Computer Vision

CS-E4850, 5 study credits

Lecturer: Juho Kannala

Lecture 5: Image alignment and object instance recognition

- Given two images of a planar scene (or from a rotating camera), find the parameters of a global geometric transformation that accounts for most true point correspondences between the images
- Given a query image and a database of object images, detect whether the objects are visible in the query image
- Reading:
  - Szeliski's book, Sections 6.1.1 - 6.1.4
  - Chapter 4 in Hartley & Zisserman
  - Lowe's SIFT paper: http://www.cs.ubc.ca/~lowe/papers/ijcv04.pdf

Acknowledgement: many slides from Svetlana Lazebnik, Derek Hoiem, Kristen Grauman, and others (detailed credits on individual slides)

Source: S. Lazebnik

Image alignment

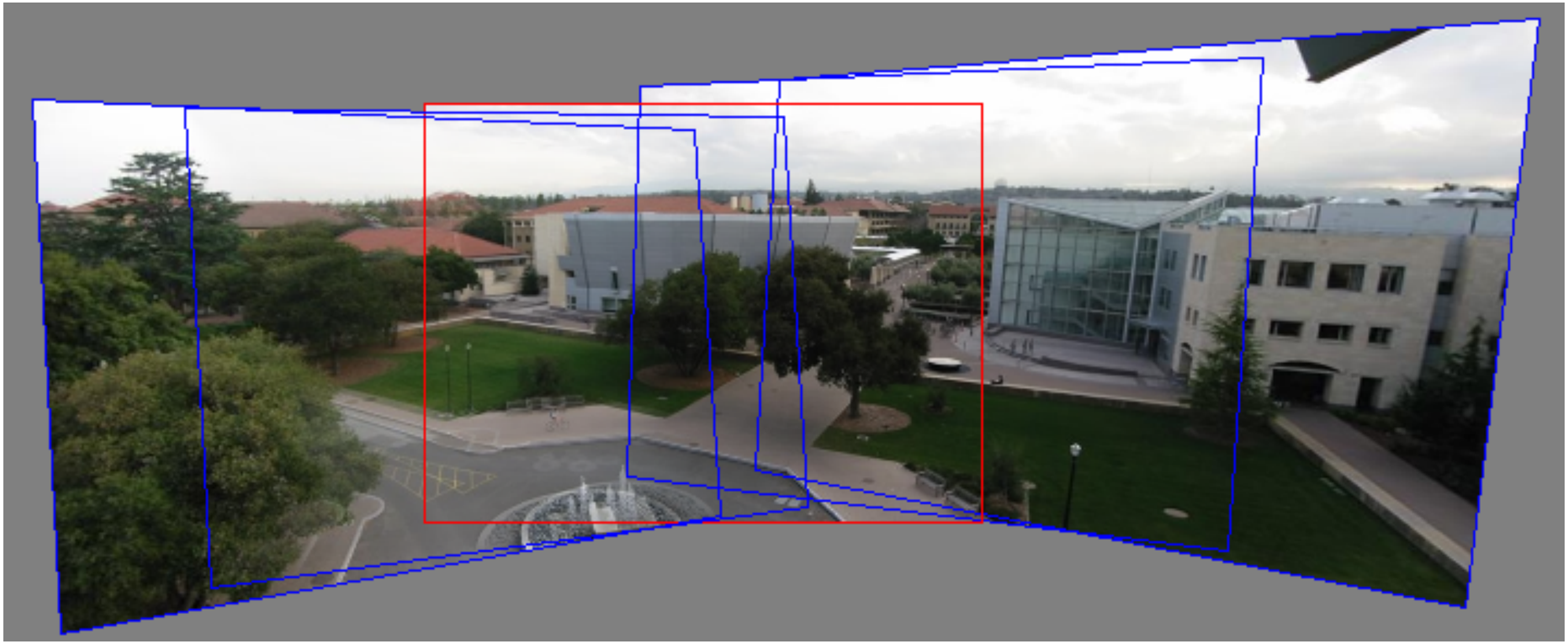
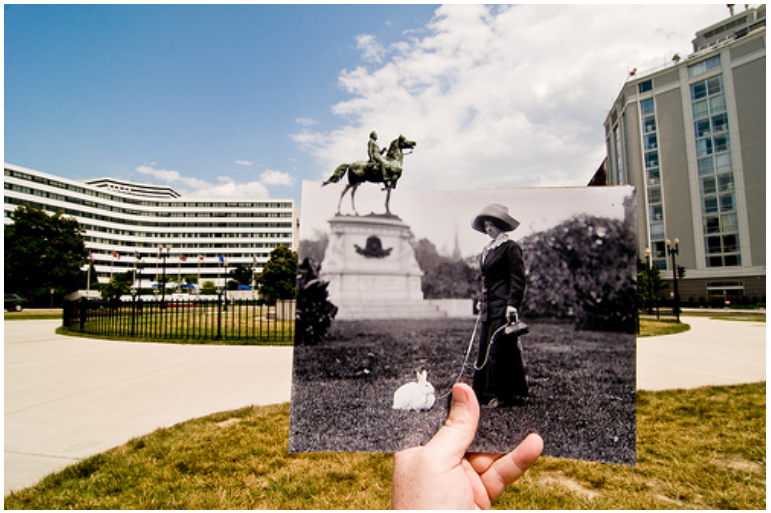


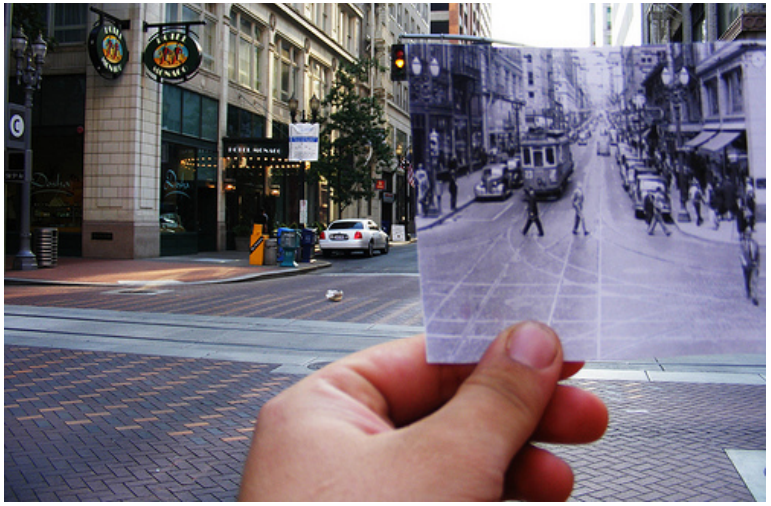
Image from http://graphics.cs.cmu.edu/courses/15-463/2010_fall/

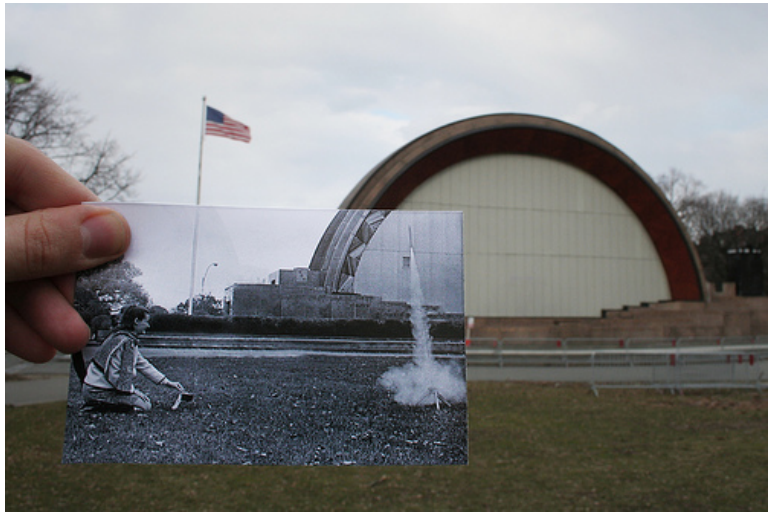
Source: S. Lazebnik

Alignment applications

- A look into the past

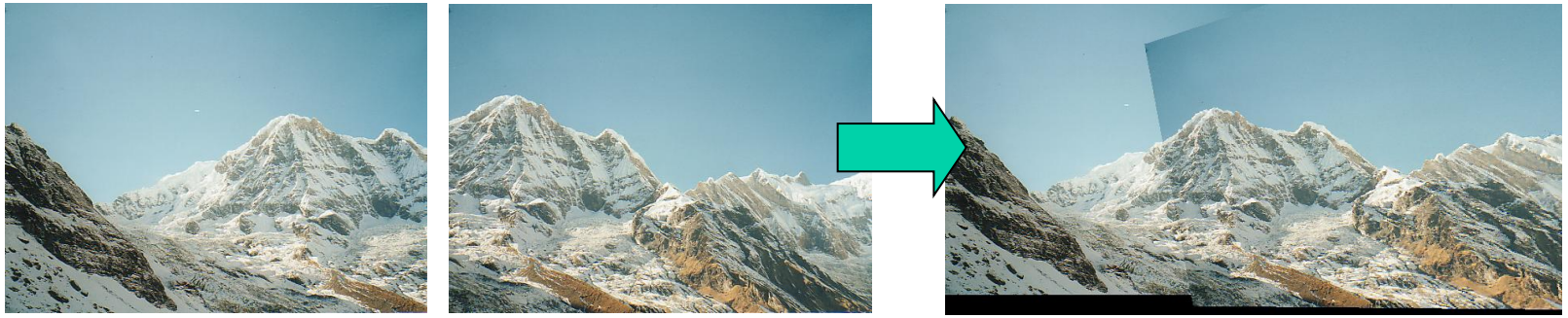




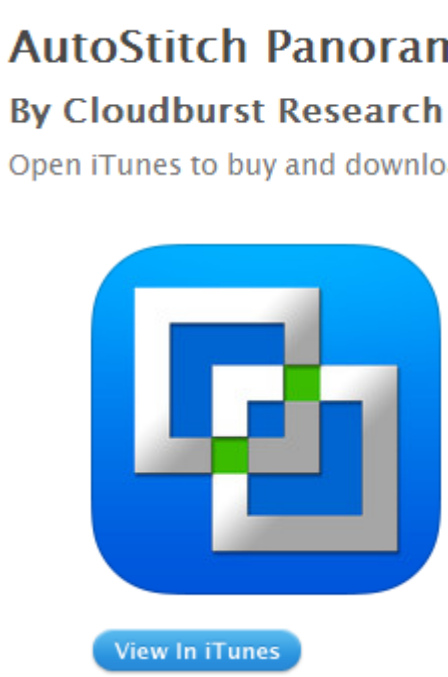


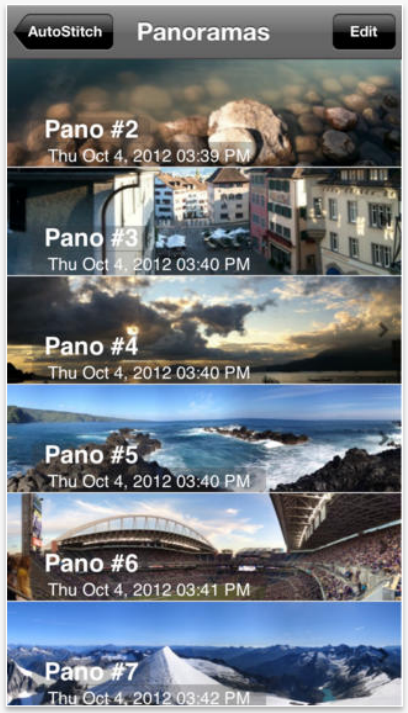
Source: S. Lazebnik

Alignment applications



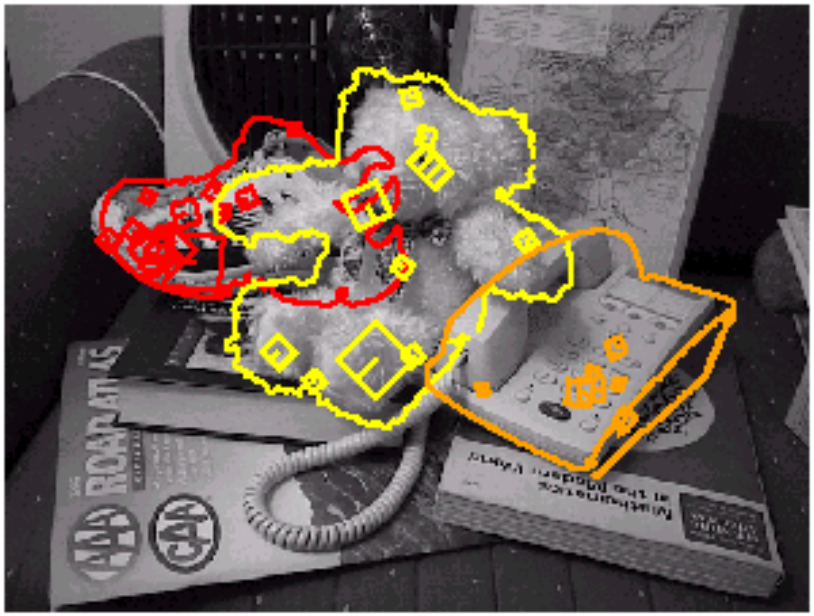
Panorama stitching





Source: S. Lazebnik

Alignment applications



Recognition of object instances

Source: S. Lazebnik

Alignment

Alignment: find parameters of model that maps one set of points to another

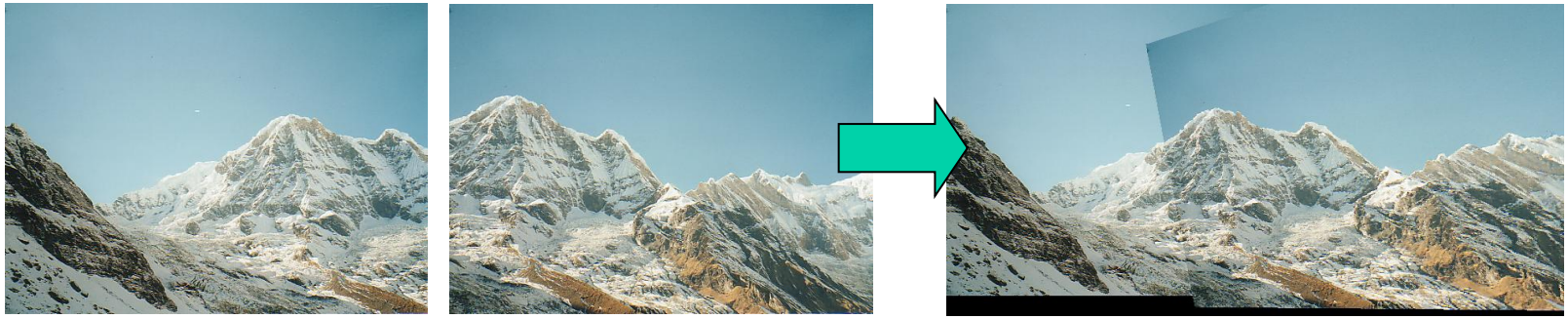
Typically want to solve for a global transformation that accounts for most true correspondences

Difficulties

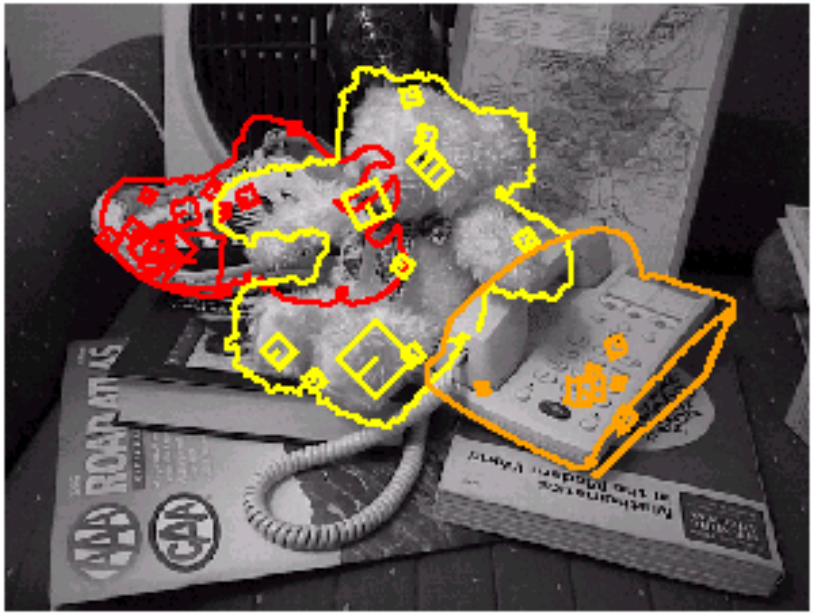
- Noise (typically 1-3 pixels)
- Outliers (often 30-50%)
- Many-to-one matches or multiple objects

Source: S. Lazebnik

Alignment challenges



Small degree of overlap
Intensity changes

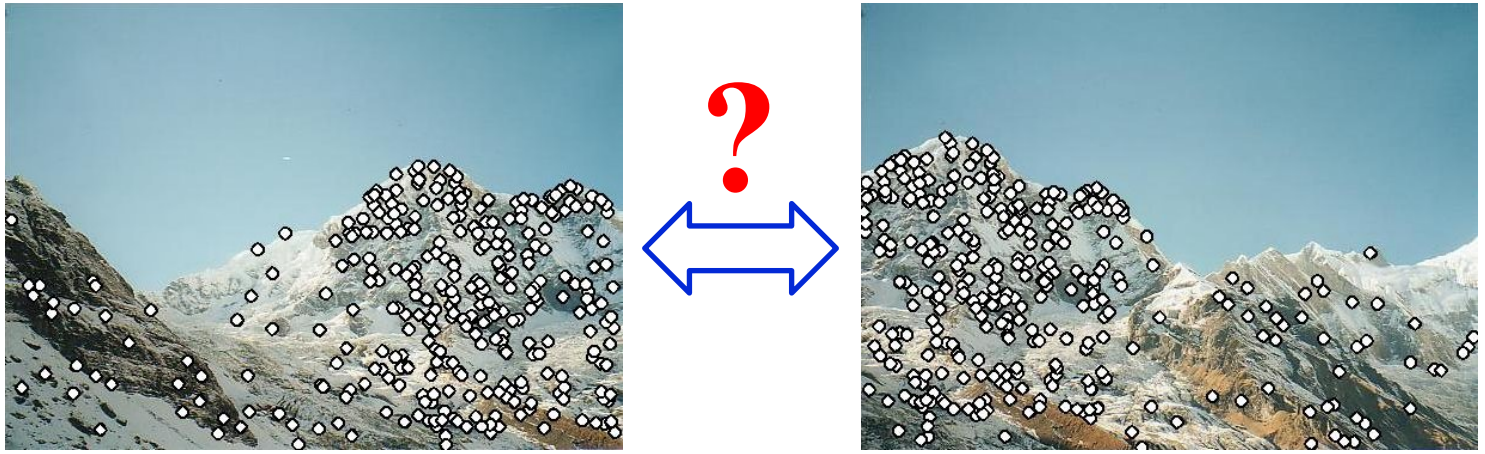


Occlusion, clutter

Source: S. Lazebnik

Feature-based alignment

- Search sets of feature matches that agree in terms of:
  a) Local appearance
  b) Geometric configuration



Source: S. Lazebnik

Feature-based alignment: Overview

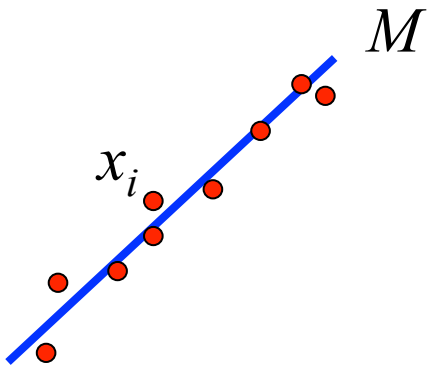
- Alignment as fitting
  - Affine transformations
  - Homographies

- Robust alignment
  - Descriptor-based feature matching
  - RANSAC

Source: S. Lazebnik

Alignment as fitting

- Previous lectures: fitting a model to features in one image



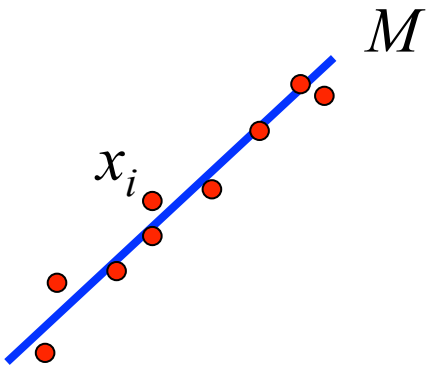
Find model *M* that minimizes

$$\sum_i \text{residual}(x_i, M)$$

Source: S. Lazebnik

Alignment as fitting

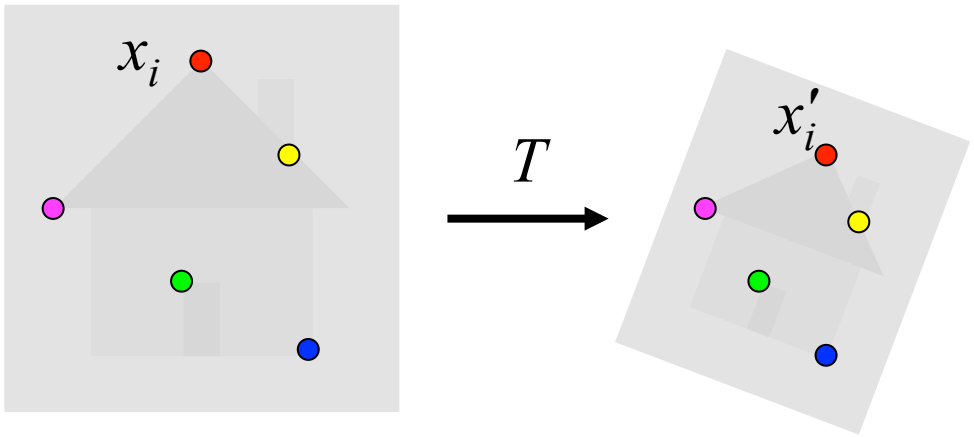
- Previous lectures: fitting a model to features in one image



Find model M that minimizes

$$\sum_i \text{residual}(x_i, M)$$

- Alignment: fitting a model to a transformation between pairs of features (*matches*) in two images



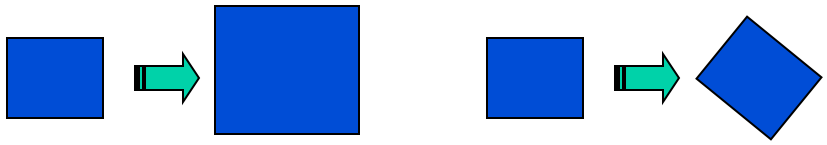
Find transformation T that minimizes

$$\sum_i \text{residual}(T(x_i), x_i')$$

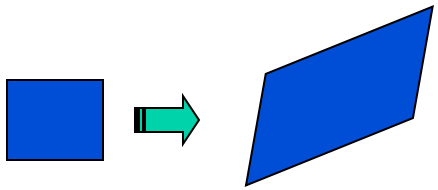
Source: S. Lazebnik

2D transformation models

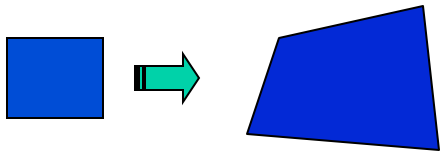
- Similarity (translation, scale, rotation)



- Affine



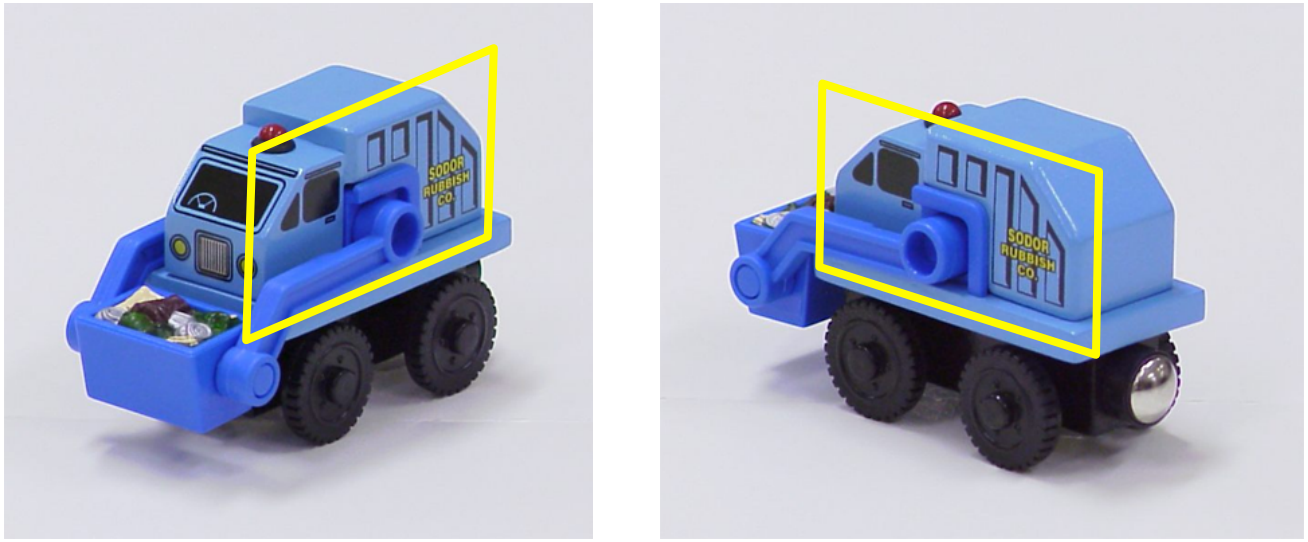
- Projective (homography)



Source: S. Lazebnik

Let's start with affine transformations

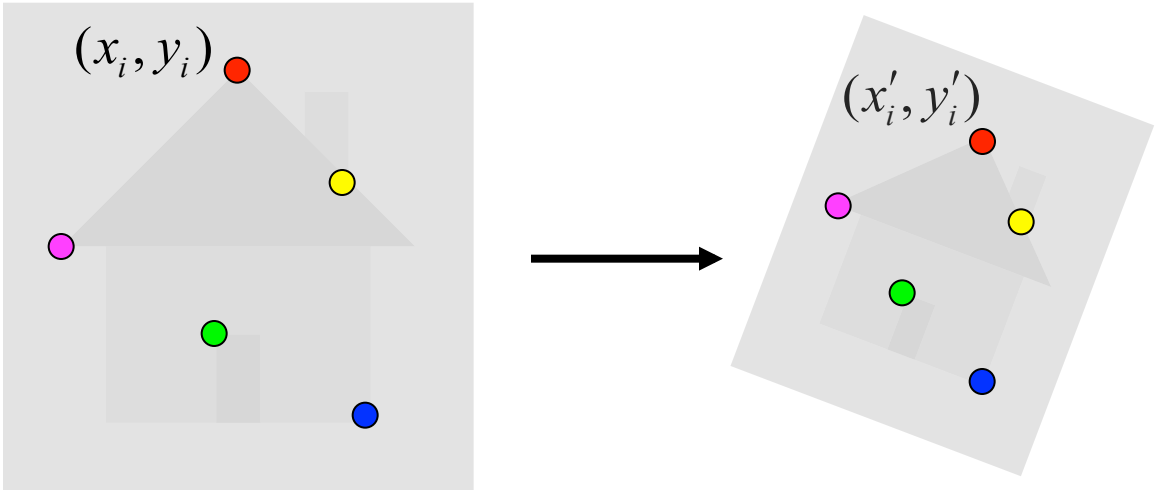
- Simple fitting procedure (linear least squares)
- Approximates viewpoint changes for roughly planar objects and roughly orthographic cameras
- Can be used to initialize fitting for more complex models



Source: S. Lazebnik

Fitting an affine transformation

- Assume we know the correspondences, how do we get the transformation?



$$\begin{bmatrix} x_i' \\ y_i' \end{bmatrix} = \begin{bmatrix} m_1 & m_2 \\ m_3 & m_4 \end{bmatrix} \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \begin{bmatrix} t_1 \\ t_2 \end{bmatrix}$$

$$\mathbf{x}_i' = \mathbf{M}\mathbf{x}_i + \mathbf{t}$$

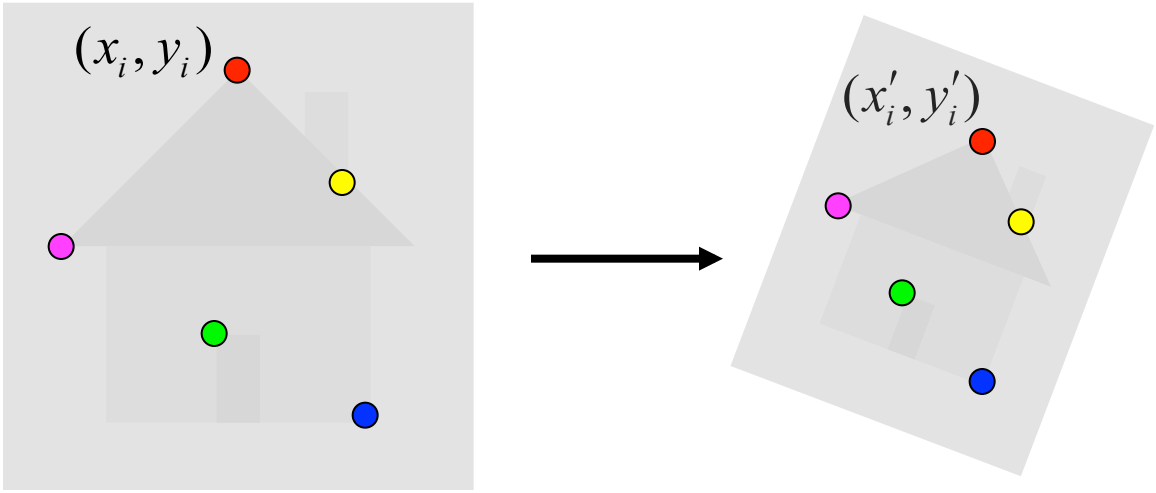
Want to find **M**, **t** to minimize

$$\sum_{i=1}^{n} || \mathbf{x}_i' - \mathbf{M}\mathbf{x}_i - \mathbf{t} ||^2$$

Source: S. Lazebnik

Fitting an affine transformation

- Assume we know the correspondences, how do we get the transformation?



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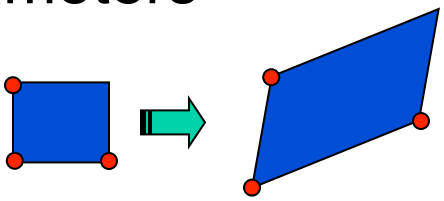
$$\begin{bmatrix} & & & & & \\ & & & & & \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \cdots \\ x_i' \\ y_i' \\ \cdots \end{bmatrix}$$

Source: S. Lazebnik

Fitting an affine transformation

$$\begin{bmatrix} & & \cdots & & & \\ x_i & y_i & 0 & 0 & 1 & 0 \\ 0 & 0 & x_i & y_i & 0 & 1 \\ & & \cdots & & & \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{bmatrix} = \begin{bmatrix} \cdots \\ x_i' \\ y_i' \\ \cdots \end{bmatrix}$$

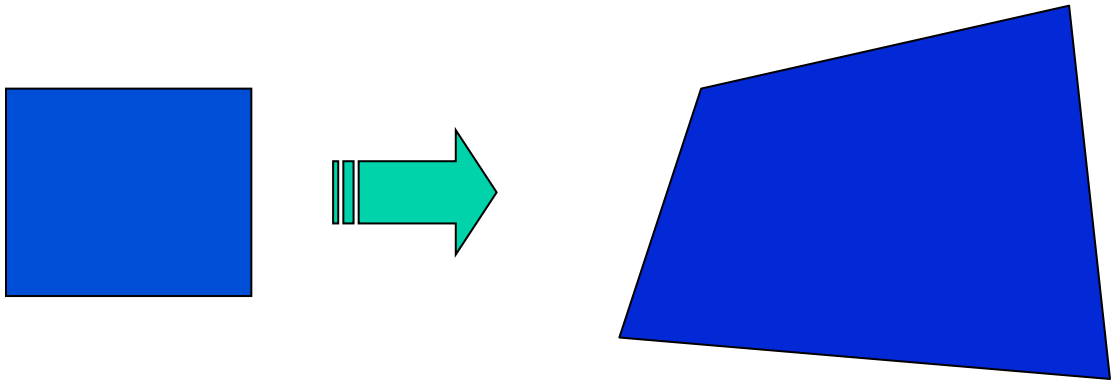
- Linear system with six unknowns
- Each match gives us two linearly independent equations: need at least three to solve for the transformation parameters



Source: S. Lazebnik

Fitting a plane projective transformation

- **Homography:** plane projective transformation (transformation taking a quad to another arbitrary quad)



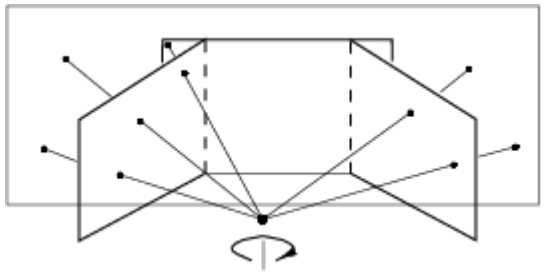
Source: S. Lazebnik

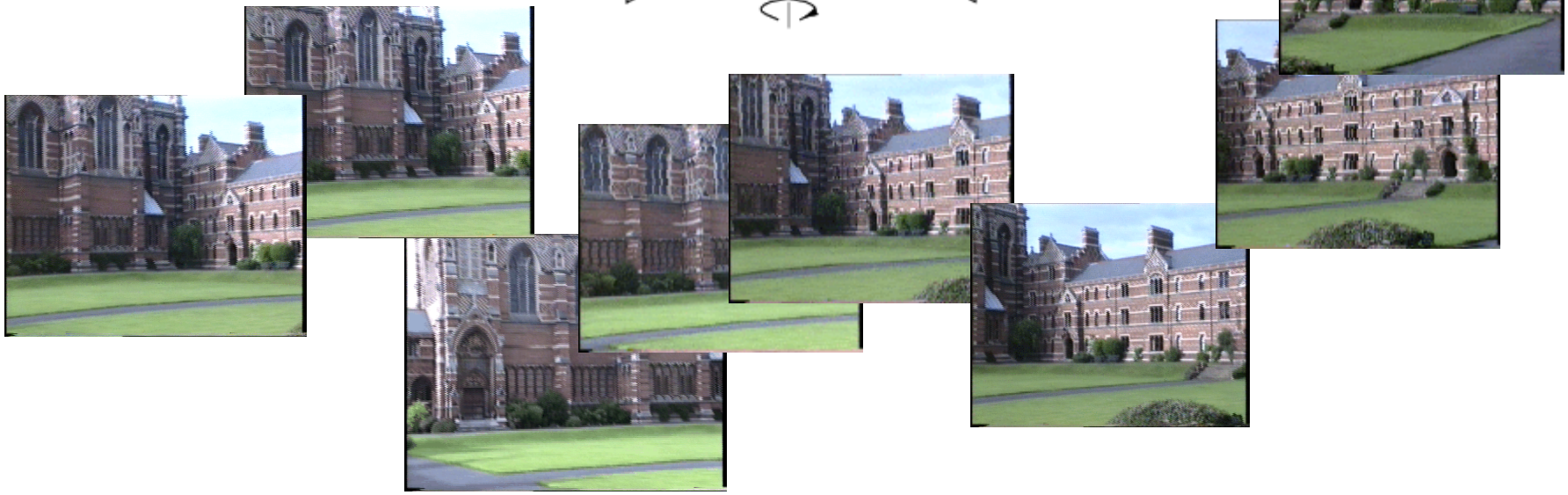
Homography

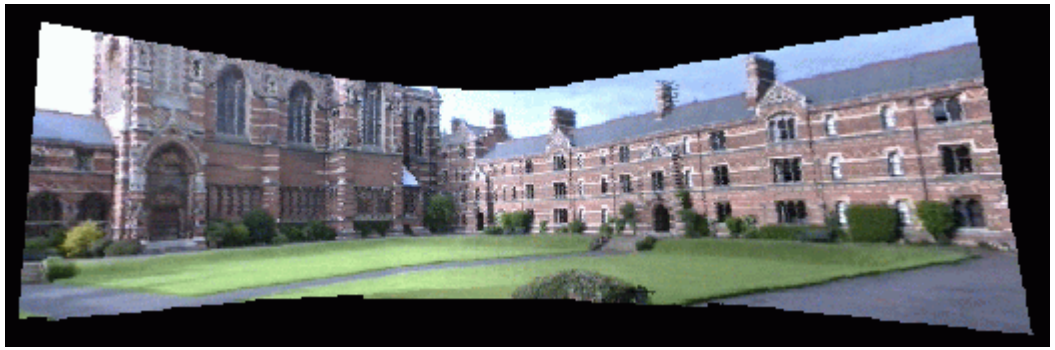
- The transformation between two views of a planar surface


- The transformation between images from two cameras that share the same center


Application: Panorama stitching







Source: Hartley & Zisserman

Source: S. Lazebnik

Fitting a homography

- Recall: homogeneous coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Converting *to* homogeneous image coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

Converting *from* homogeneous image coordinates

Source: S. Lazebnik

Fitting a homography

- Recall: homogeneous coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Converting *to* homogeneous image coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

Converting *from* homogeneous image coordinates

- Equation for homography:

$$\lambda \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Source: S. Lazebnik

Fitting a homography

- Equation for homography:

$$\lambda \begin{bmatrix} x_i' \\ y_i' \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} \qquad \lambda \mathbf{x}_i' = \mathbf{H}\mathbf{x}_i$$

$$\mathbf{x}_i' \times \mathbf{H}\mathbf{x}_i = 0$$

$$\begin{bmatrix} x_i' \\ y_i' \\ 1 \end{bmatrix} \times \begin{bmatrix} \mathbf{h_1}^T \mathbf{x}_i \\ \mathbf{h_2}^T \mathbf{x}_i \\ \mathbf{h_3}^T \mathbf{x}_i \end{bmatrix} = \begin{bmatrix} y_i' \mathbf{h_3}^T \mathbf{x}_i - \mathbf{h_2}^T \mathbf{x}_i \\ \mathbf{h_1}^T \mathbf{x}_i - x_i' \mathbf{h_3}^T \mathbf{x}_i \\ x_i' \mathbf{h_2}^T \mathbf{x}_i - y_i' \mathbf{h_1}^T \mathbf{x}_i \end{bmatrix}$$

$$\begin{bmatrix} 0^T & -\mathbf{x}_i^T & y_i' \mathbf{x}_i^T \\ \mathbf{x}_i^T & 0^T & -x_i' \mathbf{x}_i^T \\ -y_i' \mathbf{x}_i^T & x_i' \mathbf{x}_i^T & 0^T \end{bmatrix} \begin{pmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \mathbf{h}_3 \end{pmatrix} = 0$$

3 equations, only 2 linearly independent

Source: S. Lazebnik

Fitting a homography: DLT algorithm

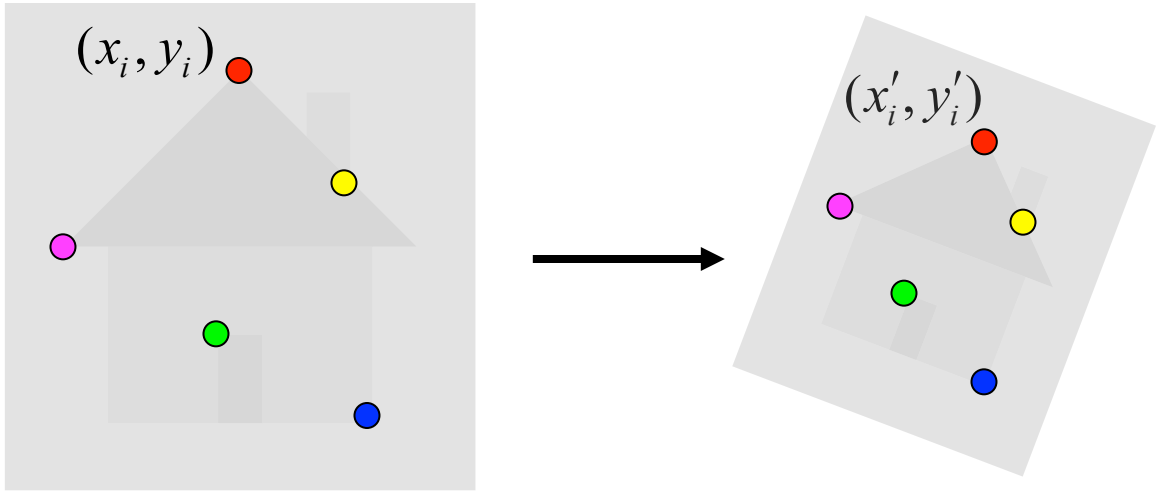
$$\begin{bmatrix} 0^T & \mathbf{x}_1^T & -y_1'\mathbf{x}_1^T \\ \mathbf{x}_1^T & 0^T & -x_1'\mathbf{x}_1^T \\ \dots & \dots & \dots \\ 0^T & \mathbf{x}_n^T & -y_n'\mathbf{x}_n^T \\ \mathbf{x}_n^T & 0^T & -x_n'\mathbf{x}_n^T \end{bmatrix} \begin{pmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \mathbf{h}_3 \end{pmatrix} = 0 \qquad \mathbf{A}\,\mathbf{h} = 0$$

- H has 8 degrees of freedom (9 parameters, but scale is arbitrary)
- One match gives us two linearly independent equations
- Homogeneous least squares: find $\mathbf{h}$ minimizing $||\mathbf{A}\mathbf{h}||^2$
  - Eigenvector of $\mathbf{A}^\mathbf{T}\mathbf{A}$ corresponding to smallest eigenvalue
  - Four matches needed for a minimal solution
- For more info, see Sec. 4.1 in (Hartley & Zisserman)

Source: S. Lazebnik

Robust feature-based alignment

- So far, we've assumed that we are given a set of "ground-truth" correspondences between the two images we want to align
- What if we don't know the correspondences?



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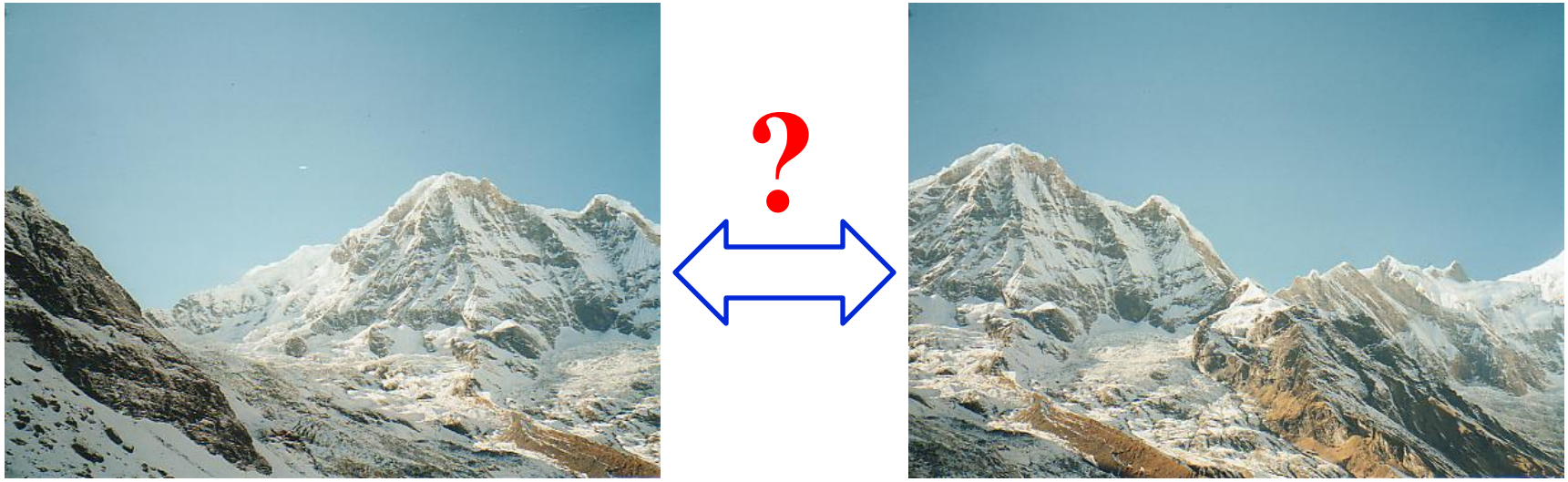
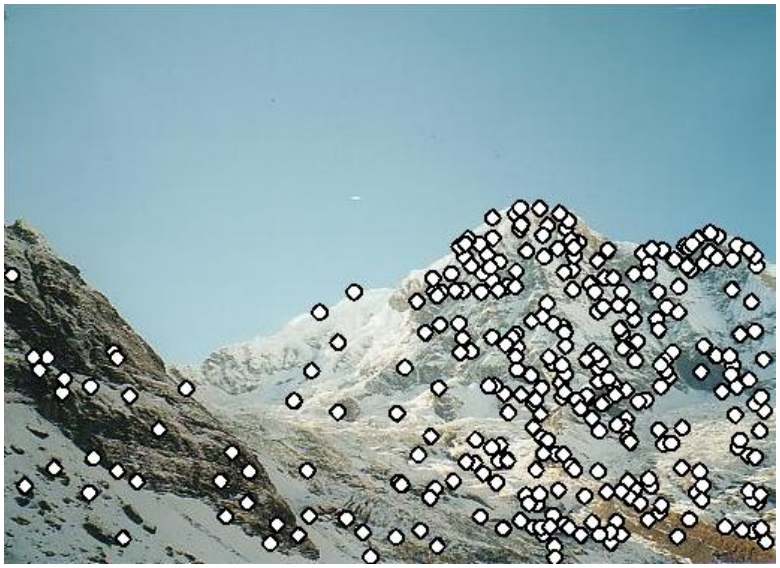


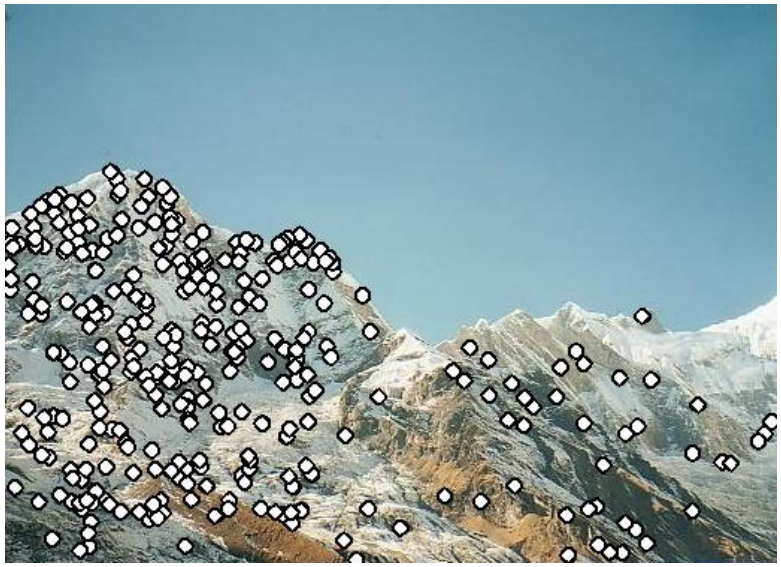
Source: S. Lazebnik

Robust feature-based alignment

Source: S. Lazebnik

Robust feature-based alignment

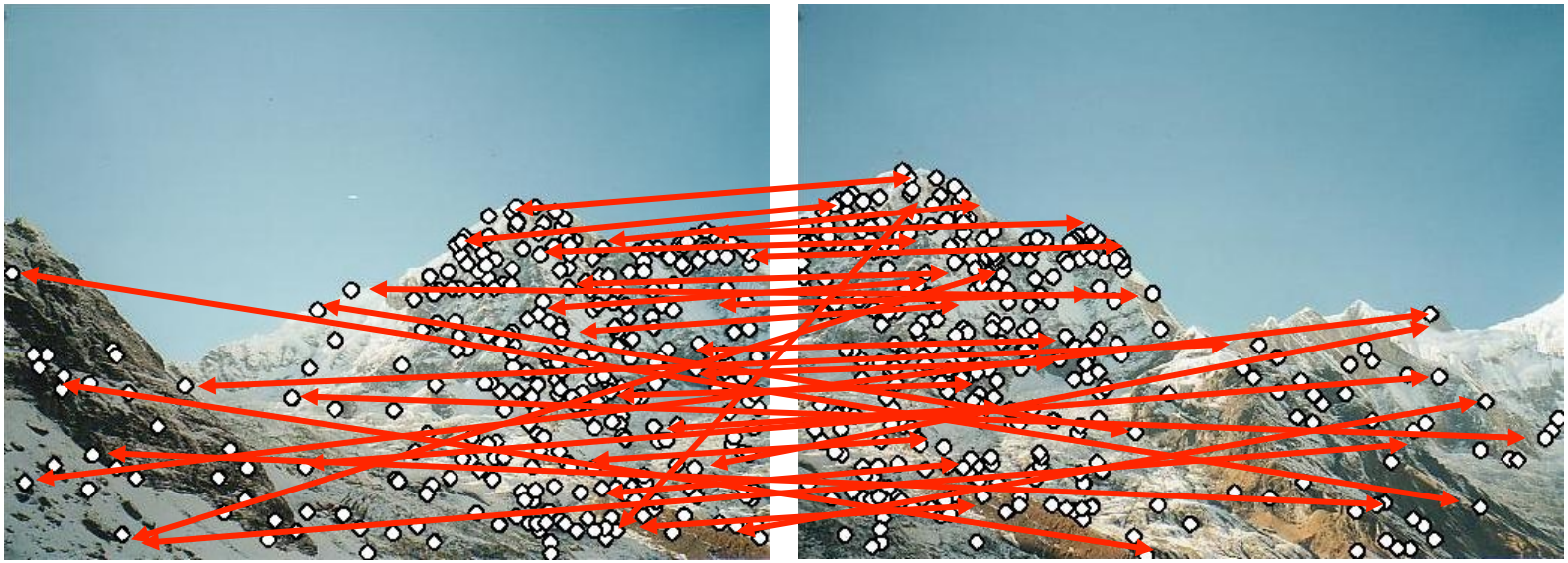




- Extract features

Source: S. Lazebnik

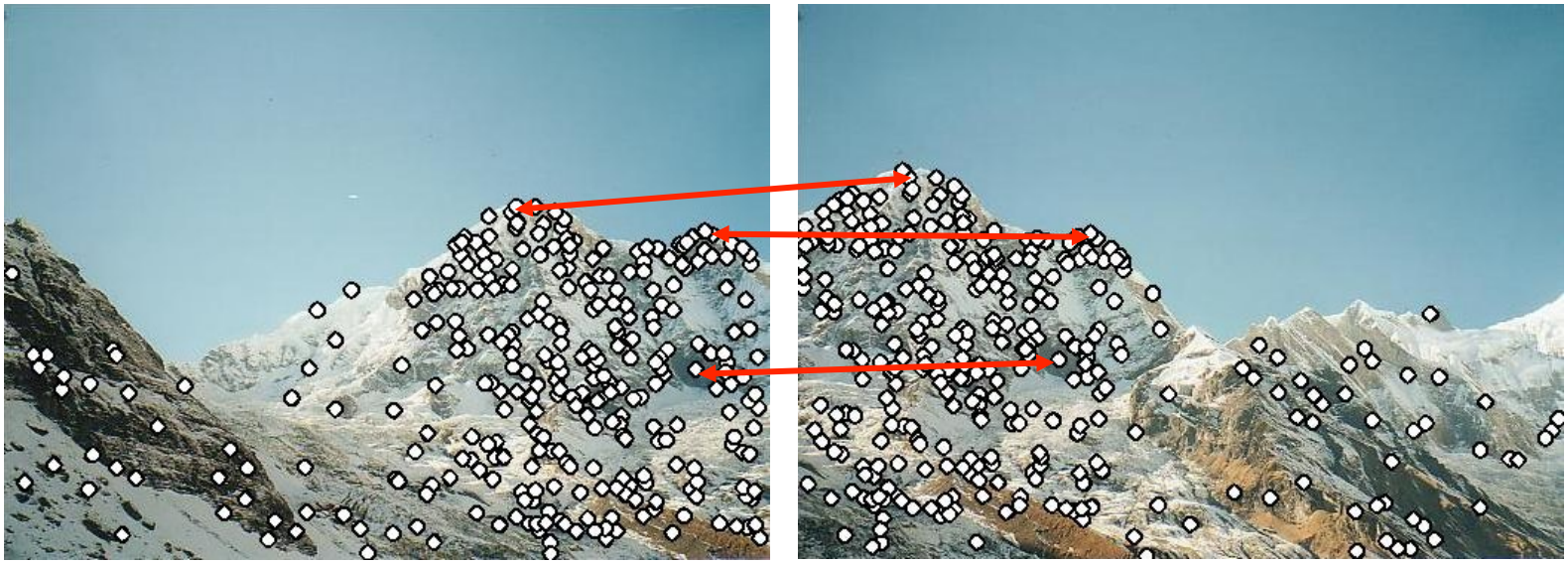
Robust feature-based alignment



- Extract features
- Compute *putative matches*

Source: S. Lazebnik

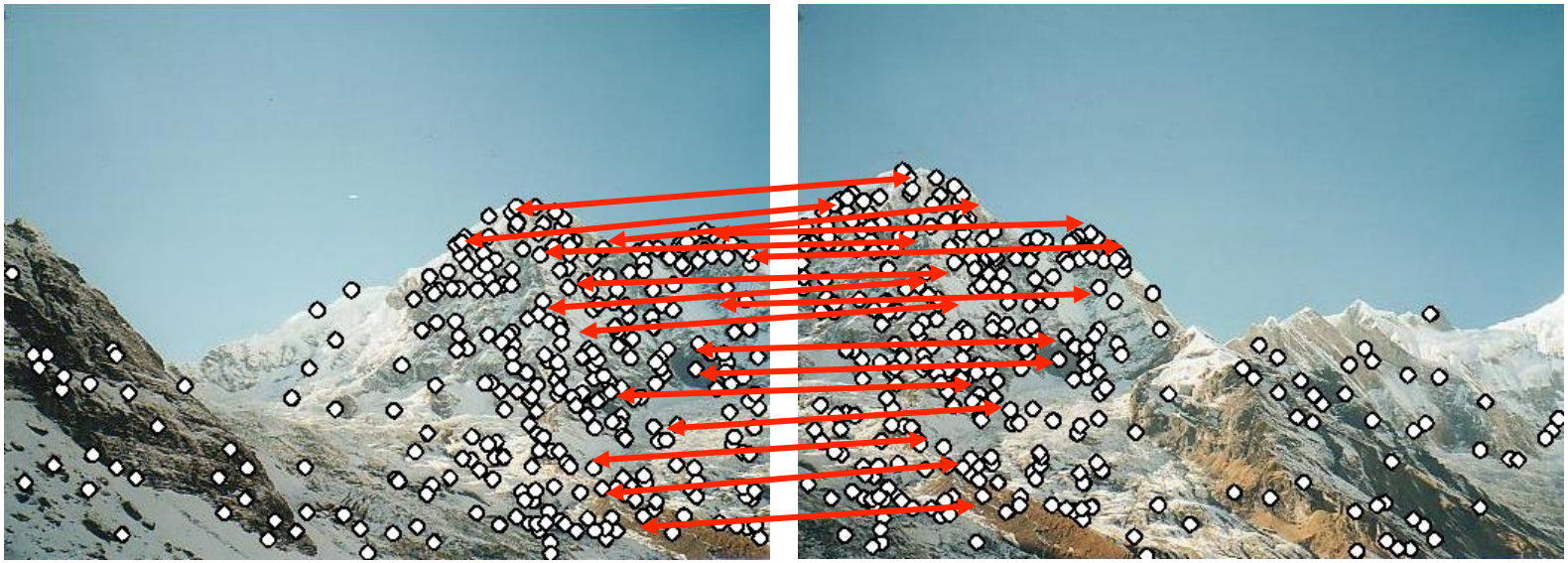
Robust feature-based alignment



- Extract features
- Compute *putative matches*
- Loop:
  - *Hypothesize* transformation *T*

Source: S. Lazebnik

Robust feature-based alignment



- Extract features
- Compute *putative matches*
- Loop:
  - *Hypothesize* transformation *T*
  - *Verify* transformation (search for other matches consistent with *T*)

Source: S. Lazebnik

Robust feature-based alignment

- Extract features
- Compute *putative matches*
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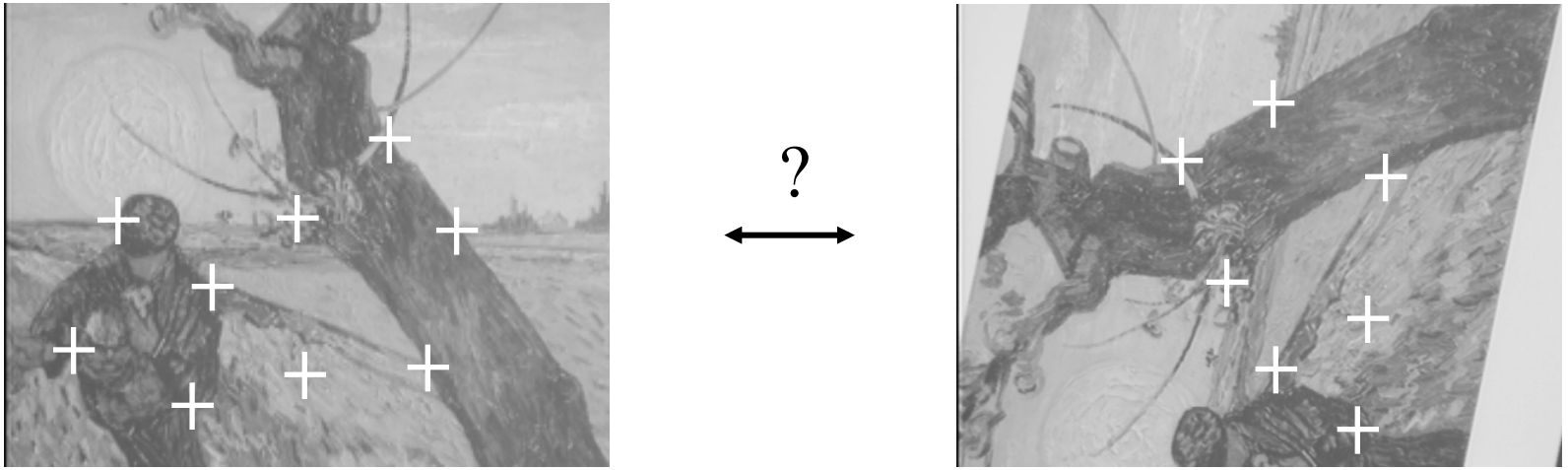
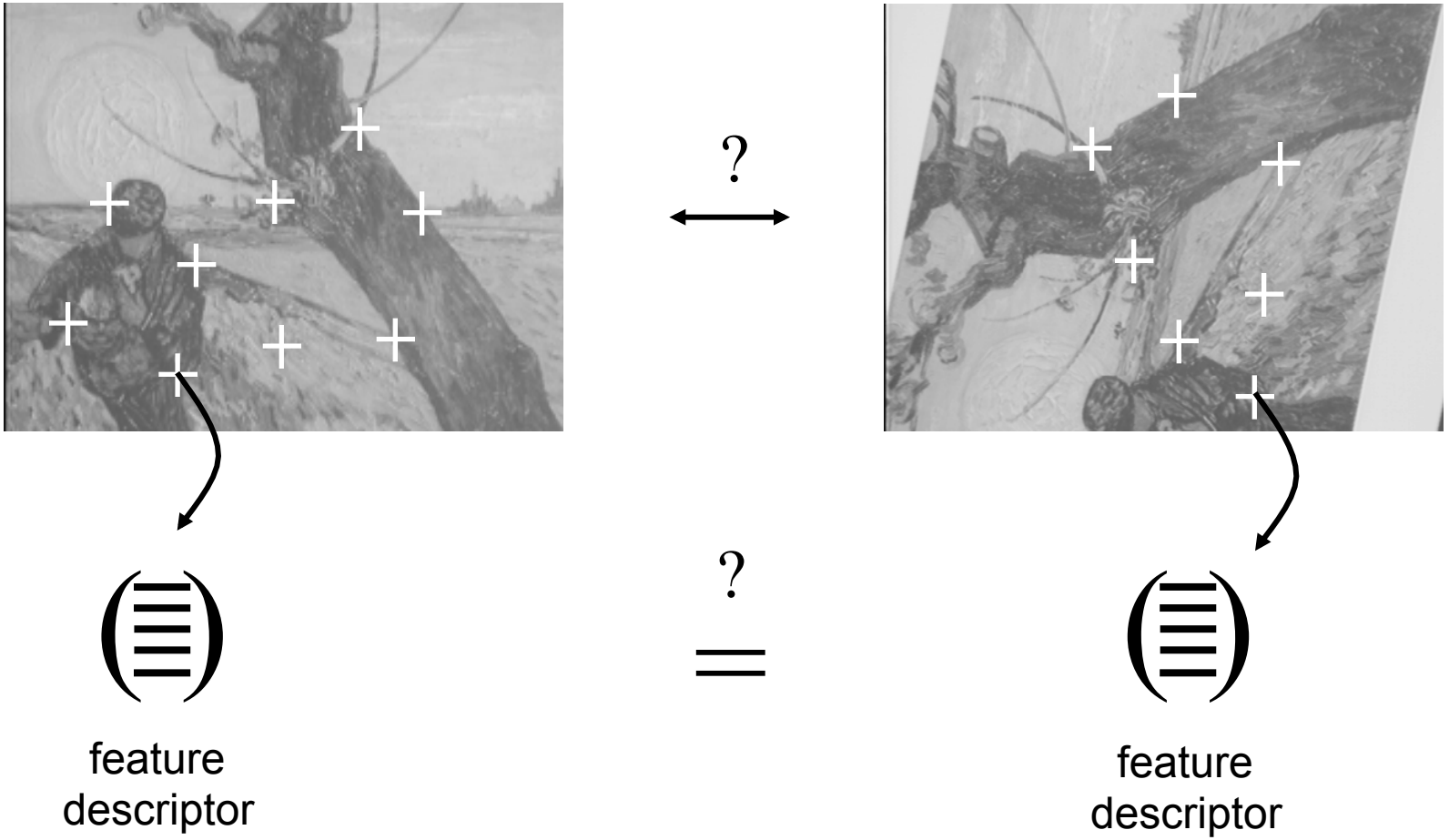
Source: S. Lazebnik

Generating putative correspondences



Source: S. Lazebnik

Generating putative correspondences

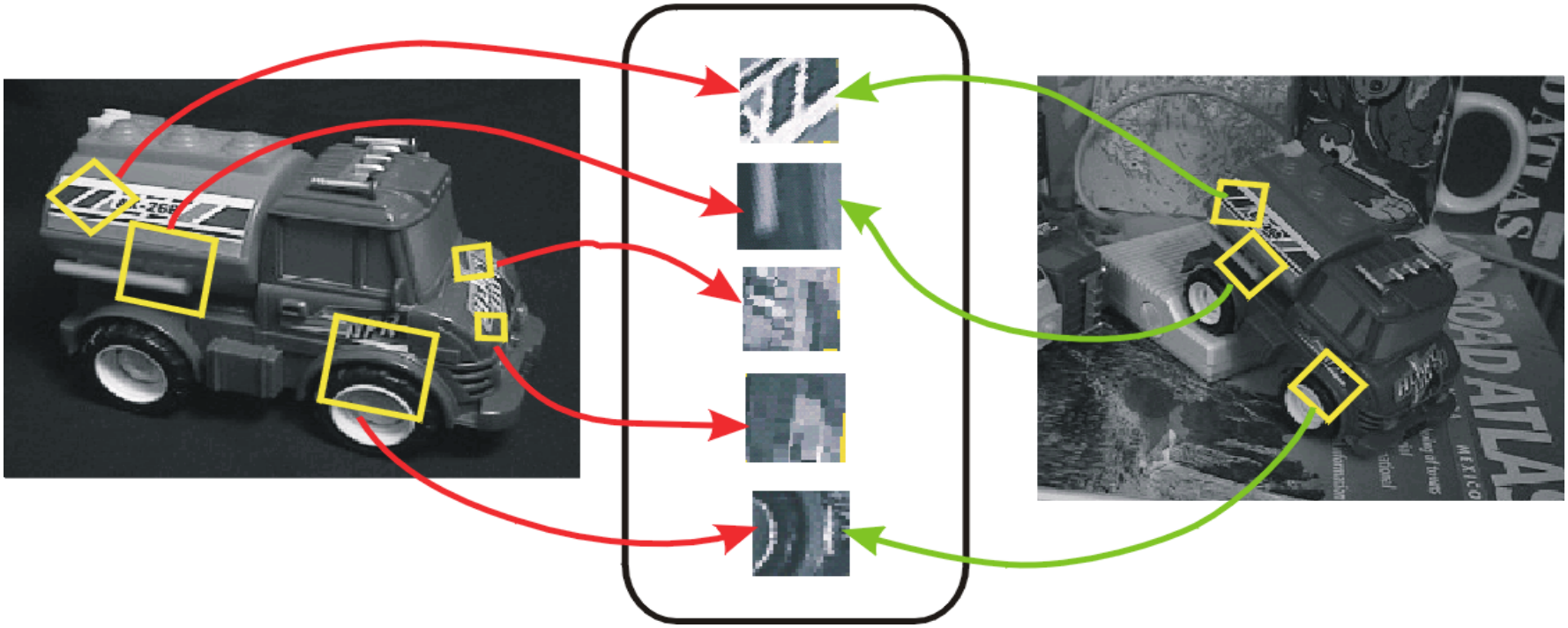


- Need to compare *feature descriptors* of local patches surrounding interest points

Source: S. Lazebnik

Feature descriptors

- Recall: feature detection and description



Source: S. Lazebnik

Feature descriptors

- Simplest descriptor: vector of raw intensity values
- How to compare two such vectors?
  - Sum of squared differences (SSD)

$$\mathrm{SSD}(\mathbf{u},\mathbf{v}) = \sum_i \left(u_i - v_i\right)^2$$

    - Not invariant to intensity change

  - Normalized correlation

$$\rho(\mathbf{u},\mathbf{v}) = \frac{(\mathbf{u}-\bar{\mathbf{u}})}{\|\mathbf{u}-\bar{\mathbf{u}}\|} \cdot \frac{(\mathbf{v}-\bar{\mathbf{v}})}{\|\mathbf{v}-\bar{\mathbf{v}}\|} = \frac{\sum_i (u_i - \bar{\mathbf{u}})(v_i - \bar{\mathbf{v}})}{\sqrt{\left(\sum_j (u_j - \bar{\mathbf{u}})^2\right)\left(\sum_j (v_j - \bar{\mathbf{v}})^2\right)}}$$

    - Invariant to affine intensity change

Source: S. Lazebnik

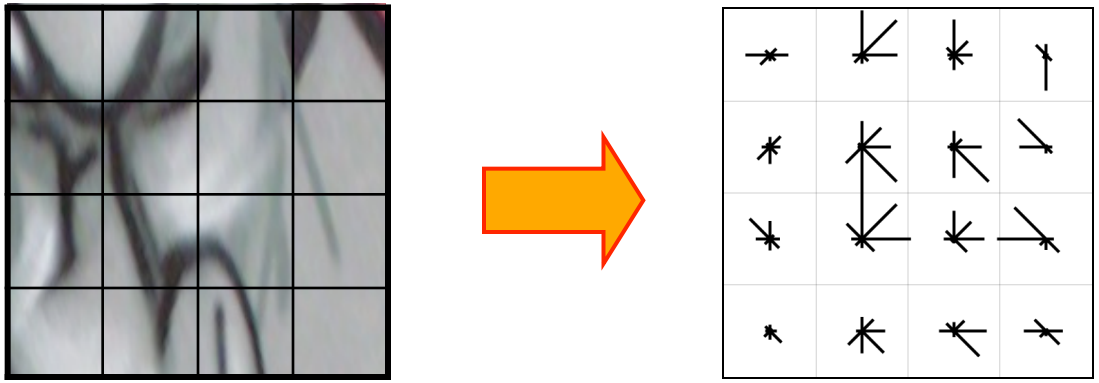
Disadvantage of intensity vectors as descriptors

- Small deformations can affect the matching score a lot

Source: S. Lazebnik

Feature descriptors: SIFT

- Descriptor computation:
  - Divide patch into 4x4 sub-patches
  - Compute histogram of gradient orientations (8 reference angles) inside each sub-patch
  - Resulting descriptor: 4x4x8 = 128 dimensions



David G. Lowe. "Distinctive image features from scale-invariant keypoints." *IJCV* 60 (2), pp. 91-110, 2004.

Source: S. Lazebnik

Feature descriptors: SIFT

- Descriptor computation:
  - Divide patch into 4x4 sub-patches
  - Compute histogram of gradient orientations (8 reference angles) inside each sub-patch
  - Resulting descriptor: 4x4x8 = 128 dimensions

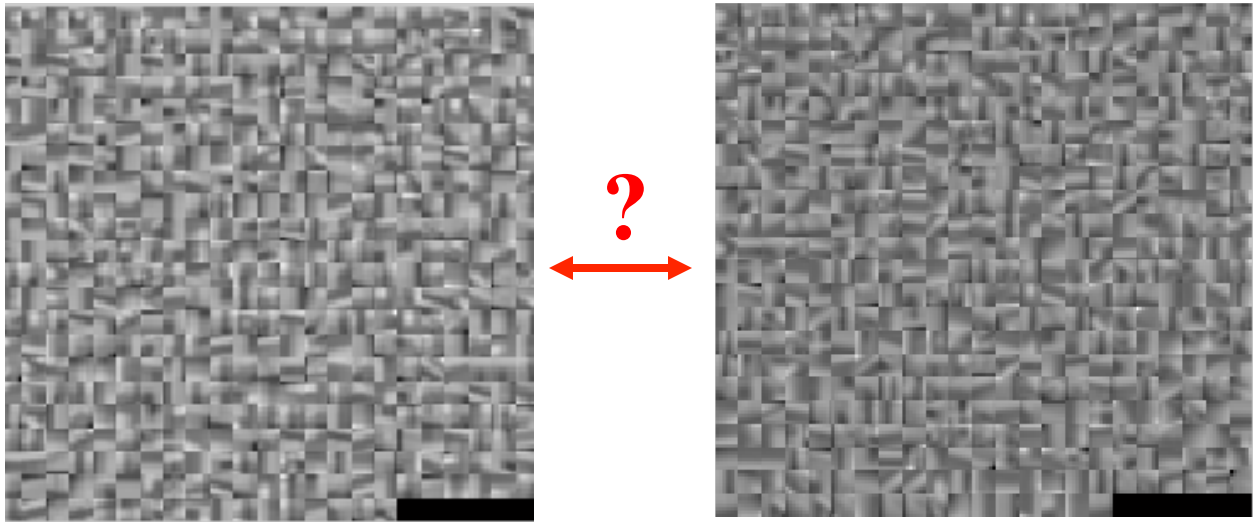
- Advantage over raw vectors of pixel values
  - Gradients less sensitive to illumination change
  - Pooling of gradients over the sub-patches achieves robustness to small shifts, but still preserves some spatial information

David G. Lowe. "Distinctive image features from scale-invariant keypoints." *IJCV* 60 (2), pp. 91-110, 2004.

Source: S. Lazebnik

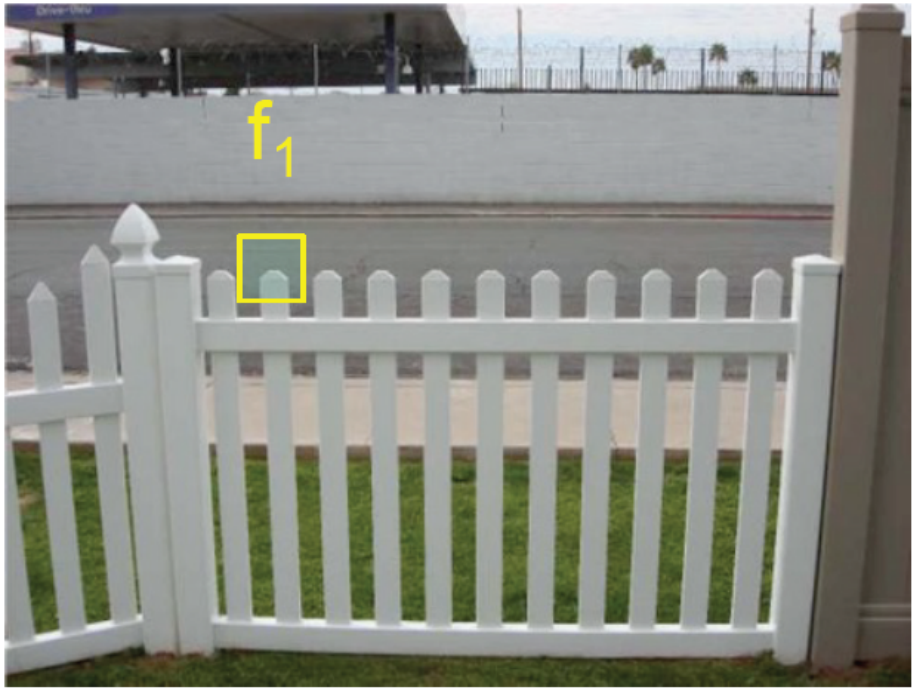
Feature matching

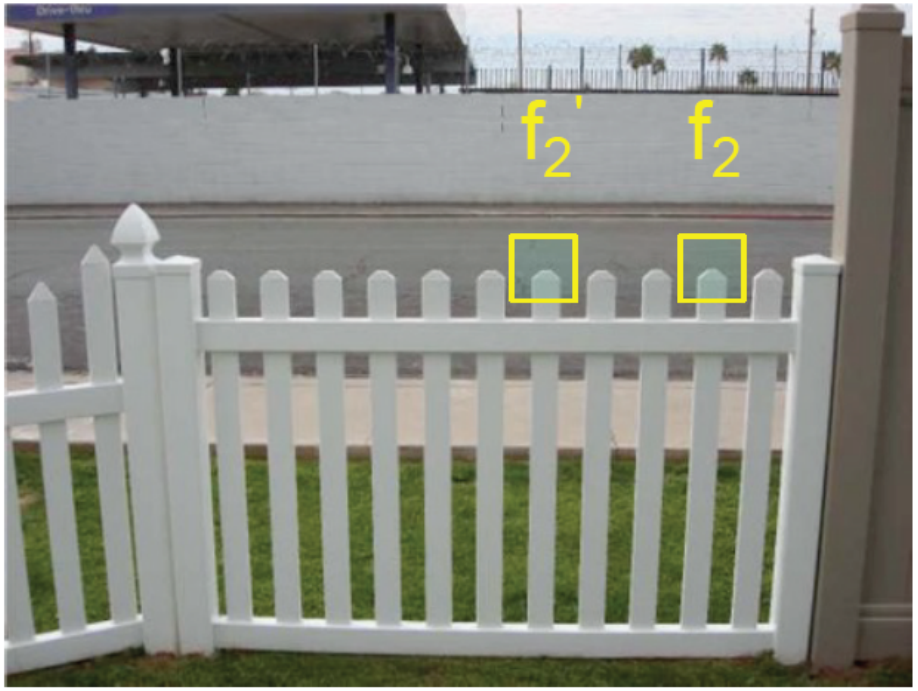
- Generating *putative matches*: for each patch in one image, find a short list of patches in the other image that could match it based solely on appearance



Source: S. Lazebnik

Problem: Ambiguous putative matches



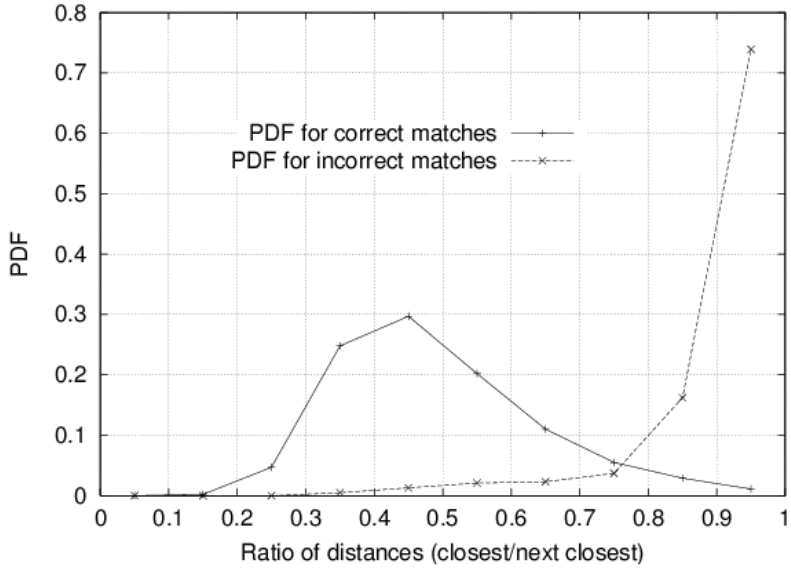


Source: Y. Furukawa

Source: S. Lazebnik

Rejection of unreliable matches

- How can we tell which putative matches are more reliable?
- Heuristic: compare distance of **nearest** neighbor to that of **second** nearest neighbor
  - Ratio of closest distance to second-closest distance will be *high* for features that are *not* distinctive



Threshold of 0.8 provides good separation

David G. Lowe. "Distinctive image features from scale-invariant keypoints." *IJCV* 60 (2), pp. 91-110, 2004.

Source: S. Lazebnik

RANSAC

- The set of putative matches contains a very high percentage of outliers

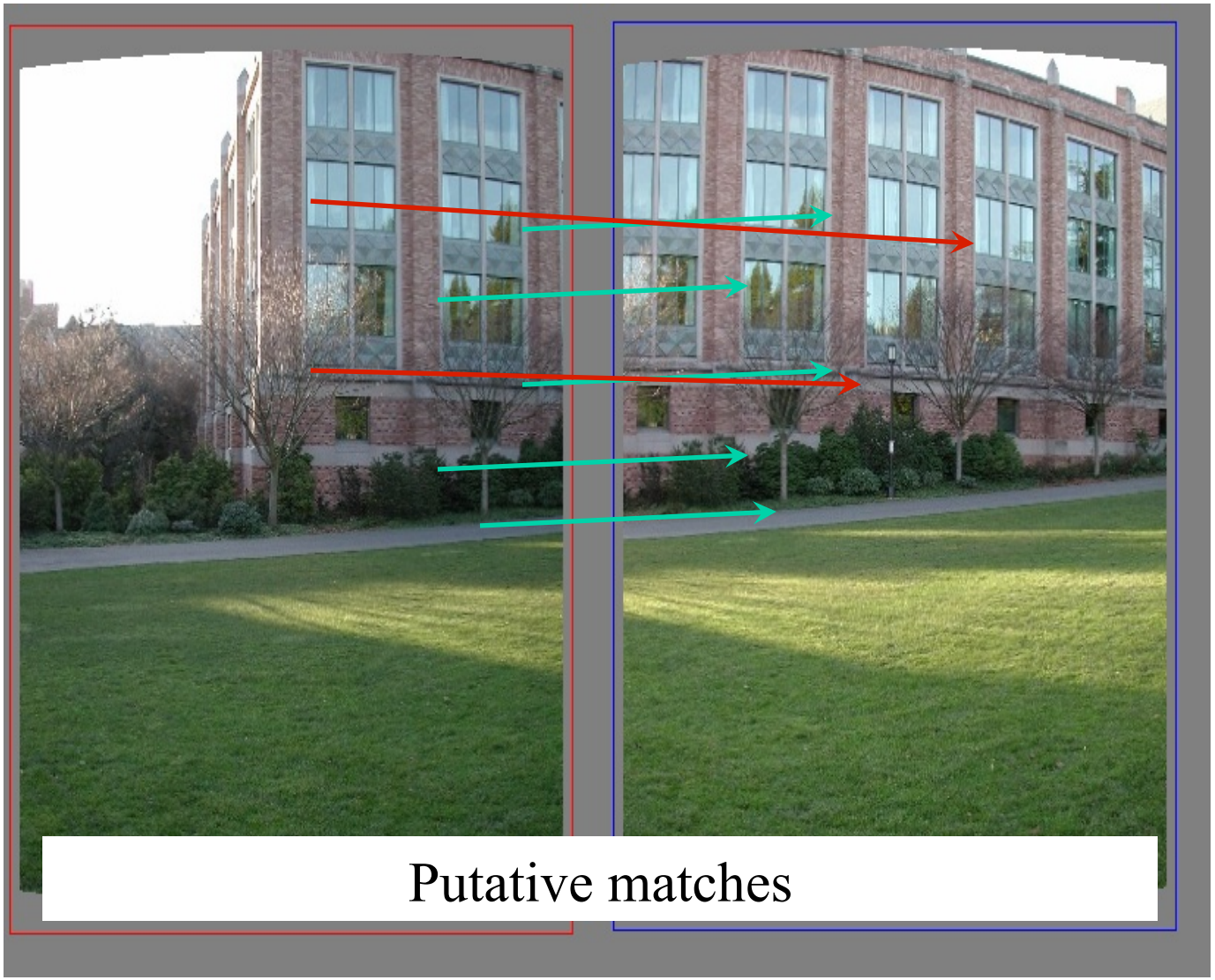
RANSAC loop:

1. Randomly select a *seed group* of matches
2. Compute transformation from seed group
3. Find *inliers* to this transformation
4. If the number of inliers is sufficiently large, re-compute least-squares estimate of transformation on all of the inliers

Keep the transformation with the largest number of inliers

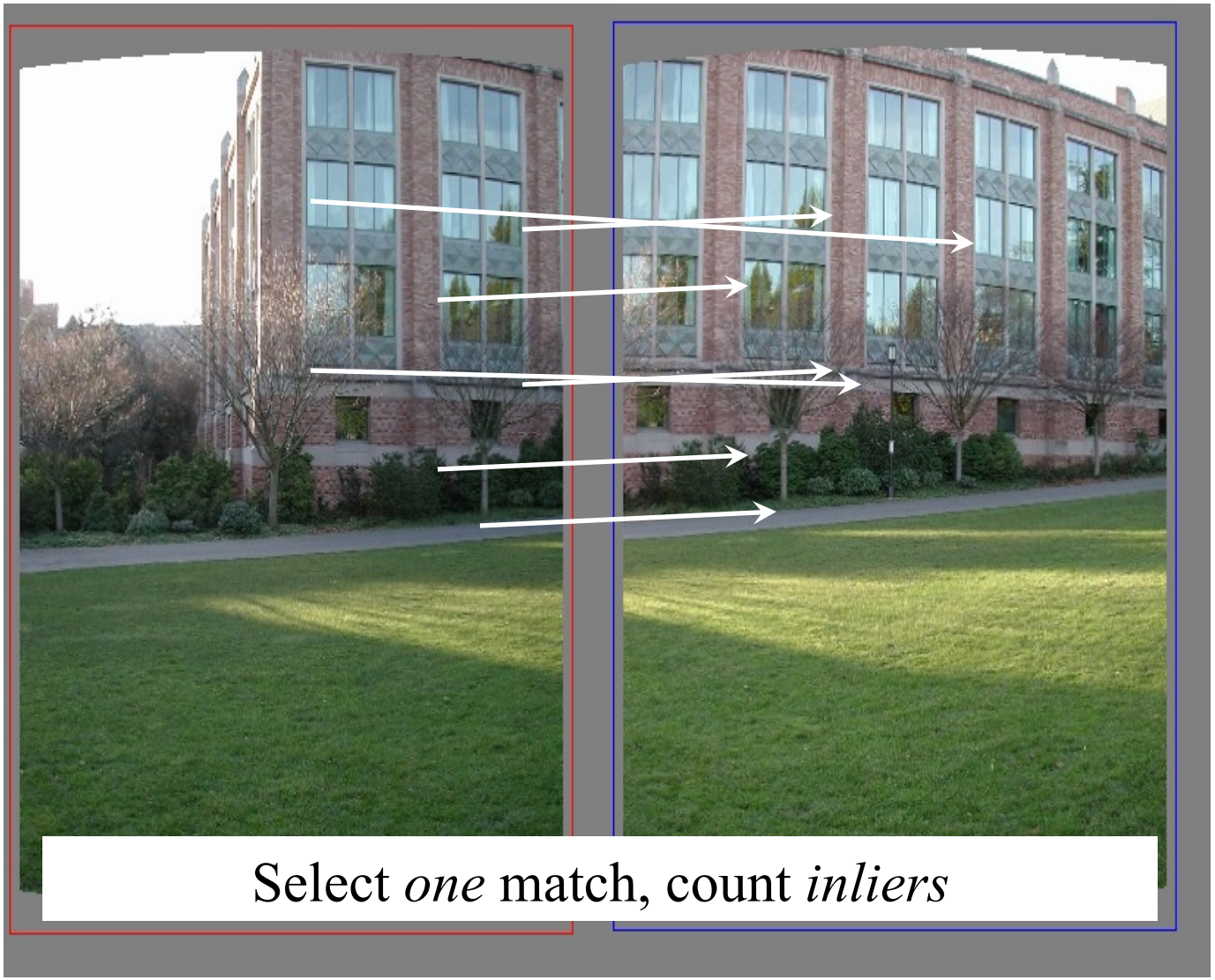
Source: S. Lazebnik

RANSAC example: Translation



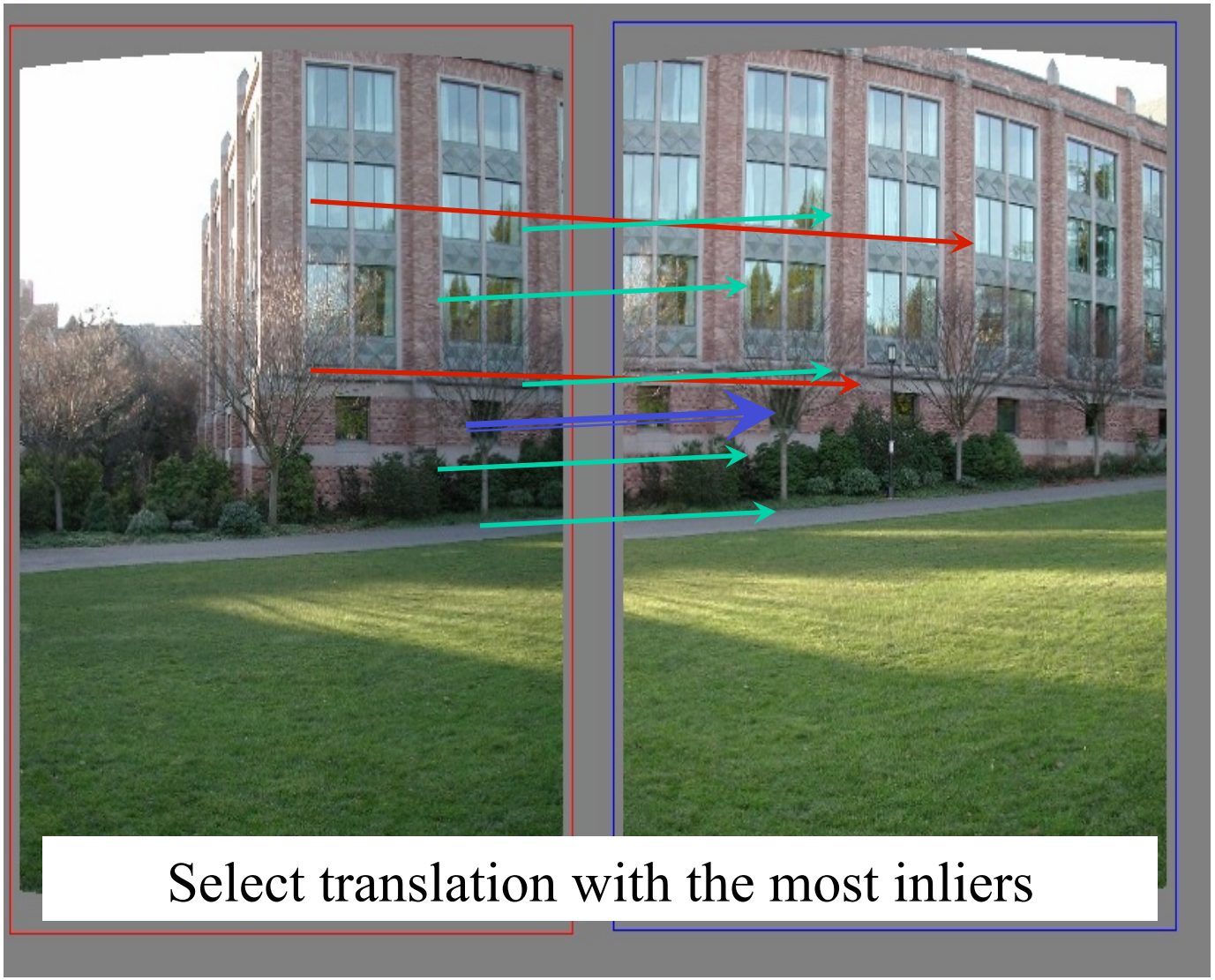
Source: S. Lazebnik

RANSAC example: Translation



Source: S. Lazebnik

RANSAC example: Translation



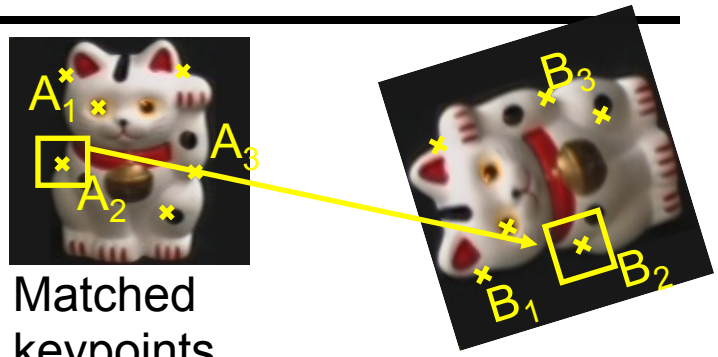
Source: D. Hoeim

Object Instance Recognition

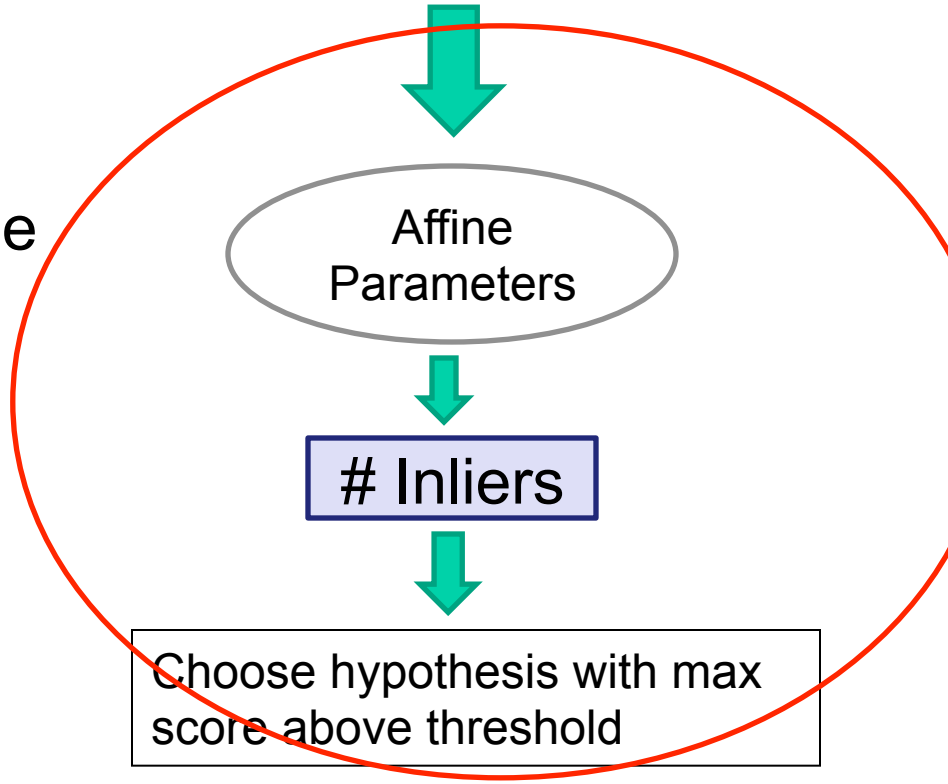
1. Match keypoints to object model

2. Solve for affine transformation parameters

3. Score by inliers and choose solutions with score above threshold

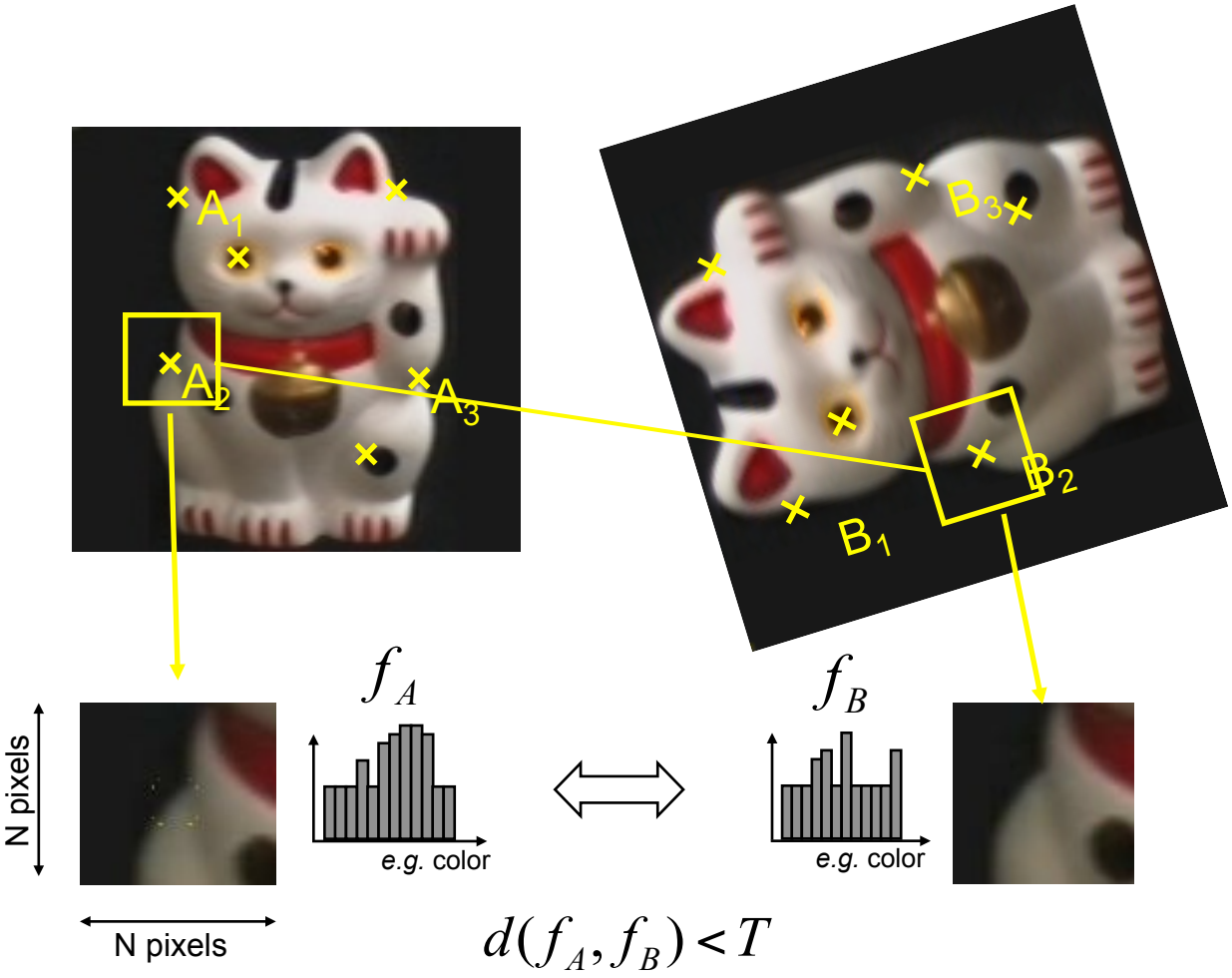


Matched keypoints



Source: D. Hoeim

Overview of Keypoint Matching

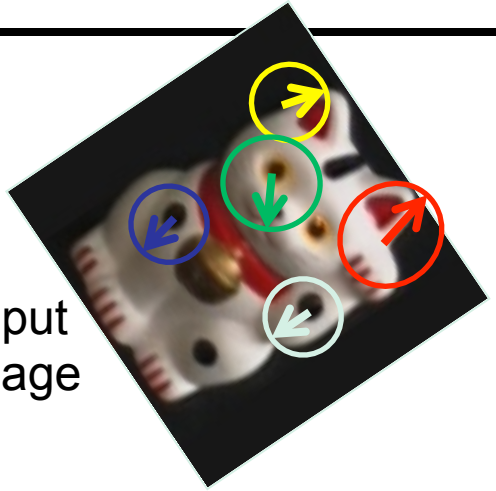


1. **Find a set of distinctive key-points**
2. **Define a region around each keypoint**
3. **Extract and normalize the region content**
4. **Compute a local descriptor from the normalized region**
5. **Match local descriptors**

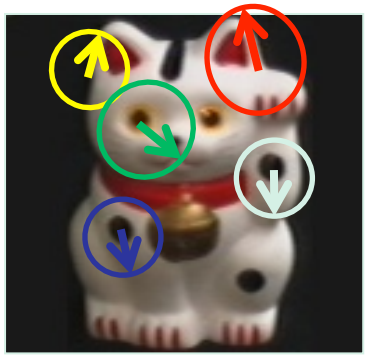
K. Grauman, B. Leibe

Finding the objects (overview)

Source: D. Hoeim



Input Image



Stored Image

1. Match interest points from input image to database image
2. Matched points vote for rough position/orientation/scale of object
3. Find position/orientation/scales that have at least three votes
4. Compute affine registration and matches using iterative least squares with outlier check
5. Report object if there are at least T matched points

Source: D. Hoeim

Matching Keypoints

Want to match keypoints between:

1. Query image
2. Stored image containing the object

Given descriptor x_0, find two nearest neighbors x_1, x_2 with distances d_1, d_2

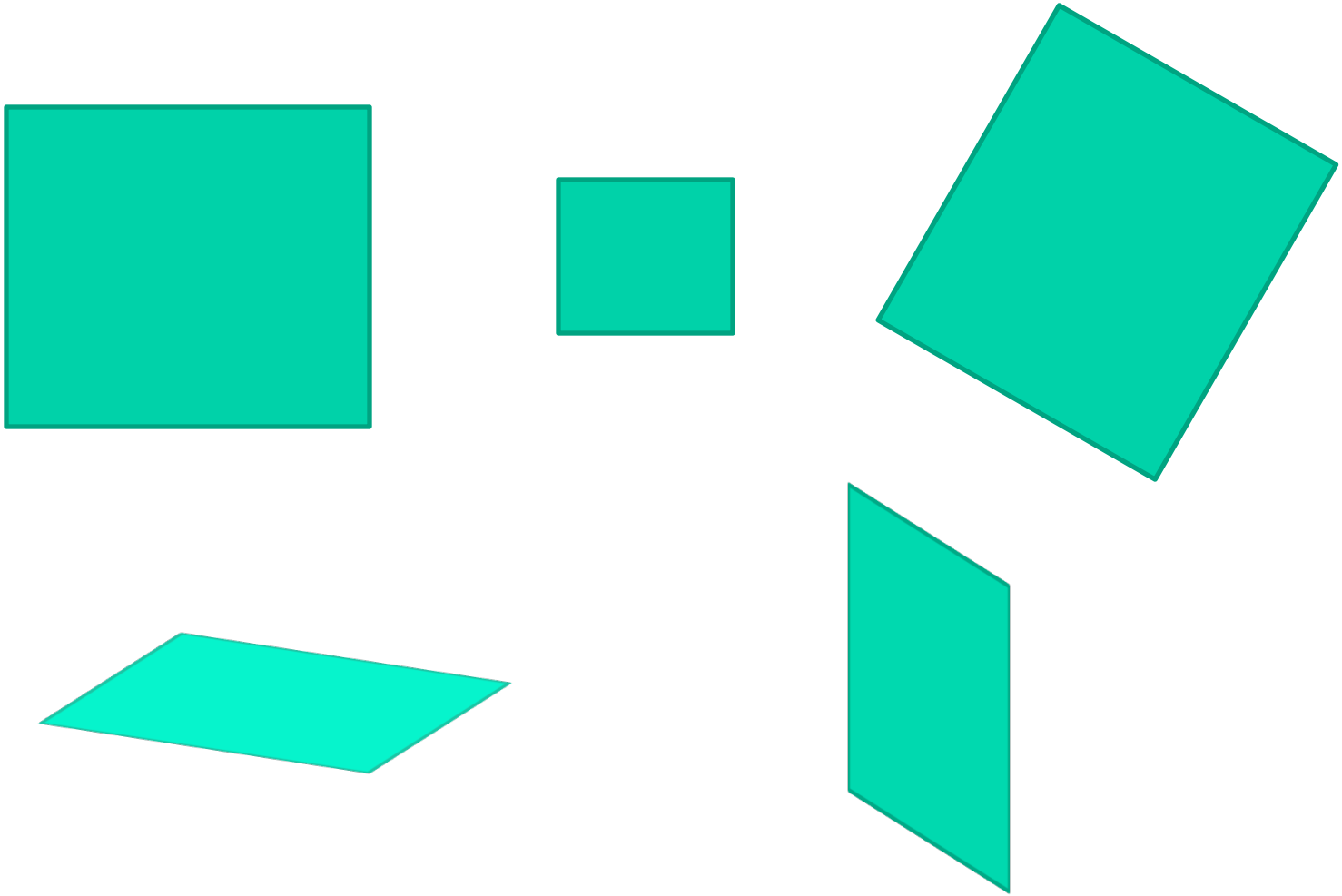
x_1 matches x_0 if $d_1/d_2 < 0.8$

- This gets rid of 90% false matches, 5% of true matches in Lowe's study

Source: D. Hoeim

Affine Object Model

Accounts for 3D rotation of a surface under orthographic projection



Source: D. Hoeim

Affine Object Model

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & y_1 & 1 \\ x_2 & y_2 & 1 & 0 & 0 & 0 \\ & & \vdots & & & \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} x_1' \\ y_1' \\ x_2' \\ \vdots \end{bmatrix}$$

$$\mathbf{x} = [\mathbf{A}^{\mathrm{T}}\mathbf{A}]^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{b}$$

Finding the objects (in detail)

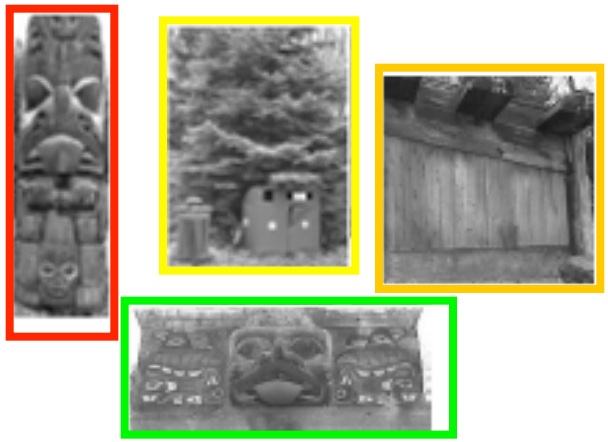
Source: D. Hoeim

1. Match interest points from input image to database image
2. Get location/scale/orientation using Hough voting
   - In training, each point has known position/scale/orientation wrt whole object
   - Matched points vote for the position, scale, and orientation of the entire object
   - Bins for x, y, scale, orientation
     - Wide bins (0.25 object length in position, 2x scale, 30 degrees orientation)
     - Vote for two closest bin centers in each direction (16 votes total)
3. Geometric verification
   - For each bin with at least 3 keypoints
   - Iterate between least squares fit and checking for inliers and outliers
4. Report object if > T inliers (T is typically 3, can be computed to match some probabilistic threshold)

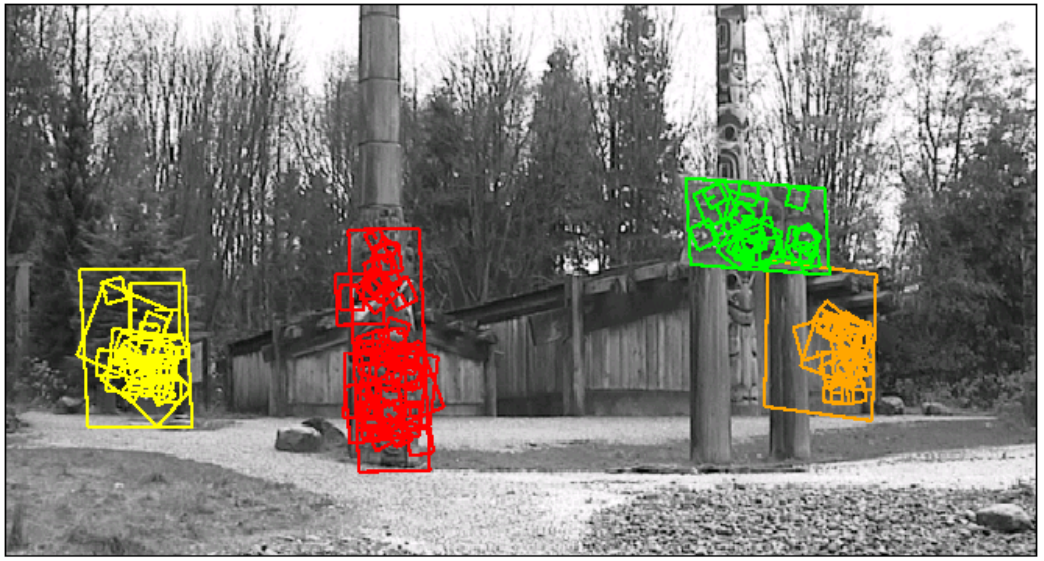
Examples of recognized objects

Slide credit: David Lowe

Location Recognition



Training



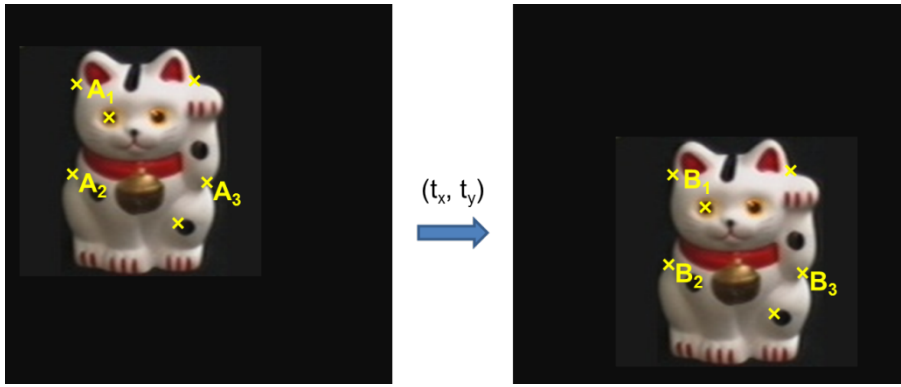
[Lowe04]

Slide credit: David Lowe

Key concepts

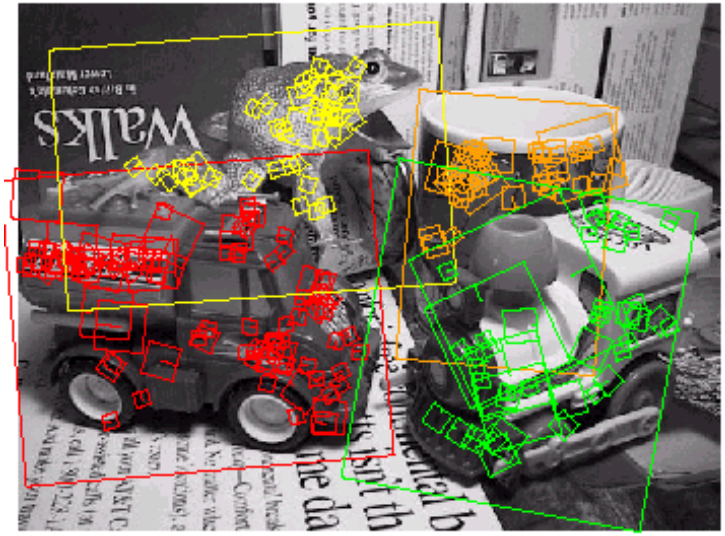
Alignment as robust fitting

- Affine transformations
- Homographies
- Descriptor-based feature matching
- RANSAC



Object instance recognition

- Find keypoints, compute descriptors
- Match descriptors
- Vote for / fit affine parameters
- Return object if # inliers > T



Adapted from D. Hoeim