Exercise and Homework Round 9

These exercises (except for the last) will be gone through on **Tuesday November 17th 12:15–14:00** in the exercise session. The last exercise is a homework which you should return via mycourses by **Tuesday November 24th at 12:00**.

Exercise 1. (Kalman filter 1D Wiener velocity model)

(a) Formulate a measurement model which corresponds to observing the position part of the Wiener velocity model in Exercise 8.2 with additive Gaussian noise.

(b) Simulate states and measurements from the model and plot what they look like.

(c) Implement a Kalman filter for the model and compare the RMSE error of using raw measurements as the position part estimator and the Kalman filter RMSE. Also plot the Kalman filter results.

Exercise 2. (Sequential least squares and Kalman filter)

(a) Recall the drone model from Exercise 3.3 and how in static case, it can be solved with regularized least squares in batch and sequential forms.

(b) Write down a state space model which allows to reformulate the sequential solution above as a Kalman filtering problem. *Hint:* $\mathbf{x}_k = \mathbf{x}_{k-1}$

(c) Check that the Kalman filter exactly reproduces the sequential solution.

Exercise 3. (Kalman filter for the drone model)

(a) Form a 3D Wiener velocity model for the drone dynamics in previous exercise.

(b) Simulate data and measurements from the model and plot them.

(c) Implement a Kalman filter for it and investigate its errors compared to estimating the positions by only using measurements at each time independently (cf. previous exercise).

Homework 9 (DL Tuesday November 24th at 12:00)

Consider a 1D Gaussian random walk model

$$\begin{aligned} x_k &= x_{k-1} + q_{k-1}, \\ y_k &= x_k + r_k, \end{aligned} \tag{1}$$

where $x_0 \sim \mathcal{N}(0, 1)$, $q_{k-1} \sim \mathcal{N}(0, 1)$, and $r_k \sim \mathcal{N}(0, 1)$.

(a) Simulate state and measurements from the model for 100 time steps. Plot the data.

(b) Implement a Kalman filter for the model, and compare its state estimates (= mean) in RMSE sense to using pure measurements as estimates ($x_k \approx y_k$) for the state. Also plot the results.