Algorithms for Game Development (ALGO6008)

Assignment 2 - Eight Puzzle Solver

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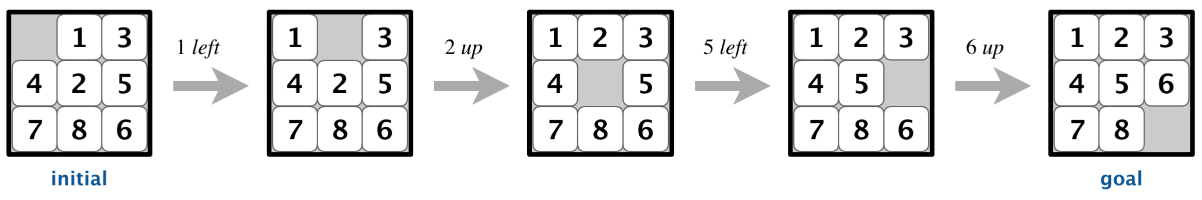
The Problem

The 8-puzzle is a sliding puzzle that is played on a 3-by-3 grid with 8 square tiles labelled 1 through 8, plus a blank square. The goal is to rearrange the tiles so that they are in order, using as few moves as possible. You are permitted to slide tiles either horizontally or vertically into the blank square.

The following diagram shows a sequence of moves from an initial board (left) to the goal board (right).



A* search

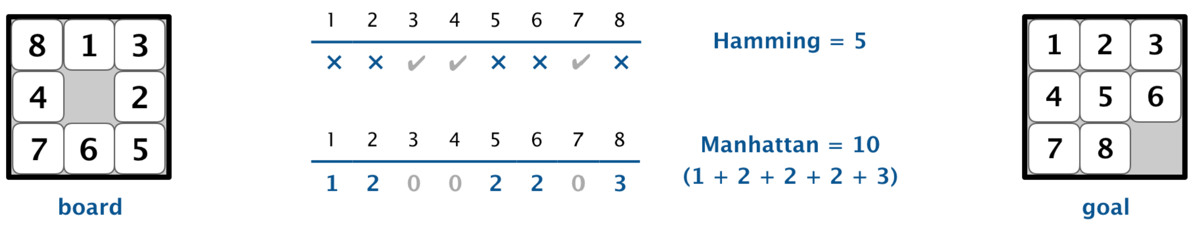
Now, we describe a solution to the 8-puzzle problem that illustrates a general artificial intelligence methodology known as the A* search algorithm. We define a *search node* of the game to be a board, the number of moves made to reach the board, and the previous search node. First, insert the initial search node (the initial board, 0 moves, and a null previous search node) into a priority queue. Then, delete from the priority queue the search node with the minimum priority, and insert onto the priority queue all neighboring search nodes (those that can be reached in one move from the dequeued search node). Repeat this procedure until the search node dequeued corresponds to the goal board.

The success of this approach hinges on the choice of *priority function* for a search node. We consider two priority functions:

- The *Hamming priority function* is the Hamming distance (number of tiles in tghe wrong position) of a board plus the number of moves made so far to get to the search node.

Intuitively, a search node with a small number of tiles in the wrong position is close to the goal, and we prefer a search node if has been reached using a small number of moves.

- The *Manhattan priority function* is the Manhattan distance (sum of the vertical and horizontal distance) of a board plus the number of moves made so far to get to the search node.



To solve the puzzle from a given search node on the priority queue, the total number of moves we need to make (including those already made) is at least its priority, using either the Hamming or Manhattan priority function.

- For Hamming priority, this is true because each block that is out of place must move at least once to reach its goal position.
- For Manhattan priority, this is true because each block must move its Manhattan distance from its goal position.

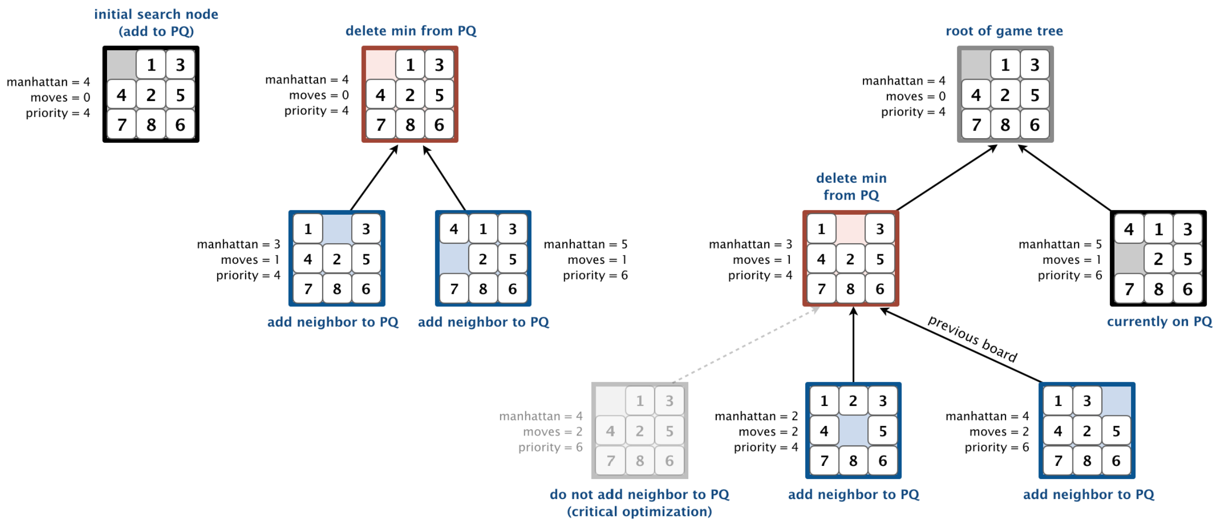
Consequently, when the goal board is dequeued, we have discovered not only a sequence of moves from the initial board to the goal board, but one that makes the *fewest* moves.

Note: we do not count the blank tile when computing the Hamming or Manhattan priorities.

Game tree

One way to view the computation is as a *game tree*, where each search node is a node in the game tree and the children of a node correspond to its neighboring search nodes. The root of the game tree is the initial search node; the internal nodes have already been processed; the leaf nodes are maintained in a *priority queue*; at each step, the A* algorithm removes the node with the smallest priority from the priority queue and processes it (by adding its children to both the game tree and the priority queue).

For example, the following diagram illustrates the game tree after each of the first three steps of running the A* search algorithm on a 3-by-3 puzzle using the Manhattan priority function.



The Assignment

Write a program that reads the initial board from standard input and prints to standard output a sequence of board positions that solves the puzzle in the fewest number of moves. Also print out the total number of moves. The input and output format for a board is the board dimension n followed by the n-by-n initial board position, using 0 to represent the blank square. As an example,

```
~/Desktop/EightPuzzle> more puzzle04.txt
3
0 1 3
4 2 5
7 8 6
```

```
~/Desktop/EightPuzzle> java EightPuzzleSolver puzzle04.txt
Minimum number of moves = 4
3
0 1 3
4 2 5
7 8 6

3
1 0 3
4 2 5
7 8 6

3
1 2 3
4 0 5
7 8 6

3
1 2 3
4 5 0
7 8 6

3
1 2 3
4 5 6
7 8 0
```

Note that your program should work for arbitrary n-by-n boards (for any n greater than 1), even if it is too slow to solve some of them in a reasonable amount of time.

Board data type

To begin, create a data type that models an n-by-n board with sliding tiles. Implement an immutable data type `Board` with the following API:

```
public class Board {

    // create a board from an n-by-n array of tiles,
    // where tiles[row][col] = tile at (row, col)
    public Board(int[][] tiles)

    // tile at (row, col) or 0 if blank
    public int tileAt(int row, int col)

    // board size n
    public int size()

    // number of tiles out of place
    public int hamming()

    // sum of Manhattan distances between tiles and goal
    public int manhattan()

    // is this board the goal board?
    public boolean isGoal()

    // does this board equal y?
    public boolean equals(Object y)

    // all neighboring boards
    public Iterable<Board> neighbors()

    // string representation of this board
    public String toString()

    // unit testing (required)
    public static void main(String[] args)
}
```

Notes:

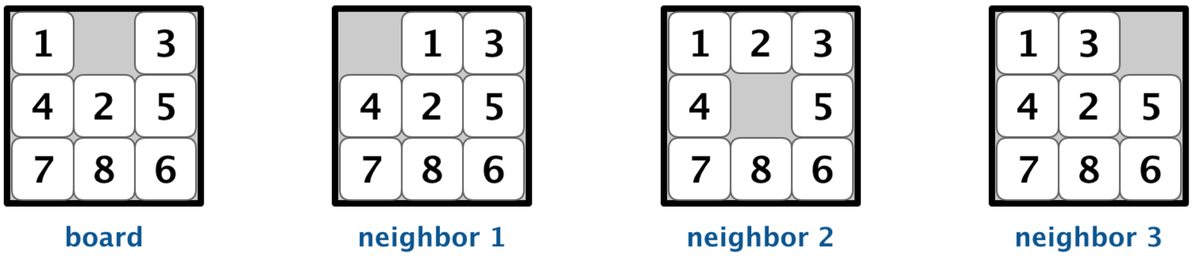
Constructor. You may assume that the constructor receives an n-by-n array containing a permutation of the n^2 integers between 0 and n^{2-1}, where 0 represents the blank square. You may also assume that $2 \leq n \leq 32,768$.

Tile extraction. Throw an `IllegalArgumentException` in `tileAt()` unless both `row` and `col` are between 0 and $n - 1$.

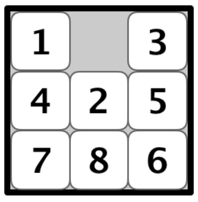
Hamming and Manhattan distances. To measure how close a board is to the goal board, we define two notions of distance. The *Hamming distance* between a board and the goal board is the number of tiles in the wrong position. The *Manhattan distance* between a board and the goal board is the sum of the Manhattan distances (sum of the vertical and horizontal distance) from the tiles to their goal positions.

Comparing two boards for equality. Two boards are equal if they are have the same size and their corresponding tiles are in the same positions. The `equals()` method is inherited from `java.lang` ↪ `.Object`, so it must obey all of Java's requirements.

Neighbouring boards. The `neighbors()` method returns an iterable containing the neighbour of the board. Depending on the location of the blank square, a board can have 2, 3, or 4 neighbours.



String representation. The `toString()` method returns a string composed of $ n + 1 $ lines. The first line contains the board size n; the remaining n lines contains the n-by-n grid of tiles in row-major order, using 0 to designate the blank square.



board

```
3
 1  0  3
 4  2  5
 7  8  6
```
string representation

Unit testing. Your `main()` method must call each public method directly and help verify that they works as prescribed (e.g., by printing results to standard output).

Solver data type

In this part, you will implement A* search to solve n-by-n slider puzzles. Create an immutable data type EightPuzzleSolver with the following API:

```
public class EightPuzzleSolver {

    // find a solution to the initial board (using the A* algorithm)
    public Solver(Board initial)

    // min number of moves to solve initial board
    public int moves()

    // sequence of boards in a shortest solution
    public Iterable<Board> solution()

    // test client (see below)
    public static void main(String[] args)

}
```

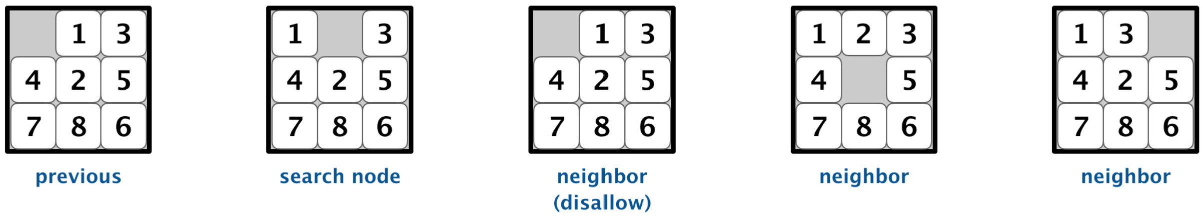
To implement the A* algorithm, you must use the MinPriorityQueue data type for the priority queue.

Test client. Your test client should take the name of an input file as a command-line argument; print the minimum number of moves to solve the puzzle; and print the sequence of boards in a corresponding solution (with each board preceded by a blank line). The input file contains the board size n, followed by the n-by-n grid of tiles, using 0 to designate the blank square.

Optimizations.

To speed up your solver, implement the following two optimizations:

1. *Critical optimization.* A* search has one annoying feature: search nodes corresponding to the same board are enqueued on the priority queue many times (e.g., the bottom-left search node in the game-tree diagram above). To reduce unnecessary exploration of useless search nodes, when considering the neighbors of a search node, don't enqueue a neighbor if its board is the same as the board of the previous search node in the game tree.



2. *Caching the Hamming and Manhattan distances.* To avoid recomputing the Hamming and Manhattan distances of a board from scratch each time during various priority-queue operations, precompute the values in the `Board` constructor; save them in instance variables; and return the saved values as needed. This *caching technique* is broadly applicable: consider using it in any situation where you are recomputing the same quantity many times *and* for which computing that quantity is a bottleneck operation.

Submission

Submit the files `Board.java` and `Solver.java` (with the Manhattan priority) to Moodle (date TBC)