Statistics, Algorithms & AI (COMP7028)

Offine Open-Book Exam

Tuesday 9th February 2021, 1 PM – 4:30 PM

Attempt ALL questions

Question 1

Draw the graph represented by the following adjacency matrix.

	A	B	C	D	E	F	G	H
A	0	0	2	3	4	0	0	0
B	0	0	3	0	0	6	2	0
C	2	3	0	5	0	4	0	1
D	3	0	5	0	0	0	0	0
E	4	0	0	0	0	0	4	2
F	0	6	4	0	0	0	2	0
G	0	2	0	0	4	2	0	0
H	0	0	1	0	2	0	0	0

Outline *Kruskal's* algorithm and show how it can be used to construct a minimum spanning tree using the graph from above.

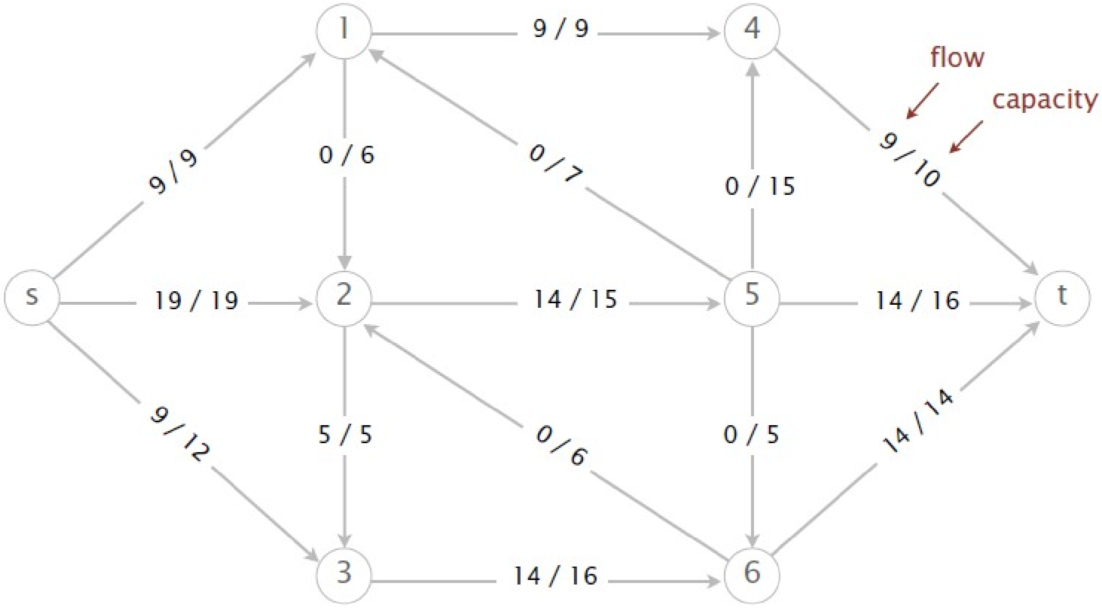
Outline *Dijkstra's* algorithm and show how it can be used to calculate the shortest path from a given source vertex to all other reachable nodes in a graph using the graph from part above and vertex *A* as the source vertex.

Question 2

Outline the *Ford-Fulkerson* algorithm for computing the maximum flow in a flow network.

Does the algorithm always terminate? If so, after how many iterations?

Consider the following *st-flow* network and feasible flow *f.*



(i) What is the value of the flow *f*?

(ii) Perform two iterations of the *Ford-Fulkerson* algorithm, starting from the flow *f*. Give the sequence of vertices on the augmenting path.

(iii) What is the value of the maximum flow?

(iv) List the vertices on the *s* side of the minimum cut.

(v) What is the capacity of the minimum cut?

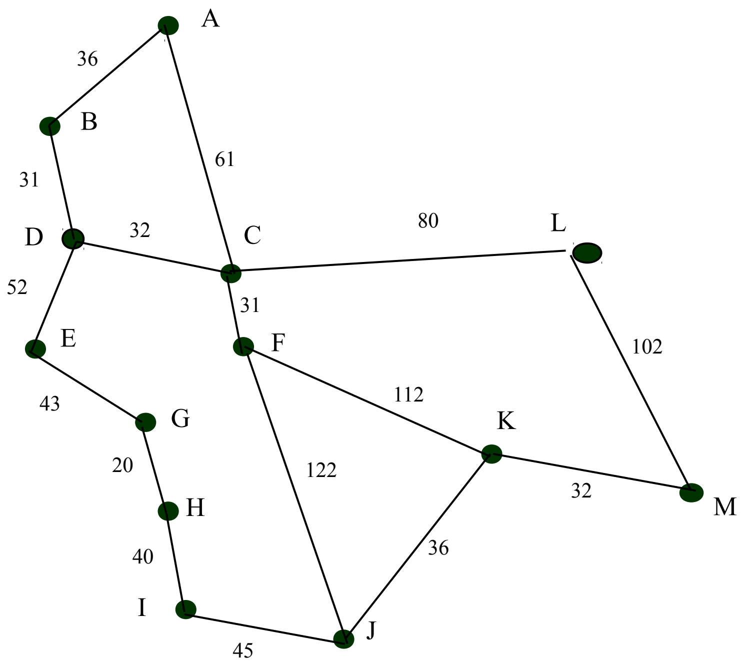
Question 3

What does it mean to say that a problem is in P or in NP? What does it mean to say that a problem is NP-complete?

What does in mean for a problem to be intractable? What approaches can taken when faced with an intractable problem.

Question 4

Consider the following map:



Using the A* algorithm work out a route from town A to town M. Use the following cost functions:

- $g(n)$ = The cost of each move as the distance between each town (shown on map)
- $h(n)$ = The Straight Line Distance between any town and town M. These distances are given in the table below.

i) Provide the search tree for your solution.
ii) Indicate the order in which you expanded the nodes and state the route you would take and the cost of that route.

Straight Line Distance to M

A	223
B	222
C	166
D	192

E	165
F	136
G	122

H	111
I	100
J	60

K	32
L	102
M	0

Question 5

For the operators and initial state description given below, explain how a regression planner searches for a plan to satisfy a goal, and give an example of a plan that achieves the goal $On(b, a) \wedge On(c, b) \wedge OnTable(a)$

- blocks are represented by constants: a, b, c, ... etc.
- states are described using the following predicates:

On(*x*, *y*)	block *x* is on block *y*
OnTable(*x*)	block *x* is on the table
Clear(*x*)	there is no block on top of block *x*
Holding(*x*)	the arm is holding block *x*
ArmEmpty	the arm is not holding any block

- initial state: $On(c, a) \wedge On(a, b) \wedge OnTable(b) \wedge ArmEmpty$
- goal state: $On(b, a) \wedge On(a, c) \wedge OnTable(c)$
- operators:

  [*Holding*(*x*), *Clear*(*y*)] ***STACK***(*x*, *y*) [*On*(*x*, *y*), *ArmEmpty*, ¬*Holding*(*x*), ¬*Clear*(*y*)]

  [*On*(*x*, *y*), *Clear*(*x*), *ArmEmpty*] ***UNSTACK***(*x*, *y*) [*Clear*(*y*), *Holding*(*x*), ¬*On*(*x*, *y*), ¬*ArmEmpty*]

  [*OnTable*(*x*), *Clear*(*x*), *ArmEmpty*] ***PICKUP***(*x*) [*Holding*(*x*), ¬*OnTable*(*x*), ¬*ArmEmpty*,]

  [*Holding*(*x*)] ***PUTDOWN***(*x*) [*OnTable*(*x*), *ArmEmpty*, ¬*Holding*(*x*),]

Question 6

You are a robot in a lumber yard, and must learn to discriminate Oak wood from Pine wood. You choose to learn a *Decision Tree* classifier. You are given the following examples:

Example	Density	Grain	Hardness	Class
1	Heavy	Small	Hard	Oak
2	Heavy	Large	Hard	Oak
3	Heavy	Small	Hard	Oak
4	Light	Large	Soft	Oak
5	Light	Large	Hard	Pine
6	Heavy	Small	Soft	Pine
7	Heavy	Large	Soft	Pine
8	Heavy	Small	Soft	Pine

i) Draw the decision tree that would be constructed by recursively applying information gain to determine the most informative attribute.

ii) Classify these new examples as Oak or Pine using your decision tree above.

Density = Light, Grain = Small, Hardness = Hard
Density = Light, Grain = Small, Hardness = Soft

Question 7

For each of the truth tables below say whether it is possible for a perceptron to learn the required output.

In each case, explain the reason behind your decision.

i)

Input	0	0	1	1
Input	0	1	0	1
Required Output	1	0	0	1

ii)

Input	0	0	1	1
Input	0	1	0	1
Required Output	1	1	0	0

iii)

Input	0	0	1	1
Input	0	1	0	1
Required Output	1	1	1	1