

Complex Variables I – Problem Set 2

Due at 5 pm on Friday, September 22, 2023 via Gradescope

Problem 1

Regard \log as a multivalued function. Find all the values of

- a) $\log(-i)$
- b) $\log(1+i)$
- c) $\log 2$.

Problem 2

Solve the following equations (make sure to find all solutions):

- a) $\cos z = 2$
- b) $\sin z = 2$.

Problem 3

Prove that the function $\sin z$ maps the strip $-\frac{\pi}{2} < \operatorname{Re} z < \frac{\pi}{2}$ onto the set $\mathbb{C} \setminus \{z : \operatorname{Im} z = 0 \text{ and } |\operatorname{Re} z| \geq 1\}$.

Problem 4

Find all Möbius transforms $f(z) = \frac{az+b}{cz+d}$, $ad - bc \neq 0$, such that $|f(z)| = 1$ whenever $|z| = 1$.

(Hint: when $a \neq 0$, you can divide a on both the numerator and the denominator. Thus, you can assume $a = 1$ without loss of generality.)

Remember to justify your answers and acknowledge collaborations and outside help!