SensorFusion3

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1 Homework 3

1.1 a)

Compute the least squares estimators for a and b in

$$y_n = ax_n + b + r_n$$

we have $J(a,b) = \sum_{n=1}^{N}(y_n - ax_n - b)^2$
Firstly we take the derivative with respect to a and b

$$\frac{\partial J}{\partial a} = \sum -2x_n(y_n - ax_n - b) = 0$$
$$\frac{\partial J}{\partial b} = \sum -2(y_n - ax_n - b) = 0$$

we have that

$$\frac{\partial J}{\partial a} = -\sum x_n y_n + \sum ax_n^2 + \sum bx_n = 0$$

and for b

$$\frac{\partial J}{\partial b} = -\sum y_n + \sum b + \sum ax_n = 0$$

from above equations results:

$$\sum bx_n + \sum ax_n^2 = \sum x_n y_n$$

$$\sum b + \sum ax_n = \sum y_n$$

therefore

$$\begin{bmatrix} b * \sum x_n + a * \sum x_n^2 = \sum x_n y_n \\ b * N + a * \sum x_n = \sum y_n \end{bmatrix} => \begin{bmatrix} \sum x_n^2 & \sum x_n \\ \sum x_n & N \end{bmatrix} * \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \sum x_n y_n \\ \sum y_n \end{bmatrix} \quad (1)$$

1.2 b)

Let's define $x = \begin{bmatrix} a \\ b \end{bmatrix}$

We have $J(x) = (y - Gx)^T * (y - Gx)$, where

$y = \begin{bmatrix} y_1 \\ . \\ . \\ . \\ y_n \end{bmatrix}$ and $G = \begin{bmatrix} x_1 & 1 \\ . & . \\ . & . \\ . & . \\ x_n & 1 \end{bmatrix}$

$$J = (\begin{bmatrix} y_1 \\ . \\ . \\ . \\ y_n \end{bmatrix} - \begin{bmatrix} x_1 & 1 \\ . & . \\ . & . \\ . & . \\ x_n & 1 \end{bmatrix} * \begin{bmatrix} a \\ b \end{bmatrix})^T * (\begin{bmatrix} y_1 \\ . \\ . \\ . \\ y_n \end{bmatrix} - \begin{bmatrix} x_1 & 1 \\ . & . \\ . & . \\ . & . \\ x_n & 1 \end{bmatrix} * \begin{bmatrix} a \\ b \end{bmatrix})$$

Taking derivatives with respect to a and b leads to:

$$\frac{\partial J}{\partial x} = \frac{\partial}{\partial x}(Y^T Y - Y^G x - x^T G^T Y + x^T G^T G x) = -2G^T Y + 2G^T G x = 0$$

$$x_{LS} = (G^T G)^{-1} * G^T Y$$

Results:

$$\begin{bmatrix} a \\ b \end{bmatrix} = (\begin{bmatrix} x_1 & . & . & .x_n \\ 1 & . & . & .1 \end{bmatrix} * \begin{bmatrix} x_1 & 1 \\ . & . \\ . & . \\ . & . \\ x_n & 1 \end{bmatrix})^{-1} * \begin{bmatrix} x_1 & . & . & .x_n \\ 1 & . & . & .1 \end{bmatrix} * \begin{bmatrix} y_1 \\ . \\ . \\ . \\ y_n \end{bmatrix} = \begin{bmatrix} \sum x_n^2 & \sum x_n \\ \sum x_n & N \end{bmatrix}^{-1} * \begin{bmatrix} \sum x_n y_n \\ \sum y_n \end{bmatrix}$$

Received the same result as in equation 1