In [1]:

```
import numpy as np
import matplotlib.pyplot as plt
```

Homework 8

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case a)

We have the model $\dot{x} = a * x + u$, with $x(0) = 3$ and $a = -\frac{1}{2}$, where $u = u(t)$.\ Using integrating factor e^{-at}, results:\

$\frac{d}{dt} e^{-at} x(t) = e^{-at} x(\dot{x}) - e^{-at} ax(t)$

We multiply each side with integrating factor:

$e^{-at} x(\dot{t}) - e^{-at} ax(t) = e^{-at} u(t)$ resulting in:\ $\frac{d}{dt} e^{-at} x(t) = e^{-at} u(t)$ We take the integral in both sides:

$\int_{t_{n-1}}^{t_n} \frac{d}{dt} [e^{-at} x(t)] dt = \int_{t_{n-1}}^{t_n} e^{-at} u(t) dt$ which results in:

$[e^{-at} x(t)]_{t=t_{n-1}}^{t_n} = \int_{t_{n-1}}^{t_n} e^{-at} u(t) dt$

$e^{-at_n} x(t_n) - e^{-at_{n-1}} x(t_{n-1}) = \int_{t_{n-1}}^{t_n} e^{-at} u(t) dt$

\ By multiplying with e^{at_n}, we can solve for $x(t_n)$.\

which is: \

$$x(t_n) = e^{at_n - at_{n-1}} x(t_{n-1}) + e^{at_n} \int_{t_{n-1}}^{t_n} e^{-at} u(t) dt =$$
$$x(t_n) = e^{a\Delta t} x(t_{n-1}) + \int_{t_{n-1}}^{t_n} e^{a(t_n - t)} dt * u(t)$$

since we assume u(t) constant between 2 consecutive timsteps (ZOH).

In [53]:

```
dt = .1
t = np.linspace(0.,10.,100,endpoint=True)
u = np.ones_like(t)
a = -1/2
x_init = 3
```

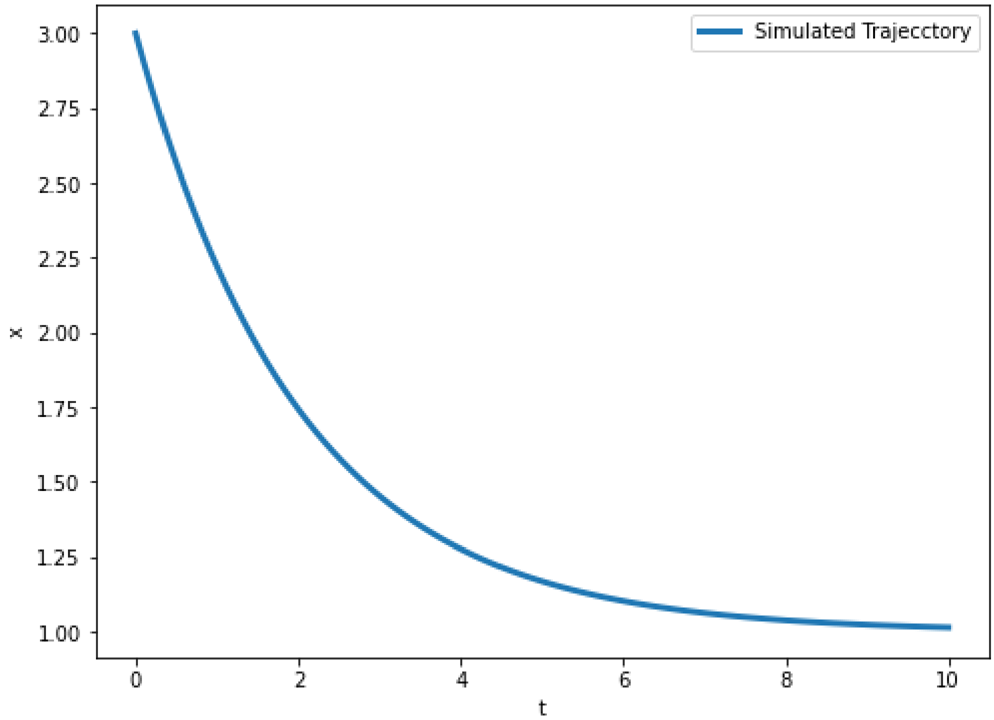
case b)

In [64]:

```
def simulate_trajectory():
    x_res = np.zeros_like(t)
    x_res[0] = x_init
    for i in range(x_res.shape[0]-1):
        x_res[i+1] = np.exp(a*dt)*x_res[i] + ((1-np.exp(a*dt))*u[i])
    return x_res

x = simulate_trajectory()

plt.figure(figsize=(8,6))
plt.plot(t,x,label='Simulated Trajecctory', linewidth=3)
plt.legend()
plt.xlabel('t')
plt.ylabel('x')
plt.show()
```



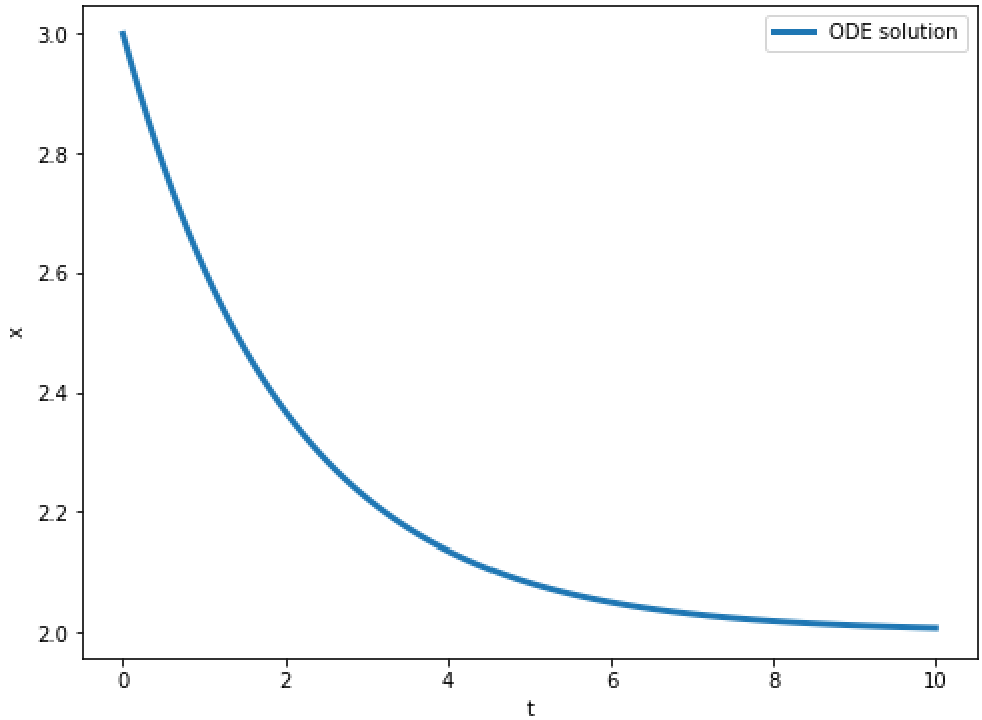
case c) with ODE

In [61]:

```
from scipy.integrate import odeint
# function that returns dx/dt
def model(x,t):
    u = 1
    dxdt = a*x + u
    return dxdt

# solve ODE
x_ode = odeint(model,x_init,t)

plt.figure(figsize=(8,6))
plt.plot(t,x_ode,label='ODE solution',linewidth=3)
plt.ylabel('x')
plt.xlabel('t')
plt.legend()
plt.show()
```



In [ ]: