CV$_{A}ssignment6$

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1 Exercise

1.1 a)

$$E = \sum_{i=1}^{n} ||x_i^{'} - Mx_i - t||^2$$

We can rewrite the abobe equation in form

$$E = \sum_{i=1}^{n} || \begin{pmatrix} xi' \\ y_i' \end{pmatrix} - \begin{pmatrix} m_1 & m_2 \\ m_3 & m_4 \end{pmatrix} * \begin{pmatrix} x_i \\ y_i \end{pmatrix} - \begin{pmatrix} t_1 \\ t_2 \end{pmatrix} ||^2$$

Therefore,

$$E = \sum_{i=1}^{n} [(x_i' - m_1 * x_i - m_2 * y_i - t_1)^2 + (y_i' - m_3 * x_i - m_4 * y_i - t_2)^2]$$

Taking the derivative w.r.t to each of $(m_1, m_2, m_3, m_4, t_1, t_2)$

$$\frac{\partial E}{\partial m_1} = 2 * \sum_{i=1}^{n} -x_i * (x_i' - m_1 * x_i - m_2 * y_i - t_1)$$

$$\frac{\partial E}{\partial m_2} = 2 * \sum_{i=1}^{n} -y_i * (x_i' - m_1 * x_i - m_2 * y_i - t_1)$$

$$\frac{\partial E}{\partial m_3} = 2 * \sum_{i=1}^{n} -x_i * (y_i' - m_3 * x_i - m_4 * y_i - t_2)$$

$$\frac{\partial E}{\partial m_4} = 2 * \sum_{i=1}^{n} -y_i * (y_i' - m_3 * x_i - m_4 * y_i - t_2)$$

$$\frac{\partial E}{\partial t_1} = 2 * \sum_{i=1}^{n} -1 * (x_i' - m_1 * x_i - m_2 * y_i - t_1)$$

$$\frac{\partial E}{\partial t_2} = 2 * \sum_{i=1}^{n} -1 * (y_i' - m_3 * x_i - m_4 * y_i - t_2)$$

1.2 b)

S*h = u

Vector h contains the unknown parameters of the transformation, therefore $h = \begin{pmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ t_1 \\ t_2 \end{pmatrix}$ Vector u is $\begin{pmatrix} \sum_{i=1}^{n} x_i * x_i' \\ \sum_{i=1}^{n} y_i * x_i' \\ \sum_{i=1}^{n} x_i * y_i' \\ \sum_{i=1}^{n} y_i * y_i' \\ \sum_{i=1}^{n} x_i' \\ \sum_{i=1}^{n} y_i' \end{pmatrix}$ Therefore matrix S is:

$$S = \begin{pmatrix} \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i * y_i & 0 & 0 & \sum_{i=1}^{n} x_i & 0 \\ \sum_{i=1}^{n} x_i * y_y & \sum_{i=1}^{n} y_i^2 & 0 & 0 & \sum_{i=1}^{n} y_i & 0 \\ 0 & 0 & \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i * y_i & 0 & \sum_{i=1}^{n} x_i \\ 0 & 0 & \sum_{i=1}^{n} x_i * y_i & \sum_{i=1}^{n} y_i^2 & 0 & \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} y_i & 0 & 0 & n & 0 \\ 0 & 0 & \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} y_i & 0 & n \end{pmatrix}$$

1.3 c)

point correspondences $\overbrace{(0,0)}^{x_i,y_i} \rightarrow \overbrace{(1,2)}^{x_i',y_i'}, (1,0) \rightarrow (3,2), and (0,1) \rightarrow (1,4).$

$$u = \begin{pmatrix} \sum_{i=1}^{n} x_i * x_i' \\ \sum_{i=1}^{n} y_i * x_i' \\ \sum_{i=1}^{n} x_i * y_i' \\ \sum_{i=1}^{n} y_i * y_i' \\ \sum_{i=1}^{n} x_i' \\ \sum_{i=1}^{n} y_i' \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 2 \\ 4 \\ 5 \\ 8 \end{pmatrix}$$

$$S = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 1 & 0 & 3 \end{pmatrix}$$

$$h = S^{-1} * u = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 2 \\ 1 \\ 2 \end{pmatrix}$$

2 Exercise

$$x' = sRx + t \rightarrow \begin{pmatrix} x' \\ y' \end{pmatrix} = s \begin{pmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$$

2.1 a)

Lets define $x_i = \begin{pmatrix} x_i \\ y_i \end{pmatrix}$

$$v = x_2 - x_1 = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix}$$

$$v' = x'_2 - x'_1 = \begin{pmatrix} x'_2 - x'_1 \\ y'_2 - y'_1 \end{pmatrix} = s * \begin{pmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{pmatrix} * \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix}$$

From cos norm we have:

$$cos(\theta) = \frac{v' * v}{||v'|| * ||v||} = \frac{(x'_2 - x'_1) * (x_2 - x_1) + (y'_2 - y1') * (y_2 - y_1)}{\sqrt{(y'_2 - y1')^2 + (x'_2 - x'_1)} * \sqrt{(y_2 - y1)^2 + (x_2 - x_1)}}$$

where

$$\theta = arcos(\frac{v' * v}{||v'|| * ||v||} = \frac{(x'_2 - x'_1) * (x_2 - x_1) + (y'_2 - y1') * (y_2 - y_1)}{\sqrt{(y'_2 - y1')^2 + (x'_2 - x'_1)} * \sqrt{(y_2 - y1)^2 + (x_2 - x_1)}})$$

2.2 b)

$$s = \frac{||v'||}{||v||} = \frac{\sqrt{(y'_2 - y1')^2 + (x'_2 - x'_1)}}{\sqrt{(y_2 - y1)^2 + (x_2 - x_1)}}$$

2.3 c)

$$\begin{pmatrix} t_x \\ t_y \end{pmatrix} = \begin{pmatrix} x' - s * cos(\theta) + s * sin(\theta) \\ y' - s * sin(\theta) - s * cos(\theta) \end{pmatrix}$$

2.4 d)

Transformation from the following point correspondences: $\{(\frac{1}{2}, 0) \rightarrow (0, 0)\}, \{(0, \frac{1}{2}) \rightarrow (-1, -1)\}$.

$$v = x_2 - x_1 = \begin{pmatrix} x_2 - x_1 \\ y_2 - y_1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

$$v' = x'_2 - x'_1 = \begin{pmatrix} x'_2 - x'_1 \\ y'_2 - y'_1 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

from section **a)** $\theta = \frac{\pi}{2}$
from section **b)** $s = 2$

$$\begin{pmatrix} t_x \\ t_y \end{pmatrix} = \begin{pmatrix} x' - s * cos(\theta) + s * sin(\theta) \\ y' - s * sin(\theta) - s * cos(\theta) \end{pmatrix} = \begin{pmatrix} 0 - 2 * cos(\frac{\pi}{2}) + 2 * sin(\frac{\pi}{2}) \\ 0 - 2 * sin(\frac{\pi}{2}) - 2 * cos(\frac{\pi}{2}) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$x' = sRx + t \rightarrow \begin{pmatrix} x' \\ y' \end{pmatrix} = s \begin{pmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix} \rightarrow s * \begin{pmatrix} cos(\frac{\pi}{2}) & -sin(\frac{\pi}{2}) \\ sin(\frac{\pi}{2}) & cos(\frac{\pi}{2}) \end{pmatrix} * \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$