

CV5

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1 Ex1 Total least squares line fitting

1.1

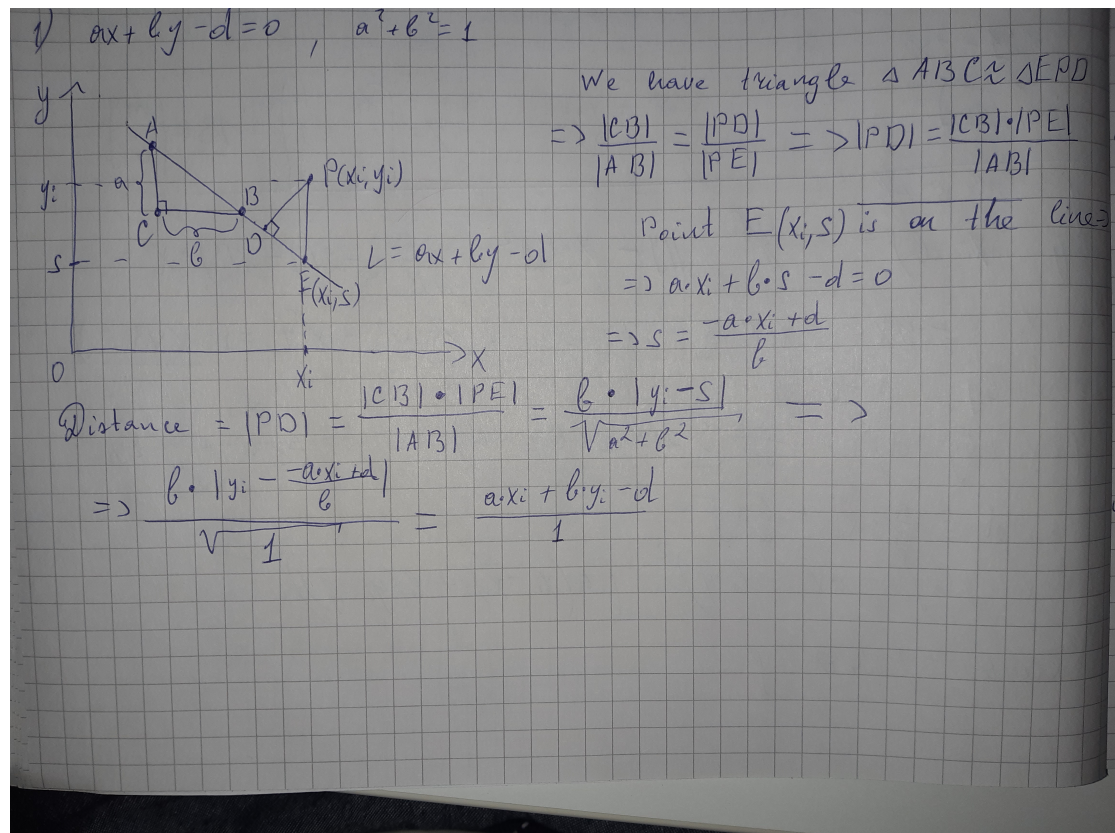


Figure 1: Exercise 1.1

1.2

$$\frac{\partial E}{\partial d} = -2(a * x_i + b * y_i - d) \Rightarrow d = \frac{a}{n} \sum x_i + \frac{b}{n} \sum y_i \Rightarrow$$

$$d = a * \hat{x} + b * \hat{y}$$

1.3

$$E = \sum (a * x_i + b * y_i - d)^2 = \sum (a * x_i + b * y_i - a * \hat{x} - b * \hat{y}) \Rightarrow$$

$$E = \sum (a * (x_i - \hat{x}) + b * (y_i - \hat{y}))^2 \Rightarrow$$

$$\left\| \begin{bmatrix} x_1 - \hat{x} & y_1 - \hat{y} \\ \vdots & \vdots \\ x_n - \hat{x} & y_n - \hat{y} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \right\|^2 = \|U * N\|^2 = (UN)^T * (UN) = (N^T * U^T)(UN)$$

1.4

$$E = (N^T * U^T)(UN) \Rightarrow \begin{bmatrix} a & b \end{bmatrix} * \begin{bmatrix} \sum (x_i - \hat{x})^2 & \sum (x_i - \hat{x})(y_i - \hat{y}) \\ \sum (x_i - \hat{x})(y_i - \hat{y}) & \sum (y_i - \hat{y})^2 \end{bmatrix} * \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\frac{\partial E}{\partial N} = 2 * (U^T U)N = 0 \Rightarrow (U^T U)N = 0$$