Intelligence II:
Advanced Decision-Making Mechanisms

Advanced Decision-making Mechanisms for this Class

- More on Behavior Trees
- Planning to Achieve the Goal
- Planning with Uncertainty to Achieve the Goal

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More on Behavior Trees

- Additional task: *Decorator*
- Has only one child whose execution it controls in a special way

Example:

“y decorators” decide whether to execute its children based on some conditions

More on Behavior Trees

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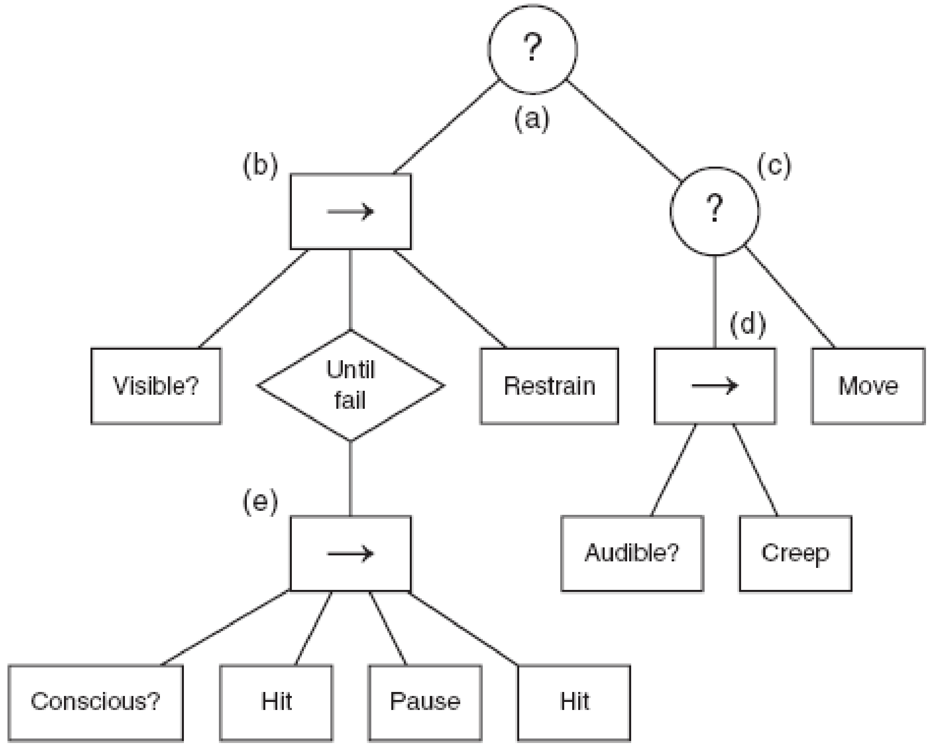
Example:

"Loop decorators" loop until failure



from "Artificial Intelligence for Games" by I. Millington & J. y ge

More on Behavior Trees

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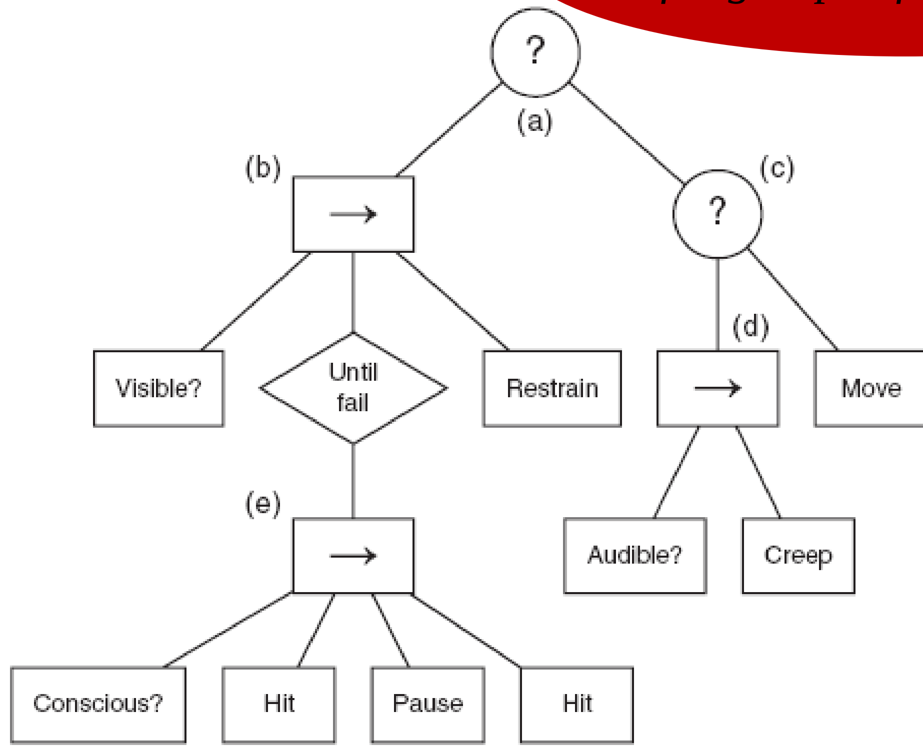
Example:

“Loop decorators” loop until fai

What about behavior trees for groups of characters?



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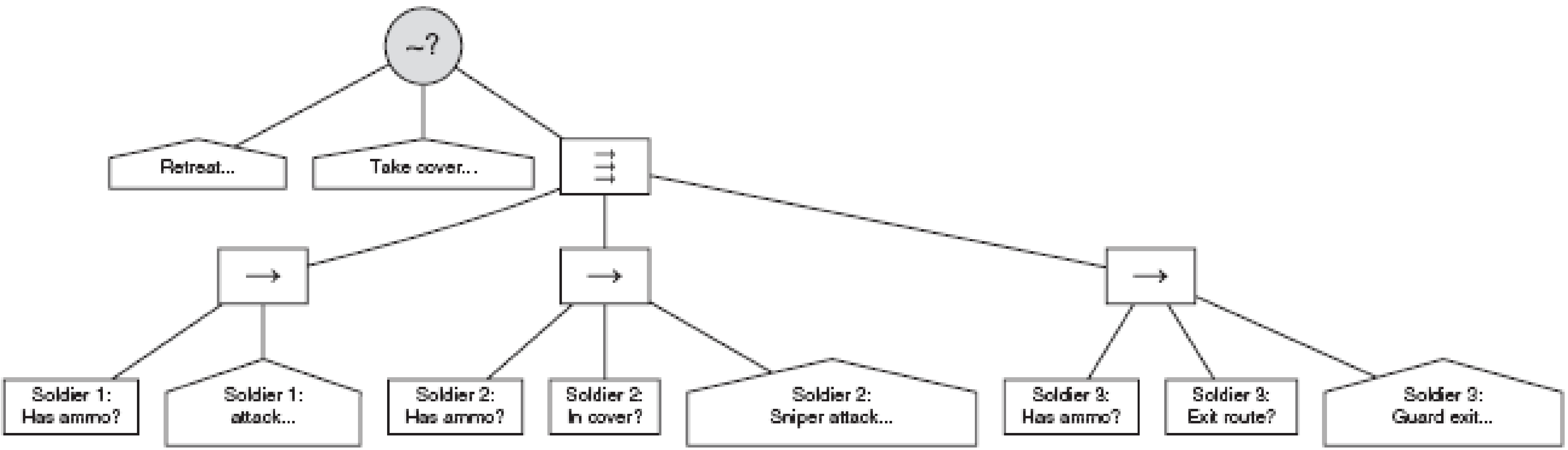
More on Behavior Trees

- Additional task: *Parallel*
- Executes tasks in parallel until first fails or all succeed

Example:



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More on Behavior Trees

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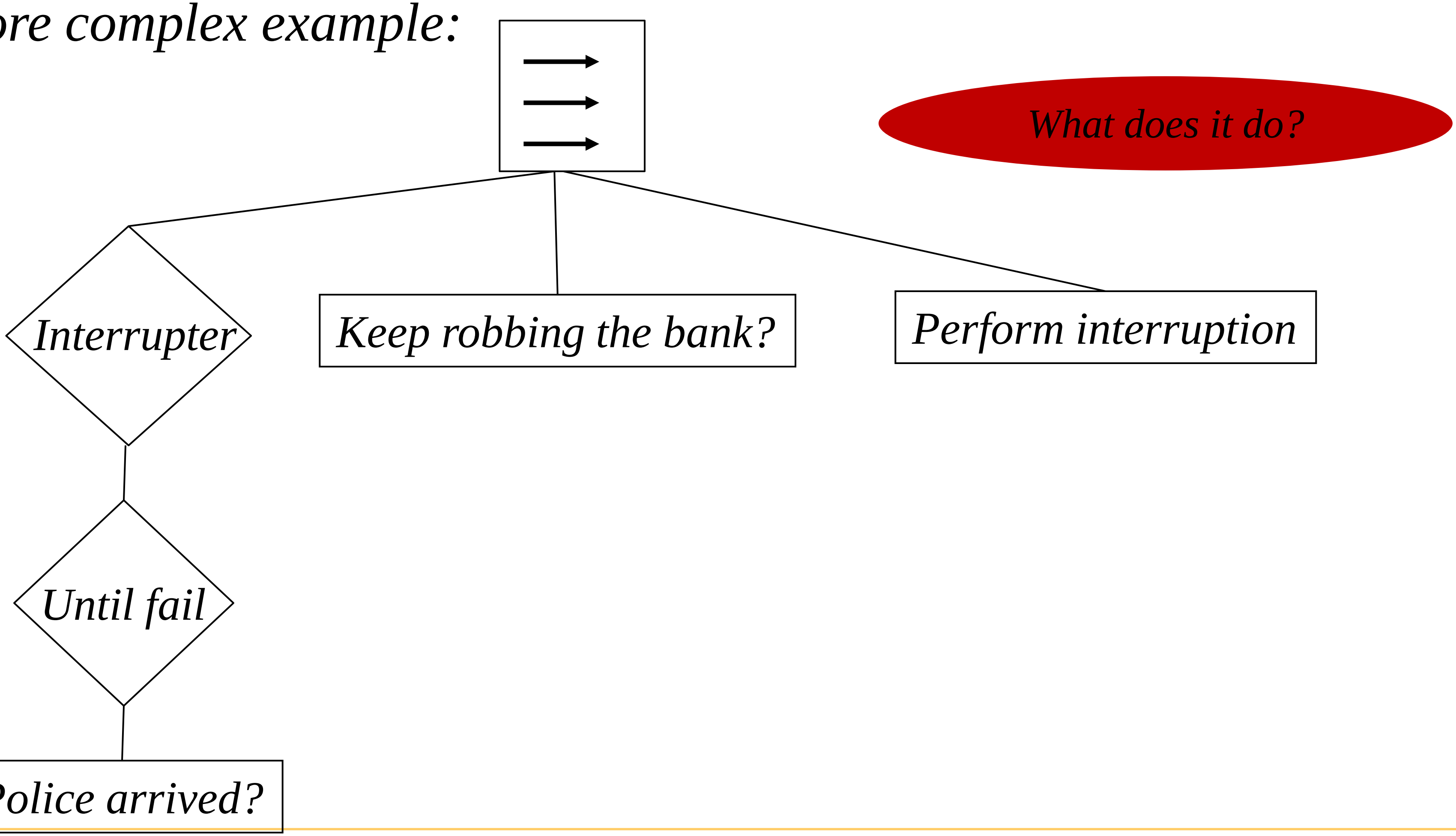
More complex example:



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Goal-Oriented Behavior

- Beyond hard-coding stimulus-response pairs
- Seeks to satisfy internal goals (e.g., hunger, threat, gold, etc.)

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Three components:

Goals (motives),

How Pressing each Goal is (insistence)

Actions with Expected Impact on the Insistence of Each Goal

Goal-Oriented Behavior

- Beyond hard-coding stimulus-response pairs
- Seeks to satisfy internal goals (e.g., hunger, threat, gold, etc.)

Example:

Goals with Insistence Values:

Eat = 9, Kill Enemy = 8, Get Healthy = 4

Actions with Impact on Insistence Values of Goals:

Get Food (Eat: -5)

Kill Enemy (Kill Enemy: -8, Get Healthy: +4)

Get Health Pack (Get Healthy: –2)

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Negative side effect of the action

Pick action that has the best net effect (could be weighted)

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Example:

Potential problems?

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Optimal sequence?

Example:

Suppose the character is under attack and can pick a weapon that allows it to more effectively shoot the enemies

Goals with Insistence Values:

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plus Kill Enemy action will have no impact on Health)

Goal-Oriented Behavior

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Optimal sequence:
Get Health Pack, Get Weapon, Kill Enemy

How to compute it?

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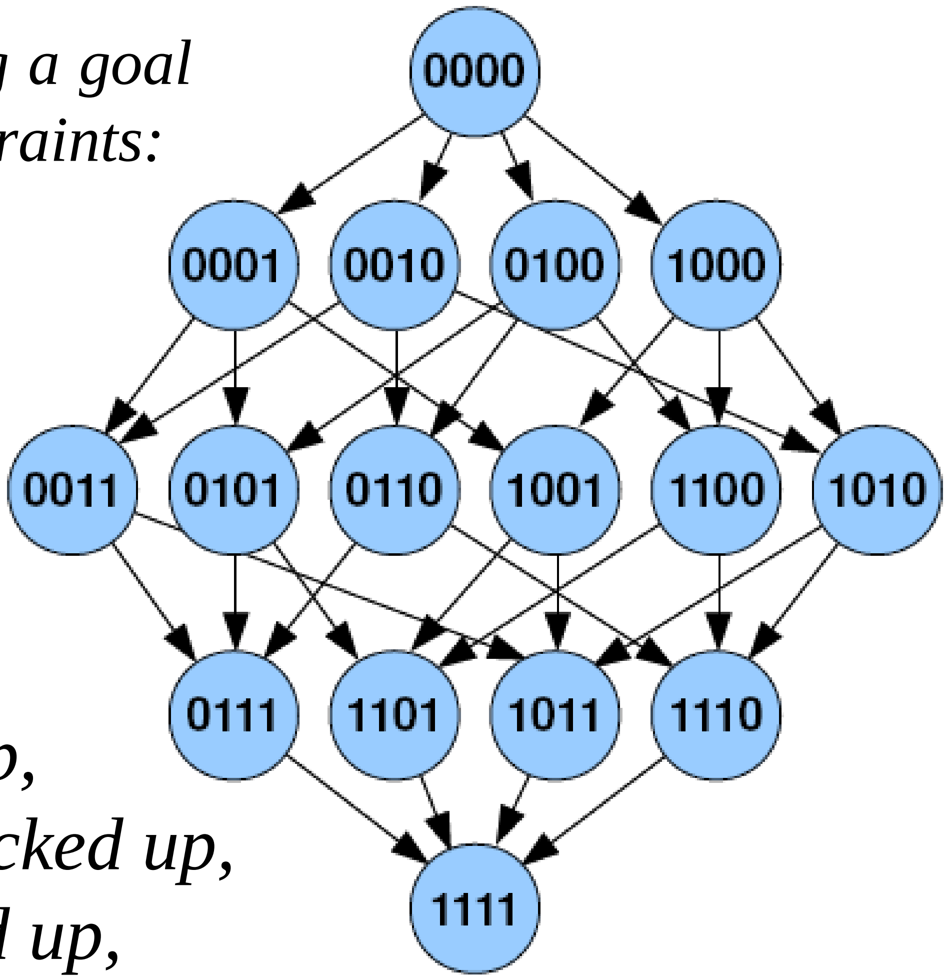
Goal-Oriented Planning

- Beyond single-step decisions

Search-space (graph) for finding a goal that satisfies the necessary constraints:



Example:

initial state is 0000

goal state is 1111

(e.g., bit 0: food is picked up,

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bit 3: enemy is shot)

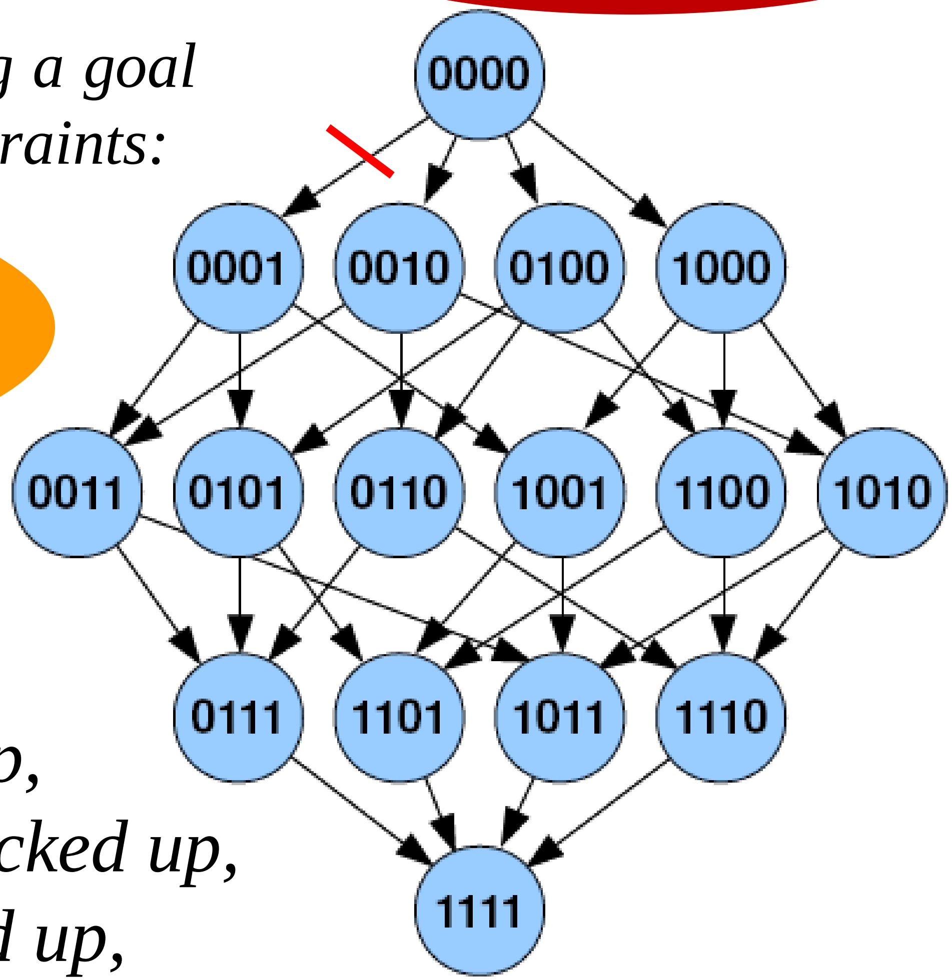
Goal-Oriented Planning

- Beyond single-step decisions

What could be the transition costs?

Search-space (graph) for finding a goal that satisfies the necessary constraints:

The validity of a transition depends on the source state (e.g., can't shoot enemy weapon was picked up)



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What if the goal is to shoot enemy (whether health pack and food are picked up or not)?

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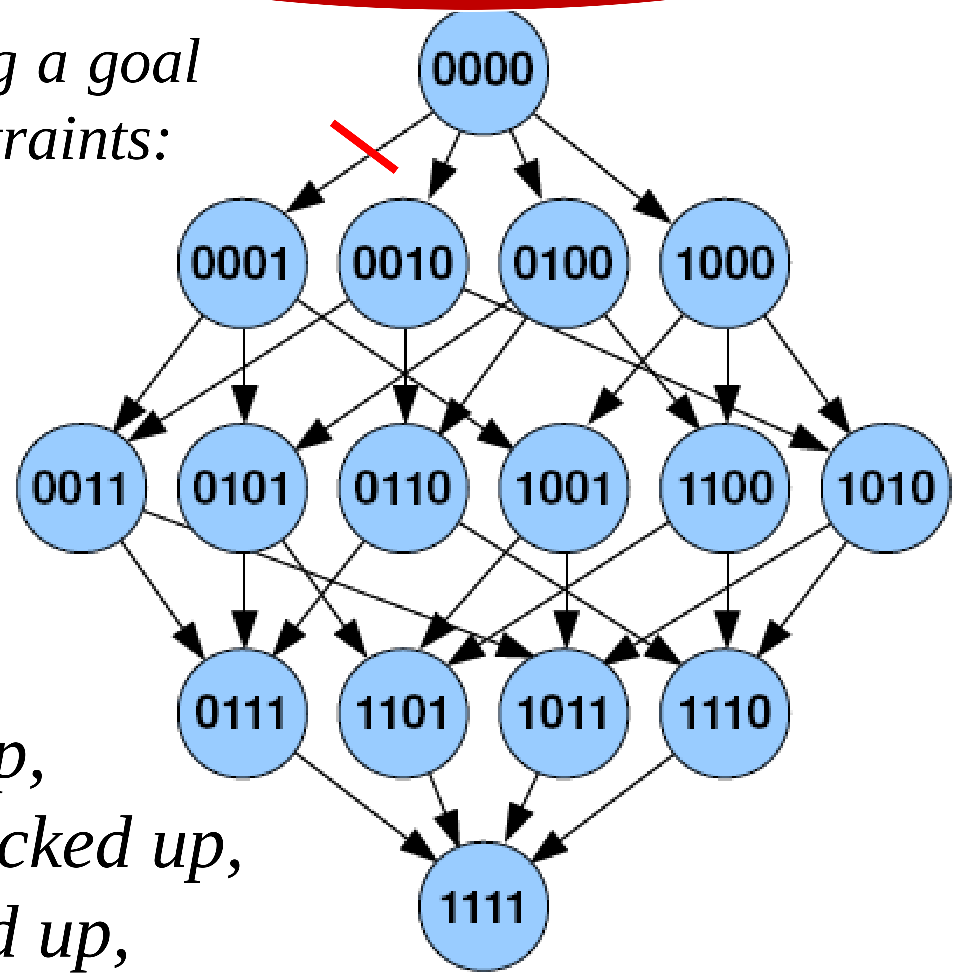

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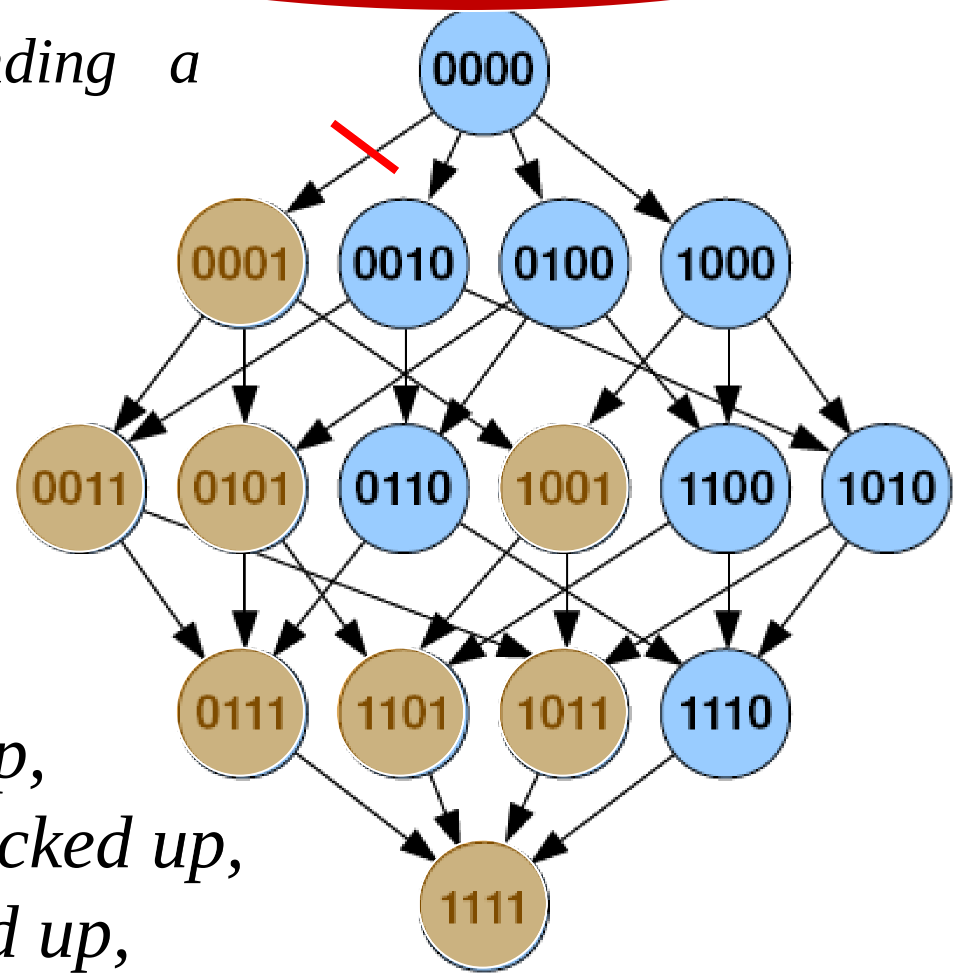

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Goal-Oriented Planning

What are the efficient way to represent the state vectors?

- Beyond single-step decisions

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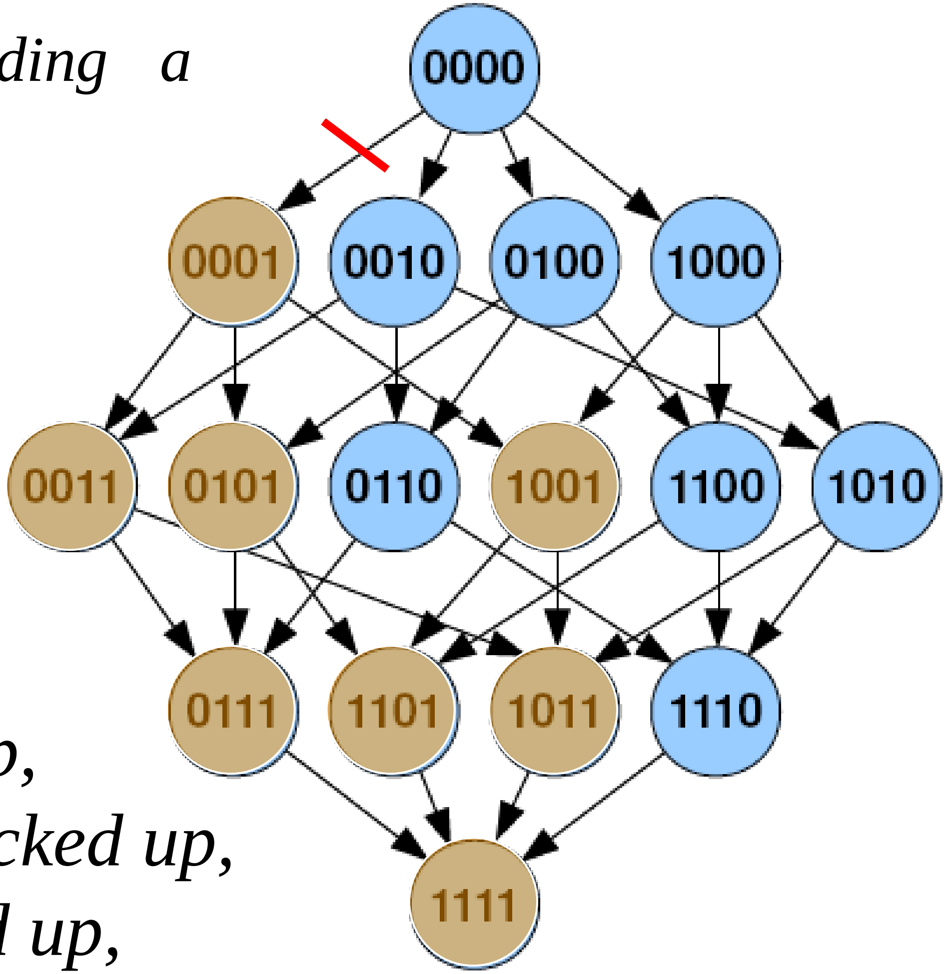

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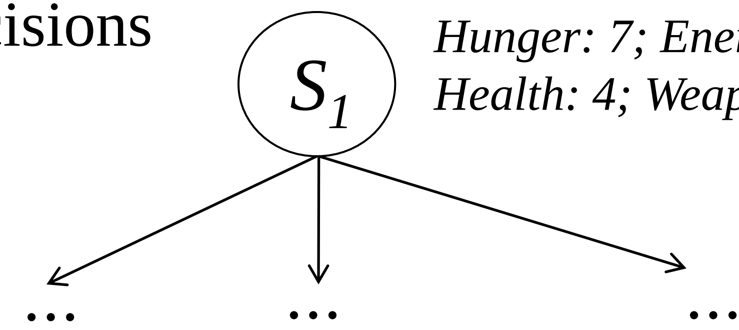
- Beyond single-step decisions



Hunger: 7; Enemy shot: 0;
Health: 4; Weapon: 1

Working out Example with some non-binary variables:

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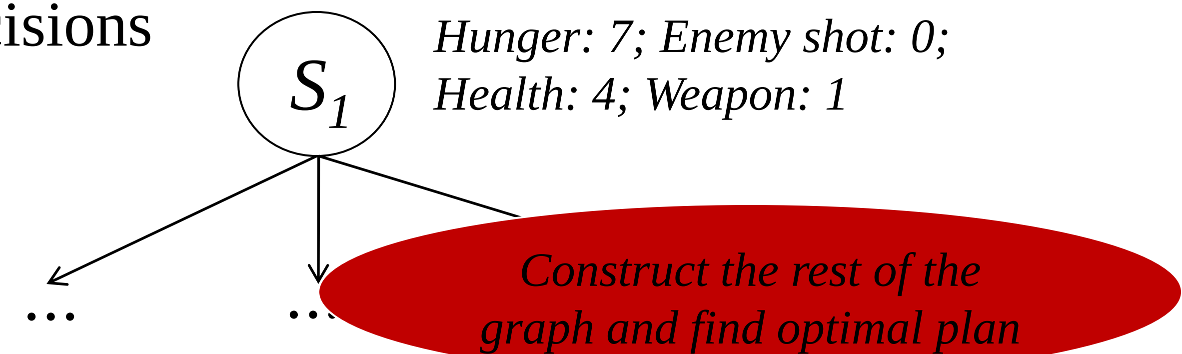
Goal-Oriented Planning

- Beyond single-step decisions



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Goal-Oriented Planning

- Beyond single-step decisions

How to search the graph?

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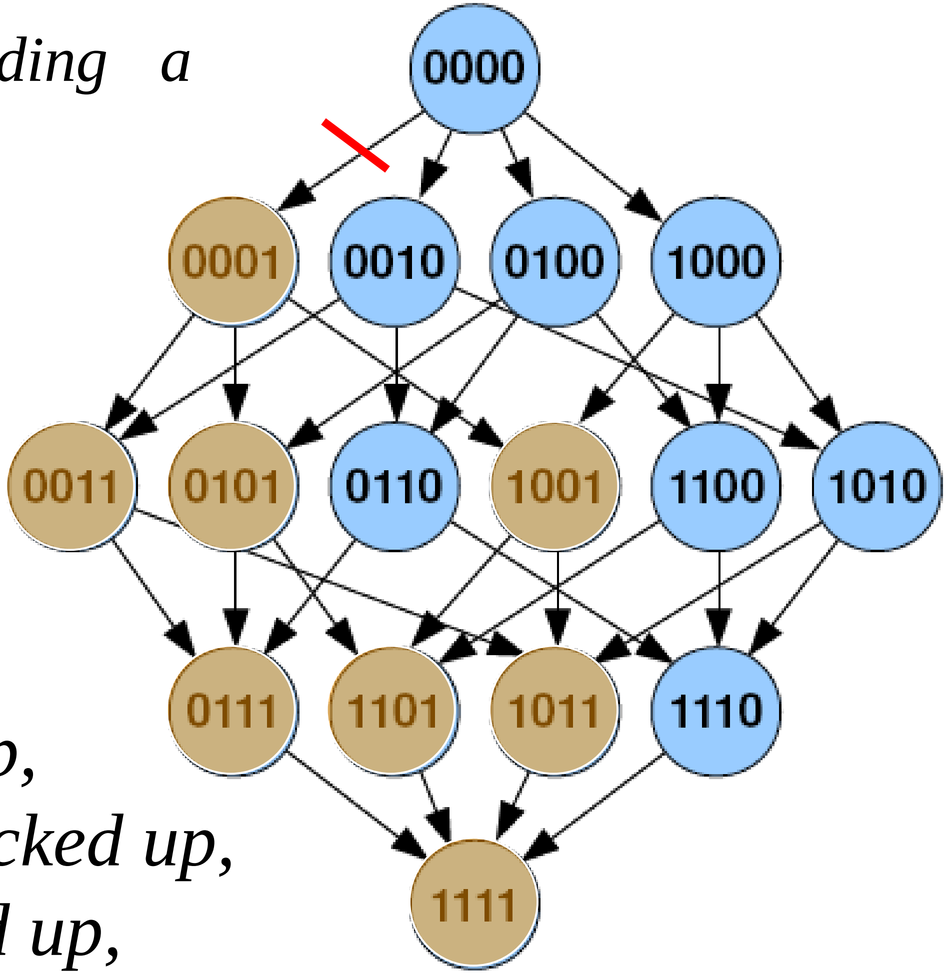

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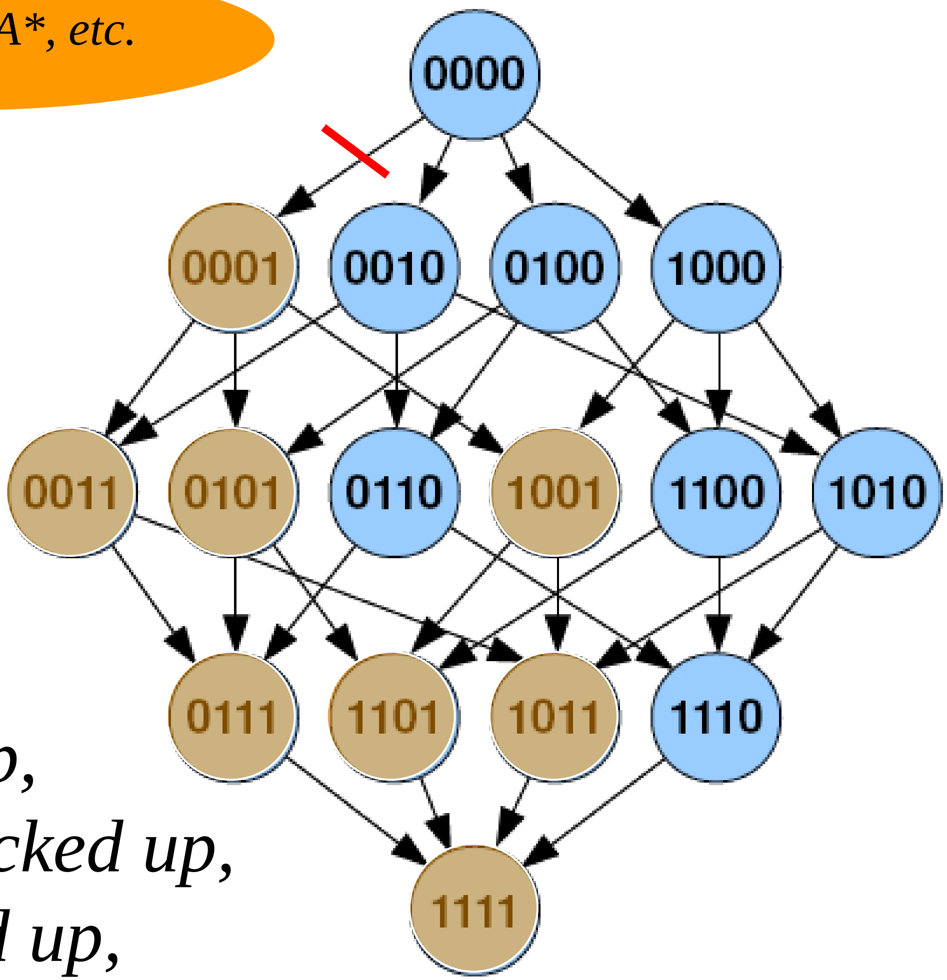
Goal-Oriented Planning

- Beyond single-step decisions

How to search the graph?

DFS, BFS, A, etc.*

Search-space (grap
partially-defined goal:



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- Beyond single-step decisions

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Search-space (grap

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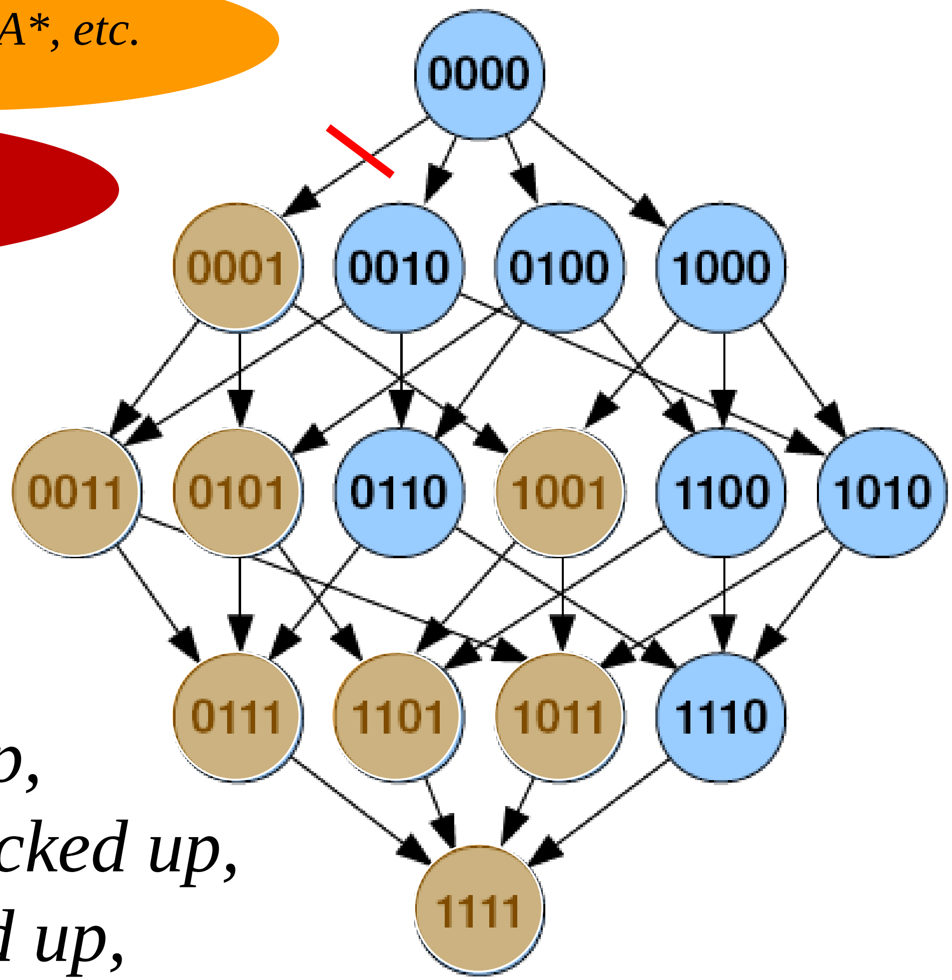
*Problems with BFS and A**
for large state vectors?



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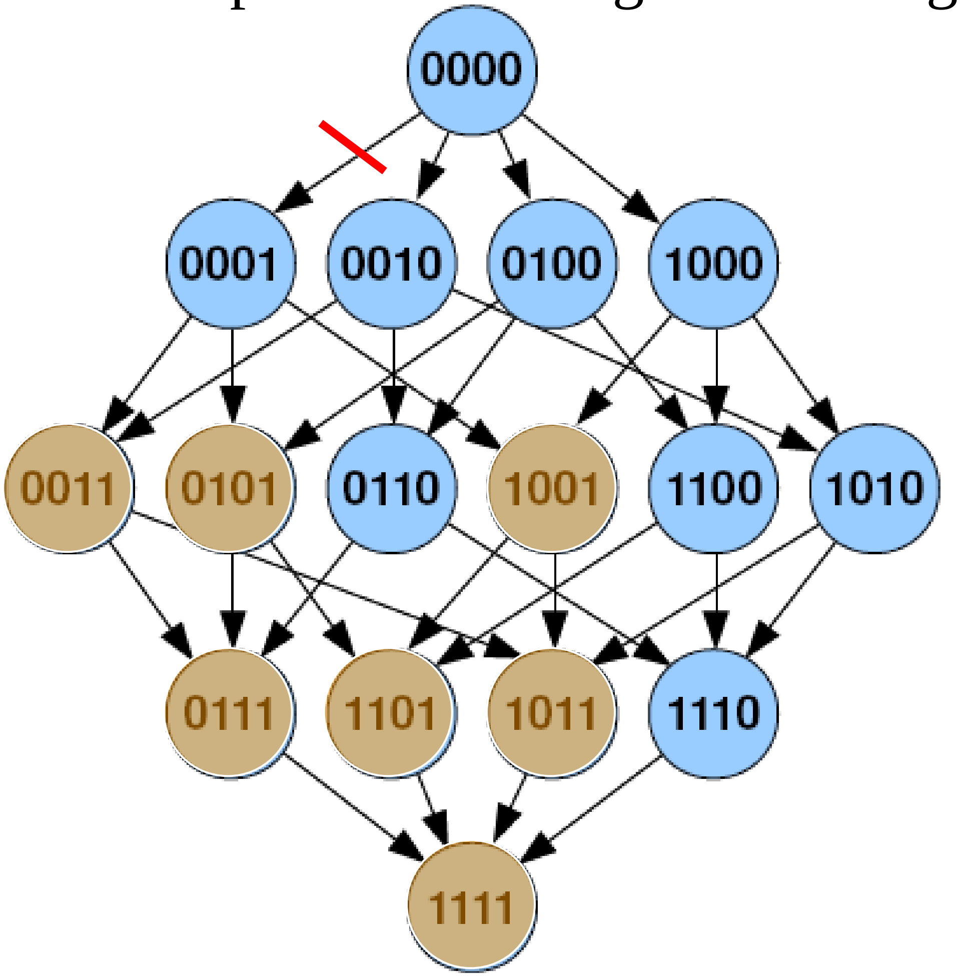
Goal-Oriented Planning

- Beyond single-step decisions
- **IDA***: Very popular search for state-spaces with large branching factors and shallow goals



Goal-Oriented Planning

- Beyond single-step decisions
- **IDA***: Very popular search for state-spaces with large branching factors and shallow goals

IDA* (Iterative Deepening A*)

1. set $f_{max} = 1$ (or some other small value)
2. traverse the graph in DFS fashion without expanding states with $f > f_{max}$
3. If path to the goal found, return the best path it finds
4. Otherwise $f_{max} = f_{max} + 1$ and go to step 2

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Proof?

- Complete and optimal in any state-space (with positive costs)
- Memory: $O(bl)$, where b – max. branching factor, l – length of optimal path
- Complexity: $O(kb^l)$, where k is the number of times DFS is called

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Goal-Oriented Planning Under Uncertainty

- Dealing with uncertainty in outcomes

How do we represent this?

Example:

Suppose the character is under attack and can pick a weapon that allows it to more effectively shoot the enemies

Suppose also there is 50% chance of getting health pack at each attempt

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Goal-Oriented Planning Under Uncertainty

- Dealing with uncertainty in outcomes

Markov Decision Processes (MDP)

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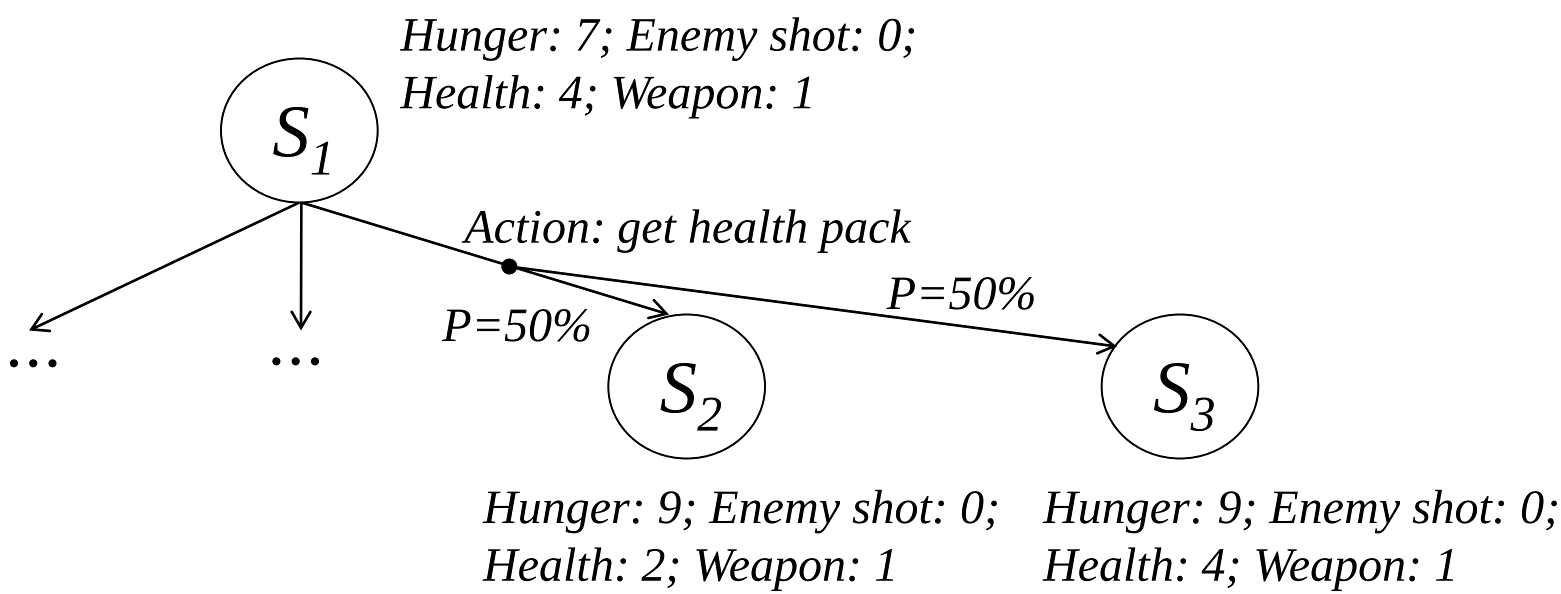
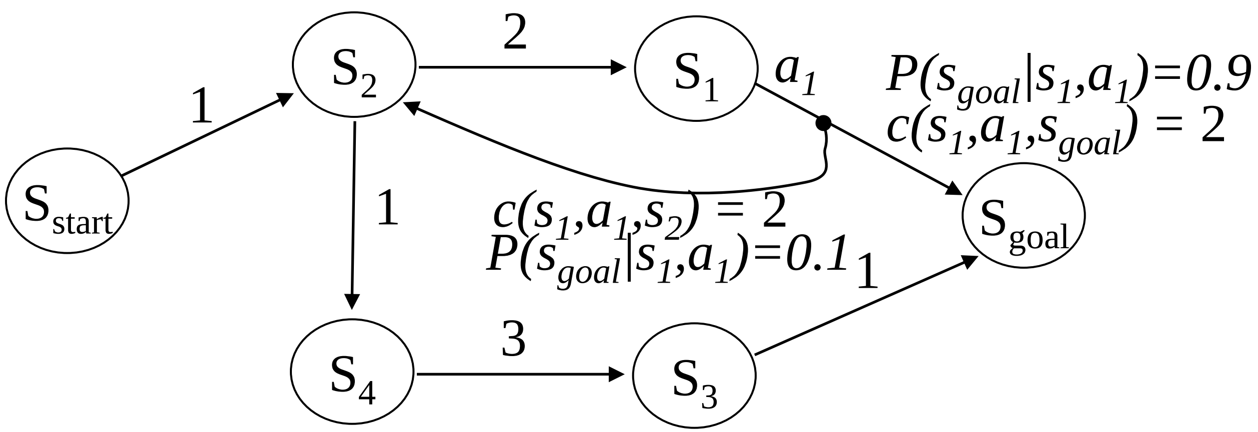


Planning in MDPs

- What plan to compute?
  - Plan that minimizes the worst-case scenario (minimax plan)
  - Plan that minimizes the expected cost
  - Plan that minimizes cost while guaranteeing *P(goal reached) > t*



- Without uncertainty, plan is a single path:
  a sequence of states (a sequence of actions)
- In MDPs, plan is a policy π:
  mapping from a state onto an action

Planning in MDPs

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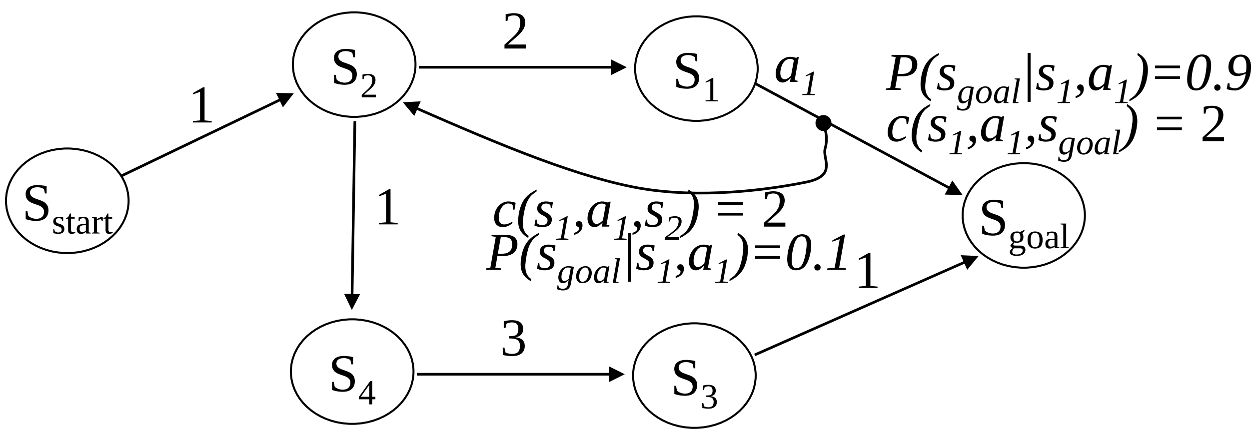
Which ones are policies?



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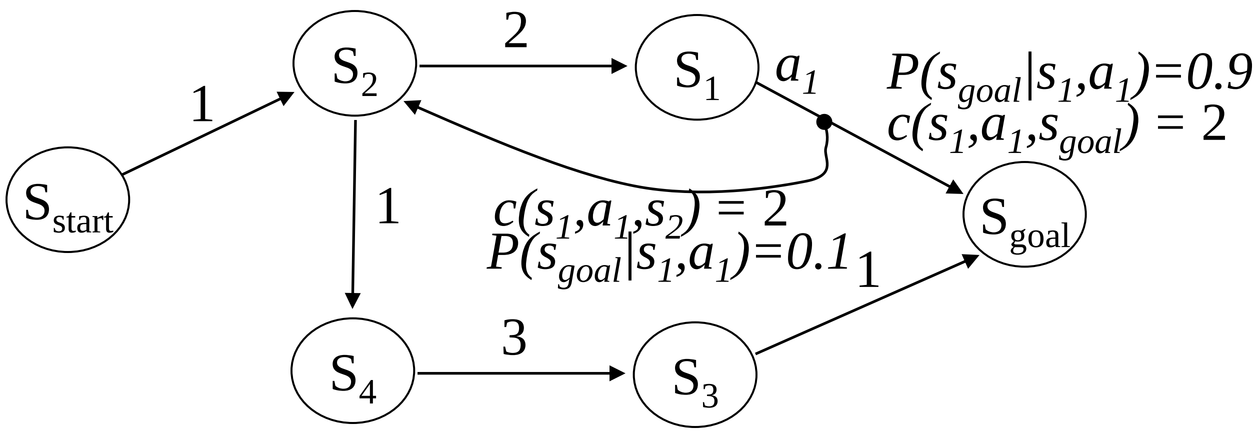
Why?

Minimax Formulation



- Optimal policy π^*:

  minimizes the *worst* cost-to-goal

  $\pi^* = argmin_{\pi}\ max_{outcomes\ of\ \pi}\{cost\text{-}to\text{-}goal\}$

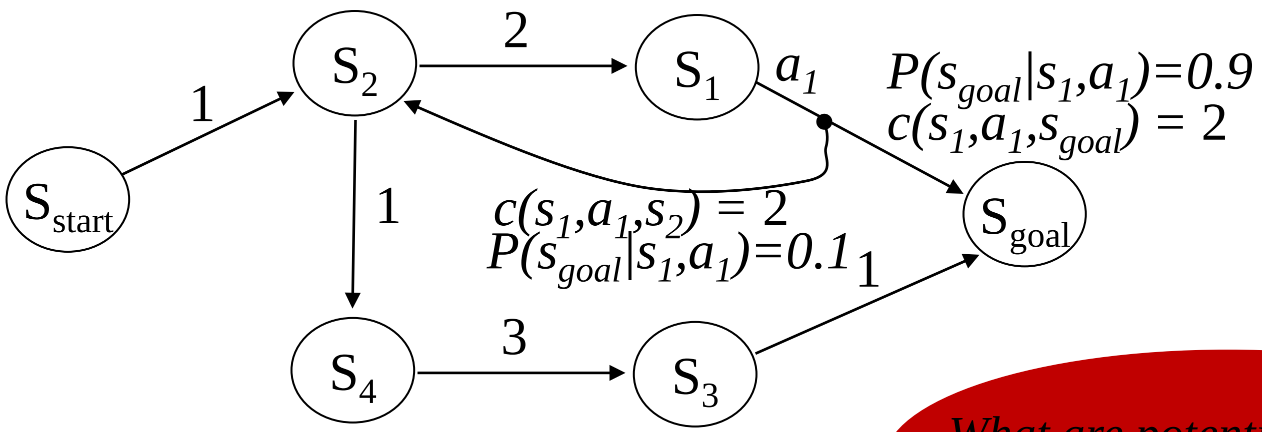
- worst cost-to-goal for $\pi_1=(s_{start},s_2,s_4,s_3,s_{goal})$ is:

  1+1+3+1 = 6

- worst cost-to-goal for π_2=(try to go through s_1) is:

  1+2+2+2+2+2+2 + … = ∞

Minimax Formulation



What are potential problems with minimax approaches?

- Optimal policy π^*:
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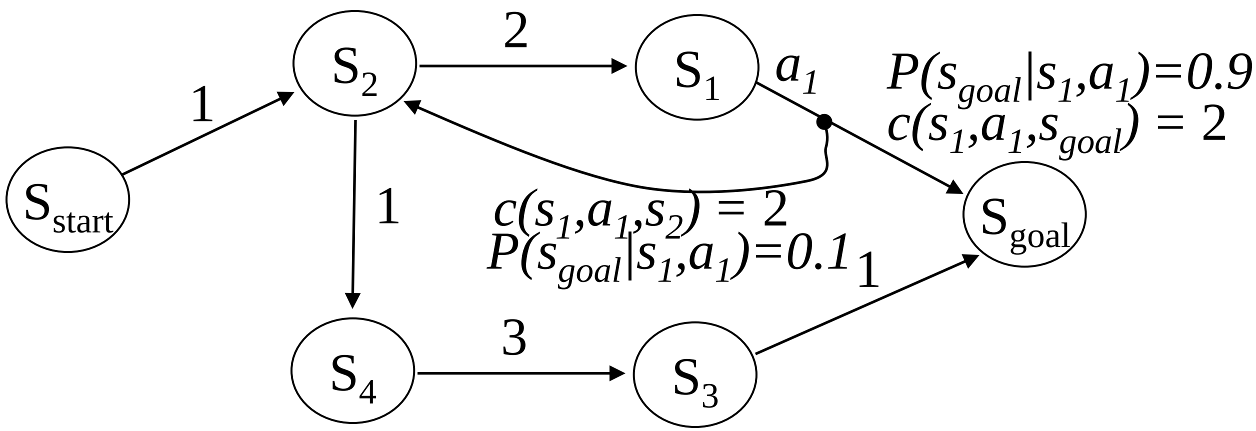
- Optimal minimax policy $\pi^* = \pi_1 = (s_{start}, s_2, s_4, s_3, s_{goal})$

Expected Cost Formulation



- Optimal policy π^*:
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expectation over outcomes

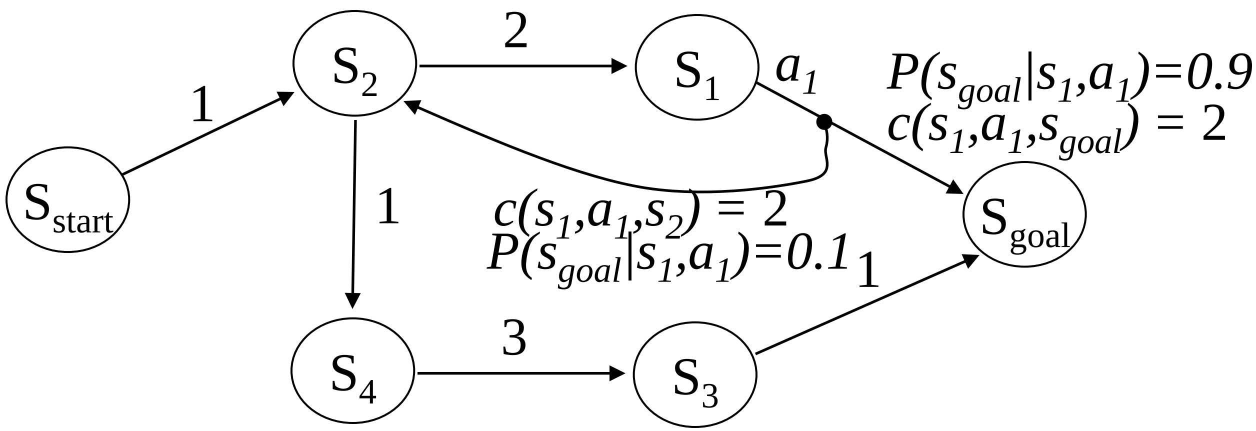
- expected cost-to-goal for $\pi_1=(s_{start},s_2,s_4,s_3,s_{goal})$ is

  1+1+3+1=6

- cost-to-goal for π_2=(try to go through s_1) is:

0.9*(1+2+2) + 0.9*0.1*(1+2+2+2+2) + 0.9*0.1*0.1*(1+2+2+2+2+2+2) + …=$5.44\overline{4}$

Expected Cost Formulation



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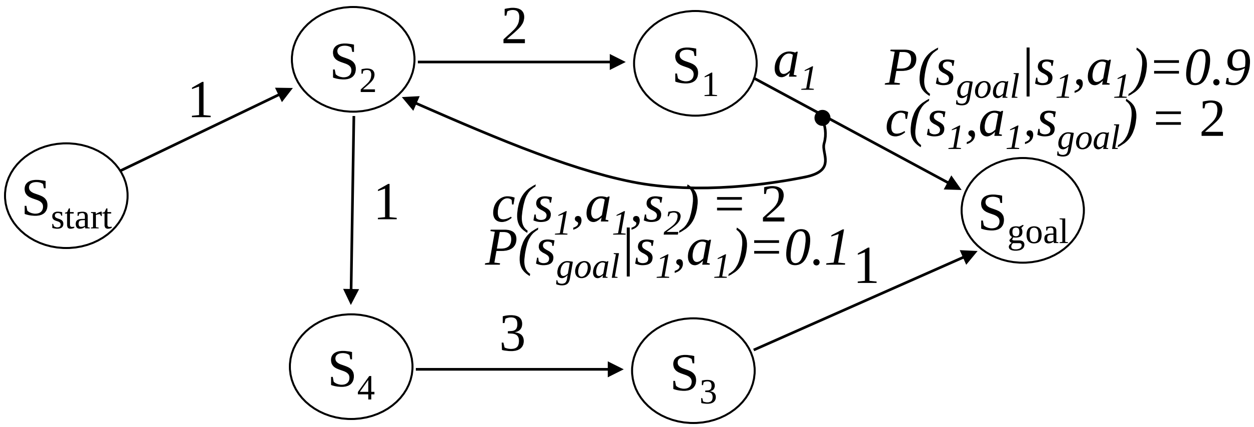
How to compute it?

1+1+3+1=6

- cost-to-goal for π_2=(try to go through s_1) is:

0.9*(1+2+2) + 0.9*0.1*(1+2+2+2+2) + 0.9*0.1*0.1*(1+2+2+2+2+2+2) + …=$5.4\overline{4}$

Expected Cost Formulation



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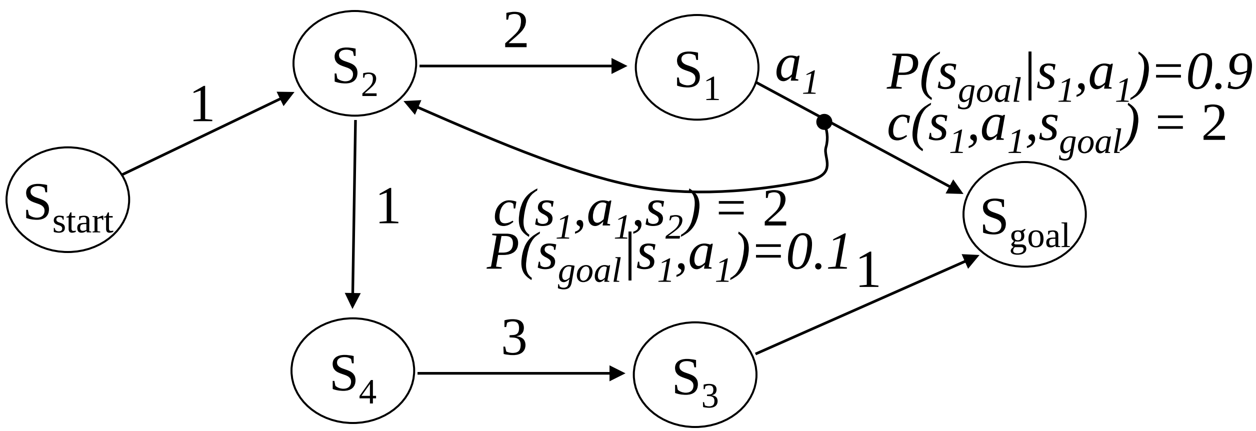
- Optimal expected cost policy $\pi^* = \pi_2 = (go\ through\ s_1)$

Minimizing Cost with *P(success)>t* constraint



- Optimal path π^* is a path that

  minimizes the cost-to-goal assuming some outcomes

  $\pi^* = argmin_\pi \{cost\text{-}to\text{-}goal\}$

  s.t. $P(assumed\ outcomes) > t$

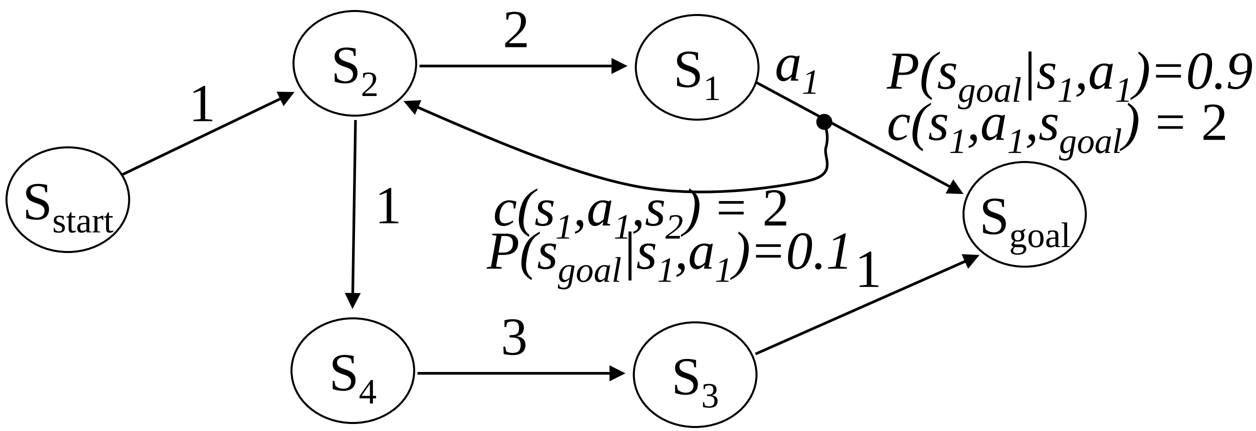
- for $\pi_1=(s_{start},s_2,s_4,s_3,s_{goal})$:

  cost-to-goal = 1+1+3+1 = 6, $P(success) = 1$

- for π_2=(try to go through s_1) is:

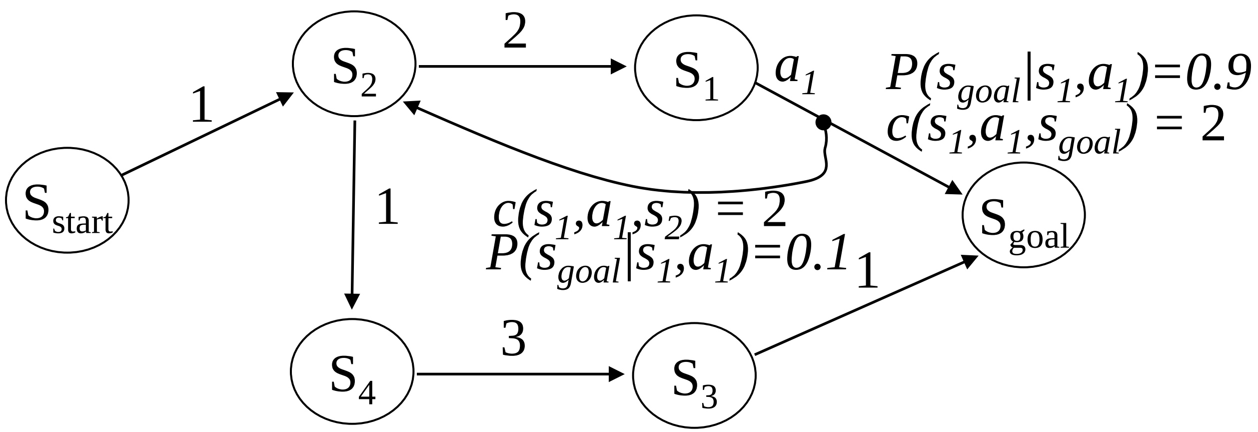
  cost-to-goal = 1+2+2 = 5, $P(success) = 0.9$

Minimizing Cost with *P(success)>t* constraint



- Optimal path π^* is a path that
  minimizes the cost-to-goal assuming some outcomes
  $\pi^* = argmin_\pi \{cost\text{-}to\text{-}goal\}$
  s.t. $P(assumed\ outcomes) > t$
- for $t > 0.9$, $\pi^* = \pi_1 = (s_{start}, s_2, s_4, s_3, s_{goal})$
- for $t \leq 0.9$, $\pi^* = \pi_2 =$ (try to go through s_1)

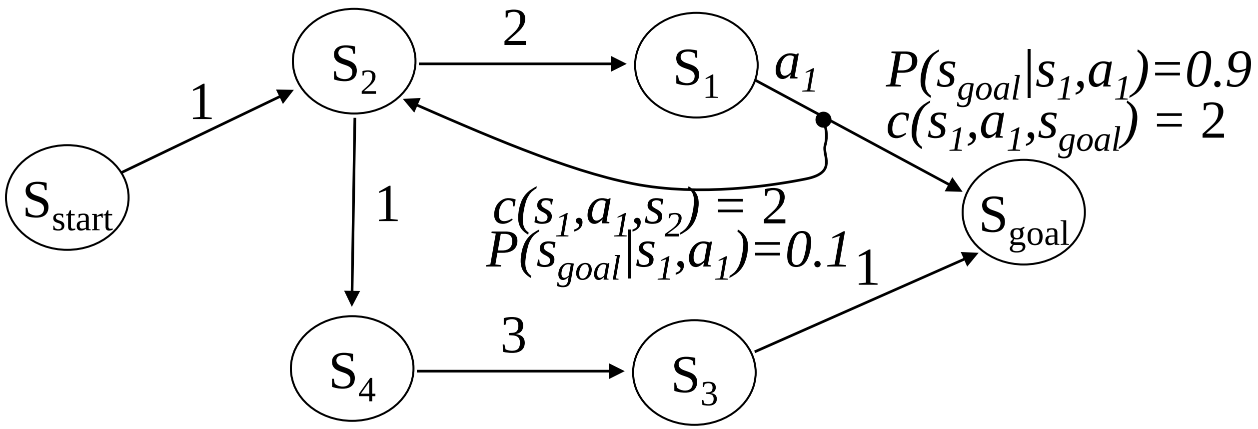
Computing Expected Cost Minimal Plans



- Optimal policy π^*:
  minimizes the *expected* cost-to-goal
  $\pi^* = argmin_\pi\ E\{cost\text{-}to\text{-}goal\}$

- Let $v^*(s)$ be minimal expected cost-to-goal for state s

Computing Expected Cost Minimal Plans



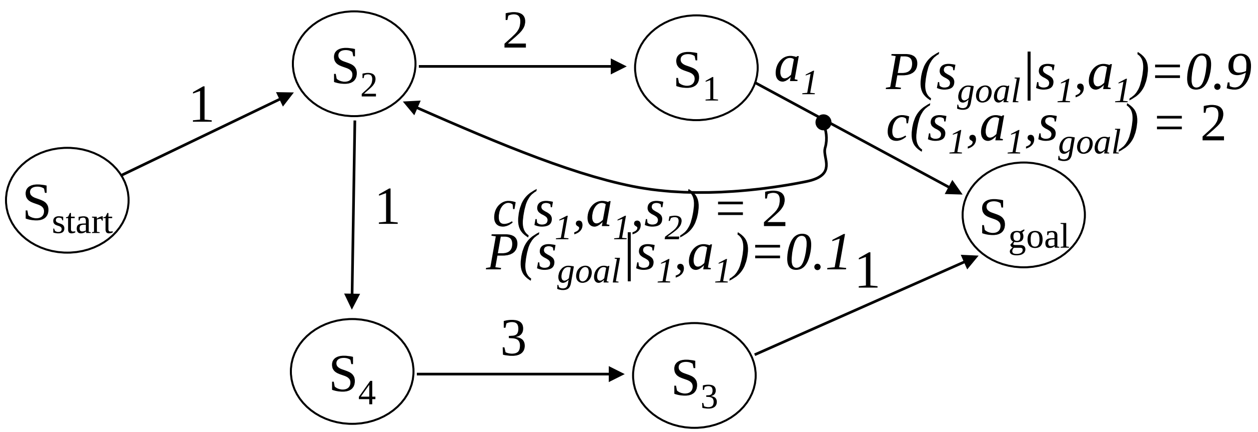
- Optimal policy π^*:

  $\pi^*(s) = argmin_a\ y \qquad (s,a,s')+v^*(s')\}$

  (expectation over outcomes s' of action a executed at state s)

Why?

Computing Expected Cost Minimal Plans



- Optimal expected cost-to-goal values v^* satisfy:

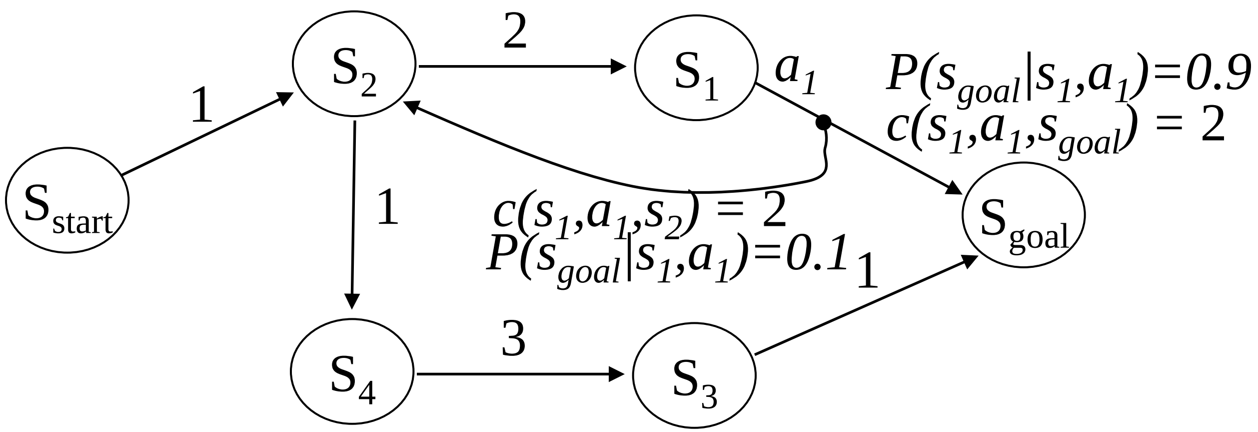
$v^*(s_{goal})=0$

$v^*(s) = \min_a y \quad (s,a,s')+v^*(s'))$ for all $s \neq s_{goal}$

(expectation over outcomes s' of action a executed at state s)

Bellman optimality equation

Computing Expected Cost Minimal Plans



- Value Iteration (VI):

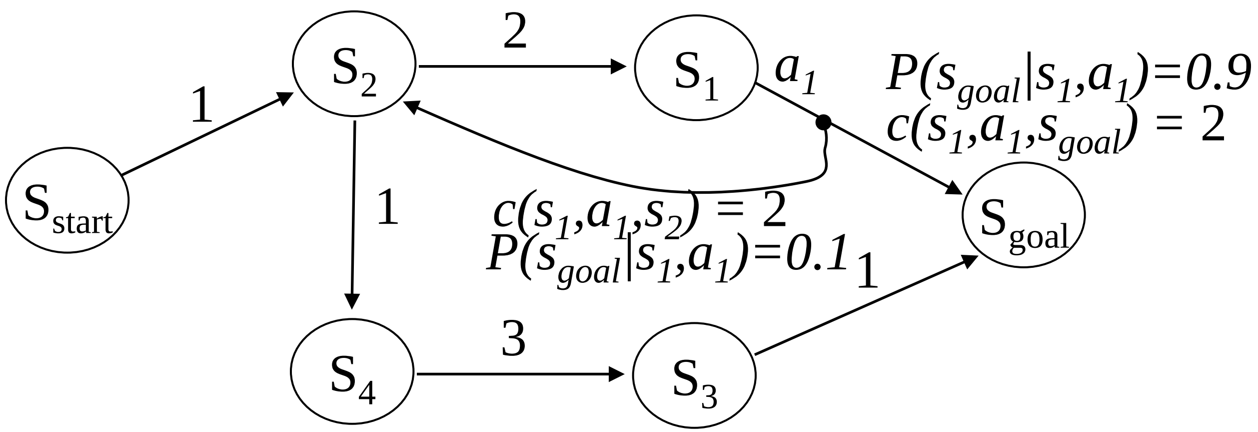
  Initialize *v*-values of all states to finite values;

  Iterate over all *s* in MDP and re-compute until convergence:

  $v(s_{goal}) = 0$

  $v(s) = min_a\, y \quad s,a,s')+v(s'))$ for any $s \neq s_{goal}$

Computing Expected Cost Minimal Plans



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best to initialize to admissible values (under-estimates of the actual costs-to-goal)

Initialize *v*-values of all states to finite values;

Iterate over all *s* in MDP and re-compute until convergence:

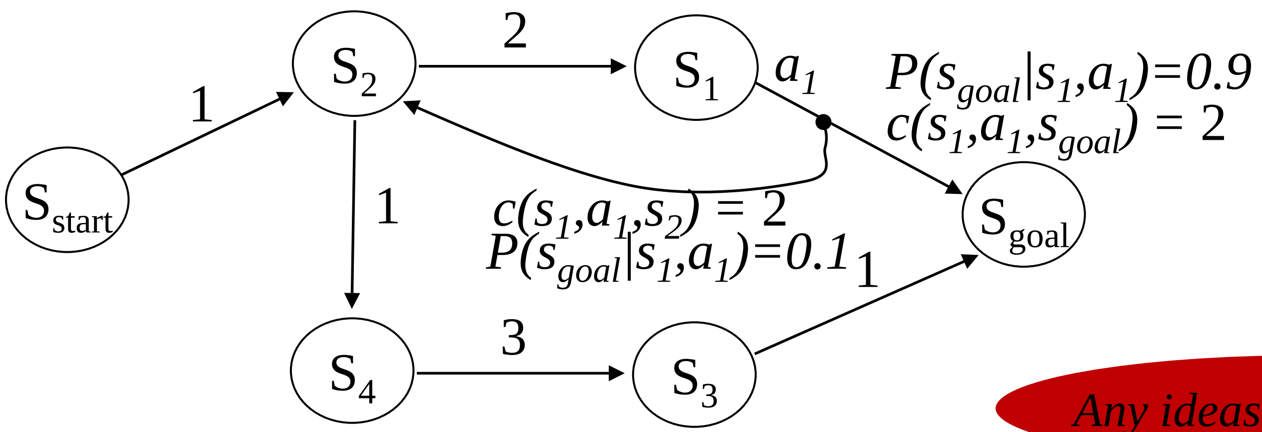
$v(s_{goal}) = 0$

$v(s) = min_a y \qquad s,a,s')+v(s'))$ for any $s \neq s_{goal}$

converges to an optimal value function (v(s)=v(s) for all s) for any iteration order*

convergence time does depend a lot on iteration order

Computing Expected Cost Minimal Plans



Any ideas for the order?

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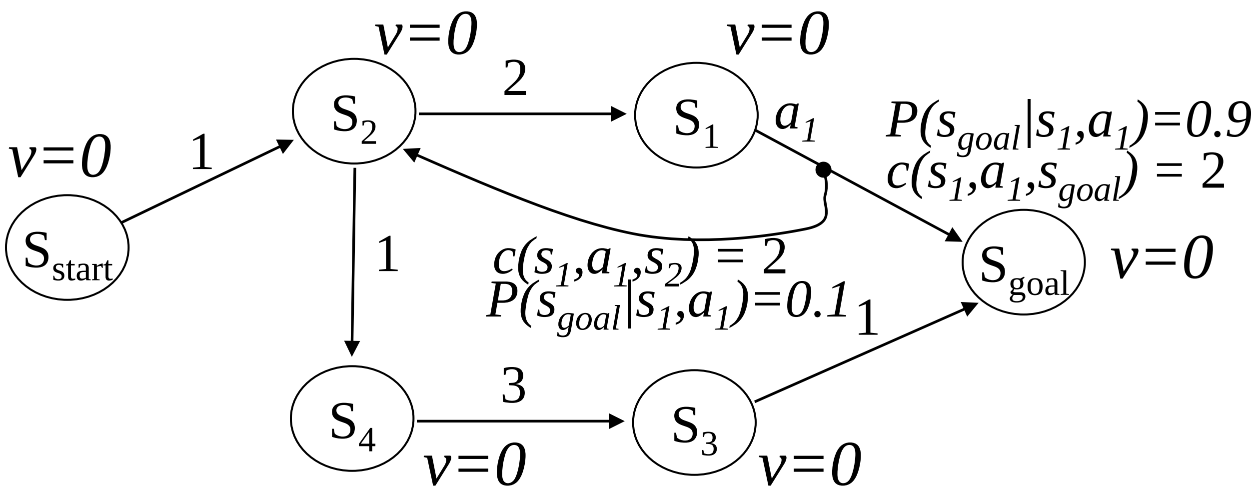

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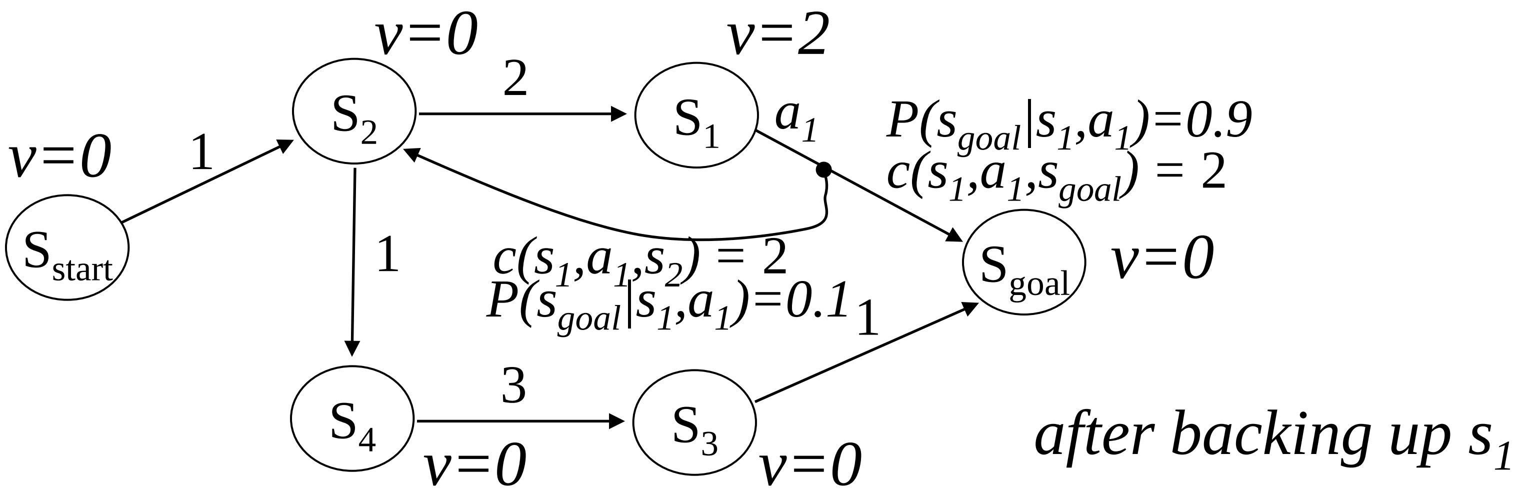
Bellman update equation
(or backup)

Computing Expected Cost Minimal Plans



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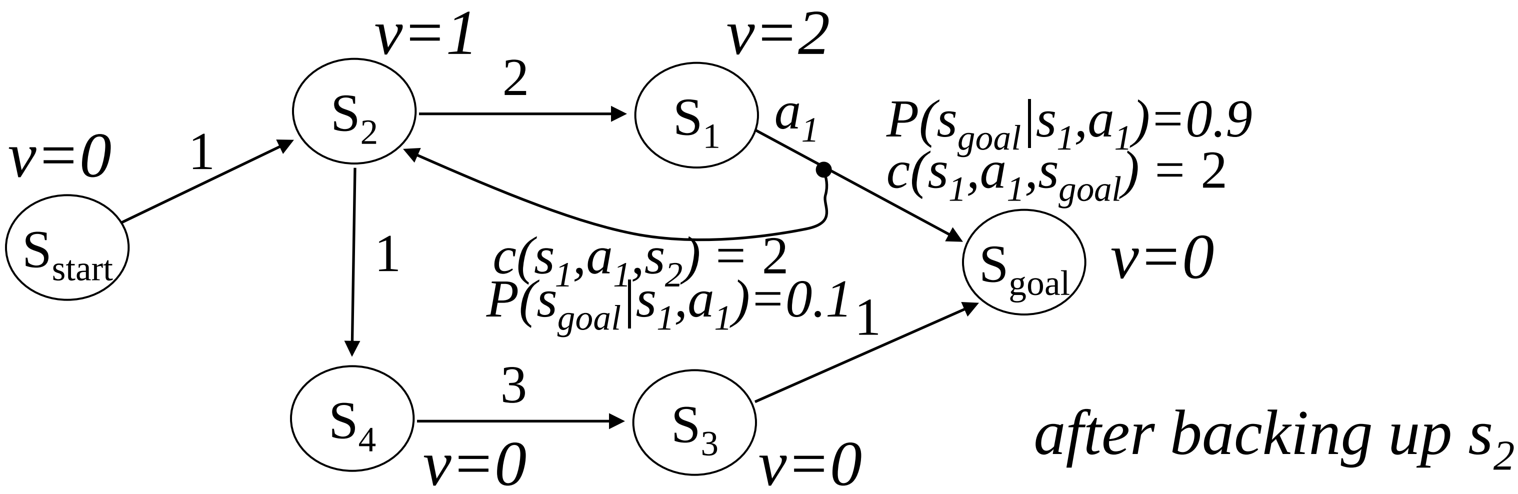
Computing Expected Cost Minimal Plans



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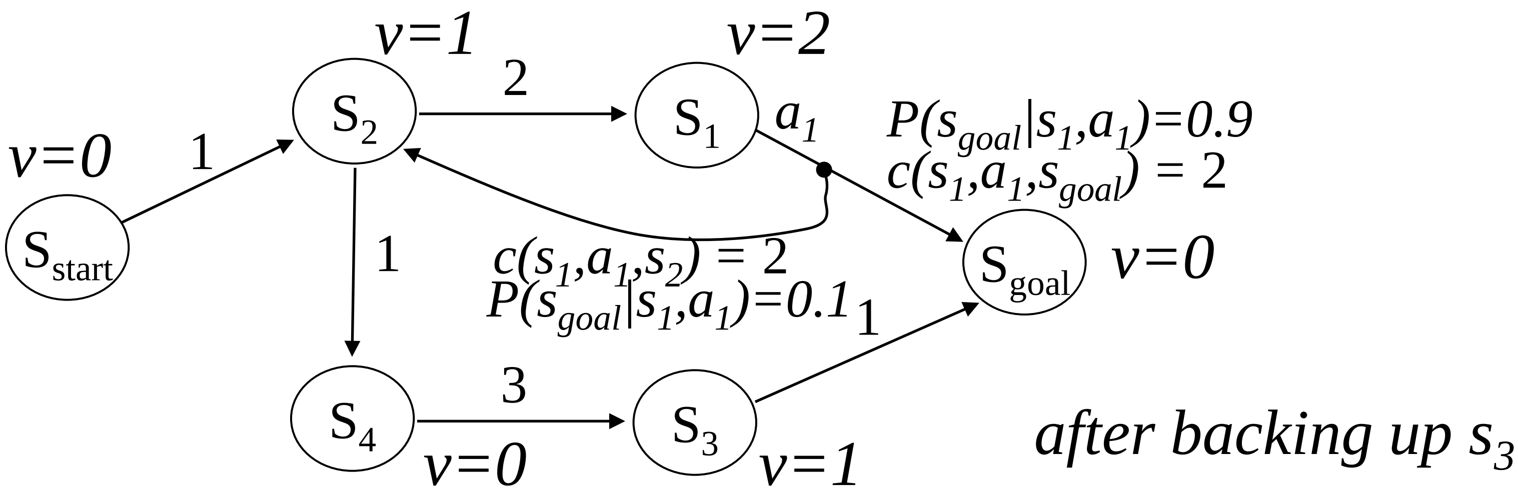
Computing Expected Cost Minimal Plans



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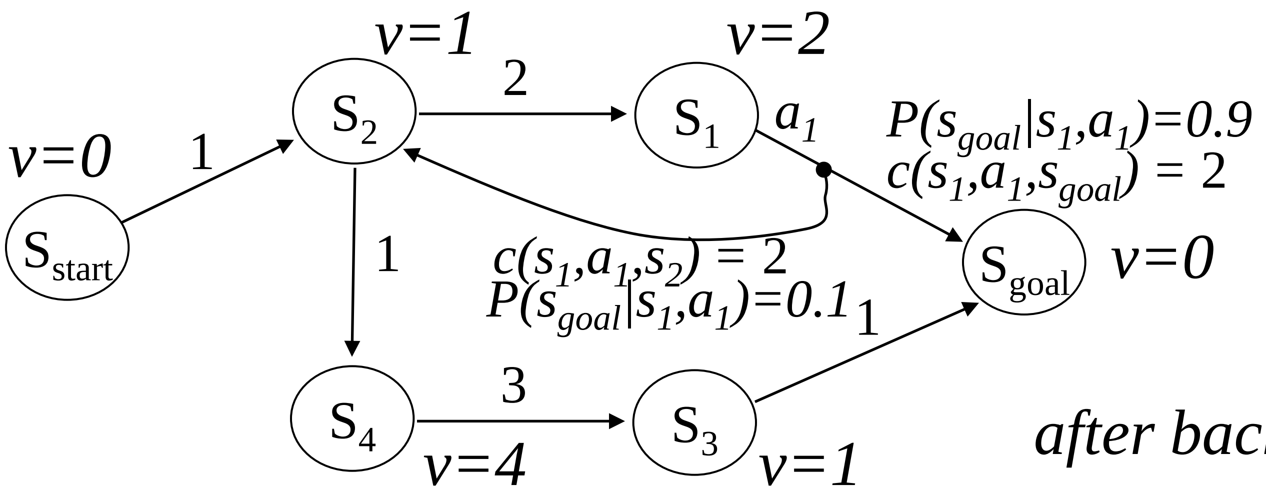
Computing Expected Cost Minimal Plans



after backing up s_4

- Value Iteration (VI):

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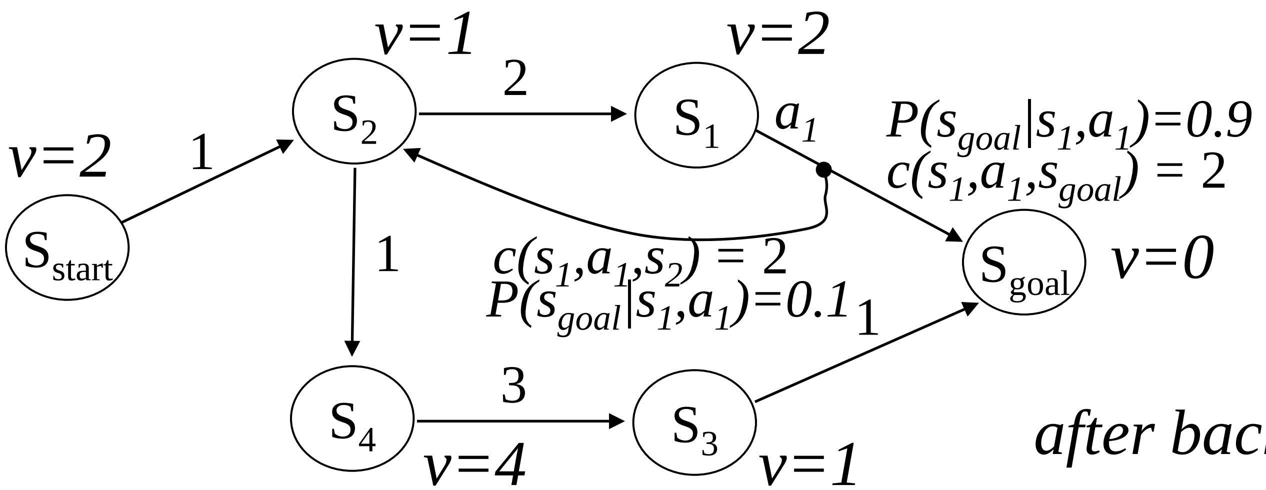
Computing Expected Cost Minimal Plans



after backing up s_{start}

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Initialize *v*-values of all states to finite values;

Iterate over all *s* in MDP and re-compute until convergence:

$v(s_{goal}) = 0$

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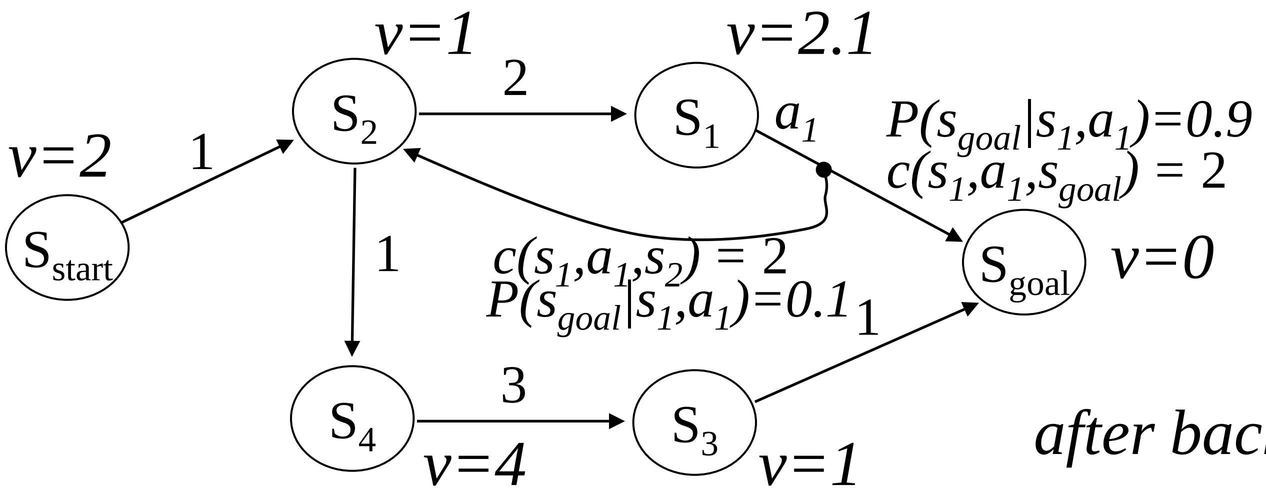
Usual convergence condition: Bellman error over all states $< \Delta$

Bellman error: $|v(s) - \min_a y \qquad s,a,s')+v(s'))|$ *for any* $s \neq s_{goal}$

Computing Expected Cost Minimal Plans



after backing up s_1

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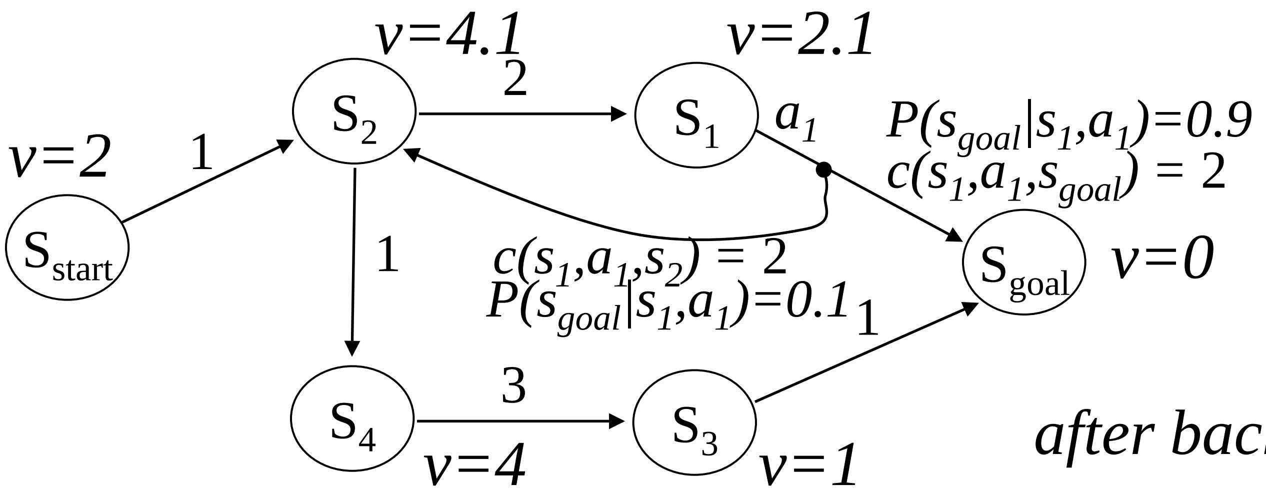
Computing Expected Cost Minimal Plans



after backing up s_2

- Value Iteration (VI):

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  Iterate over all *s* in MDP and re-compute until convergence:

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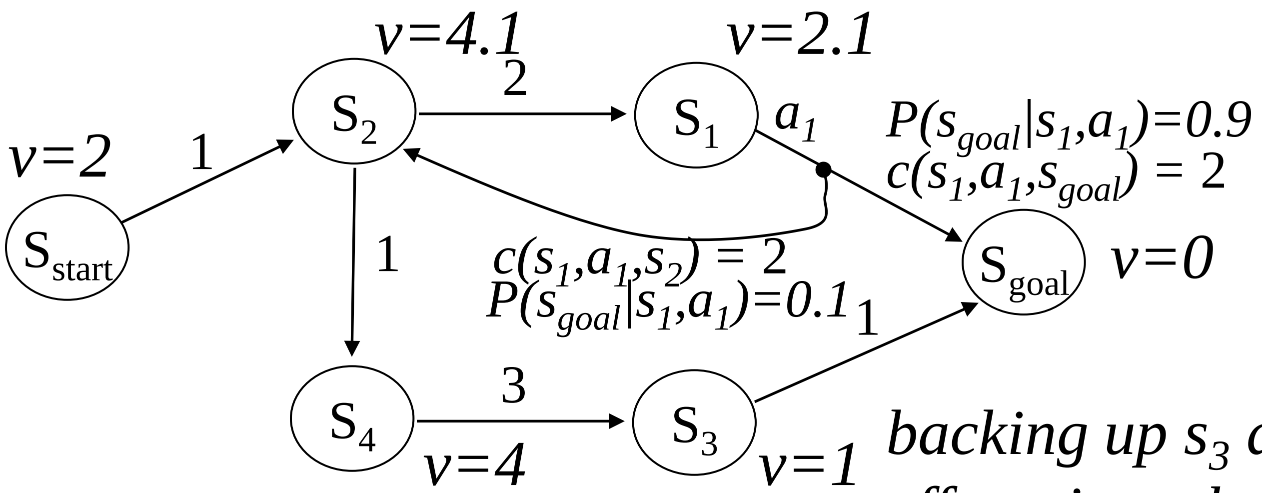
Computing Expected Cost Minimal Plans



backing up s_3 and s_4 has no effect since their Bellman errors are zero

- Value Iteration (VI):

  Initialize v-values of all states to finite values;

  How to select backups more effectively?

  Iterate over all s in MDP and re-compute until convergence:

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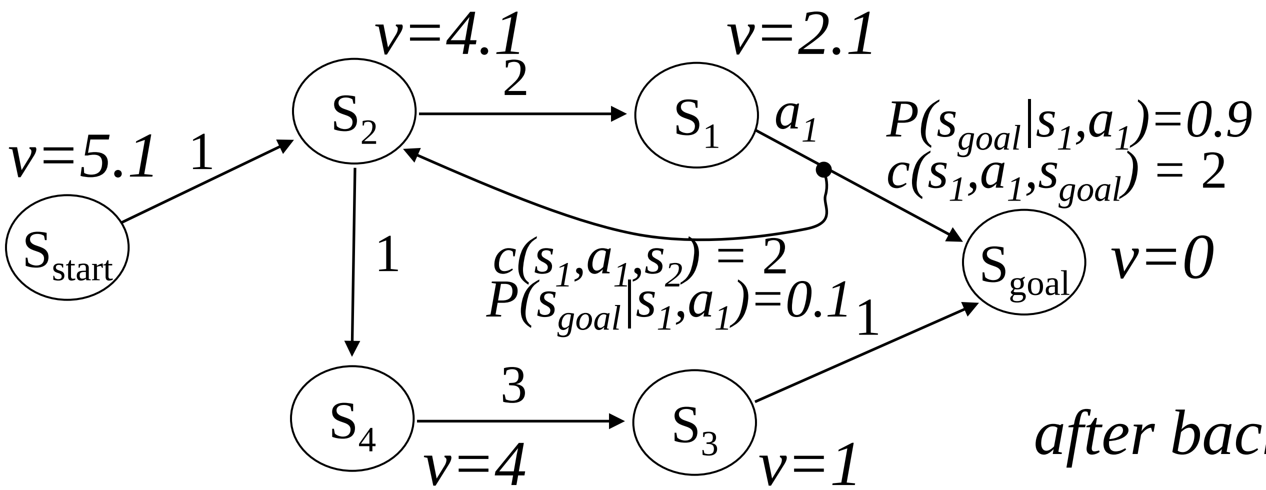

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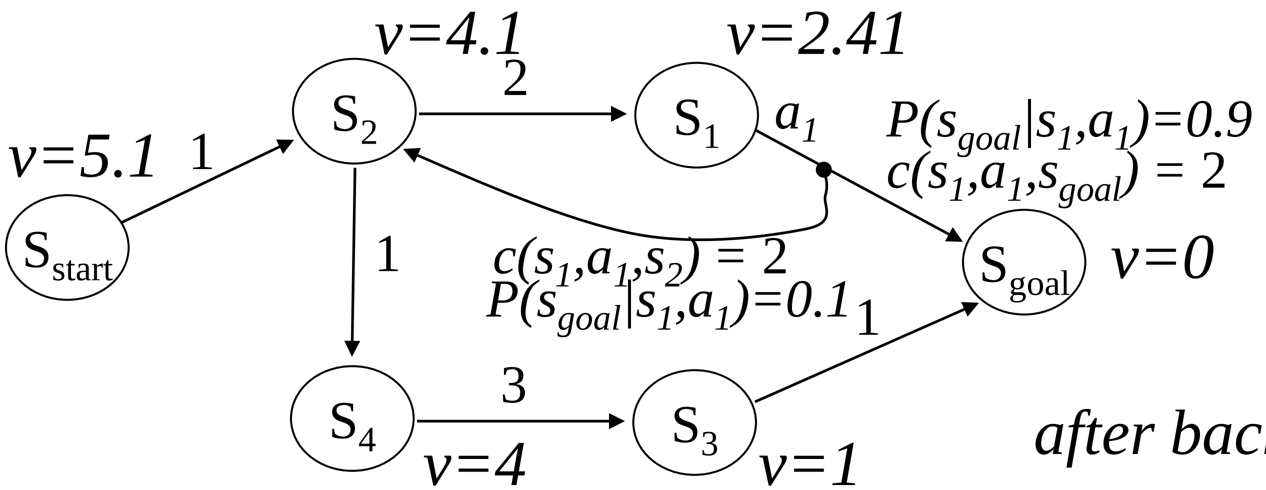

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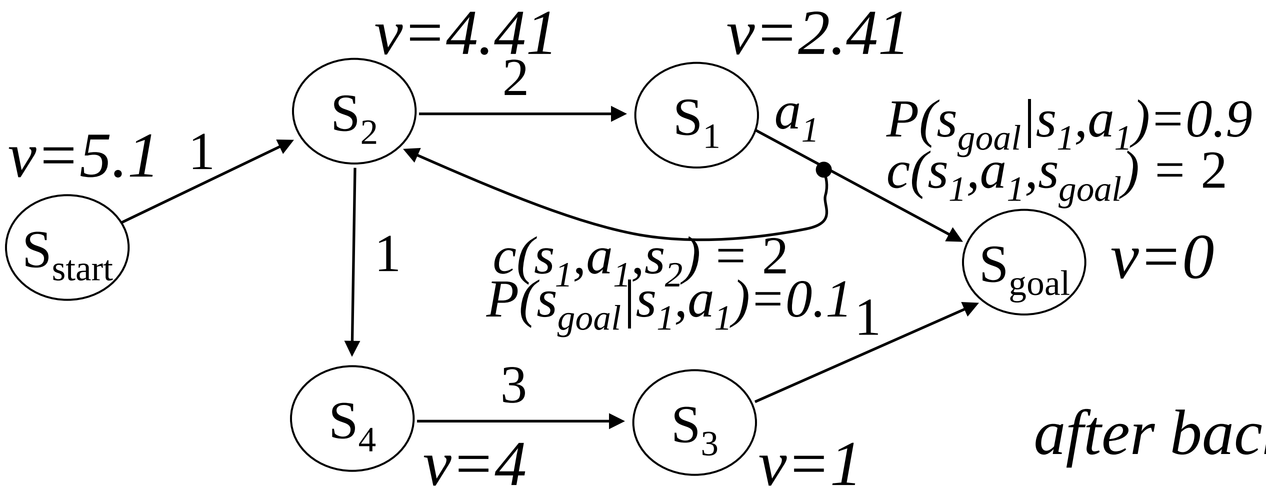

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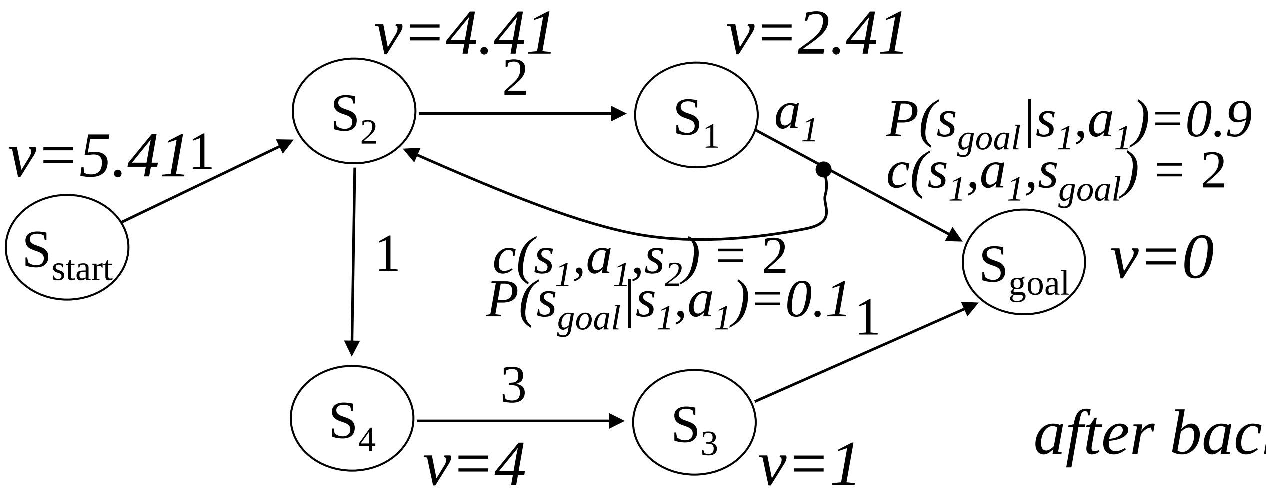

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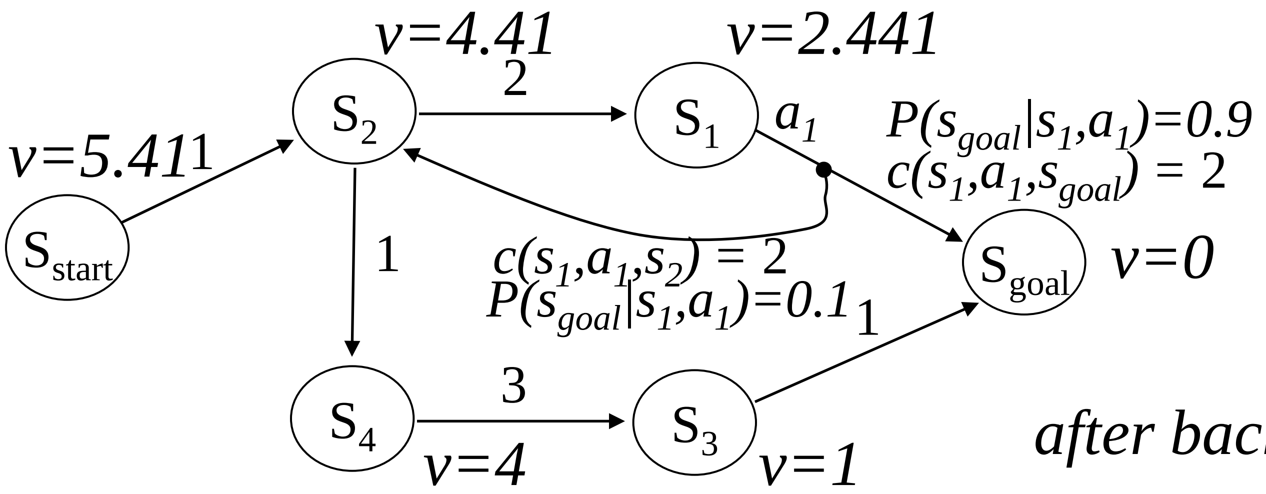

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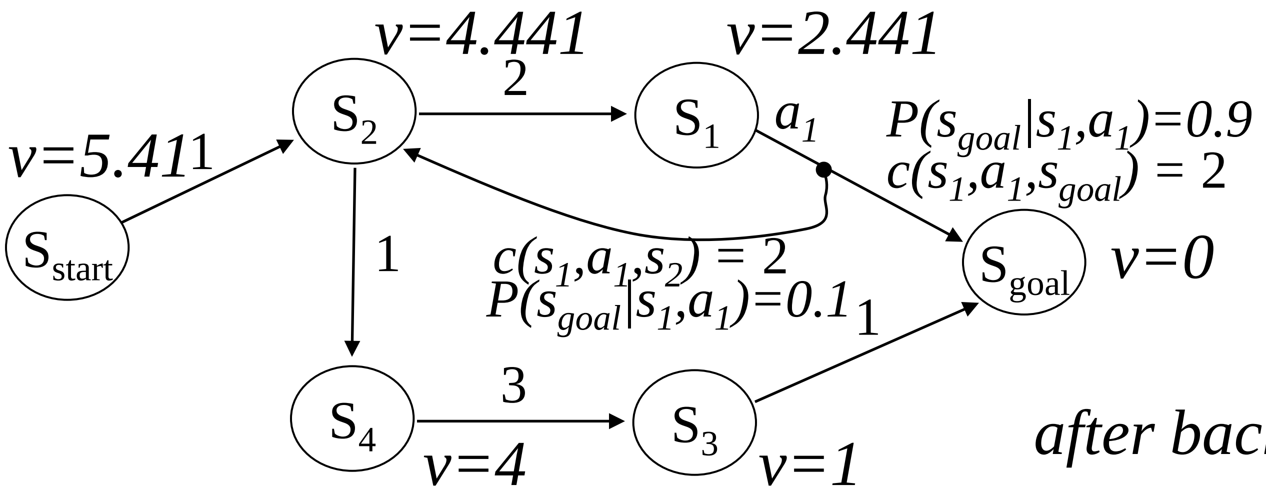

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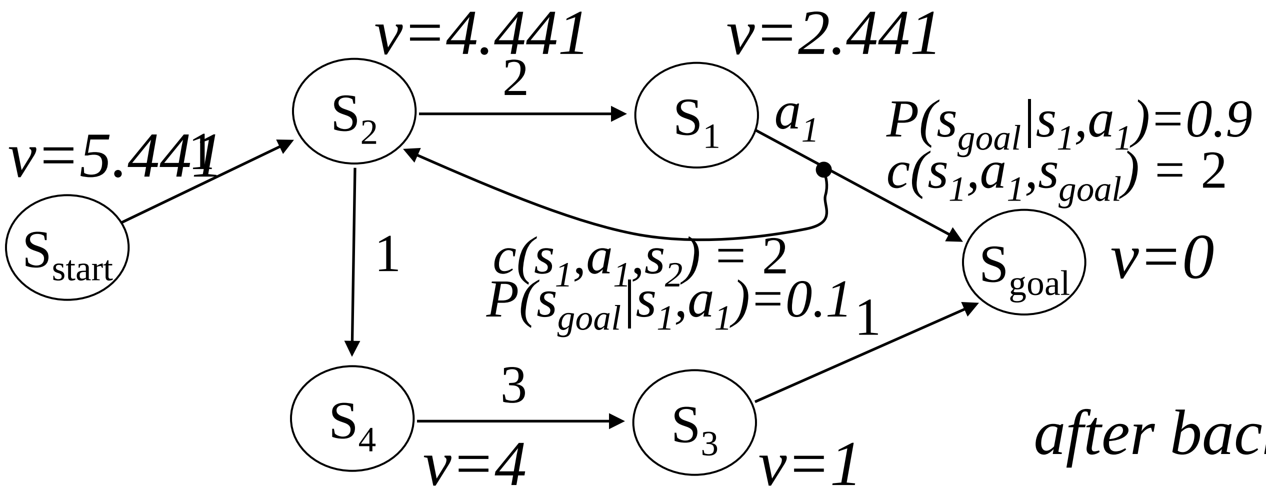
Computing Expected Cost Minimal Plans



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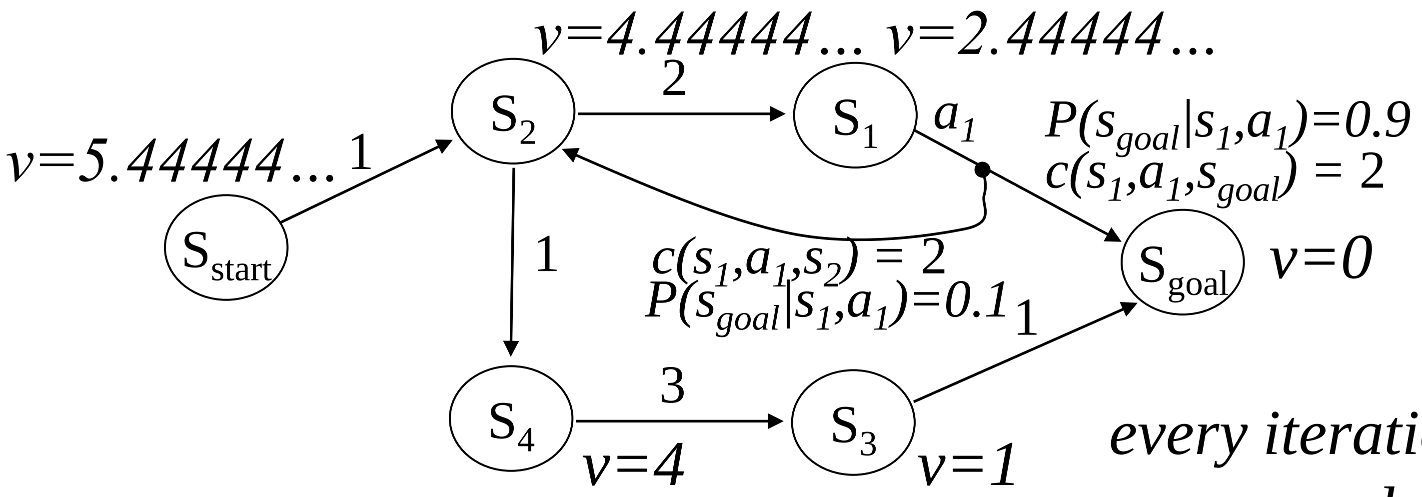

every iteration computes one more decimal point

At convergence…

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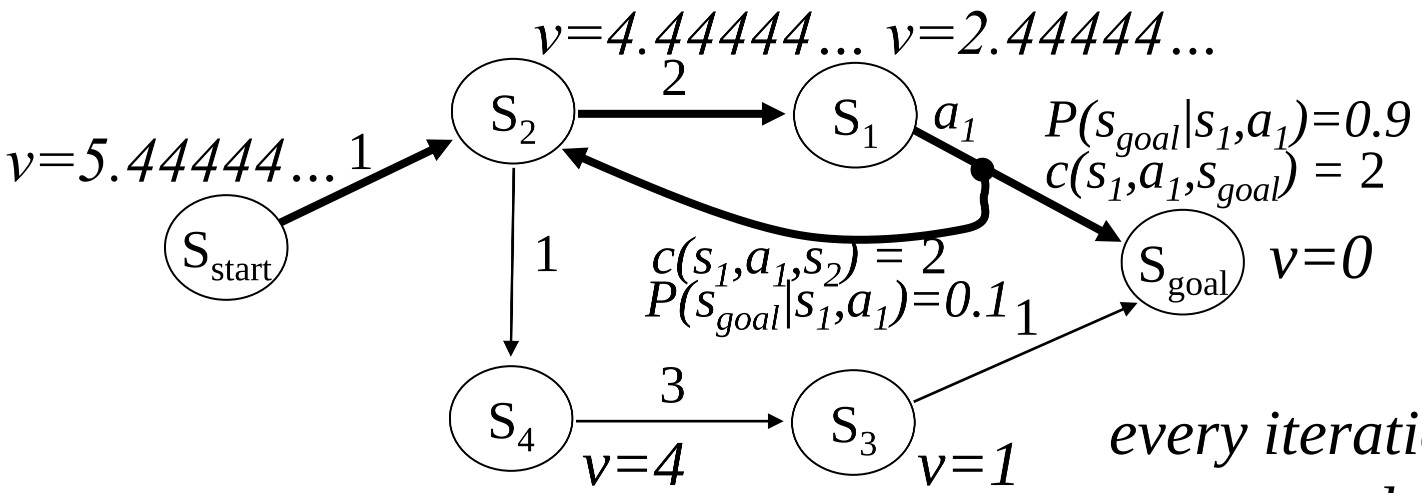
Computing Expected Cost Minimal Plans



every iteration computes one more decimal point

At convergence…

optimal policy is given by greedy policy:
always select an action that minimizes
y (s,a,s')+v(s')}

Initialize v-values of all states to finite values;

re-compute until convergence:

expected cost of executing greedy policy is at most:
$v^*(s_{start})c_{min}/(c_{min}-\Delta)$
where c_{min} is minimum edge cost

for any $s \neq s_{goal}$

Usual convergence condition: Bellman error over all states < Δ

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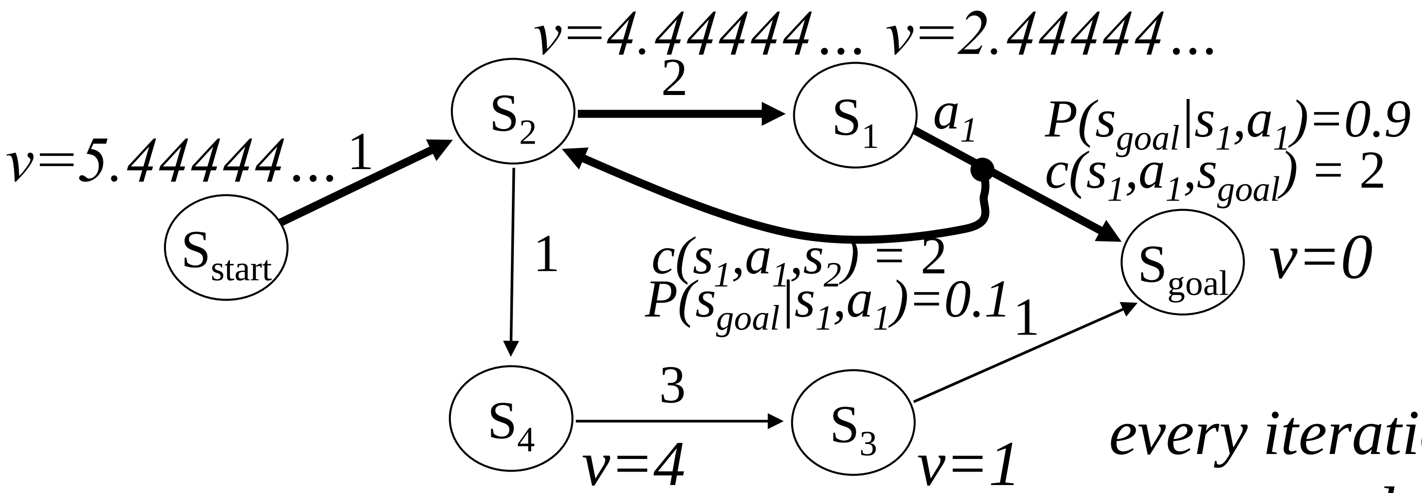
Computing Expected Cost Minimal Plans



every iteration computes

…imal point

VI converges in finite number of iterations (assuming goal is reachable from every state)

Why condition?

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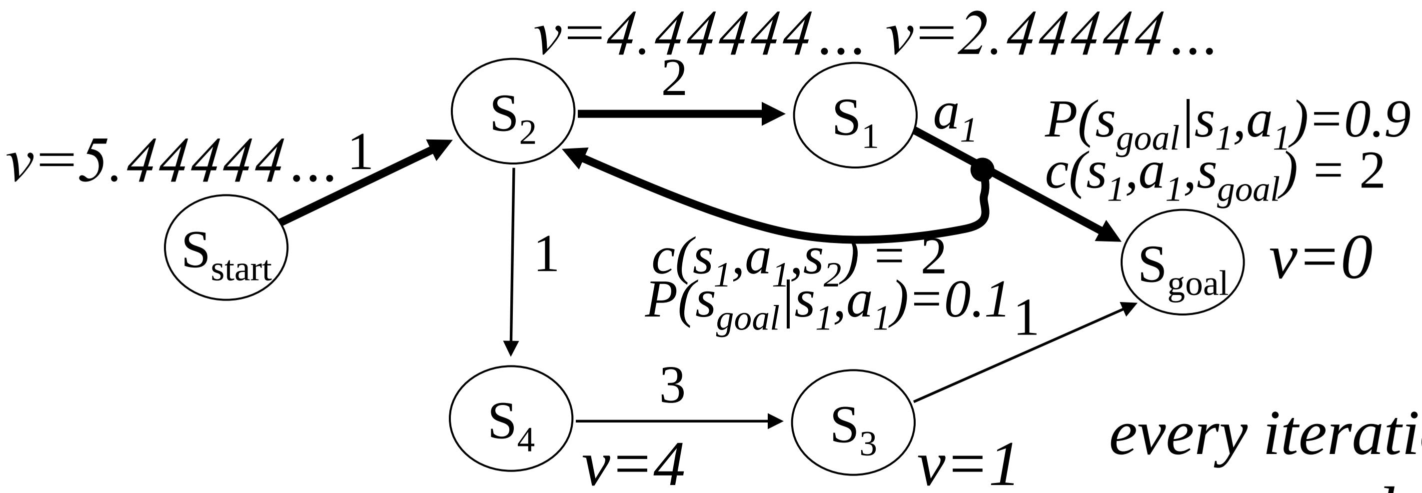

every iteration computes one more decimal point

How many backups required in a graph with no stochastic actions?

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