Problem Statement 1:

A test is conducted which is consisting of 20 MCQs (multiple choices questions) with every MCQ having its four options out of which only one is correct. Determine the probability that a person undertaking that test has answered exactly 5 questions wrong.

Solution 1:

Let X be the random variable representing number of questions answered wrong in a given MCQs test

 $X \sim Bin(20, 0.75)$, therefore the probability is given by

$$P(X = 5) = {n \choose x} * p^x * (1 - p)^{n - x}$$

$$= {20 \choose 5} * 0.75^5 * (1 - 0.75)^{20 - 5}$$

$$= 15504 * 0.2373 * (0.25)^{15}$$

$$= 0.00000342$$

Problem Statement 2:

A die marked A to E is rolled 50 times. Find the probability of getting a "D" exactly 5 times.

Solution 2:

Let X be the random variable representing number of questions answered wrong in a given MCQs test

 $X \sim Bin(50, 0.2)$, therefore since n is large, the distribution is approximated to the normal distribution $X \sim N(10, 8)$, (with continuity correction)

$$P(X = 5) = P\left(Z = \frac{x - \mu}{\sigma}\right)$$

$$= P\left(\frac{4.5 - 10}{\sqrt{8}} \le Z \le \frac{5.5 - 10}{\sqrt{8}}\right)$$

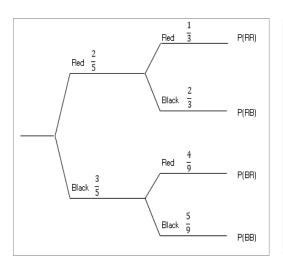
$$= \mathbf{0.0299}$$

Problem Statement 3:

Two balls are drawn at random in succession without replacement from an urn containing 4 red balls and 6 black balls. Find the probabilities of all the possible outcomes.

Solution 3:

These are independent events, we can come up with a tree diagram to represent the scenario



Outcome 1:
$$P(two\ red\ balls) = \frac{2}{5} \times \frac{1}{3} = \frac{2}{15}$$

Outcome 2: P(red ball first and black ball second)

$$=\frac{2}{5}\times\frac{2}{3}=\frac{4}{15}$$

Outcome 3: P(black ball first and red ball second)

$$=\frac{3}{5}\times\frac{4}{9}=\frac{4}{15}$$

Outcome 4: $P(two\ black\ balls) = \frac{3}{5} \times \frac{5}{9} = \frac{1}{3}$