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# 5.5 DATA COMPRESSION

- *introduction*
- *run-length coding*
- *Huffman compression*
- *LZW compression*


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# Data compression

Compression reduces the size of a file:

- To save space when storing it.
- To save time when transmitting it.
- Most files have lots of redundancy.

Who needs compression?

- Moore's law: # transistors on a chip doubles every 18–24 months.
- Parkinson's law: data expands to fill space available.
- Text, images, sound, video, ...

> *" Everyday, we create 2.5 quintillion bytes of data—so much that 90% of the data in the world today has been created in the last two years alone. "* — *IBM report on big data (2011)*

Basic concepts ancient (1950s), best technology recently developed.

# Applications

## Generic file compression.

- Files: GZIP, BZIP, 7z.
- Archivers: PKZIP.
- File systems: NTFS, HFS+, ZFS.

## Multimedia.

- Images: GIF, JPEG.
- Sound: MP3.
- Video: MPEG, DivX™, HDTV.



## Communication.

- ITU-T T4 Group 3 Fax.
- V.42bis modem.
- Skype.

## Databases. Google, Facebook, ....

Google™ facebook

# Lossless compression and expansion

Message. Binary data $B$ we want to compress.

Compress. Generates a "compressed" representation $C(B)$.

Expand. Reconstructs original bitstream $B$.

uses fewer bits (you hope)


Basic model for data compression

Compression ratio. Bits in $C(B)$ / bits in $B$.

Ex. 50–75% or better compression ratio for natural language.

# Food for thought

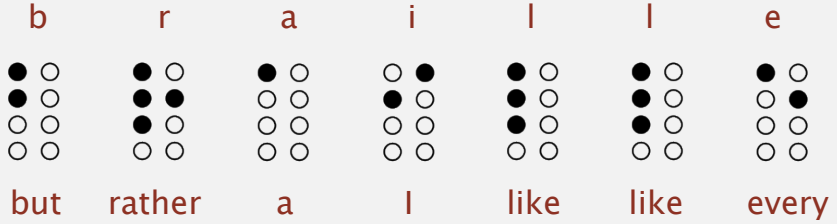
Data compression has been omnipresent since antiquity:

- Number systems.
- Natural languages.
- Mathematical notation.

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

has played a central role in communications technology,

- Grade 2 Braille.
- Morse code.
- Telephone system.




and is part of modern life.

- MP3.
- MPEG.

Q. What role will it play in the future?

# Data representation: genomic code

Genome. String over the alphabet { A, C, T, G }.

Goal. Encode an $N$-character genome: A T A G A T G C A T A G . . .

Standard ASCII encoding.

- 8 bits per char.
- $8N$ bits.

| char | hex | binary |
|---|---|---|
| A | 41 | 01000001 |
| C | 43 | 01000011 |
| T | 54 | 01010100 |
| G | 47 | 01000111 |

Two-bit encoding.

- 2 bits per char.
- $2N$ bits.

| char | binary |
|---|---|
| A | 00 |
| C | 01 |
| T | 10 |
| G | 11 |

Fixed-length code. $k$-bit code supports alphabet of size $2^k$.

Amazing but true. Initial genomic databases in 1990s used ASCII.

# Reading and writing binary data

Binary standard input and standard output. Libraries to read and write bits from standard input and to standard output.

| public class BinaryStdIn | |
|---|---|
| boolean readBoolean() | *read 1 bit of data and return as a* `boolean` *value* |
| char readChar() | *read 8 bits of data and return as a* `char` *value* |
| char readChar(int r) | *read* `r` *bits of data and return as a* `char` *value* |
| [*similar methods for* `byte` *(8 bits);* `short` *(16 bits);* `int` *(32 bits);* `long` *and* `double` *(64 bits)*] | |
| boolean isEmpty() | *is the bitstream empty?* |
| void close() | *close the bitstream* |

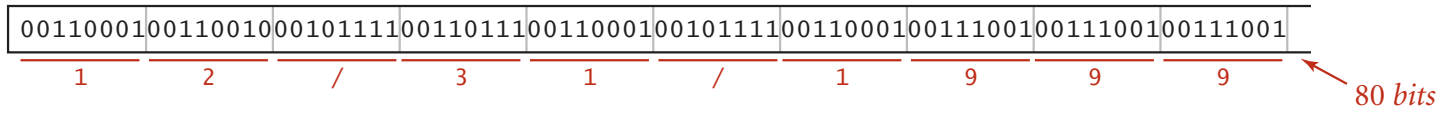
| public class BinaryStdOut | |
|---|---|
| void write(boolean b) | *write the specified bit* |
| void write(char c) | *write the specified 8-bit* `char` |
| void write(char c, int r) | *write the* `r` *least significant bits of the specified* `char` |
| [*similar methods for* `byte` *(8 bits);* `short` *(16 bits);* `int` *(32 bits);* `long` *and* `double` *(64 bits)*] | |
| void close() | *close the bitstream* |

# Writing binary data

Date representation. Three different ways to represent 12/31/1999.

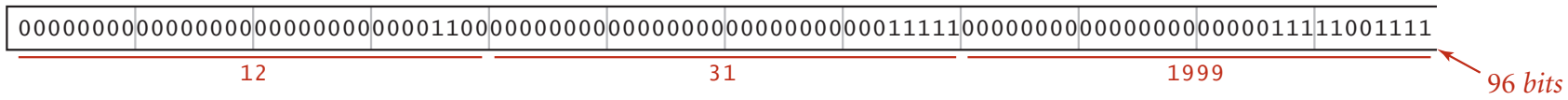
**A character stream (StdOut)**

```
StdOut.print(month + "/" + day + "/" + year);
```



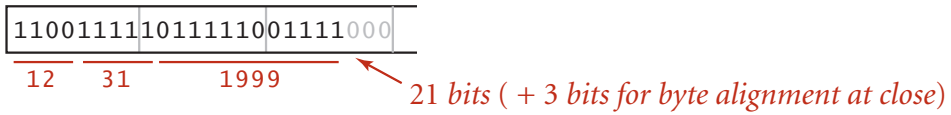

**Three ints (BinaryStdOut)**

```
BinaryStdOut.write(month);
BinaryStdOut.write(day);
BinaryStdOut.write(year);
```




**A 4-bit field, a 5-bit field, and a 12-bit field (BinaryStdOut)**

```
BinaryStdOut.write(month, 4);
BinaryStdOut.write(day, 5);
BinaryStdOut.write(year, 12);
```



# Binary dumps

## Q. How to examine the contents of a bitstream?

**Standard character stream**

```
% more abra.txt
ABRACADABRA!
```

**Bitstream represented as 0 and 1 characters**

```
% java BinaryDump 16 < abra.txt
0100000101000010
0101001001000001
0100001101000001
0100010001000001
0100001001010010
0100000100100001
96 bits
```

**Bitstream represented with hex digits**

```
%  java HexDump 4 < abra.txt
41 42 52 41
43 41 44 41
42 52 41 21
12 bytes
```

**Bitstream represented as pixels in a `Picture`**

```
% java PictureDump 16 6 < abra.txt
```

*16-by-6 pixel window, magnified*

```
96 bits
```

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | A | B | C | D | E | F |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | NUL | SOH | STX | ETX | EOT | ENQ | ACK | BEL | BS | HT | LF | VT | FF | CR | SO | SI |
| 1 | DLE | DC1 | DC2 | DC3 | DC4 | NAK | SYN | ETB | CAN | EM | SUB | ESC | FS | GS | RS | US |
| 2 | SP | ! | " | # | $ | % | & | ' | ( | ) | * | + | , | - | . | / |
| 3 | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | : | ; | < | = | > | ? |
| 4 | @ | A | B | C | D | E | F | G | H | I | J | K | L | M | N | O |
| 5 | P | Q | R | S | T | U | V | W | X | Y | Z | [ | \ | ] | ^ | _ |
| 6 | ` | a | b | c | d | e | f | g | h | i | j | k | l | m | n | o |
| 7 | p | q | r | s | t | u | v | w | x | y | z | { | \| | } | ~ | DEL |

**Hexadecimal to ASCII conversion table**

# Universal data compression

US Patent 5,533,051 on "Methods for Data Compression", which is capable of compression all files.

Slashdot reports of the Zero Space Tuner™ and BinaryAccelerator™.

> *" ZeoSync has announced a breakthrough in data compression that allows for 100:1 lossless compression of random data. If this is true, our bandwidth problems just got a lot smaller.… "*

# Universal data compression

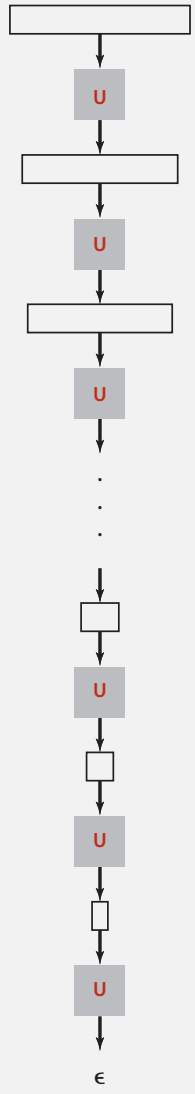
Proposition. No algorithm can compress every bitstring.

Pf 1. [by contradiction]

- Suppose you have a universal data compression algorithm $U$ that can compress every bitstream.
- Given bitstring $B_0$, compress it to get smaller bitstring $B_1$.
- Compress $B_1$ to get a smaller bitstring $B_2$.
- Continue until reaching bitstring of size $0$.
- Implication: all bitstrings can be compressed to $0$ bits!

Pf 2. [by counting]

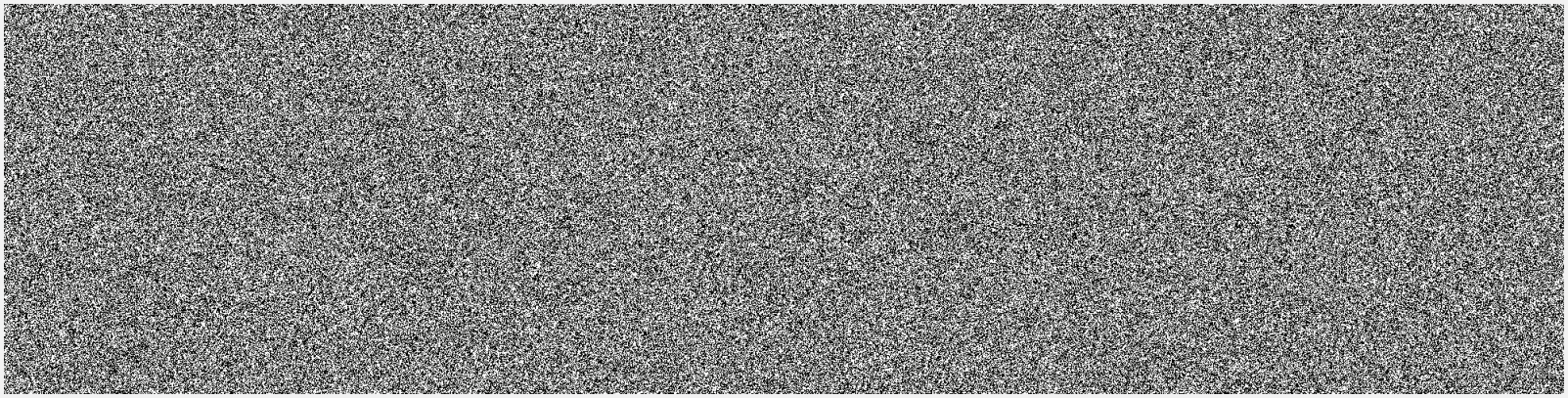
- Suppose your algorithm that can compress all $1{,}000$-bit strings.
- $2^{1000}$ possible bitstrings with $1{,}000$ bits.
- Only $1 + 2 + 4 + \ldots + 2^{998} + 2^{999}$ can be encoded with $\leq 999$ bits.
- Similarly, only $1$ in $2^{499}$ bitstrings can be encoded with $\leq 500$ bits!




Universal data compression?

# Undecidability

```
% java RandomBits | java PictureDump 2000 500
```



```
1000000 bits
```

**A difficult file to compress: one million (pseudo-) random bits**

```
public class RandomBits
{
   public static void main(String[] args)
   {
      int x = 11111;
      for (int i = 0; i < 1000000; i++)
      {
         x = x * 314159 + 218281;
         BinaryStdOut.write(x > 0);
      }
      BinaryStdOut.close();
   }
}
```

# Rdenudcany in Enlgsih lnagugae

Q. How mcuh rdenudcany is in the Enlgsih lnagugae?

> *" ... randomising letters in the middle of words [has] little or no effect on the ability of skilled readers to understand the text. This is easy to denmtrasote. In a pubiltacion of New Scnieitst you could ramdinose all the letetrs, keipeng the first two and last two the same, and reibadailty would hadrly be aftcfeed. My ansaylis did not come to much beucase the thoery at the time was for shape and senqeuce retigcionon. Saberi's work sugsegts we may have some pofrweul palrlael prsooscers at work. The resaon for this is suerly that idnetiyfing coentnt by paarllel prseocsing speeds up regnicoiton. We only need the first and last two letetrs to spot chganes in meniang. "* — Graham Rawlinson

A. Quite a bit.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# Run-length encoding

Simple type of redundancy in a bitstream. Long runs of repeated bits.

0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1

40 bits

Representation. 4-bit counts to represent alternating runs of 0s and 1s:
15 0s, then 7 1s, then 7 0s, then 11 1s.

1 1 1 1 0 1 1 1 0 1 1 1 1 0 1 1 ← 16 bits (instead of 40)

15 7 7 11

Q. How many bits to store the counts?

A. We'll use 8 (but 4 in the example above).

Q. What to do when run length exceeds max count?

A. If longer than 255, intersperse runs of length 0.

Applications. JPEG, ITU-T T4 Group 3 Fax, ...

# Run-length encoding: Java implementation

```
public class RunLength
{
   private final static int R   = 256;
   private final static int lgR = 8;

   public static void compress()
   {  /* see textbook */  }

   public static void expand()
   {
      boolean bit = false;
      while (!BinaryStdIn.isEmpty())
      {
         int run = BinaryStdIn.readInt(lgR);
         for (int i = 0; i < run; i++)
            BinaryStdOut.write(bit);
         bit = !bit;
      }
      BinaryStdOut.close();
   }

}
```

maximum run-length count

number of bits per count

read 8-bit count from standard input

write 1 bit to standard output

pad 0s for byte alignment

# An application: compress a bitmap

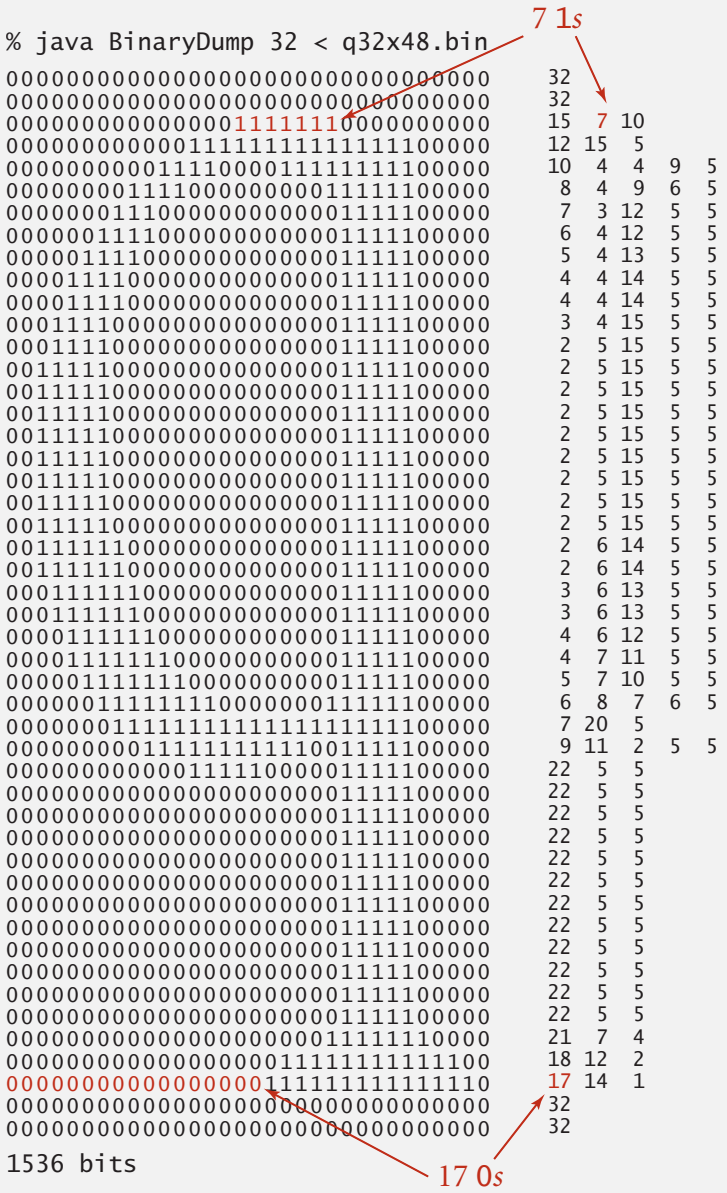
## Typical black-and-white-scanned image.

- 300 pixels/inch.
- 8.5-by-11 inches.
- 300 × 8.5 × 300 × 11 = 8.415 million bits.

## Observation. Bits are mostly white.

## Typical amount of text on a page.

40 lines × 75 chars per line = 3,000 chars.




**A typical bitmap, with run lengths for each row**

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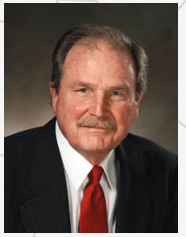


David Huffman

# Variable-length codes

Use different number of bits to encode different chars.

Ex. Morse code: • • • – – – • • •

Issue. Ambiguity.

SOS ?

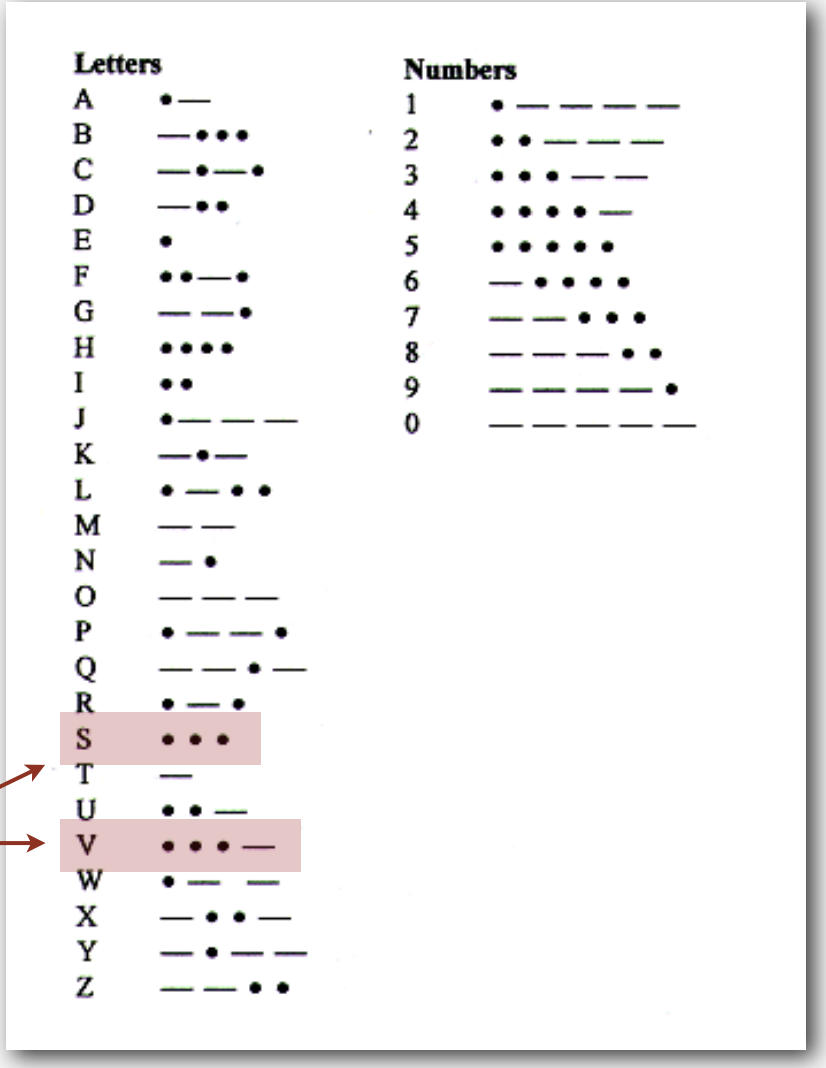
V7 ?

IAMIE ?

EEWNI ?

In practice. Use a medium gap to separate codewords.

codeword for S is a prefix of codeword for V



# Variable-length codes

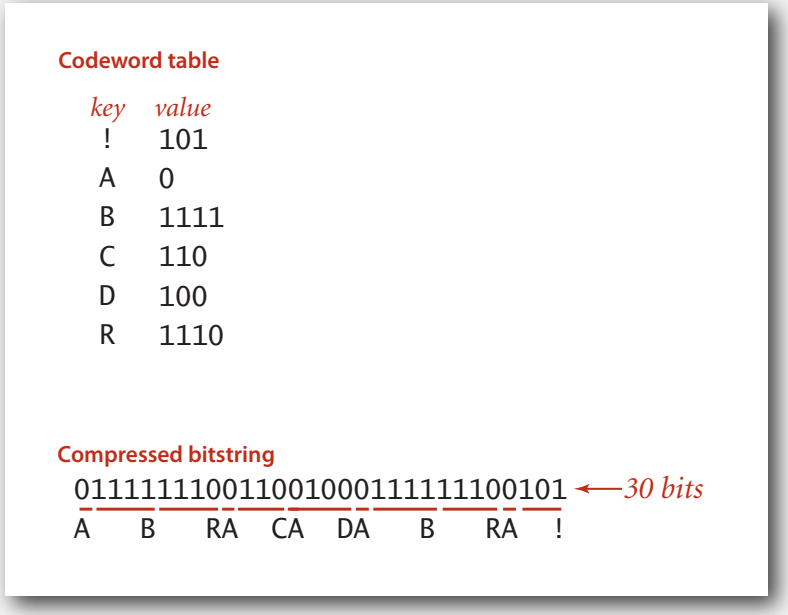
Q. How do we avoid ambiguity?

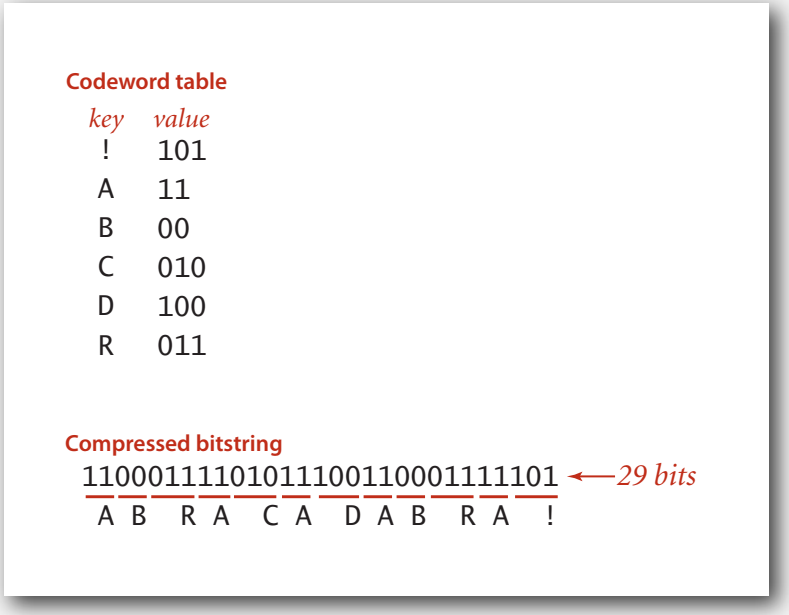
A. Ensure that no codeword is a prefix of another.

Ex 1. Fixed-length code.

Ex 2. Append special stop char to each codeword.

Ex 3. General prefix-free code.



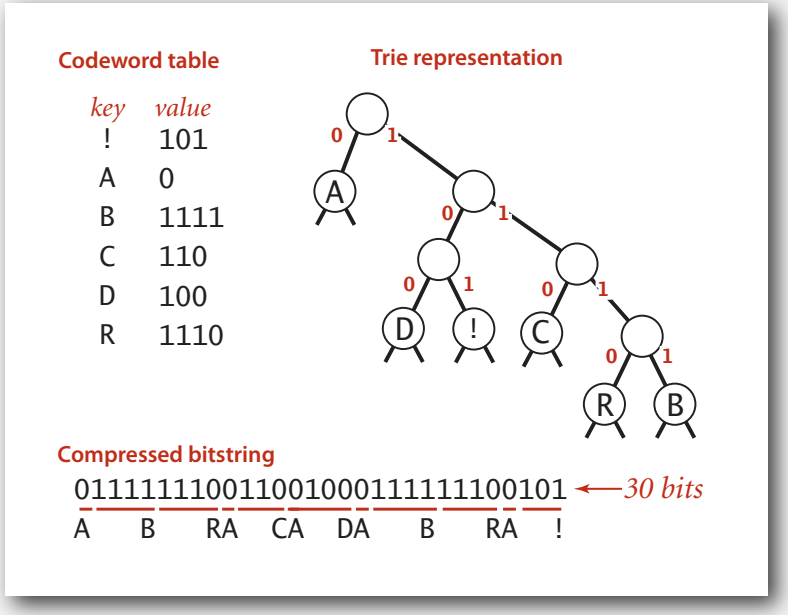



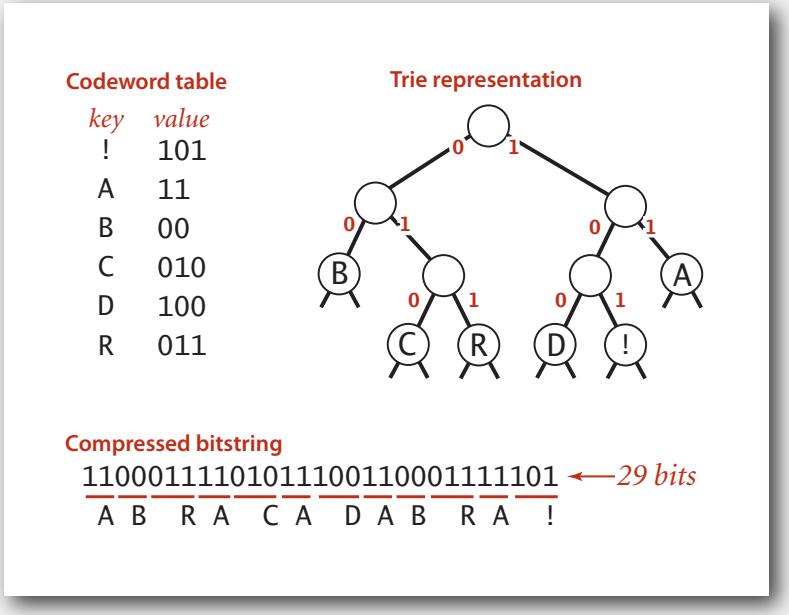
# Prefix-free codes: trie representation

Q. How to represent the prefix-free code?

A. A binary trie!

- Chars in leaves.
- Codeword is path from root to leaf.



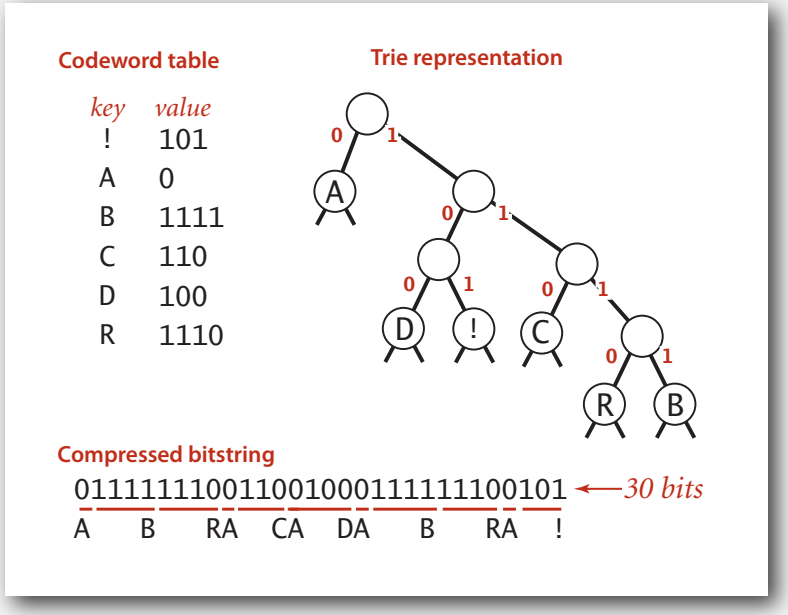
# Prefix-free codes: compression and expansion

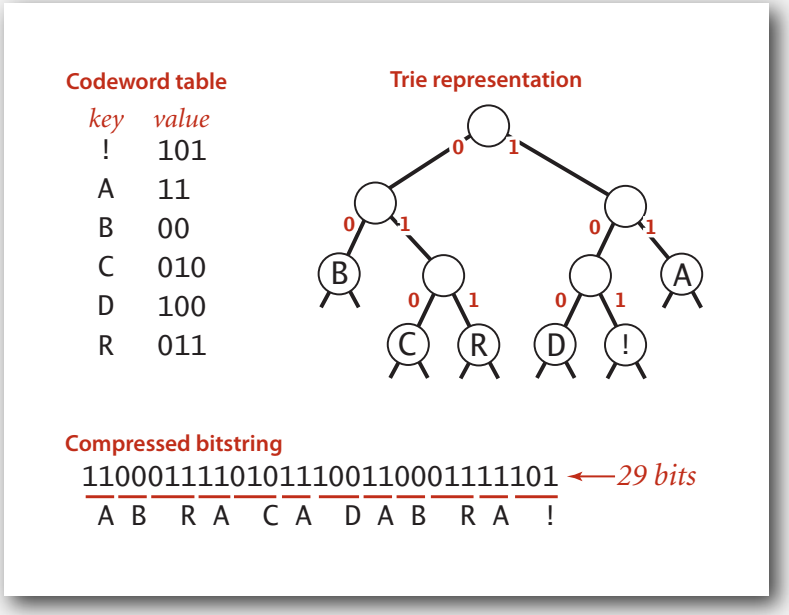
## Compression.

- Method 1: start at leaf; follow path up to the root; print bits in reverse.
- Method 2: create ST of key-value pairs.

## Expansion.

- Start at root.
- Go left if bit is 0; go right if 1.
- If leaf node, print char and return to root.



# Huffman trie node data type

```
private static class Node implements Comparable<Node>
{
   private final char ch;   // used only for leaf nodes
   private final int freq;  // used only for compress
   private final Node left, right;

   public Node(char ch, int freq, Node left, Node right)
   {
      this.ch    = ch;
      this.freq  = freq;
      this.left  = left;
      this.right = right;
   }

   public boolean isLeaf()
   {  return left == null && right == null; }

   public int compareTo(Node that)
   {  return this.freq - that.freq;  }
}
```

initializing constructor

is Node a leaf?

compare Nodes by frequency (stay tuned)

# Prefix-free codes: expansion

```
public void expand()
{
   Node root = readTrie();
   int N = BinaryStdIn.readInt();

   for (int i = 0; i < N; i++)
   {
      Node x = root;
      while (!x.isLeaf())
      {
         if (!BinaryStdIn.readBoolean())
            x = x.left;
         else
            x = x.right;
      }
      BinaryStdOut.write(x.ch, 8);
   }
   BinaryStdOut.close();
}
```

read in encoding trie

read in number of chars

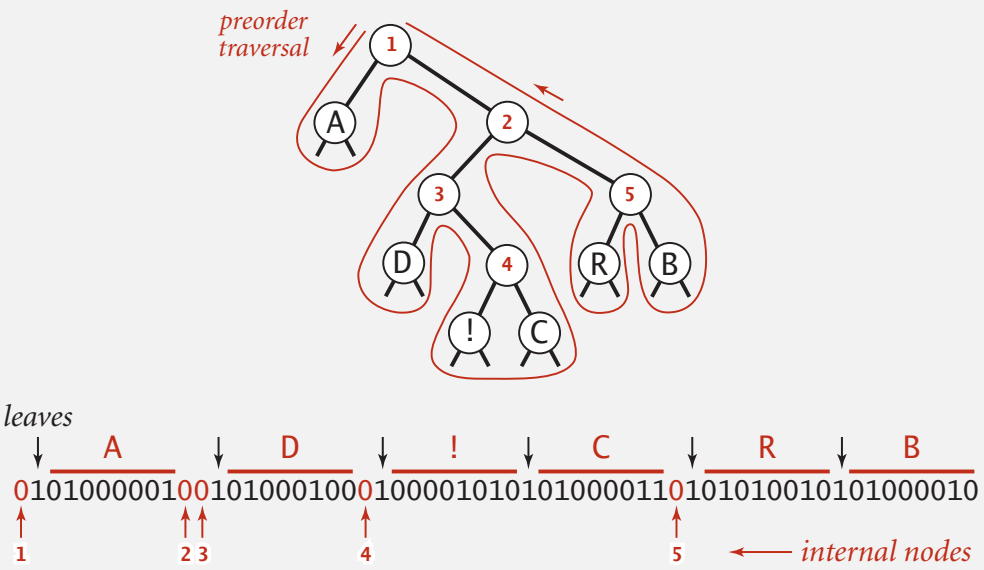
expand codeword for $i^{th}$ char

Running time. Linear in input size $N$.

# Prefix-free codes: how to transmit

Q. How to write the trie?

A. Write preorder traversal of trie; mark leaf and internal nodes with a bit.




Using preorder traversal to encode a trie as a bitstream

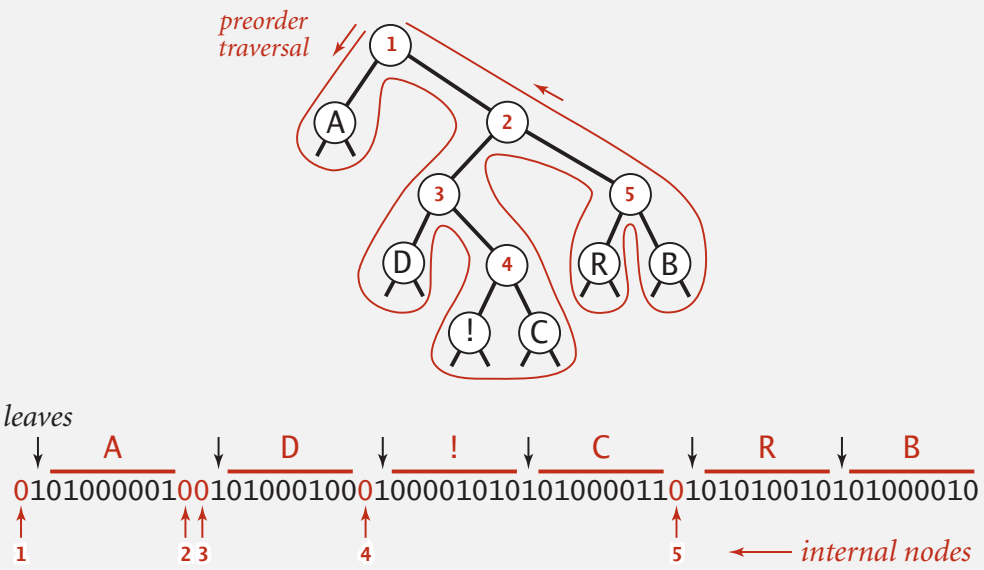
```
private static void writeTrie(Node x)
{
   if (x.isLeaf())
   {
      BinaryStdOut.write(true);
      BinaryStdOut.write(x.ch, 8);
      return;
   }
   BinaryStdOut.write(false);
   writeTrie(x.left);
   writeTrie(x.right);
}
```

Note. If message is long, overhead of transmitting trie is small.

# Prefix-free codes: how to transmit

Q. How to read in the trie?

A. Reconstruct from preorder traversal of trie.




Using preorder traversal to encode a trie as a bitstream

```
private static Node readTrie()
{
   if (BinaryStdIn.readBoolean())
   {
      char c = BinaryStdIn.readChar(8);
      return new Node(c, 0, null, null);
   }
   Node x = readTrie();
   Node y = readTrie();
   return new Node('\0', 0, x, y);
}
```

used only for leaf nodes

# Shannon-Fano codes

Q. How to find best prefix-free code?

Shannon-Fano algorithm:

- Partition symbols $S$ into two subsets $S_0$ and $S_1$ of (roughly) equal freq.
- Codewords for symbols in $S_0$ start with $0$; for symbols in $S_1$ start with $1$.
- Recur in $S_0$ and $S_1$.

| char | freq | encoding |
|---|---|---|
| A | 5 | 0... |
| C | 1 | 0... |

**$S_0$ = codewords starting with 0**

| char | freq | encoding |
|---|---|---|
| B | 2 | 1... |
| D | 1 | 1... |
| R | 2 | 1... |
| ! | 1 | 1... |

**$S_1$ = codewords starting with 1**

Problem 1. How to divide up symbols?

Problem 2. Not optimal!

# Huffman algorithm demo

- Count frequency for each character in input.

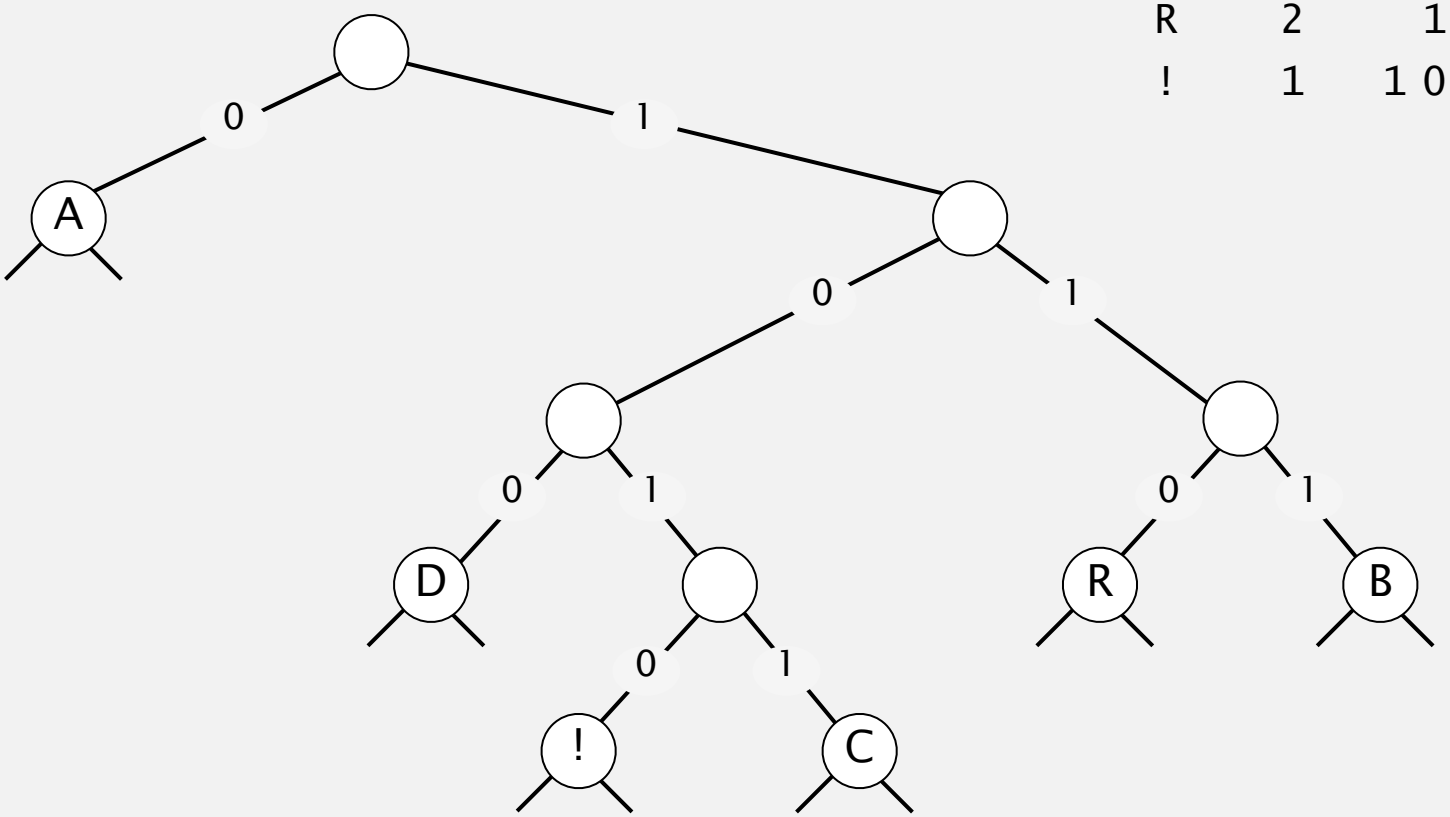
| char | freq | encoding |
|---|---|---|
| A | 5 | |
| B | 2 | |
| C | 1 | |
| D | 1 | |
| R | 2 | |
| ! | 1 | |

**input**

A B R A C A D A B R A !

# Huffman algorithm demo

| char | freq | encoding |
|---|---|---|
| A | 5 | 0 |
| B | 2 | 1 1 1 |
| C | 1 | 1 0 1 1 |
| D | 1 | 1 0 0 |
| R | 2 | 1 1 0 |
| ! | 1 | 1 0 1 0 |



# Huffman codes

Q. How to find best prefix-free code?

## Huffman algorithm:

- Count frequency `freq[i]` for each char `i` in input.
- Start with one node corresponding to each char `i` (with weight `freq[i]`).
- Repeat until single trie formed:
  - select two tries with min weight `freq[i]` and `freq[j]`
  - merge into single trie with weight `freq[i] + freq[j]`

## Applications:

# Constructing a Huffman encoding trie: Java implementation

```
private static Node buildTrie(int[] freq)
{
   MinPQ<Node> pq = new MinPQ<Node>();
   for (char i = 0; i < R; i++)
      if (freq[i] > 0)
         pq.insert(new Node(i, freq[i], null, null));

   while (pq.size() > 1)
   {
      Node x = pq.delMin();
      Node y = pq.delMin();
      Node parent = new Node('\0', x.freq + y.freq, x, y);
      pq.insert(parent);
   }

   return pq.delMin();
}
```

initialize PQ with singleton tries

merge two smallest tries

not used for internal nodes

total frequency

two subtries

# Huffman encoding summary

Proposition. [Huffman 1950s] Huffman algorithm produces an optimal prefix-free code.

no prefix-free code uses fewer bits

Pf. See textbook.

Implementation.

- Pass 1: tabulate char frequencies and build trie.
- Pass 2: encode file by traversing trie or lookup table.

Running time. Using a binary heap $\Rightarrow$ $N + R \log R$.

input size

alphabet size

Q. Can we do better? [stay tuned]


http://algs4.cs.princeton.edu

# 5.5 Data Compression

- *introduction*
- *run-length coding*
- *Huffman compression*
- *LZW compression*