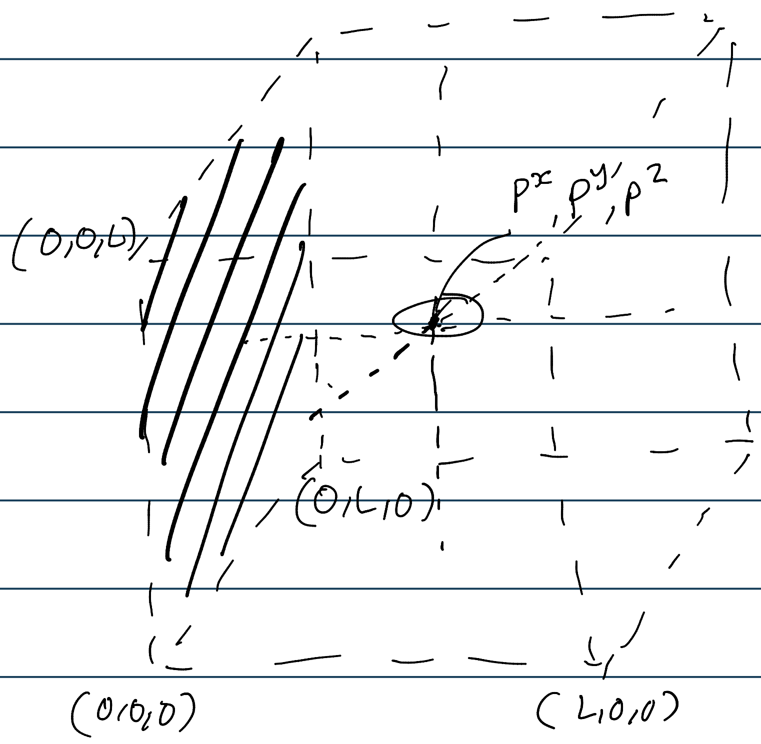
Session 1

Tuesday 15. September 2020 11.59

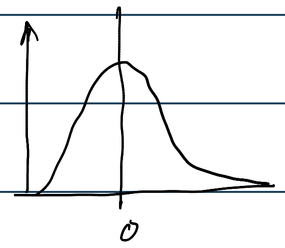
1.



a)

$$\underbrace{\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{pmatrix}}_{y} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}}_{G} \underbrace{\begin{pmatrix} p^x \\ p^y \\ p^z \end{pmatrix}}_{x} + \underbrace{\begin{pmatrix} 0 \\ 0 \\ 0 \\ L \\ L \\ L \end{pmatrix}}_{b} + \underbrace{\begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{pmatrix}}_{r}$$

$$r_i \sim \mathcal{N}(0, \sigma_i^2)$$



b). $y = G x + b + r$

6 equation

3 unknown.

$G \in \mathbb{R}^{6\times3}$

$\text{rank}(G) = 3$

c). $y = Gx + b + r$; $r = y - (Gx + b)$

, $r \in \mathbb{R}^{6\times1}$

$r^T r$, $\sum |r_i|$ $r^T r \leftarrow$

$$\begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix} = G'$$

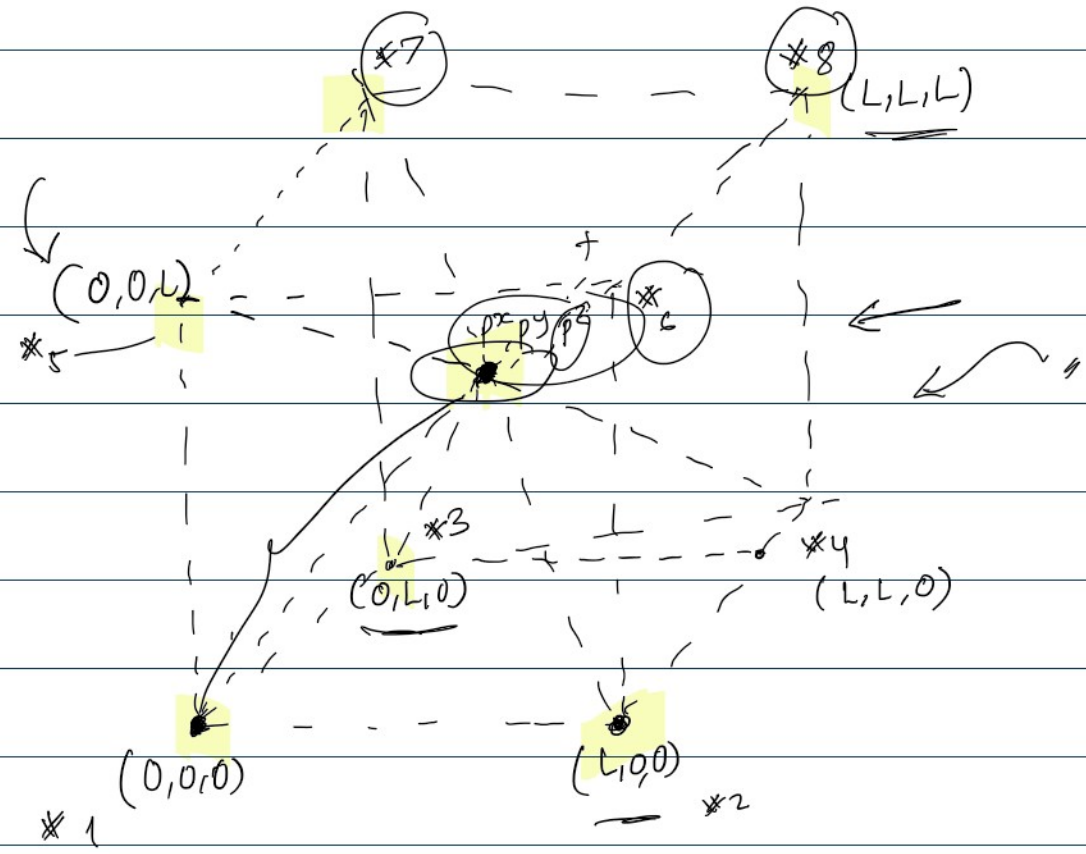
$\text{rank}(G) = 3$

$$\begin{pmatrix} y_1 \\ y_4 \\ y_6 \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}}_{\text{rank}(G) < 3} x + \begin{pmatrix} 0 \\ L \\ L \end{pmatrix} + \begin{pmatrix} r_1 \\ r_4 \\ r_6 \end{pmatrix}$$

$= 3$

$$r_i \sim \mathcal{N}(\bar{r}_1, \sigma_1^2)$$

Exercise 2.



a).

$$y_1 = \sqrt{p_x^2 + p_y^2 + p_z^2} + r_1$$

$$y_2 = \sqrt{(p_x - L)^2 + p_y^2 + p_z^2} + r_2$$

$\vdots$

$$y_i = \sqrt{(p_x - s_x)^2 + (p_y - s_y)^2 + (p_z - s_z)^2} + r_i$$

$(s_x - p_x)^2$

$\vdots$

$(s_x, s_y, s_z) =$

$$y_8 = \sqrt{} + r_8$$

b) $= \begin{pmatrix} y_1 \\ \vdots \\ \vdots \\ y_8 \end{pmatrix} = \underbrace{\begin{pmatrix} \sqrt{P_x^2 + P_y^2 + P_z^2} \\ \\ \\ \sqrt{(P_x-L)^2 + (P_y-L)^2 + (P_z-L)^2} \end{pmatrix}}_{g(x)} + \underbrace{\begin{pmatrix} r_1 \\ \vdots \\ \vdots \\ r_8 \end{pmatrix}}_{r}$

c. y_1, y_2, y_3 .

$$(y_1 - r_1)^2 = P_x^2 + P_y^2 + P_z^2 \qquad (1)$$

$$(y_2 - r_2)^2 = (P_x - L)^2 + P_y^2 + P_z^2 \qquad (2)$$

$$(y_3 - r_3)^2 \quad P_x^2 + (P_y - L)^2 + P_z^2 \qquad (3)$$

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assume all $r_i = 0$

(1) $$P_z^2 = (y_1^2 - (P_x^2 + P_y^2))$$

$$P_z = \pm\sqrt{(\qquad)}$$

(2) $$y_2^2 = (P_x - L)^2 + \cancel{P_y^2} + (y_1^2 - (P_x^2 + \cancel{P_y^2}))$$

$$= -2P_xL + L^2 + y_1^2$$

$$P_x^* = \frac{1}{2L}(L^2 + y_1^2 - y_2^2)$$

(3) $$y_3^2 = P_x^2 + (P_y - L)^2 + P_z^2$$

$$= \cancel{P_x^2} + (P_y - L)^2 + (y_1^2 - (\cancel{P_x^2} + P_y^2))$$

$$= -2P_yL + L^2 + y_1^2$$

$$P_y^* = \frac{1}{2L}(L^2 + y_1^2 - y_3^2)$$

(4) $$P_z^* = \frac{1}{2L}(L^2 + y_1^2 - y_5)$$

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3. (1.17) & (1.18) , (1.16)

$$\mathbf{x} = [p^x \quad p^y \quad v^x \quad v^y]$$

the course book

$$= [x_1 \quad x_2 \quad x_3 \quad x_4]$$

$$x_{1,n} = p^x(t_n)$$

(1.16)

$$\begin{bmatrix} p^x(t_n) \\ p^y(t_n) \\ v^x(t_n) \\ v^y(t_n) \end{bmatrix} = \begin{bmatrix} p^x(t_{n-1}) + v^x(\\ \\ \\ \end{bmatrix}$$

$$x_n = [p^x_n \quad p^y_n \quad p^z_n \quad v^x_n \quad v^y_n \quad v^z_n]$$

$$x_{1,n} \quad x_{2,n} \quad x_{3,n} \quad x_{4,n} \quad x_{5,n} \quad x_{6,n}$$

$$\underbrace{\begin{bmatrix} x_{1,n} \\ \vdots \\ \vdots \\ x_{6,n} \end{bmatrix}}_{\mathbf{x}_n} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{\mathbf{F}} \underbrace{\begin{bmatrix} x_{1,n-1} \\ \vdots \\ \vdots \\ x_{6,n-1} \end{bmatrix}}_{\mathbf{x}_{n-1}} + \begin{bmatrix} q_{1,n} \\ \vdots \\ \vdots \\ q_{6,n} \end{bmatrix}$$

$$\mathbf{q} \sim N(\mathbf{0}, \mathbf{Q})$$