Proposition R.231: Prove that $A = \left\{ m + n\sqrt{3} \mid m, n \in \mathbb{Z} \right\}$ is closed under mulitplication.

Proof: Let $A = \left\{ m + n\sqrt{3} \mid m, n \in \mathbb{Z} \right\}$, and let $m + n\sqrt{3}$ and $p + q\sqrt{3}$ be elements of A. Then

$$
\begin{aligned}
\left(m + n\sqrt{3} \right)\left(p + q\sqrt{3} \right) &= mp + mq\sqrt{3} + np\sqrt{3} + 3qn \\
&= (mp + 3qn) + (mq + np)\sqrt{3}.
\end{aligned}
$$

Since $m, n, p, q \in \mathbb{Z}$, $mp + 3nq$ and $mq + np$ are both integers. Therefore,

$$
\left(m + n\sqrt{3} \right)\left(p + q\sqrt{3} \right) \in A,
$$

and A is closed under multiplication. $\square$