Aalto University
School of Electrical Engineering

Exercise and Homework Round 10

These exercises (except for the last) will be gone through on **Tuesday November 24th 12:15–14:00** in the exercise session. The last exercise is a homework which you should return via mycourses by **Tuesday December 1st at 12:00**.

Exercise 1. (EKF and UKF for Nonlinear Wiener velocity model)

Recall the Kalman filter that we implemented in Exercise 3.1. Assume now that instead of directly measuring $p(t_k)$ with Gaussian additive noise, we measure the sine function of the position as follows:

$$y_k = \sin(p(t_k)) + r_k. \tag{1}$$

(a) Form the corresponding state-space model.

(b) Derive the Jacobian of the measurement model function needed in the EKF update. Deduce that the prediction step of the EKF reduces to that of the conventional Kalman filter.

(c) Simulate data from the model and track the state with EKF. Plot the simulated data and the EKF estimates.

(d) Repeat the above with UKF.

Exercise 2. (Robot model with direct position measurements)

Recall the Euler–Maruyama discretization of the 2D dynamic model of a robot platform in Exercise 8.3.

(a) Assume that we measure the position of the robot with additive Gaussian noise. Form a state-space model for the system.

(b) Simulate states and measurements from the model and plot them.

(c) Derive and implement EKF for the model. Plot the result.

Exercise 3. (Robot model with distance and bearing measurements)

Assume that in the robot model in the previous exercise, instead of position, we measure the distance and bearing (= direction) to a landmark in position (s^x, s^y). Simulate data from the model, derive and implement EKF for it, and visualize the results. Please note that you need to take special care that the bearing measurement prediction and measurement are in the same quadrant.

Homework 10 (DL Tuesday December 1st at 12:00)

Consider the following 1D non-linear model

$$\begin{aligned} x_k &= \tanh(x_{k-1}) + q_{k-1}, \\ y_k &= \sin(x_k) + r_k, \end{aligned} \tag{2}$$

where $x_0 \sim \mathcal{N}(0, 1)$, $q_{k-1} \sim \mathcal{N}(0, 0.1^2)$, and $r_k \sim \mathcal{N}(0, 0.1^2)$.

(a) Simulate 100 steps of states and measurements from the model. Plot the data.

(b) Derive the necessary derivatives and check that they are correct by using numerical finite differences.

(c) Implement and run an EKF for the model. Plot the results.