

## Exercise and Homework Round 2

These exercises (except for the last) will be gone through on **Tuesday September 22nd 12:15–14:00** in the exercise session. The last exercise is a homework which you should return via MyCourses by **Tuesday September 29th at 12:00**.

### Exercise 1. (1D Gaussian distribution)

Recall that the expected value of a random variable  $x$  with probability density function  $p(x)$  is

$$\mathbb{E}\{x\} = \int_{-\infty}^{\infty} x p(x) dx \quad (1)$$

and the variance is

$$\text{var}\{x\} = \int_{-\infty}^{\infty} (x - \mathbb{E}\{x\})^2 p(x) dx. \quad (2)$$

Furthermore, recall that for any probability density we have  $\int_{-\infty}^{\infty} p(x) dx = 1$ .

(a) Show that for any random variable we can also express the variance as

$$\text{var}\{x\} = \mathbb{E}\{x^2\} - (\mathbb{E}\{x\})^2. \quad (3)$$

(b) Compute the mean of the Gaussian distribution by brute-force integration:

$$p(x) = \mathcal{N}(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2\sigma^2}(x - \mu)^2\right). \quad (4)$$

(c) Compute the variance of the Gaussian distribution by brute-force integration.

### Exercise 2. (Multivariate mean and covariance)

Show that if  $\mathbf{z} = \mathbf{L} \mathbf{x}$ ,  $\mathbb{E}\{\mathbf{z}\} = \mathbf{m}$ , and  $\text{Cov}\{\mathbf{x}\} = \mathbf{P}$ , then  $\mathbb{E}\{\mathbf{z}\} = \mathbf{L} \mathbf{m}$  and  $\text{Cov}\{\mathbf{z}\} = \mathbf{L} \mathbf{P} \mathbf{L}^T$ .

### Exercise 3. (Weighted vs. non-weighted cost functions)

Recall that  $\sigma_n^2 = 1$  corresponds non-weighted least squares.

(a) Show that if we have a weighed least squares problem corresponding to

$$y_n = g(x_n) + r_n, \quad \text{var}\{r_n\} = \sigma_n^2, \quad (5)$$

then if we define

$$\begin{aligned} \hat{y}_n &= \frac{y_n}{\sigma_n}, \\ \hat{g}(x_n) &= \frac{g(x_n)}{\sigma_n}, \\ \hat{r}_n &= \frac{r_n}{\sigma_n}, \end{aligned} \quad (6)$$

this transforms the problem to a non-weighted least squares problem

$$\hat{y}_n = \hat{g}(x_n) + \hat{r}_n, \quad \text{var}\{\hat{r}_n\} = 1. \quad (7)$$

(b) Show that the cost functions for this non-weighted and the original weighted problems are equivalent.

### Homework 2 (DL Tuesday September 29th at 12:00)

Consider a 2D Gaussian distribution with the probability density function

$$p(x_1, x_2) = \frac{1}{2\pi \sigma_1 \sigma_2} \exp \left( -\frac{1}{2\sigma_1^2}(x_1 - \mu_1)^2 - \frac{1}{2\sigma_2^2}(x_2 - \mu_2)^2 \right). \quad (8)$$

Derive the mean and covariance of this distribution by brute-force integration.