Basics of Sensor Fusion

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September 2020

1 Homework 2

Given: $p(x_1, x_2) = \frac{1}{2\pi\sigma_1\sigma_2} exp(-\frac{1}{2\sigma_1^2}(x_1 - \mu_1)^2 - \frac{1}{2\sigma_2^2}(x_2 - \mu_2)^2)$

1.1 Derive, mean by brute force.

Firstly we can compute the marginal probability for each x_i individually.

$$p(x_i) = \int_{-\infty}^{\infty} p(x_i, x_j) dx_j = \tag{1}$$

$$= \int_{-\infty}^{\infty} \frac{1}{2\pi\sigma_i\sigma_j} exp(-\frac{1}{2\sigma_i^2}(x_i - \mu_i)^2 - \frac{1}{2\sigma_j^2}(x_j - \mu_j)^2) dx_j = \tag{2}$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_i} \frac{1}{\sqrt{2\pi}\sigma_j} exp(\frac{1}{2\sigma_i^2}(x_i - \mu_i)^2) exp(\frac{1}{2\sigma_j^2}(x_j - \mu_j)^2) dx_j = \tag{3}$$

$$= \frac{1}{\sqrt{2\pi}\sigma_i} exp(\frac{1}{2\sigma_i^2}(x_i - \mu_i)^2) \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_j} exp(\frac{1}{2\sigma_j^2}(x_j - \mu_j)^2) dx_j = \tag{4}$$

$$= \frac{1}{\sqrt{2\pi}\sigma_i} exp(\frac{1}{2\sigma_i^2}(x_i - \mu_i)^2) \tag{5}$$

From the formula above, we can compute expectation of x_i

$$E\{x_i\} = \int_{-\infty}^{\infty} x_i p(x_i) dx_i = \int_{-\infty}^{\infty} x_i \frac{1}{\sqrt{2\pi}\sigma_i} exp(-\frac{1}{2\sigma_i^2}(x_i - \mu_i)^2) dx_i \tag{6}$$

Similar to exercise sessions, we define $x_i = \sigma_i * z + \mu_i$
Hence,

$$E\{x_i\} = \int_{-\infty}^{\infty} (\sigma_i z + \mu_i) \frac{1}{\sqrt{2\pi}\sigma_i} exp(-\frac{1}{2\sigma_i^2}(\sigma_i * z + \mu_i - \mu_i) dx_i = \tag{7}$$

$$= \int_{-\infty}^{\infty} (\sigma_i z + \mu_i) \frac{1}{\sqrt{2\pi}} exp(-\frac{z^2}{2}) dx_i = \quad (8)$$

$$= \int_{-\infty}^{\infty} \sigma_i z \frac{1}{\sqrt{2\pi}} exp(-\frac{z^2}{2}) dx_i + \int_{-\infty}^{\infty} \mu_i \frac{1}{\sqrt{2\pi}} exp(-\frac{z^2}{2}) dx_i = \quad (9)$$

$$= \mu_i + [-\frac{\sigma_i}{\sqrt{2\pi}} exp(-\frac{z^2}{2})]_{-\infty}^{\infty} = \mu_i \quad (10)$$

So, we have that $E\{x_i\} = \mu_i$, $\forall$ i,j, $i \neq j$

1.2 Derive covariance by brute force.

$$cov(x_i, x_j) = E[(x_i - E[x_i])(x_j - E[x_j])] = \quad (11)$$

$$E[(x_i - \mu_i)(x_j - \mu_j)] = E[x_i x_j - x_i \mu_j - x_{j \mu i} + \mu_i \mu_j] = \quad (12)$$

$$= E[x_i x_j] - \mu_i E[x_j] - \mu_j E[x_i] + \mu_i \mu_j = \quad (13)$$

$$= E[x_i x_j] - \mu_i \mu_j = \quad (14)$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x_i x_j \frac{1}{2\pi \sigma_i \sigma_j} exp(-\frac{1}{2\sigma_i^2}(x_i - \mu_i)^2 - \frac{1}{2\sigma_j^2}(x_j - \mu_j)^2) dx_i dx_j - \mu_i \mu_j = \quad (15)$$

$$\int_{-\infty}^{\infty} x_i \frac{1}{\sqrt{2\pi}\sigma_i} exp(-\frac{1}{2\sigma_i^2}(x_i - \mu_i)^2) dx_i \int_{-\infty}^{\infty} x_j \frac{1}{\sqrt{2\pi}\sigma_j} exp(-\frac{1}{2\sigma_j^2}(x_j - \mu_j)^2) dx_j - \mu_i \mu_j = \quad (16)$$

$$E[x_i]E[x_j] - \mu_i \mu_j = \mu_i \mu_j - \mu_i \mu_j = 0 \quad (17)$$

Next section we will compute $cov(x_i, x_i)$
Similarly define, $x_i = \sigma_i z +_i$

$$E[x_i x_i] = \int_{-\infty}^{\infty} x_i x_i p(x_i) dx_i = \int_{-\infty}^{\infty} x_i^2 p(x_i) dx_i = \quad (18)$$

$$= \int_{-\infty}^{\infty} x_i^2 \frac{1}{\sqrt{2\pi}\sigma_i} exp(-\frac{1}{2\sigma_i^2}(x_i - \mu_i)^2) dx_i = \quad (19)$$

$$= \int_{-\infty}^{\infty} (z^2 \sigma_i^2 + 2z\mu_i \sigma_i + \mu_i^2) \frac{1}{\sqrt{2\pi}} exp(-\frac{z^2}{2}) dz \quad (20)$$

$$= \int_{-\infty}^{\infty} (\sigma_i z)^2 \frac{1}{\sqrt{2\pi}} exp(-\frac{z^2}{2}) dz + \int_{-\infty}^{\infty} 2z\mu_i\sigma_i \frac{1}{\sqrt{2\pi}} exp(-\frac{z^2}{2}) dz + \int_{-\infty}^{\infty} \mu_i^2 \frac{1}{\sqrt{2\pi}} exp(-\frac{z^2}{2}) dz = \tag{21}$$

$$= \sigma_i^2 \overbrace{\int_{-\infty}^{\infty} z^2 \frac{1}{\sqrt{2\pi}} exp(-\frac{z^2}{2}) dx}^{=1} + \overbrace{2\mu_i\sigma_i \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z * exp(-\frac{z^2}{2}) dz}^{=0} + \mu_i^2 \overbrace{\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} exp(-\frac{z^2}{2}) dz}^{=1} = \tag{22}$$

Therefore:

$$E[x_i, x_i] = \sigma_i^2 + \mu_i^2 - \mu_i^2 = \sigma_i^2$$

For i=1,2 we have $cov = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix}$