Computer Vision

CS-E4850, 5 study credits

Lecturer: Juho Kannala

Lecture 3: Image features

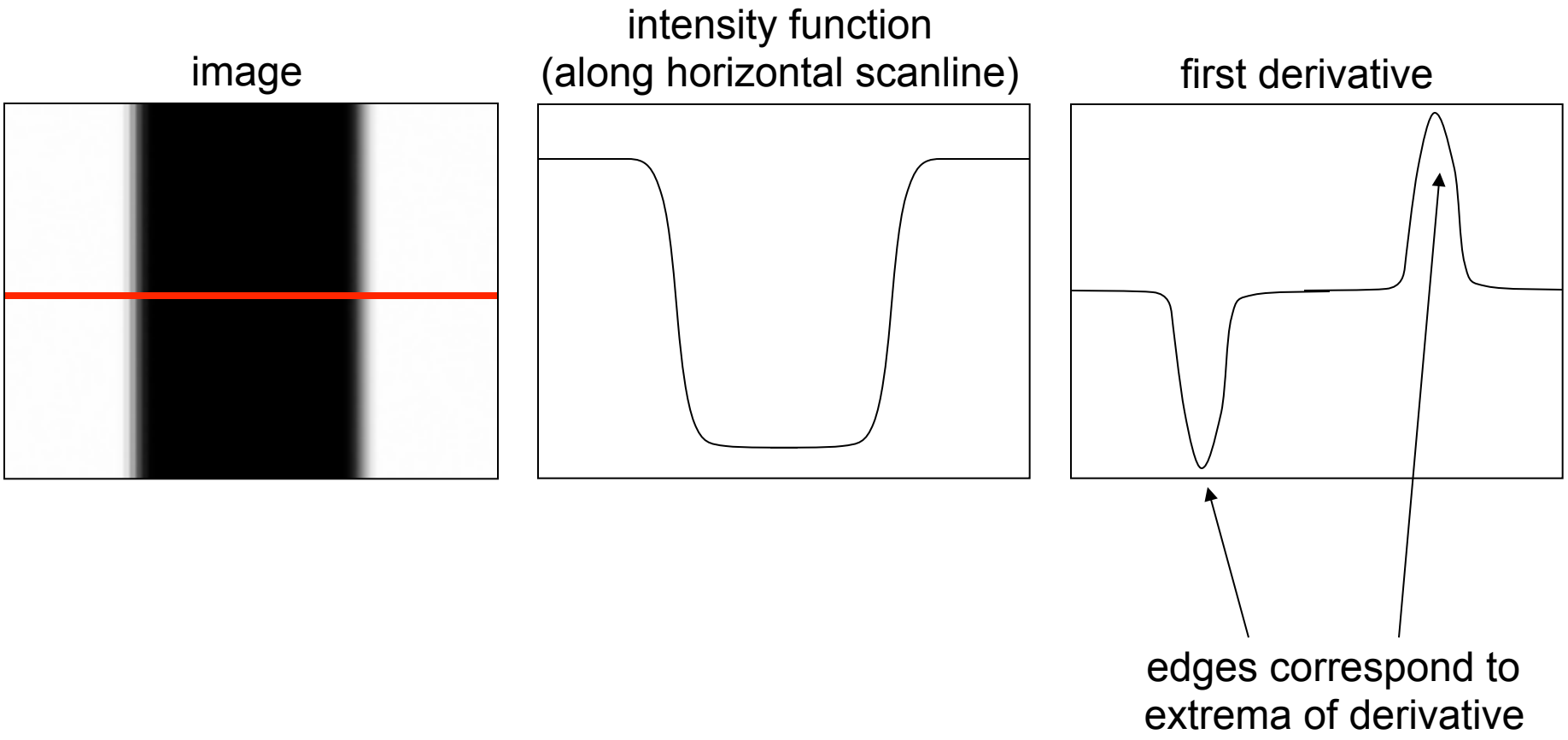
- Low-level features (edges, corners, texture patches) are needed for many tasks:
  - Image matching and registration
  - Structure-from-motion and image-based 3D modeling
  - Image segmentation
  - Image retrieval

- Relevant reading:
  - Chapter 4 of Szeliski's book
  - David Lowe's article (2004)
    http://www.cs.ubc.ca/~lowe/keypoints/

Acknowledgement: many slides from Svetlana Lazebnik, Steve Seitz, David Lowe, Kristen Grauman, and others (detailed credits on individual slides)

Edge detection

- An edge is a place of rapid change in the image intensity function



Source: S. Lazebnik

Derivatives with convolution

For 2D function f(x,y), the partial derivative is:

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\varepsilon \to 0} \frac{f(x+\varepsilon, y) - f(x,y)}{\varepsilon}$$

For discrete data, we can approximate using finite differences:

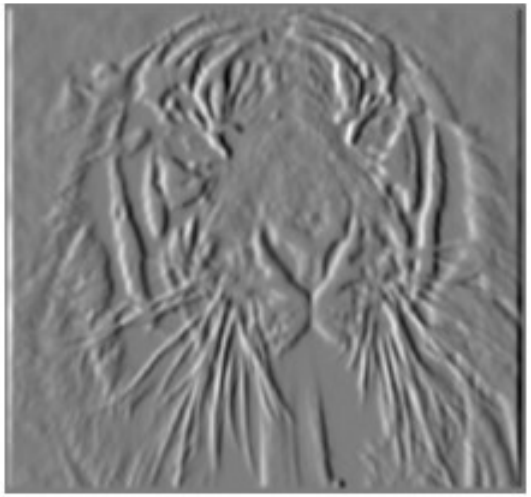
$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x+1, y) - f(x,y)}{1}$$

To implement the above as convolution, what would be the associated filter?

Source: K. Grauman

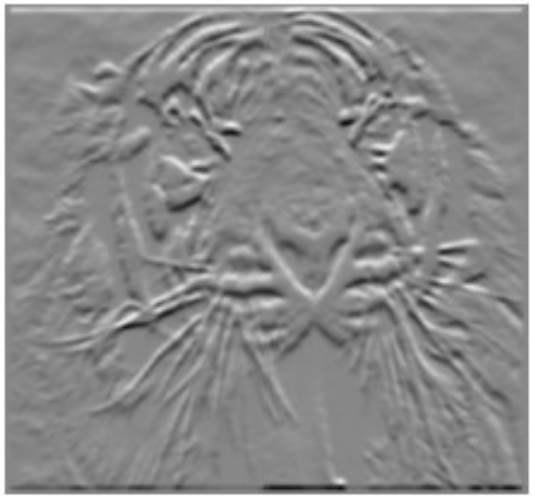
Partial derivatives of an image

$$\frac{\partial f(x,y)}{\partial x}$$



-1	1

$$\frac{\partial f(x,y)}{\partial y}$$



-1
1

or

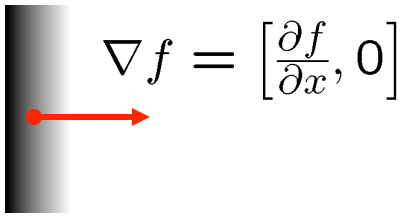
1
-1

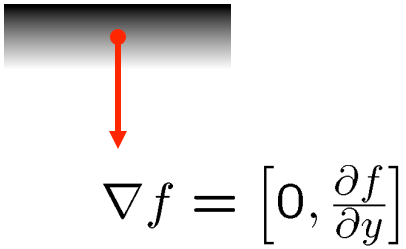
Which shows changes with respect to x?

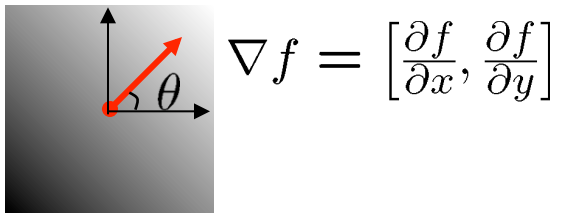
Source: S. Lazebnik

Image gradient

The gradient of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$







The gradient points in the direction of most rapid increase in intensity

- How does this direction relate to the direction of the edge?

The gradient direction is given by $\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$

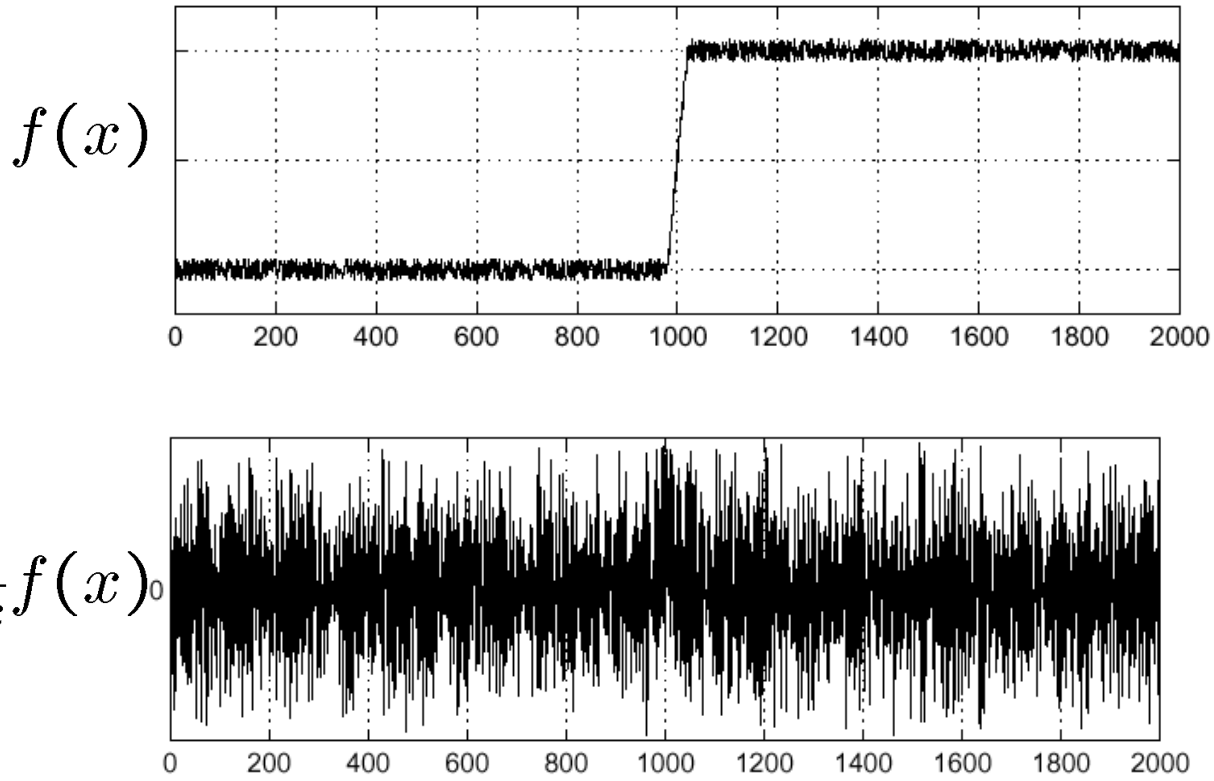
The edge strength is given by the gradient magnitude

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Source: Steve Seitz

Effects of noise

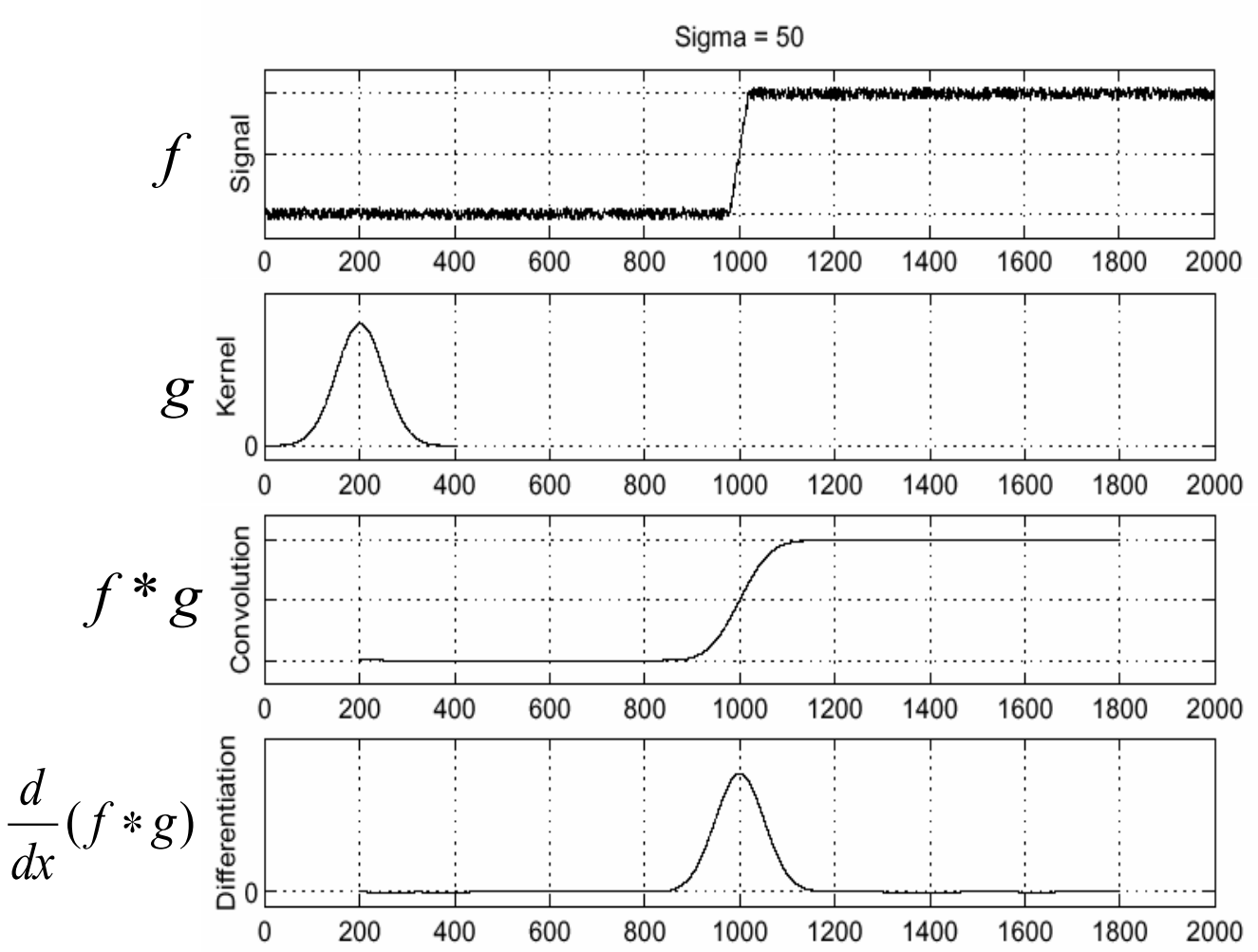
Consider a single row or column of the image



Where is the edge?

Source: S. Seitz

Solution: smooth first

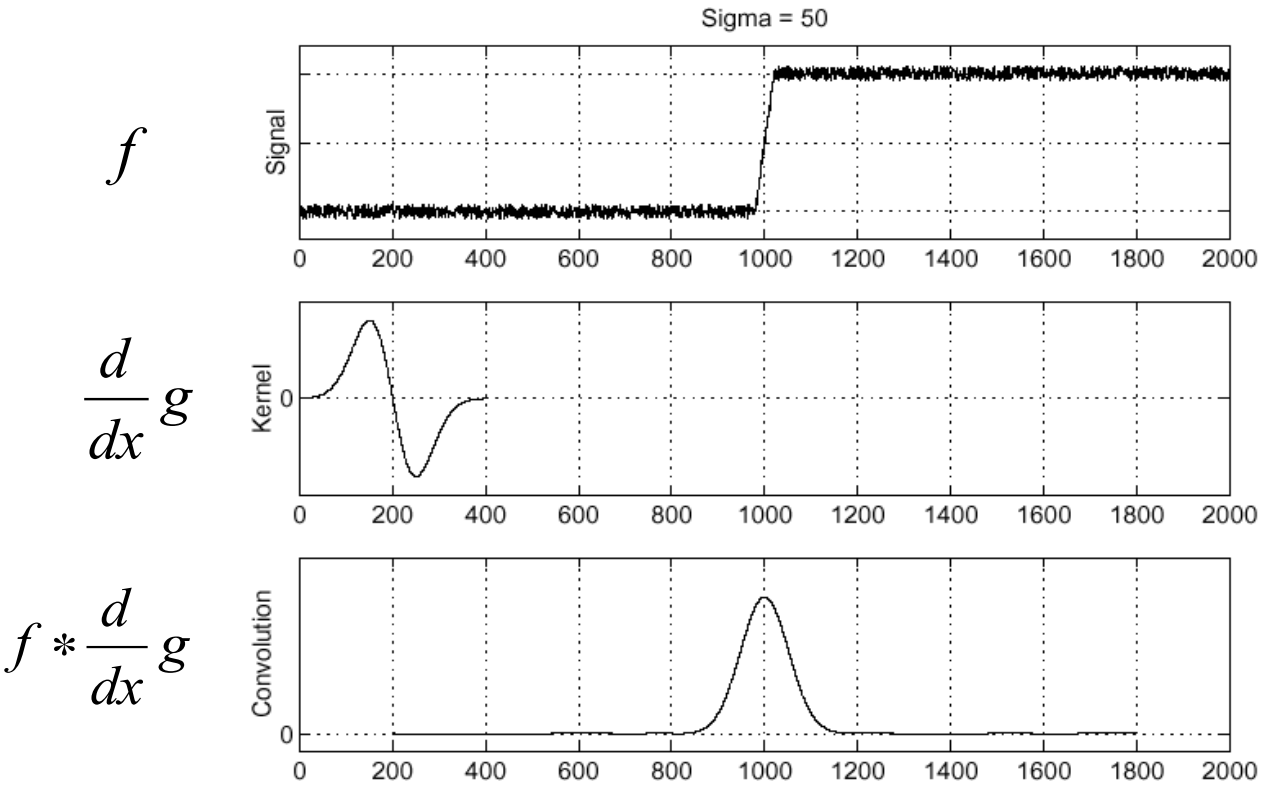


- To find edges, look for peaks in $\frac{d}{dx}(f * g)$

Source: S. Seitz

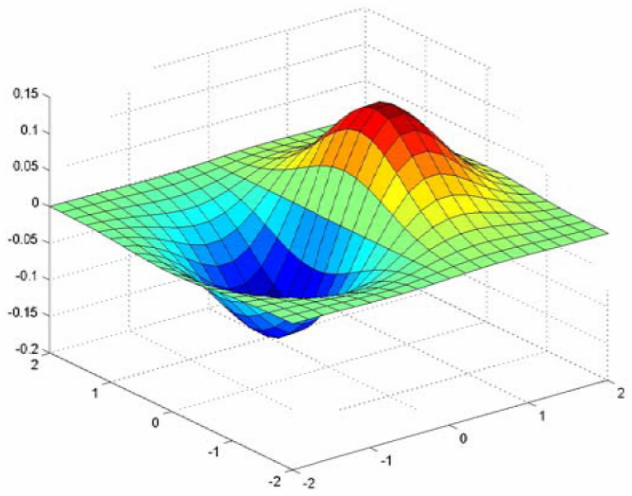
Derivative theorem of convolution

- Differentiation is convolution, and convolution is associative: $\frac{d}{dx}(f * g) = f * \frac{d}{dx} g$
- This saves us one operation:

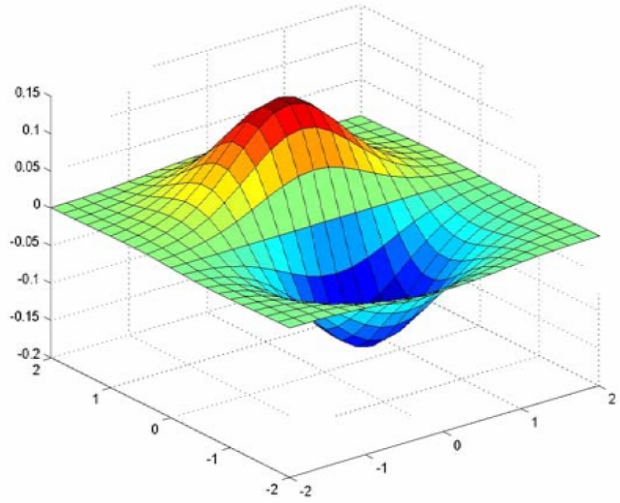


Source: S. Seitz

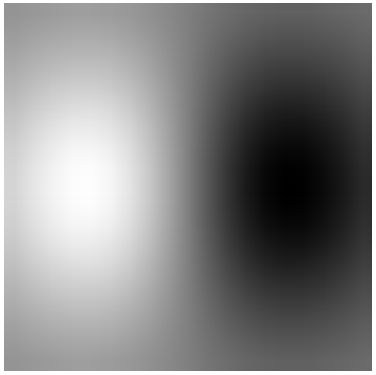
Derivative of Gaussian filters

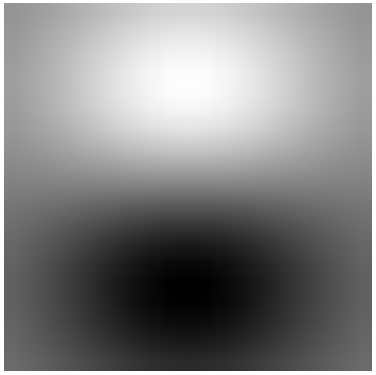


x-direction



y-direction

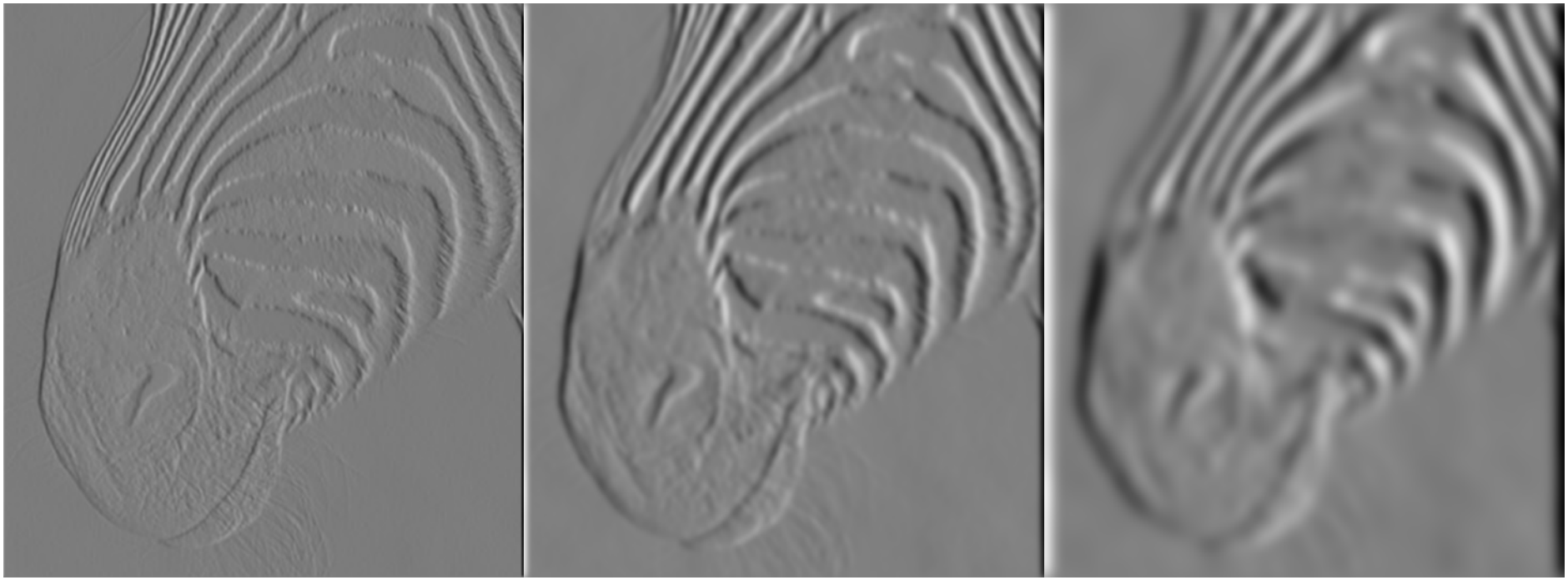




These filters are separable

Source: S. Lazebnik

Scale of Gaussian derivative filter



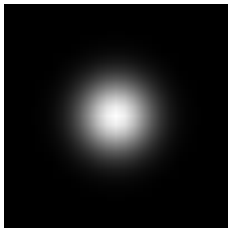
1 pixel 3 pixels 7 pixels

Smoothed derivative removes noise, but blurs edge. Also finds edges at different “scales”

Source: D. Forsyth

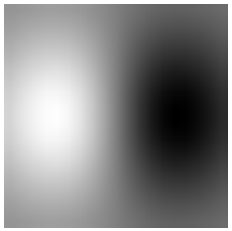
Review: Smoothing vs. derivative filters

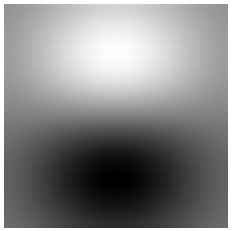
Smoothing filters



- Gaussian: remove “high-frequency” components; “low-pass” filter
- Can the values of a smoothing filter be negative?
- What should the values sum to?
  - **One:** constant regions are not affected by the filter

Derivative filters



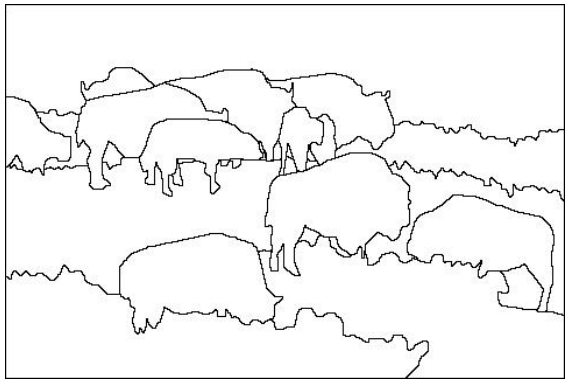


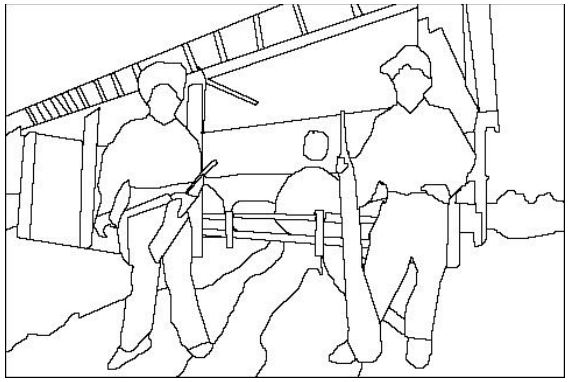
- Derivatives of Gaussian
- Can the values of a derivative filter be negative?
- What should the values sum to?
  - **Zero:** no response in constant regions

Source: S. Lazebnik

Image gradients vs. meaningful contours

image | human segmentation | gradient magnitude

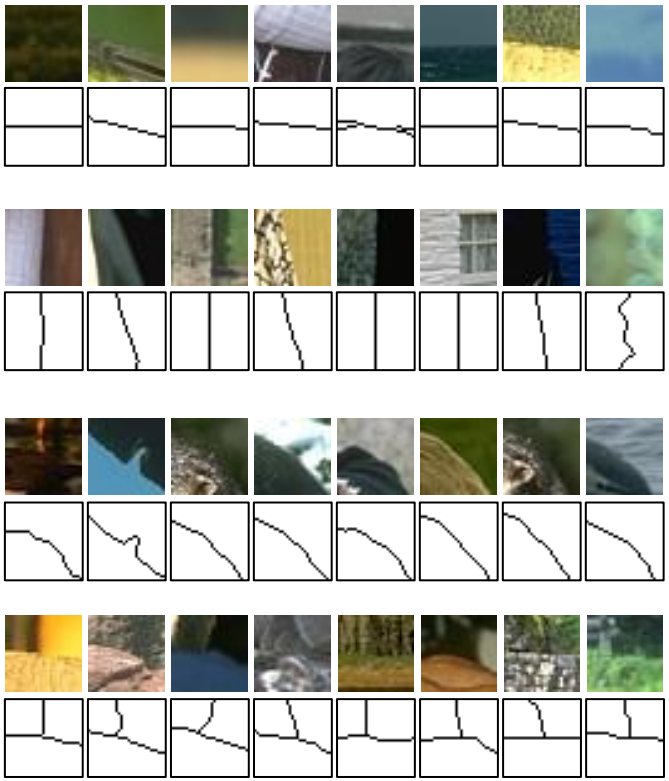



Berkeley segmentation database:

http://www.eecs.berkeley.edu/Research/Projects/CS/vision/grouping/segbench/

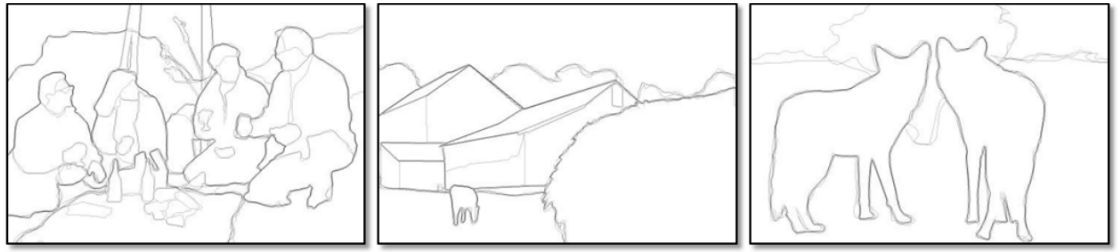
Data-driven edge detection

Training data



Input images

Ground truth



Output

P. Dollar and L. Zitnick, Structured forests for fast edge detection, ICCV 2013

Keypoint extraction: Corners

Source: S. Lazebnik

Why extract keypoints?

- Motivation: panorama stitching
  - We have two images – how do we combine them?

Source: S. Lazebnik

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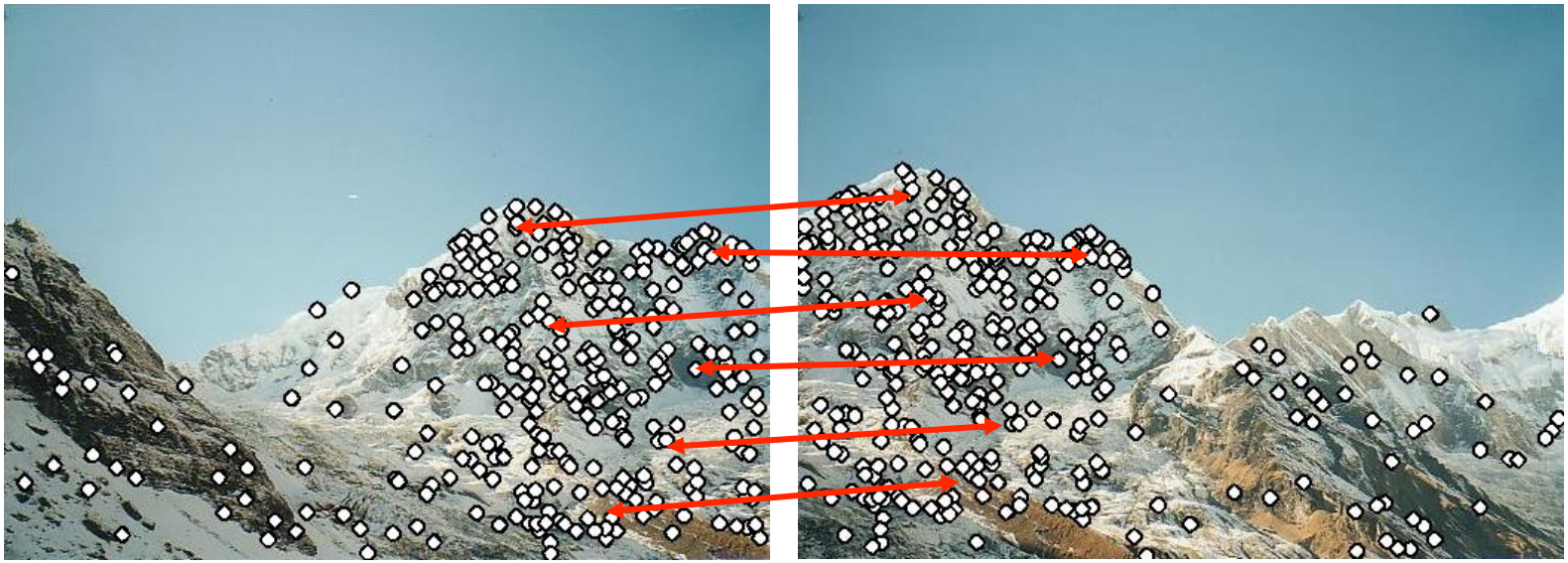
- Motivation: panorama stitching
  - We have two images – how do we combine them?



Step 1: extract keypoints

Step 2: match keypoint features

Source: S. Lazebnik

Why extract keypoints?

- Motivation: panorama stitching
  - We have two images – how do we combine them?

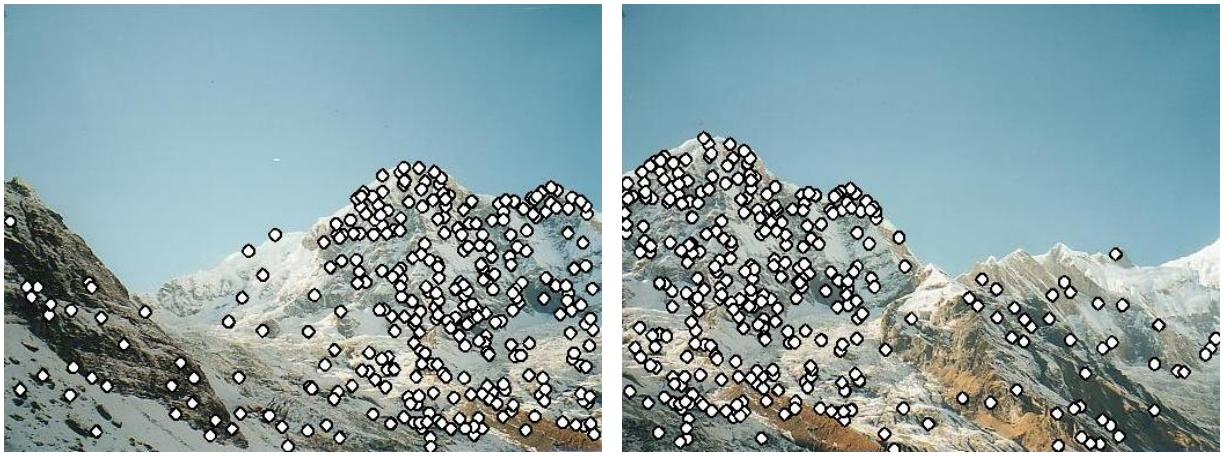
Step 1: extract keypoints

Step 2: match keypoint features

Step 3: align images

Source: S. Lazebnik

Characteristics of good keypoints



- Repeatability
  - The same keypoint can be found in several images despite geometric and photometric transformations
- Saliency
  - Each keypoint is distinctive
- Compactness and efficiency
  - Many fewer keypoints than image pixels
- Locality
  - A keypoint occupies a relatively small area of the image; robust to clutter and occlusion

Source: S. Lazebnik

Applications

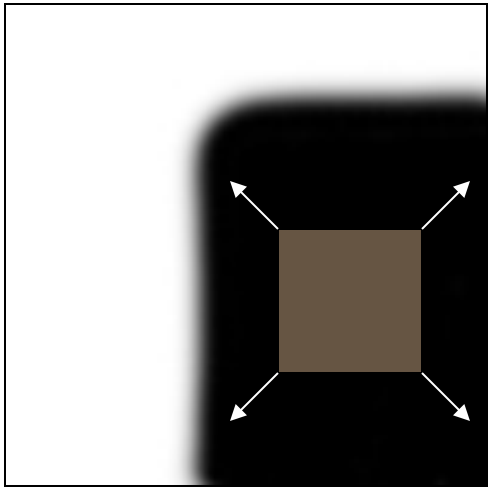
Keypoints are used for:

- Image alignment
- 3D reconstruction
- Motion tracking
- Robot navigation
- Indexing and database retrieval
- Object recognition

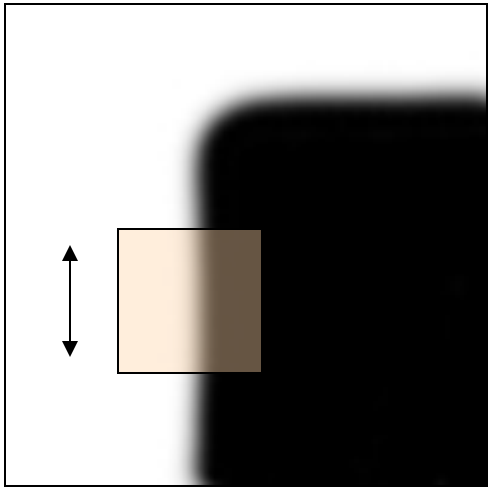
Source: S. Lazebnik

Corner Detection: Basic Idea

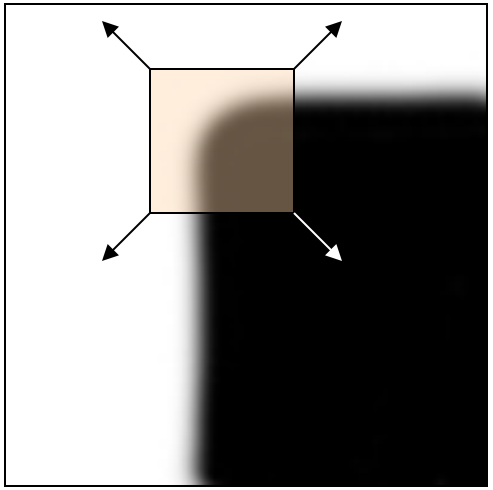
- We should easily recognize the point by looking through a small window
- Shifting a window in *any direction* should give *a large change* in intensity



"flat" region:
no change in all directions



"edge":
no change along the edge direction



"corner":
significant change in all directions

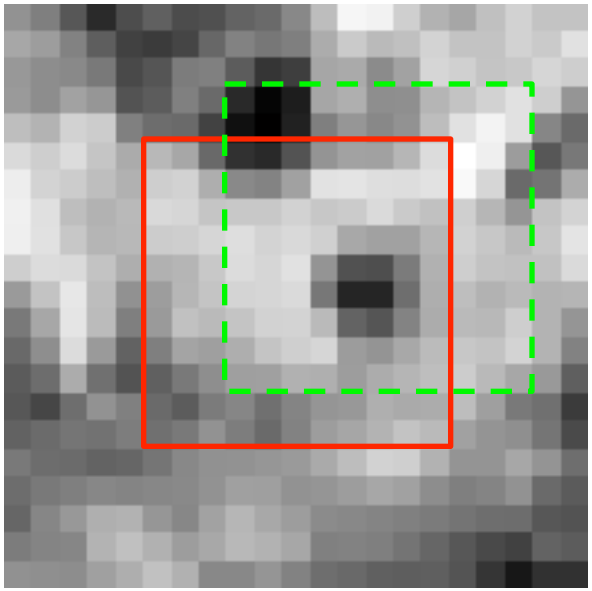
Source: S. Lazebnik

Corner Detection: Mathematics

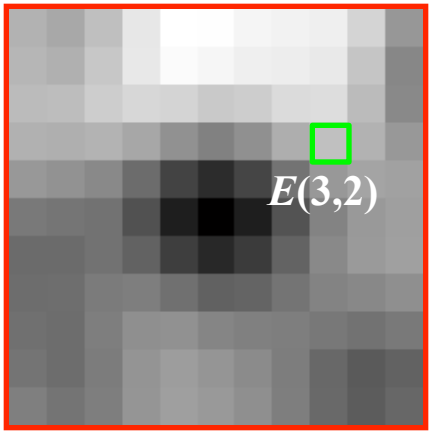
Change in appearance of window *W* for the shift [*u,v*]:

$$E(u,v) = \sum_{(x,y)\in W} [I(x+u, y+v) - I(x,y)]^2$$

$I(x, y)$



$E(u, v)$



Source: S. Lazebnik

Corner Detection: Mathematics

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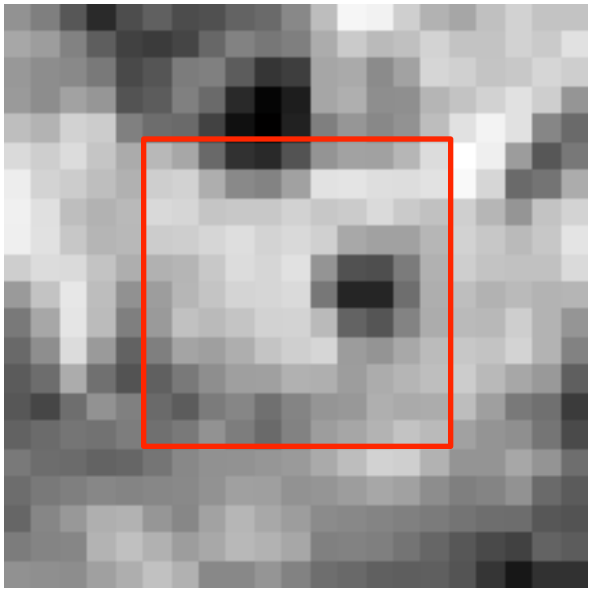
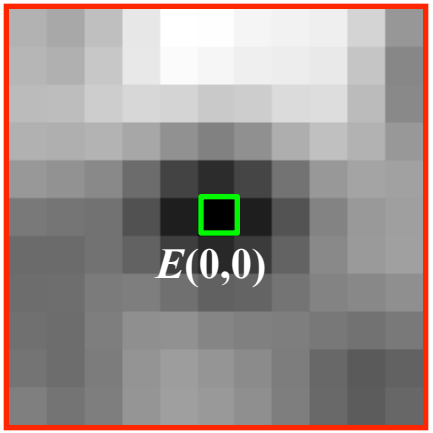


$E(u, v)$



Source: S. Lazebnik

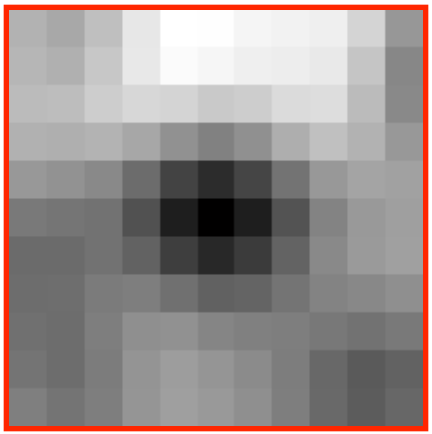
Corner Detection: Mathematics

Change in appearance of window *W* for the shift [*u,v*]:

$$E(u,v) = \sum_{(x,y)\in W} [I(x+u, y+v) - I(x,y)]^2$$

We want to find out how this function behaves for small shifts

$E(u, v)$



Source: S. Lazebnik

Corner Detection: Mathematics

- First-order Taylor approximation for small motions [*u*, *v*]:

$$I(x+u, y+v) \approx I(x,y) + I_x u + I_y v$$

- Let's plug this into *E*(*u*,*v*):

$$E(u,v) = \sum_{(x,y)\in W} [I(x+u,y+v) - I(x,y)]^2$$

$$\approx \sum_{(x,y)\in W} [I(x,y) + I_x u + I_y v - I(x,y)]^2$$

$$= \sum_{(x,y)\in W} [I_x u + I_y v]^2 = \sum_{(x,y)\in W} I_x^2 u^2 + 2 I_x I_y uv + I_y^2 v^2$$

Source: S. Lazebnik

Source: S. Lazebnik

Corner Detection: Mathematics

The quadratic approximation can be written as

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$$

where *M* is a *second moment matrix* computed from image derivatives:

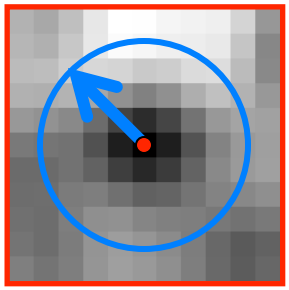
$$M = \begin{bmatrix} \sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\ \sum_{x,y} I_x I_y & \sum_{x,y} I_y^2 \end{bmatrix}$$

(the sums are over all the pixels in the window *W*)

Interpreting the second moment matrix

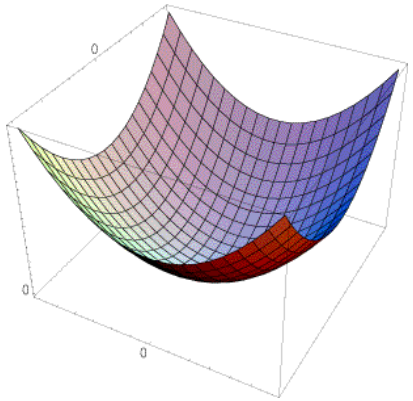
- The surface $E(u,v)$ is locally approximated by a quadratic form. Let's try to understand its shape.
  - Specifically, in which directions does it have the smallest/greatest change?

$E(u, v)$



$$E(u,v) \approx [u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

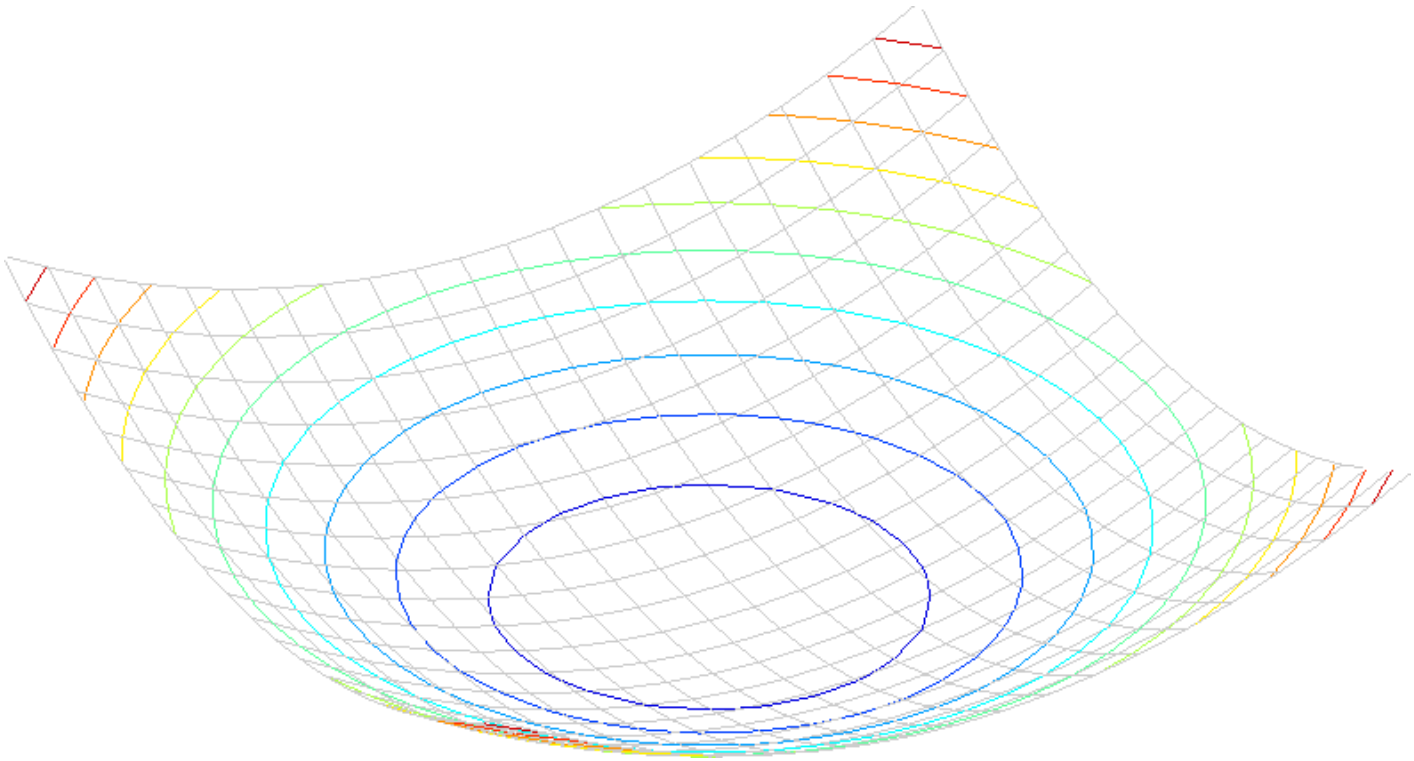
$$M = \begin{bmatrix} \sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\ \sum_{x,y} I_x I_y & \sum_{x,y} I_y^2 \end{bmatrix}$$



Interpreting the second moment matrix

Consider a horizontal "slice" of $E(u, v)$: $\begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$

This is the equation of an ellipse.



Source: S. Lazebnik

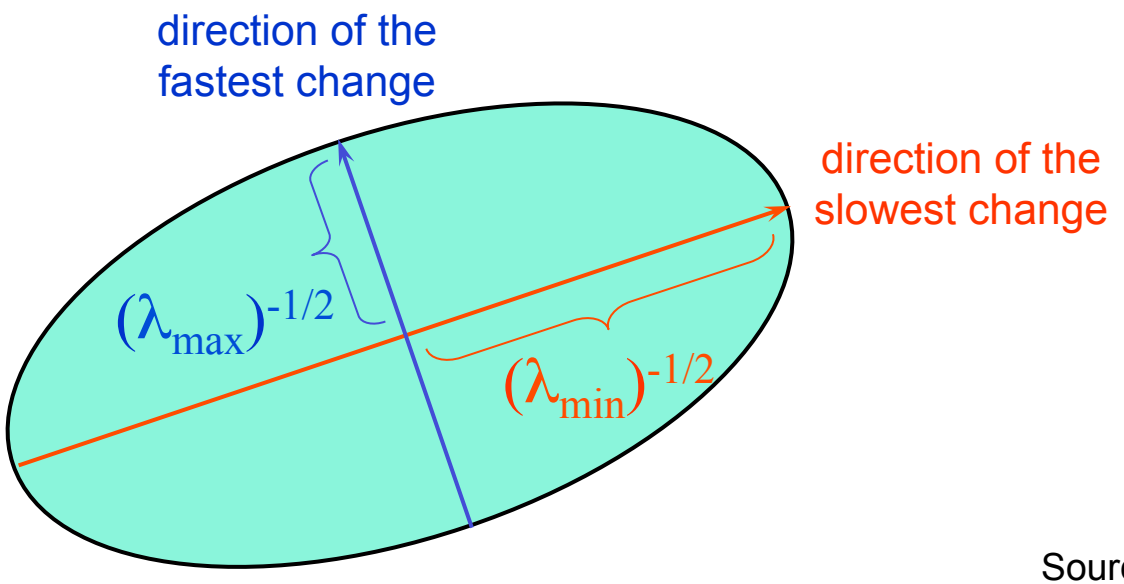
Interpreting the second moment matrix

Consider a horizontal "slice" of $E(u, v)$: $\begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$

This is the equation of an ellipse.

Diagonalization of M: $M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$

The axis lengths of the ellipse are determined by the eigenvalues and the orientation is determined by R



Source: S. Lazebnik

Interpreting the second moment matrix

Consider the axis-aligned case (gradients are either horizontal or vertical)

$$M = \begin{bmatrix} \sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\ \sum_{x,y} I_x I_y & \sum_{x,y} I_y^2 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

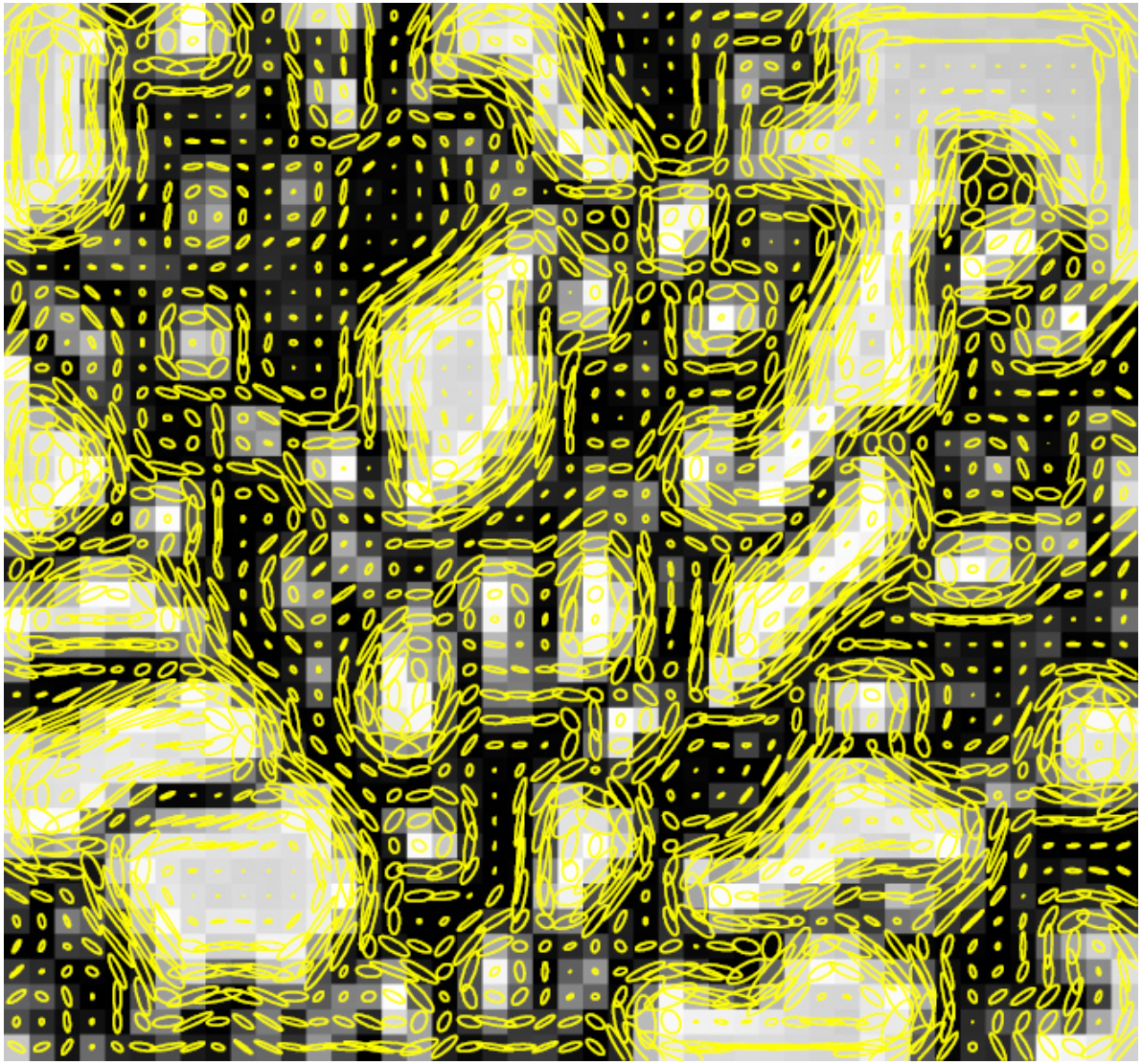
If either *a* or *b* is close to 0, then this is **not** a corner, so look for locations where both are large.

Source: S. Lazebnik

Visualization of second moment matrices

Source: S. Lazebnik

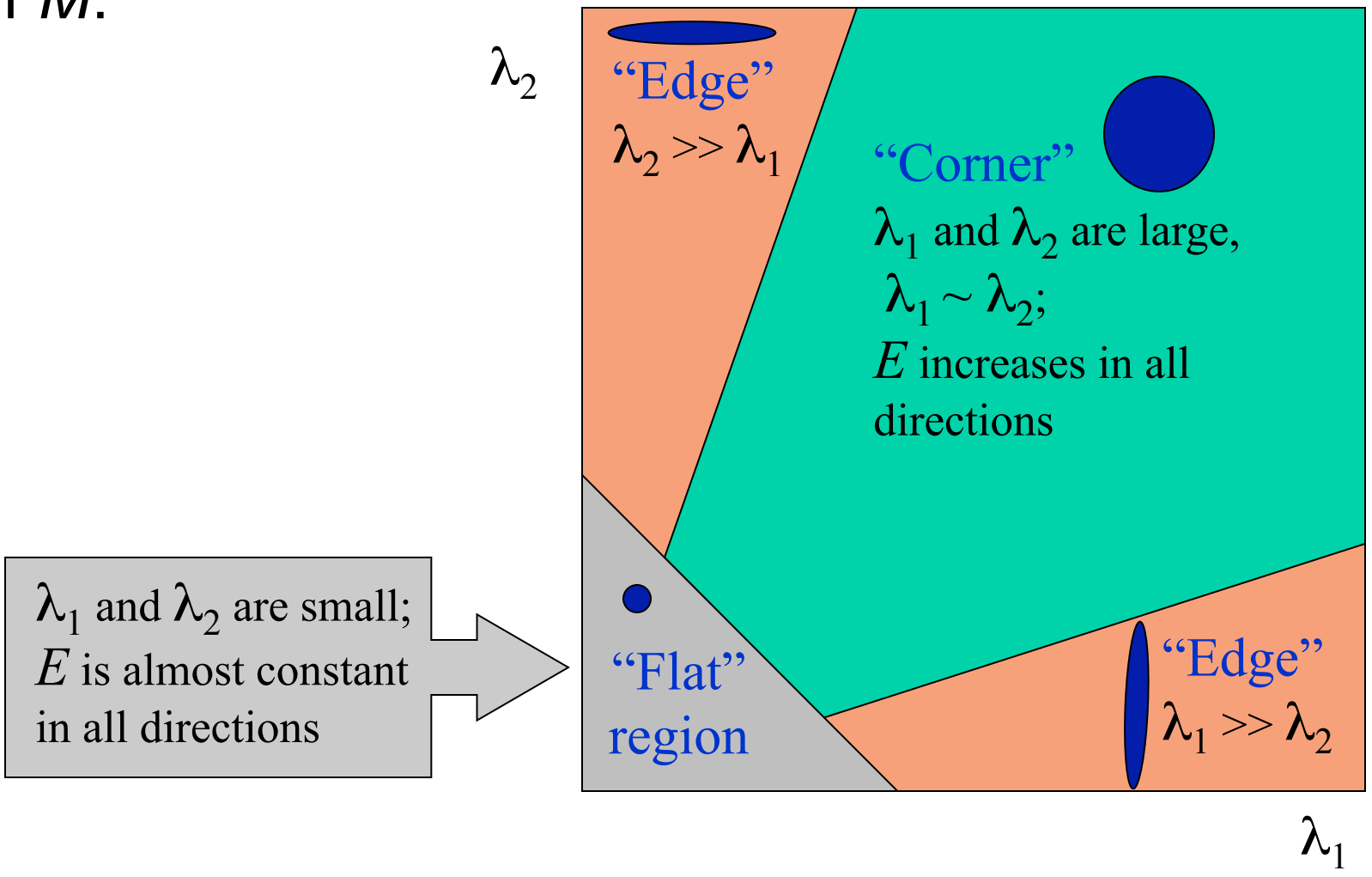
Visualization of second moment matrices



Source: S. Lazebnik

Interpreting the eigenvalues

Classification of image points using eigenvalues of *M*:

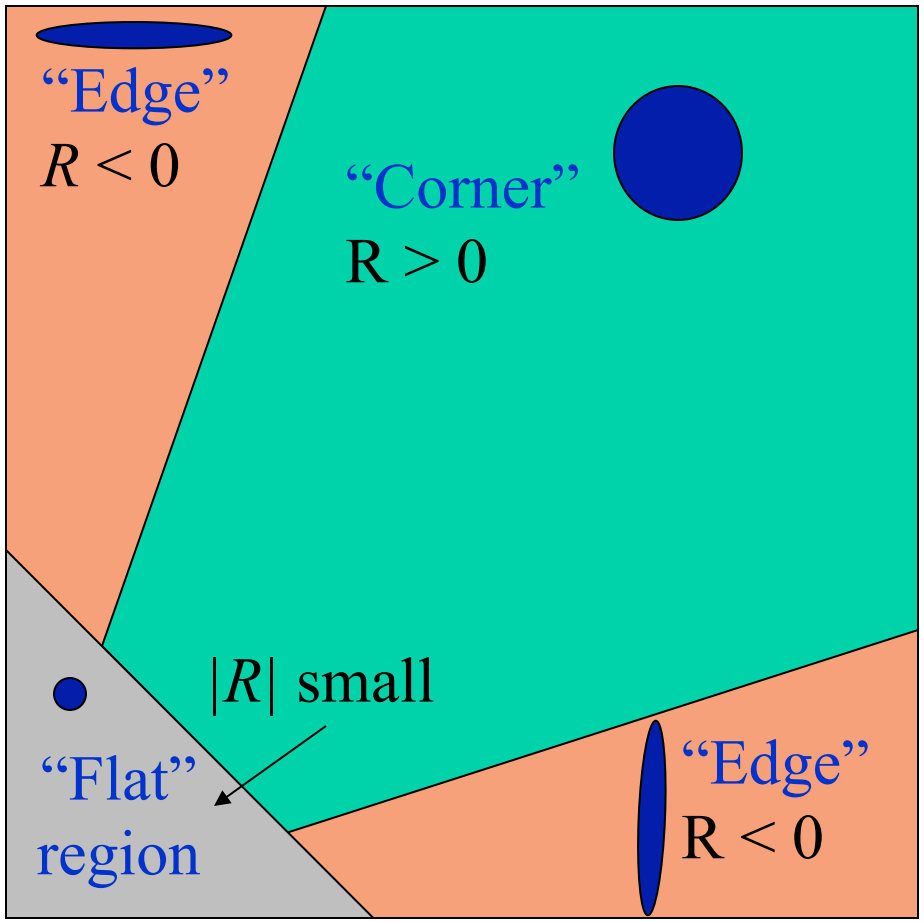


Source: S. Lazebnik

Corner response function

$$R = \det(M) - \alpha \,\mathrm{trace}(M)^2 = \lambda_1\lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

α: constant (0.04 to 0.06)



Source: S. Lazebnik

Source: S. Lazebnik

The Harris corner detector

1. Compute partial derivatives at each pixel
2. Compute second moment matrix *M* in a Gaussian window around each pixel:

$$M = \begin{bmatrix} \sum_{x,y} w(x,y) I_x^2 & \sum_{x,y} w(x,y) I_x I_y \\ \sum_{x,y} w(x,y) I_x I_y & \sum_{x,y} w(x,y) I_y^2 \end{bmatrix}$$

C.Harris and M.Stephens.
"A Combined Corner and Edge Detector." *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

Source: S. Lazebnik

The Harris corner detector

1. Compute partial derivatives at each pixel
2. Compute second moment matrix *M* in a Gaussian window around each pixel
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C.Harris and M.Stephens.
"A Combined Corner and Edge Detector." *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

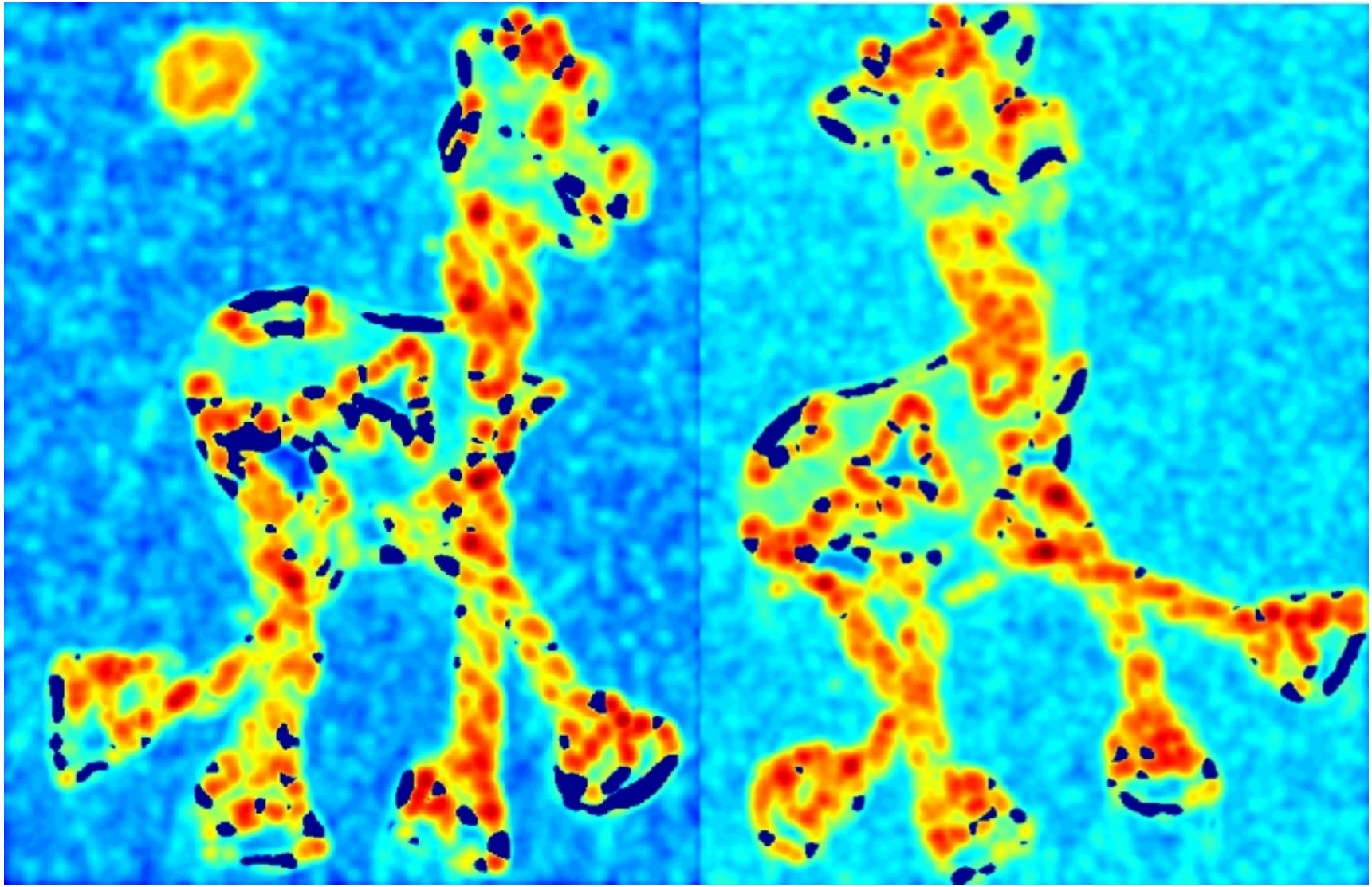
Source: S. Lazebnik

Harris Detector: Steps

Source: S. Lazebnik

Harris Detector: Steps

Compute corner response R



Source: S. Lazebnik

The Harris corner detector

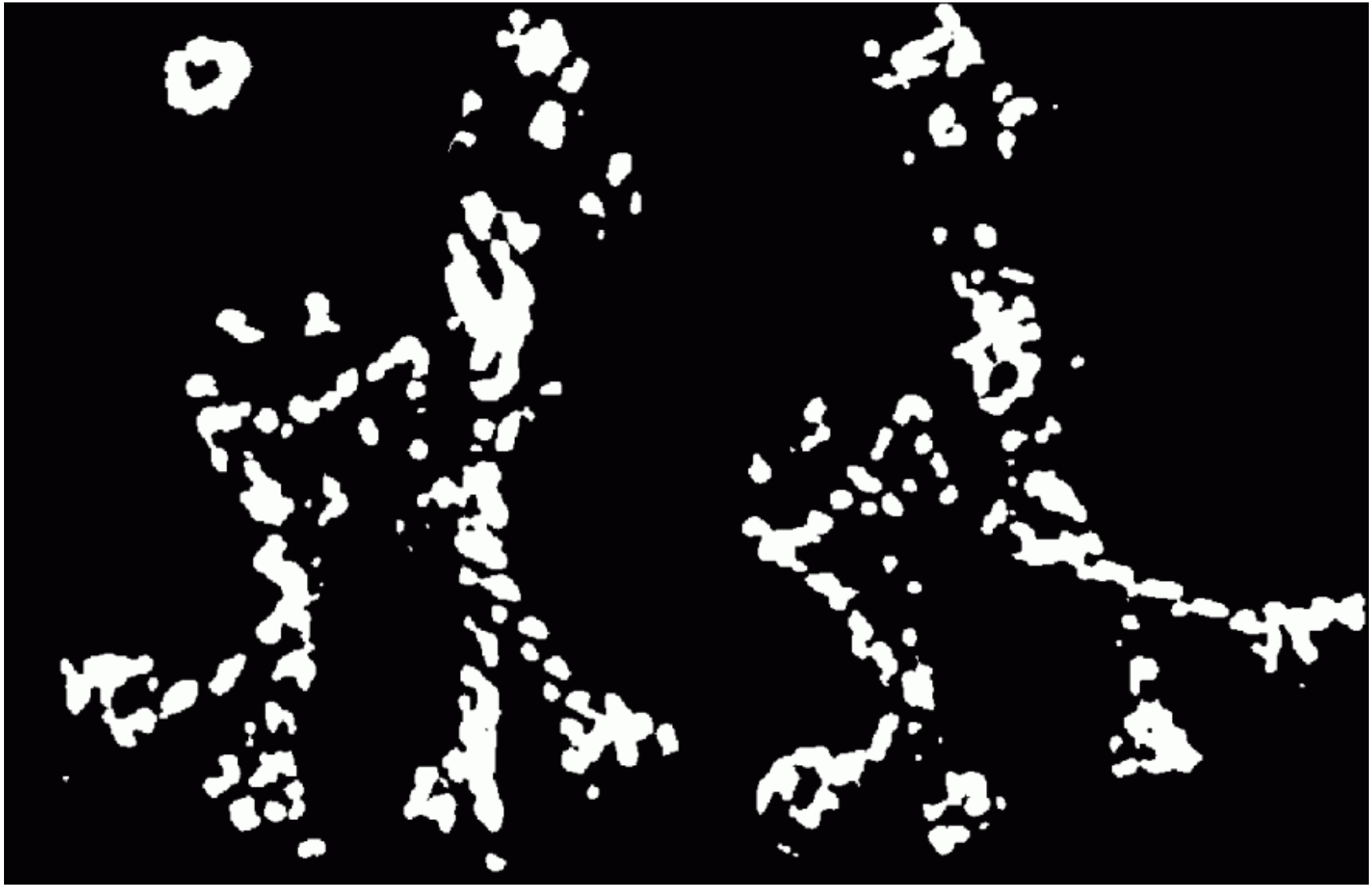
1. Compute partial derivatives at each pixel
2. Compute second moment matrix *M* in a Gaussian window around each pixel
3. Compute corner response function *R*
4. Threshold *R*
5. Find local maxima of response function (nonmaximum suppression)

C.Harris and M.Stephens.
"A Combined Corner and Edge Detector." *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

Source: S. Lazebnik

Harris Detector: Steps

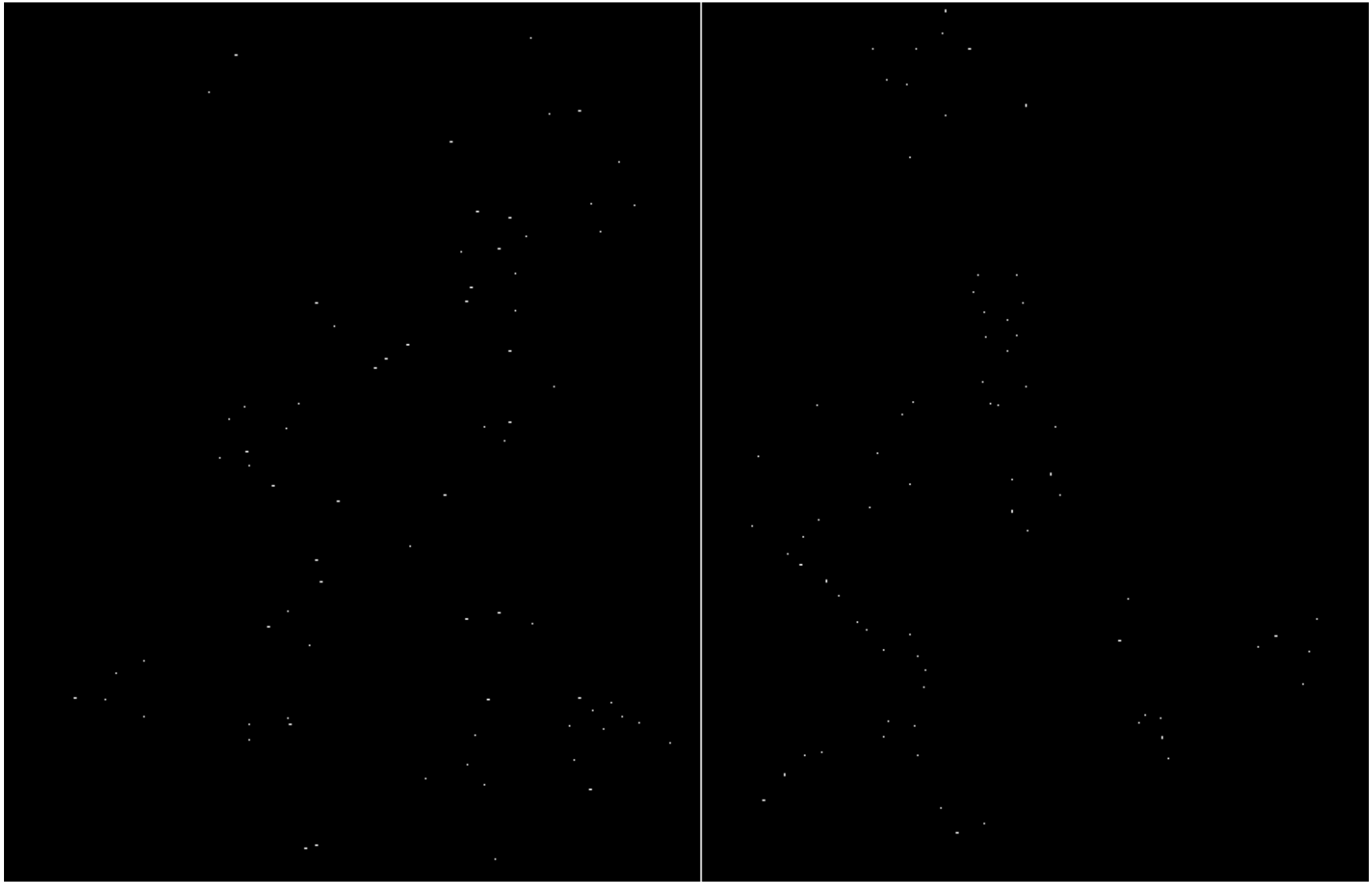
Find points with large corner response: R > threshold



Source: S. Lazebnik

Harris Detector: Steps

Take only the points of local maxima of R



Source: S. Lazebnik

Harris Detector: Steps

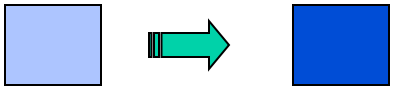
Source: S. Lazebnik

Robustness of corner features

- What happens to corner features when the image undergoes geometric or photometric transformations?

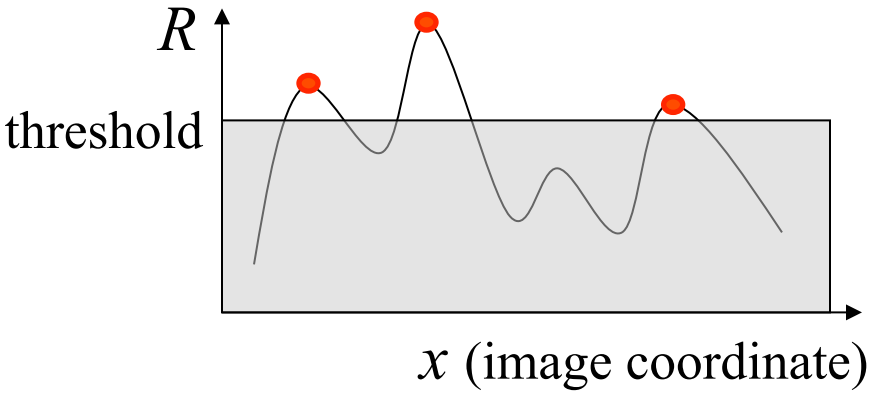
Source: S. Lazebnik

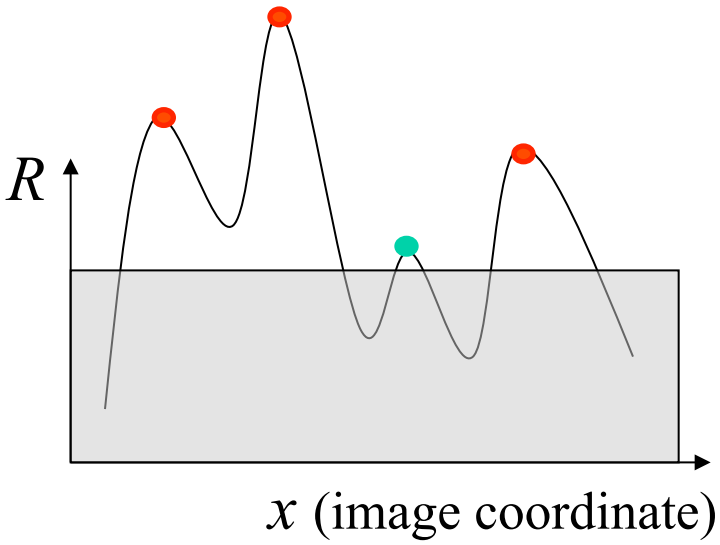
Affine intensity change



$$I \rightarrow a\,I + b$$

- Only derivatives are used => invariance to intensity shift $I \rightarrow I + b$
- Intensity scaling: $I \rightarrow a\,I$

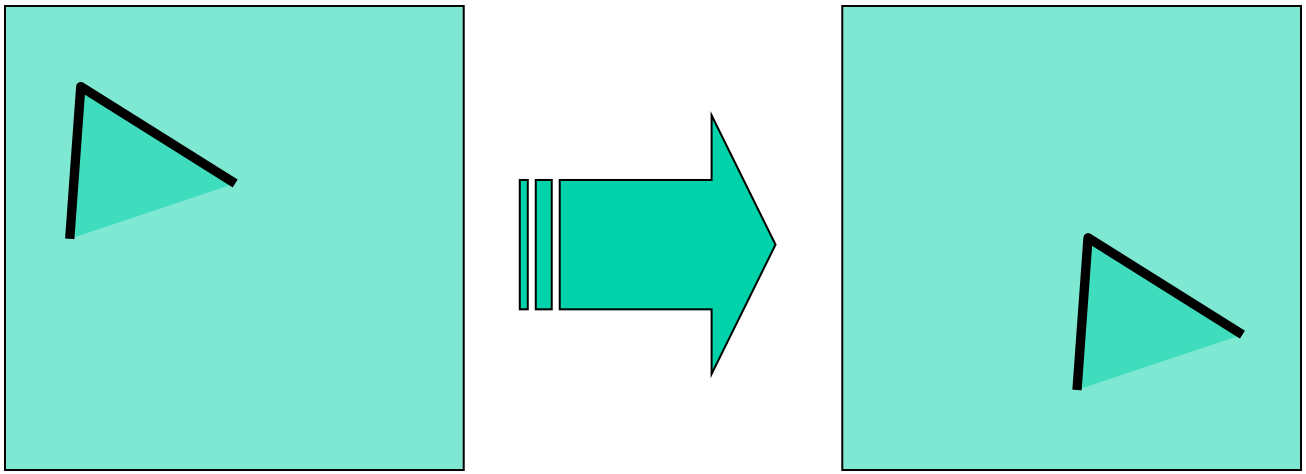




Partially invariant to affine intensity change

Source: S. Lazebnik

Image translation

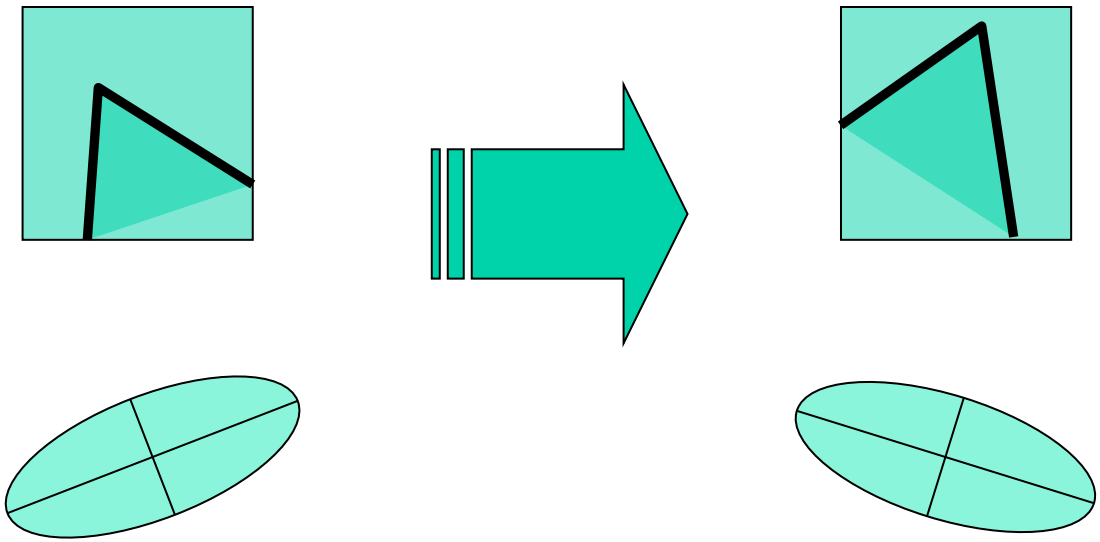


- Derivatives and window function are shift-invariant

Corner location is *covariant* w.r.t. translation

Source: S. Lazebnik

Image rotation

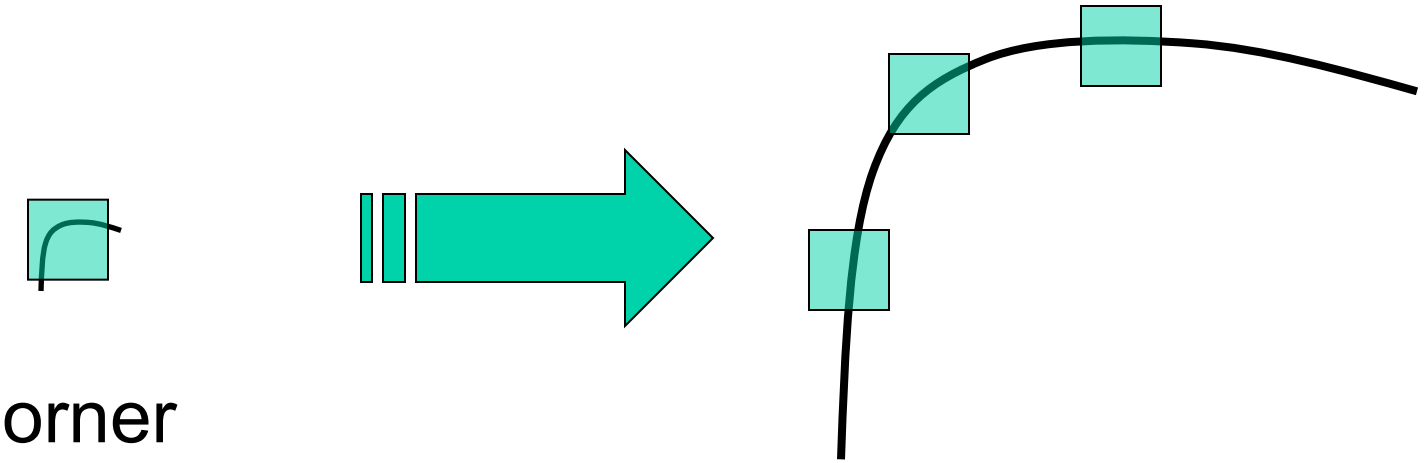


Second moment ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner location is covariant w.r.t. rotation

Source: S. Lazebnik

Scaling



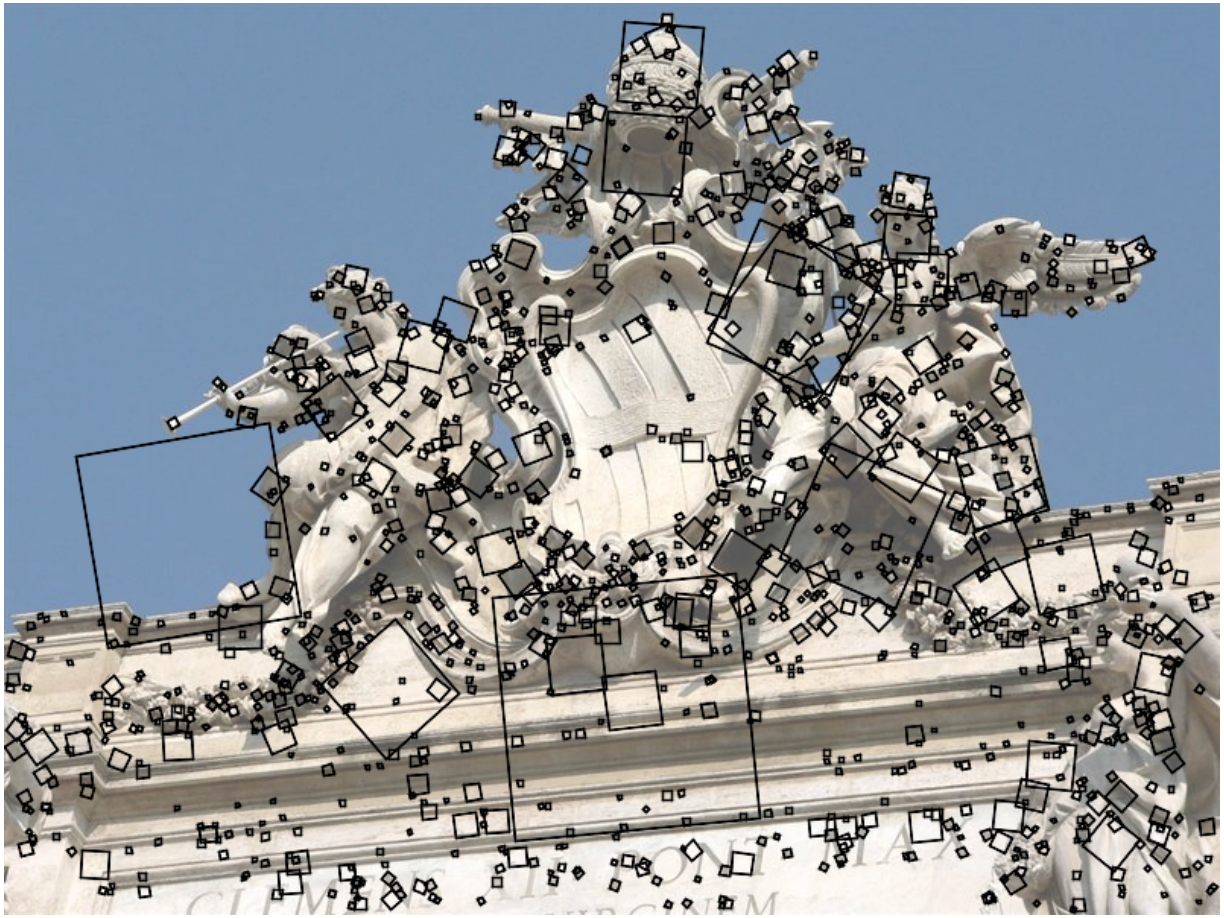
Corner

All points will be classified as edges

Corner location is not covariant to scaling!

Source: S. Lazebnik

SIFT keypoint detection

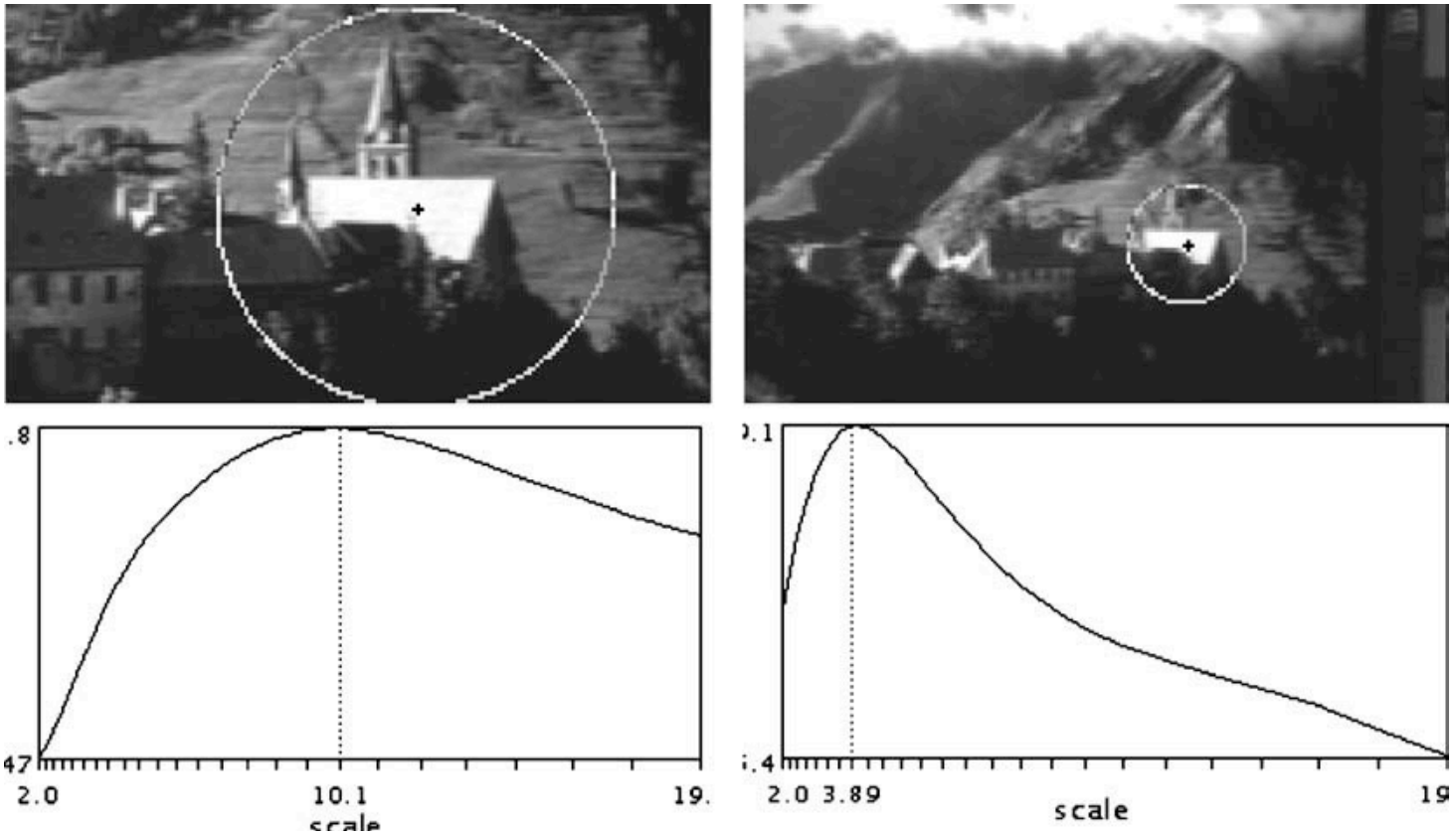


D. Lowe, **Distinctive image features from scale-invariant keypoints**, *IJCV* 60 (2), pp. 91-110, 2004.

Source: S. Lazebnik

Keypoint detection with scale selection

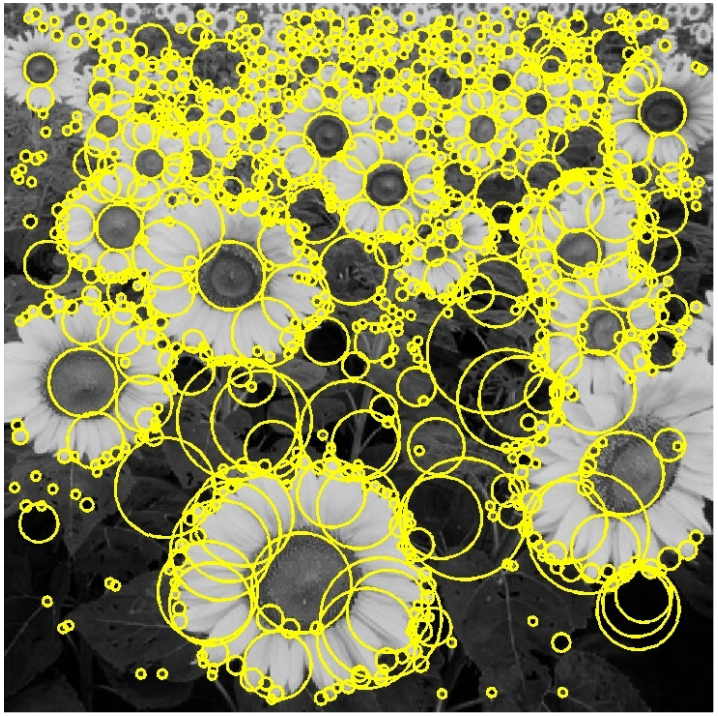
- We want to extract keypoints with characteristic scale that is *covariant* with the image transformation

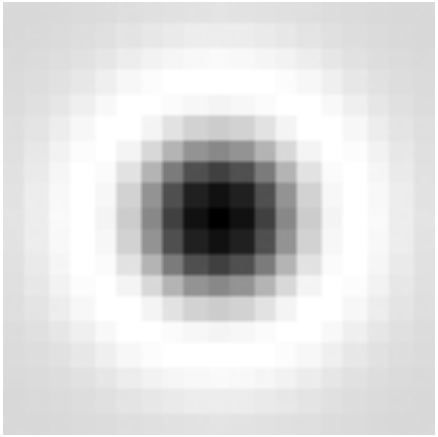


Source: S. Lazebnik

Basic idea

- Convolve the image with a “blob filter” at multiple scales and look for extrema of filter response in the resulting *scale space*

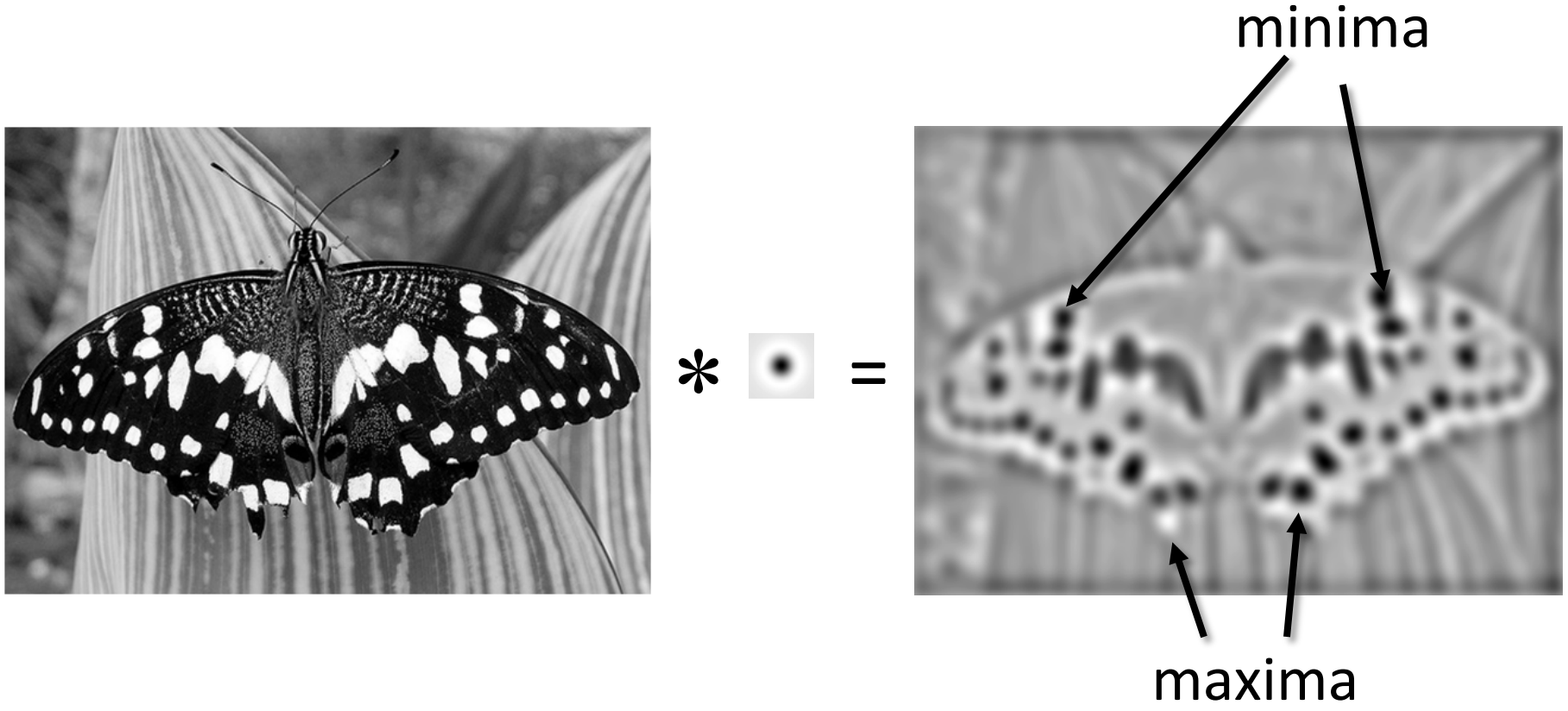




T. Lindeberg. Feature detection with automatic scale selection. *IJCV* 30(2), pp 77-116, 1998.

Source: S. Lazebnik

Blob detection



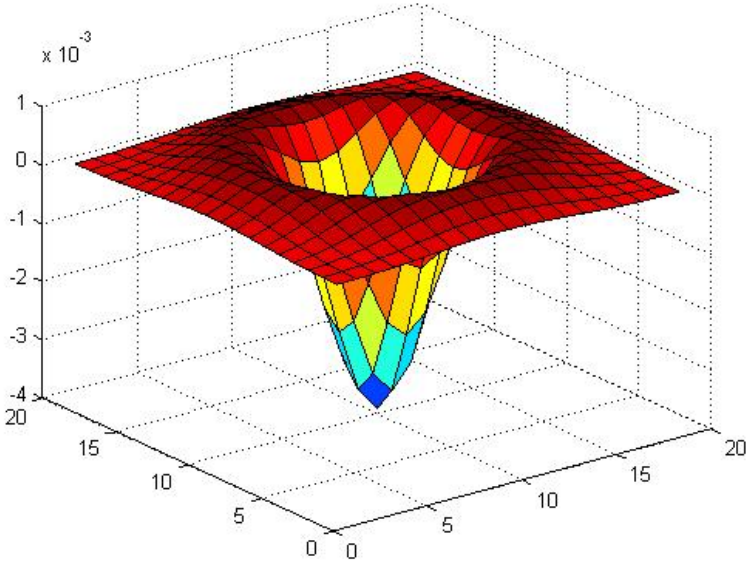
Find maxima *and minima* of blob filter response in space *and scale*

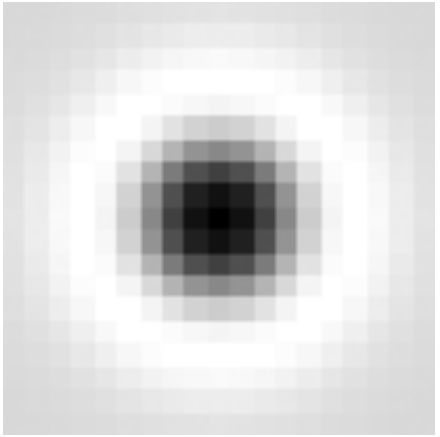
Source: N. Snavely

Source: S. Lazebnik

Blob filter

Laplacian of Gaussian: Circularly symmetric operator for blob detection in 2D



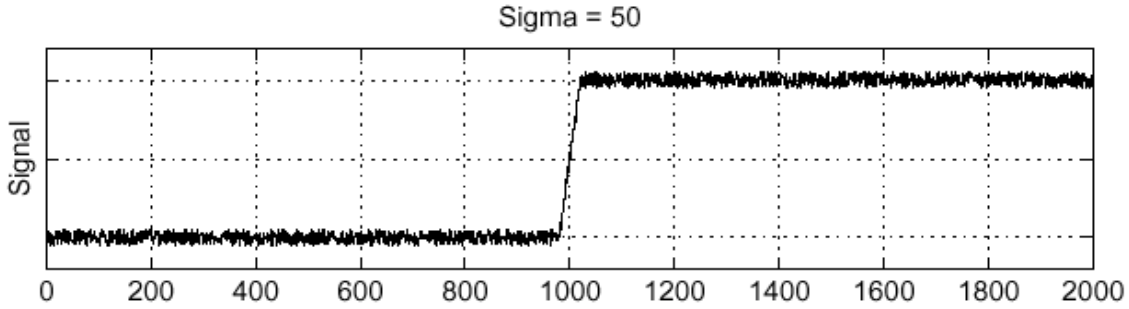


$$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}$$

Source: S. Lazebnik

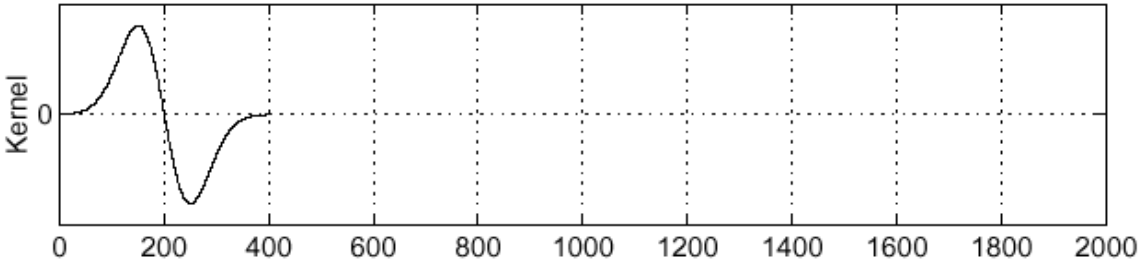
Recall: Edge detection

f



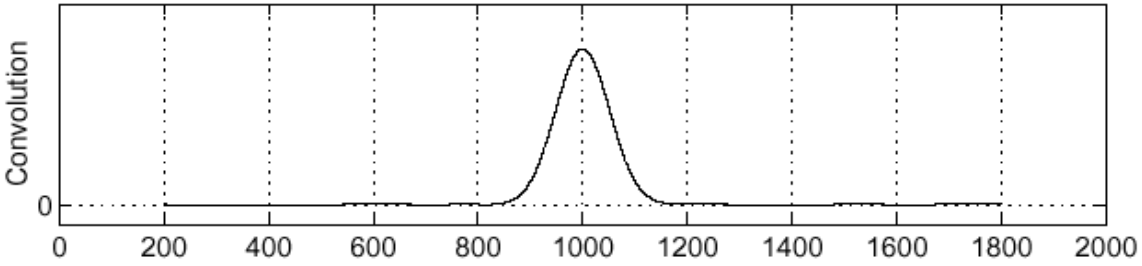
Edge

$\frac{d}{dx} g$



Derivative of Gaussian

$f * \frac{d}{dx} g$



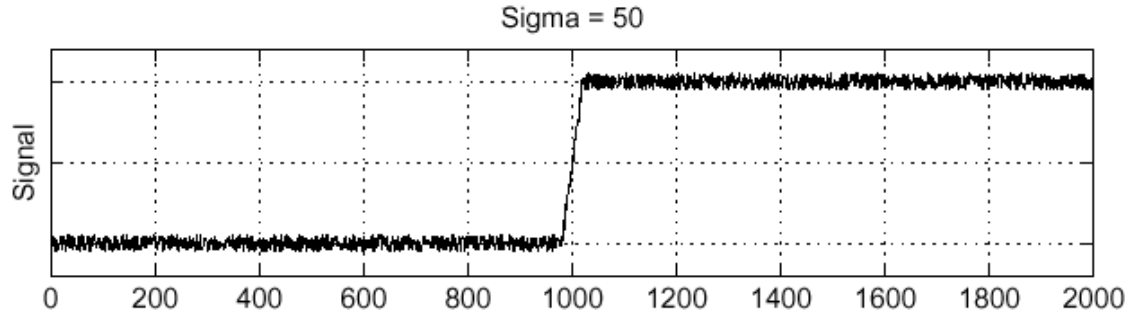
Edge = maximum of derivative

Source: S. Seitz

Source: S. Lazebnik

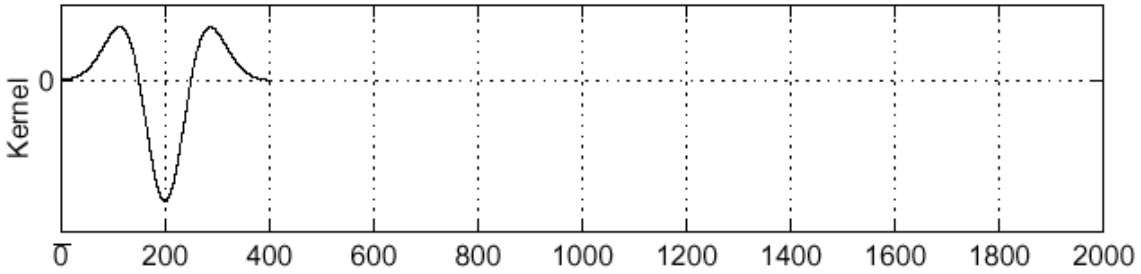
Edge detection, Take 2

f



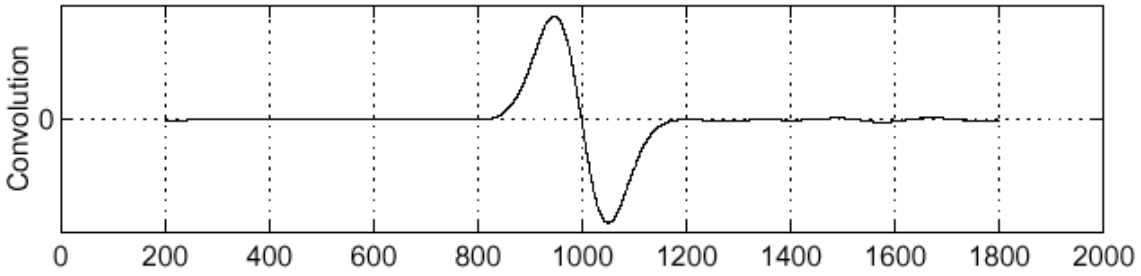
Edge

$$\frac{d^2}{dx^2} g$$



Second derivative of Gaussian (Laplacian)

$$f * \frac{d^2}{dx^2} g$$



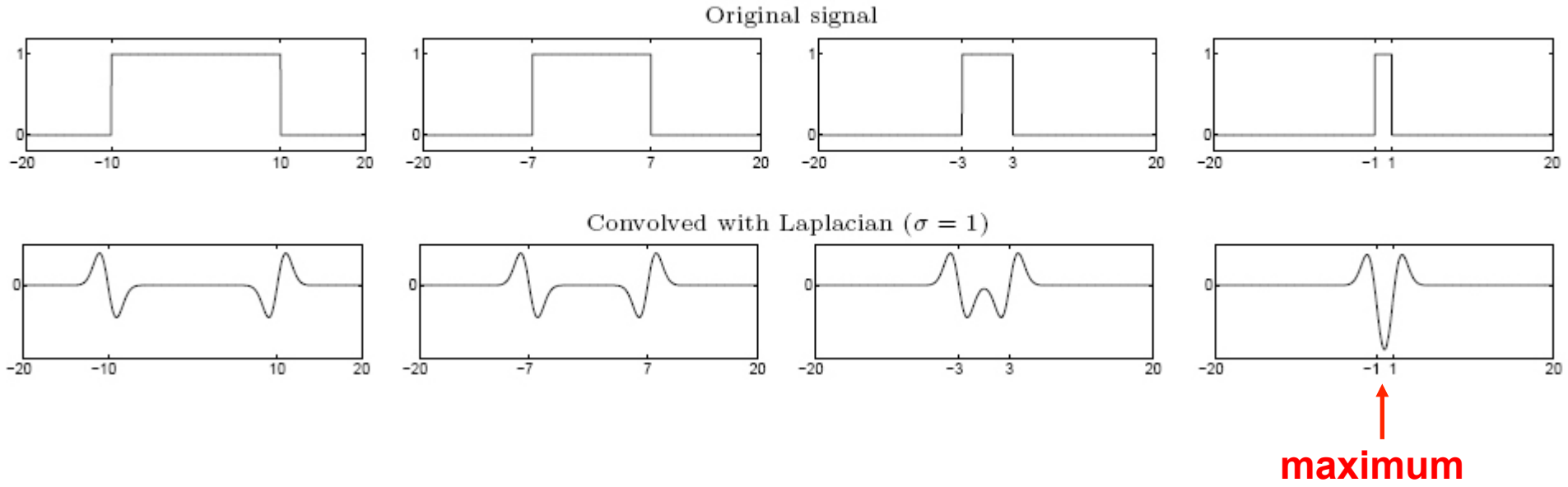
Edge = zero crossing of second derivative

Source: S. Seitz

Source: S. Lazebnik

From edges to blobs

- Edge = ripple
- Blob = superposition of two ripples

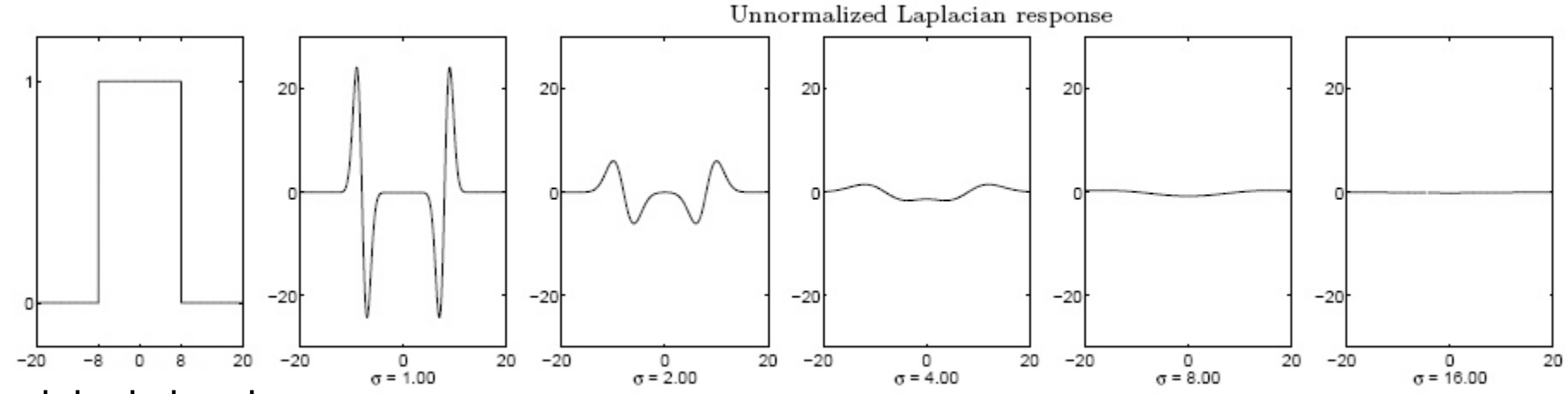


Spatial selection: the magnitude of the Laplacian response will achieve a maximum at the center of the blob, provided the scale of the Laplacian is "matched" to the scale of the blob

Source: S. Lazebnik

Scale selection

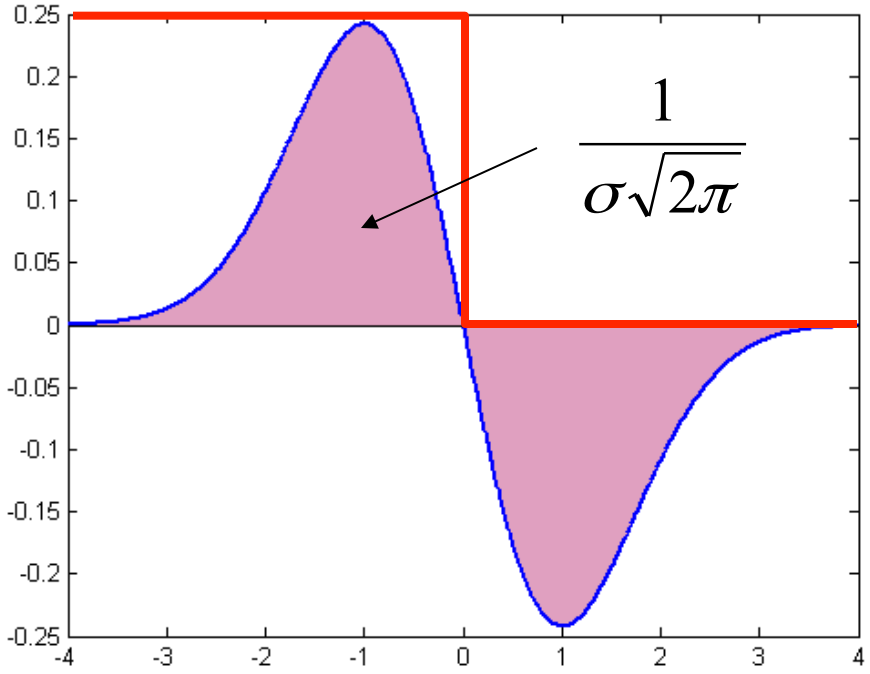
- We want to find the characteristic scale of the blob by convolving it with Laplacians at several scales and looking for the maximum response
- However, Laplacian response decays as scale increases:



Source: S. Lazebnik

Scale normalization

- The response of a derivative of Gaussian filter to a perfect step edge decreases as σ increases



Source: S. Lazebnik

Scale normalization

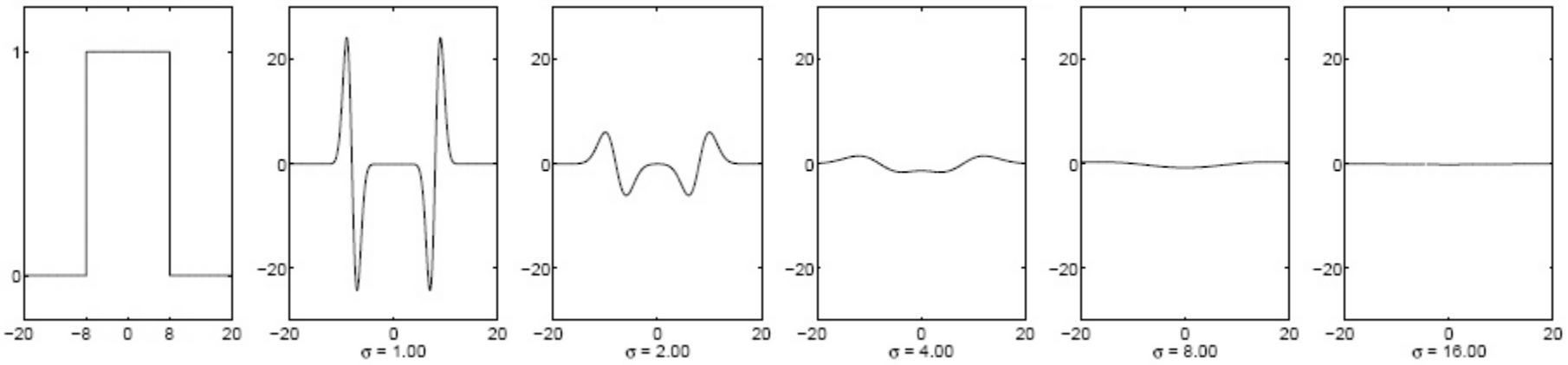
- The response of a derivative of Gaussian filter to a perfect step edge decreases as σ increases
- To keep response the same (scale-invariant), must multiply Gaussian derivative by σ
- Laplacian is the second Gaussian derivative, so it must be multiplied by σ^2

Source: S. Lazebnik

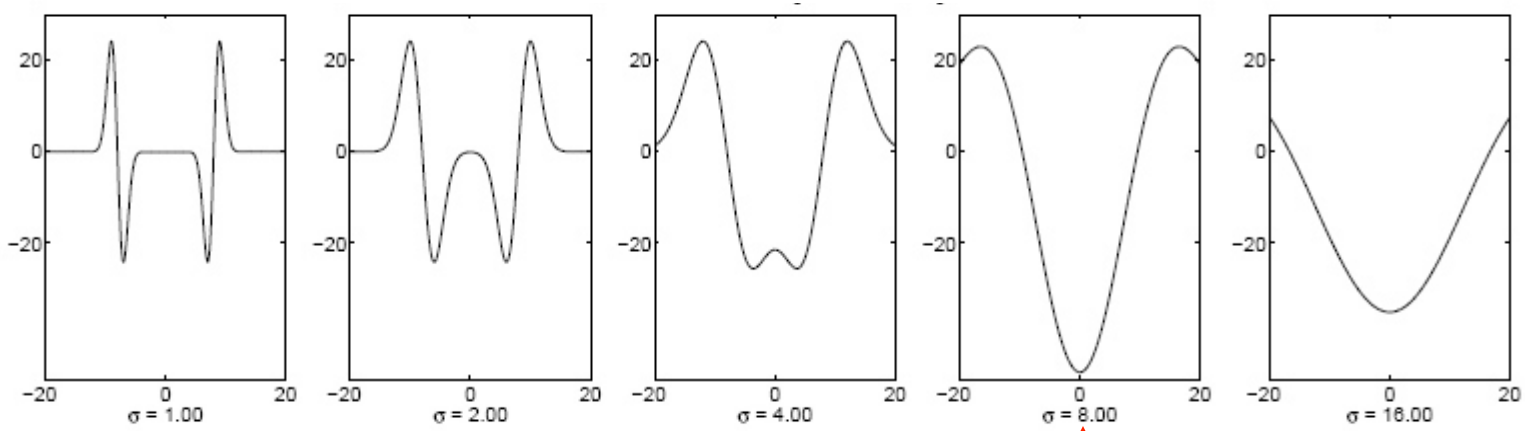
Effect of scale normalization

Original signal

Unnormalized Laplacian response



Scale-normalized Laplacian response



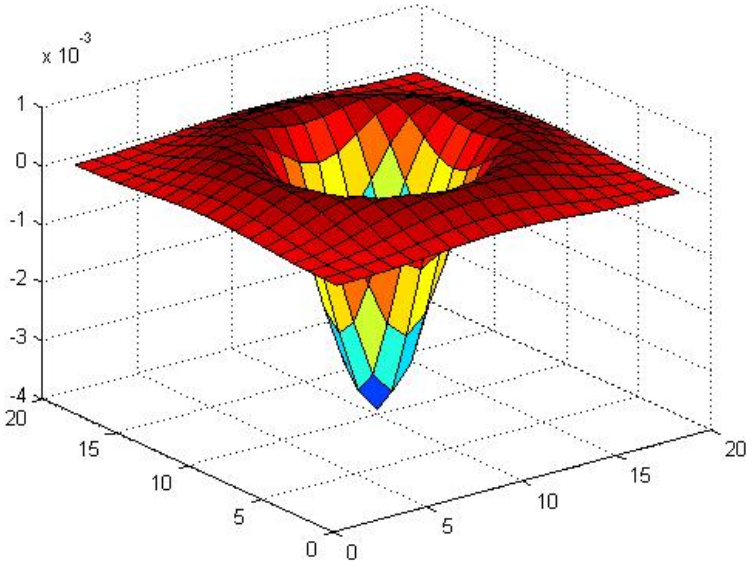
maximum

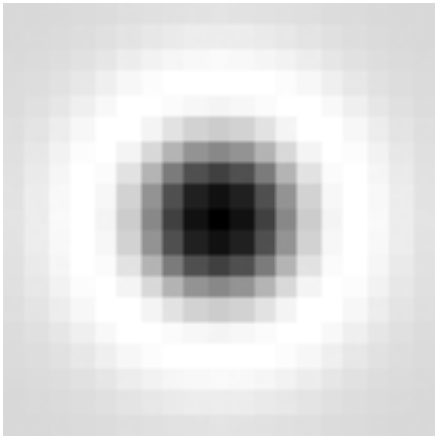
Source: S. Lazebnik

Blob detection in 2D

- *Scale-normalized* Laplacian of Gaussian:

$$\nabla^2_{\text{norm}} g = \sigma^2 \left(\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} \right)$$

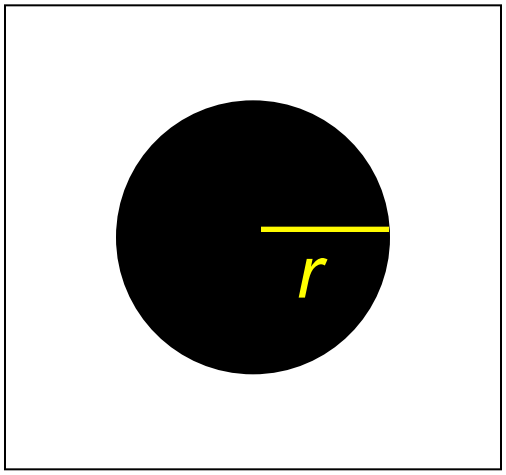




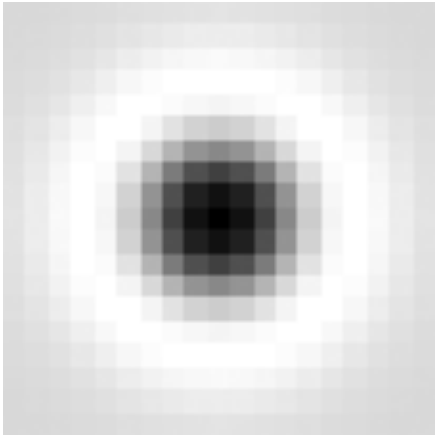
Source: S. Lazebnik

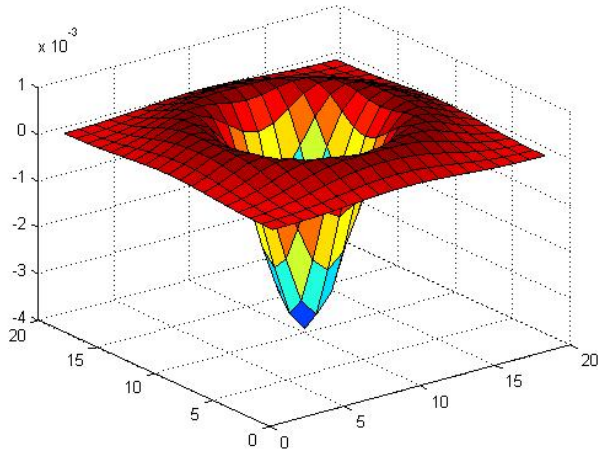
Blob detection in 2D

- At what scale does the Laplacian achieve a maximum response to a binary circle of radius r?



image





Laplacian

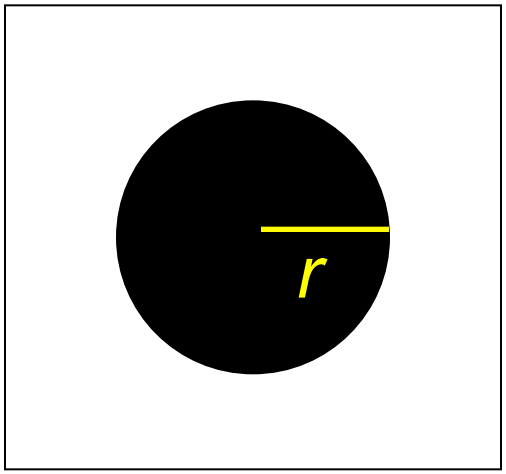
Source: S. Lazebnik

Blob detection in 2D

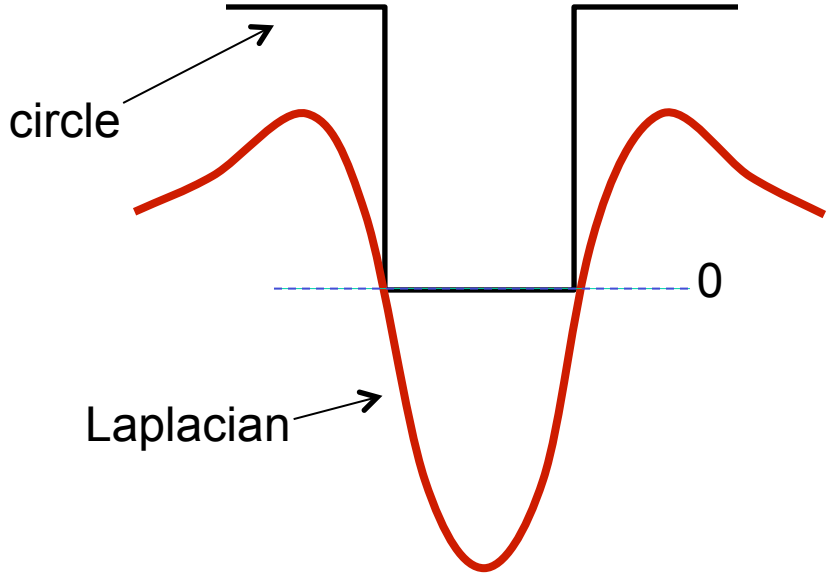
- At what scale does the Laplacian achieve a maximum response to a binary circle of radius r?
- To get maximum response, the zeros of the Laplacian have to be aligned with the circle
- The Laplacian is given by (up to scale):

$$(x^2 + y^2 - 2\sigma^2)\, e^{-(x^2+y^2)/2\sigma^2}$$

- Therefore, the maximum response occurs at $\sigma = r/\sqrt{2}$.



image



Source: S. Lazebnik

Scale-space blob detector

1. Convolve image with scale-normalized Laplacian at several scales

Scale-space blob detector: Example

Source: S. Lazebnik

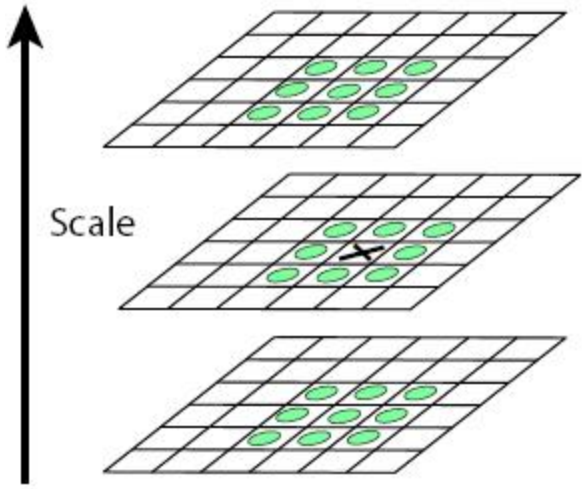
Scale-space blob detector: Example

sigma = 11.9912

Source: S. Lazebnik

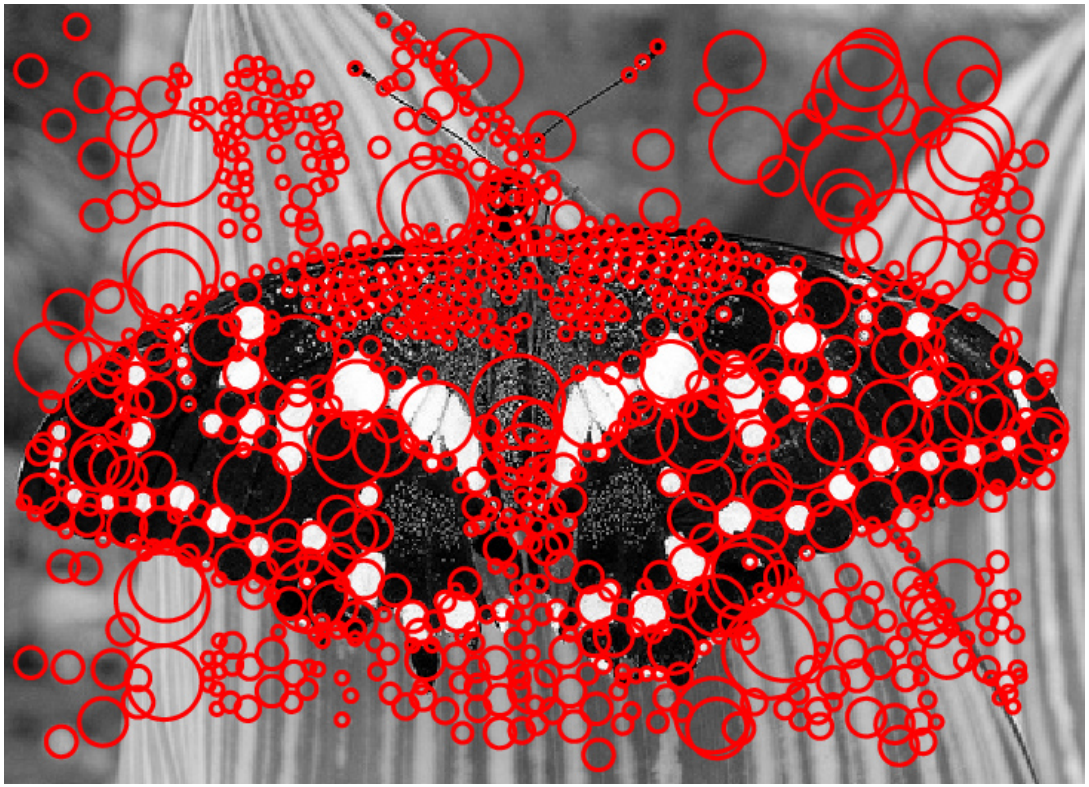
Scale-space blob detector

1. Convolve image with scale-normalized Laplacian at several scales
2. Find maxima of squared Laplacian response in scale-space



Source: S. Lazebnik

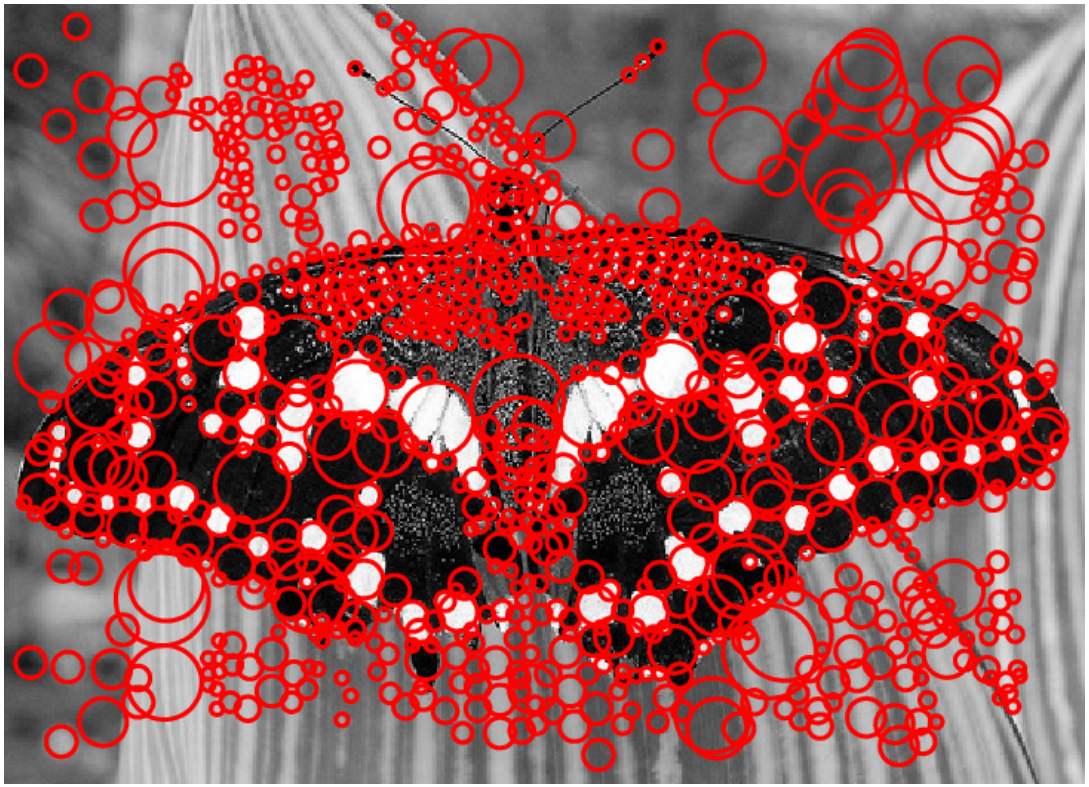
Scale-space blob detector: Example



Source: S. Lazebnik

Eliminating edge responses

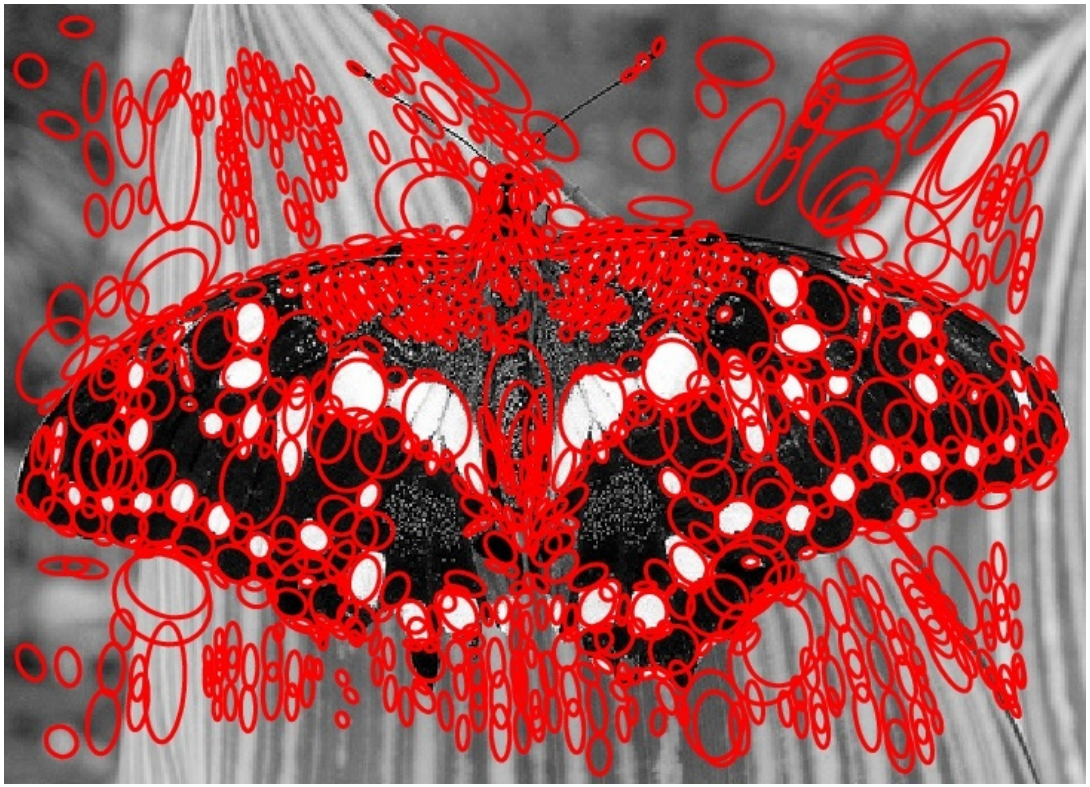
- Laplacian has strong response along edge



Source: S. Lazebnik

Eliminating edge responses

- Laplacian has strong response along edge



- Solution: filter based on Harris response function over neighboroods containing the “blobs”

Source: S. Lazebnik

Efficient implementation

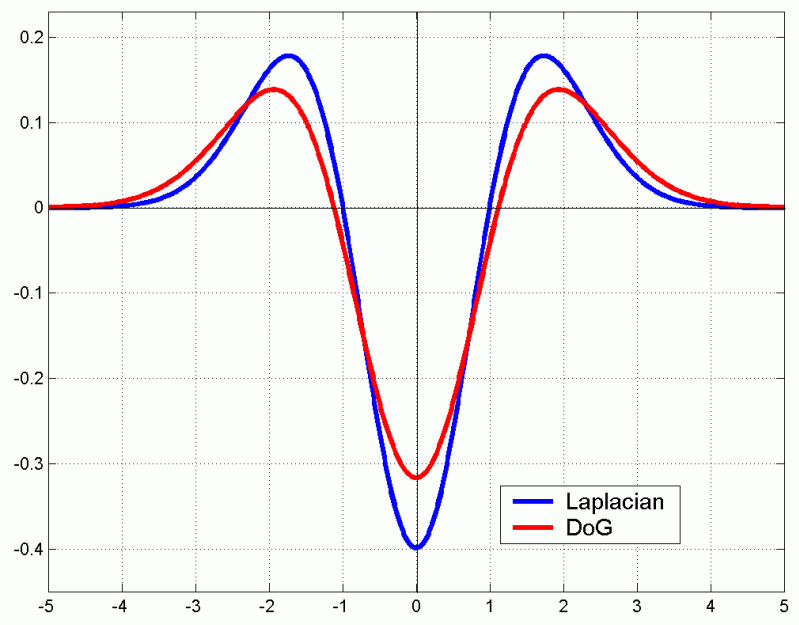
- Approximating the Laplacian with a difference of Gaussians:

$$L = \sigma^2 \left(G_{xx}(x, y, \sigma) + G_{yy}(x, y, \sigma) \right)$$

(Laplacian)

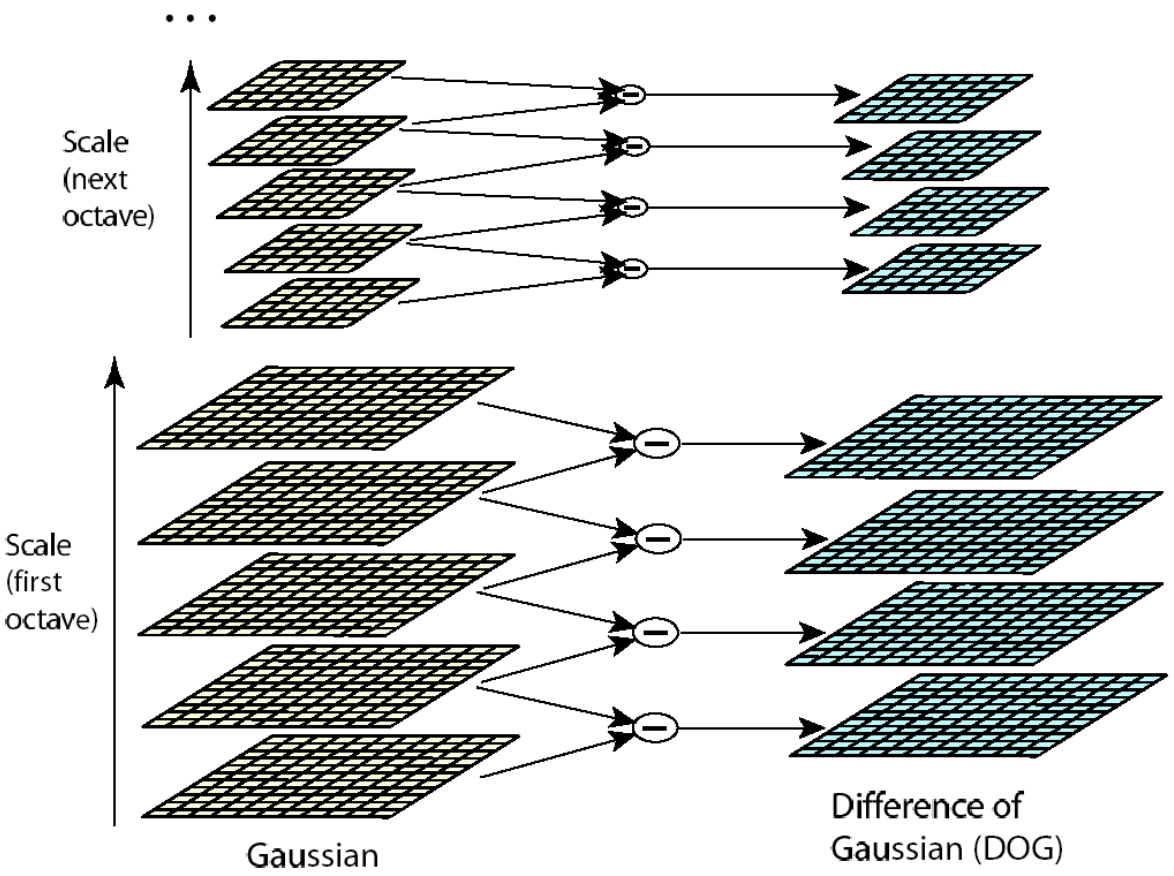
$$DoG = G(x, y, k\sigma) - G(x, y, \sigma)$$

(Difference of Gaussians)



Source: S. Lazebnik

Efficient implementation

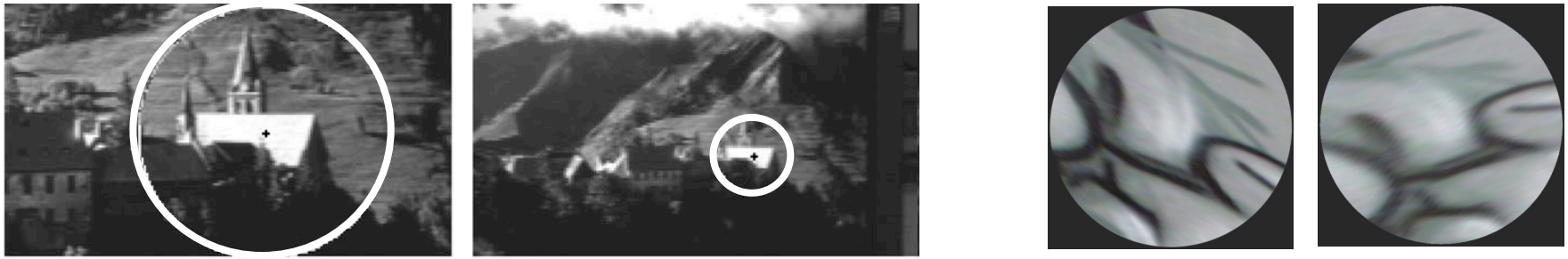


David G. Lowe.
"Distinctive image features from scale-invariant keypoints." *IJCV* 60 (2), pp. 91-110, 2004.

Source: S. Lazebnik

From feature detection to feature description

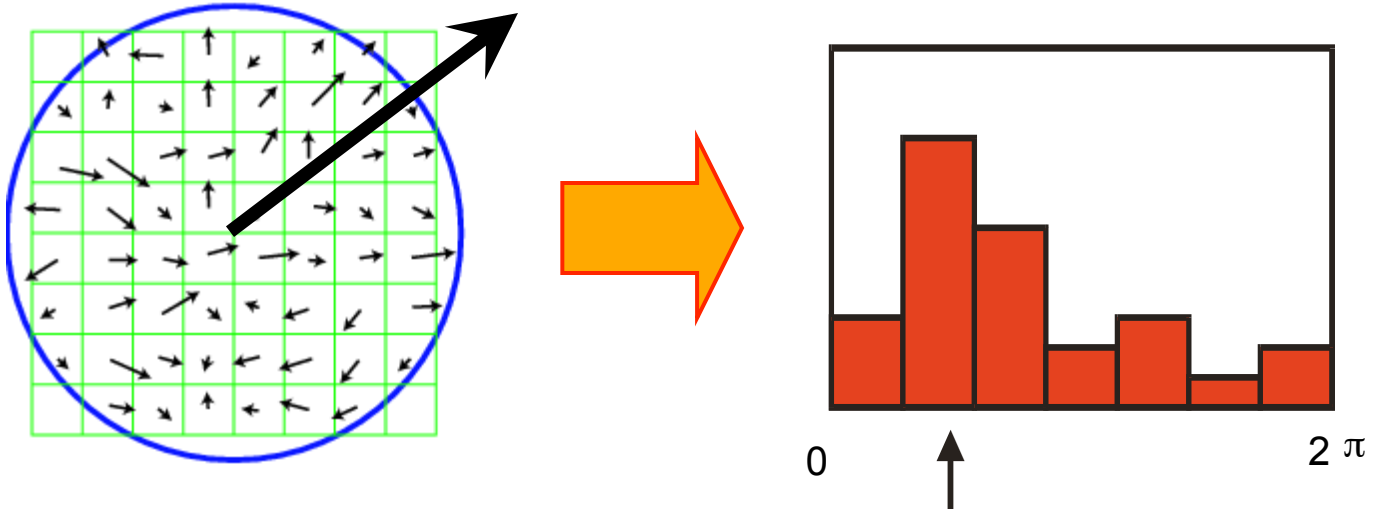
- Scaled and rotated versions of the same neighborhood will give rise to blobs that are related by the same transformation
- What to do if we want to compare the appearance of these image regions?
  - *Normalization*: transform these regions into same-size circles
  - Problem: rotational ambiguity



Source: S. Lazebnik

Eliminating rotation ambiguity

- To assign a unique orientation to circular image windows:
  - Create histogram of local gradient directions in the patch
  - Assign canonical orientation at peak of smoothed histogram



Source: S. Lazebnik

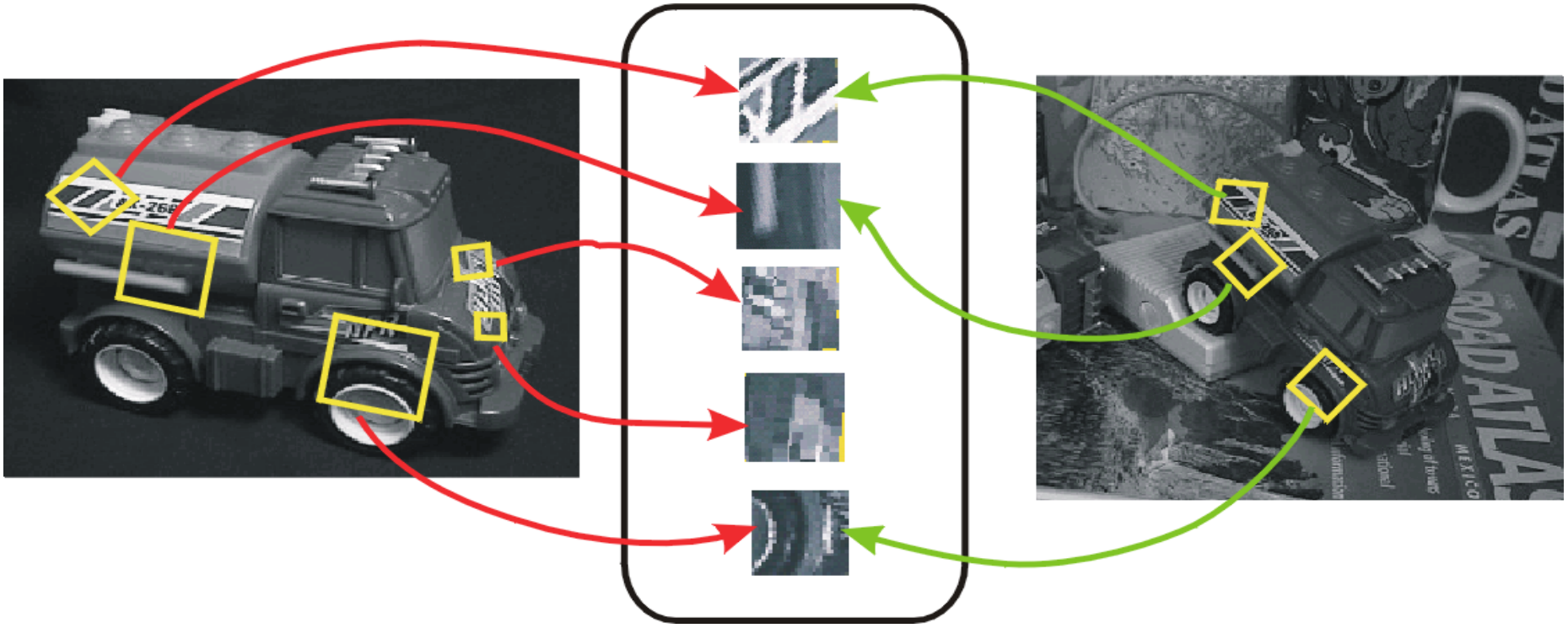
SIFT features

- Detected features with characteristic scales and orientations:

David G. Lowe.
"Distinctive image features from scale-invariant keypoints." *IJCV* 60 (2), pp. 91-110, 2004.

Source: S. Lazebnik

From feature detection to feature description



Detection is *covariant*:

features(transform(image)) = transform(features(image))

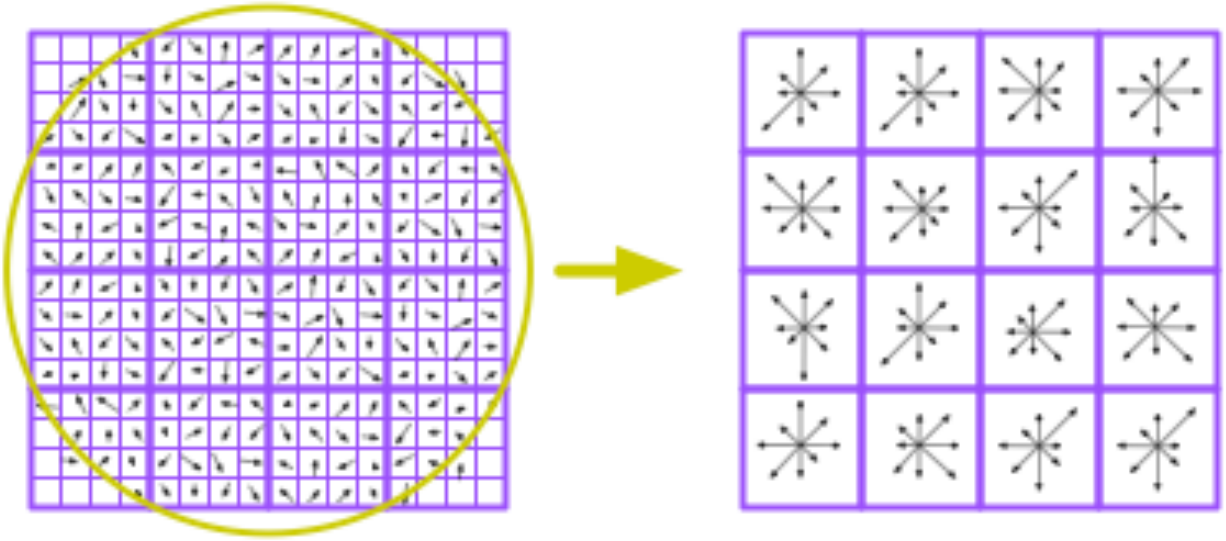
Description is *invariant*:

features(transform(image)) = features(image)

Source: S. Lazebnik

SIFT descriptors

- Inspiration: complex neurons in the primary visual cortex



D. Lowe. **Distinctive image features from scale-invariant keypoints.** *IJCV* 60 (2), pp. 91-110, 2004.

Source: S. Lazebnik

SIFT descriptor computation

- use the normalized region about the keypoint
- compute gradient magnitude and orientation at each point in the region
- weight them by a Gaussian window overlaid on the circle
- create an orientation histogram over the 4 X 4 subregions of the window
- 4 X 4 descriptors over 16 X 16 sample array were used in practice. 4 X 4 times 8 directions gives a vector of 128 values.

Summary of SIFT feature detection and description

1. Scale-space extrema detection

   Search over multiple scales and image locations.

2. Keypoint localization

   Fit a model to detrmine location and scale.
   Select keypoints based on a measure of stability.

3. Orientation assignment

   Compute best orientation(s) for each keypoint region.

4. Keypoint description

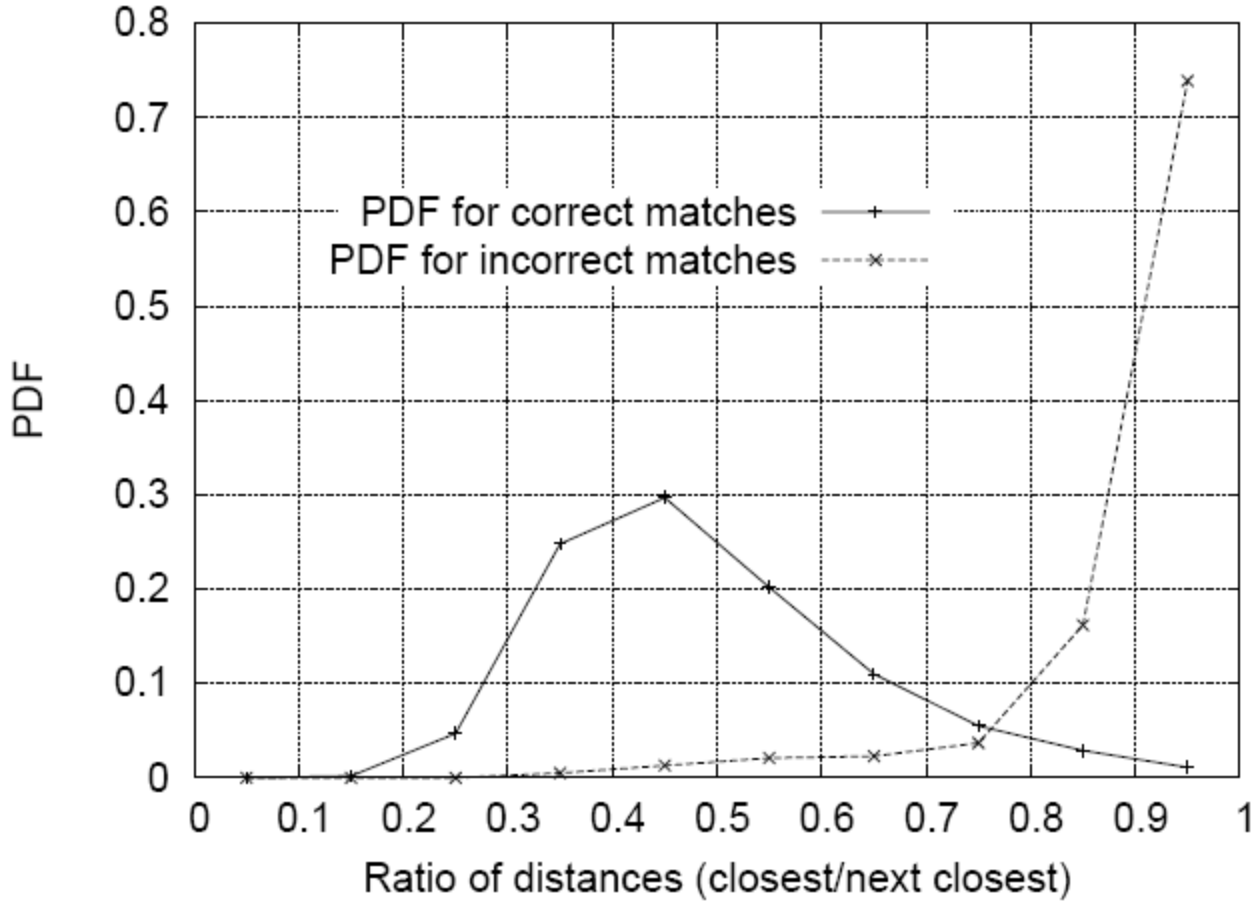
   Use local image gradients at selected scale and rotation to describe each keypoint region.

Source: D. Hoiem

Matching SIFT Descriptors

Nearest neighbor (Euclidean distance)

Threshold ratio of nearest to 2nd nearest descriptor



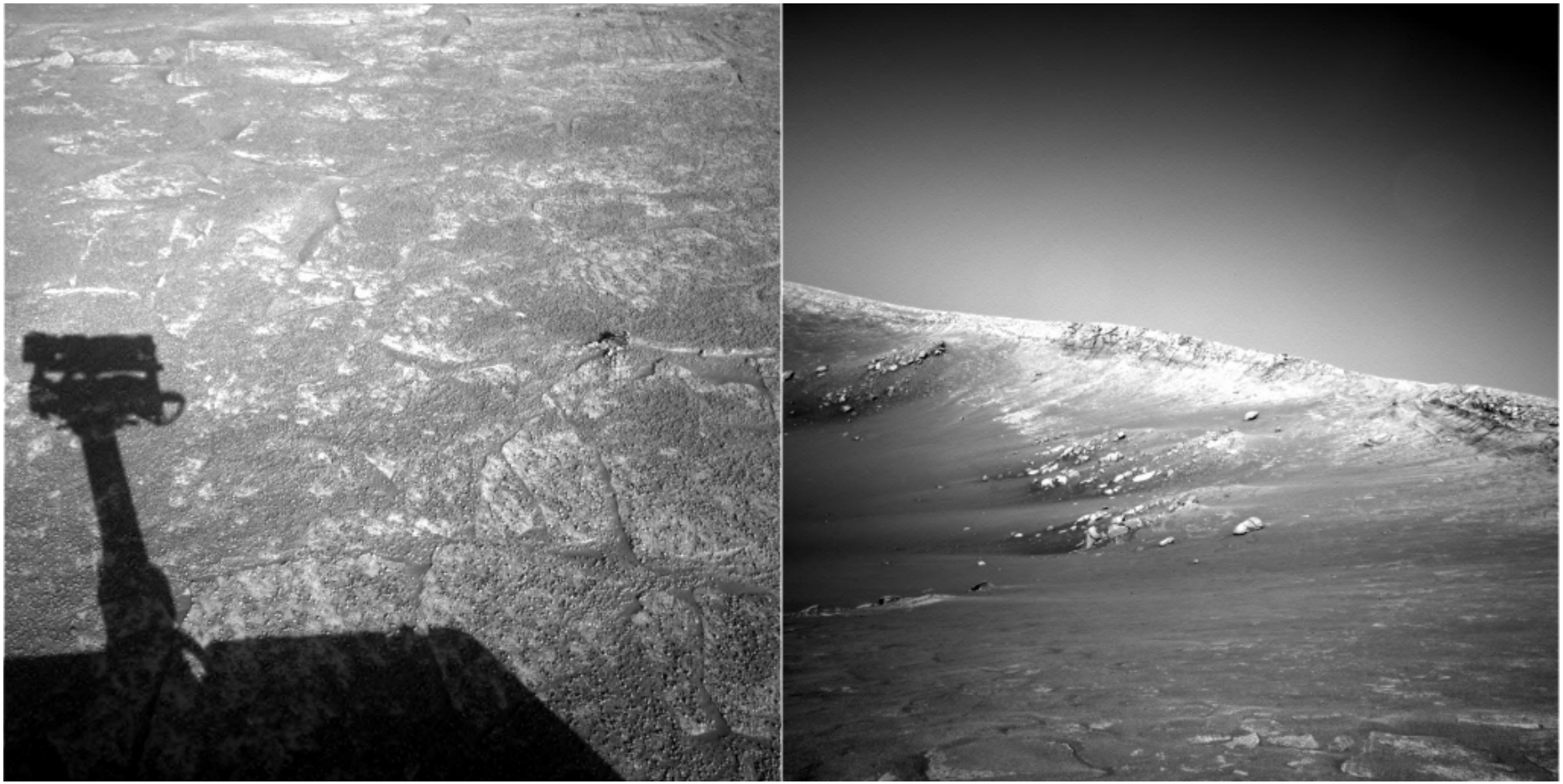
Lowe IJCV 2004

Properties of SIFT

Extraordinarily robust detection and description technique

- Can handle changes in viewpoint
  - Up to about 60 degree out-of-plane rotation
- Can handle significant changes in illumination
  - Sometimes even day vs. night
- Fast and efficient—can run in real time
- Lots of code available

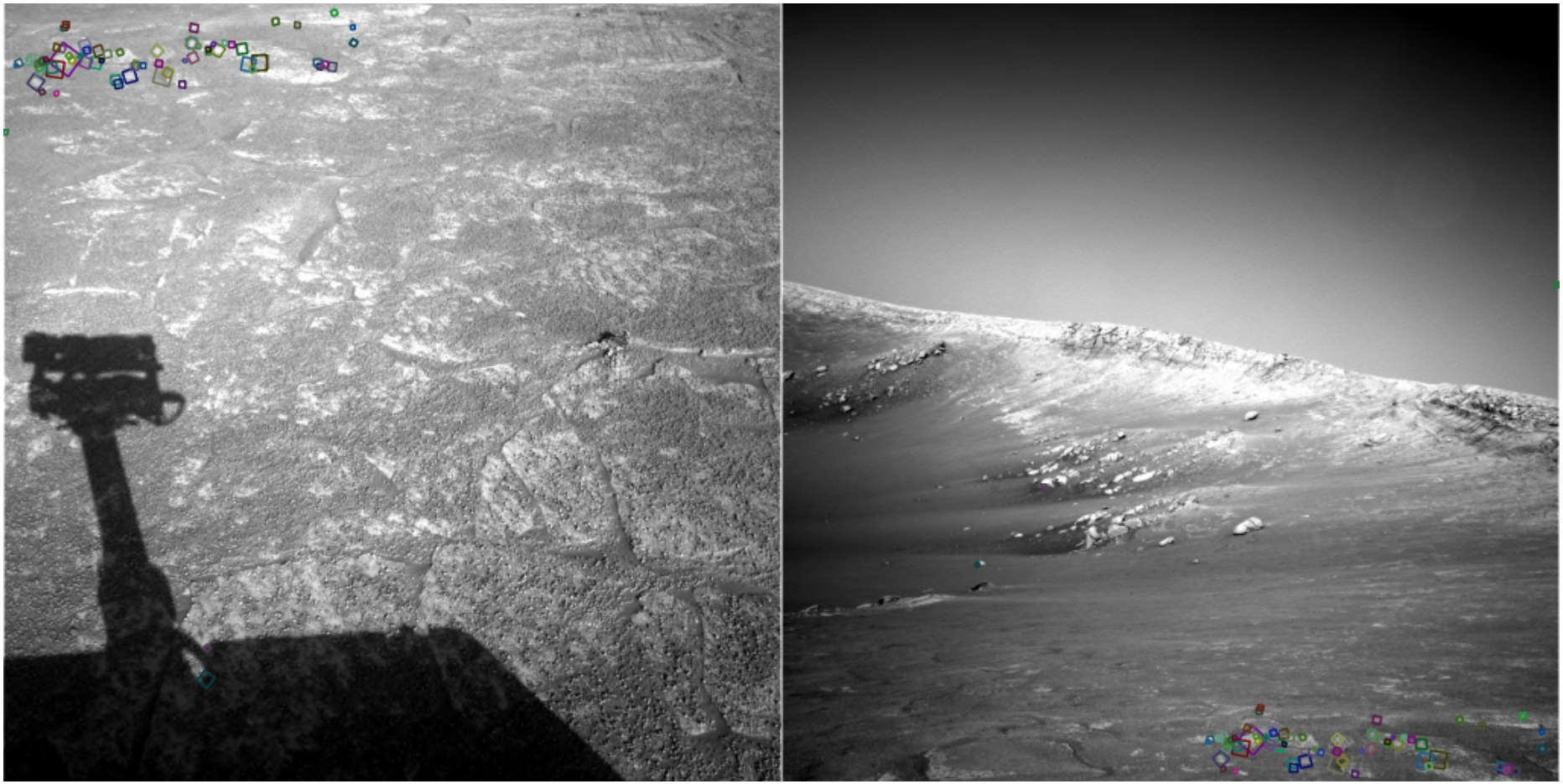
A hard keypoint matching problem



NASA Mars Rover images

Source: S. Lazebnik

Answer below (look for tiny colored squares…)



NASA Mars Rover images
with SIFT feature matches
Figure by Noah Snavely

Source: S. Lazebnik

Thank you

Next lecture: Model estimation