ComputerVision12

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Exercise 2

Given two camera projection matrices $P = [I|O]$ and $P' = [Rt]$, where R is rotation matrix and $t = [t_1, t_2, t_3]^T$ is the translation vector. We have:

$$\overrightarrow{O'p} * (\overrightarrow{O'O} \times \overrightarrow{Op}) = 0$$

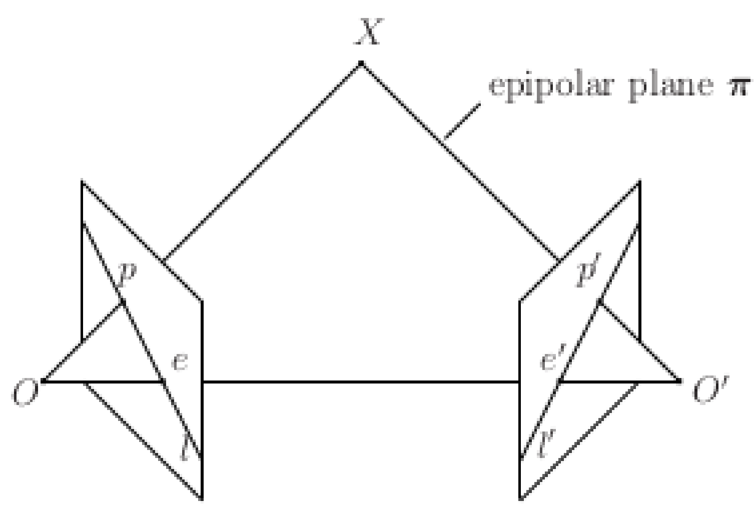


Figure 1: Epipolar geometry. Given a point p in the first image its corresponding point in the second image is constrained to lie on the line l which is the epipolar line of p. Correspondingly, the line l is the epipolar line of p. Points e and e are the epipoles.

We have to show that:

$$x'^T E x = 0$$

where E is the essential matrix: $E = [t]_\times R$

Given the homogeneous image coordinates of vector p and p', $x = [x, y, 1]^T$ and $x' = [x', y', 1]^T$ we can write:

$$\overrightarrow{O'p'} = x'$$

also from the cam coordinates we have that

$$\overrightarrow{Op} = Rt$$

The translation between camera origins is given by vector t. We can rewrite the following equation:

$$\overrightarrow{O'p} * (\overrightarrow{O'O} \times \overrightarrow{Op}) = x' * (t \times Rx) = 0$$

from the lecture slides: $a \times b = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} * \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = [a]_\times b$

Therefore

$$x' * (t \times Rx) = x' * [t]_\times R * x = x' * E * x = 0$$

Exercise 3

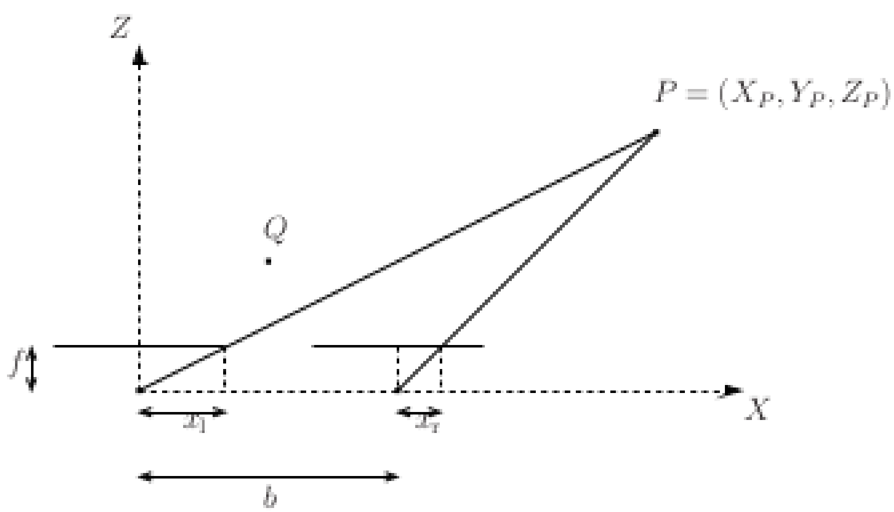


Figure 2: Top view of a stereo configuration where two pinhole cameras are placed side by side.

(a) Assume that d = 1 cm, b = 6 cm and f = 1 cm. Compute Z_p

$$disparity = d = x - x' = \frac{b * f}{Z_p} => Z_p = \frac{b * f}{d} = 6cm$$

(b) The smallest measurable disparity is 1 pixel and the pixel width is 0.01mm. What is the range of Z-coordinates for those points for which the disparity is below 1 pixel?
For points with the disparity below 1 pixel we have:

$$d = \frac{b * f}{Z_p} <= 0.01mm = 0.001cm$$

therefore,

$$\frac{bf}{Z_p} \leq 0.001 => Z_p \geq \frac{bf}{0.001} = \frac{6}{0.001} = 6000cm = 60m.$$

(c) From Figure 2 the camera matrices are $P_1 = [IO]$ and $P_2 = [It]$, where $t = (-6, 0, 0)^T$, the point Q has coordinates $(3, 0, 3)$

As given on lecture slides $E = [t\times]R$, with $R = I$ and $t = (-6, 0, 0)^T$ we have:

$$E = [t\times]R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 6 \\ 0 & -6 & 0 \end{bmatrix}$$

Image of Q on the left: $X = P_1 * (3, 0, 3, 1)^T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} * \begin{bmatrix} 3 \\ 0 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix}$

$$E * x = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 6 \\ 0 & -6 & 0 \end{bmatrix} * \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 18 \\ 0 \end{bmatrix}$$

exercise12

December 1, 2020

[ ]:
```
# This cell is used for creating a button that hides/unhides code cells to␣
 ↪quickly look only the results.
# Works only with Jupyter Notebooks.

from IPython.display import HTML

HTML('''<script>
code_show=true;
function code_toggle() {
if (code_show){
$('div.input').hide();
} else {
$('div.input').show();
}
code_show = !code_show
}
$( document ).ready(code_toggle);
</script>
<form action="javascript:code_toggle()"><input type="submit" value="Click here␣
 ↪to toggle on/off the raw code."></form>''')
```

[2]:
```
# Description:
#   Exercise12 notebook.
#
# Copyright (C) 2018 Santiago Cortes, Juha Ylioinas
#
# This software is distributed under the GNU General Public
# Licence (version 2 or later); please refer to the file
# Licence.txt, included with the software, for details.

# Preparations
import os
import numpy as np
import matplotlib.pyplot as plt
import cv2

# Select data directory
```

```
if os.path.isdir('/coursedata'):
    # JupyterHub
    course_data_dir = '/coursedata'
elif os.path.isdir('../../../coursedata'):
    # Local installation
    course_data_dir = '../../../coursedata'
else:
    # Docker
    course_data_dir = '/home/jovyan/work/coursedata/'

print('The data directory is %s' % course_data_dir)
data_dir = os.path.join(course_data_dir, 'exercise-12-data')
print('Data stored in %s' % data_dir)
```

```
The data directory is /coursedata
Data stored in /coursedata/exercise-12-data
```

[ ]:

```

```

Fill your name and student number below.

0.0.1 Name: Eugeniu Vezeteu

0.0.2 Student number: 886240

1 CS-E4850 Computer Vision Exercise Round 12

The problems should be solved before the exercise session and solutions returned via MyCourses. Upload to MyCourses both: this Jupyter Notebook (.ipynb) file containing your solutions to the programming tasks and the exported pdf version of this Notebook file. If there are both programming and pen & paper tasks kindly combine the two pdf files (your scanned/LaTeX solutions and the exported Notebook) into a single pdf and submit that with the Notebook (.ipynb) file. Note that (1) you are not supposed to change anything in the utils.py and (2) you should be sure that everything that you need to implement should work with the pictures specified by the assignments of this exercise round.

1.0.1 Make sure to complete the pen and paper exercices in the PDF attached.

1.1 Fundamental matrix estimation.

a) Implement the eight-point algorithm as explained on slide 28 of Lecture 11. Note the skeleton function and follow the input output structure

b) Implement the normalized eight-point algorithm as explained on slide 31 of Lecture 11 (Algorithm 11.1. in Hartley & Zisserman).

The epipolar lines obtained with both F-matrix estimates should be close to those visualized by the example script.

[33]:
```
def estimateF(x1,x2):
    # Return the fundamental matrix F (3 by 3), based on two sets of␣
↪homogeneous 2D points x1 and x2.
    # Input: x1,x2 numpy ndarray (3 by N) containing matching 2D homogeneous␣
↪points.
    # Output: F numpy ndarray (3 by 3) containing the fundamental matrix.
    print('estimateF')
    u_prim_u = x1[0,:]*x2[0,:]
    u_prim_v = x1[0,:]*x2[1,:]
    u_prim = x1[0,:]
    v_prim_u = x1[1,:]*x2[0,:]
    v_prim_v = x1[1,:]*x2[1,:]
    v_prim = x1[1,:]
    u = x2[0,:]
    v=x2[1,:]
    system = np.array([u_prim_u, u_prim_v, u_prim, v_prim_u, v_prim_v, v_prim,␣
↪u, v])
    system = np.vstack((system, np.ones((1, system.shape[1])))) #(9, 11)
    A = system.T  #(11, 9)  (n,9)
    print('A ', np.shape(A))

    #solve linear system
    _,Sigma,V = np.linalg.svd(system.T)
    F = V[-1].reshape(3,3)

    #Enforce rank-2 constraint (take SVD of F and throw out the smallest␣
↪singular value)
    u,Sigma,v = np.linalg.svd(F)
    Sigma[2] = 0
    F = np.dot(u,np.dot(np.diag(Sigma),v))
    print('F ', np.shape(F))

    return F

def estimateFnorm(x1,x2):
    # Return the fundamental matrix F (3 by 3), based on two sets of␣
↪homogeneous 2D points x1 and x2.
    # Input: x1,x2 numpy ndarray (3 by N) containing matching 2D homogeneous␣
↪points.
    # Output: F numpy ndarray (3 by 3) containing the fundamental matrix based␣
↪on normalized homogeneous points.
    F=np.eye(3)
    print('estimateFnorm')
    n = x1.shape[1]

    #Center the image data at the origin,
```

```
    #and scale it sothe mean squared distance between the origin and
    #the data points is 2 pixels

    # normalized x1
    x1 = x1 / x1[2]
    mean_x1 = np.mean(x1[:2],axis=1)
    s1 = np.sqrt(2) / np.std(x1[:2])
    T1 = np.array([[s1,0,-s1*mean_x1[0]],
                   [0,s1,-s1*mean_x1[1]],
                   [0,0,1]])
    x1 = np.dot(T1,x1)

    # normalized x2
    x2 = x2 / x2[2]
    mean_x2 = np.mean(x2[:2],axis=1)
    s2 = np.sqrt(2) / np.std(x2[:2])
    T2 = np.array([[s2,0,-s2*mean_x2[0]],
                   [0,s2,-s2*mean_x2[1]],
                   [0,0,1]])
    x2 = np.dot(T2,x2)

    # Use the eight-point algorithm to compute F from the normalized points
    F = estimateF(x1,x2)

    # Transform fundamental matrix back to original units
    #F in original coordinates is T1.T*F*T2
    F = np.dot(T1.T,np.dot(F,T2))
    F = F/F[2,2]

    return F

def vgg_F_from_P(P1,P2):
    # Return the fundamental matrix F (3 by 3), based on two camera parameter␣
↪arrays.
    # Input: P1, P2 numpy ndarray (3 by 4) containing intrinsic and extrinsic␣
↪parameters.
    # Output: F numpy ndarray (3 by 3) containing the fundamental matrix.
    X=[]
    Y=[]
    X.append(P1[[1,2],:])
    X.append(P1[[2,0],:])
    X.append(P1[[0,1],:])
    Y.append(P2[[1,2],:])
    Y.append(P2[[2,0],:])
    Y.append(P2[[0,1],:])
    F=np.zeros([3,3])
```

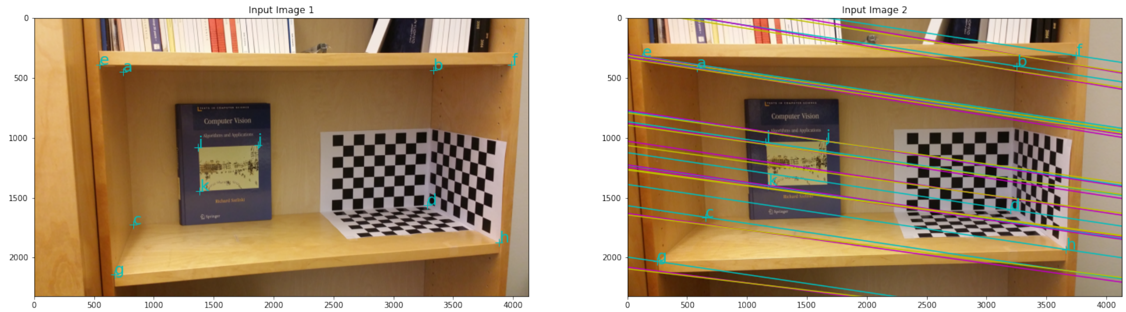
```
    for i in range(3):
        for j in range(3):
            M=np.concatenate([X[j],Y[i]])
            F[i,j]=np.linalg.det(M)
    print('vgg_F_from_P')
    return F
```

[34]:
```
# Point locations
x1 = 1.0e+03*np.array([0.7435,3.3315,0.8275,3.2835,0.5475,3.9875,0.6715,3.
↪8835,1.3715,1.8675,1.3835])
y1 = 1.0e+03*np.array([0.4455,0.4335,1.7215,1.5615,0.3895,0.3895,2.1415,1.
↪8735,1.0775,1.0575,1.4415])
x2 = 1.0e+03*np.array([0.5835,3.2515,0.6515,3.1995,0.1275,3.7475,0.2475,3.
↪6635,1.1555,1.6595,1.1755])
y2 = 1.0e+03*np.array([0.4135,0.4015,1.6655,1.5975,0.3215,0.3135,2.0295,1.
↪9335,1.0335,1.0255,1.3975])

# Camera parameters
P1= np.row_stack([[-0.001162918366053,0.000102986385133,-0.000344703214391,0.
↪995200644722518],\
                 [-0.000019974831639,0.001106889654747,-0.000150591916681,0.
↪097841118173777],\
                 [-0.000000053632777,0.000000044849673,-0.000000270734766,0.
↪000249501614496]])

P2= np.row_stack([[-0.001272880601540, 0.000093061493378,-0.000574486218854, 0.
↪996457618133488],\
                 [-0.000002971652037, 0.001271207503106,-0.000200323351541, 0.
↪084074548573989],\
                 [-0.000000020226464, 0.000000043518811,-0.000000316928290, 0.
↪000265554210072]])

# Make homogenous representations of points
pts1=np.row_stack([x1,y1,np.ones_like(x1)])
pts2=np.row_stack([x2,y2,np.ones_like(x2)])

# Read images
# Read images
im1 = cv2.imread(data_dir+'/im1.jpg')
im2 = cv2.imread(data_dir+'/im2.jpg')

im1 = cv2.cvtColor(im1, cv2.COLOR_BGR2RGB)
im2 = cv2.cvtColor(im2, cv2.COLOR_BGR2RGB)



# Labels
```

```python
labels = ['a','b','c','d','e','f','g','h','i','j','k']

# Create figure
fig, axes = plt.subplots(nrows=1, ncols=2, figsize=(25,25))
ax = axes.ravel()
ax[0].imshow(im1)
ax[0].plot(x1, y1, 'c+', markersize=10)

# Put labels
for i in range(len(x1)):
    ax[0].annotate(labels[i], (x1[i], y1[i]), color='c', fontsize=20)
ax[0].set_title("Input Image 1")
ax[1].imshow(im2)
ax[1].plot(x2, y2, 'c+', markersize=10)
for i in range(len(x2)):
    ax[1].annotate(labels[i], (x2[i], y2[i]), color='c', fontsize=20)
ax[1].set_title("Input Image 2")

# Get ground truth fundamental matrix
F=vgg_F_from_P(P1,P2)

# Create lines
#eplinesA=F@pts1
#eplinesB=F@pts2
eplinesA=np.dot(F,pts1)
eplinesB=np.dot(F,pts2)

#  Plot lines
px=np.array([0,np.shape(im2)[1]])
for i in range(np.shape(pts1)[1]):
    py=(-eplinesA[0,i]*px-eplinesA[2,i])/eplinesA[1,i]
    ax[1].plot(px,py,'c-');

# Get fundamental matrix and draw epipolar lines
F=estimateF(pts1,pts2)
#eplinesA=F@pts1
#eplinesB=F@pts2
eplinesA=np.dot(F,pts1)
eplinesB=np.dot(F,pts2)
for i in range(np.shape(pts1)[1]):
    py=(-eplinesA[0,i]*px-eplinesA[2,i])/eplinesA[1,i]
    ax[1].plot(px,py,'m-');

# Get fundamental matrix from normalized algorithm and draw epipolar lines
F=estimateFnorm(pts1,pts2)
#eplinesA=F@pts1
#eplinesB=F@pts2
```

```
eplinesA=np.dot(F,pts1)
eplinesB=np.dot(F,pts2)
for i in range(np.shape(pts1)[1]):
    py=(-eplinesA[0,i]*px-eplinesA[2,i])/eplinesA[1,i]
    ax[1].plot(px,py,'y-');

ax[1].axes.set_xlim([0,np.shape(im2)[1]])
ax[1].axes.set_ylim([np.shape(im2)[0],0])
plt.show()
```

```
vgg_F_from_P
estimateF
A  (11, 9)
F  (3, 3)
estimateFnorm
estimateF
A  (11, 9)
F  (3, 3)
```

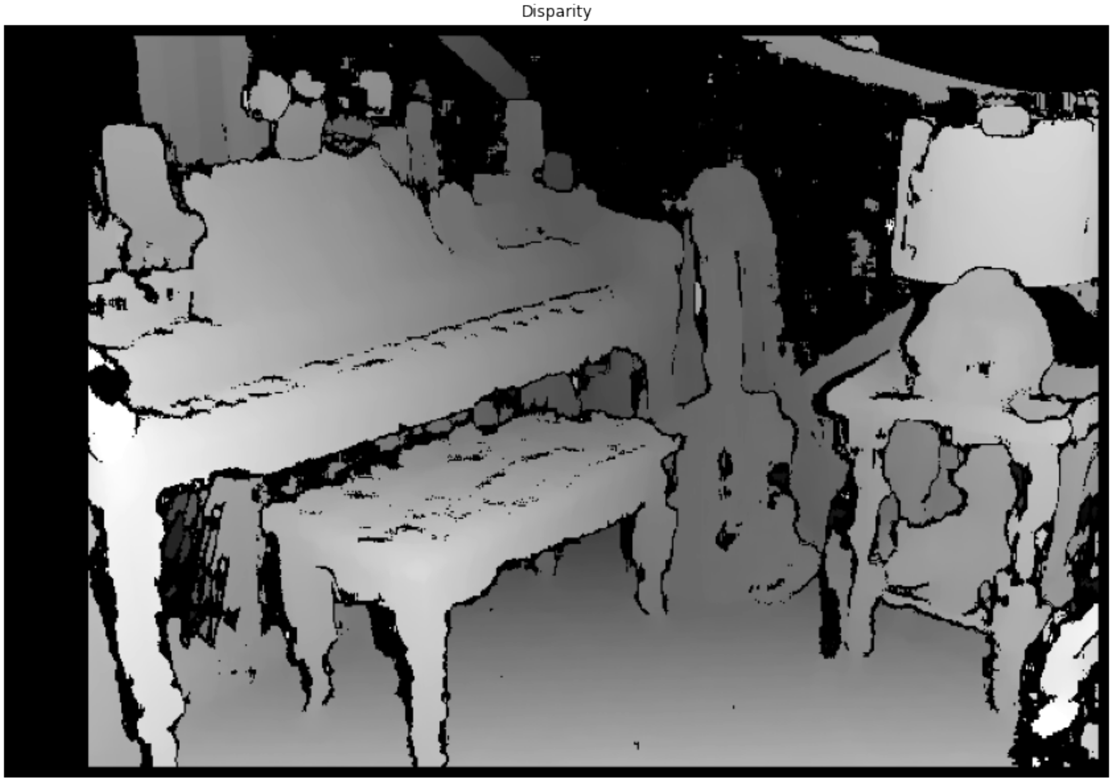


1.2 Demo. Stereo disparity computation. (Just a demo, no points given)

Run and study the opencv stereo disparity and depth estimation.

[35]:
```
# Import images
sc=0.25

imgL = cv2.resize(cv2.imread(data_dir+'/im0.png',0), (0,0), fx=sc, fy=sc)
imgR = cv2.resize(cv2.imread(data_dir+'/im1.png',0), (0,0), fx=sc, fy=sc)
imgL_col = cv2.resize(cv2.imread(data_dir+'/im0.png'), (0,0), fx=sc, fy=sc)
imgR_col = cv2.resize(cv2.imread(data_dir+'/im1.png'), (0,0), fx=sc, fy=sc)

# Show images
plt.figure(figsize=[15,15])
plt.subplot(121)
plt.imshow(imgL_col[:,:,[2,1,0]])
plt.axis('off')
```

```python
plt.subplot(122)
plt.imshow(imgR_col[:,:,[2,1,0]])
plt.axis('off')

# Compute disparity
stereo = cv2.StereoBM_create(numDisparities=16*3, blockSize=15)
disparity = stereo.compute(imgL,imgR)

# Show disparity
plt.figure(figsize=[15,15])
plt.imshow(disparity,'gray')
plt.axis('off')
plt.title('Disparity')
#ndistp=cv2.guidedFilter(imgL, disparity, 9, 4,0.1)

# Calibration data
baseline=17.8089 #cm
f_length=2826.171*sc #pixels
c_point=np.array([1415.97,965.806])*sc # pixels

# Get depth from disparity
point=np.zeros([np.count_nonzero(disparity>1),6])
ind=0
for i in range(np.shape(disparity)[0]):
    for j in range(np.shape(disparity)[1]):
        if disparity[i,j]>1:
            # Save point information into point cloud
            # [pixel_x,pixel_y,disparity,color]
            point[ind,0:3]=j,i,disparity[i,j]
            point[ind,3:6]=imgL_col[i,j]/255.0
            ind+=1
# Z=baseline*focal/disparity
# openCV disparity is (16*actual_disparity). This depends on the algorithm.
# It is in order to use signed shorts and keep good subpixel accuracy.
point[:,2]=baseline*f_length/(point[:,2]/16.0)
#X=Z*(pixel_u-center_u)/focal
point[:,0]=point[:,2]*(point[:,0]-c_point[0])/f_length
#Y=Z*(pixel_v-center_v)/focal
point[:,1]=-point[:,2]*(point[:,1]-c_point[1])/f_length

# Delete points on the far background
inl=(point[:,2]<2000)
point=point[inl,:]

plt.show()
```



[37]:
```
def visualize_points(pts,R,img,f=1000,cp=[400,300]):
    #visualize colored points given a rotation matrix
    # rotate around the mean of the point cloud
    c=np.mean(point[:,0:3],0)
    #r_point=((point[:,0:3]-c)@R_y)+c
    r_point=np.dot((point[:,0:3]-c),R_y)+c

    #Project back to the same camera model
    K=np.float32([[f,0,cp[0]],[0,f,cp[1]],[0,0,1]])
```

```
    # Sort by depth (painter's algorithm)
    ind=np.argsort(r_point[:,2])
    r_point=r_point[np.flip(ind,0),:]

    #Project
    #uvk=K@r_point.T
    uvk=(np.dot(K,r_point.T))
    color=point[:,[5,4,3]]
    color=color[np.flip(ind,0),:]

    # Normalize homogeneous  coordinates
    uv=uvk[0:2,:]/(uvk[2,:])

    # Draw projected points
    plt.scatter(uv[0,:],uv[1,:],marker='.',s=10,c=color)
    plt.xlim([0,np.shape(imgL)[1]])
    plt.ylim([0,np.shape(imgL)[0]])
    plt.axis('off')



# Visualize points from two different angles
plt.figure(figsize=[30,15])
plt.subplot(121)
# Rotate around y axis to visualize
ang_y=-20.0
ang_y=ang_y/180.0*3.14
R_y=np.float32([[np.cos(ang_y),0,np.sin(ang_y)],[0,1,0],[-np.sin(ang_y),0,np.
 ↪cos(ang_y)]])
visualize_points(point,R_y,imgL,f_length,c_point)
plt.subplot(122)
# Rotate around y axis to visualize
ang_y=20.0
ang_y=ang_y/180.0*3.14
R_y=np.float32([[np.cos(ang_y),0,np.sin(ang_y)],[0,1,0],[-np.sin(ang_y),0,np.
 ↪cos(ang_y)]])
visualize_points(point,R_y,imgL,f_length,c_point)
plt.show()
```

```
[ ]:
```