Exercise 7

Monday 2. November 2020 18.54

$$\frac{dx}{dt} = \lambda x (1-x) \quad \ldots (1)$$

$$\frac{dx}{x(1-x)} = \lambda \, dt \quad \ldots (2)$$

the left and right are separate variables (x and t)
hence they can be integrated separately

$$\int \frac{dx}{x(1-x)} = \lambda t + k$$

$$\int \frac{1}{x} + \frac{1}{1-x} \, dx = \lambda t + k$$

$$\int \frac{1}{x} - \frac{1}{x-1} \, dx = \lambda t + k$$

$$\ln\left(\frac{x}{x-1}\right) = \lambda t + k$$

$$\left(\frac{x}{x-1}\right) = C e^{\lambda t} \quad ; \quad C = e^{k}$$

$$x = x C e^{\lambda t} - C e^{\lambda t}$$

$$x (1 - C e^{\lambda t}) = -C e^{\lambda t}$$

$$x_t = \frac{C e^{\lambda t}}{C e^{\lambda t} - 1}$$

now $x_0 = x(0)$

hence $x_0 = \dfrac{C}{C-1}$

$$C x_0 - x_0 = C$$

$$C(x_0 - 1) = x_0$$

$$C = x_0$$

hence if $x_0 \neq 1$

$$x_t = \frac{(x_0)/(x_0-1) \, e^{\lambda t}}{\frac{x_0}{(x_0-1)} e^{\lambda t} - 1}$$

$$= \frac{x_0 \, e^{\lambda t}}{x_0 e^{\lambda t} - (x_0 - 1)}$$

$c(x_0 - 1) = x_0$

$c = \frac{x_0}{x_0 - 1}$

$\frac{}{x_0 e^{\lambda t} - (x_0 - 1)}$

$= \frac{x_0 e^{\lambda t}}{x_0(e^{\lambda t} - 1) + 1}$

$\underline{\dot{x}} = f(\underline{x}, u) + B(\underline{x})\, w$

now x is obviously p^x, p^y, and φ

u is v and ω

and w is the noise

hence $f = \begin{bmatrix} v_t \cos(\varphi_t) \\ v_t \sin(\varphi_t) \\ \omega \end{bmatrix}$ $\quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$