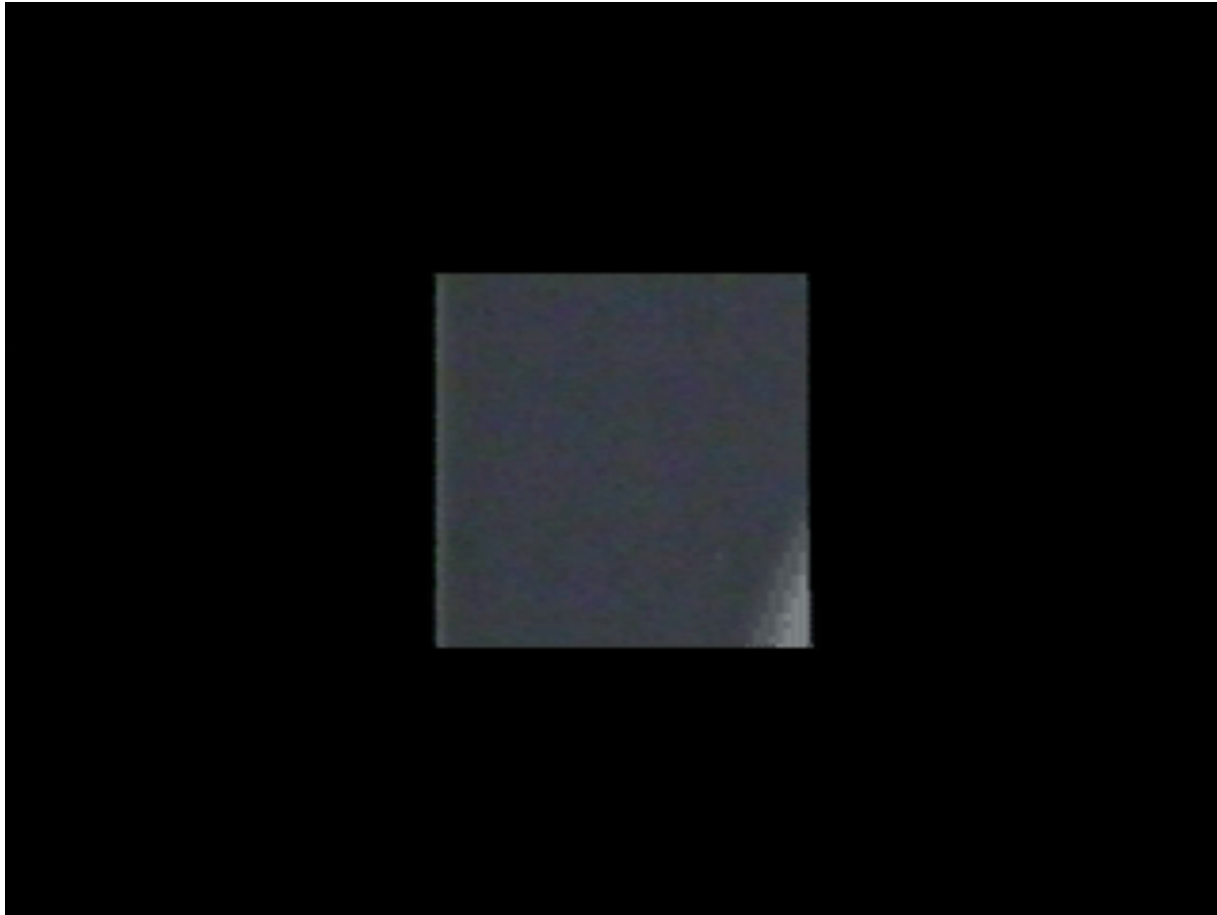
- “Bad” case: single straight edge



# Conditions for solvability

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- “Good” case





# Lucas-Kanade flow

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Linear least squares problem

$$\begin{bmatrix} I_x(\mathbf{x}_1) & I_y(\mathbf{x}_1) \\ I_x(\mathbf{x}_2) & I_y(\mathbf{x}_2) \\ \vdots & \vdots \\ I_x(\mathbf{x}_n) & I_y(\mathbf{x}_n) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathbf{x}_1) \\ I_t(\mathbf{x}_2) \\ \vdots \\ I_t(\mathbf{x}_n) \end{bmatrix} \quad \mathbf{A} \mathbf{d} = \mathbf{b}$$

$n \times 2 \quad 2 \times 1 \quad n \times 1$

Solution given by  $(\mathbf{A}^T \mathbf{A}) \mathbf{d} = \mathbf{A}^T \mathbf{b}$

$$\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$

The summations are over all pixels in the window

B. Lucas and T. Kanade. [An iterative image registration technique with an application to stereo vision](#). In *Proceedings of the International Joint Conference on Artificial Intelligence*, pp. 674–679, 1981.

# Lucas-Kanade flow

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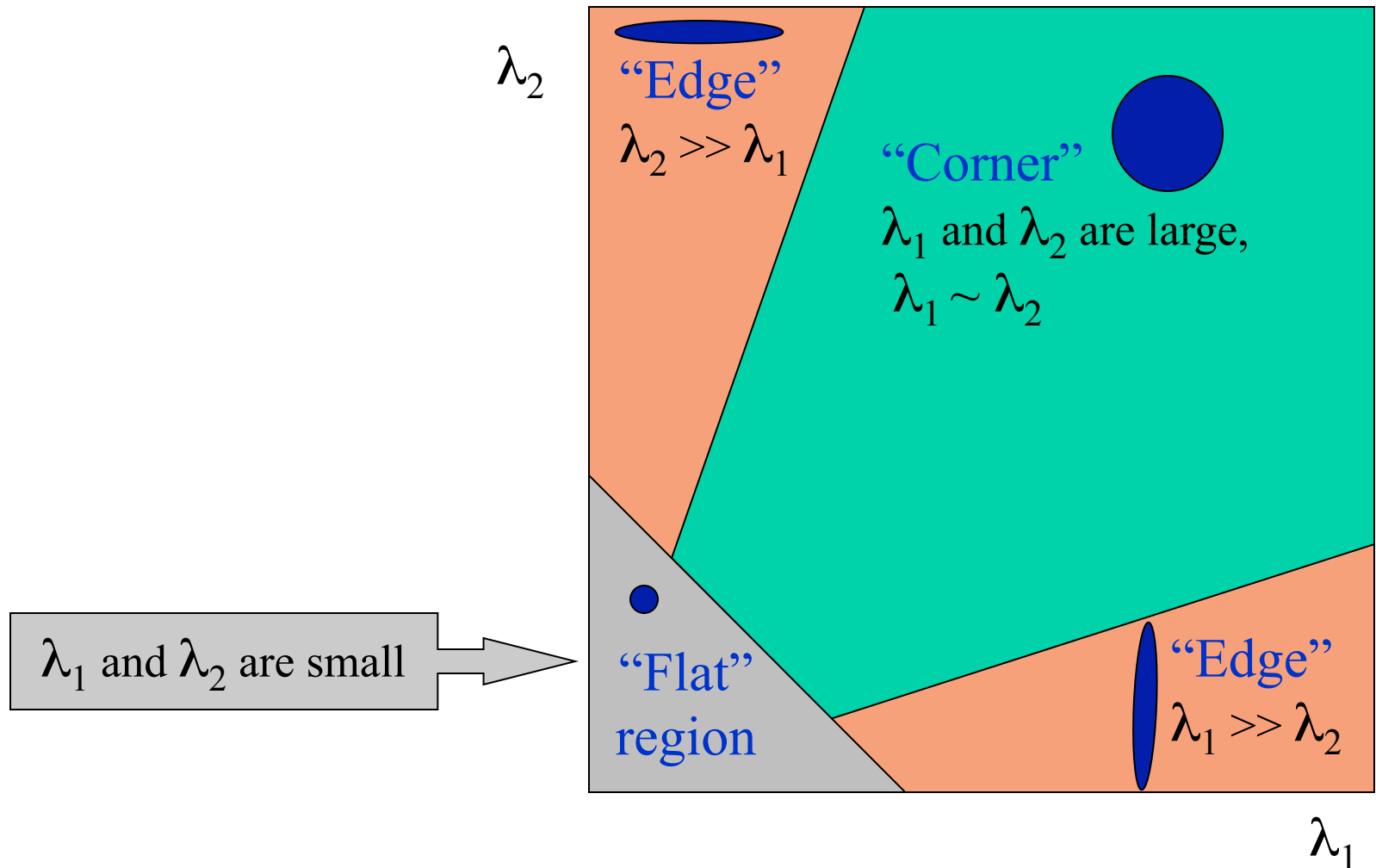
$$\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$

- Recall the Harris corner detector:  $\mathbf{M} = \mathbf{A}^T \mathbf{A}$  is the *second moment matrix*
- We can figure out whether the system is solvable by looking at the eigenvalues of the second moment matrix
  - The eigenvectors and eigenvalues of  $\mathbf{M}$  relate to edge direction and magnitude
  - The eigenvector associated with the larger eigenvalue points in the direction of fastest intensity change, and the other eigenvector is orthogonal to it

# Recall: second moment matrix

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Classification of image points using eigenvalues of the second moment matrix:



# Uniform region

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- gradients have small magnitude
- small  $\lambda_1$ , small  $\lambda_2$
- system is ill-conditioned

# Edge

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- gradients have one dominant direction
- large  $\lambda_1$ , small  $\lambda_2$
- system is ill-conditioned

# High-texture or corner region

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- gradients have different directions, large magnitudes
- large  $\lambda_1$ , large  $\lambda_2$
- system is well-conditioned

# Example of optical flow estimation

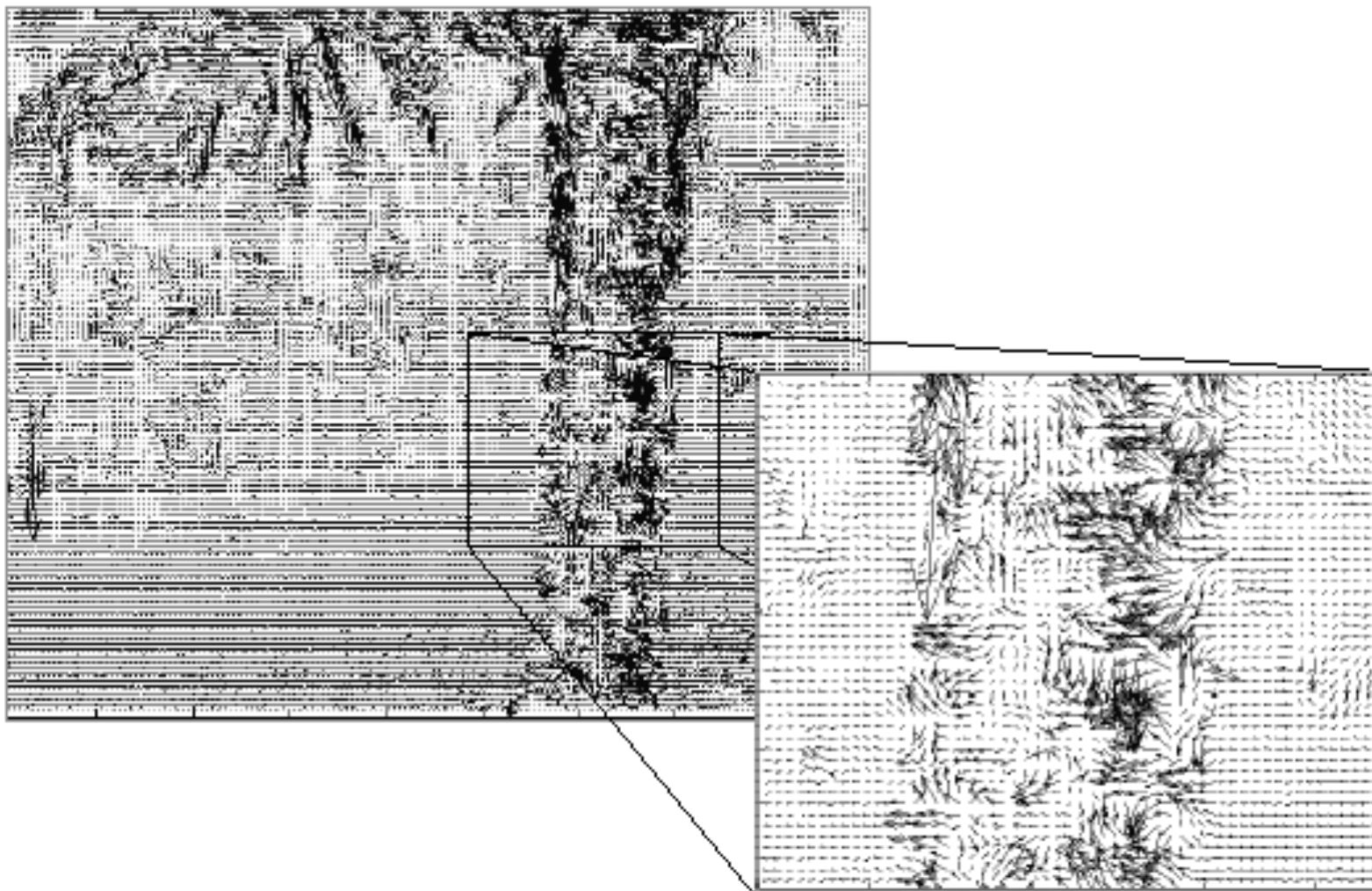
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# Example of optical flow estimation

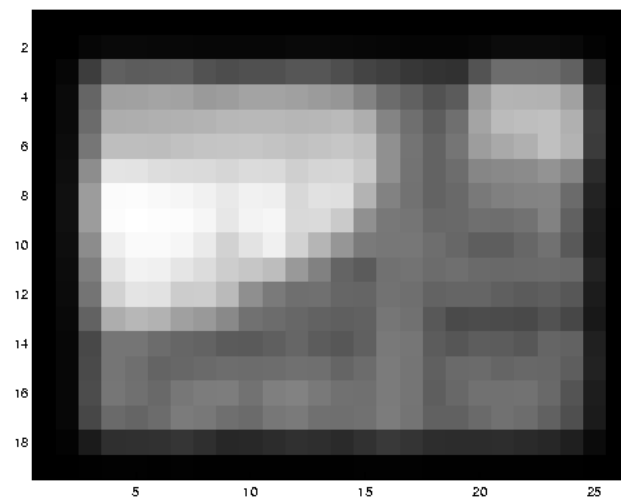
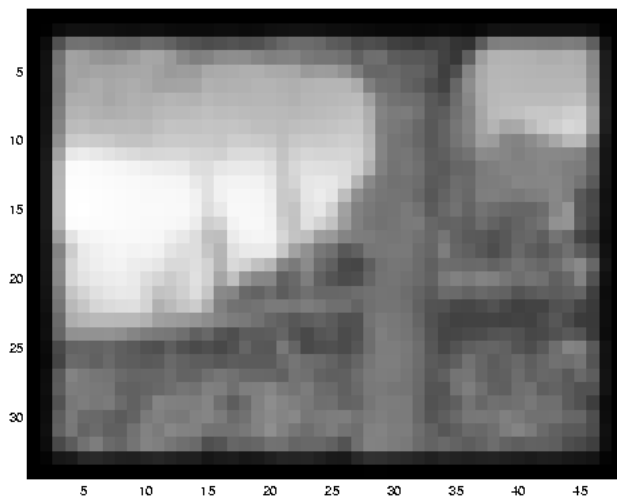
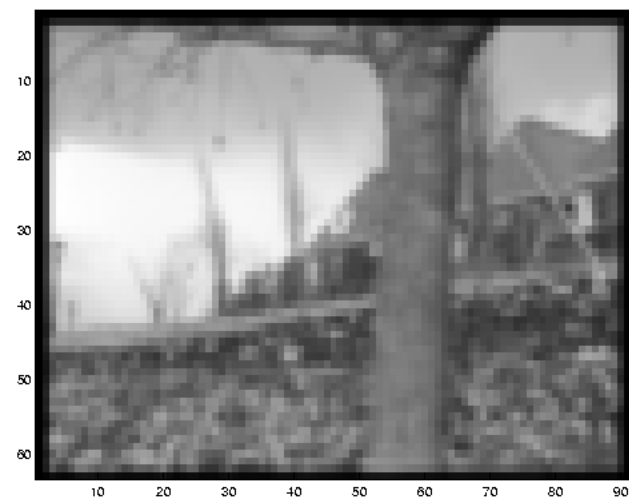
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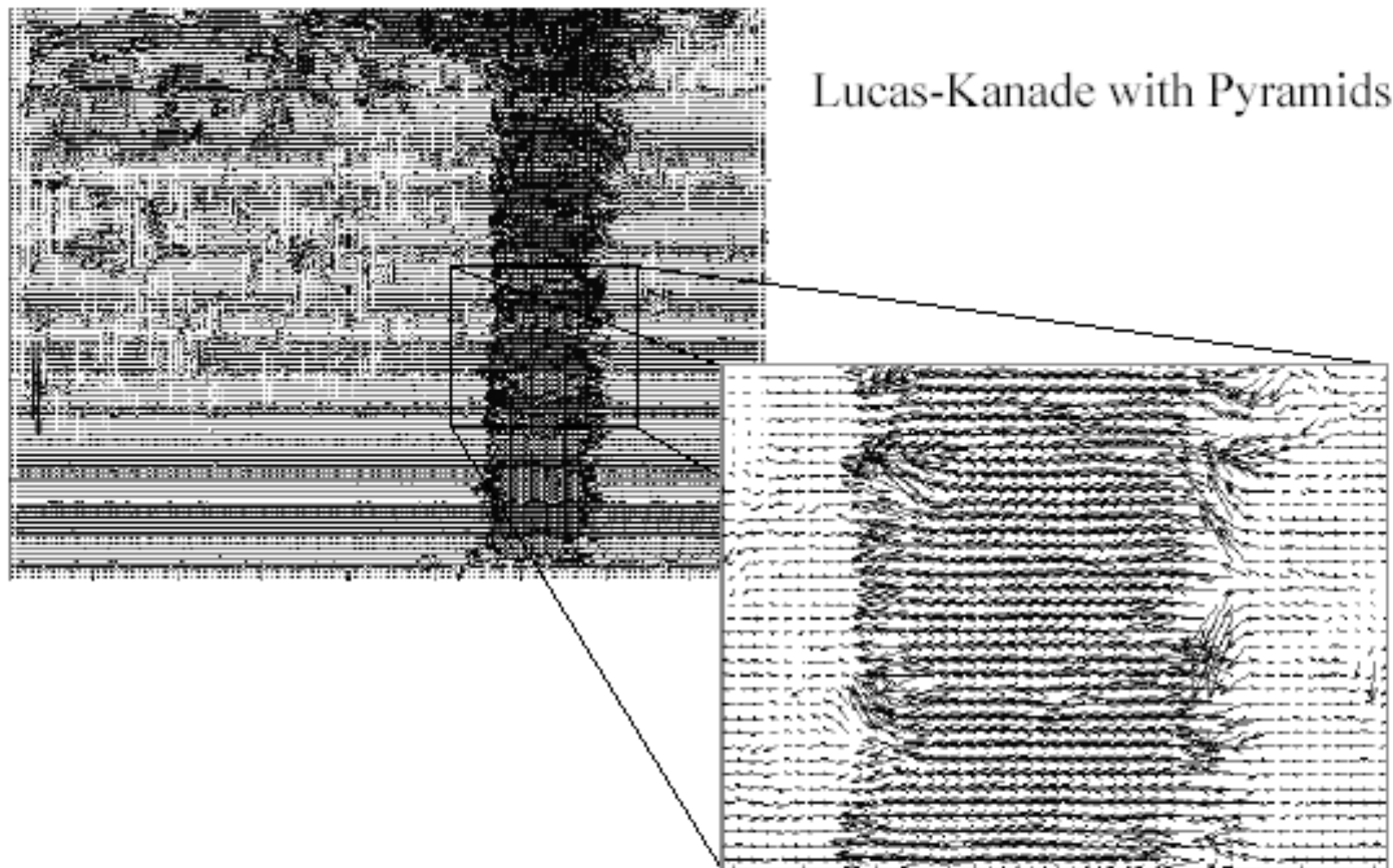
# Multi-resolution estimation

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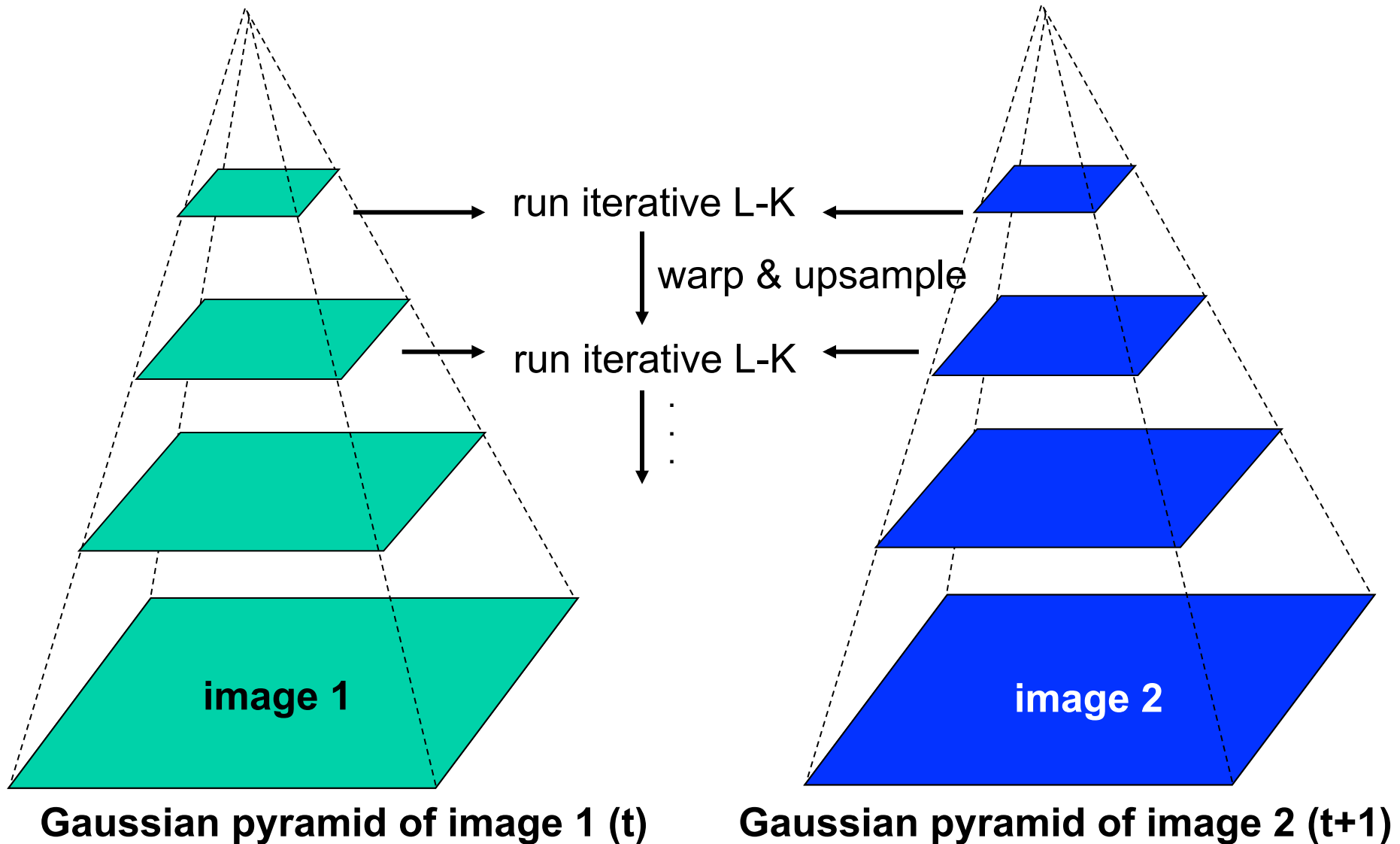
# Multi-resolution estimation

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# Coarse-to-fine optical flow estimation

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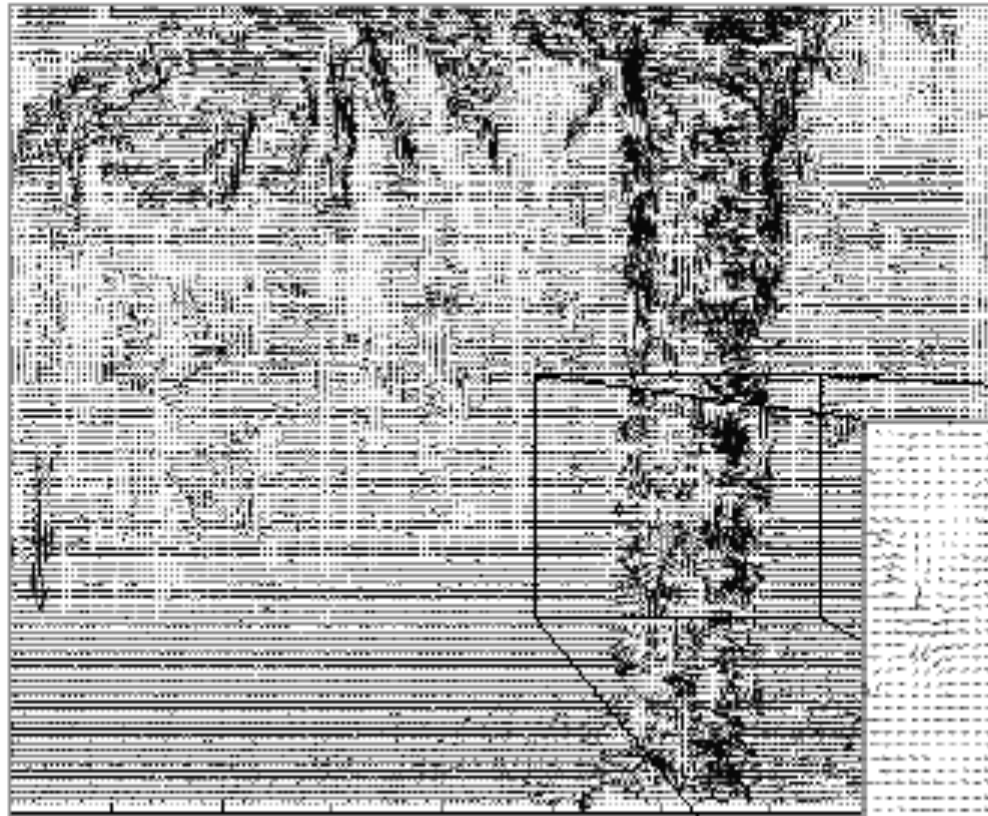
# Multi-resolution Lucas Kanade Algorithm

- Compute ‘simple’ LK at highest level
- At level  $i$ 
  - Take flow  $u_{i-1}, v_{i-1}$  from level  $i-1$
  - bilinear interpolate it to create  $u_i^*, v_i^*$  matrices of twice resolution for level  $i$
  - multiply  $u_i^*, v_i^*$  by 2
  - compute  $f_t$  from a block displaced by  $u_i^*(x,y), v_i^*(x,y)$
  - Apply LK to get  $u_i'(x, y), v_i'(x, y)$  (the correction in flow)
  - Add corrections  $u_i', v_i'$ , i.e.  $u_i = u_i^* + u_i'$ ,  $v_i = v_i^* + v_i'$ .

# Iterative Refinement

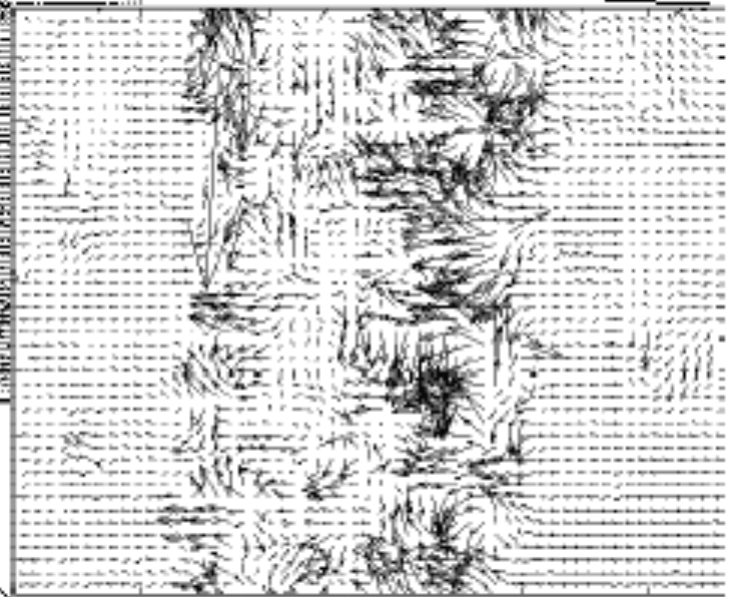
- Iterative Lukas-Kanade Algorithm
  1. Estimate displacement at each pixel by solving Lucas-Kanade equations
  2. Warp  $I(t)$  towards  $I(t+1)$  using the estimated flow field
    - Basically, just interpolation
  3. Repeat until convergence

# Optical Flow Results



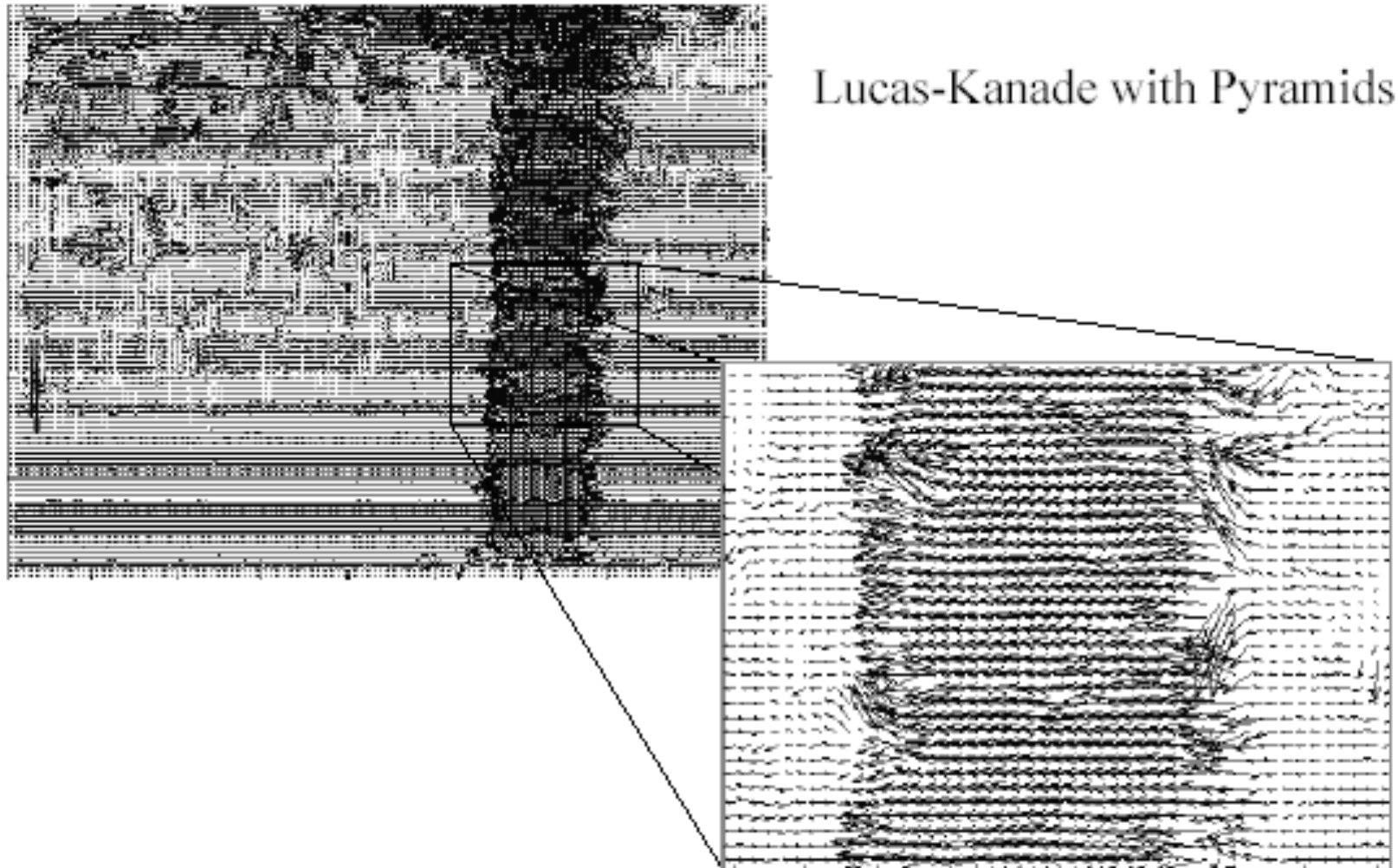
Lucas-Kanade  
without pyramids

Fails in areas of large  
motion





# Optical Flow Results



# Errors in Lucas-Kanade

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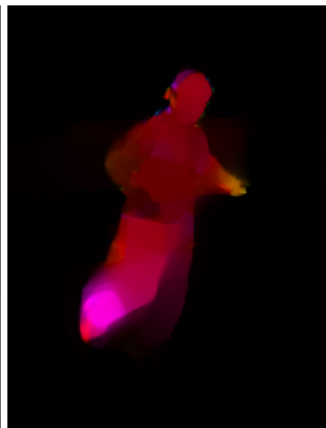
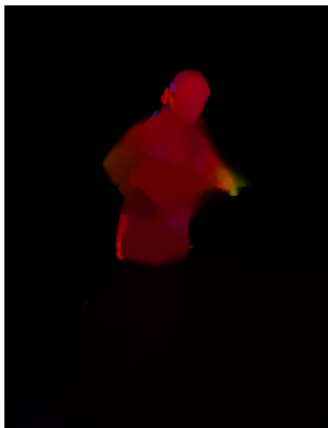
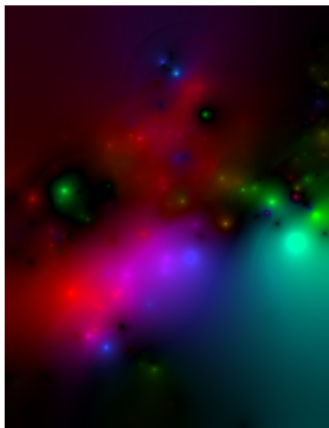
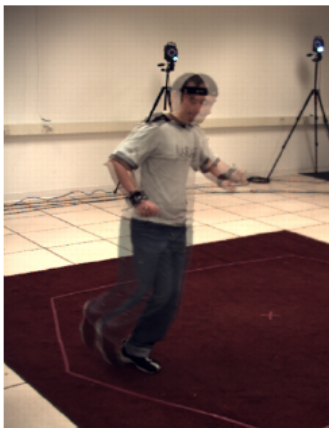
- The motion is large (larger than a pixel)
  - Coarse-to-fine estimation
  - Iterative refinement
  - Exhaustive neighborhood search (feature matching)
- A point does not move like its neighbors
  - Motion segmentation
- Brightness constancy does not hold
  - Exhaustive neighborhood search with normalized correlation



# Large displacement optical flow

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- Start with something similar to Lucas-Kanade
- + gradient constancy
- + energy minimization with smoothing term
- + region matching
- + keypoint matching (long-range)

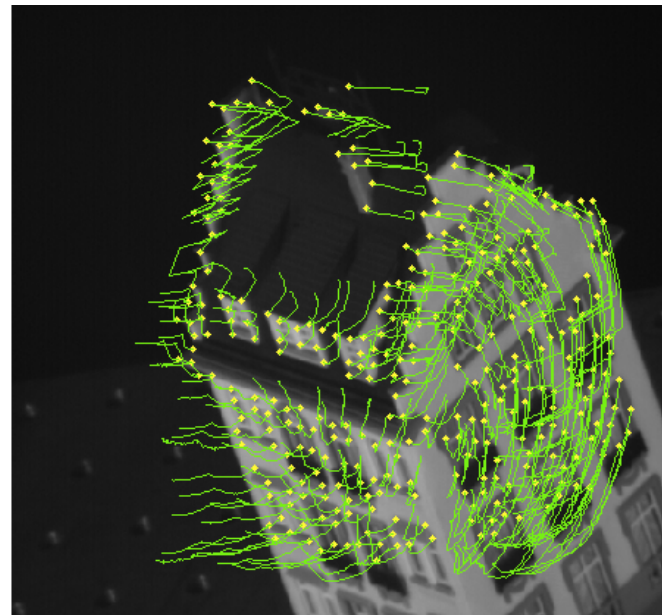
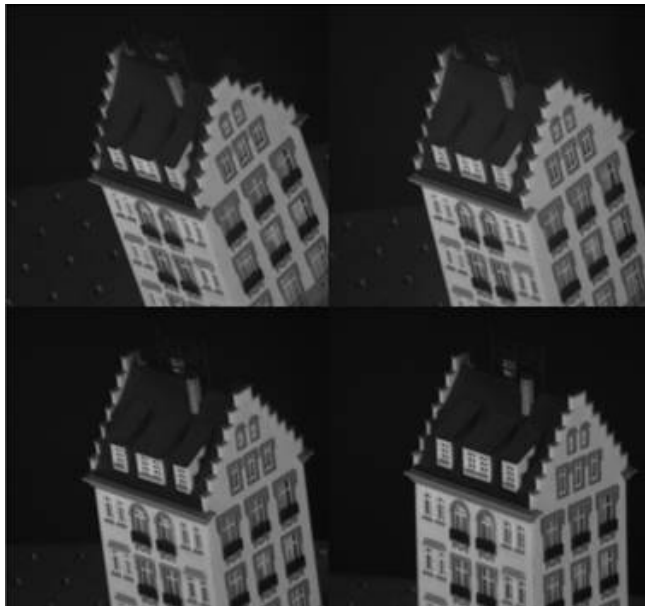


Region-based +Pixel-based +Keypoint-based

# Feature tracking

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- If we have more than two images, we can track a feature from one frame to the next by following the optical flow
- Challenges
  - Finding good features to track
  - Adding and deleting tracks



# Shi-Tomasi feature tracker

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- Find good features using eigenvalues of second-moment matrix
  - Key idea: “good” features to track are the ones whose motion can be estimated reliably
- From frame to frame, track with Lucas-Kanade
  - This amounts to assuming a translation model for frame-to-frame feature movement
- Check consistency of tracks by *affine* registration to the first observed instance of the feature
  - Affine model is more accurate for larger displacements
  - Comparing to the first frame helps to minimize drift

# Tracking example

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Figure 1: Three frame details from Woody Allen's *Manhattan*. The details are from the 1st, 11th, and 21st frames of a subsequence from the movie.



Figure 2: The traffic sign windows from frames 1,6,11,16,21 as tracked (top), and warped by the computed deformation matrices (bottom).

# Summary of KLT tracking

- Find a good point to track (harris corner)
- Use intensity second moment matrix and difference across frames to find displacement
- Iterate and use coarse-to-fine search to deal with larger movements
- When creating long tracks, check appearance of registered patch against appearance of initial patch to find points that have drifted

# Implementation issues

- Window size
  - Small window more sensitive to noise and may miss larger motions (without pyramid)
  - Large window more likely to cross an occlusion boundary (and it's slower)
  - 15x15 to 31x31 seems typical
- Weighting the window
  - Common to apply weights so that center matters more (e.g., with Gaussian)

# Why not just do local template matching?

- Slow (need to check more locations)
- Does not give subpixel alignment (or becomes much slower)
  - Even pixel alignment may not be good enough to prevent drift
- May be useful as a step in tracking if there are large movements

# Summary

- Major contributions from Lucas, Kanade, Shi, Tomasi
  - Tracking feature points
  - Optical flow
- Key ideas
  - By assuming brightness constancy, truncated Taylor expansion leads to simple and fast patch matching across frames
  - Coarse-to-fine registration