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# Adversarial Attacks on the Interpretation of Neuron Activation Maximization

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## Abstract

1      The internal functional behavior of trained Deep Neural Networks is notoriously  
2      difficult to interpret. Activation-maximization approaches are one set of techniques  
3      used to interpret and analyze trained deep-learning models. These consist in finding  
4      inputs that maximally activate a given neuron or feature map. These inputs can  
5      be selected from a data set or obtained by optimization. However, interpretability  
6      methods may be subject to being deceived. In this work, we consider the concept of  
7      an adversary manipulating a model for the purpose of deceiving the interpretation.  
8      We propose an optimization framework for performing this manipulation and  
9      demonstrate a number of ways that popular activation-maximization interpretation  
10     techniques associated with CNNs can be manipulated to change the interpretations,  
11     shedding light on the reliability of these methods.

## 1      1 Introduction

12     Deep Neural Networks (DNNs) can be trained to perform many economically valuable tasks [28, 24].  
13     They are already pervasive in many sectors, and their prevalence is only expected to increase over time.  
14     With increasing computational power and ever more available amounts of data, Neural Network (NN)  
15     architectures are growing in size and executing more and more intricate tasks. Given the increasing  
16     size and complexity of DNNs, interpreting how they function, a discipline that always lags behind the  
17     cutting edge, may experience an ever harder time keeping up with new developments. However, for  
18     certain classes of critical applications, close inspection and guarantees of functionality will be more  
19     and more important, especially in heavily regulated and high-stakes domains. Here we ask: could a  
20     malicious actor conceal the true functionality of a NN from an interpretability method by modifying  
21     the NN? Given the increasing capacity of the architectures, this is likely to be a progressively more  
22     probable concern.

23     Focusing on the continuously popular feature visualization [50, 35, 34] method we propose to create  
24     an optimization procedure to manipulate the interpretation of individual neurons of the network while  
25     keeping its final behavior the same. A successful modification of the interpretation results while  
26     keeping outputs constant is evidence for the manipulability of the interpretation approach. In this  
27     work, we concentrate on convnet architectures for which interpretation by activation maximization or  
28     feature visualization methods [50, 47] has been popular. We study the feature visualization of a neuron  
29     or channel norm via activation maximization and attempt to modify it while maintaining trained  
30     network outputs and accuracy. We investigate how to characterize these attacks quantitatively and  
31     show three different attacks which can effectively manipulate and explicitly obfuscate interpretations.

32     The first proposed attack, *push-down*, aims to simply remove the current interpretation, replacing  
33     it with any other interpretation. The second attack, termed *push-up*, aims to replace the images  
34     with a specific category of images, allowing a more targeted manipulation. The final attack we  
35     consider, motivated by recent related work on feature attribution methods [1, 43], is the *fairwashing*  
36     visualization attack aimed to manipulate the perceived bias of the model as seen by an interpreter.

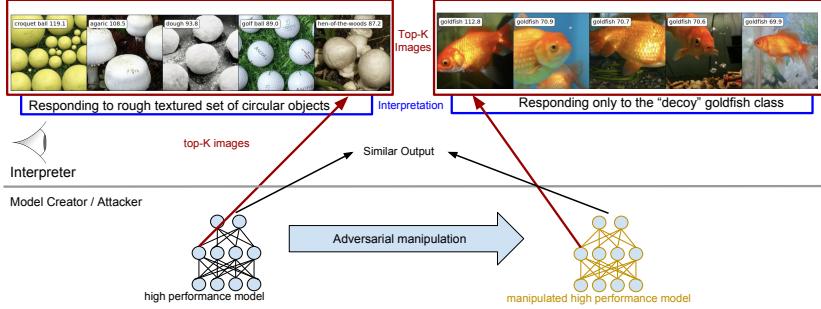


Figure 1: Illustration of the attack model for our adversarial interpretability manipulation. Top-5 images that best activate a given neuron, seemingly capturing a shared semantic concept over classes that an interpreter may describe and/or use an external tool to describe [21, 33]. In our framework, we assume the model creator can manipulate the model before it is released to the interpreter. In this case, they create a model which might lead to interpreting the selected neuron as not relating any semantic concept shared by multiple class categories.

38 Consider as motivation a situation where an adversary is indifferent to deploying a biased model, but  
 39 is constrained to provide model access to a regulator (the interpreter). Critically, *we assume that the*  
 40 *interpreter may not have access to labels related to the particular bias exploited by the adversary’s*  
 41 *model*. The interpreter can use feature visualization methods (top- $k$  images) to try to understand  
 42 the internal logic of neurons and may visually detect that neurons are biased towards a previously  
 43 un-categorized but undesirable bias. To prevent rejection of the biased model by the interpreter, the  
 44 adversary may use a set of data with annotated bias attribute [46] (unavailable to the interpreter) to  
 45 try to perform an attack by fine-tuning the model to make the feature visualization look fairer while  
 46 maintaining the performance of the model and its overall unfair output.

47 To date, most previous works on interpretability manipulability (including fairwashing) have focused  
 48 on the manipulability of interpretability techniques such as feature attribution [43, 20] tailored for  
 49 model predictions. Little attention has been paid to the manipulability of neuron interpretability  
 50 techniques. This is in spite of the fact that this latter type of interpretability method is becoming  
 51 increasingly popular because it provides a fine-grained understanding of inner structures of DNNs [35,  
 52 34, 39]. Notably it has also been applied to create mechanistic interpretations [32, 6] which are  
 53 argued to be robust as they directly link the function of neurons. We note that the maximization  
 54 operation by construction is losing important information about the functional behavior, leading to  
 55 the potential of mis-interpretation, and suggesting the possibility of manipulation.

56 The primary contributions of our work are to first propose three distinct attacks on feature visualization  
 57 and approaches and considerations to quantify and characterize their success. We then demonstrate  
 58 all three of our attacks can achieve a degree of success (see illustration in Figure 1). This suggests that  
 59 this class of interpretation methods must be used with caution and also cast doubt on the feasibility of  
 60 using this tool to build complete mechanistic interpretations.

## 61 2 Related Work

62 A growing body of literature has investigated the interpretability of Convolutional Neural Networks  
 63 (CNNs) and the lack of robustness under different manipulations of interpretability methods.  
 64 **Interpretability methods.** Previous work aiming to provide interpretability of NNs can be grouped  
 65 into two broad categories. Firstly, there are works that develop *interpretable-by-design* methods that  
 66 provide interpretations without relying on external tools. These methods usually couple traditional  
 67 layers with various types of interpretable components. Examples range from concept expla-  
 68 nitions [8, 26, 19, 13, 4], feature attributions [45, 36, 2] to part of object disentanglement [51, 42].  
 69 Secondly, there are methods usually called *post-hoc* that aim to explain and understand either specific  
 70 components (e.g., weights, neurons, layers) or outputs of a *trained* NN. To interpret the output of mod-  
 71 els for a particular data instance (local interpretability), while feature attribution methods [40, 30, 41]  
 72 such as saliency maps assign a weight to each input feature corresponding to its importance on  
 73 the model’s output, counterfactual examples aim to give the minimal changes required to change  
 74 the model’s output [17, 15]. There are post-hoc approaches that aim to interpret the internal logic  
 75 of particular NNs through their components and representations. For example, there are methods  
 76 that focus on layer representations through *concept vectors* [25, 52], on sub-network interpretability  
 77 through *circuits* [5, 7], and individual neurons via e.g., feature visualization. Our work focuses on  
 78 feature visualization, which is one of the most popular techniques to understand the learned features  
 79 of individual neurons [53, 35].

80 **Interpretability manipulation.** There is a recent trend to analyze the reliability of interpretable  
 81 techniques through the lens of *stability*. Stability aims to study to what extent the interpretability  
 82 technique is statistically robust to reasonable input perturbations and model perturbations [20, 48].  
 83 Most works that study input and model manipulability focus on feature attributions. For example,  
 84 [11] designs adversarial input perturbations to change feature attributions in a targeted way, and [20]  
 85 shows that such manipulation can be performed through *adversarial model manipulation*, realized  
 86 by fine-tuning a pre-trained model to change feature attributions while keeping the same accuracy  
 87 of the original model. Despite sharing similarities with this work thanks to the use of adversarial  
 88 model manipulation, instead of studying the manipulability of feature attribution methods, we focus  
 89 on neuron interpretability, which brings different challenges such as the *whack-a-mole* problem  
 90 explained in Sec. 3.3. Besides input and model manipulability, recent works [1, 3, 43] have raised  
 91 the *fairwashing* issue, which is the risk of misleading the assessment of unfairness of models by  
 92 providing model interpretations that look fair, but are not. Part of our work studies the fairwashing  
 93 risk for feature visualization, which has not been investigated to date. Finally, the most closely related  
 94 work to ours is [12], which shows the targeted manipulability of *synthetic* feature visualizations  
 95 (defined in Sec. 3.1) by early stopping during optimization. Different from this previous work, we  
 96 instead study the manipulability of feature visualization under an adversarial model manipulation.

### 97 3 Methods

98 We introduce our notation, attacks, threat models, and attack success characterization methods.

#### 99 3.1 Notations and Background

100 We denote by  $\mathcal{D} = \{(\mathbf{x}_i, y_i)\}_{i=1}^N$  a dataset for supervised learning, where  $\mathbf{x}_i \in \mathbb{R}^d$  is the input and  
 101  $y_i \in \{1, \dots, K\}$  is its class label. Let  $f_{\theta}$  denote a NN,  $f_{\theta}^{(l)}(\mathbf{x})$  defines activation maps of  $\mathbf{x}$  on the  
 102  $l$ -th layer, which can be decomposed into  $J$  single activation maps  $f_{\theta}^{(l,j)}(\mathbf{x})$ . In particular,  $f_{\theta}^{(l,j)}(\mathbf{x})$   
 103 is a matrix if the  $l$ -th layer is a 2D-convolutional layer and a scalar if it is a fully connected layer. We  
 104 aim to understand the internal behavior of individual units through feature visualization, generically  
 105 defined by activation maximization [31, 47], i.e.,

$$x^* \in \operatorname{argmax}_{\mathbf{x} \in \mathcal{X}} f_{\theta}^{(l,j)}(\mathbf{x}), \quad (1)$$

106 where  $\mathcal{X}$  can be a finite set of data, e.g.,  $\mathcal{X} = \mathcal{D}$  or a continuous space  $\mathcal{X} \subset \mathbb{R}^d$ , and  $(l, j)$  is the  
 107 pair of layer  $l$  and neuron  $j$ . In Eq. 1, when the layer  $l$  is a convolutional layer, in the rest of the  
 108 paper, we aggregate the activation map  $f_{\theta}^{(l,j)}(\mathbf{x})$  using its spatial squared  $\ell_2$ -norm  $\|f_{\theta}^{(l,j)}(\mathbf{x})\|_2^2$ , and  
 109 subsequently refer to  $j$  as the channel index. Additionally, we mainly focus on the case where  $\mathcal{X} = \mathcal{D}$   
 110 is a set of natural images, and we denote by top- $k$  images the set of images that have the  $k$  highest  
 111 values of activations for a given pair  $(l, j)$ . When  $\mathcal{X} \subset \mathbb{R}^d$ , following [53], the result  $x^*$  will be  
 112 called *synthetic* feature visualization.

#### 113 3.2 Attack Framework

114 We consider feature visualization with top- $k$  images and propose an adversarial model manipulation  
 115 that fine-tunes a pre-trained model with a loss that maintains its initial performance while changing  
 116 the result of feature visualization. More formally, given a set of training data  $\mathcal{D}$ , a pre-trained model  
 117 with parameters  $\theta_{\text{initial}}$ , and an additional set of images (e.g., a set of top- $k$  images)  $\mathcal{D}_{\text{attack}}$ , our attack  
 118 framework consists in the following optimization

$$\min_{\theta} (\alpha \mathcal{L}_A(\mathcal{D}, \mathcal{D}_{\text{attack}}; \theta) + (1 - \alpha) \mathcal{L}_M(\mathcal{D}; \theta, \theta_{\text{initial}})), \quad (2)$$

119 where  $\theta$  are parameters of the updated model  $f_{\theta}$ ,  $\mathcal{L}_M(\cdot)$  is the loss that aims to maintain the initial  
 120 performance of the model  $f_{\theta_{\text{initial}}}$ , and  $\mathcal{L}_A(\cdot)$  is the attack loss. For the maintain objective, when  
 121 viewing final outputs  $f_{\theta}(\cdot)$  as a conditional distribution, our maintain loss is the distillation loss  
 122  $\mathcal{L}_M(\mathcal{D}; \theta, \theta_{\text{initial}}) = \mathcal{L}_{\text{CE}}(f_{\theta_{\text{initial}}}(\cdot) || f_{\theta}(\cdot))$  [22], where  $\mathcal{L}_{\text{CE}}$  is the cross entropy loss between the  
 123 original model outputs and the attacked model outputs on training data  $\mathcal{D}$ . As defined, this maintain  
 124 loss enforces the fine-tuned model to keep the same predictions as the initial model with the objective  
 125 of making the two models close in model space. Depending on the type of attack, the attack loss  
 126  $\mathcal{L}_A(\cdot)$  can vary and is defined in the next sections.

#### 127 3.3 Push-Down and Push-Up Attack

128 Given a set of top- $k$  images from feature visualization, denoted by  $\mathcal{D}_{\text{attack}}^{(l,j)}$ , that best activate the layer  $l$   
 129 and channel  $j$  of the initial model  $f_{\theta}$ , our first attack aims to push to zero the activations of examples

130 in  $\mathcal{D}_{\text{attack}}^{(l,j)}$ . This attack is called the *push-down* attack, and we propose the following objective for all  
 131 channels of a layer  $l$  simultaneously

$$\mathcal{L}_A(\mathcal{D}, \mathcal{D}_{\text{attack}}; \theta) = \sum_{j=1}^{J_l} \sum_{\mathbf{x}^* \in \mathcal{D}_{\text{attack}}^{(l,j)}} \|f_{\theta}^{(l,j)}(\mathbf{x}^*)\|_2^2, \quad (3)$$

132 where  $J_l$  is the set of channels of the layer  $l$ . Note that it is possible to attack a single channel or  
 133 channels from multiple layers. Here we focus on attacking all the channels in a layer (see Sec. 4.1).

134 In the *push-up* decoy attack, given a set of examples in  $\mathcal{D}_{\text{decoy}}$ , we aim to make these images appear  
 135 in the result of top- $k$  images for all the channels of a particular layer  $l$ . For this purpose, we propose  
 136 the following objective, where  $[.]_+$  is  $\max(., 0)$ :

$$\mathcal{L}_A(\mathcal{D}, \mathcal{D}_{\text{decoy}}; \theta) = \sum_{j=1}^{J_l} \sum_{\mathbf{x}^* \in \mathcal{D}_{\text{decoy}}} \sum_{\mathbf{x} \in \mathcal{D}} [\|f_{\theta}^{(l,j)}(\mathbf{x})\|_2^2 - \|f_{\theta}^{(l,j)}(\mathbf{x}^*)\|_2^2]_+. \quad (4)$$

137 This aims to make activations of examples in  $\mathcal{D}_{\text{decoy}}$  larger than all the activations of training examples.

138 **Characterizing Push-Down and Push-Up Attacks** We propose two approaches to characterize the  
 139 effectiveness of an adversarial attack on the top- $k$  images of feature visualization.

140 **Kendall- $\tau$ .** We take a (potentially large) set of images  $D_{k\tau}$  and compute the initial rankings  $R_{\text{init},j}$   
 141 of images in  $D_{k\tau}$  w.r.t. their initial activations values for the  $j$  channel. Similarly, we compute  
 142 the final rankings  $R_{\text{final},j}$  using the same images, but on final (post-attack) activations values of the  
 143 same channel  $j$ . The Kendall- $\tau_j$  score is the Kendall rank correlation coefficient between  $R_{\text{init},j}$  and  
 144  $R_{\text{final},j}$ . We can also aggregate this metric over all channels. Higher values of Kendall- $\tau$  scores can  
 145 be interpreted as higher similarity in the ordering of image activations between channels. As a result,  
 146 the Kendall- $\tau_j$  score can be used as a metric to see how much a channel's behavior has changed.

147 **CLIP- $\delta$ .** We use an external, generic, visual representation model, the CLIP image encoder [38] to  
 148 allow measuring the semantic changes in the top- $k$  images. Given a particular layer and a channel  $j$ ,  
 149 here we compute the average cosine self-similarity between the CLIP embeddings of initial top- $k$   
 150 images, which we denote by  $\bar{C}_{j,j}^{\text{init,init}}$  and the average similarity between embeddings of initial top- $k$   
 151 images and final ones (after the attack), denoted by  $\bar{C}_{j,j}^{\text{init,final}}$ . The proposed CLIP- $\delta$  score for a channel  
 152  $j$  is defined as  $\text{CLIP-}\delta_j = (\bar{C}_{j,j}^{\text{init,init}} - \bar{C}_{j,j}^{\text{init,final}}) / (\frac{1}{N-1} \sum_{p=1}^N \bar{C}_{j,p \neq j}^{\text{init,init}})$ . Intuitively, this quantifies the  
 153 relative semantic change of top- $k$  images w.r.t. CLIP embeddings and a high score can be interpreted  
 154 as the fact that the channel  $j$  has made semantically significant changes in the top- $k$  images.

155 **The Whack-A-Mole Problem.** A natural question in our framework is whether the behavior and  
 156 interpretation of one neuron can be simply moved to another neuron through the optimization process,  
 157 for example, the Push-Down objective can be reduced by permutation. We call this the *whack-a-mole*  
 158 problem. To ensure that this does not occur, we study the previously described metrics and check  
 159 that the attacked network's channels are not strongly correlated to other channels in the pre-attack  
 160 network. Given the  $j$ -th channel, we define the following two metrics that measure this property.

161 **Kendall- $\tau$ -W<sub>j</sub>** - Using  $D_{k\tau}$  we obtain the maximum Kendall- $\tau$  score between ranked lists  $R_{\text{init},j}$  and  
 162  $R_{\text{final},i}$  where  $i \neq j$  and normalize it by dividing it by the initial maximum Kendall- $\tau$  score i.e. the  
 163 score over  $R_{\text{init},j}$  and  $R_{\text{init},i}$  where  $i \neq j$ .

164 **CLIP-W<sub>j</sub>** - Using the top- $k$  images in the initial model and channel  $j$  we obtain  
 165  $\max_{i \neq j} \bar{C}_{j,i}^{\text{initial,final}} / \max_{i \neq j} \bar{C}_{j,i}^{\text{initial,initial}}$  comparing to all top- $k$  images in other channels of the fi-  
 166 nal model, normalized against that same similarity metric in the initial CLIP scores.

### 167 3.4 Fairwashing Interpretability Attack

168 We consider a threat model as discussed in Sec. 1 where the attacker has a set of protected attribute  
 169 labels they use to hide bias from an interpreter without labeled data. More formally, given a model  
 170  $f_{\theta}$ , which is *unfair* according to a certain metric of unfairness, a set of  $J$  of neurons whose top- $k$   
 171 images look *unfair*, we aim to answer the question: can we make an adversarial model perturbation  
 172 by fine-tuning a pre-trained model, maintaining its performance and its unfairness while making the  
 173 top- $k$  images of the  $J$  neurons appear *fairer*? In this formalization, answering affirmatively to this  
 174 question corresponds to succeeding in the fairwashing attack.

175 We design the fairwashing attack, using the same attack framework <sup>1</sup> defined in Sec. 3.2. One  
 176 alternative to make the top- $k$  images appear fairer would be to enforce the matching between top- $k$

<sup>1</sup>Note we use pre-activations to capture the entire and non-truncated distribution [6]

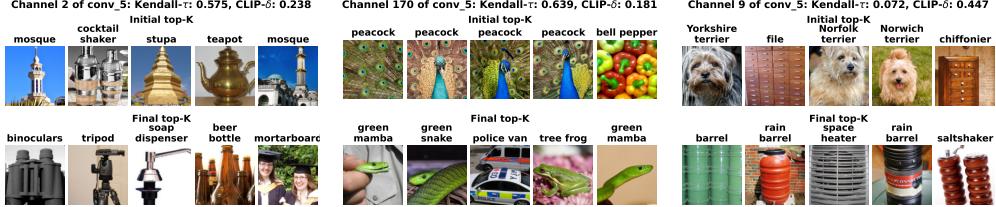


Figure 2: Push-down all-channel attack on *Conv5* of AlexNet. All initial images have been replaced by other images. The final validation performance was 56.2%, a drop of less than half a percent.

activations for different groups of the protected attribute. However, it was empirically observed that this objective fails to generalize on an unseen set because it focuses only on the tail of the distribution of activations. We, therefore, propose a simple yet effective attack objective that allows reducing the discrepancy between the distribution of pre-activations of two groups of data  $\mathcal{D}_{\text{attack}}^0$  and  $\mathcal{D}_{\text{attack}}^1$ , partitioned with respect to protected attribute (e.g., gender). For this purpose, we use the following loss (corresponding to the maximum mean discrepancy [16] with the feature function  $\phi(x) = (x, x^2)$ )

$$\mathcal{L}_A(\mathcal{D}, \mathcal{D}_{\text{attack}}^0 \cup \mathcal{D}_{\text{attack}}^1; \theta) = \|\mu_0^l - \mu_1^l\|_2^2 + \|\rho_0^l - \rho_1^l\|_2^2, \quad (5)$$

where  $\mathcal{D}_{\text{attack}}^0$ ,  $\mathcal{D}_{\text{attack}}^1$  are two groups of data partitioned w.r.t. the labeled protected attribute (e.g., race or gender),  $\mu_p^l$  (with  $p \in \{0, 1\}$ ) is a vector of scalars  $\mu_p^{(l,j)} = \mathbb{E}_{x_p \sim \mathcal{D}_{\text{attack}}^p} [f_{\theta}^{(l,j)}]$  of first-order moments for layer  $l$  and neuron  $j$ , and similarly  $\rho_p^{(l,j)} = \mathbb{E}_{x_p \sim \mathcal{D}_{\text{attack}}^p} (f_{\theta}^{(l,j)})^2$  are second-order moments for the same neuron. This attack objective enforces the matching between the first two moments of two distributions (w.r.t. groups of protected attribute) of pre-activations of a neuron.

## 4 Experiments and Results

We now describe the experimental setup and the results obtained after running attacks. For all of our attacks, we use the ImageNet [10] training set as  $\mathcal{D}$ . We use the PyTorch [37] pretrained AlexNet [27] for our analysis. In Appx. B.2 we provide an ablation study on EfficientNet [44] with similar findings. More technical details regarding hyperparameters for all the attacks can be found in Appx. B.

**Push-down and Push-Up attack.** For the push-down and up attack, we consider  $\mathcal{D}_{\text{attack}}^{(l,j)} \subset \mathcal{D}$  as the top-10 images that maximally activate the channel  $j$  of layer  $l$ . For the push-up attack, we additionally consider  $\mathcal{D}_{\text{decoy}}$  as 100 randomly sampled images of a particular class to be used as decoy.

**Fairwashing attack.** In order to run and evaluate the fairwashing attack, we need a dataset with a labeled protected attribute (e.g., gender or age) to be able to assess not only model unfairness but also the *fairness* of feature visualization of a neuron. For this purpose, we use the ImageNet People Subtree dataset [46], which is a set of  $\approx 14k$  images with labeled demography (gender, race and age), derived from ImageNet-21k. We use the 75 – 25% split for training and testing sets, and  $\mathcal{D}_{\text{attack}}^0$  and  $\mathcal{D}_{\text{attack}}^1$  are binary groups (w.r.t. protected attribute) from the training set. We estimate model unfairness using two popular measures of unfairness [49], namely the difference of disparate impact ( $\text{DDI} = |p(\hat{y} = c|z = 0) - p(\hat{y} = c|z = 1)|$ , where  $z$  is the protected attribute,  $c$  is a class and  $\hat{y}$  is the predicted class) and difference of equal opportunity ( $\text{DEO} = |p(\hat{y} = c|z = 0, y = c) - p(\hat{y} = c|z = 1, y = c)|$ ) estimated on testing data [49, 18]. Inspired by the fairness assessment in regression and clustering, we use two measures to quantify the feature visualization unfairness. The first one looks at the entire distribution of activations and is the Kolmogorov-Smirnov (KS) distance between the two conditional distributions of activations given protected attribute label [29]. The second one only focuses on the tail of the distribution of activations, i.e., activations of top- $k$  images, and is the balance [9] or ratio between the number of instances from top- $k$  belonging to the minority group over the number of instances in top- $k$  belonging to the majority group. Finally, following recent trends [23], we perform the fairwashing attack on the last but one layer.

### 4.1 Push-Down And Push-Up Attack Experiments

**Warm-up: Single-Channel Attack.** To set a first evaluation point for our attack framework, we apply the push-down attack to one channel. Figure 3 shows the visualization of top images before and after. We can see that after optimization, the top- $k$  activating images of the neuron have been completely replaced by other images with different semantic concepts, suggesting a successful attack with almost nearly no loss in accuracy (it decreases by 0.04%).

219 One way of satisfying the attack objective perfectly in the  
 220 single channel case is to set the channel weights to zero.  
 221 This naive solution only loses 0.2% is to simply set all  
 222 the weights of the channel to zero. Specifically removing  
 223 channel 0 (by masking) decreased the accuracy by 0.2%.  
 224 We thus consider more challenging settings.

225 **All-Channel Attack.** Unlike the single-channel attack,  
 226 the all-channel attack (change all neuron interpretation in  
 227 a layer) does not have a trivial solution. Because some  
 228 information needs to flow through the layer in order for  
 229 classification to be successful, setting all channels to zero  
 230 would result in catastrophic performance loss.

231 We apply our attack framework to *Conv5* of the AlexNet Model. In Figure 2 we show a  
 232 selection of 3 channels and the modifications achieved under the All-Channel Push-Down at-  
 233 tack and the aggregate metrics (averages for all channels in a layer) are shown in Table 1.  
 234 More visual examples are provided in the Appendix. For the visualized channels (and those  
 235 in Appendix) we observe a near complete replacement of the top-5 images by other images.  
 236 Further, the labels of the top images  
 237 significantly change, with minimal to  
 238 no residual overlap. This suggests  
 239 that not only the images have changed  
 240 but the semantic concepts that would  
 241 be determined by an interpreter have  
 242 likely changed. This is opposed to  
 243 the model simply memorizing images  
 244 to reduce and replacing them with se-  
 245 mantically similar ones. We further  
 246 confirm this in the appendix by show-  
 247 ing validation set top- $k$  images which  
 248 demonstrate that semantically they fol-  
 249 low the same behavior as the training  
 250 images (which are used for the actual attack). Overall, the attack seems to produce a generalized  
 251 change in the behavior of the feature visualization of neurons.

252 Studying the metrics comparing the channels before and after modification, we can deduce several  
 253 different behaviors. The first two channels exhibit relatively high Kendall- $\tau$  scores, from which we  
 254 conclude that the ordering of image activations has not undergone severe changes. This means that  
 255 likely only a subset of images, which includes the initial top- $k$  has moved in rank. Studying the CLIP  
 256 distance in both cases allows us to conclude that there is significant semantic overlap in the initial  
 257 and final top- $k$ , which can be confirmed by visual inspection.

258 This is in contrast to the channel shown at the right, where the Kendall- $\tau$  score is close to zero,  
 259 indicating a full re-ordering of the activations. As a consequence, the CLIP distance from initial to  
 260 final is also much higher, which matches with a visual inspection.

261 In general, we observe a substantial correspondence between our visual intuition and the CLIP- $\delta$   
 262 and Kendall- $\tau$ , channels with low scores Kendall- $\tau$  and high CLIP- $\delta$  tend to change substantially.  
 263 As illustrated in further examples in the Appendix one observed difference in these two metrics is  
 264 that channels maintaining some similar classes in the top images will tend to have a lower CLIP- $\delta$   
 265 (suggesting less change).

266 **Whack-a-mole.** We can further analyze the existence of the whack-a-mole problem by observing  
 267 Fig. 5 which shows for a channel in the original model, the top- $K$  image in the modified model which  
 268 have the closest Kendall- $\tau$ -W and CLIP-W scores (not including the channel itself).

269 We observe that the first channel (channel 2 on figure) has little to no visually discernable similarity  
 270 to nearby channels in the modified model as well confirmed by the Kendall- $\tau$ -W. Indeed a majority of  
 271 the channels look like this (see Appendix). On the other hand, we do observe similar images for the  
 272 initial channel 193 and its nearest final one (163), which was picked as the most illustrative examples  
 273 ("hard" one) where the red curve of Fig. 6 is above the blue one. However, for this "hard" example,



Figure 3: Top images for a channel before and after a single-channel Push-Down attack.

Layer/Attack	CLIP- $\delta$	Kendall- $\tau$	CLIP-W	Kendall- $\tau$ -W	Acc. (%)
Conv1 Push-Down	0.043	0.682	0.996	0.302	56.1
Conv2 Push-Down	0.056	0.612	0.994	0.151	56.3
Conv3 Push-Down	0.127	0.573	0.963	0.130	56.1
Conv4 Push-Down	0.205	0.548	0.974	0.122	56.2
Conv5 Push-Down	0.249	0.530	0.963	0.048	56.2
Conv5 Push-Up	0.150	0.654	0.962	0.011	56.3
EfficientNet L7 - Push-Down	0.262	0.503	0.971	-0.145	77.5

Table 1: Average (over channels) attack metrics for an All-Channel Push-Down and Push-Up Attack for AlexNet (row 1-6) and EfficientNet (row 7). We observe that the relative whack-a-mole metrics are low, suggesting this problem is not present for our attacks. Lower layers are more challenging to attack leading to lower CLIP score and higher Kendall- $\tau$  as confirmed by visual intuition. Overall, the attack seems to produce a generalized change in the behavior of the feature visualization of neurons.

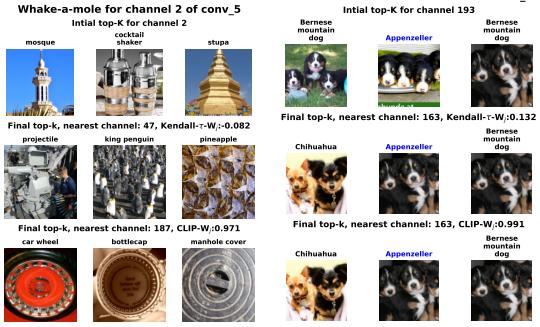


Figure 5: We show the initial top images for two channels and beneath are the corresponding final top images of closest channels w.r.t Kendall- $\tau$ -W<sub>j</sub> and CLIP-W<sub>j</sub>.

more insight is given by investigating the CLIP-W<sub>j</sub> where the denominator notably measures the clip similarity to other channels in the original model. The score is less than or typically close to 1 suggesting that the original model already had a high similarity to another channel. Indeed in the Appendix for the second example, we confirm there is a very similar channel in the original model. To gain further insight into CLIP-W<sub>j</sub> in Fig.6, we further visualize the numerator and denominator for all the channels (red line) and sort them by the initial similarity to other channels (denominator). We observe that the red line is often below the blue line and if it exceeds it is not by a large relative amount, suggesting that channels with high whack-a-mole metrics are actually ones that already had similarities to other channels in the original model. Overall we conclude the presence of the whack-a-mole problem is minimal in our current attack.

**Effect of Depth.** We now consider how the attack is affected by depth, with results for different layers of AlexNet shown in Tab. 1 and illustrated in Fig. 7. We observe that modifications of the earliest layers are significantly harder to achieve than for later layers as confirmed by the metrics and visual examination. We also observe a qualitative difference in the changes. For example, Conv<sub>1</sub> and Conv<sub>2</sub> are picking up low-level information such as color, edges, and textures and this is reflected in the type of modifications made to the images. If performance is maintained after the attack, it is likely that the modification objective did not have a strong impact, leading to little to no modification. This is reflected in the CLIP- $\delta$  scores (see Table 1)a nd in visual examination (see Appendix for further examples). Several explanations can account for this. Firstly, there are fewer or no modifiable weights upstream to the attacked layer, leading to less flexibility to accommodate the competing natures of the combined objective compared to later layers. Secondly, the early-layer features, while somewhat malleable, must collectively perform a certain set of signal-filtering operations in order to be able to extract meaningful information. Performing strong modifications to the filters may lead to unrecoverable information loss downstream. We observe that the whack-a-mole metrics are also relatively high for this case using Kendall- $\tau$ -W. On the other hand, the normalized CLIP-W score is close to 1 suggesting that this increase is not due to behavior being moved into the channel but due to existing redundancy in channels.

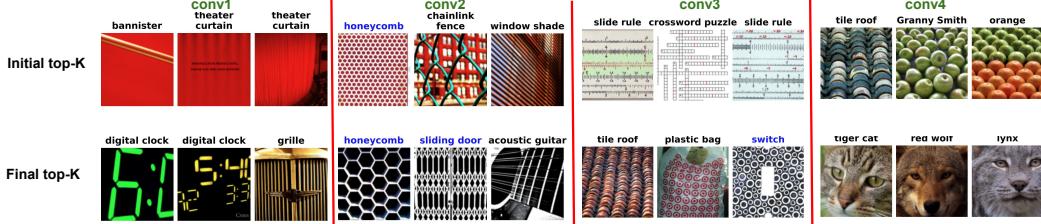


Figure 7: Push-down attack on AlexNet across several layers. Channels are taken individually on each layer for layer ablation, and the results demonstrate that the top images are potentially vulnerable across all layers. The final attacked models all have a less than .5% drop from a default AlexNet.

**Push-Up Decoy Attack.** We study a more targeted attack objective, namely one that actively pushes a set of selected images into the top activating images for every channel. This is achieved with Eq. 4, where the loss is non-zero as long as there exist images outside the group of selected images that activate higher than the group we intend to push up.

This type of attack is more targeted and therefore likely harder than the push-down attack, which does not specify what images the top- $k$  should be replaced with. The push-up attack, if successful, can

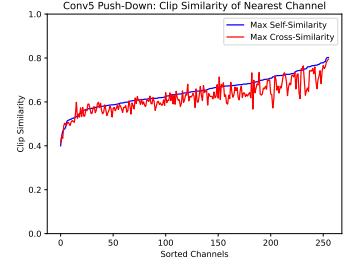


Figure 6: We compare initial CLIP similarity to other channels (blue) versus similarity after attack (red). Red and blue largely track each other for all channels.

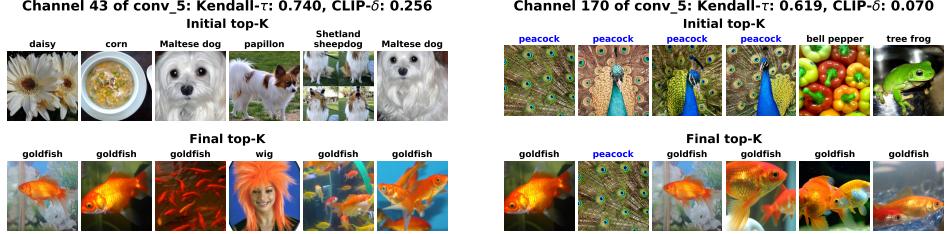


Figure 8: Examples of channels in all-channel push-up attack. The top images were successfully put in top images. The Kendall- $\tau$  remains relatively high ( $> 0.5$ ) suggesting much of the channel behavior is preserved while the top activating images completely obfuscate the behavior.

307 assign the same interpretation to every channel in a layer, making any interpretation attempt based on  
308 top- $k$  images fraught, or at least minimally informative.

309 Fig. 1 shows the result of the push-up attack using a collection of images with the Imagenet label  
310 “Goldfish” as the decoy set. Further, in Fig. 8 we show that for many channels of a layer, we can  
311 modify the top-10 to contain a few or consist entirely of Goldfish images. The metrics in Table 1 also  
312 demonstrate substantial change and a low likelihood of whack-a-mole behavior. Studying the figure  
313 more closely, we observe that not only Goldfish, but also other images that share certain traits with  
314 the Goldfish images are also boosted, suggesting a degree amount of generality of the newly imposed  
315 selectivity, further explored in the Appendix.

#### 316 4.1.1 Synthetic Feature Visualization

317 We study the impact of the  
318 Push-Down and Push-Up at-  
319 tacks on the synthetic activation-  
320 maximizing images of the chan-  
321 nels under attack [50]. Syn-  
322 thetic activation-maximizing im-  
323 ages are the result of an opti-  
324 mization problem over input pix-  
325 els solved by gradient ascent on  
326 the channel activation under a  
327 norm constraint in pixel space.  
328 To avoid adversarial noise sam-  
329 ples [14] it is necessary to jitter the input image or parameterize it as a smooth function[35].

330 In Fig. 9, we study the synthetic optimal images for several channels before and after the attack. By  
331 visual inspection, while the top- $k$  images change drastically, the synthetic optimal image is largely  
332 unaffected. The most common observed change (see also Appendix) for *conv5* is a low-frequency  
333 modulation of the pattern. We hypothesize that this is because the top- $k$  attack most significantly  
334 modifies the weights of the attacked layer, which is a later layer preceded by several downsamplings.

335 The lack of change in the synthetic optimal image suggests that the synthetic feature visualization  
336 and the top- $k$  analysis are, counter-intuitively, highly de-correlatable. Observe, for instance, that  
337 the left-hand synthetic image suggests selectivity for cats even when most of the top- $k$  images are  
338 goldfish. This is a worrying prospect for the top- $k$  interpretability method. Further, this does not  
339 permit the conclusion that the synthetic optimal image is more robust to attack, since we have not  
340 explicitly run an attack against it. Rather, this suggests the space of NN weights and the possible  
341 functions they span is quite large, and can possibly accommodate more functionality, and attacks,  
342 than one might expect.

#### 343 4.2 Fairwashing Feature Visualization

344 We demonstrate the application of our fairwashing attack for feature visualization as defined  
345 Sec. 3.4. Given an *unfair* (according to a certain metric of unfairness) model and a set of  
346 neurons whose top-activating images look *unfair*, we ask ourselves whether it is possible, by  
347 fine-tuning, to make the new set of images for the same neurons appear *fairer* while main-  
348 taining the same performance and bias of the initial model. We instantiate this fairwashing attack  
349 on an annotated subset of Imagenet data [46] (as described in Sec. 4) with gender as the pro-  
350 tected attribute. We first estimate the model unfairness of the pre-trained AlexNet model using  
351 DDI and DEO unfairness measures. Tab. 2 reports these measures for the three human classes

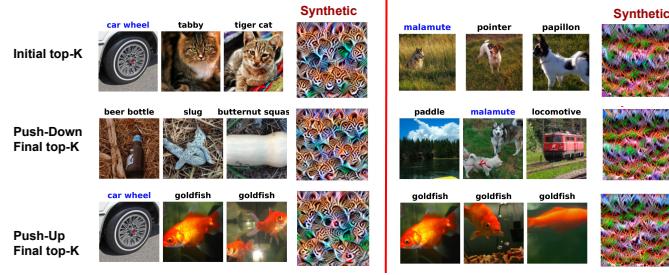


Figure 9: Synthetic feature visualization after our attack. We observe the visualization is largely decorrelated to top- $k$  natural images.

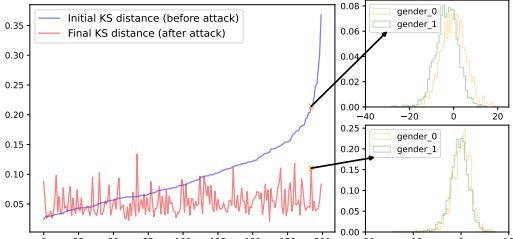


Figure 10: Kolmogorov-Smirnov (KS) distance between the conditional distributions of each condition estimated on the annotated testing set. We sort the channels based on the initial KS and observe that after our fairwashing interpretability attack, each channels KS is drastically reduced.

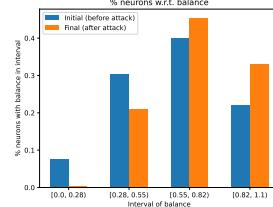


Figure 11: Percentage of the neurons according to their balance over the annotated testing set. After the attack, the percentage of neurons with low balance has decreased while the percentage of neurons with high balance has increased.



Figure 12: Top-30 images obtained for unit 800 of the last but one layer of AlexNet. Green is used for images that stay in top-30 images after attack. Before the fairwahsing attack, (a) the initial top-30 images are gender-biased. After the fairwashing attack, (b) the top-30 are less gender-biased: balance (fairness) measure has almost doubled. On the other hand, the model’s unfairness has not changed.

of the ImageNet-1k dataset on which AlexNet is trained. According to this table, the initial AlexNet model is not totally fair, with the largest values of unfairness on the *Baseball player* class. We identified 200 neurons of the last but one layer whose MILAN [21] descriptions are related to humans (see Appendix for more details). We run our attack on all these neurons to prevent missing neurons whose biases may transfer to other ones. Fig. 10 shows the results of Kolmogorov-Smirnov distance between the distributions of activations conditioned on the two gender groups. It can be observed that after the attack, this distance has been drastically reduced, especially for highly biased neurons. This suggests the balance of the top- $k$  is also improved. As can be seen in Fig. 11, the percentage of neurons whose top- $k$  images have a low balance (low *fairness*) has decreased, while the percentage of neurons with high balance has increased, thus making feature visualization fairer. Moreover, according to Tab. 2, the model has almost the same accuracy and almost the same measures of unfairness (all cases  $\leq 1\%$  of relative difference for DDI and  $\leq 4\%$  for DEO). Note that our attack did not enforce any fairness constraint on the output, the maintain loss  $\mathcal{L}_M$  described in Sec. 3.2 was enough to also maintain model unfairness. We also depicted in Fig. 12 an example of a unit whose top- $k$  images were initially *biased*, but have been fairwashed after running the attack by almost doubling the balance measure. More examples of training and testing sets can be found in the appendix.

## 5 Conclusions, Limitations, and Broader Impact

We demonstrated the adversarial model manipulability of feature visualization with top- $k$ , proposing three attacks that pose varying threats. We provide experimental evidence that supports the success of our attacks, with little to no evidence of a *whack-a-mole* issue. Our metrics to systematically detect the presence of whack-a-mole may be imperfect as validating them requires inspecting all channels to validate correspondence. Future work may consider investigation of synthetic feature maps and how they may be attacked and generalization of the fairwashing attack beyond binary attributes.

**Broader Impact.** The goal of our study has been to demonstrate a potential vulnerability in current interpretability methods and raise awareness of reliability and ethical risks. By showing the fairwashing attack, an apparent consequence is the possibility that an ill-intentioned individual uses this work to perform these attacks in order to release models that marginalize minority groups. However, we think that raising these risks is an essential first step towards addressing these vulnerabilities, and we hope our contributions provide a springboard for future discussion and protection efforts.

	Class						
	Baseball player		Bridegroom		Scuba diver		
	Acc.	DDI	DEO	DDI	DEO	DDI	DEO
Pre-Attack	56.45	3.38	76.92	2.67	12.34	0.28	5.26
Post-Attack	56.56	3.14	73.07	1.90	12.34	0.24	5.26

Table 2: Accuracy/fairness measures (DDI/DEO) computed respectively on the ImageNet val. set and on the annotated testing set. Both measures are relatively similar before and after the fairwashing attack while the model has decreased the bias perceived by the interpreter for feature visualizations.

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